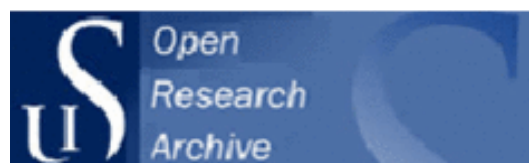




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# Exchange, raiding, and the shadow of the future

Kjell Hausken

A two-period exchange model is developed where production decisions in the first period determine the amount of resources available in the second period. Each agent allocates resources to defend its production and attack the production of the other agent. Production, conflict and exchange occur simultaneously in a dynamic model. This extends earlier exchange models, which are static and preclude defense and appropriation. The agents jointly determine price through their export decisions. Upon introducing exchange endogenously, raiding in the first relative to the second period decreases with growth, appropriation cost, and when the future becomes more important, and increases with defense cost, production cost, and usability of appropriation. Increasing the usability of appropriation and defense cost causes a transition from pure exchange via joint exchange and raiding to pure raiding. This implies that agents gradually substitute from defense to appropriation, they exchange less, and utility decreases. Utility isoquants in a usability of appropriation versus discount factor diagram are concavely increasing for joint exchange and raiding, and can be convexly decreasing for pure raiding. Cobb–Douglas utilities are assumed. The results are confirmed with CES utilities.

*Keywords:* production; exchange; trade; appropriation; defense; mutual raiding; dynamics; growth; discounting; predation; attack

## INTRODUCTION

Among many economists and classical liberals, exchange is considered important not just for the direct material benefits it gives but also for the possible reduction in frictions among potential adversaries that it may induce. Partly in this tradition, this article analyzes how exchange may reduce conflict in a dynamic setting. It departs from a similar setting that Skaperdas and Syropoulos (1996a) have examined but which did not take exchange into account. With the approach in this article, the presence of exchange reverses some of the results regarding the effect of the discount factor. Further comparative static analyses are also performed.

Determining whether the shadow of the future induces or harms cooperation or other kinds of behavior is essential. On the one hand, folk theorem (Fudenberg and Maskin, 1986) arguments, which apply for infinitely repeated games, suggest the possibility of cooperation in long-term relationships. On the other hand, Skaperdas and Syropoulos (1996a) demonstrate, in a two-period conflict model, how increased importance of the

future may harm cooperation. Different conceptualizations of strategic interaction, intertemporal linkage, and strategy sets cause different results. The two-person prisoner's dilemma assumes the two strategies, co-operation versus defection, with four combinations of utilities. Game repetition may cause temporary benefits from defecting today to be outweighed by defection by the other agent in the future (Axelrod, 1984). In contrast, Skaperdas and Syropoulos (1996a) conceptualize a game where each agent has a resource that can be allocated into production versus arms. A larger share of the total product is given to the agent investing more in arms.

Uncritical use of the folk theorem<sup>1</sup> may induce framing effects and blind less cautious users to important dimensions that are relevant for determining how different strategies impact utilities dependent on the shadow of the future. Similarly, uncritically assuming that investment in arms today causes benefits tomorrow is unwarranted. To enhance our insight into these matters, this article introduces exchange endogenously as a possible strategy in addition to production and raiding, divided into appropriation and defense.<sup>2</sup> We want to find out whether allowing agents to exchange goods voluntarily, in addition to fighting for goods, gives different results.

Throughout history, mankind has engaged in mutual raiding and voluntary exchange to acquire desired goods. The relative emphasis of each activity has changed back and forth, although scholarly work has typically analyzed each activity in separation. Pure market exchange may be supplemented by, or grow out of, activities such as war, piracy, corruption, extortion, crime, plundering, theft – referred to as mutual raiding. Exchange has a long tradition in economics. Appropriation and defense of productive resources, without reciprocal exchange, has also been analyzed. A few authors attempt to integrate production, exchange, and appropriation.<sup>3</sup> This article is the first to provide a dynamic analysis that integrates production, exchange, appropriation, and defense. The equilibrium balance between raiding and exchange depends on six parameters; namely, the costs of production, appropriation, defense, the usability of appropriation, the discount factor, and a growth factor.

Most contest models let appropriated goods be equally valuable to the appropriator and the defender. In practice, however, as Grossman and Kim (1995, p. 1279) point out, 'predation involves violence and destruction,' appropriated goods may deteriorate 'during shipment or a predator's gain needs to be processed to be usable.' Further, agents generally value differently goods appropriated from others and goods they produce or possess themselves. Distinguishing between costs of appropriation and defense allows for tuning these activities to be inferior or superior to each other, or equally costly. Clausewitz (1832, 6.1.2) argues for the 'superiority of defense over attack.' 'The defender enjoys optimum lines of communication and retreat, and can choose the place for battle.' In contrast, the attacker enjoys 'initial surprise, and the benefit of choosing the time, the nature, and the form of the attack.' Rapid world changes present scenarios where both appropriation and

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<sup>1</sup> The folk theorem asserts that any individually rational outcome can arise as a Nash equilibrium in infinitely repeated games with sufficiently little discounting. Defining  $V^*$  as 'the set of individually rational payoffs,' the Folk Theorem states: 'For any  $(v_1, \dots, v_n) \in V^*$ , if players discount the future sufficiently little, there exists a Nash equilibrium of the infinitely repeated game where, for all  $i$ , player  $i$ 's average payoff is  $v_i$ ' (Fudenberg and Maskin, 1986, p. 537).

<sup>2</sup> 'Appropriation' and 'defense' are metaphors that need not involve actual violence. They refer to rent-seeking manoeuvres for licenses and monopoly privileges, commercial efforts to raise rivals' costs, strikes and lockouts, litigation, etc.

<sup>3</sup> Anderton (1999), Anderton and Anderton (1997), Anderton *et al.* (1999), Hausken (2004), Rider (1993, 1999), Skaperdas and Syropoulos (1996b, 2001). Skaperdas and Syropoulos (2002) show that exchange may be inefficient when property rights are insecure, due to large enforcement costs. This may cause limited settlement without exchange. Conversely, more secure property rights may allow for exchange. Anderson and Marcouiller (1997) allow predation and production and show that autarky, insecure exchange, and secure exchange may occur. Secure exchange is supported only in a narrow range of security parameter values. Large poor countries are harmed by increased security.

defense are superior to each other.<sup>4</sup> The cost of production affects both relative investments in production versus raiding, and affects the utilities in each of two periods differently due to the nature of intertemporal linkage.

This article lets two agents (individuals, groups, companies, nations) specialize in producing one good each, while also attaching utility to the other good. The two agents attempt to appropriate and defend their production. At the same time, they have the opportunity to exchange goods voluntarily. Only that part of an agent's production that has been successfully defended can be exported. In return, the agent receives import which is a part of the other agent's production that has been successfully defended by the other agent. The terms of exchange are such that each agent chooses his appropriation, defense, and export, taking the other agent's appropriation, defense, and export (which is the first agent's import) as given. We assume Cobb–Douglas utilities, and test the robustness of results through CES utilities. Their resource in each of two periods is allocated between production, appropriation, and defense, allowing for exchange. Exchange and raiding occurs through time in a two-period model with intertemporal linkage. Each agent's resource in the second period depends on his production in the first period multiplied by a growth factor. Furthermore, an agent's total utility equals his first period utility plus a discount factor multiplied by his second period utility. The growth and discount factors together form the shadow of the future.

Skaperdas and Syropoulos (1996a) let two agents fight over joint production, assuming a Cobb–Douglas production function and a logistic contest success function. Introducing exchange into joint production is not straightforward since output has to be made disjoint before the two agents can exchange. Introducing exchange, and Cobb–Douglas utilities for the two goods exchanged, this article enables analytical tractability by assuming a production function that is linear in effort and a ratio contest success function.<sup>5</sup> Despite the dissimilarities, we nevertheless make some comparisons with Skaperdas and Syropoulos' (1996a) model since they offer results of a general nature. For example, they find that increasing the shadow of the future decreases efficiency, may increase fighting, and they find that the ratio of utilities in the first and second periods decreases when growth increases. It is of interest to determine to what extent such findings depend on the specific model chosen, and whether exchange generates different results.

The relationship between exchange and conflict has been disputed for centuries. Commercial liberalists claim that exchange inhibits conflict, while alternative views are that exchange promotes belligerence or is unrelated, for example, to interstate disputes.<sup>6</sup> Reuveny (2001, p. 131) suggests a 'need for richer, more microfounded models,' which this article intends to provide.

The next section presents the two-period Cobb–Douglas model. The section after solves the model. The fourth section analyzes the results and presents nine properties. The fifth section illustrates graphically while the sixth section tests the robustness of the results by applying the CES model. The seventh section concludes.

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<sup>4</sup> See Hirshleifer (2000, pp. 784–787) for discussions of offense versus defense.

<sup>5</sup> Differentiation to determine the first order conditions when using the logistic contest success function gives logarithms which, combined with the exponential Cobb–Douglas parameter, prevents analytical tractability. Fortunately, the ratio contest success function is analytically tractable and is, furthermore, more commonly used. See Hirshleifer (1989) for a comparison of the two functions, Skaperdas (1996) for an axiomatization, and Tullock (1980) for an early use of the ratio form in rent seeking.

<sup>6</sup> See Polachek (1980), Anderton and Carter (2001), Mansfield and Pevehouse (2000), Morrow (1999).

## THE TWO-PERIOD COBB–DOUGLAS MODEL

Assume a budget constraint for a resource  $R_{i1}$  (e.g. a capital good, or labor) which for agent  $i$  in period 1 is divided into productive effort  $E_{i1}$  (designed to generate good  $i$  from resources currently controlled), appropriative effort  $F_{i1}$  (aimed at acquiring the other agent's good at the expense of the other agent), and defensive effort  $D_{i1}$  (aimed at repelling the other agent as he attempts to acquire one's own good). This gives:

$$R_{i1} = a_i E_{i1} + b_i F_{i1} + c_i D_{i1}, \quad i=1,2 \quad (1)$$

where  $a_i, b_i, c_i$  are unit conversion costs of transforming resources into productive, appropriative, and defensive effort, respectively. Assume a simple production function  $E_{i1}$  where output produced equals effort  $E_{i1}$  invested, that is:<sup>7</sup>

$$E_{i1} = (R_{i1} - b_i F_{i1} - c_i D_{i1}) / a_i, \quad i=1,2 \quad (2)$$

The agents have equivalent Cobb–Douglas preferences for the two productions. The article can also be developed assuming CES preferences (see the sixth section). To ensure analytical tractability, they engage in division of labor by producing one good each, which is a special case of an Edgeworth box where each agent is endowed with one of the two goods. Rider (1999) provides an alternative exchange model where each agent produces one good. Although each agent produces only one good, he has two ways of acquiring the other good (i.e. exchange and appropriation), which allows for a rich analysis. Using the ratio formula as our contest success function (Hirshleifer, 1995; Skaperdas, 1996; Tullock, 1980), let agent 1 defend an object with  $D_{11}$ , and agent 2 appropriate the object with  $F_{21}$ . Agent 1 receives a fraction  $D_{11}/(D_{11} + F_{21})$ , and agent 2 receives the remaining fraction  $F_{21}/(D_{11} + F_{21})$ .

Hausken (2004) and Rider (1999) assume that production can be appropriated, while Anderton *et al.* (1999) and Anderton (1999) assume in a predator-prey model that the resource holding of the Defender is vulnerable to appropriation. This article assumes that the agents defend and appropriate each other's productions  $E_{11}$  and  $E_{21}$ .<sup>8</sup> Conflict is an ever-present phenomenon. The agents contemplate whether exchange can emerge endogenously despite conflict. Agent 1 exports  $X_{11}$  and imports  $X_{21}$ , and conversely for agent 2. Their von Neumann and Morgenstern utilities  $U_1$  and  $U_2$  in the first period are:

$$U_1 = \left[ \frac{D_{11}}{D_{11} + F_{21}} E_{11} - X_{11} \right]^\alpha \left[ \frac{\beta F_{11}}{F_{11} + D_{21}} E_{21} + X_{21} \right]^{1-\alpha}$$

$$U_2 = \left[ \frac{D_{21}}{D_{21} + F_{11}} E_{21} - X_{21} \right]^{1-\alpha} \left[ \frac{\beta F_{21}}{F_{21} + D_{11}} E_{11} + X_{11} \right]^\alpha, \quad X_{11} = P_{21} X_{21}, \quad 0 < \alpha < 1, \quad 0 \leq \beta \leq 1 \quad (3)$$

<sup>7</sup> Not distinguishing between offense and defense, Hirshleifer (1995, p. 31) presents a similar production function where, in our notation,  $E_{i1}$  is raised to the parameter  $h$ . Our analysis sets  $h=1$  to ensure analytical tractability. The article can also be developed for general  $h$ , for example  $h < 1$  which gives a production function with diminishing returns. Such a more general analysis gives unreasonably complex first-order conditions that cannot be solved analytically, although the phenomenon can be analyzed computationally with simulations.

<sup>8</sup> Hausken (2003, pp. 33–36) classifies the 78 possible models for how one or several of the five objects (production, resources, consumption goods, exports, and imports) can be set under attack. Raiding production means that any part of the successfully defended production can be exchanged. More detrimentally, raiding resources gives a fraction, typically less than one, of successfully defended resources to be divided between production, appropriation, and defense, before exchange can occur. Hence, Hausken (2003) finds that raiding resources tends to increase the prevalence of raiding. Possessing more resources is advantageous with low usability of appropriated resources and superiority of defense over attack, especially when resources rather than production are raided.

where  $\alpha$  is the relative preference parameter for good 1 for both agents, and  $P_{21}$  is an interior terms-of-exchange price denoting the price of good 2 in terms of good 1 in the first period.

Equation (3) states that the two agents attempt to appropriate and defend their production. At the same time they have the opportunity to exchange goods voluntarily. No fraction of the production is exempt from appropriation. Only that part of an agent's production that has been successfully defended can be exported. In return, the agent receives an import which is a part of the other agent's production that has been successfully defended by the other agent. Without the possibility of exchange, which means that we exogenously set  $X_{11}=X_{21}=X_2=X_{12}=0$ , equation (3) reduces to a pure conflict model. One objective of this article is to determine the conditions under which the agents endogenously choose positive  $X_{11}, X_{21}, X_2, X_{12}$ .

The price equation  $P_{21}=X_{11}/X_{21}$  means that as agent 1 exports more ( $X_{11}$  increases), the price of good 1 in terms of good 2, that is  $1/P_{21}$ , decreases. That is, as agent 1 exports more, the price of those exports decreases. A conventional supply-demand consideration explains this. As good 1 becomes more readily available for agent 2, the price of good 1 decreases. Despite such a reduction in the price of agent 1's exports, we show the conditions under which agent 1 is willing to increase his exports voluntarily. One essential parameter is  $\beta$ , which is the usability or non-destruction parameter denoting that fraction of the appropriated production that the appropriating agent can make use of in the same manner as his own production (that is, the appropriator gains a fraction  $\beta$  of what the defender loses).<sup>9</sup> Agent  $i$ 's free choice variables in the first period are  $F_{i1}, D_{i1}, X_{i1}$ , and similarly  $F_{21}, D_{21}, X_{21}$  for agent 2, determined endogenously.

Skaperdas and Syropoulos (1996a, p. 361) 'think that there are few societies where the resources of individual agents are not positively related to their past incomes or wealth.' They accordingly assume that the second period resource is proportional to the first period payoff which is in units of output.<sup>10</sup> They state that the proportionality growth 'parameter  $\gamma$  can be thought to reflect investment choices in period 1, which for simplicity are effectively assumed to be a constant fraction of period 1 payoffs.' Letting the second period resource be proportional to the first period resource  $R_{i1}$  is not interesting since that provides no linkage between the two periods. That would render the discount factor  $\delta$  irrelevant, and would give equivalent solutions for the two periods, merely substituting the resource  $R_{i1}$  in the first period solution with  $\gamma R_{i2}$  in the second period solution. To capture the logic of the linkage between the two periods, the second period resource must be proportional to the first period resource where the proportion is endogenous to the decisions made in the first period. We accordingly offer a defense burden story of diminished growth. The greater diversion of resources relative to the first period resource  $R_{i1}$ , the smaller is the growth of the resource from the first to the second period.<sup>11</sup> The burden of appropriation and defense in the first period is reflected by the expenditure  $b_i F_{i1}$  for appropriation and the expenditure  $c_i D_{i1}$  for defense. Subtracting these two expenditures from the first period resource  $R_{i1}$ , to reflect the diversion of resources, gives an expression that is proportional to the first period production expressed in equation (2). Hence, we let the second period resource be proportional to the first period production. That provides linkage between the two periods so that the shadow of the future can be assessed. Accordingly, the second period resource for agent  $i$  is  $\gamma E_{i1}$ , where  $\gamma$  is a growth parameter and  $E_{i1}$  is agent  $i$ 's first period production determined by equation (2). Production  $E_{i1}$  generated in the first period is a good indicator of growth. It allows an economy to grow in the second period. This is not the case for resources converted into appropriation and defense, which determine how much can be consumed in the first period.<sup>12</sup>

<sup>9</sup> We require  $F_{i1} \geq 0$  and  $D_{i1} \geq \varepsilon > 0$ , where  $\varepsilon$  is arbitrarily small but positive, assuming  $c_i < \infty$ . The slightly different bounds are necessary to avoid '0/0' ratios in equation (3), which happens when the agents prefer pure exchange. Of course, we explicitly test for corner solutions where  $F_{i1}=0$  or  $D_{i1}=\varepsilon$ .

<sup>10</sup> Our utilities in equation (3) capture consumption goods.

<sup>11</sup> I thank an anonymous referee of this journal for this suggestion.

Analogously to equation (2), the agents' production function  $E_{i2}$  for the second period is:

$$E_{i2} = (\gamma E_{i1} - b_i F_{i2} - c_i D_{i2}) / a_i, \quad i=1,2 \quad (4)$$

where  $F_{i2}$  and  $D_{i2}$  are the appropriative and defensive efforts by agent  $i$  in the second period. The agents' utilities  $V_1$  and  $V_2$  in the second period are:

$$V_1 = \left[ \frac{D_{12}}{D_{12} + F_{22}} E_{12} - X_{12} \right]^\alpha \left[ \frac{\beta F_{12}}{F_{12} + D_{22}} E_{22} + X_{22} \right]^{1-\alpha}$$

$$V_2 = \left[ \frac{D_{22}}{D_{22} + F_{12}} E_{22} - X_{22} \right]^{1-\alpha} \left[ \frac{\beta F_{22}}{F_{22} + D_{12}} E_{12} + X_{12} \right]^\alpha, \quad X_{12} = P_{22} X_{22} \quad (5)$$

where agent 1 exports  $X_{12}$  and imports  $X_{22}$ , and conversely for agent 2.  $P_{22}$  is the price of good 2 in terms of good 1 in the second period. With a discount factor  $\delta$ , agent  $i$ 's total utility  $W_i$  over the two periods is:

$$W_i = U_i + \delta V_i, \quad i=1,2 \quad (6)$$

## SOLVING THE MODEL

Assuming a subgame perfect equilibrium, we first solve for equilibrium in the second period, conditional on the resources  $\gamma E_{1i}$  available in the second period. We thereafter find the optimal solution in the first period, taking into account that the agents' choices in the second period must be in equilibrium. Agent  $i$  has the three free choice variables  $F_{ij}, D_{ij}, X_{ij}$  in period  $j$ . For two agents and two periods this gives the 12 FOCs:

$$\partial W_i / \partial F_{ij} = 0, \quad \partial W_i / \partial D_{ij} = 0, \quad \partial W_i / \partial X_{ij} = 0, \quad i, j=1,2 \quad (7)$$

Most exchange models assume that the agents are price takers and a Walrasian equilibrium emerges as the price that balances exchange offers (see for example Kreps, 1990, for a treatment of classical exchange). In bilateral monopoly treatments of exchange, the price is indeterminate. As problematic as the Walrasian equilibrium may be, it gives rise to a determinate price. In most exchange models, a price equation is necessary to close the model. This suggests a need for two exchange balance equations (one for each period) in addition to the 12 FOCs listed in equation (7).<sup>13</sup>

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<sup>13</sup> Another possibility is to let the second period resource be proportional to that portion of the first period production that is successfully defended, but that would be inappropriate since some of the production of the other agent is also appropriated. A further possibility is to let the second period resource be proportional to the portion of the first period production that is successfully defended, plus the portion of the first period production of the other agent that is successfully appropriated. That possibility gives first-order conditions that can be solved computationally, but without analytical solutions. A final possibility is to let the second-period resource be proportional to the first-period utility, so that the second-period resource for agent  $i$  is  $\gamma U_i$ . That interpretation can also be given a defense burden story of diminished growth. The argument would then be that future resources depend on how well one has done in the past in terms of actual consumption or utility which determines fitness in the future. However, in many cases when something is consumed, the good that is consumed does not exist any more. Think for example of food, leisure activities, travel, or wasteful consumption. It is not clear that such consumption, which then does not exist any more, shall determine the availability of future resources. The author has analyzed this case. The results are qualitatively similar to the results in this article, and are available from the author upon request.



To see that the exchange balance equations are already built into the mathematics in this model, we reason as follows. The price in the first period is  $P_{21}=X_{11}/X_{21}$ . Since  $X_{11}$  and  $X_{21}$  are free choice variables determined by the 12 FOCs in equation (7),  $P_{21}$  is a dependent variable that follows from  $X_{11}$  and  $X_{21}$ . This means that the agents jointly control the price. No factors external to the agents control the price. Also, no agent unilaterally controls the price. Agent 1 controls his export  $X_{11}$  and agent 2 controls his export  $X_{21}$ . The terms of exchange are such that each agent chooses his appropriation, defense, and export optimally taking the other agent's appropriation, defense, and export (which is the first agent's import) as given. More specifically, when agent 1 chooses  $X_{11}$  optimally, he takes  $X_{21}$  as given. Together, since agent 1 chooses  $X_{11}$  and agent 2 chooses  $X_{21}$ , the two agents jointly choose the first period price  $P_{21}=X_{11}/X_{21}$ . The second period price is analogously defined as  $P_{22}=X_{12}/X_{22}$  where agent 1 controls his export  $X_{12}$  and agent 2 controls his export  $X_{22}$ . In other words, the terms of exchange or international price  $P_{21}$  and  $P_{22}$  is implicitly determined by the choices of  $X_{11}$ ,  $X_{21}$ ,  $X_{12}$ ,  $X_{22}$ . That the prices are not taken as given, but depend on both agents' free choice variables, is realistic in a duopoly. Both agents influence the price. In the optimization problem agent 1 replaces  $X_{21}$  with  $X_{11}/P_{21}$  and replaces  $X_{22}$  with  $X_{12}/P_{22}$  in equations (3) and (4). After these two replacements, however, the prices  $P_{21}$  and  $P_{22}$  are taken as given in the optimization. The four last FOCs in equation (7) can be written as:

$$\begin{aligned}
X_{11} &= \frac{(1-\alpha)(R_{11}-b_1F_{11}-c_1D_{11})(D_{11}D_{21}-\beta^2F_{11}F_{21})}{a_1(D_{11}+F_{21})(D_{21}+\beta F_{11})} \\
X_{21} &= \frac{\alpha(R_{21}-b_2F_{21}-c_2D_{21})(D_{11}D_{21}-\beta^2F_{11}F_{21})}{a_2(D_{21}+F_{11})(D_{11}+\beta F_{21})}, \quad P_{21} = \frac{X_{11}}{X_{21}} \\
X_{12} &= \frac{(1-\alpha)[\gamma(R_{11}-b_1F_{11}-c_1D_{11})/a_1-b_1F_{12}-c_1D_{12}](D_{12}D_{22}-\beta^2F_{12}F_{22})}{a_1(D_{12}+F_{22})(D_{22}+\beta F_{12})} \\
X_{22} &= \frac{\alpha[\gamma(R_{21}-b_2F_{21}-c_2D_{21})/a_2-b_2F_{22}-c_2D_{22}](D_{12}D_{22}-\beta^2F_{12}F_{22})}{a_2(D_{22}+F_{12})(D_{12}+\beta F_{22})}, \quad P_{22} = \frac{X_{12}}{X_{22}} \quad (8)
\end{aligned}$$

For the symmetric case  $F_{ij}=F_j$ ,  $D_{ij}=D_j$ ,  $X_{ij}=X_j$ ,  $W_i=W$ ,  $R_{i1}=R$ ,  $a_i=a$ ,  $b_i=b$ ,  $c_i=c$ , and  $\alpha=1/2$  so that the agents have equal preferences for the two goods, the solution of equation (7) when exchange occurs ( $X_j > 0$ ), which means joint exchange and raiding, is:

$$\begin{aligned}
F_1 &= \frac{acR\beta^2}{ab(b+c\beta(2+\beta))+b(b+c\beta)\gamma\delta}, \quad D_1 = \frac{bF_1}{c\beta}, \quad X_1 = \frac{R(b-c\beta^2)(a+\gamma\delta)}{2a[a(b+c\beta(2+\beta))+b(b+c\beta)\gamma\delta]} \\
F_2 &= \frac{cR\beta^2(b+c\beta)\gamma(a+\gamma\delta)}{ab(b+c\beta(2+\beta))[a(b+c\beta(2+\beta))+b(b+c\beta)\gamma\delta]}, \quad D_2 = \frac{bF_2}{c\beta}, \quad W = \frac{R(b+c\beta^2)(a+\gamma\delta)}{2a^2(b+c\beta(2+\beta))} \\
X_2 &= \frac{R(b-c\beta^2)(b+c\beta)\gamma(a+\gamma\delta)}{2a^2(b+c\beta(2+\beta))[a(b+c\beta(2+\beta))+b(b+c\beta)\gamma\delta]}, \quad \frac{F_1}{F_2} = \frac{D_1}{D_2} = \frac{a^2(b+c\beta(2+\beta))}{(b+c\beta)\gamma(a+\gamma\delta)} \\
\frac{X_1}{X_2} &= \frac{a(b+c\beta(2+\beta))}{(b+c\beta)\gamma}, \quad X_j > 0 \Leftrightarrow b > c\beta^2 \quad (9)
\end{aligned}$$

<sup>13</sup>I thank an anonymous referee for these formulations and for requesting an elaboration of how the exchange balance equations are built into the mathematics.



When exchange does not occur ( $X_j = 0$ ), which means pure raiding, the special case  $\gamma^2\delta + a\gamma = 2a^2$ , one example of which is  $a=\delta=\gamma=1$ , has proven possible to solve, giving:

$$F_1 = F_2 = \frac{R\gamma}{\sqrt{b}(\sqrt{b} + \sqrt{c})(2a + \gamma)}, \quad D_1 = D_2 = \sqrt{\frac{b}{c}}F_1, \quad W = \frac{R\sqrt{\beta}(bc)^{1/4}(2a + \gamma\delta)}{a(\sqrt{b} + \sqrt{c})(2a + \gamma)}$$

$$X_j = 0, \quad b < c\beta^2, \quad \gamma^2\delta + a\gamma = 2a^2 \quad (10)$$

Joint exchange and raiding occurs in both periods when  $b > c\beta^2$ , which means that the defense cannot be too costly compared with appropriation ( $b / \beta^2 > c$ ), or, for the case that the defense is more costly than appropriation, that the appropriated production is less valuable to the appropriator than to the defender ( $\sqrt{b/c} > \beta$ ). Conversely, pure raiding occurs when  $b \leq c\beta^2$ . Provided that  $\beta > 0$ , equation (9) implies for the symmetric case that pure exchange is impossible and appropriation and defense are guaranteed in both periods. This stands in contrast to Skaperdas and Syropoulos' (1996a) model, which gives no investment in arms in any of the two periods if the growth factor  $\gamma$  is below a certain value.

Let us determine the efficient utility  $W_{ie}$  for the non-symmetric case when there is no appropriation and no defense. Inserting  $F_{ij}=0$  and  $D_{ij}=\varepsilon$  into equation (6), applying equations (2)–(5) and (8) gives:

$$W_{1e} = \alpha \left( \frac{R_{11}}{a_1} \right)^\alpha \left( \frac{R_{21}}{a_2} \right)^{1-\alpha} + \alpha\delta\gamma \left( \frac{R_{11}}{a_1^2} \right)^\alpha \left( \frac{R_{21}}{a_2^2} \right)^{1-\alpha}, \quad e_1 = \frac{W_1}{W_{1e}}$$

$$W_{2e} = (1-\alpha) \left( \frac{R_{11}}{a_1} \right)^\alpha \left( \frac{R_{21}}{a_2} \right)^{1-\alpha} + (1-\alpha)\delta\gamma \left( \frac{R_{11}}{a_1^2} \right)^\alpha \left( \frac{R_{21}}{a_2^2} \right)^{1-\alpha}, \quad e_2 = \frac{W_2}{W_{2e}} \quad (11)$$

where the efficiency  $e_i$  is the ratio of the utilities with and without appropriation and defense.  $W_{ie}$  is naturally independent of  $\beta$ . For the symmetric case, equation (11) simplifies to:

$$W_e = \frac{R(a + \delta\gamma)}{2a^2}, \quad e = \frac{W}{W_e} = \begin{cases} \frac{b + c\beta^2}{b + c\beta(2 + \beta)} & \text{when } b > c\beta^2 \\ \frac{\sqrt{\beta}(bc)^{1/4}}{(\sqrt{b} + \sqrt{c})} & \text{when } b \leq c\beta^2 \quad \text{and} \quad \gamma^2\delta + a\gamma = 2a^2 \end{cases} \quad (12)$$

For joint exchange and raiding the efficiency  $e$  is independent of  $\delta$ ,  $\gamma$ , and the cost  $a$  of production.  $\beta=0$  makes appropriation valueless, causing pure exchange and efficiency  $e=1$ . The efficiency decreases convexly in  $\beta$ , giving  $e = (b + c)/(b + 3c)$  when  $\beta=1$ .

The ratio between the first period and second period utilities is:

$$\frac{U}{V} = \begin{cases} \frac{a(b + c\beta(2 + \beta))}{(b + c\beta)\gamma} & \text{when } b > c\beta^2 \\ \frac{2a}{\gamma} & \text{when } b \leq c\beta^2 \quad \text{and} \quad \gamma^2\delta + a\gamma = 2a^2 \end{cases} \quad (13)$$

which increases in  $a$  and decreases in  $\gamma$ . For joint exchange and raiding,  $U/V$  increases in  $\beta, b, c$ .

In order to determine utility isoquants, solving the utility  $W$  in equation (9) with respect to the discount factor  $\delta = \delta'$  for joint exchange and raiding when  $b > c\beta^2$  gives:

$$\delta = \frac{a[b(2aW/R-1) + c\beta(2a(2+\beta)W/R-\beta)]}{(b+c\beta^2)\gamma}, \quad \frac{\partial \delta}{\partial \gamma} = \frac{-\delta'}{\gamma}, \quad \frac{\partial^2 \delta'}{\partial \gamma^2} = \frac{2\delta'}{\gamma^2}$$

$$\frac{\partial \delta}{\partial \beta} = \frac{4a^2c(b-c\beta^2)W/R}{(b+c\beta^2)^2\gamma}, \quad \frac{\partial^2 \delta'}{\partial \beta^2} = -\frac{8a^2c^2\beta(3b-c\beta^2)W/R}{(b+c\beta^2)^3\gamma}, \quad b > c\beta^2 \quad (14)$$

$\delta$  decreases convexly in  $\gamma$  and increases concavely in  $\beta$ . The utility  $W$  increases in  $\delta$  and  $\gamma$ , as the future becomes more important or growth increases. Hence the utility isoquants in a growth factor  $\gamma$  versus discount factor  $\delta$  diagram are convexly decreasing throughout the first quadrant, in contrast to Skaperdas and Syropoulos' (1996a, p. 369) 'iso-efficiency contours'. Assuming the growth factor  $\gamma$  is above a certain value so that raiding occurs, their contours are vertical for large discount factors  $\delta$  (raiding only in the first period), convex for intermediate  $\delta$  (raiding in both periods), and concave in the transition to small  $\delta$  (raiding only in the second period).

As appropriation becomes more usable in equation (9), fewer resources are allocated to production. Hence, the utility  $W$  decreases convexly in  $\beta$ . This means that a higher discount factor  $\delta$  is needed to maintain the same utility as  $\beta$  increases. Hence, for joint exchange and raiding the utility isoquants are concavely increasing in a usability of appropriation  $\beta$  versus discount factor  $\delta$  diagram.

Solving the utility  $W$  in equation (10) with respect to  $\delta = \delta'$  for pure raiding when  $b \leq c\beta^2$  and  $\gamma^2\delta + a\gamma = 2a^2$  gives:

$$\delta'' = \frac{a[(\sqrt{b} + \sqrt{c})(2a + \gamma)W/R - 2\sqrt{\beta}(bc)^{1/4}]}{\sqrt{\beta}(bc)^{1/4}\gamma}$$

$$\frac{\partial \delta'}{\partial \gamma} = -\frac{2a[(\sqrt{b} + \sqrt{c})aW/R - \sqrt{\beta}(bc)^{1/4}]}{\sqrt{\beta}(bc)^{1/4}\gamma^2}, \quad \frac{\partial^2 \delta'}{\partial \gamma^2} = \frac{4a[(\sqrt{b} + \sqrt{c})aW/R - \sqrt{\beta}(bc)^{1/4}]}{\sqrt{\beta}(bc)^{1/4}\gamma^3}$$

$$\frac{\partial \delta'}{\partial \beta} = -\frac{a(\sqrt{b} + \sqrt{c})(2a + \gamma)W/R}{2\beta^{3/2}(bc)^{1/4}\gamma}, \quad \frac{\partial^2 \delta'}{\partial \beta^2} = \frac{3a(\sqrt{b} + \sqrt{c})(2a + \gamma)W/R}{4\beta^{5/2}(bc)^{1/4}\gamma} \quad (15)$$

$\delta$  decreases convexly in both  $\gamma$  and  $\beta$ . The utility isoquants in a growth factor  $\gamma$  versus discount factor  $\delta$  diagram are convexly decreasing in both equations (14) and (15) since higher emphasis on the future and growth are beneficial for both joint exchange and raiding, and for pure raiding. In equation (9),  $F_1$  and  $F_2$  increase in  $\beta$ , and  $W$  decreases convexly in  $\beta$ . In equation (10), in contrast,  $F_1, F_2, D_1, D_2$  are independent of  $\beta$ . The reason is the nature of the Cobb–Douglas utility where an agent also needs the other agent's good. In equation (10), the agents endogenously choose pure raiding without exchange to obtain the other good. Since increasing  $\beta$  does not cause more allocation to appropriation and defense, but instead causes the appropriation to be more usable, the utility  $W$  in equation (10) increases in  $\beta$ , and it increases concavely due to  $\beta$ . Consequently, a lower emphasis  $\delta$  on the future is sufficient to maintain the same utility as  $\beta$  increases. Hence, for pure raiding the utility isoquants are convexly decreasing in a usability of appropriation  $\beta$  versus discount factor  $\delta$  diagram.

## ANALYZING THE RESULTS

This section confines attention to joint exchange and raiding where  $b > c\beta^2$ , acknowledging that pure raiding where  $b \leq c\beta^2$  has been analyzed in earlier literature. The proofs of Properties

1–8 follow from differentiating equations (9) and (12), and determining the signs of the derivatives. The proof of Property 9 follows from equation (13).

*Property 1*

$$\begin{aligned} \partial F_1/\partial \delta < 0, \partial D_1/\partial \delta < 0, \partial X_1/\partial \delta > 0, \partial U/\partial \delta > 0, \partial F_2/\partial \delta > 0, \partial D_2/\partial \delta > 0, \\ \partial X_2/\partial \delta > 0, \partial V/\partial \delta > 0, \partial W/\partial \delta > 0, \partial W_e/\partial \delta > 0, \partial e/\partial \delta = 0 \end{aligned}$$

First, increasing the shadow of the future decreases appropriation and defense, and increases exchange, in the first period. Strikingly, this result is the opposite of Skaperdas and Syropoulos' (1996a) result in a model allowing fighting and production, but not exchange. Introducing exchange endogenously as a possible strategy in addition to fighting (appropriation and defense) and production, reverses the result when only fighting and production are possible. Second, increasing the shadow of the future increases appropriation, defense, and exchange, in the second period. The incentive to constrain appropriation and defense is not the same in the second period as in the first period. Third, increasing the shadow of the future increases all the four utilities  $U, V, W, W_e$  since the pie gets enlarged. Fourth, increasing the shadow of the future has no impact on the efficiency  $e$  since the term  $a + \delta\gamma$  is equivalently present in equations (9) and (12). This contrasts with Skaperdas and Syropoulos' (1996a) finding that increasing the shadow of the future decreases efficiency. The reason is the first point where the shadow of the future in their model increases investment in arms today.

*Property 2*

$$\begin{aligned} \partial F_1/\partial \gamma < 0, \partial D_1/\partial \gamma < 0, \partial X_1/\partial \gamma > 0, \partial U/\partial \gamma > 0, \partial F_2/\partial \gamma > 0, \partial D_2/\partial \gamma > 0, \\ \partial X_2/\partial \gamma > 0, \partial V/\partial \gamma > 0, \partial W/\partial \gamma > 0, \partial W_e/\partial \gamma > 0, \partial e/\partial \gamma = 0 \end{aligned}$$

Properties 1 and 2 reveal the same signs for the derivatives with respect to  $\delta$  and  $\gamma$ . Inspecting equation (9) reveals that  $\delta$  and  $\gamma$  always occur multiplicatively as  $\gamma\delta$  in  $F_1, D_1, X_1$ . When located in the first period, increasing the shadow  $\delta$  of the future and increasing the growth  $\gamma$  in resources from the first to the second period have the same impact. A high (low) value for  $\delta$  may compensate for a low (high) value for  $\gamma$ , and vice versa. If  $\delta$  and  $\gamma$  are both low or both high, their joint impact reinforces each other. The parameters  $\delta$  and  $\gamma$  occur multiplicatively as  $\gamma\delta$  once in the numerator and once in the denominator of  $F_2, D_2, X_2$ . Additionally,  $\gamma$  occurs alone as a proportionality factor in  $F_2, D_2, X_2$ . The reason is that discounting occurs proportionally with the second period utility  $V$  as viewed from the first period, but not proportionally as viewed from the second period. In equation (5),  $V$  depends on  $\delta$  and  $\gamma$  as  $F_2, D_2, X_2$  do since  $\alpha=1/2$  and the ratios are dimensionless. This is clarified by equation (6) where  $\delta$  is multiplied by the second period utility  $V$  and added to  $U$  to yield the total utility  $W$ . Hence,  $\delta$  and  $\gamma$  occur multiplicatively as  $\gamma\delta$  in  $W$  in equation (9).

*Property 3*

$$\begin{aligned} \partial F_1/\partial b < 0, \partial D_1/\partial b < 0, \partial X_1/\partial b > 0, \partial U/\partial b > 0, \partial F_2/\partial b < 0, \partial D_2/\partial b < 0, \\ \partial X_2/\partial b > 0, \partial V/\partial b > 0, \partial W/\partial b > 0, \partial W_e/\partial b = 0, \partial e/\partial b > 0 \end{aligned}$$

Of all the parameters,  $b$  has the most straightforward impact when operating equivalently on both agents. The derivatives are the same as for  $\delta$  and  $\gamma$  except that the second period

appropriation and defense also decrease in  $b$ , and the impact on  $W_e$  and  $e$  differs. That is, increasing the cost  $b$  of appropriation has the universally beneficial effect of reducing appropriation and defense in both periods, increasing exchange in both periods, increasing all utilities except  $W_e$  (which is not influenced by  $b$ ), which thus increases the efficiency  $e$ .

*Property 4*

$$\begin{aligned} \partial F_1 / \partial c > 0, \partial D_1 / \partial c < 0, \partial X_1 / \partial c < 0, \partial U / \partial c < 0, \partial F_2 / \partial c > 0, \partial D_2 / \partial c < 0, \\ \partial X_2 / \partial c < 0, \partial V / \partial c < 0, \partial W / \partial c < 0, \partial W_e / \partial c = 0, \partial e / \partial c < 0 \end{aligned}$$

The three parameters  $c, a, \beta$  are the most detrimental to increase, although with different impacts. Increasing the cost  $c$  of defense increases appropriation in both periods, decreases defense and exchange in both periods, decreases all utilities except  $W_e$  (which is not influenced by  $c$ ), and decreases efficiency.

*Property 5*

$$\begin{aligned} \partial F_1 / \partial a > 0, \partial D_1 / \partial a > 0, \partial X_1 / \partial a < 0, \partial U / \partial a < 0, \partial F_2 / \partial a < 0, \partial D_2 / \partial a < 0, \\ \partial X_2 / \partial a < 0, \partial V / \partial a < 0, \partial W / \partial a < 0, \partial W_e / \partial a < 0, \partial e / \partial a = 0 \end{aligned}$$

As the cost  $a$  of production increases, agents switch from production to raiding in the first period, as they would in a one-shot game. They also do so in the second period, but raiding in the second period decreases in  $a$  since reduced production in the first period causes fewer resources to be available for conversion into raiding and production in the second period. Hence, appropriation and defense increase in  $a$  in the first period, and decrease in  $a$  in the second period. All utilities decrease in  $a$ . Increased raiding in the first period compensates for reduced raiding in the second period, causing  $a$  to have no impact on the efficiency  $e$ , where  $a$  operates equivalently in  $W$  and  $W_e$ .

*Property 6*

$$\begin{aligned} \partial F_1 / \partial \beta > 0, \partial D_1 / \partial \beta > 0, \partial^2 D_1 / \partial \beta^2 < 0, \partial X_1 / \partial \beta < 0, \partial^2 U / \partial \beta^2 > 0, \partial F_2 / \partial \beta > 0, \\ \partial^2 D_2 / \partial \beta^2 < 0, \partial X_2 / \partial \beta < 0, \partial V / \partial \beta < 0, \partial W / \partial \beta < 0, \partial^2 W / \partial \beta^2 > 0, \partial W_e / \partial \beta = 0, \\ \partial e / \partial \beta < 0, \partial^2 e / \partial \beta^2 > 0 \end{aligned}$$

Increasing the usability  $\beta$  of appropriation naturally increases appropriation in both periods, and it increases the defense in the first period. This causes the first period utility derivative  $\partial U / \partial \beta$  to be positive or negative, but  $\partial^2 U / \partial \beta^2 > 0$  is positive. Hence,  $U$  is U-shaped. Viewing the first period in isolation, an intermediate  $\beta$  is most detrimental on the utility  $U$ . When  $\beta$  is increased above this detrimental level, appropriation becomes more usable. This causes appropriation and defense to increase, and exchange to decrease. The second period defense derivative  $\partial D_2 / \partial \beta$  can be positive or negative, but  $\partial^2 D_2 / \partial \beta^2 < 0$  is negative. Hence,  $D_2$  is inverse U-shaped. The defenses in both periods are concave. Agents substitute from defense to appropriation when appropriation becomes more usable, illustrated with the

ratio  $D_j / F_j = b/(c\beta)$  in equation (9). The total utility  $W$  and efficiency  $e$  decrease convexly in  $\beta$ .

*Property 7*

$$\begin{aligned} \partial(F_1 / F_2) / \partial\delta = \partial(D_1 / D_2) / \partial\delta < 0, \quad \partial(F_1 / F_2) / \partial\gamma < 0, \quad \partial(F_1 / F_2) / \partial b < 0, \\ \partial(F_1 / F_2) / \partial c > 0, \quad \partial(F_1 / F_2) / \partial a > 0, \quad \partial(F_1 / F_2) / \partial\beta > 0 \end{aligned}$$

Introducing exchange endogenously as a possible strategy in addition to fighting (appropriation and defense) and production, induces agents to restrain their appropriation  $F_1$  and defense  $D_1$  in the first period, relative to  $F_2$  and  $D_2$  in the second period, when the parameters  $\delta, \gamma, b$  increase (i.e. when the future becomes more important), when there is growth, and when the cost of appropriation increases. They allocate resources to production and engage in exchange rather than costly raiding to equip themselves well with resources for the second period. This stands in contrast to Skaperdas and Syropoulos' (1996a) result that increasing the shadow of the future may increase fighting. Conversely, the ratios  $F_1/F_2 = D_1/D_2$  increase in  $c, a, \beta$  (i.e. when the cost of defense, or the cost of production, or the usability of appropriation increases).

*Property 8*

$$\begin{aligned} \partial(X_1 / X_2) / \partial\delta = 0, \quad \partial(X_1 / X_2) / \partial\gamma < 0, \quad \partial(X_1 / X_2) / \partial b < 0, \quad \partial(X_1 / X_2) / \partial c > 0, \\ \partial(X_1 / X_2) / \partial a > 0, \quad \partial(X_1 / X_2) / \partial\beta > 0 \end{aligned}$$

The importance  $\delta$  of the future has no impact on the exchange  $X_1$  in the first period relative to  $X_2$  in the second period. The other five parameters  $\gamma, b, c, a, \beta$  have the same impact on  $X_1/X_2$  as on  $F_1/F_2 = D_1/D_2$ .

*Property 9*

$U/V > 1$  when  $\gamma < a(b + c\beta(2 + \beta))/(b + c\beta)$  which simplifies to  $\gamma < a$  when  $\beta=0$  and  $\gamma < a(b + 3c)/(b + c)$  when  $\beta=1$

Applying equation (13), the ratio  $U/V$  of utilities in the first and second periods first decreases in  $\gamma$ , which causes benefit in the second period. This result is also found by Skaperdas and Syropoulos (1996a, p. 370). Second,  $U/V$  increases in the cost  $a$  of production. The reason is that the first period production in equation (2) is determined by dividing the resource  $R$  (deducting appropriation and defense) with  $a$ , while the second period production in equation (4) is determined by dividing the resource  $\gamma E_{i1}$  (deducting appropriation and defense) with  $a$ , with the added impact that  $U$  decreases in  $a$ . Increasing  $a$  is thus more detrimental for the second period utility than for the first period utility. Third, the term  $(b + c\beta(2 + \beta))/(b + c\beta)$  increases from 1 to  $(b + 3c)/(b + c)$  as  $\beta$  increases from 0 to 1. While  $\beta=0$  causes pure exchange in both periods, and  $\beta=1$  causes joint exchange and raiding when  $b > c$ , and pure raiding when  $b \leq c$ , in both periods,  $\beta=1$  is more detrimental for the second period utility since the resource  $\gamma E_{i1}$  available in the second period gets reduced by the first period raiding. That is, the high usability  $\beta$  of appropriation causes costly raiding in the second period, while the agents impose restraints on their raiding in the first period due to the nature of time dependence from the first to the second period.

## GRAPHIC ILLUSTRATIONS

Standardizing the variables by dividing with the resource  $R_{i1}$  or  $R$ ,

$$f_{ij} = \frac{F_{ij}}{R_{i1}}, d_{ij} = \frac{D_{ij}}{R_{i1}}, x_{ij} = \frac{X_{ij}}{R_{i1}}, w_i = \frac{W_i}{R_{i1}}, f_j = \frac{F_j}{R}, d_j = \frac{D_j}{R}, x_j = \frac{X_j}{R}, u = \frac{U}{R}, v = \frac{V}{R}, w = \frac{W}{R} \quad (16)$$

the dependence on the usability  $\beta$  of appropriation is shown in Figure 1 where,  $a=b=\delta=\gamma=1$ ,  $c=2$ , illustrating Property 6. Joint exchange and raiding occurs when  $\beta < \sqrt{1/2}$ , and pure raiding occurs when  $\beta \geq \sqrt{1/2}$ . The growth  $\gamma=1$  and cost  $a=1$  of production imply, due to equations (2) and (4), that the resource  $\gamma E_{i1}$  available in the second period is lower than the resource  $R$  available in the first period. Despite this, the second period appropriation  $f_2$  and defense  $d_2$  are higher than the first period appropriation  $f_1$  and defense  $d_1$ . Conversely, the second period exchange  $x_2$  is lower than the first period exchange  $x_1$ .

Figure 2 shows the dependence on the cost  $c$  of defense. The parameters are as in Figure 1, although  $\beta=1$ , and  $\gamma$  has been doubled to  $\gamma=2$  to illustrate a substantial increase for  $f_2$  and  $d_2$  in the second period. Joint exchange and raiding occurs when  $c < 1$ , and pure raiding occurs when  $c \geq 1$ , solved numerically since  $\gamma^2 \delta + a\gamma \neq 2a^2$ . The curves exemplify Property 4.

Figure 3 shows the convexly decreasing utility isoquants in a growth factor  $\gamma$  versus discount factor  $\delta$  diagram where  $a=b=1$ ,  $c=2$ ,  $\beta=0.6$ , which satisfies  $b > c\beta^2$  ensuring joint exchange and raiding. Figure 4 shows the utility isoquants in a usability of appropriation  $\beta$  versus discount factor  $\delta$  diagram where  $a=b=\gamma=1$ ,  $c=2$ , giving a concave increase for joint exchange and raiding when  $\beta < \sqrt{1/2}$ , and a convex decrease for pure raiding when  $\beta \geq \sqrt{1/2}$ .

Symmetry facilitates exchange more easily than asymmetry (Hausken, 2004). Figure 5 tests the sensitivity to symmetry by equipping the agents with different resources  $R_{11}$  and  $R_{21}$ . The FOCs in equation (7) are applied to determine the variables as functions of the resource ratio  $R_{11}/R_{21}$ ,  $a_1=a_2=b_1=b_2=c_1=c_2=\delta=\gamma=1$ ,  $\alpha=\beta=1/2$ , where  $w$  and  $e$  are the average utilities and efficiencies. All variables except the prices and efficiencies are, according to equation (16), divided by  $R_{11}$  for agent 1 and  $R_{21}$  for agent 2. The convexly decreasing curve proportional to  $y=1/(R_{11}/R_{21})$ , where  $y$  is measured along the vertical axis, demarcates whether the absolute variables

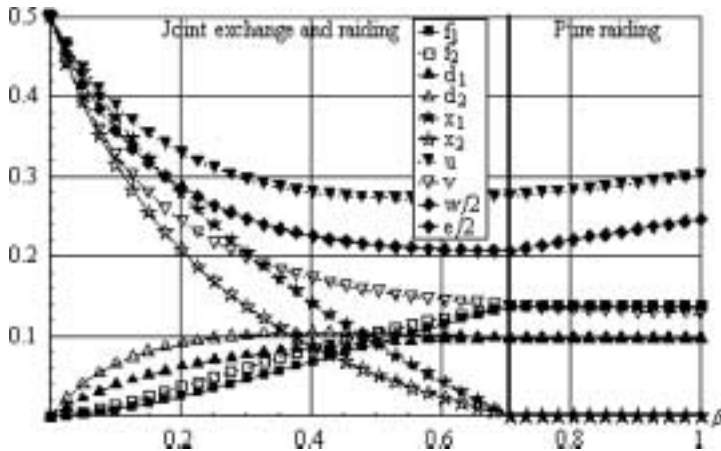


FIGURE 1 Variables as functions of the usability  $\beta$  of appropriation. Division by 2 in  $w/2$  and  $e/2$  are for scaling purposes,  $a=b=\delta=\gamma=1$ ,  $c=2$ .

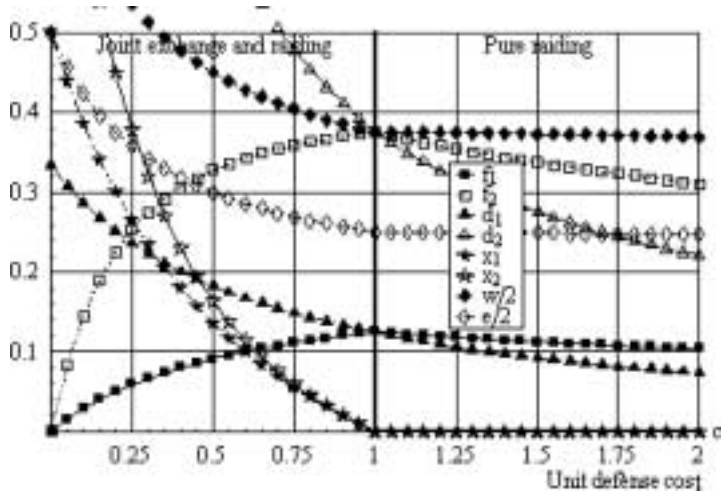


FIGURE 2 Variables as functions of the unit defense cost  $c$ ,  $a=b=\delta=\beta=1$ ,  $\gamma=2$ .

for agent 1 increases or decreases in  $R_{11}/R_{21}$ . Figure 5(a) shows that the agent receiving more resources increases his appropriation and decreases his defense in relative terms.  $d_{1j}$  decreases less than proportional to  $\gamma=1/(R_{11}/R_{21})$ , so  $D_{1j}$  increases in absolute terms. The agent receiving fewer resources increases his appropriation considerably, consistently with Hirshleifer's (1991) paradox of power, and also increases his defense. The behavioral trends are the same in both periods. Figure 5(b) shows that agent 1's export  $x_{1j} = X_{1j}/R_{11}$  decreases moderately, which means that  $X_{1j}$  increases, while agent 2's export  $x_{2j} = X_{2j}/R_{21}$  decreases more, as  $R_{11}/R_{21}$  increases. The prices  $P_{2j}$  of good 2 in terms of good 1 increases in  $R_{11}/R_{21}$ , which reflects that agent 1's high resource  $R_{11}$  translates into higher production of good 1, causing good 1 to be more readily available and cheaper than the scarce expensive good 2. The utility  $w_1 = W_1/R_{11}$  for agent 1 decreases in  $R_{11}/R_{21}$ , while  $W_1$  increases slightly, and  $w_2 = W_2/R_{21}$  for agent 2 is inverse U shaped. The average utility  $w = (w_1 + w_2)/2$  and efficiency  $e$  decrease in  $R_{11}/R_{21}$ , as asymmetry causes more inefficient appropriation and defense.

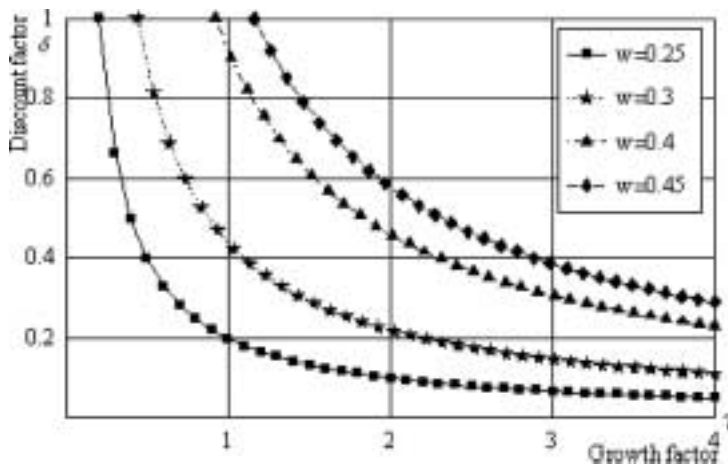


FIGURE 3 Utility isoquants in a growth factor  $\gamma$  versus discount factor  $\delta$  diagram,  $a=b=1$ ,  $c=2$ ,  $\beta=0.6$ .



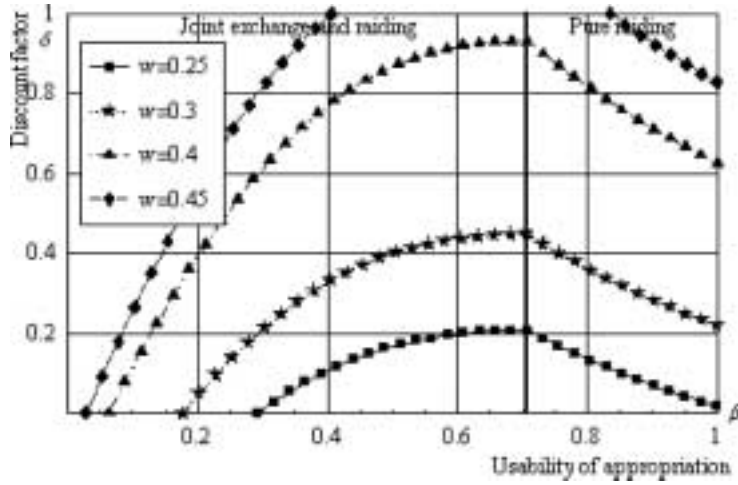


FIGURE 4 Utility isoquants in a usability of appropriation  $\beta$  versus discount factor  $\delta$  diagram,  $a=b=\gamma=1$ ,  $c=2$ .

## THE CES MODEL

To analyze the robustness of the results, let us substitute the Cobb–Douglas utilities in equations (3) and (5) and the total utility in equation (6) with:

$$\begin{aligned}
 U_{1CES} &= \left( \lambda \left[ \frac{D_{11}}{D_{11} + F_{21}} E_{11} - X_{11} \right]^{\xi} + (1-\lambda) \left[ \frac{\beta F_{11}}{F_{11} + D_{21}} E_{21} + X_{21} \right]^{\xi} \right)^{\frac{1}{\xi}} \\
 U_{2CES} &= \left( \lambda \left[ \frac{D_{21}}{D_{21} + F_{11}} E_{21} - X_{21} \right]^{\xi} + (1-\lambda) \left[ \frac{\beta F_{21}}{F_{21} + D_{11}} E_{11} + X_{11} \right]^{\xi} \right)^{\frac{1}{\xi}}, \quad 0 < \xi < 1, \quad 0 \leq \lambda \leq 1 \quad (19) \\
 V_{1CES} &= \left( \lambda \left[ \frac{D_{12}}{D_{12} + F_{22}} E_{12} - X_{12} \right]^{\xi} + (1-\lambda) \left[ \frac{\beta F_{12}}{F_{12} + D_{22}} E_{22} + X_{22} \right]^{\xi} \right)^{\frac{1}{\xi}} \\
 V_{2CES} &= \left( \lambda \left[ \frac{D_{22}}{D_{22} + F_{12}} E_{22} - X_{22} \right]^{\xi} + (1-\lambda) \left[ \frac{\beta F_{22}}{F_{22} + D_{12}} E_{12} + X_{12} \right]^{\xi} \right)^{\frac{1}{\xi}}, \quad W_{iCES} = U_{iCES} + \partial V_{iCES} \quad (20)
 \end{aligned}$$

keeping equations (1),(2),(4) as before, setting  $\lambda=1/2$  for symmetric equilibria. Applying the same method of analysis as in the third section, and assuming symmetry, remarkably, gives, for the case of joint exchange and raiding where  $b > c\beta^2$ , exactly the same results for the Cobb–Douglas model and the CES model, independently of  $\xi$ . That is, the CES FOCs reproduce all the equations in equation (9) for the two periods exactly. This suggests exceptionally good robustness of the results. For pure raiding where  $b \leq c\beta^2$ , no general analytical solution exists. However, it has been possible to use the CES FOCs to reproduce the equations in equation (10) with the assumption  $\gamma^2\delta + a\gamma = 2a^2$ , and additionally assuming  $\xi=1/2$  and  $b = c\beta^2$ , which gives the transition point between joint exchange and raiding, and pure raiding, where  $X_j = 0$ .

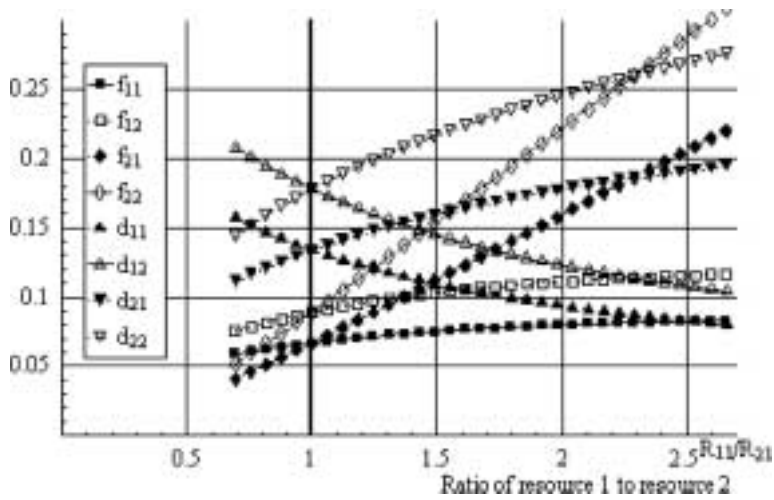


FIGURE 5 (a) Variables as functions of the resource ratio  $R_{11}/R_{21}$ ,  $a_1=a_2=b_1=b_2=c_1=c_2=\delta=\gamma=1$ ,  $\alpha=\beta=1/2$ .

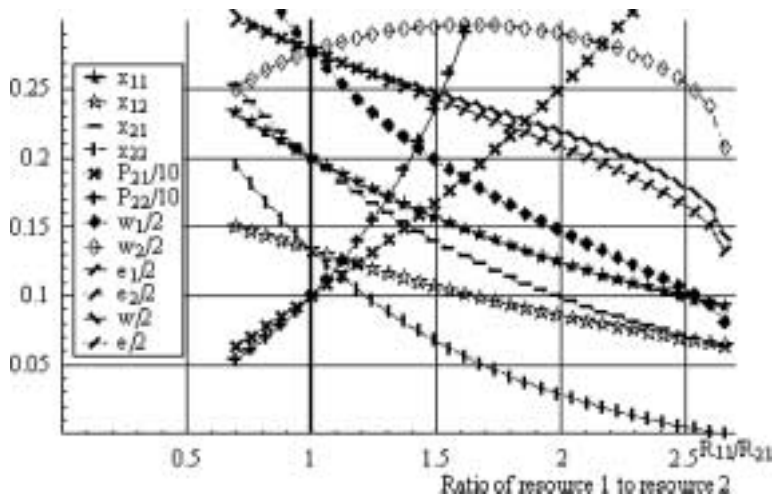


FIGURE 5 (b) Variables as functions of the resource ratio  $R_{11}/R_{21}$ ,  $a_1=a_2=b_1=b_2=c_1=c_2=\delta=\gamma=1$ ,  $\alpha=\beta=1/2$ .

## CONCLUSION

Although the folk theorem is generally formulated, it is often taken to imply that increasing the shadow of the future may suppress conflict and induce cooperation. Skaperdas and Syropoulos (1996a) demonstrate, in a model that allows production and fighting, the opposite effect – that agents may choose conflict today to reap benefits tomorrow, so that the increased importance of the future may harm cooperation. To broaden our insight, this article introduces exchange endogenously as a possible strategy in addition to production, appropriation, and defense. Exchange and raiding (appropriation and defense) have traditionally been analyzed separately within economics, which this article intends to overcome through joint treatment. As an example, each side to some extent looted the other in the American Civil War, although some reciprocal exchange was conducted by “blockage-runners”, where the South exported cotton and imported manufactured goods.

A two-period model with time dependence is analyzed. Cobb–Douglas utilities are assumed, and the results are confirmed with CES utilities. Agents discount the future, and there is growth from the first to the second period. Increasing the shadow of the future has four impacts. First, it decreases appropriation and defense, and increases exchange, in the first period. This result is the opposite of Skaperdas and Syropoulos' (1996a) result and shows that agents voluntarily reduce raiding and increase voluntary exchange when given that option. Second, appropriation, defense, and exchange, in the second period increase. Third, all utilities increase. Fourth, there is no impact on efficiency.

Increasing appropriation cost reduces appropriation and defense, and increases exchange and utility, in both periods. Increasing defense cost increases appropriation, and decreases defense, exchange, and utility, in both periods. Increasing production cost increases appropriation and defense in the first period, decreases appropriation and defense in the second period, since fewer resources become available in the second period, and decreases exchange and utility in both periods. From a collective welfare point of view, increasing appropriation cost and decreasing defense cost and production cost is beneficial.

Appropriated goods can be less valuable to the appropriator than to the defender due to destruction and differential preferences. No usability of appropriation gives pure exchange and 100% efficiency, and exhaustive usability of appropriation when appropriation and defense costs are equal gives pure raiding and 50% efficiency. Increasing the usability of appropriation increases appropriation in both periods, causes concave defense, and decreases exchange, in both periods. Agents substitute from defense to appropriation when appropriation becomes more usable, and utility decreases. However, as the usability of appropriation increases into the range where exchange no longer occurs, which is possible when the defense cost exceeds the appropriation cost, the special case of pure raiding analyzed when an analytical solution exists reveals that the utility increases while agents find a mutual interest in constraining appropriation and defense. The implication is that utility isoquants in a usability of appropriation versus discount factor diagram are concavely increasing for joint exchange and raiding, and convexly decreasing for the special case analyzed of pure raiding.

Introducing exchange endogenously as a possible strategy, in addition to fighting, decreases appropriation, defense, and exchange in the first relative to the second period when there is growth, when the cost of appropriation increases, and for appropriation and defense when the future becomes more important, in contrast to Skaperdas and Syropoulos' (1996a) result that increasing the shadow of the future may increase fighting. Conversely, appropriation, defense, and exchange increase in the first relative to the second period when the cost of defense, the cost of production, or the usability of appropriation increases.

The ratio of utilities in the first and second periods has three characteristics. First, it intuitively decreases in the growth factor. Second, it increases in the cost of production, which is more detrimental for the second period due to time dependence. Third, it increases in the usability of appropriation.

Exchange (trade) has increased over the centuries, caused by factors such as improved transport, transmission of information, preservation techniques, human rights, and education. A hypothesis is that exchange has increased in periods with lower usability of appropriated goods, when the defense was cheaper than appropriation, and when farsightedness was possible, increasing the shadow of the future. The development of more sophisticated tastes over the last millennia, and improved capacity for destruction, have likely decreased the usability of appropriation. The development of more lawful societies has given the defending parties additional resort opportunities. A population explosion has moved dispersed communities closer, and long-term relationships have developed. As farsightedness in interaction has developed, and the shadow of the future has increased, possibilities for exchange have emerged in addition to raiding.

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