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HIGHER ORDER AND ADAPTIVE DISCONTINUOUS GALERKIN METHODS FOR 3D AERODYNAMIC FLOWS

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Discontinuous Galerkin (DG) methods allow high-order flow solutions on unstructured as well as locally refined meshes by increasing the polynomial degree and using curved instead of straight-sided elements. In this talk, we give an extension of the adjoint consistency analysis of DG discretizations of the compressible Euler and Navier-Stokes equations. While in previous publications [1, 3, 7] only *one* specific discretization based on normal wall boundary fluxes and an associated discretization of aerodynamic force coefficients (like drag and lift coefficients) was known to be adjoint consistent, we provide (cf. [5]) a discretization of the force coefficients which results in an adjoint consistent discretization for *any* discretization of wall boundary fluxes provided by the baseline DG discretization is adjoint consistent on interior faces (like e.g. the SIPG or BR2 scheme). For an adjoint consistent discretization the discrete adjoint problem is a consistent discretization of the continuous adjoint problem. The discrete adjoint solution can be used in adjoint-based error estimation providing an estimate of the discretization error in the computed force coefficient. Furthermore, the error estimate can be decomposed in a sum of local indicators consisting of the primal residuals multiplied by the discrete adjoint solution. Adaptive mesh refinement using these so-called adjoint-based indicators targets at the accurate and efficient approximation of the force coefficient under consideration. Note, that an extension of this approach to the treatment of multiple force coefficients is given in [2]. In contrast to that, adaptive mesh refinement using indicators which include the primal residuals but are independent of any target quantity targets at the resolution of the overall flow field. Furthermore, it turned out that these so-called residual-based indicators are particularly well suited for the resolution and tracking of vortical systems (cf. [4]).

In the European project IDIHOM [6] on the industrialization of high-order methods for aeronautical applications all developments are targeted at improving the applicability of the methods to so-called "Underlying flow cases" with moderate complexity and "Application challenges" with significantly higher complexity. These test cases pose major difficulties on high-order flow solvers in terms of stability as well as due to the sheer size

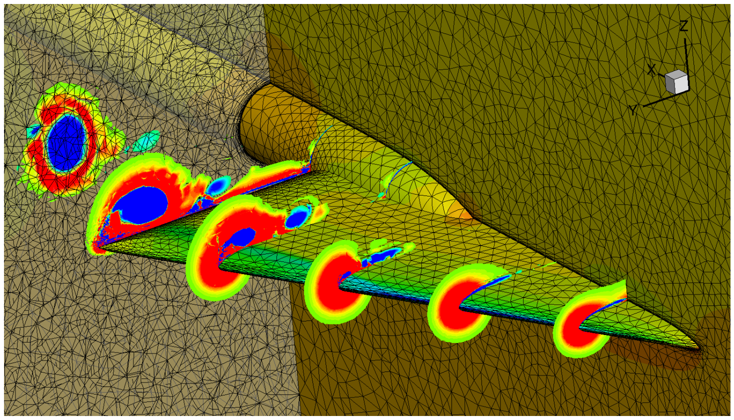

Figure 1: Turbulent flow around the VFE-2 delta wing configuration. 4^{th}-order solution on an unstructured grid of about 1.15×10^6 tetrahedral elements, c_p-distribution and slices of the λ_2-criterion.

of the discrete flow problems to be solved. As an example, the underlying test case U1.b consists of a subsonic turbulent flow around the VFE-2 delta wing configuration with medium rounded leading edge (cf. [5] and the references cited therein). Figure 1 shows the c_p-distribution of a 4^{th}-order flow solution of the RANS-$k\omega$ equations of almost $23 \cdot 10^6$ degrees of freedoms per equation on an unstructured high-order grid (courtesy of Oubay Hassan [8]) with grid lines being represented by polynomials of degree 3.

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