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Input-Output Finite-Time Stability of Discrete-Time Impulsive Switched Linear Systems with State Delays

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Abstract This paper is concerned with the problem of input-output finite-time stability (IO-FTS) for discrete impulsive switched systems with state delays. Sufficient conditions are presented for the existence of IO-FTS for such systems under the cases of certain switching, arbitrary switching, and uncertain switching. All the obtained results are formulated in a set of linear matrix inequalities (LMIs). Two numerical examples are given to illustrate the effectiveness of the proposed results.

Keywords Impulsive systems · Switched systems · State delays · Input-output stability · Finite-time stability · Linear matrix inequality

1 Introduction

Impulsive switched systems are gaining momentum due to their extensive applications in many fields, such as mechanical systems, the automotive industry, aircraft, air traffic control, networked control, chaotic-based secure communication, quality of service in the Internet, and video coding [13]. Impulsive switched systems are a special class of hybrid systems with isolated discrete switching events, and whose state will jump at switching instants. The problems of controllability, observability,

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stability, and stabilization of these systems have been successfully investigated, and a rich body of literature is available [11, 15, 18, 25]. For example, [18] established necessary and sufficient conditions for controllability and observability with respect to a given switching time sequence. Some results on stability and stabilization were developed in [11, 15, 25]. Because time delays exist widely in practical environments and often cause undesirable performance, recently many research efforts have been devoted to the study of time-delay systems [17, 19, 23, 26–30, 32, 33]. For instance, the problems of control and filtering for networked systems with delays were investigated in [26–30]. In [17, 19, 23, 32, 33], the stability of impulsive switched systems with time delays was researched. It should be pointed out that most of the reported results have been concerned with Lyapunov stability.

On the other hand, as an important issue of stability theory, finite-time stability has received considerable research attention because of its engineering applications in some practical dynamics such as missile systems and certain aircraft maneuvers, etc., which are only required to perform satisfactorily on a fixed time horizon. Compared with classical Lyapunov stability, finite-time stability is a more practical concept, and it is useful to study the behavior of the system over a finite time interval. More specifically, it describes the phenomenon that the system state is not asymptotically stable, but stays within an acceptable bound during a short period of time. Some useful results on this problem have appeared (see, e.g., [2, 4, 5, 10, 12, 16, 21, 22, 31] and the references therein).

Recently, Amato et al. [3] proposed a notion called input-output finite-time stability (IO-FTS). This concept is used to quantify the input-output behavior of the dynamics within a prescribed finite time interval. Roughly speaking, a system is said to be input-output finite-time stable if, given a class of norm-bounded input signals defined over a specified time interval T, the outputs of the system do not exceed an assigned threshold during T. It has been stated in [14] that IO-FTS and classic L_p IO stability [6] are two independent concepts because there are three main differences between them. Indeed, the latter involves signals defined over a finite time interval and does not necessarily require the inputs and outputs to belong to the same class; also quantitative bounds on both inputs and outputs must be specified. So far, some results on this topic have been reported. For example, IO-FTS of discrete-time linear systems was studied in [8]. The problem of IO-FTS of hybrid systems was solved in [1, 7]. In [9], necessary and sufficient conditions for the existence of IO-FTS of linear systems for the class of L_2 input signals were given. In [20], the issues of input-output finite-time stability and stabilization of stochastic Markovian jump systems were investigated. IO-FTS of singular linear systems was analyzed in [24]. However, note that the aforementioned results are mainly concerned with delay-free systems. To the best of our knowledge, the issue of IO-FTS of discrete impulsive switched systems with state delays has not yet been investigated, which motivates our present study.

In the paper, we are interested in investigating the IO-FTS problem for a class of discrete impulsive switched systems with state delays. The main contributions of this paper can be summarized as follows. (i) State delay is first considered for the analysis of IO-FTS. (ii) Sufficient conditions which guarantee that a given discrete impulsive switched system with state delay is input-output finite-time stable over a specified time interval are presented for two different input classes. (iii) Three cases including known switching instants, arbitrary switching (no knowledge about the switching instants), and uncertain switching (the switching instants are known within a given uncertainty) are taken into account.

This paper is organized as follows. In Sect. 2, the problem formulation and the definition of IO-FTS are introduced. In Sect. 3, sufficient conditions for the existence of IO-FTS of the underlying system under these considered cases (known switching instants, arbitrary switching, and uncertain switching) are developed. Two numerical examples are provided to illustrate the effectiveness of the proposed results in Sect. 4. The concluding remarks are given in Sect. 5.

Notation Throughout this paper, the superscript "T" denotes the transpose, and the notation $X \geq Y (X > Y)$ means that the matrix $X - Y$ is positive semidefinite (positive definite, respectively). X^{-1} denotes the inverse of X. The asterisk $*$ in a matrix is used to denote the term that is induced by symmetry. I represents the identity matrix with an appropriate dimension. The set of all nonnegative integers is represented by Z. The set of all positive integers is represented by Z^+. Given a set $\Omega \subseteq Z$, a symmetric positive definite matrix R and a discrete-time signal $w(\cdot) : \Omega \mapsto R^q$, the weighted norm

$$\left(\sum_{k \in \Omega} w^T(k) R w(k) \right)^{\frac{1}{2}}$$

will be denoted by $\|w\|_{\Omega,R}$.

2 Problem Formulation and Preliminaries

Consider the following discrete time impulsive switched systems with state delays:

$$x(k+1) = A_{\sigma(k)}(k)x(k) + A_{d\sigma(k)}(k)x(k-d) + G_{\sigma(k)}(k)w(k),$$
$$k \neq k_b - 1,\ b \in Z^+, \tag{1a}$$
$$x(k+1) = J(k)x(k), \quad k = k_b - 1,\ b \in Z^+, \tag{1b}$$
$$x(\theta) = \phi(\theta), \quad \theta \in [-d, 0], \tag{1c}$$
$$y(k) = C_{\sigma(k)}(k)x(k) + D_{\sigma(k)}w(k), \tag{1d}$$

where $x(k) \in R^n$ is the state vector, $w(k) \in R^q$ is the disturbance, and $y(k) \in R^\vartheta$ is the output. $\phi(\theta)$ is a discrete vector-valued initial function defined on the interval $[-d, 0]$. d is the discrete constant delay. $\sigma(k)$ is a switching signal which takes its values in the finite set $\Upsilon := \{1, \ldots, l\}$. l denotes the number of subsystems. $k_0 = 0$ is the initial time. k_b ($b \in Z^+$) denotes the bth switching instant or impulsive jump. Moreover, $\sigma(k) = i \in \Upsilon$ means that the ith subsystem is active. $\sigma(k-1) = j$ and $\sigma(k) = i$ $(i \neq j)$ indicate that k is a switching instant at which the system is switched from the j-th subsystem to the ith subsystem. At switching instants, there exist impulsive jumps described by (1b). $A_i(k)$, $A_{di}(k)$, $G_i(k)$, $C_i(k)$, $D_i(k)$ $(i \in \Upsilon)$ and $J(k)$ are discrete-time matrix-valued functions of k.

Before giving the definition of IO-FTS, we first introduce some classes of discrete-time signals, which have been given in Ref. [8].

Let $D(R^n)$ denote the vector space of R^n-valued sequences on the set Z. The subspace L_p of $D(R^n)$, with $p < +\infty$, consists of all the sequences v such that

$$\left(\sum_{k=0}^{\infty} \|v(k)\|^p\right)^{\frac{1}{p}} < +\infty.$$

The left-hand side is defined to be the norm in L_p and it is denoted with $\|v\|_p$.

In this paper, given a subset $\Omega \subseteq Z$, we indicate by $L_{p,\Omega}$ the restriction of L_p over the interval $\Omega = \{0, 1, \ldots, N\}$. In particular, all the sequences $v \in L_{p,\Omega}$ verify

$$\left(\sum_{\Omega} \|v(k)\|^p\right)^{\frac{1}{p}} < +\infty.$$

Finally, we indicate by L_∞ the subspace of $D(R^n)$ composed of all the sequences v such that

$$\|v(k)\|^2 < +\infty, \quad \forall k \in Z.$$

According to the previous definitions, we indicate by $L_{\infty,\Omega}$ the restriction of L_∞ over the set Ω.

Definition 1 ([8]) Given an integer $N \in Z$, a class of input signals W defined over a set $\Omega = \{0, 1, \ldots, N\}$, and a positive definite discrete-time matrix-valued function $Q(\cdot)$, system (1a)–(1d) is said to be input-output finite-time stable with respect to $(W, Q(\cdot), N)$ if for $\phi(\theta) = 0$, $\theta \in [-d, 0]$,

$$w(\cdot) \in W \Rightarrow y^T(k)Q(k)y(k) < 1, \quad k \in \Omega. \tag{2}$$

In this paper, we shall consider two different cases (as was done in [8]), since different classes of signals may require different analysis techniques. Now, we denote a positive definite symmetric matrix R.

(i) The set W coincides with the set of signals with bounded weighted L_2 norm over $\Omega = \{0, 1, \ldots, N\}$, i.e.,

$$W_2(N, R) := \left\{w(\cdot) \in L_{2,\Omega} : \|w\|_{\Omega,R} \le 1\right\}.$$

(ii) The set W coincides with the set of uniformly bounded signals over $\Omega = \{0, 1, \ldots, N\}$, i.e.,

$$W_\infty(N, R) := \left\{w(\cdot) \in L_{\infty,\Omega} : w^T(k)Rw(k) \le 1, k \in \Omega\right\}.$$

Le Γ denote the set of switching instants of system (1a)–(1d). Since we are interested in the behavior of switched linear systems with impulsive jumps in a given time interval, we assume that

$$\Omega \cap \Gamma = \{k_1, k_2, \ldots, k_m\} \tag{3}$$

i.e., only a finite number of switches occur in (1a)–(1d).

In the rest of the paper, in order to simplify the notation, we will drop the dependency of σ on k, and of the class of input W on N and R, except when those terms could introduce misunderstanding.

3 Stability Analysis

In this section, we provide sufficient conditions for the existence of IO-FTS of system (1a)–(1d). We consider three different cases, depending on the knowledge of the switching instants over the set $\Omega = \{0, 1, \dots, N\}$. In particular, in Sect. 3.1, we assume that the switching instants are perfectly known. Afterwards, we derive a sufficient condition for the existence of IO-FTS when no information about the switching instants is available, i.e., the case of arbitrary switching. In Sect. 3.3, we consider the case of uncertain switching instants.

3.1 Known Switching Instants Case

Let us first consider the case of known switching instants with W_2 input signals. The following theorem presents a sufficient condition for the existence of IO-FTS of the underlying system in this case.

Theorem 1 *Consider system* (1a)–(1d), *for a given positive definite discrete-time matrix-valued function* $Q(\cdot)$ *defined over a set* $\Omega = \{0, 1, \dots, N\}$. *If there exist positive definite matrix-valued functions* $P(\cdot)$, $\theta(\cdot)$, $T_1(\cdot)$, *and* $T_2(\cdot)$, *and a positive definite matrix* S, *such that*

$$\begin{bmatrix} S - P(k) & 0 & A_\sigma^T(k)P(k+1) & \Xi_1(k) & 0 \\ * & -S & A_{d\sigma}^T(k)P(k+1) & 0 & \Xi_2(k) \\ * & * & -P(k+1) & 0 & 0 \\ * & * & * & -T_1(k) & 0 \\ * & * & * & * & -T_2(k) \end{bmatrix} < 0,$$

$$k \in \{0, 1, \dots, N-1\}, \quad k \neq k_b - 1, \tag{4}$$

$$\begin{bmatrix} T_1(k) + T_2(k) - R & G_\sigma^T(k) \\ * & -P(k+1) \end{bmatrix} < 0, \quad k \in \{0, 1, \dots, N-1\},\ k \neq k_b - 1, \tag{5}$$

$$J^T(k)P(k+1)J(k) - P(k) < 0, \quad k = k_b - 1, \tag{6}$$

$$2\theta(k)C_\sigma^T(k)Q(k)C_\sigma(k) - P(k) \leq 0, \quad k \in \{0, 1, \dots, N\}, \tag{7}$$

$$\theta(k)R - R \geq 2\theta(k)D_\sigma^T(k)Q(k)D_\sigma(k), \quad k \in \{0, 1, \dots, N\}, \tag{8}$$

$$\theta(k) > 1, \quad k \in \{0, 1, \dots, N\}, \tag{9}$$

hold, then the system is input-output finite-time stable with respect to $(W_2, Q(\cdot), N)$, *where* $\Xi_1(k) = A_\sigma^T(k)P(k+1)G_\sigma(k)$, $\Xi_2(k) = A_{d\sigma}^T(k)P(k+1)G_\sigma(k)$.

Proof Consider the following Lyapunov function candidate:

$$V(k) = V_1(k) + V_2(k),$$

with

$$V_1(k) = x^T(k)P(k)x(k),$$
$$V_2(k) = \sum_{r=k-d}^{k-1} x^T(r)Sx(r).$$

When $k \neq k_b - 1$, along the trajectory of system (1a)–(1d), we have

$$\begin{aligned}
\Delta V_1(k) &= x^T(k+1)P(k+1)x(k+1) - x^T(k)P(k)x(k) \\
&= x^T(k)A_\sigma^T(k)P(k+1)A_\sigma(k)x(k) + 2x^T(k)A_\sigma^T(k)P(k+1)A_{d\sigma}(k)x(k-d) \\
&\quad + x^T(k)A_\sigma^T(k)P(k+1)G_\sigma(k)w(k) + w^T(k)G_\sigma^T(k)P(k+1)A_\sigma(k)x(k) \\
&\quad + x^T(k-d)A_{d\sigma}^T(k)P(k+1)A_{d\sigma}(k)x(k-d) \\
&\quad + w^T(k)G_\sigma^T(k)P(k+1)A_{d\sigma}(k)x(k-d) \\
&\quad + x^T(k-d)A_{d\sigma}^T(k)P(k+1)G_{d\sigma}(k)w(k) \\
&\quad + w^T(k)G_\sigma^T(k)P(k+1)G_\sigma(k)w(k) - x^T(k)P(k)x(k), \\
\Delta V_2(k) &= \sum_{r=k-d+1}^{k} x^T(r)Sx(r) - \sum_{r=k-d}^{k-1} x^T(r)Sx(r) \\
&= x^T(k)Sx(k) - x^T(k-d)Sx(k-d).
\end{aligned}$$

Denote

$$\begin{aligned}
v_1(k) &= T_1^{1/2}(k)w(k) - T_1^{-1/2}(k)G_\sigma^T(k)P(k+1)A_\sigma(k)x(k), \\
v_2(k) &= T_2^{1/2}(k)w(k) - T_2^{-1/2}(k)G_\sigma^T(k)P(k+1)A_{d\sigma}(k)x(k-d).
\end{aligned}$$

Then, we have

$$\begin{aligned}
& v_1^T(k)v_1(k) \\
&\quad = w^T(k)T_1(k)w(k) - x^T(k)A_\sigma^T(k)P(k+1)G_\sigma(k)w(k) \\
&\qquad + x^T(k)A_\sigma^T(k)P(k+1)G_\sigma(k)T_1^{-1}(k)G_\sigma^T(k)P(k+1)A_\sigma(k)x(k) \\
&\qquad - w^T(k)G_\sigma^T(k)P(k+1)A_\sigma(k)x(k), \\
& v_2^T(k)v_2(k) \\
&\quad = w^T(k)T_2(k)w(k) - x^T(k-d)A_{d\sigma}^T(k)P(k+1)G_\sigma(k)w(k) \\
&\qquad + x^T(k-d)A_{d\sigma}^T(k)P(k+1)G_\sigma(k)T_2^{-1}(k)G_\sigma^T(k)P(k+1)A_{d\sigma}(k)x(k-d) \\
&\qquad - w^T(k)G_\sigma^T(k)P(k+1)A_{d\sigma}(k)x(k-d).
\end{aligned}$$

It follows that

$$\begin{aligned}
& \Delta V(k) + v_1^T(k)v_1(k) + v_2^T(k)v_2(k) \\
&\quad = x^T(k)\big[A_\sigma^T(k)P(k+1)A_\sigma(k) - P(k) + S\big]x(k) \\
&\qquad + x^T(k)A_\sigma^T(k)P(k+1)G_\sigma(k)T_1^{-1}(k)G_\sigma^T(k)P(k+1)A_\sigma(k)x(k) \\
&\qquad + 2x^T(k)A_\sigma^T(k)P(k+1)A_{d\sigma}(k)x(k-d) \\
&\qquad + x^T(k-d)\big[A_{d\sigma}^T(k)P(k+1)A_{d\sigma}(k) - S\big]x(k-d) \\
&\qquad + x^T(k-d)A_{d\sigma}^T(k)P(k+1)G_\sigma(k)T_2^{-1}(k)G_\sigma^T(k)P(k+1)A_{d\sigma}(k)x(k-d) \\
&\qquad + w^T(k)\big[G_\sigma^T(k)P(k+1)G_\sigma(k) + T_2(k) + T_1(k)\big]w(k). \qquad (10)
\end{aligned}$$

Denoting $X(k) = [x^T(k)\ x^T(k-d)]^T$, (10) can be expressed as

$$\begin{aligned}&\Delta V(k)+v_1^T(k)v_1(k)+v_2^T(k)v_2(k)\\&\quad=X^T(k)\Phi_1(k)X(k)+w^T(k)\Phi_2(k)w(k),\end{aligned}$$

where

$$\begin{aligned}\Phi_1(k)&=\begin{bmatrix}\Phi_{11}(k)&\Phi_{12}(k)\\ *&\Phi_{22}(k)\end{bmatrix},\\
\Phi_{11}(k)&=A_\sigma^T(k)P(k+1)A_\sigma(k)-P(k)+S\\&\quad+A_\sigma^T(k)P(k+1)G_\sigma(k)T_1^{-1}(k)G_\sigma^T(k)P(k+1)A_\sigma(k),\\
\Phi_{12}(k)&=A_\sigma^T(k)P(k+1)A_{d\sigma}(k),\\
\Phi_{22}(k)&=A_{d\sigma}^T(k)P(k+1)A_{d\sigma}(k)-S\\&\quad+A_{d\sigma}^T(k)P(k+1)G_\sigma(k)T_2^{-1}(k)G_\sigma^T(k)P(k+1)A_{d\sigma}(k),\\
\Phi_2(k)&=G_\sigma^T(k)P(k+1)G_\sigma(k)+T_2(k)+T_1(k).\end{aligned}$$

By Schur's complement, (4) is equivalent to

$$\Phi_1(k)<0, \tag{11}$$

and (5) is equivalent to the following inequality:

$$\Phi_2(k)<R. \tag{12}$$

Then it is not difficult to get

$$\Delta V(k)+v_1^T(k)v_1(k)+v_2^T(k)v_2(k)<w^T(k)Rw(k),\quad k\neq k_b-1. \tag{13}$$

It follows that

$$\Delta V(k)<w^T(k)Rw(k),\quad k\neq k_b-1. \tag{14}$$

Note that the state jumps $g\le m$ times over the set $\Omega=\{0,1,\dots,N\}$. Summing (14) from 0 to k_1-1 and taking into account that $x(0)=0$ and that $w(\cdot)$ belongs to W_2, we obtain

$$x^T(k_1-1)P(k_1-1)x(k_1-1)-x^T(0)P(0)x(0)\le\sum_{h=0}^{k_1-2}w^T(h)Rw(h). \tag{15}$$

Similarly, we obtain

$$\begin{aligned}&x^T(k_2-1)P(k_2-1)x(k_2-1)-x^T(k_1)P(k_1)x(k_1)\le\sum_{h=k_1}^{k_2-2}w^T(h)Rw(h),\\&\vdots\\&x^T(k)P(k)x(k)-x^T(k_g)P(k_g)x(k_g)\le\sum_{h=k_g}^{k-1}w^T(h)Rw(h),\quad k\le N\end{aligned} \tag{16}$$

From (15)–(16), it readily follows that

$$\begin{aligned}
& x^T(k)P(k)x(k) + \sum_{b=1}^{g}\big[x^T(k_b-1)P(k_b-1)x(k_b-1) - x^T(k_b)P(k_b)x(k_b)\big] \\
& \le \sum_{h=0}^{k_1-2} w^T(h)Rw(h) + \sum_{h=k_1}^{k_2-2} w^T(h)Rw(h) + \cdots + \sum_{h=k_g}^{k-1} w^T(h)Rw(h) \\
& < \sum_{h=0}^{k-1} w^T(h)Rw(h).
\end{aligned} \tag{17}$$

Since

$$\begin{aligned}
& x^T(k_b)P(k_b)x(k_b) - x^T(k_b-1)P(k_b-1)x(k_b-1) \\
& \quad = x^T(k_b-1)\big[J^T(k_b-1)P(k_b)J(k_b-1) - P(k_b-1)\big]x(k_b-1),
\end{aligned}$$

for all $b \le m$, condition (6) implies that

$$\sum_{b=1}^{g}\big[x^T(k_b-1)P(k_b-1)x(k_b-1) - x^T(k_b)P(k_b)x(k_b)\big] \ge 0, \tag{18}$$

hence it holds that

$$\begin{aligned}
x^T(k)P(k)x(k) & < \sum_{h=0}^{k-1} w^T(h)Rw(h) \\
& = \|w\|^2_{\{0,1,\ldots,k-1\},R} < \|w\|^2_{\Omega,R} \le 1.
\end{aligned} \tag{19}$$

Finally, let us consider the output weighted norm $y^T(k)Q(k)y(k)$. According to the output equation (1d), we have

$$\begin{aligned}
& y^T(k)Q(k)y(k) \\
& \quad = \big[C_\sigma(k)x(k) + D_\sigma(k)w(k)\big]^T Q(k)\big[C_\sigma(k)x(k) + D_\sigma(k)w(k)\big].
\end{aligned} \tag{20}$$

Denote

$$v_3(k) = Q^{1/2}(k)C_\sigma(k)x(k) - Q^{1/2}(k)D_\sigma(k)w(k). \tag{21}$$

It follows from (21) that

$$\begin{aligned}
& v_3^T(k)v_3(k) \\
& \quad = x^T(k)C_\sigma^T(k)Q(k)C_\sigma(k)x(k) + w^T(k)D_\sigma^T(k)Q(k)D_\sigma(k)w(k) \\
& \qquad - x^T(k)C_\sigma^T(k)Q(k)D_\sigma(k)w(k) - w^T(k)D_\sigma^T(k)Q(k)C_\sigma(k)x(k).
\end{aligned} \tag{22}$$

Substituting (22) into (20), we have

$$\begin{aligned}
& y^T(k)Q(k)y(k) \\
& \quad = 2x^T(k)C_\sigma^T(k)Q(k)C_\sigma(k)x(k) + 2w^T(k)D_\sigma^T(k)Q(k)D_\sigma(k)w(k) - v_3^T(k)v_3(k) \\
& \quad \le 2\big[x^T(k)C_\sigma^T(k)Q(k)C_\sigma(k)x(k) + w^T(k)D_\sigma^T(k)Q(k)D_\sigma(k)w(k)\big].
\end{aligned}$$

Exploiting conditions (7), (8), it follows that

$$y^T(k)Q(k)y(k) \le 2\left(\frac{1}{2\theta(k)}x^T(k)P(k)x(k) + \frac{\theta(k)-1}{2\theta(k)}w^T(k)Rw(k)\right). \quad (23)$$

Considering that $x^T(k)P(k)x(k) < 1$, $w(\cdot) \in W_2$, and $\theta(k) > 1$, we can conclude that

$$y^T(k)Q(k)y(k) < 1, \quad k \in \Omega.$$

According to Definition 1, we can obtain that system (1a)–(1d) is input-output finite-time stable with respect to $(W_2, Q(\cdot), N)$. This completes the proof. □

When $A_{di}(k) = 0$, $i \in \Upsilon$, we can get the following result.

Corollary 1 *Consider system* (1a)–(1d) *with* $A_{di}(k) = 0$, $i \in \Upsilon$. *Given a positive definite discrete-time matrix-valued function* $Q(\cdot)$ *defined over a set* $\Omega = \{0, 1, \dots, N\}$, *if there exist positive definite matrix-valued functions* $P(\cdot)$, $\theta(\cdot)$, *and* $T_1(\cdot)$, *and a positive definite matrix* S, *such that* (6)–(9) *and*

$$\begin{bmatrix} A_\sigma^T(k)P(k+1)A_\sigma(k) - P(k) & \Xi_1(k) \\ * & -T_1(k) \end{bmatrix} < 0,$$
$$k \in \{0, 1, \dots, N-1\}, \; k \ne k_b - 1, \quad (24)$$

$$\begin{bmatrix} T_1(k) - R & G_\sigma^T(k) \\ * & -P(k+1) \end{bmatrix} < 0, \quad k \in \{0, 1, \dots, N-1\}, \; k \ne k_b - 1, \quad (25)$$

hold, then the system is input-output finite-time stable with respect to $(W_2, Q(\cdot), N)$.

Remark 1 Note that if there is no state jump or switching occurs over the set $\Omega = \{0, 1, \dots, N\}$, then the condition (6) is no longer required, and the result presented in Corollary 1 can generate the one proposed in Theorem 1 of Ref. [8].

By exploiting similar arguments to those in [3], the next theorem states a sufficient condition for the existence of IO-FTS of system (1a)–(1d) with respect to W_∞ signals.

Theorem 2 *Consider system* (1a)–(1d), *given a positive definite discrete-time matrix-valued function* $Q(\cdot)$ *defined over a set* $\Omega = \{0, 1, \dots, N\}$. *If there exist positive definite matrix-valued functions* $P(\cdot)$, $\theta(\cdot)$, $T_1(\cdot)$, *and* $T_2(\cdot)$, *and a positive definite matrix* S, *such that* (4)–(6), (8), (9) *and*

$$2\theta(k)C_\sigma^T(k)\tilde{Q}(k)C_\sigma(k) - P(k) < 0, \qquad k \in \{0, 1, \dots, N\}, \quad (26)$$

hold, then the system is input-output finite-time stable with respect to $(W_\infty, Q(\cdot), N)$, *where* $\tilde{Q}(k) = (k-g)Q(k)$, *and* g *denotes the number of switchings over the set* $\{0, 1, \dots, k\}$.

Proof By using the same arguments exploited in Theorem 1, it turns out that inequality (14) holds. Since $w(\cdot) \in W_\infty$, it follows that

$$\Delta V(k) < 1, \quad k \ne k_b - 1. \quad (27)$$

Summing (27) from 0 to $k_1 - 1$, and taking into account that $x(0) = 0$ and that $w(\cdot)$ belongs to W_∞, we obtain

$$x^T(k_1 - 1)P(k_1 - 1)x(k_1 - 1) \le k_1 - 1. \tag{28}$$

Similarly, we get

$$\begin{aligned} x^T(k_2 - 1)P(k_2 - 1)x(k_2 - 1) - x^T(k_1)P(k_1)x(k_1) &\le k_2 - k_1 - 1, \\ &\vdots \\ x^T(k)P(k)x(k) - x^T(k_g)P(k_g)x(k_g) &\le k - k_g. \end{aligned} \tag{29}$$

Then we have

$$\begin{aligned} & x^T(k)P(k)x(k) + \sum_{b=1}^{g}\left[x^T(k_b - 1)P(k_b - 1)x(k_b - 1) - x^T(k_b)P(k_b)x(k_b)\right] \\ & \quad \le k_1 - 1 + k_2 - 1 - k_1 + \cdots k - k_g \\ & \quad = k - g. \end{aligned} \tag{30}$$

From inequality (6), the following inequality can be derived:

$$x^T(k)P(k)x(k) < k - g. \tag{31}$$

Now, let us consider Eq. (20). Exploiting conditions (8) and (26), it follows that

$$\begin{aligned} & y^T(k)Q(k)y(k) \\ & \quad \le 2\left(\frac{1}{2(k-g)\theta(k)}x^T(k)P(k)x(k) + \frac{\theta(k) - 1}{2\theta(k)}w^T(k)Rw(k)\right). \end{aligned} \tag{32}$$

Taking into account that $w^T(k)Rw(k) < 1$, $\theta(k) > 1$ and (31), we can conclude that

$$y^T(k)Q(k)y(k) < 1.$$

Thus, the system is input-output finite-time stable with respect to $(W_\infty, Q(\cdot), N)$. This completes the proof. □

Remark 2 When $A_{di}(k) = 0$, $i \in \Upsilon$, if there is no state jump or switching occurs over the set $\Omega = \{0, 1, \ldots, N\}$, then the result in Theorem 2 can be reduced to the one presented in Ref. [8].

3.2 Arbitrary Switching Case

The case of no knowledge of the switching instants, i.e., arbitrary switching, is examined in this subsection.

Theorem 3 *Consider system* (1a)–(1d), *given a positive definite discrete-time matrix-valued function* $Q(\cdot)$ *defined over the set* $\Omega = \{0, 1, \ldots, N\}$. *If there exist four positive definite matrix-valued functions* $P(\cdot)$, $\theta(\cdot)$, $T_1(\cdot)$, *and* $T_2(\cdot)$, *and a positive definite matrix* S, $\forall i \in \Upsilon$, *such that* (9) *and*

$$\begin{bmatrix} S-P(k) & 0 & A_i^T(k)P(k+1) & \tilde{\Xi}_1(k) & 0 \\ * & -S & A_{di}^T(k)P(k+1) & 0 & \tilde{\Xi}_2(k) \\ * & * & -P(k+1) & 0 & 0 \\ * & * & * & -T_1(k) & 0 \\ * & * & * & * & -T_2(k) \end{bmatrix} < 0,$$

$$k \in \{0, 1, \dots, N-1\}, \tag{33}$$

$$\begin{bmatrix} T_1(k)+T_2(k)-R & G_i^T(k) \\ * & -P(k+1) \end{bmatrix} < 0, \quad k \in \{0, 1, \dots, N-1\}, \tag{34}$$

$$J^T(k)P(k+1)J(k) - P(k) < 0, \quad k \in \{0, 1, \dots, N-1\}, \tag{35}$$

$$2\theta(k)C_i^T(k)Q(k)C_i(k) - P(k) \le 0, \quad k \in \{0, 1, \dots, N\}, \tag{36}$$

$$\theta(k)R - R \ge 2\theta(k)D_i^T(k)Q(k)D_i(k), \quad k \in \{0, 1, \dots, N\}, \tag{37}$$

hold, then the system is input-output finite-time stable with respect to $(W_2, Q(\cdot), N)$, *where* $\tilde{\Xi}_1(k) = A_i^T(k)P(k+1)G_i(k)$, $\tilde{\Xi}_2(k) = A_{di}^T(k)P(k+1)G_i(k)$.

Proof The proof readily follows by exploiting similar arguments to those in Theorem 1, and considering the set $\Omega = \{0, 1, \dots, N\}$ for each subsystem in the family (1a)–(1d). □

Remark 3 Although conditions (33)–(37) are similar to the ones given in Theorem 1, they have to be checked for each subsystem in (1a)–(1d) over the set $\{0, 1, \dots, N-1\}$. This unavoidably leads to more conservatism. Due to the lack of knowledge of switching instants, it is necessary to have stable switching laws in order to guarantee the IO-FTS of the system.

From Theorem 3, a similar result can be obtained for W_∞ signals by considering the matrix

$$\tilde{Q}(k) = (k-g)Q(k)$$

in place of $Q(k)$.

Theorem 4 *Consider system* (1a)–(1d), *given a positive definite discrete-time matrix-valued function* $Q(\cdot)$ *defined over a set* $\Omega = \{0, 1, \dots, N\}$. *If there exist four positive definite matrix-valued functions* $P(\cdot)$, $\theta(\cdot)$, $T_1(\cdot)$, *and* $T_2(\cdot)$, *and a positive definite matrix* S, $\forall i \in \Upsilon$, *such that* (9), (33)–(35), (37) *and the following inequality hold*:

$$2\theta(k)C_i^T(k)\tilde{Q}(k)C_i(k) - P(k) < 0, \quad k \in \{0, 1, \dots, N\}, \tag{38}$$

then the system is input-output finite-time stable with respect to $(W_\infty, Q(\cdot), N)$.

3.3 Uncertain Switching Case

Let us now consider system (1a)–(1d) with uncertain switching, i.e., the case where the b-th switching instant is known with a given uncertainty $\pm\Delta k_b$, $b = 1, \dots, m$.

Even in the uncertain switching case, the sufficient condition to be checked to assess IO-FTS turns out to be more conservative with respect to the one derived in Theorem 1. Furthermore, a trade-off between uncertainty on switching instants and additional constraints to be added in order to check IO-FTS clearly appears. In particular, the less the uncertainty on the switching instants, the fewer additional constraints to be verified.

Since we still consider $\sigma(k)$ piecewise constant with discontinuities in correspondence of k_b, $b = 1, \ldots, m$, in the uncertain switching case, it is useful to introduce the following definitions to describe the uncertainty on the switching instants:

$$\Psi_1 = \begin{cases} 0, k_1 < 2; \\ [0, k_1 + \Delta k_1 - 2], \quad k_1 \geq 2 \end{cases}$$
$$\Psi_b = [k_{b-1} - \Delta k_{b-1}, k_b + \Delta k_b - 2], \quad b = 2, \ldots, m,$$
$$\Psi_{m+1} = [k_m - \Delta k_m, N - 1],$$
$$\Gamma_1 = \begin{cases} [0, k_1 + \Delta k_1 - 1], & k_1 < 2; \\ [k_1 - \Delta k_1 - 1, k_1 + \Delta k_1 - 1], & k_1 \geq 2 \end{cases}$$
$$\Gamma_b = [k_b - \Delta k_b - 1, k_b + \Delta k_b - 1], \quad b = 2, \ldots, m,$$

Furthermore, in the following, we assume that

$$\bigcap_{b=1}^{m} \Gamma_b = \varnothing \tag{39}$$

which implies the knowledge of the switching instants order.

Theorem 5 *Consider system (1a)–(1d), given a positive definite discrete-time matrix-valued function $Q(\cdot)$ defined over a set $\Omega = \{0, 1, \ldots, N\}$. If there exist four positive definite matrix-valued functions $P(\cdot)$, $\theta(\cdot)$, $T_1(\cdot)$, and $T_2(\cdot)$, and a positive definite matrix S, such that (7)–(9) and*

$$\begin{bmatrix} S - P(k) & 0 & A^T_{\sigma(k_{b-1})}(k)P(k+1) & \bar{\Xi}_1(k) & 0 \\ * & -S & A^T_{d\sigma(k_{b-1})}(k)P(k+1) & 0 & \bar{\Xi}_2(k) \\ * & * & -P(k+1) & 0 & 0 \\ * & * & * & -T_1(k) & 0 \\ * & * & * & * & -T_2(k) \end{bmatrix} < 0,$$
$$k \in \Psi_b, \ b = 1, \ldots, m+1, \tag{40}$$
$$\begin{bmatrix} T_1(k) + T_2(k) - R & G^T_{\sigma(k_{b-1})}(k) \\ * & -P(k+1) \end{bmatrix} < 0, \quad k \in \Psi_b, \ b = 1, \ldots, m+1, \tag{41}$$
$$J^T(k)P(k+1)J(k) - P(k) < 0, \quad k \in \Gamma_b, \ b = 1, \ldots, m, \tag{42}$$

hold, then the system is input-output finite-time stable with respect to $(W_2, Q(\cdot), N)$, where $\bar{\Xi}_1(k) = A^T_{\sigma(k_{b-1})}(k)P(k+1)G_{\sigma(k_{b-1})}(k)$, $\bar{\Xi}_2(k) = A^T_{d\sigma(k_{b-1})}(k)P(k+1) \times G_{\sigma(k_{b-1})}(k)$.

Proof Following a similar proof to that of Theorem 1, Theorem 5 can be derived; it is omitted here. □

Remark 4 Notice that conditions (40) and (41) have to be verified in Ψ_b, with $b = 1, \ldots, m+1$, i.e., the time interval in which the correspondent subsystem could be active. Furthermore, condition (42) has to be checked in Γ_b, with $b = 1, \dots, m$, i.e., the time interval in which the state jump could occur. Note also that the lengths of Ψ_b and Γ_b decrease when the uncertainties become smaller, leading us to the same result of Theorem 1 when $\Delta k_b = 0$ for all b.

Also in the uncertain switching case, the sufficient condition for W_∞ input signals can be obtained by considering $\tilde{Q}(k)$ instead of $Q(k)$.

4 Numerical Examples

In this section, we present two examples to illustrate the effectiveness of the proposed results.

Example 1 Consider system (1a)–(1d) with parameters as follows:

$$A_1(k) = \begin{bmatrix} 0.6+0.03k & 0.3 \\ 0.08 & 0.9+0.04k \end{bmatrix}, \qquad A_{d1}(k) = \begin{bmatrix} 0.1 & 0 \\ 0.05 & 0.02 \end{bmatrix},$$

$$G_1(k) = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad C_1(k) = \begin{bmatrix} 0 & 1+0.1k \end{bmatrix}, \qquad D_1(k) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$

$$A_2(k) = \begin{bmatrix} 0.2+0.03k & 0.1 \\ 0.04 & 0.6+0.02k \end{bmatrix}, \qquad A_{d2}(k) = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.012 \end{bmatrix},$$

$$G_2(k) = \begin{bmatrix} 0.3 & 0 \\ 0.4 & 0.3 \end{bmatrix}, \qquad C_2(k) = \begin{bmatrix} 0 & 1.2+0.05k \end{bmatrix},$$

$$D_2(k) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \qquad J(k) = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

As for the IO-FTS, we consider

$$R = I, \qquad N = 10.$$

The value of the switching signal $\sigma(k)$ over the set $\Omega = \{0, 1, \dots, 10\}$ is shown in Fig. 1; then

$$\Omega \cap \Gamma = \{3, 6, 8\}.$$

(1) Known switching instants case

Given $Q(k) = 2.5$, solving the matrix inequalities (4)–(9) in Theorem 1 gives rise to

$$P(0) = 10^3 \times \begin{bmatrix} 0.5380 & 0.0127 \\ 0.0127 & 1.2594 \end{bmatrix}, \qquad P(1) = \begin{bmatrix} 61.9502 & -21.1379 \\ -21.1379 & 77.7172 \end{bmatrix},$$

$$P(2) = \begin{bmatrix} 14.7485 & -4.6637 \\ -4.6637 & 17.3572 \end{bmatrix}, \qquad P(3) = \begin{bmatrix} 127.0138 & -44.5380 \\ -44.5380 & 385.2233 \end{bmatrix},$$

$$P(4) = \begin{bmatrix} 108.6290 & -45.6095 \\ -45.6095 & 66.5117 \end{bmatrix}, \qquad P(5) = \begin{bmatrix} 81.7310 & -32.3226 \\ -32.3226 & 27.8263 \end{bmatrix},$$

$$P(6) = \begin{bmatrix} 295.7496 & -135.8413 \\ -135.8413 & 419.6071 \end{bmatrix}, \qquad P(7) = \begin{bmatrix} 32.5749 & -11.2842 \\ -11.2842 & 37.9155 \end{bmatrix},$$

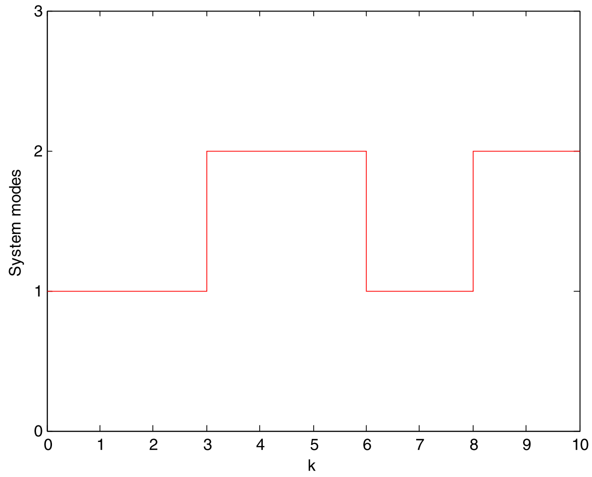


Fig. 1 Switching signal for Example 1

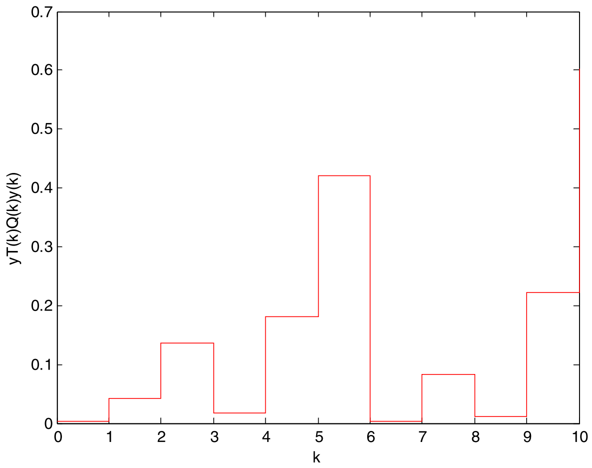


Fig. 2 Weighted output for the system with known switching instants when $Q(k) = 2.5$

$$P(8) = \begin{bmatrix} 209.6623 & -64.4468 \\ -64.4468 & 833.7396 \end{bmatrix}, \qquad P(9) = \begin{bmatrix} 97.8596 & -44.0130 \\ -44.0130 & 85.6677 \end{bmatrix},$$
$$P(10) = \begin{bmatrix} 48.5900 & -22.0357 \\ -22.0357 & 26.5591 \end{bmatrix}, \qquad S = \begin{bmatrix} 29.3048 & -12.1189 \\ -12.1189 & 10.4339 \end{bmatrix}.$$

Choosing $w(k) = \left[0.2\ 0.2\right]^T \in W_2$, the weighted output is shown in Fig. 2 when $Q(k) = 2.5$. One can see that the considered system with known switching instants is input-output finite-time stable with respect to $(W_2, 2.5, 10)$.

(2) Arbitrary switching case

Let us now suppose that Γ is totally unknown. In this case, we cannot find the feasible solution of inequalities in Theorem 3 for any $Q(k) > 0$. Therefore, we cannot conclude that the system is input-output finite-time stable in this case.

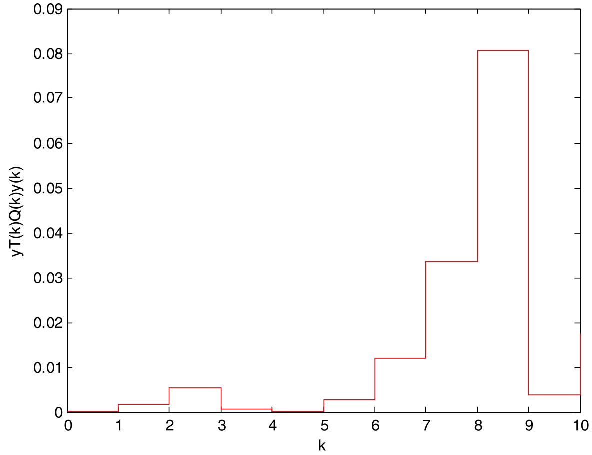


Fig. 3 Worst-case weighted output for uncertain switching when $Q(k)=0.1$

(3) Uncertain switching case

Let us now consider the case of uncertain switching with $\Delta k_b = 1$ for all $b \geq 1$. Given $Q(k) = 0.1$, solving the matrix inequalities in Theorem 5, we get the following solutions:

$$P(0) = 10^3 \times \begin{bmatrix} 3.2639 & -0.1538 \\ -0.1538 & 4.8252 \end{bmatrix}, \qquad P(1) = \begin{bmatrix} 20.3078 & -18.6017 \\ -18.6017 & 30.8850 \end{bmatrix},$$

$$P(2) = \begin{bmatrix} 10.0268 & -7.9125 \\ -7.9125 & 11.8735 \end{bmatrix}, \qquad P(3) = \begin{bmatrix} 23.6924 & -11.4169 \\ -11.4169 & 10.4753 \end{bmatrix},$$

$$P(4) = \begin{bmatrix} 19.0160 & -7.6390 \\ -7.6390 & 7.6728 \end{bmatrix}, \qquad P(5) = \begin{bmatrix} 25.6653 & -10.1435 \\ -10.1435 & 8.2754 \end{bmatrix},$$

$$P(6) = \begin{bmatrix} 6.5670 & -3.2493 \\ -3.2493 & 3.8129 \end{bmatrix}, \qquad P(7) = \begin{bmatrix} 4.1515 & -2.3118 \\ -2.3118 & 2.4959 \end{bmatrix},$$

$$P(8) = \begin{bmatrix} 2.3966 & -1.5246 \\ -1.5246 & 1.6029 \end{bmatrix}, \qquad P(9) = \begin{bmatrix} 7.3984 & -3.8027 \\ -3.8027 & 2.5512 \end{bmatrix},$$

$$P(10) = \begin{bmatrix} 13.9435 & -6.2606 \\ -6.2606 & 3.3917 \end{bmatrix}, \qquad S = \begin{bmatrix} 0.3378 & -0.1219 \\ -0.1219 & 0.1080 \end{bmatrix}.$$

Choosing $w(k) = \begin{bmatrix} 0.2 \; 0.2 \end{bmatrix}^T \in W_2$, Fig. 3 shows the worst-case output when $Q(k) = 0.1$. It can be seen that the considered system with uncertain switching is input-output finite-time stable with respect to $(W_2, 0.1, 10)$.

Example 2 Consider system (1a)–(1d) with parameters as follows:

$$A_1(k) = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & -0.3 \end{bmatrix}, \qquad A_{d1}(k) = \begin{bmatrix} 0.04 & 0.01 \\ 0 & 0.04 \end{bmatrix}, \qquad G_1(k) = \begin{bmatrix} 0 & 0.3 \\ 0.3 & 0 \end{bmatrix},$$

$$C_1(k) = \begin{bmatrix} 0.2 + 0.2k & 0.4 \end{bmatrix}, \qquad D_1(k) = \begin{bmatrix} 0.15 & 0.12 \end{bmatrix},$$

$$A_2(k) = \begin{bmatrix} 0.15 & 0.2 \\ 1 & -0.2 \end{bmatrix}, \qquad A_{d2}(k) = \begin{bmatrix} 0.02 & 0 \\ 0.02 & 0 \end{bmatrix},$$

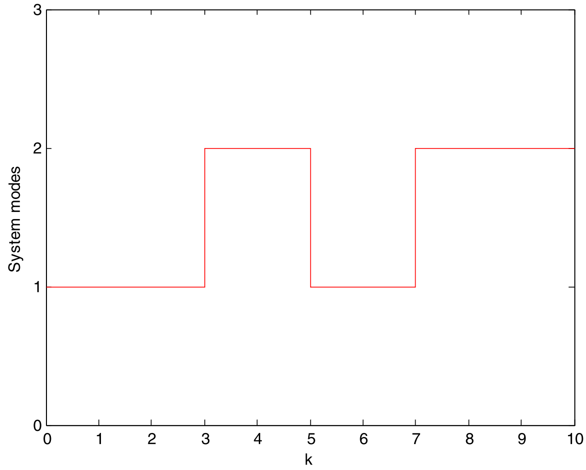


Fig. 4 Switching signal for Example 2

$$G_2(k) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \qquad C_2(k) = \begin{bmatrix} 0.2 + 0.2k & 0.3 \end{bmatrix},$$

$$D_2(k) = \begin{bmatrix} 0.15 & 0.12 \end{bmatrix}, \qquad J(k) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

As for the IO-FTS, we consider

$$R = I, \qquad N = 10.$$

Given $Q(k) = 0.2$, solving the inequalities in Theorem 4 gives rise to

$$P(0) = \begin{bmatrix} 167.0156 & -12.5404 \\ -12.5404 & 88.1469 \end{bmatrix}, \qquad P(1) = \begin{bmatrix} 47.6633 & -4.2341 \\ -4.2341 & 17.4439 \end{bmatrix},$$

$$P(2) = \begin{bmatrix} 26.9380 & -1.6513 \\ -1.6513 & 9.6787 \end{bmatrix}, \qquad P(3) = \begin{bmatrix} 21.4352 & -1.0913 \\ -1.0913 & 7.0907 \end{bmatrix},$$

$$P(4) = \begin{bmatrix} 18.1411 & -0.7058 \\ -0.7058 & 6.1257 \end{bmatrix}, \qquad P(5) = \begin{bmatrix} 17.1762 & -0.6961 \\ -0.6961 & 5.3652 \end{bmatrix},$$

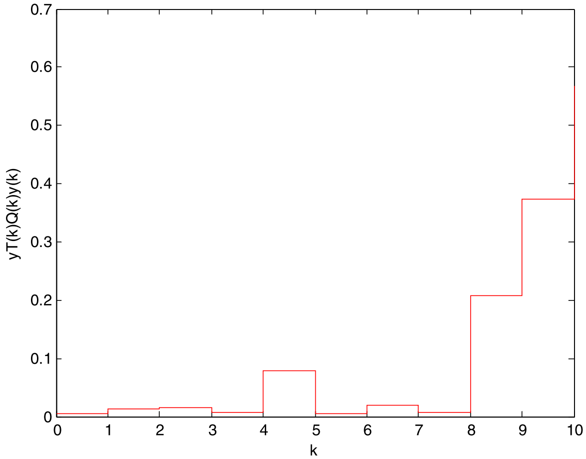
$$P(6) = \begin{bmatrix} 15.4913 & -0.3963 \\ -0.3963 & 5.3680 \end{bmatrix}, \qquad P(7) = \begin{bmatrix} 15.3431 & -0.2772 \\ -0.2772 & 4.7424 \end{bmatrix},$$

$$P(8) = \begin{bmatrix} 14.9546 & -0.0303 \\ -0.0303 & 4.5873 \end{bmatrix}, \qquad P(9) = \begin{bmatrix} 14.7370 & 0.2313 \\ 0.2313 & 4.4941 \end{bmatrix},$$

$$P(10) = \begin{bmatrix} 15.3262 & 0.3776 \\ 0.3776 & 4.1120 \end{bmatrix}, \qquad S = \begin{bmatrix} 1.5471 & -0.1430 \\ -0.1430 & 0.5838 \end{bmatrix}.$$

Thus, according to Theorem 4, we can obtain that the considered system is input-output finite-time stable with respect to $(W_\infty, 0.2, 10)$ under arbitrary switching. Figure 4 depicts the switching signal, and Fig. 5 plots the weighted output when $Q(k) = 0.2$, where $w(k) = \begin{bmatrix} 0.9 \; 0.1 \end{bmatrix}^T \in W_\infty$.

Fig. 5 Weighted output for the system when $Q(k) = 0.2$



5 Conclusions

This paper has investigated the problem of input-output finite-time stability (IO-FTS) for a class of discrete impulsive switched systems with state delays. For two different class input signals, IO-FTS criteria for three cases have been presented in terms of a set of LMIs. Two examples have also been given to illustrate the applicability of the proposed results. Our future work will focus on investigating the input-output finite-time stabilization problem for the considered systems and extending the proposed results to other kinds of systems, such as positive systems.

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