A Study of the Influence of Shadowing on the Statistical Properties of the Capacity of Mobile Radio Channels

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Abstract. This paper[1] studies the influence of shadowing on the statistical properties of the channel capacity. The problem is addressed by using a Suzuki process as an appropriate statistical channel model for land mobile terrestrial channels. Using this model, exact solutions for the probability density function (PDF), cumulative distribution function (CDF), level-crossing rate (LCR), and average duration of fades (ADF) of the channel capacity are derived. The results are studied for different levels of shadowing, corresponding to different terrestrial environments. It is observed that the shadowing effect has a significant influence on the variance and the maximum value of the PDF and LCR of the channel capacity, but it has almost no impact on the mean capacity of the channel. The correctness of the theoretical results is confirmed by simulation using a stochastic channel simulator based on the sum-of-sinusoids principle.

Keywords: Land mobile terrestrial channels, channel capacity, shadowing effects, lognormal process, Suzuki process, level-crossing rate, average duration of fades.

1. Introduction

The channel capacity can be considered as a measure of how much information can be transmitted over a channel with a negligible probability of error [1]. The precise knowledge of the statistical properties of the channel capacity is indispensable for the development of future mobile communication systems. While studying the capacity of mobile fading channels, the dynamic behaviour of the channel is generally ignored. However, for the development of future optimized heterogeneous multiuser communication networks, it is important to know how fast the channel capacity changes with time. Therefore, studies pertaining to unveil the dynamics of the channel capacity can be very helpful to

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achieve higher data rates while keeping the probability of error as low as possible. In mobile communication systems, the LCR and ADF of the channel capacity are important characteristic quantities which provide insight into the dynamic behaviour of the channel capacity [2], [3]. The LCR of the channel capacity describes the average number of up-crossings (or down-crossings) of the capacity through a fixed level within a time interval of one second. Analogously, the ADF of the channel capacity is the expected value of the length of the time intervals in which the capacity is below a given level [2–4].

The random amplitude fluctuations of the received signal can be modelled using an appropriate stochastic process. It has widely been accepted that for urban and suburban areas, where the line-of-sight (LOS) signal component is blocked by obstacles, the Rayleigh process is a suitable stochastic process to model the channel [5–7]. In rural regions, however, the LOS component is often a part of the received signal, so that the Rice process is an appropriate choice for modelling such channels. The validity of Rayleigh and Rice channel models is limited to small areas having dimensions in the order of a few tens of wavelengths. The local mean of the received signal envelope remains approximately constant in these areas [8]. The local mean, however, fluctuates in large areas due to shadowing effects. It has widely been reported in the literature that shadowing can adequately be modelled by a lognormal process [9–12]. Therefore, for the case of land mobile terrestrial channels, a Suzuki process is considered to be a more suitable statistical channel model [13]. The Suzuki process is generated by taking the product of a Rayleigh and a lognormal process [14]. The analysis of the PDF, CDF, LCR, and ADF of the channel capacity of fast fading channels, like Rayleigh channels can be found, e.g., in [2–4], [15], [16]. However, there is a lack of information regarding the combined effects of shadowing and fast fading on the channel capacity. The purpose of this paper is to close this gap by using a Suzuki process as an appropriate channel model.

The paper studies the influence of shadowing on the channel capacity. In particular, we have derived analytical expressions for the PDF, CDF, LCR, and ADF of the capacity of Suzuki channels. Previous studies show that the shadow standard deviation can have a wide range of values depending on the terrestrial environment [9]. Therefore, it is important to study the statistical properties of the channel capacity for different values of the shadow standard deviation. Our analysis has revealed that the variance and the maximum value of the PDF and LCR of the channel capacity, respectively, are highly influenced by the shadow standard deviation. However, this parameter has nearly no effect on the mean channel capacity.

The rest of the paper is organized as follows. In Section 2, we review briefly the Suzuki process. The statistical properties of the channel capacity for this model are analyzed in Section 3. Section 4 introduces some special cases of the presented channel model. The simulation model used to verify the theoretical results is introduced in Section 5. In Section 6, the theoretical and simulation results are discussed. Finally, the conclusions are given in Section 7.

2. The Suzuki Channel Model

In this section, we will describe the Suzuki process $\eta(t)$, which is considered as a proper statistical channel model for our problem. The Suzuki process is defined by [14]

$$\eta(t) = \zeta(t) \cdot \lambda(t) \tag{1}$$

where $\zeta(t)$ represents a Rayleigh process and $\lambda(t)$ denotes a lognormal process.

The Rayleigh process $\zeta(t)$ can be described as

$$\zeta(t) = |\mu(t)| \tag{2}$$

where $\mu(t)$ is a complex Gaussian process, i.e.,

$$\mu(t) = \mu_1(t) + j\mu_2(t). \tag{3}$$

In (3), $\mu_1(t)$ and $\mu_2(t)$ are uncorrelated zero-mean real-valued Gaussian processes with identical variances σ_0^2. Under the assumption of isotropic scattering, the autocorrelation function (ACF) $r_{\mu\mu}(\tau)$ of the complex Gaussian process $\mu(t)$ is given by [20]

$$r_{\mu\mu}(\tau) = r_{\mu_1\mu_1}(\tau) + r_{\mu_2\mu_2}(\tau) \tag{4}$$

where

$$r_{\mu_i\mu_i}(\tau) = \sigma_0^2 J_0\left(2\pi f_{\max}\tau\right), \quad i = 1, 2. \tag{5}$$

In (5), $J_0(\cdot)$ denotes the 0th-order Bessel function of the first kind, $r_{\mu_i\mu_i}(\tau)$ is the ACF of the process $\mu_i(t)$, and $f_{\max}$ represents the maximum Doppler frequency.

The lognormal process $\lambda(t)$ in (1) can be expressed as

$$\lambda(t) = 10^{[\sigma_L \nu_3(t) + m_L]/20} \tag{6}$$

where $\nu_3(t)$ is a zero-mean real-valued Gaussian process with unit variance. The third Gaussian process $\nu_3(t)$ is statistically independent of the other two Gaussian processes $\mu_1(t)$ and $\mu_2(t)$. The parameters σ_L and m_L are called the shadow standard deviation and the area mean, respectively. It has been observed that the shadow standard deviation depends on the terrestrial environment [9]. Specifically, it has been shown in [9] that $\sigma_L = 4.3$ dB can be chosen as a suitable value for urban environments, whereas $\sigma_L = 7.5$ dB is an appropriate value for suburban areas. The PDF $p_\lambda(z)$ of the lognormal process $\lambda(t)$ is given by

$$p_\lambda(z) = \frac{20}{\sqrt{2\pi}\ln(10)\sigma_L z} e^{-\frac{(20\log(z)-m_L)^2}{2\sigma_L^2}}, \quad z \geq 0. \tag{7}$$

For the spectral shape of the process $\nu_3(t)$ in (6), we have assumed a Gaussian power spectral density (PSD) given by [13], [17]

$$S_{\nu_3\nu_3}(f) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{f^2}{2\sigma_c^2}} \tag{8}$$

where σ_c is related to the 3 dB cutoff frequency f_c by $f_c = \sigma_c\sqrt{2\ln(2)}$. It is assumed that f_c is much smaller than $f_{\max}$, i.e., $\kappa_c = f_{\max}/f_c \gg 1$. The inverse Fourier transform of $S_{\nu_3\nu_3}(f)$ in (8) results in the ACF $r_{\nu_3\nu_3}(\tau)$ of the process $\nu_3(t)$ as

$$r_{\nu_3\nu_3}(\tau) = e^{-2(\pi\sigma_c\tau)^2}. \tag{9}$$

The time derivative of the Suzuki process $\eta(t)$ is represented by $\dot{\eta}(t)$.[1] In order to analyze the statistical properties of the Suzuki channel capacity (see Section 3), it is necessary to find the joint PDF $p_{\eta^2\dot{\eta}^2}(z,\dot{z})$ of $\eta^2(t)$ and $\dot{\eta}^2(t)$. This problem can be solved by first finding the joint PDF $p_{\eta\dot{\eta}}(z,\dot{z})$ of $\eta(t)$ and $\dot{\eta}(t)$ at the same time t. Thereafter, using the obtained expression for $p_{\eta\dot{\eta}}(z,\dot{z})$, the joint PDF $p_{\eta^2\dot{\eta}^2}(z,\dot{z})$ can be found by applying the concept of transformation of random variables [18]. After some algebraic manipulations on the results found in [13], the PDF $p_\eta(z)$ of $\eta(t)$ can be written as

$$p_\eta(z) = \frac{20 \cdot z}{\ln(10)\sqrt{2\pi}\sigma_0^2\sigma_L} \int_0^\infty \frac{1}{y^3} \cdot e^{-\left(\frac{z}{\sqrt{2}\sigma_0 y}\right)^2} e^{-\left(\frac{20\log(y)-m_L}{\sqrt{2}\sigma_L}\right)^2} dy, \; z \geq 0. \tag{10}$$

[1] Throughout this paper, we will represent the time derivative of a process by an overdot.

Similarly, the joint PDF $p_{\eta\dot{\eta}}(z,\dot{z})$ can be expressed as [13]

$$p_{\eta\dot{\eta}}(z,\dot{z}) = \frac{20 \cdot z}{2\pi \ln(10)\sqrt{\beta}\sigma_0^2\sigma_L} \int_0^\infty \frac{e^{-\left(\frac{z}{\sqrt{2}\sigma_0 y}\right)^2} e^{-\left(\frac{20\log(y)-m_L}{\sqrt{2}\sigma_L}\right)^2}}{y^4 K(z,y)} \times e^{-\left(\frac{\dot{z}}{\sqrt{2\beta}yK(z,y)}\right)^2} dy, \quad z \geq 0, |\dot{z}| < \infty \tag{11a}$$

where

$$K(z,y) = \sqrt{1 + \frac{\gamma}{\beta}\left(\frac{z\sigma_L \ln(10)}{20y}\right)^2} \tag{11b}$$

and

$$\beta = -\ddot{r}_{\mu_i\mu_i}(0) = 2\left(\pi f_{\max}\sigma_0\right)^2, \quad i = 1,2 \tag{11c}$$

$$\gamma = -\ddot{r}_{\nu_3\nu_3}(0) = (2\pi\sigma_c)^2. \tag{11d}$$

Here, β represents the negative curvature of the ACF $r_{\mu_i\mu_i}(\tau)$ of $\mu_i(t)$ at the origin [20], i.e.,

$$\beta = -\frac{d^2}{d\tau^2} r_{\mu_i\mu_i}(\tau)\bigg|_{\tau=0} = -\ddot{r}_{\mu_i\mu_i}(0), \quad i = 1,2. \tag{12}$$

In accordance with (12), the parameter γ is defined.

In order to find the joint PDF $p_{\eta^2\dot{\eta}^2}(z,\dot{z})$, the concept of transformation of random variables [18] is applied. Hence, by using the relationship $p_{\eta^2\dot{\eta}^2}(z,\dot{z}) = (1/4z) \times p_{\eta\dot{\eta}}(\sqrt{z}, \dot{z}/2\sqrt{z})$, we can write by using (11a)

$$p_{\eta^2\dot{\eta}^2}(z,\dot{z}) = \frac{5}{\sqrt{z}2\pi \ln(10)\sqrt{\beta}\sigma_0^2\sigma_L} \int_0^\infty \frac{e^{-\left(\frac{\sqrt{z}}{\sqrt{2}\sigma_0 y}\right)^2} e^{-\left(\frac{20\log(y)-m_L}{\sqrt{2}\sigma_L}\right)^2}}{y^4 K(\sqrt{z},y)} \times e^{-\left(\frac{\dot{z}}{\sqrt{8\beta z}yK(\sqrt{z},y)}\right)^2} dy, \quad z \geq 0, |\dot{z}| < \infty. \tag{13}$$

The expression for $p_{\eta^2\dot{\eta}^2}(z,\dot{z})$ in (13) will be used in the next section for the calculation of the LCR of the channel capacity. From (13), it can be observed that $\eta^2(t)$ and $\dot{\eta}^2(t)$ are not statistically independent processes, since their joint PDF cannot be written as a product of the marginal PDFs $p_{\eta^2}(z)$ and $p_{\dot{\eta}^2}(\dot{z})$. By using (13) in $p_{\eta^2}(z) = \int_{-\infty}^{\infty} p_{\eta^2\dot{\eta}^2}(z,\dot{z})d\dot{z}$, the PDF $p_{\eta^2}(z)$ of $\eta^2(t)$ can be written as

$$p_{\eta^2}(z) = \frac{10}{\sqrt{2\pi}\ln(10)\sigma_0^2\sigma_L} \int_0^\infty \frac{e^{-\left(\frac{\sqrt{z}}{\sqrt{2}\sigma_0 y}\right)^2} e^{-\left(\frac{20\log(y)-m_L}{\sqrt{2}\sigma_L}\right)^2}}{y^3} dy, \quad z \geq 0. \tag{14}$$

The formula presented above will be used in the next section to calculate the PDF of the channel capacity.

3. Statistical Properties of the Capacity of Suzuki Channels

In this section, we will first introduce the capacity of the channel described by Suzuki processes. Thereafter, the expressions for the statistical properties of the channel capacity will be derived using the results obtained in the previous section.

The expression for the channel capacity $C(t)$ of an additive white Gaussian noise (AWGN) channel can be written using the Shannon capacity formula [19] as

$$C(t) = \log_2\left(1 + \gamma_s\,|H(t)|^2\right) \quad \text{(bits/sec/Hz)} \tag{15}$$

where the quantity γ_s is the signal-to-noise ratio (SNR). In (15), $H(t)$ represents the random complex channel gain described using any suitable stochastic channel model. In this article, we have represented the random channel $H(t)$ by a Suzuki process $\eta(t)$. From the fact that the Suzuki process $\eta(t)$ is a real-valued random process, the instantaneous capacity of the Suzuki channel in (15) can be expressed as

$$C(t) = \log_2\left(1 + \gamma_s\eta^2(t)\right). \tag{16}$$

The expression presented in (16) can be considered as a mapping of the random process $\eta(t)$ to another random process $C(t)$. Therefore, by applying the concept of transformation of random variables [18], the PDF $p_{C,\eta}(r)$ of the channel capacity $C(t)$ can be written by substituting (14) in the expression $p_{C,\eta}(r) = (2^r \ln(2)/\gamma_s) \times p_{\eta^2}\left(2^r - 1/\gamma_s\right)$ as

$$p_{C,\eta}(r) = \frac{2^r \ln(2) 10}{\sqrt{2\pi}\,\ln(10)\gamma_s\sigma_0^2\sigma_L} \int_0^\infty \frac{e^{-\left(\frac{\sqrt{2^r-1}}{\sqrt{2\gamma_s}\sigma_0 y}\right)^2} e^{-\left(\frac{20\log(y)-m_L}{\sqrt{2}\sigma_L}\right)^2}}{y^3} dy, \quad r \geq 0. \tag{17}$$

The CDF $F_{C,\eta}(r)$ of the channel capacity $C(t)$ can now be expressed by using $F_{C,\eta}(r) = \int_0^r p_{C,\eta}(x)dx$ as

$$F_{C,\eta}(r) = \frac{20}{\sqrt{2\pi}\,\ln(10)\sigma_L} \int_0^\infty \frac{1}{y} \cdot e^{-\left(\frac{20\log(y)-m_L}{\sqrt{2}\,\sigma_L}\right)^2} \left[1 - e^{-\left(\frac{\sqrt{2^r-1}}{\sqrt{2\gamma_s}\sigma_0 y}\right)^2}\right] dy, \quad r \geq 0. \tag{18}$$

The LCR $N_{C,\eta}(r)$ of the channel capacity $C(t)$ is defined as [2]

$$N_{C,\eta}(r) = \int_0^\infty \dot{z} p_{C\dot{C},\eta}(r,\dot{z}) d\dot{z}, \quad r \geq 0. \tag{19}$$

Thus, in order to find the LCR $N_{C,\eta}(r)$, the joint PDF $p_{C\dot{C},\eta}(z,\dot{z})$ of $C(t)$ and $\dot{C}(t)$ is required. Applying the concept of transformation of random variables [18], $p_{C\dot{C},\eta}(z,\dot{z})$ can be expressed after substituting (13) in $p_{C\dot{C},\eta}(z,\dot{z}) = \left(2^z \ln(2)/\gamma_s\right)^2 \times p_{\eta^2\dot{\eta}^2}\left(2^z - 1/\gamma_s, 2^z \dot{z} \ln(2)/\gamma_s\right)$ as

$$\begin{aligned} p_{C\dot{C},\eta}(z,\dot{z}) \;=\; & \frac{5 \times \left(2^z \ln(2)\right)^2}{2\pi\sqrt{2^z - 1}\,\ln(10)\gamma_s^{\frac{3}{2}}\sqrt{\beta}\sigma_0^2\sigma_L} \int_0^\infty \frac{e^{-\left(\frac{\sqrt{2^z-1}}{\sqrt{2\gamma_s}\sigma_0 y}\right)^2}}{y^4 K\left(\sqrt{\frac{2^z-1}{\gamma_s}}, y\right)} \\ & \times e^{-\left(\frac{20\log(y) - m_L}{\sqrt{2}\sigma_L}\right)^2} e^{-\left(\frac{\dot{z}\left(2^z \ln(2)\right)}{y\sqrt{8\beta\gamma_s\left(2^z-1\right)}\,K\left(\sqrt{\frac{2^z-1}{\gamma_s}}, y\right)}\right)^2} dy, \\ & z \geq 0, |\dot{z}| < \infty \end{aligned} \tag{20}$$

where $K(\cdot,\cdot)$ is the function introduced in (11b).

After substituting (20) in (19) and carrying out some algebraic calculations, we obtain

$$\begin{aligned} N_{C,\eta}(r) \;=\; & \frac{20\sqrt{\beta\left(2^r - 1\right)}}{2\pi \ln(10)\sqrt{\gamma_s}\sigma_0^2\sigma_L} \int_0^\infty \frac{e^{-\left(\frac{\sqrt{2^r-1}}{\sqrt{2\gamma_s}\sigma_0 y}\right)^2} e^{-\left(\frac{20\log(y) - m_L}{\sqrt{2}\sigma_L}\right)^2}}{y^2} \\ & \times K\left(\sqrt{\frac{2^r - 1}{\gamma_s}}, y\right) dy, \quad r \geq 0. \end{aligned} \tag{21}$$

Due to the quantity β, appearing in the numerator of (21), it is observed that the LCR $N_{C,\eta}(r)$ of the channel capacity $C(t)$ is proportional to the maximum Doppler frequency $f_{\max}$. This can be seen by replacing β in (21) by the expression presented in (11c). Thus, by normalizing $N_{C,\eta}(r)$ on $f_{\max}$, the influence of the mobile speed on the LCR can be removed.

Finally, from (18) and (21), the ADF $T_{C,\eta}(r)$ of the channel capacity $C(t)$ can be obtained using [2]

$$T_{C,\eta}(r) = \frac{F_{C,\eta}(r)}{N_{C,\eta}(r)}. \tag{22}$$

4. Special Cases of the Suzuki Channel Model

In this section, we will derive the statistical properties of the channel capacity of Rayleigh and lognormal processes. It will be shown that the corresponding statistical quantities like the PDF, CDF, LCR, and ADF of the channel capacity can be obtained as special cases of the respective results derived for Suzuki processes in the previous section. The detailed discussion on the relationships between the statistical properties of channel capacity of Suzuki, Rayleigh, and lognormal processes can be found in Section 6.

4.1. The Channel Capacity of Rayleigh Processes

Let $\sigma_L \to 0$ and $m_L = 1$ (unit area mean), then the PDF, CDF, and LCR of the channel capacity of Suzuki channels can be written as

$$p_{C,\eta}(r)\Big|_{\substack{\sigma_L \to 0 \\ m_L=1}} = \frac{2^r \ln(2)}{2\gamma_s \sigma_0^2} e^{-\left(\frac{2^r-1}{2\gamma_s\sigma_0^2}\right)}, \quad r \geq 0 \tag{23}$$

$$F_{C,\eta}(r)\Big|_{\substack{\sigma_L \to 0 \\ m_L=1}} = 1 - e^{-\left(\frac{2^r-1}{2\gamma_s\sigma_0^2}\right)}, \quad r \geq 0 \tag{24}$$

and

$$N_{C,\eta}(r)\Big|_{\substack{\sigma_L \to 0 \\ m_L=1}} = \frac{1}{\sigma_0^2}\sqrt{\frac{\beta\left(2^r-1\right)}{2\pi\gamma_s}} e^{-\left(\frac{2^r-1}{2\gamma_s\sigma_0^2}\right)}, \quad r \geq 0 \tag{25}$$

respectively. It can be observed that the expressions presented above correspond to those known for the channel capacity of Rayleigh channels [2]. Hence, the Rayleigh process is a special case of the Suzuki process when $\sigma_L \to 0$ and $m_L = 1$.

4.2. The Channel Capacity of Lognormal Processes

In order to derive the expressions for the statistical properties of the capacity of lognormal channels, a similar procedure can be applied as developed here for Suzuki channels. By using the result for the joint PDF $p_{\lambda\dot{\lambda}}(z,\dot{z})$ of $\lambda(t)$ and $\dot{\lambda}(t)$ in [13] and following similar steps from (13) to (21), the expressions for the PDF, CDF, and LCR of the capacity of lognormal channels can be expressed as

$$p_{C,\lambda}(r) = \frac{2^r \ln(2) 10}{\sqrt{2\pi}\ln(10)\left(2^r-1\right)\sigma_L} e^{-\left(\frac{20\log\left(\sqrt{\frac{2^r-1}{\gamma_s}}\right)-m_L}{\sqrt{2}\sigma_L}\right)^2}, \quad r \geq 0 \tag{26}$$

$$F_{C,\lambda}(r) = \frac{\ln(2)10}{\sqrt{2\pi}\ln(10)\sigma_L} \int_0^r \frac{2^x}{(2^x-1)} \cdot e^{-\left(\frac{20\log\left(\sqrt{\frac{2^x-1}{\gamma_s}}\right)-m_L}{\sqrt{2}\sigma_L}\right)^2} dx, \quad r \geq 0 \qquad (27)$$

and

$$N_{C,\lambda}(r) = \frac{\sqrt{\gamma}}{2\pi} e^{-\left(\frac{20\log\left(\sqrt{\frac{2^r-1}{\gamma_s}}\right)-m_L}{\sqrt{2}\sigma_L}\right)^2}, \quad r \geq 0 \qquad (28)$$

respectively. Furthermore, the ADF of the capacity of lognormal channels can be found using (22), (27), and (28). Alternatively, one can show that the expressions (26)–(28) can be obtained by setting $\sigma_0^2 = 0$ in (17)–(21).

5. The Simulation Model

In this section, the analytical results derived in the previous section will be verified by simulation. We have employed a stochastic channel simulator based on the sum-of-sinusoids principle [20]. The resulting structure of the simulation model for the analysis of the capacity of Suzuki channels is shown in Fig. 1. Here, the hat ($\hat{}$) symbolizes the fact that the underlying stochastic processes are modelled by applying

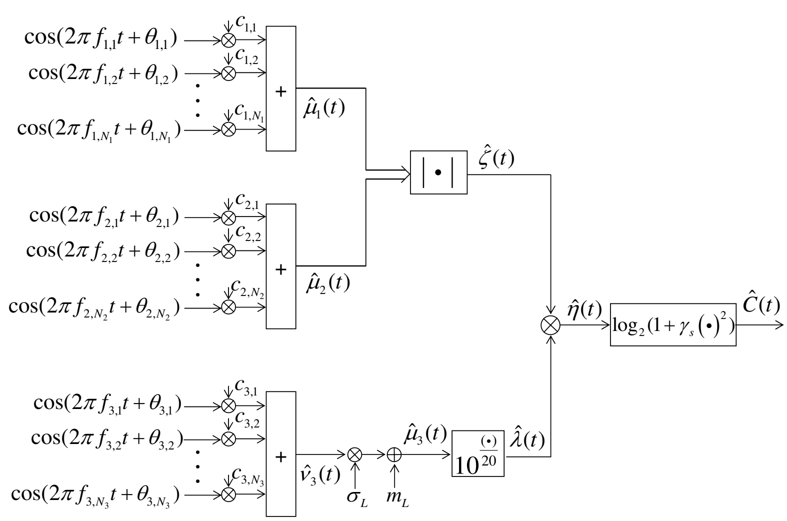

Figure 1. The stochastic simulation model for the capacity analysis of Suzuki channels.

the sum-of-sinusoids method with constant gains $c_{i,n}$, constant frequencies $f_{i,n}$, and random phases $\theta_{i,n}$, respectively. The phases $\theta_{i,n}$ are independent and identically distributed (i.i.d.) random variables, each having a uniform distribution over the interval $(0, 2\pi]$. For the stochastic processes $\hat{\mu}_1(t)$ and $\hat{\mu}_2(t)$ in Fig. 1, the parameters $f_{i,n}$ and $c_{i,n}$ are calculated using the generalized method of exact Doppler spread (GMEDS_1) [21]. Whereas, for the stochastic process $\hat{\nu}_3(t)$, these parameters are calculated by applying the modified method of equal areas (MMEA) [17]. In Fig. 1, the mapping of the Suzuki process $\hat{\eta}(t)$ to the capacity $\hat{C}(t)$ is also shown. Finally, by using this model, all simulation results presented in the next section are obtained by averaging over 15 sample functions of the capacity $\hat{C}(t)$.

6. Numerical Results

In this section, we will discuss the analytical and simulation results for the statistical properties of the channel capacity. In order to illustrate the influence of the shadowing effect on the statistics of the channel capacity, we have taken into account different values of σ_L, ranging from 1 dB to 10 dB. We have also included some special cases, e.g., Rayleigh fading ($\sigma_L \to 0$ dB) and lognormal fading ($\sigma_0^2 = 0$), in our study for comparison purposes. Moreover, the results obtained for $\sigma_L = 4.3$ dB (urban environment [9]) and $\sigma_L = 7.5$ dB (suburban environment [9]) are also shown. For the simulation model presented in Fig. 1, we have used $N_1 = 30$, $N_2 = 31$, and $N_3 = 32$. The maximum Doppler frequency $f_{\max}$ was chosen to be 91 Hz. The value for the parameter κ_c was taken as 5 and the value for σ_0^2 and area mean m_L was set to unity. Unless otherwise stated, the value of the SNR γ_s was set to 25 dB.

Firstly, the PDFs of the lognormal and Suzuki processes are shown in Figs. 2 and 3, respectively. These figures demonstrate that the shadow standard deviation σ_L has a dominant effect on the spread and the peak value of the PDFs of these processes. The PDF and CDF of the capacity of the Suzuki process are presented in Figs. 4 and 5, respectively. Results for the special cases, i.e., for $\sigma_L \to 0$ dB and $\sigma_0^2 = 0$, are also shown in these figures. From these results it can be observed that, as the value of σ_L approaches 0 dB, the statistics of the capacity of the Suzuki process approaches the statistics of the capacity of the Rayleigh process.

The mean capacity $E[C(t)]$ of the Suzuki process is shown in Fig. 6 for different values of the SNR γ_s. From Figs. 4 and 6, it can easily be seen that the shadow standard deviation σ_L has nearly no influence on the mean capacity of the Suzuki process. The variance of the channel

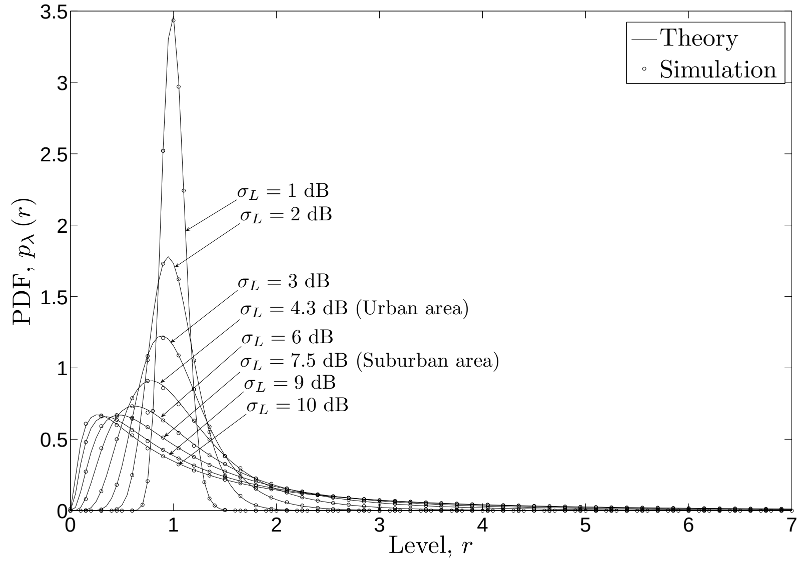

Figure 2. The PDF of lognormal processes.

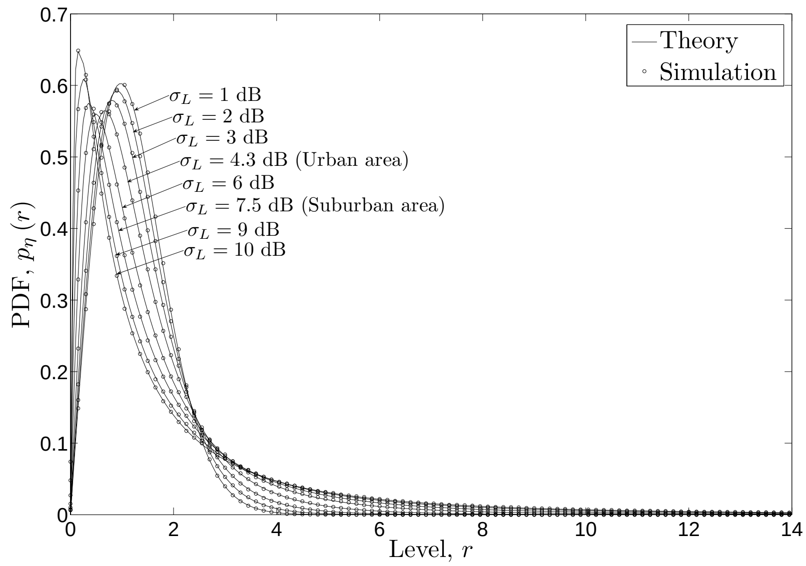

Figure 3. The PDF of Suzuki processes.

capacity is presented in Fig. 7. It is quite evident that an increase in the shadow standard deviation σ_L increases the variance of the channel capacity. This observation is in accordance with the results presented in Fig. 4. Similarly, the same effect can be observed by increasing the value of the SNR γ_s. In Figs. 8 and 9, the normalized LCR and ADF

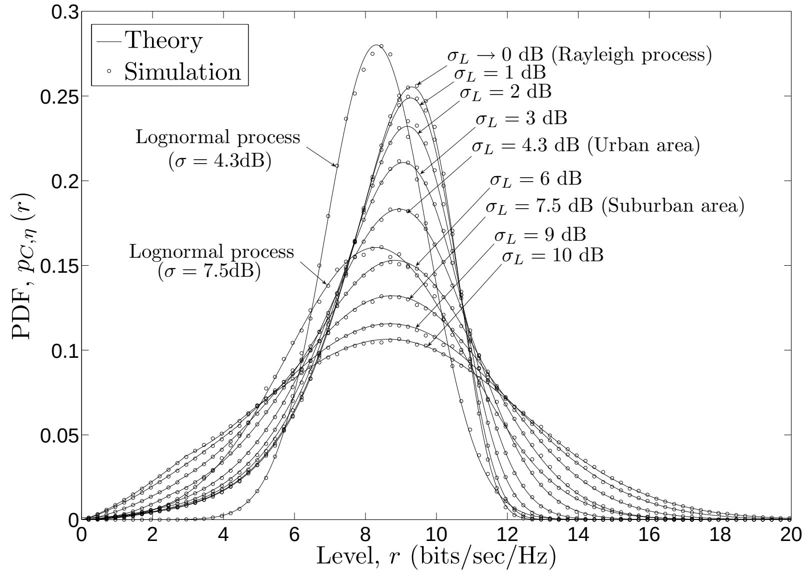

Figure 4. The PDF of the capacity of Suzuki and lognormal channels.

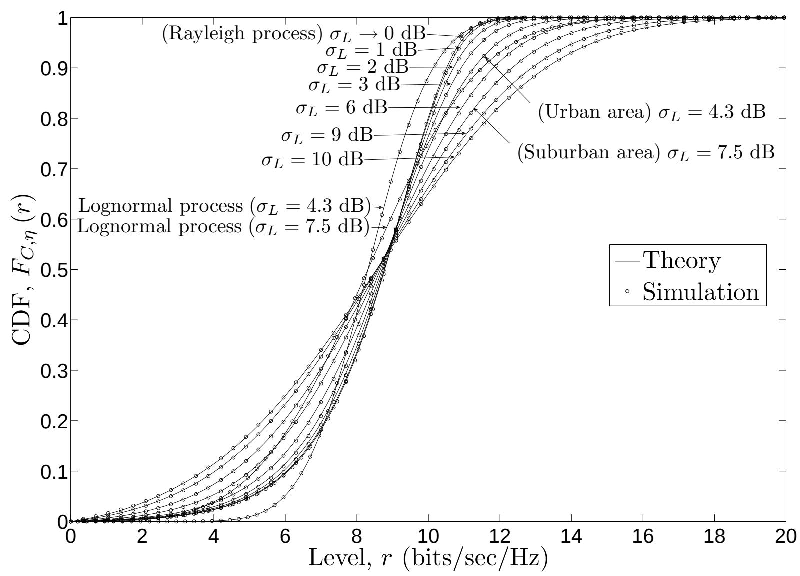

Figure 5. The CDF of the capacity of Suzuki and lognormal channels.

of the channel capacity are shown, respectively. It can be observed that the Rayleigh and lognormal processes set an upper and a lower bound, respectively, on the LCR of the capacity of the Suzuki process. For low and medium signal levels r, the LCR of the capacity of the lognormal process is lower than the LCR of the capacity of the Suzuki

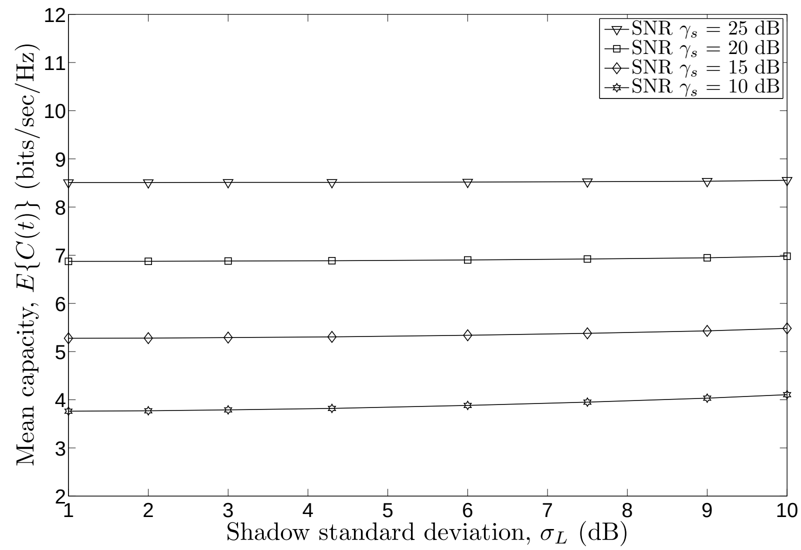

Figure 6. The mean capacity of Suzuki channels.

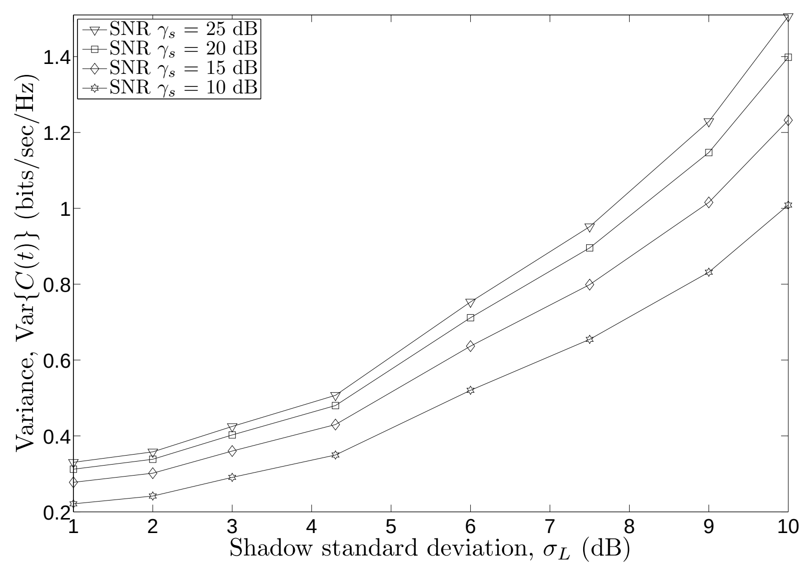

Figure 7. The variance of the capacity of Suzuki channels.

and Rayleigh processes. However, for high signal levels r, the LCR of the capacity of the lognormal process is higher than that of the Suzuki and Rayleigh processes. Analogously, the converse statement is true for the ADF of the channel capacity. It is also observed from Figs. 4 and 8 that increasing the value of the shadow standard deviation σ_L

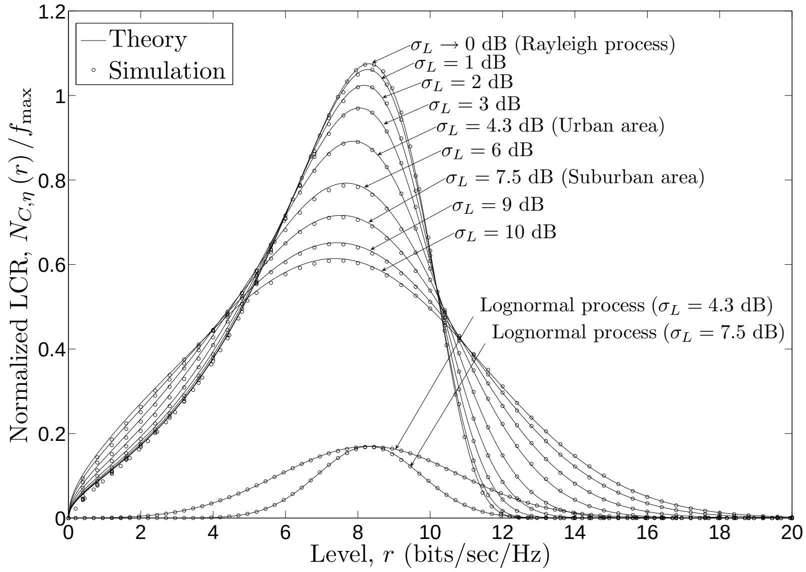

Figure 8. The normalized LCR of the capacity of Suzuki and lognormal channels.

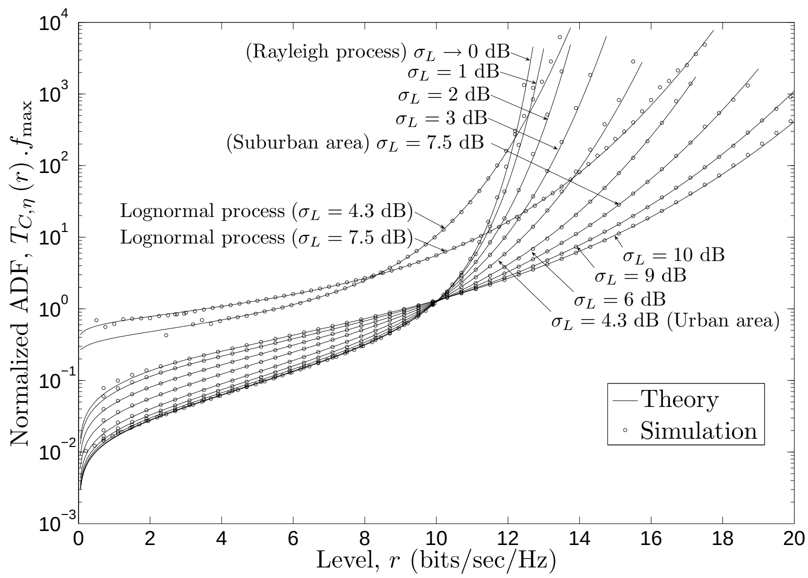

Figure 9. The normalized ADF of the capacity of Suzuki and lognormal channels.

results in an increase of the spread of the PDF and LCR of the channel capacity, where the peak value of these quantities decreases. From the results presented in Figs. 4–9, we gain an insight into how the statistics of the Suzuki channel capacity approaches those of the Rayleigh and lognormal channels.

7. Conclusion

In all results presented here, the simulation results are found to be in a very good correspondence with the analytical results. In this paper, we have studied the statistical properties of the capacity of Suzuki channels. We have derived exact analytical expressions for the PDF, CDF, LCR, and ADF of the channel capacity. Moreover, the influence of shadowing on the statistics of the channel capacity has been investigated. It has been observed that the variance and the maximum value of the PDF and LCR of the channel capacity, respectively, are highly influenced by the shadow standard deviation. It has been shown that as the value of shadow standard deviation increases the variance of the channel capacity increases. However, this parameter has only a minor effect on the mean channel capacity. It has also been observed that as the shadow standard deviation approaches 0 dB, the statistics of the channel capacity of Suzuki channels approaches to that of Rayleigh channels. Findings of this paper are helpful for analyzing the dynamic behaviour of the channel capacity for land mobile terrestrial channels in different terrestrial environments. The theoretical results obtained have been verified by simulations, where the simulation results match the theoretical expectations very closely.

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Author's Vitae

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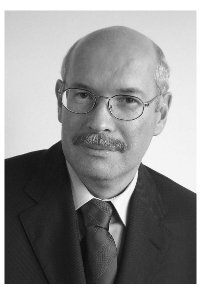

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