Network analysis of 3D complex plasma clusters

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Outline

Measures on networks

Degree

$$k_\nu = \sum_{i=0}^{n-1} A_{\nu,i}$$

How many nodes are connected to node ν?
(local scale)

Clustering coefficient

$$C_\nu = \frac{\sum_{i,j=0}^{n-1} A_{\nu,i} A_{i,j} A_{j,\nu}}{k_\nu (k_\nu - 1)}$$

Which fraction of neighbors of node ν are themselves connected?
(intermediate scale)

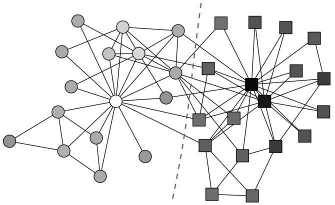
Average path length

$$L_\nu$$

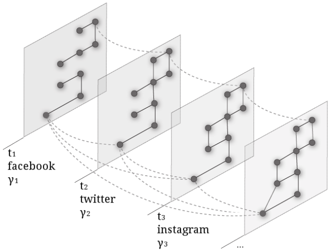
How long is the average path from node ν to any other node?
(global scale)

Community finding in networks

- Community: group of nodes that are more connected to each other than to rest of network.
- $Q \sim \sum_{ij} \left(A_{ij} - \gamma A_{ij}^{\text{null}} \right) \delta(g_i, g_j)$ measure for quality of a partition g.
- Multislice network: each node in one slice is connected with itself in other slices.

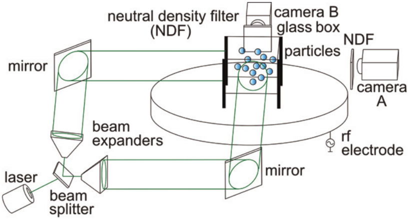


Newman et al. (2004)



Mucha et al. (2010)

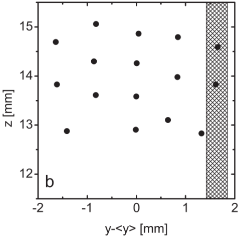
Experimental setup



Wörner et al. (2012)

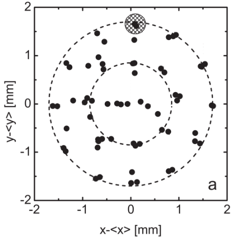
- 3D cluster in glass box
- holographic 3D imaging
- external rotating electric field drives clusters

Particle strings:



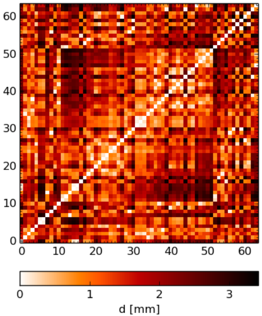
- How to detect them?

Competing geometries:

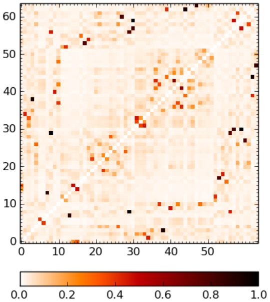


- How to quantify them?

Particle strings: Adjacency matrix

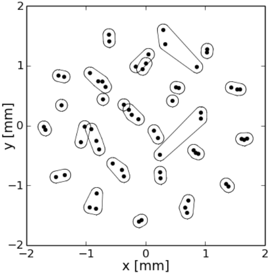


Horizontal distance matrix d_{ij}.



Adjacency matrix

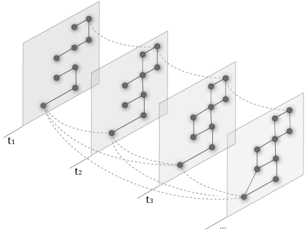
$$A_{ij} = \begin{cases} d_{\min}/d_{ij}, i \neq j \\ 0 \text{ otherwise.} \end{cases}$$

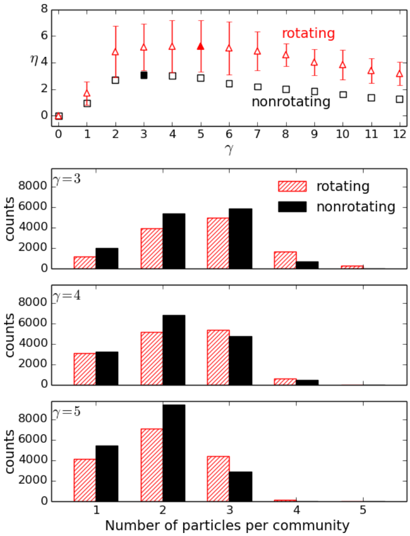


Communities as vertical particle strings.

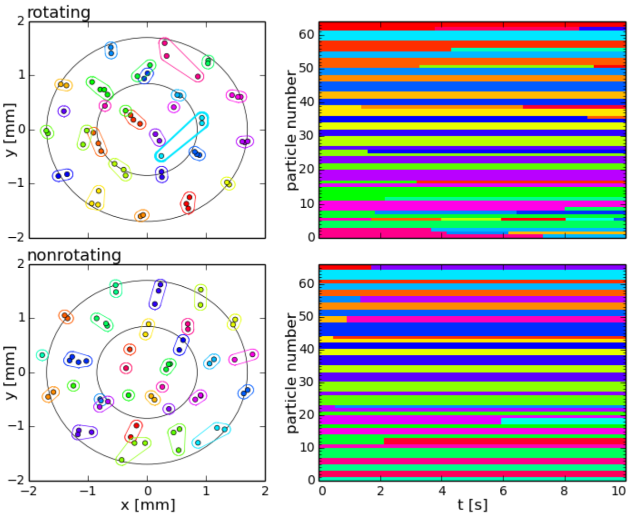
Particle strings: Resolution parameter

- Find optimal resolution parameter γ where inter-community distance is large compared to average extent of the communities, $\eta = \langle d_{i,\mathrm{next}} / d_{i,\mathrm{same}} \rangle$.
- Connect particle at time t_i with itself at t_{i+1} in a multislice network.

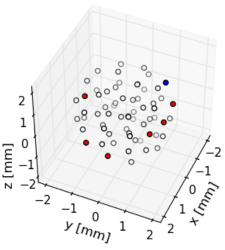


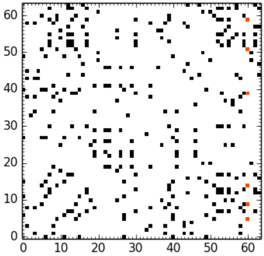


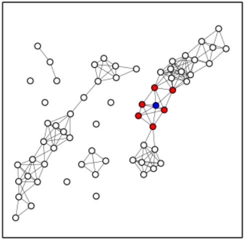
Particle strings: Time evolution



Cluster geometry: Create networks



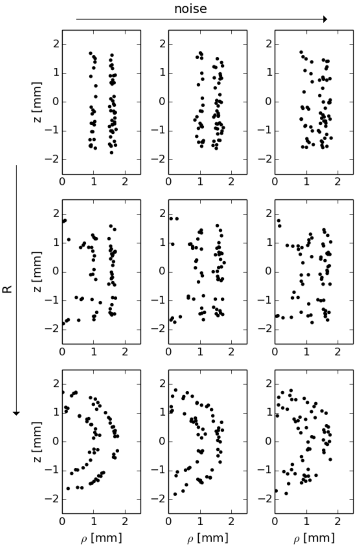




- Connect nodes, if difference in cylindrical radius is smaller than a threshold ϵ:
  $A_{ij}^{\text{cyl}}(\epsilon) = \Theta\left(\epsilon - |\rho_i - \rho_j|\right) - \delta_{ij}$
- Calculate network measures.
- Compare to *null models* of known geometry.

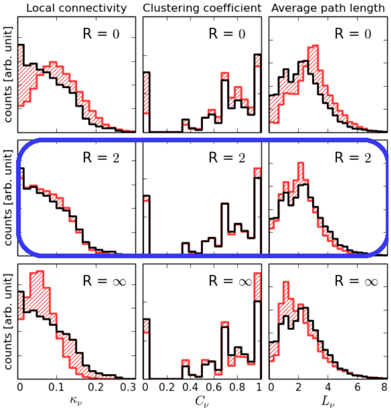
Cluster geometry: Null models

- Artificial structures with ratios $R = n^{\mathrm{sph}}/n^{\mathrm{cyl}}$
- Uniform noise is added.

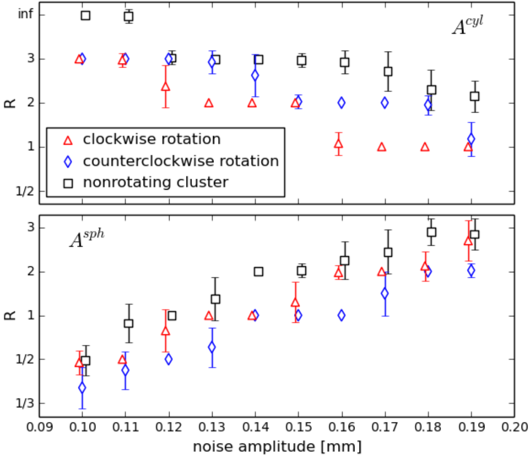


Cluster geometry: Find best ratio R

- Compare network measures of data to null models with $R = 0,\ 1/3,\ 1/2,\ 1,\ 2,\ 3,\ \infty$ to find the best ratio.
- Repeat for spherical adjacency matrix $A_{ij}^{\mathrm{sph}}(\epsilon) = \Theta\left(\epsilon - |r_i - r_j|\right) - \delta_{ij}$



Cluster geometry: Result



- Strong dependency on noise amplitude of null models.
- Rotating clusters systematically “more cylindrical”.

Cluster geometry: Estimate particle confinement

Dynamical force balance

$$\langle \mathbf{F}^{\text{rep}} \rangle + \langle \mathbf{F}^{\text{fr}} \rangle + \langle \mathbf{F}^{\text{in}} \rangle + \langle \mathbf{F}^{\text{conf}} \rangle = 0.$$

- Yukawa interaction $\langle \mathbf{F}^{\text{rep}} \rangle$ with estimated particle charge $Z = 50000$ and screening length $\lambda = 0.4\,\text{mm}$.
- Neglect friction $\langle \mathbf{F}^{\text{fr}} \rangle$ and inertial forces $\langle \mathbf{F}^{\text{in}} \rangle$ due to slow particle movement and cluster rotation.
- $\langle \mathbf{F}^{\text{conf}} \rangle \simeq -\langle \mathbf{F}^{\text{rep}} \rangle$

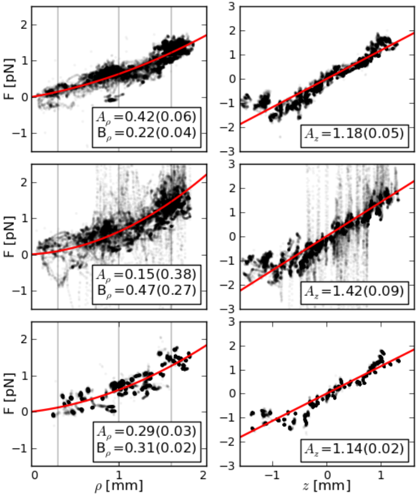
Cluster geometry: Estimate particle confinement

Calculate vertical and radial confinement parameters from force profile:

$F_\rho^{\text{conf}} \simeq -M\Omega_\rho^2 \rho \equiv -A_\rho \rho$

$F_z^{\text{conf}} \simeq -M\Omega_z^2 z \equiv -A_z z$

Rotation	Ω_ρ [s^{-1}]	Ω_z [s^{-1}]
clockwise	19 ± 2	33 ± 1
counter-clockwise	12 ± 15	36 ± 2
nonrotating	16.2 ± 0.8	32.0 ± 0.3



Conclusion

With networks, you can ...

- focus on different aspects of your data.
- find stable units in complicated structures.
- measure global properties of the structure.
- ...

Thank you for your attention.