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RECONSTRUCTION OF TURBULENCE PROPERTIES FOR STOCHASTICAL TURBULENCE MODELING

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Abstract. *The correct modeling of turbulent and transient flow is still a major task for computational fluid dynamics. This is even more a topic concerning the CFD-simulations on wind turbines, which face already highly turbulent inflow conditions. Thus motivated the improvement of stochastic methods of turbulence modeling is the scope of this work.*
Stochastic turbulence model rely on an estimation of the statistical distribution of the flow properties at a certain point or time. E.g. Bakosi et. al.[1] *have proposed a model using the probability density functions (PDFs) of the flow properties on an unstructured grid. The method however relies on the knowledge of the PDFs in the flow on the grid. Here we will present a method to gain such PDFs of the flow.*
We performed a 3D DNS simulation using spectral/hp method on an fx79-w151a airfoil at a Reynolds number of Re=5000. In the wake of the airfoil an inhomogeneous turbulent field evolved. Within this field a time series of the flow properties has been gathered at certain points. As an example the data of the time series at one point has been analyzed using a multi-point correlation method on incremental statistics to gain the Kramers-Moyal coefficients.[2] *Using these coefficients it is possible to reconstruct the time series at the point itself to gain a PDF function artificially.*
The resulting PDFs did not completely match the statistical properties of the original points. The main reason were to high values of the high order statistical moments and the short time series, that served as a base. However the results were surprisingly good, reproducing the main shapes of the PDFs, even though the underlying function for the reconstruction was a Langevin equation using only Gaussian white noise in the diffusion. Thus further progress is to be expected in the method.

1 INTRODUCTION

Turbulence modeling for computational fluid dynamics (CFD) remains a difficult task. Different models for all kinds of applications have been proposed for Reynolds Averaged Navier-Stockes (RANS) solvers as well as for large eddy simulations (LES). A lot of improvements have been undertaken during recent years. While the $k-\omega SST$ model by Menter is one of the most popular every day approach for RANS calculations for example in the wind industry, as could be observed at the eurpoean wind energy conference 2009[3–6] , a lot of open accuracy questions on RANS calculations remain.[7] Therefore DES and LES models have become more popular in use, with each specific types of models. After the classical model of Smagorinsky,[8] other models based on eddy viscosity have been introduced.[9] Other approaches used a self-similarity concept.[10,11]However the research activity indicates still a desire for more fitting models for specific problems.[12,13]
Due to the difficulties in describing turbulence correctly, one branch of turbulence research has focussed on the statistical description and stochastic reconstruction of turbulence rather than on physical models. Thus stochastic turbulence models have also been introduced into CFD research. Pope or Fox have introduced stochastic models based on particle tracking in a Langrangian framework.[14–16]
One model that has been especially of interest for this work has been developed by Bakosi et. al.[1] They used probability density functions (PDFs) for a stochastic model on an unstructured Eulerian grid on particles. The method has been implemented in a finite element solver and tested for some cases. The PDFs are calculated under modeling assumptions dynamically using Bayes' rule.
Motivated by the recent research on the reconstruction of time series we present a different approach of achieving PDFs for the use of CFD turbulence modeling. Nawroth et. al.[2] showed good results using a Fokker-Planck equation on multi-point correlation approach on incremental statistics on a time series of a turbulent flow experiment. By extracting the necessary Kramers-Moyal coefficients for many scales they were able reconstruct the time series gain PDF functions for the scales artificially without the estimation of any additional parameters.
The method is attractive for the reconstruction of time series, this is a base for also extending the method further by using the underlying Kramers-Moyal coefficients for a reconstruction in space. Thus in a previously calculated DNS flow field at an airfoil a time series reconstruction has been done. Therefore we will present the method of reconstruction first, give in the following a short overview over the used simulation and present first results for the reconstruction of the PDFs.

2 INCREMENT STATISTICS AND MARKOV PROPERTIES

For the analysis of turbulence it is of central interest to characterize the correlations in time and space of a investigated magnitude $\phi(x_i, t)$. Thus increments

$$\phi(\gamma) = \phi(y) - \phi(y - \Delta y) \tag{1}$$

are preferred measure for the statistical analysis of the magnitude at a scale *gamma*. Such increments for fluid problems can be regarded in terms of time or spacial scales. The probability of a certain state ϕ_1 of the magnitude described by $p(\phi_1(\gamma_1), \phi_2(\gamma_2), ..., \phi_n(\gamma_n))$, which is hard to compute due to the high dimensionality. From the conditioned probability

$$p(\phi_1(\gamma_1)|\phi_2(\gamma_2), ..., \phi_n(\gamma_n)) = \frac{p(\phi_1(\gamma_1), \phi_2(\gamma_2), ..., \phi_n(\gamma_n))}{p(\phi_2(\gamma_2), ..., phi_n(\gamma_n))} \tag{2}$$

however, in case of Markov properties

$$p(\phi_i(\gamma_i)|\phi_{i+1}(\gamma_{i+1}), ..., \phi_n(\gamma_n)) = p(\phi_i(\gamma_i)|\phi_{i+1}(\gamma_{i+1})) \tag{3}$$

the joint PDF reduces to

$$p(\phi_1(\gamma_1), \phi_2(\gamma_2), ..., \phi_n(\gamma_n)) = p(\phi_1(\gamma_1)|\phi_2(\gamma_2))...p(\phi_{n-1}(\gamma_{n-1})|\phi_n(\gamma_n)) \cdot p(\phi_n(\gamma_n)) \tag{4}$$

Here and in the following we will assume for convenience $\gamma_{i+1} \geq \gamma_i$.
As presented in[17,18] such PDFs can be reconstructed using a Kramers-Moyal expansion

$$-\gamma \frac{\partial p(\phi, \gamma|\phi_1, \gamma_1)}{\partial \gamma} = \sum_{k=1}^{\infty} \left(-\frac{\partial}{\partial \phi}\right)^k D^{(k)}(\phi, \gamma) p(\phi, \gamma|\phi_1, \gamma_1), \tag{5}$$

with $D^{(k)}$ being the Kramers-Moyal coefficients given by

$$D^{(k)}(\phi, r) = \lim_{\Delta\gamma \to 0} \frac{1}{k!\Delta\gamma} \int_{-\infty}^{\infty} (\phi' - \phi)^k p(\phi'(\gamma - \Delta\gamma)|\phi(\gamma)) d\phi' \tag{6}$$

from a given dataset. In case of $D^{(4)} = 0$ the series reduces to the it first two elements which gives leads to the Fokker-Planck equation.[19] The Fokker-Planck equation can be reformulated as a Langevin equation following Itô to:

$$-\frac{\partial \phi(\gamma)}{\partial \gamma} = \frac{1}{\gamma} D^{(1)}(\phi, \gamma) + \sqrt{\frac{1}{\gamma} D^{(2)}(\phi, \gamma)} \Gamma(\gamma). \tag{7}$$

$\Gamma(r)$ is the Gaussian distributed white noise.
The method has been under investigation for time and spatial scales.[17,20,21] Here we present the results obtained for an analysis of a flow field in the wake of an airfoil generated by a direct numerical simulation.

3 THE SIMULATION

3.1 Simulation setup

A direct numerical simulation (DNS) has been done for the flow around an fx79w-151a airfoil at an angle of attack $\alpha = 12°$ and a Reynolds number of Re=5000 in respect to the chord length L_c. For the simulation the spectral finite element code $\mathcal{N}\varepsilon\kappa\mathcal{T}\alpha r$[22] has been used to gain a time series of a flow field. The method combines finite elements with the use of Jacobi polynomials for the spatial resolution.[23] Here we use the incompressible, dimensionless Navier-Stokes equations as

$$\partial_t U_i + (U_j \partial_j) U_i = \partial_i p + \frac{1}{Re} \partial_j^2 U_i + f_i, \quad (8)$$
$$\partial_i U_i = 0.$$

With p denoting the pressure, Re the Reynolds number, U_i the velocity field and f_i the forces.
For the discretisation a hybrid 2D mesh of 2116 triangular and quadrilateral elements has been used. The polynomial expansion within the elements has been done with a polynomial order of n=9. The homogeneous third dimensions has been modeled using periodic boundary conditions and a Fourier expansion with $N_z = 64$ Fourier planes at a span width of $L_z = \pi$. Validations of the Fourier expansion for the code have been undertaken for a cylinder flow by Ma and Dong.[24,25]
The domain of the simulation expanded in the streamwise direction from an inflow at $-6L_c$ to an outflow with Neuman boundary conditions at $20L_c$. A sponge layer was introduced increasing the viscosity $3L_c$ before the outflow for stability reasons.[26] In the cross flow direction the domain expanded from $-10L_c$ to $10L_c$. A hybrid mesh constisting of triangles and quadrilaterals was used for the finite element discretisation.
Due to small geometric structures of the airfoil at the trailing edge, the time step was reduced to $\Delta t \approx 10^{-5}$, leading to rather costly computations, as the parallelization of the code is based on the Fourier expansion.[27] This limited the parallel computation to 32 possible parallel computing cores.

3.2 Simulation results

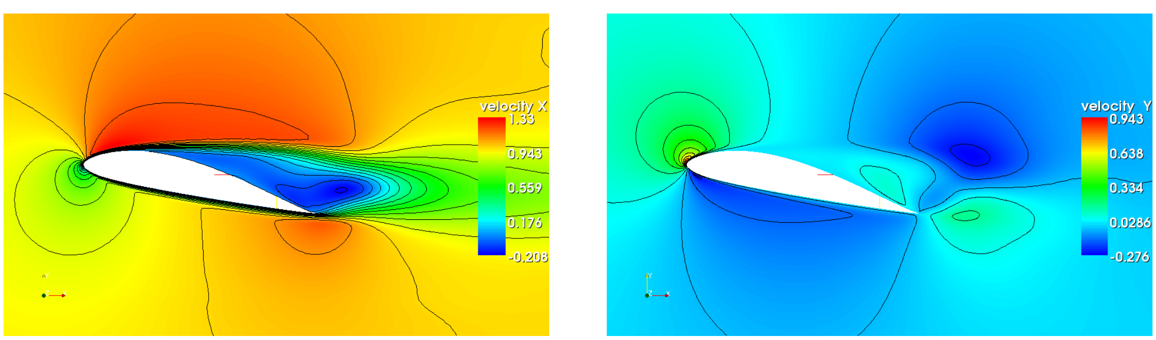

Figure 1: Contours of the average velocity of the flow field, showing the streamwise velocity component U_x on the left and U_y right.

At this Reynolds number the flow show a laminar separation at about $0.3L_c$ from the tip. The this leads to a shear flow above the airfoil which triggers a van Karman street like vortex shedding. This vortex shedding leads in the average to reattachment of the flow at the tail of the airfoil (see velocity contours in fig. 1).

Just above the trailing edge of the airfoil there is in average a recirculation area, where the flow is oriented toward the airfoil. Here a zone starts where the flow shows turbulent patterns. Fig. 2 shows the distribution of the rms magnitude of the flow. At certain

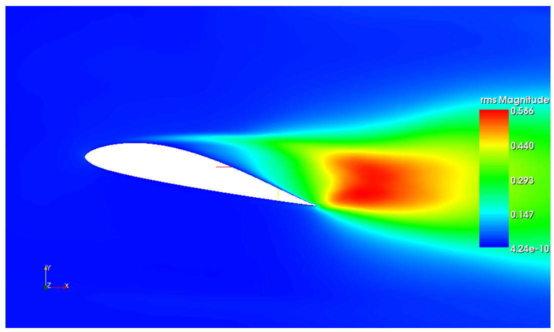

Figure 2: Contours of the average velocity of the flow field, showing the streamwise velocity component U_x on the left and U_y right.

observation points in the region of the turbulent flow the characteristic fields of the flow have been recorded of a period of t=27 leading to a dataset of 10922 recordings. This data has been used for the statistical flow analysis and the stochastic reconstruction.

4 STOCHASTIC ANALYSIS AND RECONSTRUCTION

4.1 Analysis of the time series

In a first step the velocity field at a single base point within the described turbulent region above the trailing edge has been analyzed. Table 4.1 describes the mean velocity components $\overline{U}_i$, its standard deviations σ_i, the integral, dissipation and Taylor length scale L_i, η_i and λ_i. All length scales have been calculated as described by Pope in.[14] The streamwise, cross flow and spanwise direction are denoted with x, y and z respectively.

To show that the estimation of Markov properties is valid for the velocity-increment

Component	$\overline{U}$	σ	L	λ	η
x	-0.14	0.253	0.18	0.014	0.0005
y	-0.12	0.216	0.15	0.009	0.0004
z	-0.006	0.157	0.33	0.005	0.0003

Table 1: Basic statistical properties of the analyzed flow field at the base point

field, we have to proof that

$$p(u_i(\tau_0)|u_i(\tau_1), u_i(\tau_2)) = p(u_i(\tau_0)|u_i(\tau_1)). \tag{9}$$

is valid, with

$$u_i(\tau_0) = U_i(t+\tau) - U_i(t). \tag{10}$$

Here we use $\tau_2 = \tau_1 + \Delta\tau = \tau + 2\Delta\tau$ with $\Delta\tau = 0.1375$. For the further evaluation the pdfs have been binned. As the dataset consisted of only of slightly more than 10000 data points was used, only 15 bins were used for the comparison. Fig. 3 shows the contour plots of the single and double conditioned PDFs of equation 9 for the u (streamwise) and v (cross flow) velocities. For brevity we refrain here from showing the similar results for w. The contours show rather good agreement of the contours although the outer areas show some deviances. Such deviances can however be explained by the lack of data in the outer bins of the conditioned PDFs. Thus we conclude for now that the assumption of Markov properties in this area of the flow seems to be rather good.

4.2 Reconstruction of the time series

Under the estimation of Markov properties it is thus possible to do a Kramers-Moyal expansion from the data for different τ. Here the expansion has been done in the range of $0.0125 \leq \tau \leq 0.45$ for all τ with a step size of $\tau_{i+1} - \tau_i = 0.0125$, giving us coefficients on 36 scales. As $D^{(4)} \leq\leq D^{(2)}$ the estimation of the validity of the Fokker-Planck equation seemed reasonable. Thus we proceeded with the reconstruction using the Langevin equation (7).

In Fig. 4 the PDFs of the original time series of u, v and w is plotted against the reconstructed one. Although especially for the u component some larger deviances can be observed. The main reason for such deviances lay most likely in the shortness of the time series as such reconstructions are mostly done using datasets being more than two orders larger. However the results seem to be quite promising for further reconstruction.

5 CONCLUSIONS

A DNS simulation of the flow over an fx79w-151a airfoil has been done, using the the spectral finite element code $\mathcal{N}\varepsilon\kappa\mathcal{T}\alpha r$. The vortices in the wake of the flow separation over the airfoil cause a turbulent field. Within this field a time series has been analyzed at one point by statistical means. From the data available the assumption of Markov properties at this position within the flow field was made.

Using this assumption the time series has been reconstructed using Kramers-Moyal coefficients on a Langevin equation. The results given by the PDFs showed in general a good agreement with the data, however for the u-velocity still some deviances exist, which are most likely caused by the shortness time series.

In the future it shall be tested if such a reconstruction can be transfered into a spatial reconstruction for LES modeling.

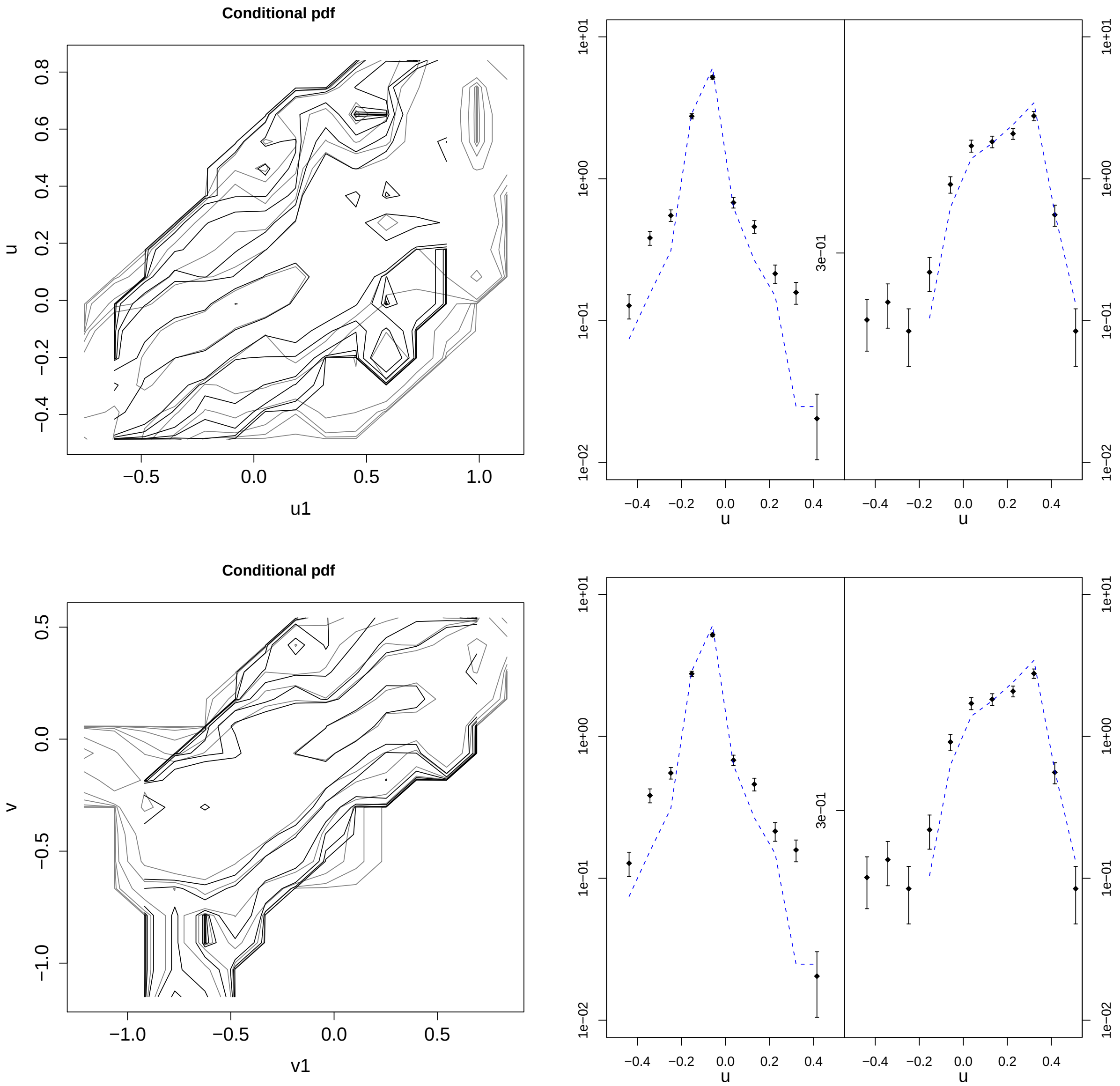

Figure 3: The left hand figures show the contour plots of the pdfs of $p(u_i(\tau)|u_{i,1}(\tau_1))$ (gray) and $p(u_i(\tau)|u_{i,1}(\tau_1), u_{i,2}(\tau_2))$ (black) for $\tau = 0.1375$ for u- and v-increments respectively. Slices at about $+/-0.8\sigma$ for u- and $+/-0.9\sigma$ for v-increments respectively are given on the right hand side and show a high coherence between the two distributions, where the dotted line represents the data from $p(u_i(\tau)|u_{i,1}(\tau_1), u_{i,2}(\tau_2))$ and the errorbars are given by $\sqrt{N}$ with the number of events N in the bin.

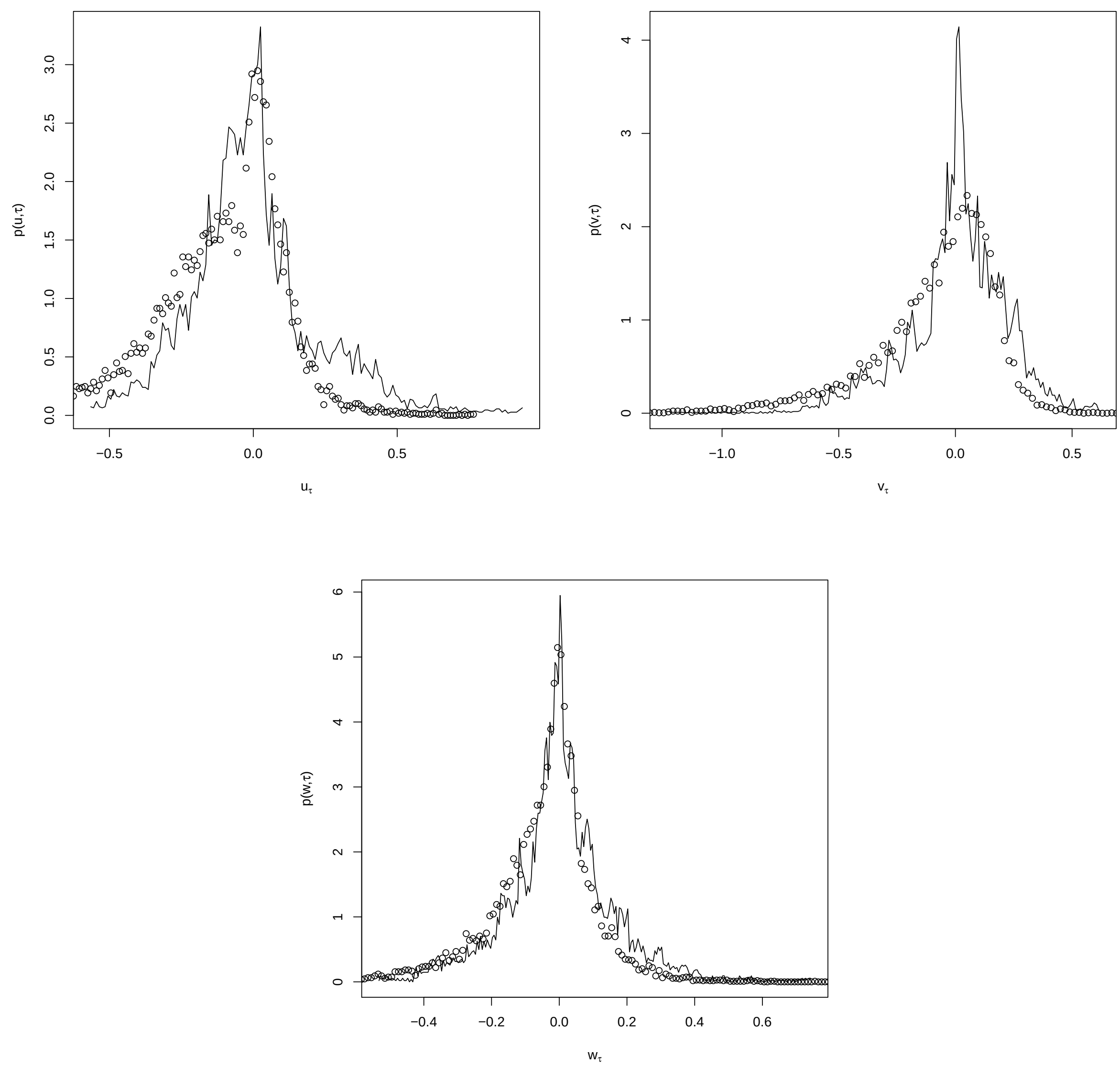

Figure 4: Plot of the non conditioned pdfs $p(u, \tau)$ (line) and $p(u_{rec}, \tau)$ dots at $\tau = 0.1375$ showing good general coincidence however some differences in particular due to the different skewness of the u distribution.

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