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Research Article

Fuzzy Sliding Mode Controller Design Using Takagi-Sugeno Modelled Nonlinear Systems

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Adaptive fuzzy sliding mode controller for a class of uncertain nonlinear systems is proposed in this paper. The unknown system dynamics and upper bounds of the minimum approximation errors are adaptively updated with stabilizing adaptive laws. The closed-loop system driven by the proposed controllers is shown to be stable with all the adaptation parameters being bounded. The performance and stability of the proposed control system are achieved analytically using the Lyapunov stability theory. Simulations show that the proposed controller performs well and exhibits good performance.

1. Introduction

Recent research on fuzzy logic control has, therefore, been devoted to model based fuzzy control systems that guarantee not only stability, but also performance of closed-loop fuzzy control systems [1–6]. For a systematic control design of nonlinear systems, the Takagi-Sugeno (T-S) fuzzy model [4, 5, 7–12] has been a popular choice in industrial processes due to its ability to represent the nonlinear system only for input-output data without complex mathematical equations.

In an effort to improve the robustness of the adaptive fuzzy control system, many works have been published on the design of adaptive fuzzy sliding mode controller [13–18], which integrates the sliding mode controller [16, 19–23] design technique into the adaptive fuzzy control to improve the stability and the robustness of the control system. Conventionally, adaptive fuzzy sliding control systems (AFSCSs) design is based on the assumption that the system states are available for measurement, so the adaptive laws of AFSCS are formulated as functions of the tracking error of the system [21, 24–27].

However, some problems on the algorithm convergence and conditions stabilities remain with no response. To resolve this problem, first is the need for accurate information on the evolution of the system in the state space, upper bounds of uncertainties and disturbances. The second is the knowledge of the upper bound of the minimum approximation error. We know that the uncertain nature of nonlinear systems makes it difficult to have an analytical description of the dynamics of the system. Moreover, the knowledge of the upper bound of the minimum approximation error leaves the control law still restrictive. In the further study involving a perturbed large-scale system with a time-varying interconnection, an adaptive algorithm for estimating an uncertain upper bound based on a variable sliding control frame was proposed in [28].

In this note, based on the variable surface, a fuzzy sliding model controller is developed for guaranteeing the tracking performance, in particular, time-varying uncertain parameters are approximated by fuzzy system, and the adaptive sliding mode control is designed so as to compensate for any unknown reconstruction error, through parameter adaptation law. In this way, the actual system can follow the reference signal even in the event of a hard nonlinearity, and fuzzy sliding mode control gives the unknown upper bound of uncertainties that are adaptively updated with stabilizing adaptive laws. It is proved that the closed-loop system is globally stable in the Lyapunov sense if the signals involved are bounded and the system output can track the desired reference input asymptotically.

This paper is organized as follows: some preliminaries are provided in Section 2. Following the introduction, the fuzzy logic system is reviewed briefly in Section 3. The design and stability analysis for the proposed adaptive fuzzy sliding mode controller is included in Section 4. Simulation examples to demonstrate the performance of the proposed method are provided in Section 5. Finally, in Section 6, we give a brief conclusion.

2. Preliminaries

Consider the nth-order nonlinear dynamical system of the form:

$$x^n = f(\mathbf{x}) + g(\mathbf{x})u + d_s, \qquad y = x, \tag{1}$$

where $\mathbf{x} = [x, \dot{x}, \ldots, x^{n-1}]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is vector of the system that is assumed to be available for measurements, f and g are unknown but bounded nonlinear functions, $u \in R$ and $y \in R$ control input and output of the system, respectively, and d_s is external disturbance. As system (1) is required to be controllable, the nonzero condition of input gain $g(\mathbf{x}) \neq 0$ is necessary.

The system (1) can be rewritten in the following form:

$$0 = -g^{-1}(\mathbf{x})x^n + g^{-1}(\mathbf{x})f(\mathbf{x}) + u + g^{-1}(\mathbf{x})d_s. \tag{2}$$

By adding x^n to both sides, we get

$$x^n = x^n - g^{-1}(\mathbf{x})x^n + g^{-1}(\mathbf{x})f(\mathbf{x}) + u + g^{-1}(\mathbf{x})d_s. \tag{3}$$

Equation (3) can be rewritten as

$$x^n = F(\mathbf{x}) + u + d(\mathbf{x}) \tag{4}$$

such that

$$F(\mathbf{x}) = \left(1 - g^{-1}(\mathbf{x})\right)x^n + g^{-1}(\mathbf{x})f(\mathbf{x}), \qquad d(\mathbf{x}) = g^{-1}(\mathbf{x})d_s. \tag{5}$$

Assumption 1 (see [29, 30]). Assume that $f(\mathbf{x})$, $g(\mathbf{x})$, and d_s satisfy $|f(\mathbf{x})| \leq \mu < \infty$, $0 < g_{\min} \leq g(\mathbf{x}) \leq g_{\max} < \infty$, and $|d| \leq \kappa$, respectively, for all $\mathbf{x} \in \mathbf{U_x} \subset \Re^n$.

Where μ, $g_{\min}$, $g_{\max}$, and κ are known constants. The control problem is to force the system output y to follow a given bounded reference signal y_d.

Define the tracking error as

$$e = y_d - y. \tag{6}$$

3. Takagi-Sugeno (T-S) Fuzzy Model

Fuzzy logic systems address the imprecision of the input and output variables directly by defining them with fuzzy numbers (and fuzzy sets) that can be expressed in linguistic terms. The basic configuration of the Takagi and Sugeno [5, 8, 31] system includes a rule base, which consists of a collection of fuzzy IF-THEN rules in the following form:

Plant Rule r:

IF x_1 is B_1^r and $\cdots$ and x_n is B_n^r,

$$\text{THEN} \quad y_r = a_0^r + a_1^r x_1 + \cdots + a_n^r x_n = \theta_r^T \mathbf{x}, \tag{7}$$

where B_i^r are fuzzy sets and $\theta_r^T = [a_0^r, a_1^r, \ldots, a_n^r]$ is a vector of the adjustable factors on the consequence part of the fuzzy rule, and the input vector $\mathbf{x} = [1, x_1, \ldots, x_n] \in R^n$. Let $i = 1, 2, \ldots, n$ denote the number of input for fuzzy logic system, and let $r = 1, 2, \ldots, m$ denote the number of the fuzzy IF-THEN rules. By using the singleton fuzzification, product inference and centre average defuzzication, the output value of the fuzzy system is

$$y(\mathbf{x}) = \frac{\sum_{r=1}^{m} y^r \left(\prod_{i=1}^{n} \mu_{B_i^r}(x_i)\right)}{\sum_{r=1}^{m} \left(\prod_{i=1}^{n} \mu_{B_i^r}(x_i)\right)}, \tag{8}$$

where $\mu_{B_i^r}(x_i)$ is the membership function value of the fuzzy variable x_i and $\prod_{i=1}^{n} \mu_{B_i^r}(x_i)$ is the true value of the rth implication. Equation (8) can be rewritten as

$$y(\mathbf{x}) = \theta^T \xi(\mathbf{x}), \tag{9}$$

where $\theta^T = [\theta_1^T, \theta_2^T, \ldots, \theta_m]$ is an adjustable parameter vector, $\xi(\mathbf{x})^T = [\xi^1(\mathbf{x}), \ldots, \xi^m(\mathbf{x})]$ is a fuzzy basis function vector in which, $\xi^r(\mathbf{x})$, $r = 1, 2, \ldots, m$,

$$\xi^r(\mathbf{x}) = \frac{\prod_{i=1}^{n} \mu_{B_i^r}(x_i)}{\sum_{r=1}^{m} \prod_{i=1}^{n} \mu_{B_i^r}(x_i)}. \tag{10}$$

The aforementioned fuzzy system has been shown to be capable of universally approximating well-defined functions over a compact set to arbitrary degree of accuracy. For smooth nonlinear functions $F(\mathbf{x})$, $d(\mathbf{x})$, they can be approximated by

$$F(\mathbf{x}) = \theta_f^{*T} \Psi(\mathbf{x}) + \varepsilon, \qquad d(\mathbf{x}) = \theta_d^{*T} \Xi(\mathbf{x}) + \sigma, \tag{11}$$

where ε and σ are the fuzzy approximations and θ_f^*, and θ_d^* are optimal weight vectors.

And whose estimates are given by

$$F\left(\frac{\mathbf{x}}{\widehat{\theta}_f}\right) = \widehat{\theta}_f^T \Psi(\mathbf{x}), \qquad d\left(\frac{\mathbf{x}}{\widehat{\theta}_d}\right) = \widehat{\theta}_d^T \Xi(\mathbf{x}). \tag{12}$$

4. Adaptive Fuzzy Sliding Mode Controller Design

In this section, a systematic methodology is presented for the design of stable adaptive fuzzy sliding mode controller, the control law and the weight adaptation rules are developed, guaranteeing the uniform ultimate boundedness of the tracking error with respect to an arbitrary small set around the origin. Additionally, the boundedness of all signals involved in the closed-loop configuration is ensured. The resetting scheme is introduced, performing on the weight estimates θ_f, θ_d to guarantee the validity of the control law.

If we consider the system given by (4), the sliding surface can be defined by

$$S = a_{n-1}e + \cdots + a_1 e^{n-2} + e^{n-1}. \quad (13)$$

The elements of the sliding surface are chosen such that the polynomial $a_{n-1}\,p^{n-1} + a_{n-2}\,p^{n-2} + \cdots + a_0$ is strictly Hurwitz [32] (here p denotes the complex Laplace transform variable).

We propose to choose "a" as follows [33]:

$$a_i = \frac{M_i}{\left|e^{n-1-i}\right| + \eta_i}; \quad i = 1, \ldots, n-1, \quad (14)$$

where M_i is a given positive scalar, and η_i is positive constant low value.

Note that M/η represents the slope of sliding along the surface when it is reached by the system.

By using the tracking error defined by (6), the time derivative of (13) is

$$\begin{aligned}\dot{S} &= \sum_{i=1}^{n-1} a_{n-i}e^i + \sum_{i=1}^{n-1}\dot{a}_{n-i}e^{i-1} + e^n,\\ \dot{S} &= y_d^n - y^n + \sum_{i=1}^{n-1} a_{n-i}e^i + \sum_{i=1}^{n-1}\dot{a}_{n-i}e^{i-1}\\ &= y_d^n - F(\mathbf{x}) - u - d(\mathbf{x}) + \sum_{i=1}^{n-1} a_{n-i}e^i + \sum_{i=1}^{n-1}\dot{a}_{n-i}e^{i-1}\\ &= y_d^n - F(\mathbf{x}) - u - d(\mathbf{x}) + \mathbf{Ae},\end{aligned} \quad (15)$$

where n is the nth derivative of the system, and $\mathbf{A} = [\dot{a}_{n-1}, a_{n-1}, \dot{a}_{n-2}, \ldots, \dot{a}_1, a_1]$, $\mathbf{e} = [e, \ldots, e^{n-3}; e^{n-2}, e^{n-2}; e^{n-1}]^T$.

Assumption 2. Let $\mathbf{x}$ belong to a compact set Ω_x. The optimal weight vectors θ_f^* and θ_d^* are defined as

$$\begin{aligned}\theta_f^* &= \arg\min_{\hat{\theta}_f \in \Omega_f}\left[\sup_{x\in\Omega_x}\left[F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) - F(\mathbf{x})\right]\right],\\ \theta_d^* &= \arg\min_{\hat{\theta}_d \in \Omega_d}\left[\sup_{x\in\Omega_x}\left[d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) - d(\mathbf{x})\right]\right].\end{aligned} \quad (16)$$

And define the constraint sets that the parameters concerned belong to

$$\begin{aligned}\Omega_f &= \left\{\theta_f \mid \left\|\theta_f\right\| \le M_f\right\},\\ \Omega_d &= \left\{\theta_d \mid \left\|\theta_d\right\| \le M_d\right\},\end{aligned} \quad (17)$$

where M_f and M_g are design parameters.

We assume that $\hat{\theta}_f$, $\hat{\theta}_d$, and x never reach the boundaries Ω_f, Ω_d, and Ω_x. We can define the minimum approximation errors as

$$\begin{aligned}\varepsilon &= F(\mathbf{x}) - F\left(\frac{\mathbf{x}}{\theta_f^*}\right),\\ \sigma &= d(\mathbf{x}) - d\left(\frac{\mathbf{x}}{\theta_d^*}\right).\end{aligned} \quad (18)$$

It is assumed that minimum approximation errors are bounded for all $x \in \Omega_x$:

$$|\varepsilon| \le \bar{\varepsilon}, \quad |\sigma| \le \bar{\sigma}, \quad \forall x \in \Omega_x. \quad (19)$$

The upper bound $\bar{\varepsilon}$, $\bar{\sigma}$ can be reduced arbitrarily. But this choice is not always easy, that is our aim in this work to estimate them by adaptive laws, which guarantee the stability of the closed loop system.

The role of the fuzzy systems $F(\mathbf{x}/\hat{\theta}_f)$ and $d(\mathbf{x}/\hat{\theta}_d)$ is to represent the unknown functions using the input-output measurement of the target system. Also, a corrective controller is defined to guarantee the stability of the closed-loop control system and compensate the approximation errors. A direct adaptive control law can be chosen as

$$u = \left(\left|F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right)\right| + \left|d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right)\right| + y_d^n + \lambda + \mathbf{Ae} + \hat{\bar{\varepsilon}} + \hat{\bar{\sigma}}\right)\operatorname{sgn}(S), \quad (20)$$

where λ is a strictly positive constant, and $\hat{\bar{\varepsilon}}$, $\hat{\bar{\sigma}}$ are estimates of $\bar{\varepsilon}, \bar{\sigma}$, and

$$\operatorname{sgn}(S) = \begin{cases}1 & \text{if } S > 0\\ 0 & \text{if } S = 0\\ -1 & \text{if } S < 0.\end{cases} \quad (21)$$

Theorem 1. *Consider the nonlinear system described by* (4), *and suppose that Assumptions 1 and 2 are satisfied. The control law is provided by* (20), *and the parameters adaptation laws are given by*

$$\begin{aligned}\dot{\hat{\theta}}_f &= \gamma_f \Psi_f(\mathbf{x})\,S,\\ \dot{\hat{\theta}}_d &= \gamma_d \Xi_d(\mathbf{x})\,S,\\ \dot{\hat{\bar{\varepsilon}}} &= S\gamma_\varepsilon,\\ \dot{\hat{\bar{\sigma}}} &= S\gamma_\sigma.\end{aligned} \quad (22)$$

Then, the desired tracking performance can be achieved as S becomes asymptotically stable and all adaptation parameters remain bounded.

Proof. Taking into account the minimum approximation errors (18) and control law (20), the sliding surface (15) can be rewritten as

$$\dot{S} = y_d^n - \left(\left| F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) \right| + \left| d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) \right| + y_d^n + l + \mathbf{Ae} + \hat{\bar{\varepsilon}} + \hat{\bar{s}} \right) \operatorname{sgn}(S) - F(\mathbf{x}) - d(\mathbf{x}) + \mathbf{Ae}. \tag{23}$$

Defining the parameters errors $\tilde{\theta}_f = \hat{\theta}_f - \theta_f^*$, $\tilde{\theta}_d = \hat{\theta}_d - \theta_d^*$.

We choose the Lyapunov function candidate as follows:

$$V = \frac{1}{2}S^2 + \frac{1}{2\gamma_f}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{1}{2\gamma_d}\tilde{\theta}_d^T\tilde{\theta}_d + \frac{1}{2\gamma_e}\tilde{\bar{e}}^2 + \frac{1}{2\gamma_s}\tilde{\bar{s}}^2$$
$$\tilde{\bar{\varepsilon}} = \bar{\varepsilon} - \hat{\bar{\varepsilon}}, \qquad \tilde{\bar{\sigma}} = \bar{\sigma} - \hat{\bar{\sigma}}, \tag{24}$$

where γ_f, γ_d, γ_ε, and γ_σ are positive constants. The time derivative of (24) can be obtained as follows:

$$\begin{aligned}
\dot{V} &= S\dot{S} + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \frac{1}{\gamma_d}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d + \frac{1}{\gamma_\varepsilon}\tilde{\bar{\varepsilon}}\dot{\tilde{\bar{\varepsilon}}} + \frac{1}{\gamma_\sigma}\tilde{\bar{\sigma}}\dot{\tilde{\bar{\sigma}}} \\
&= S\left(y_d^n - F(\mathbf{x}) - \left(\left| F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) \right| + \left| d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) \right| + y_d^n + \lambda + \mathbf{Ae} + \hat{\bar{\varepsilon}} + \hat{\bar{\sigma}} \right) \operatorname{sgn}(S) - d(\mathbf{x}) + \mathbf{Ae} \right) \\
&\quad + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \frac{1}{\gamma_d}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d + \frac{1}{\gamma_\varepsilon}\tilde{\bar{\varepsilon}}\dot{\tilde{\bar{\varepsilon}}} + \frac{1}{\gamma_\sigma}\tilde{\bar{\sigma}}\dot{\tilde{\bar{\sigma}}} \\
&\le |S|\,|\mathbf{Ae}| + |S|\, y_d^n - S\left(\left| F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) \right| + \left| d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) \right| + \ddot{y}_d + \lambda + \mathbf{Ae} + \hat{\bar{\varepsilon}} + \hat{\bar{\sigma}} \right) \operatorname{sgn}(S) \\
&\quad - SF(\mathbf{x}) - Sd(\mathbf{x}) + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \frac{1}{\gamma_d}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d + \frac{1}{\gamma_\varepsilon}\tilde{\bar{\varepsilon}}\dot{\tilde{\bar{\varepsilon}}} + \frac{1}{\gamma_\sigma}\tilde{\bar{\sigma}}\dot{\tilde{\bar{\sigma}}} \\
&= -\lambda|S| - |S|\left| F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) \right| - |S|\left| d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) \right| - |S|\hat{\bar{\varepsilon}} \\
&\quad - |S|\hat{\bar{\sigma}} - SF(\mathbf{x}) - Sd(\mathbf{x}) \\
&\quad + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \frac{1}{\gamma_d}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d + \frac{1}{\gamma_\varepsilon}\tilde{\bar{\varepsilon}}\dot{\tilde{\bar{\varepsilon}}} + \frac{1}{\gamma_\sigma}\tilde{\bar{\sigma}}\dot{\tilde{\bar{\sigma}}} \\
&= -\lambda|S| - |S|\left| F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) \right| - |S|\left| d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) \right| - |S|\hat{\bar{\varepsilon}} \\
&\quad - |S|\hat{\bar{\sigma}} - S\left(F\left(\frac{\mathbf{x}}{\theta_f^*}\right) + \varepsilon \right) \\
&\quad - S\left(d\left(\frac{\mathbf{x}}{\theta_d^*}\right) + \sigma \right) + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \frac{1}{\gamma_d}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d \\
&\quad + \frac{1}{\gamma_\varepsilon}\tilde{\bar{\varepsilon}}\dot{\tilde{\bar{\varepsilon}}} + \frac{1}{\gamma_\sigma}\tilde{\bar{\sigma}}\dot{\tilde{\bar{\sigma}}} \\
&\le -\lambda|S| - |S|\left| F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) \right| - |S|\left| d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) \right| - |S|\hat{\bar{\varepsilon}} - |S|\hat{\bar{\sigma}} \\
&\quad + |S|\left| F\left(\frac{\mathbf{x}}{\hat{\theta}_f}\right) \right| - |S|\left| d\left(\frac{\mathbf{x}}{\hat{\theta}_d}\right) \right| \\
&\quad - S\tilde{\theta}_f\Psi(\mathbf{x}) - S\tilde{\theta}_d\Xi(\mathbf{x}) + |S|\bar{\varepsilon} + |S|\bar{\sigma} + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f \\
&\quad + \frac{1}{\gamma_d}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d + \frac{1}{\gamma_\varepsilon}\tilde{\bar{\varepsilon}}\dot{\tilde{\bar{\varepsilon}}} + \frac{1}{\gamma_\sigma}\tilde{\bar{\sigma}}\dot{\tilde{\bar{\sigma}}} \\
&= -\lambda|S| - S\tilde{\theta}_f\Psi(\mathbf{x}) - S\tilde{\theta}_d\Xi(\mathbf{x}) + |S|\tilde{\bar{\varepsilon}} + |S|\tilde{\bar{\sigma}} \\
&\quad + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \frac{1}{\gamma_d}\tilde{\theta}_d^T\dot{\tilde{\theta}}_d + \frac{1}{\gamma_\varepsilon}\tilde{\bar{\varepsilon}}\dot{\tilde{\bar{\varepsilon}}} + \frac{1}{\gamma_\sigma}\tilde{\bar{\sigma}}\dot{\tilde{\bar{\sigma}}} \\
&= -\lambda|S| + \tilde{\theta}_f^T\left(-S\Psi_f(\mathbf{x}) + \frac{1}{\gamma_f}\dot{\tilde{\theta}}_f \right) \\
&\quad + \tilde{\theta}_d^T\left(-S\Xi_d(\mathbf{x})\,u + \frac{1}{\gamma_d}\dot{\tilde{\theta}}_d \right) \\
&\quad + \tilde{\bar{\varepsilon}}\left(S + \frac{1}{\gamma_\varepsilon}\dot{\tilde{\bar{\varepsilon}}} \right) + \tilde{\bar{\sigma}}\left(S + \frac{1}{\gamma_d}\dot{\tilde{\bar{\sigma}}} \right).
\end{aligned} \tag{25}$$

Choosing a fuzzy rule adaptive method as

$$\begin{aligned}
\dot{\tilde{\theta}}_f &= \gamma_f\Psi_f(\mathbf{x})\,S, \\
\dot{\tilde{\theta}}_g &= \gamma_d\Xi_d(\mathbf{x})\,S, \\
\dot{\tilde{\bar{\varepsilon}}} &= -S\gamma_\varepsilon, \\
\dot{\tilde{\bar{\sigma}}} &= -S\gamma_\sigma
\end{aligned} \tag{26}$$

or equivalently, by definition

$$\dot{\hat{\theta}}_f = \gamma_f \Psi_f(\mathbf{x}) S,$$
$$\dot{\hat{\theta}}_g = \gamma_d \Xi_d(\mathbf{x}) S,$$
$$\dot{\hat{\bar{\varepsilon}}} = S\gamma_\varepsilon,$$
$$\dot{\hat{\bar{\sigma}}} = S\gamma_\sigma \quad (27)$$

yields

$$\dot{V} = -\lambda |S|. \quad (28)$$

Integrating both sides of (28), we have $\int_0^\infty \dot{V} dt \le -\int_0^\infty (1/2)|S| dt$, and thus, the following equation holds:

$$\int_0^\infty |S| \, dt \le 2(V(0) - V(\infty)). \quad (29)$$

As $V(0)$ is bounded and also $0 \le V(\infty) \le V(0)$ from (28), $\int_0^\infty |S| dt$ is also bounded from (29). Using Barbalat's lemma, [19, 34] $|S| \to 0$ for $t \to \infty$.

From the moment where the sliding surface is designed and constructed to be attractive, we can also see that $\lim_{t\to\infty} e = 0$. Therefore, the closed-loop system is asymptotically stable and the position tracking objective is achieved. The modified projection adaptive laws are given in [7]. □

5. Simulation Example

We illustrate the validity of the design approach by an example of robot arm tracking control with a single degree of freedom as Figure 1 shows.

The dynamic equations of such a system are given by

$$x^{(3)} = f(\mathbf{x}) + g(\mathbf{x}) u + d,$$
$$y = x, \quad (30)$$

where

$$f(\mathbf{x}) = -\frac{r}{L} x_3 - \left(\frac{g}{l} \cos(x_1) + \frac{K_b N^2 K_t}{Lml^2} \right) x_2 - \frac{rg}{Ll} \sin(x_1),$$
$$g(\mathbf{x}) = \frac{K_t N}{Lml^2},$$
$$\mathbf{x} = [x_1, x_2, x_3]^T, \quad (31)$$

where x_1 is the angular position (rad), x_2 is the angular velocity (rad/s), x_3 is the angular acceleration (rad/s^2), $u(t)$ is the applied force (control signal) (N), and d is the external disturbance. The simulation parameters are given in Table 1.

According to (30), we choose the sliding surface as $S = e^{(2)} + a_2 e^{(1)} + a_1 e$. The following parameters are chosen so that the characteristic function of the surface is the negative real part

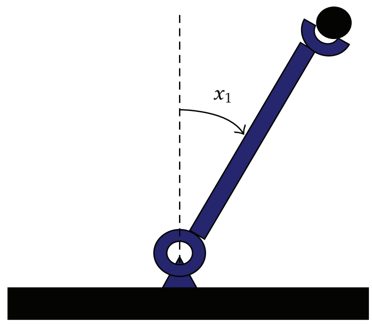

Figure 1: Robot arm.

Table 1: Simulation parameters.

Mass of the pole	$m = 5$ kg
The half-length of the pole	$l = 0.5$ m
The acceleration due to gravity	$G = 9.8$
Resistance	$r = 1.5$
Inductance	$L = 0.5$
Electromotive force constant	$K_b = 0.2$
Constant torque motor	$K_t = 0.3$
Reduction ratio	$N = 60$

$$M_1 = 8, \quad \eta_1 = 1, \quad M_2 = 15 \quad \eta_2 = 1. \quad (32)$$

To construct two fuzzy logic systems, $F(\mathbf{x}/\hat{\theta}_f)$ and $d(\mathbf{x}/\hat{\theta}_d)$ as given in (12), the initial consequent parameters of fuzzy rules are chosen randomly in the interval $[-1.2, 1.2]$. The initial values of $\mathbf{x}$ are given as $[0\ 0\ 0]^T$.

This interval will be sufficiently covered by three membership functions for position, velocity, and angular acceleration. Then, we have 27 rules.

Let the learning rate $\gamma_f = 0.05$, $\gamma_d = 20$, $\gamma_\varepsilon = 100$, $\gamma_\sigma = 500$, $k_d = 6$, and $M = 40$.

The control objective is to maintain the system to track the desired angle trajectory, $y_d = \sin(t)$, and to test the proposed control, we introduced parametric variations and external disturbances given by $\Delta m = 0.1 \sin(x)$, $d = 0.125 \times \sin(2t)$, respectively.

Figures 2–5 show the simulation results obtained in the case where the system is subjected to external disturbances and parametric variations. Figures 2, 3, and 4 show the rapid convergence of the system output to the reference signal. In Figure 5, we can see that the control signal is smooth and that the actual and desired trajectories are superposed, after a short transitional arrangement whereby the error is significant between the two outputs, this is due to disturbances, initial conditions, and initialization of adjustable parameters.

Figures 2–4 show that the effect of parametric perturbations is negligible, with less stress to the control level despite greater external disturbances. Similarly, the results obtained in [35] show that the tracking error is about 8% whereas it is less than 2, 5% in our case.

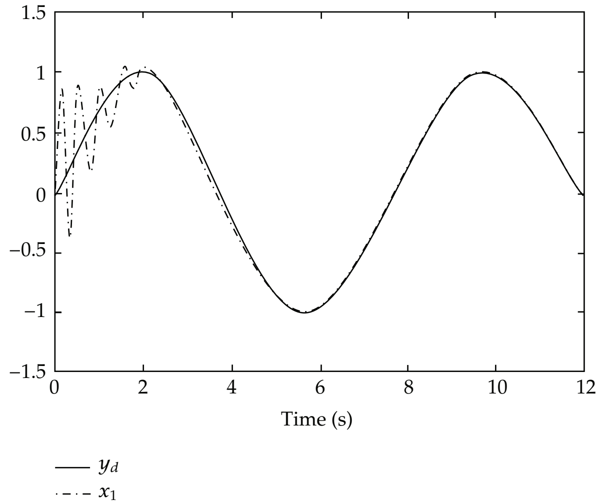

Figure 2: Trajectories of the state $x_1(t)$ of tracking control of the desired $y_d(t)$ for the robot arms.

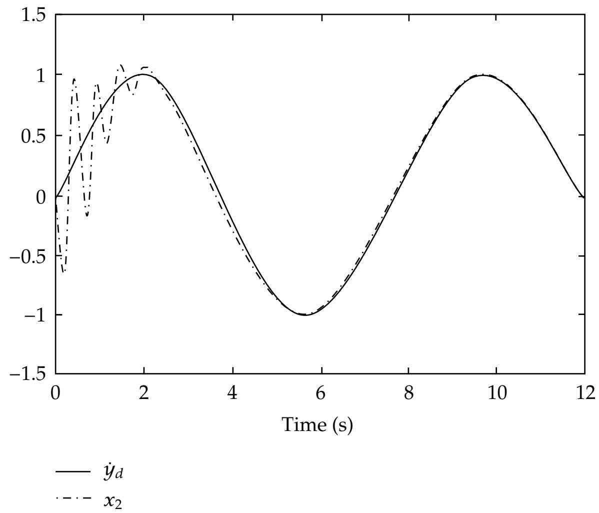

Figure 3: Trajectories of the state $x_2(t)$ of tracking control of the desired $\dot{y}_d(t)$ for the robot arm.

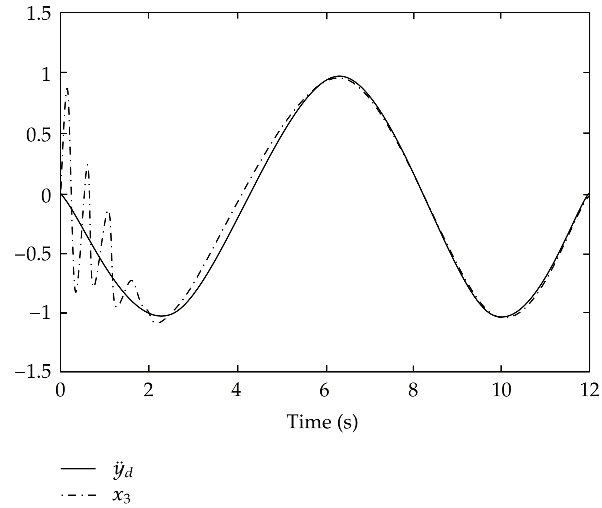

Figure 4: Trajectories of the state $x_3(t)$ of tracking control of the desired $\ddot{y}_d(t)$ for the robot arm.

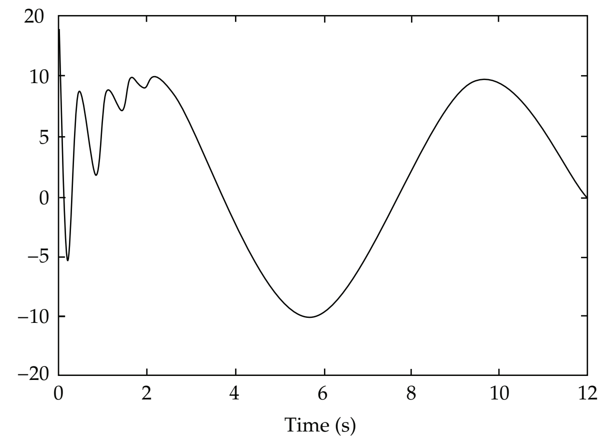

Figure 5: Trajectories of the control input $u(t)$ of the tracking control for robot arm.

It can be seen in Figures 2–5 that the advantage of our controller is its ability to eliminate the effect of fluctuations in the transient response with less effort on the control law; moreover, an estimation of the upper bound of error is performed without needing their prior knowledge, which allows the control law to be less restrictive regarding the conditions of stability.

6. Conclusion

In this paper, the output tracking control problem has been considered for a class of uncertain nonlinear systems. The unknown functions in systems are not linearly parameterized and have no a priori knowledge of the bounded functions. Fuzzy logic systems are used to approximate these unknown nonlinear functions. By sliding mode design technique, the adaptive fuzzy tracking control scheme has been developed for nonlinear systems. The proposed controllers guarantee that the outputs of the closed-loop system follow the reference signals, and achieve uniform ultimate boundedness of all the signals in the closed-loop system. It is proved in theory and shown in simulation that the closed-loop system is stable and the output tracks the given reference signal satisfactorily. Future work will deal with the delay systems in the type 2 fuzzy systems taking into account uncertainties and a novel nonlinearity sliding mode surface and an application to a real process.

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