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Sovereign Debt and Default: A Question of Ownership

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Preface

During the process of writing this thesis I have not worked in a vacuum and there are people that deserve credit for helping me finish this project. First of all I would like to give my supervisor, Professor Steinar Holden a big thank you for providing helpful feedback and leaving the door open for questions and discussion. I would also like to thank ESOP - Centre for the study of equality, social organization and performance for providing me with a scholarship and an office space while working on my thesis. The mentioned office also housed other talented young economists, thank you Kjersti, Eirik, Frikk and other visitors for a stimulating and entertaining work environment.

Summary

In this thesis I present a model featuring sovereign default in equilibrium. The government can choose a default rate between zero and one and the ownership structure of the debt matters for the decision. My main motivation is to propose an explanation of why the share of sovereign debt held by foreign investors has decreased in most countries in the European Monetary Union (EMU) since the beginning of the financial crisis. In the model decreasing foreign share is a rational response of expected utility maximizing investors when faced with increased risk of default. The model is also able to explain the negative correlation between bond yields and foreign share of bondholders found in the data. The empirical motivation for the thesis is the events in Portugal, Italy, Ireland, Greece and Spain (PIIGS), as those are the countries most affected by the crisis. From 2007 to 2011 the average foreign share of debt holdings in those countries decreased by 11 percentage points. The PIIGS countries also experienced the largest increases in interest rates on their sovereign debt. In light of the low and stable bond yields and increasing shares of foreign bond holders that characterized the beginning of the millennium, this is a dramatic change and an interesting topic for research.

Sovereign debt makes up almost one fifth of the world's financial assets and has become an important interest rate benchmark and investment option. Before the financial crisis the trade of European sovereign debt across boarders increased, probably because of increased financial integration. Government debt obligations issued by advanced countries were viewed as a safe investment and credit ratings were strong. A lot has changed since 2007. European sovereign debt markets have been in turmoil for most of the time between 2008 and 2014. Yield spreads have been huge for many countries and credit ratings are poor. Debt obligations have changed hands at a rapid rate and are increasingly sold back to investors in the issuing country. It is this last development that is the main focus of my thesis. Little is known about the determinants of - and impact on - the ownership structure of sovereign debt. Some new empirical work show some facts and trends, but little theoretical work has been done on the subject.

I suggest that the reduction in the share of foreign investors holding sovereign debt is a rational response to sovereign risk. The reduction is analyzed as a home bias in sovereign bonds only present when there is risk of default. Much of the home bias literature introduces trade costs or other market frictions to explain observations in the data. My home bias mechanism is not dependent on market failure or frictions, but is only applicable on sovereign debt. I build my model following a long tradition of modeling sovereign debt

as a non-enforceable asset with penalties for defaulting. My model differs from the main literature in that it focuses on the ownership structure of the bonds. It also allows for partial default, where most models focus on the binary choice of default or full repayment.

The model contains foreign and domestic investors in addition to a domestic government that issues debt. The home and foreign investors act as expected utility maximizers. All agents live for two periods and the second period income for the home investor is uncertain. The government's debt is not enforceable and creditors risk default in equilibrium. The interest rate on government bonds is determined in the market and depends on expectations about government behavior. Debt is repaid with tax revenue in the second period and the government chooses the default rate by maximizing the utility of their population. Domestic investors incur a loss if the government chooses to default, the size of the loss depends on the default rate and the realization income. The decision is thus a trade-of between tax-gains and default-penalties. My analysis consists of varying the risk level by changing the income distribution and debt level and then comparing the results. Increased risk leads to higher interest rates and lower share of foreign bond holdings. Specifically I find that decreased expected GDP or increased debt has about the same effect. Both increase interest rates and decrease the foreign share by about the same magnitude. Increased probability of recession on the other hand has a much larger effect on interest rates and less effect on the foreign share than the first two.

The mechanism at work is related to the government's default decision. The government finances repayment of the bond by taxing their inhabitants. Domestic investors observe that they get a lower tax in case of default and is thus partly "insured" against default risk. This effect is increasing in the risk level and induces domestic investors to take advantage of the increased interest rate to buy more of the bond. Foreign investors have income that is uncorrelated with the return on sovereign debt and thus only care about expected return and the risk level. The decline in foreign bond holdings is thus a result of increased risk. Following directly from this is also the proposal that there is no causal relationship between yields and the foreign share, both have a common driver; the default risk. I also propose mechanisms that might have contributed to the outbreak of the sovereign debt crisis. The model can describe a situation with multiple equilibria where the good, low risk, equilibrium is fragile both to expectations and the share of foreign bond holders. Expectations of default could increase interest rates and make default optimal for the government. Similarly, an increase in foreign bond holders makes default more likely, because the government only cares about domestic investors.

Solution of the model and the analysis is carried out in Matlab.

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1 Introduction

Recent data by Merler and Pisani-Ferry (2012b) indicate that the share of sovereign debt held by foreign investors in several European countries has been falling since the financial crisis in 2007. In this thesis I propose that the decrease in the foreign share of bond holdings is the result of increased default risk. I analyze a rational expectations model with the possibility of sovereign default in equilibrium and find that the decrease could be viewed as a rational response to default risk. I solve this model by the $fsolve$ function in Matlab. The government finances repayment of their outstanding debt by taxing their inhabitants. Domestic investors are thus partly insured against default as default leads to a lower tax burden. They exploit this to increase their holdings of sovereign debt when risk increases. The model also enables me to answer the question of why there is a strong negative correlation between sovereign bond yields and foreign share of bond holdings by proposing that default risk causes both high yields and low foreign share. Sovereign debt has not been studied much in the literature on asset ownership and home bias in the past. Sovereign debt is a large and important asset class in international financial markets. Tomz and Wright (2013) estimate that in 2010 sovereign debt made up about 19% of global financial assets. This, combined with the fact that not even the debt of advanced countries debt can be trusted as risk free, makes the study of this asset class a hot topic for research and I hope this thesis can contribute to the understanding of sovereign debt.

2 Background

The inspiration for the project comes from events in European sovereign debt markets during and after the financial crisis. The Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012b) brings forth an interesting topic, namely the role of the ownership structure of sovereign debt. The dataset contains information on the ownership of the sovereign debt of 10 countries in the European Monetary Union (EMU) and additionally the USA and UK. Data for all the EMU countries is available for 2002-2012, In the data presented later I drop the last year because of a break in the Greek series. It decomposes debt by ownership in several domestic and one foreign category. This kind of dataset is new and reveals several interesting topics of research, notably the question why sovereign debt ownership has changed in later years. A similar dataset was developed by Andritzky (2012), which includes fewer EMU countries, but also includes data for other G20 countries. The main features presented are similar in the two datasets.

A secondary source of interest is the negative correlation between bond yields and the share of foreign investors holding the debt. Andritzky (2012) establishes the relationship between 10 year government bond yields and the share of foreign holdings. He finds a strong negative correlation between government bond yields and the share of foreign holders. He suggests that it is the low and stable yields that attract foreign investors, but is unable to conclude.

2.1 European sovereign debt

The main empirical focus in this thesis is on European sovereign debt. In the next subsections I present an overview of previous and current bond yields and the ownership structure of European government debt in later years. I focus on recent trends and shifts in those trends around two specific events; the introduction of the euro and the recent financial crisis. The trends are apparent for most EMU-countries, but strongest in Portugal, Italy, Ireland, Greece and Spain (PIIGS). The PIIGS countries experienced the highest interest rates and the biggest shifts in the ownership structure of government debt. The model I present later on propose explanations for those shifts and will be discussed in relation to the PIIGS countries in section 6. The background given here serves both as motivation for studying sovereign debt ownership and as a reference for later discussion and comparison to the model.

2.1.1 1999-2007

The introduction of the euro in 1999 had major effects on the involved countries. Faruqee (2004) estimates a 10% increase in trade between member countries in the first three years. Lane (2006) show that trade has increased both within the euro-zone and between EMU-members and the rest of the world. There is also much evidence in support of the euro boosting foreign direct investment and the trade of financial assets across borders. One example of this is found in Lane (2006), where he investigates the share of European countries international portfolio holdings allocated to Euro-zone partners and show increases in the share held by Eurozone-partners for almost all countries. Sovereign bonds was flowing across national borders and every country in the Bruegel dataset experience an increase in the share of foreign holdings of their sovereign debt from the introduction of the euro in January 1999 and well in to the new decade. On average we see foreign

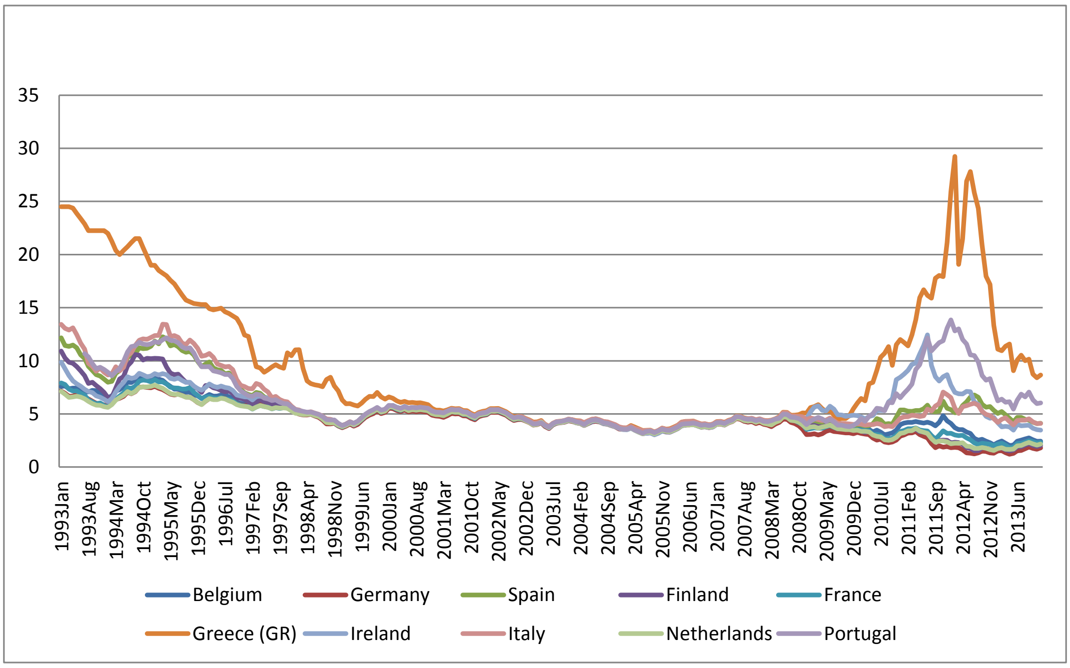


Figure 1: 10 year European bond yields 1993-2013

holdings increase from 1999 to 2007 by around 26 percentage points in the ten countries in the dataset while the median 22 percentage points[1]. The data is presented in table 1. Andritzky (2012) argues that the increased financial integration and regulatory changes fueled the large increases in foreign holdings of sovereign debt.

The interest rates in the EMU have followed an interesting pattern in later years. In figure 1 I plot interest rates on long term sovereign bonds for the same countries discussed above. With the introduction of the single currency exchange rate risk was eliminated between EMU-countries. Following this reduction in risk it was expected that interest rates would converge, and they did. The period was also characterized by an unprecedented stability of these rates. From 1999 and throughout 2007 the interest rates hardly moved outside the 4-5 % range as seen in figure 1. Cappiello et al. (2006) confirm the convergence in yields by showing that correlation coefficients between European sovereign bonds rose to almost 1 after the introduction of the monetary union and the euro.

[1]For these and later calculations I have used non-weighted averages and inserted most recent (2001,2000 and 2005) data for the countries without 1999 data.

Country\Year	1999	2007	2011
Greece	31.6%	73.8%	59.2%
Ireland	56.9%*	93.1 %	77.9%
Portugal	54.2%*	75.8%	67.0 %
Italy	29.9%	49.1 %	45.0 %
Spain	28.8%	47.7%	36.2%
Germany	34.4%	49.5%	55.8%
France	13.0%	55.0%	57.0%
Netherlands	65.9%*	69.5 %	64.1 %
Belgium	29.8%	59.7%	48.6%
Finland	49.5%	84.9%	88.9%

Table 1: Non-resident holdings of Sovereign debt. *Values from 1999 not available, the displayed numbers are from 2001, 2000 and 2005 respectively.

2.1.2 2007-2011

The most interesting trend shifts happened with the financial unrest in 2007 and the breakdown that followed in 2008. As seen in table 1 the share of foreign holdings of sovereign debt fell in most countries from 2007 to 2011. On average the share dropped just short of 6 percentage points and the median drop was around 7 percentage points. The only countries with increasing foreign share were Germany, France and Finland, but the increases were small. Looking at the most troubled economies, PIIGS, both the average and median drop in foreign share was 11 percentage points. We observe that the same countries experiencing the largest drop in the foreign share also had the largest increases in interest rates and the only countries with growing shares had low and stable rates. More formal evidence on the trend-break is presented in Merler and Pisani-Ferry (2012a) who use a sudden stop approach in identifying episodes of large decreases in the inflow of foreign capital. They identify several such episodes for all the PIIGS countries from 2007 to 2011. The previously low bond yield differentials also changed dramatically. Yields in the stable countries, namely Germany, Netherlands, France and Belgium fell and interest rates in the PIIGS countries rose. The effect seen in figure 1 is dramatic. While many, e.g. Attinasi et al. (2009), Barrios et al. (2009) and De Santis (2012), have proposed different explanations for the high yield differentials the question of why the foreign share fell remains largely unanswered. In this thesis I propose a possible mechanism.

Country	Correlation
Portugal	-0.44
Italy	-0.28
Ireland	-0.04
Greece	0.09
Spain	-0.63

Table 2: Correlation between interest rates and foreign shares of debt holdings in the PIIGS-countries between 2002 and 2011.

2.2 Investor base and Yields

Andritzky (2012) conclude that there is a strong and robust association between lower yields and larger shares of non-resident holdings of government bonds. Using the same data as in figure 1 and table 1 I calculate the correlation between interest rates and the foreign share on data from 2002 to 2011[2] for the PIIGS-countries. I find similar results for the PIIGS countries as the conclusion from Andritzky. The results are displayed in table 2. Portugal, Italy, Ireland and Spain all have negative correlation coefficients between interest rates and foreign share between January 2002 and December 2011. Greece has the only positive coefficient of 0.09. This is because there is a break in the trends around 2007, if we split the series in two parts we get coefficients of -0.39 for 2002-2006 and -0.97 for 2007-2011. The same pattern is apparent also for the other PIIGS-countries.

Andritzky (2012) examines which way the causation goes between them. He finds some evidence for the argument that low yields attract foreign investors, but remains hesitant to conclude. In this thesis I argue that there is no clear causation, but rather that they are both caused by a common factor, the risk of default.

3 Literature review

This thesis argues that the outflow of foreign investors from many European countries over the last years is the result of a rational home bias only present when there is default risk present. While I find few references on home bias in sovereign debt in the literature, there are some topics that are connected. I find it useful to examine other views on home

[2] I use monthly data for interest rates. For the foreign share I have quarterly data, except for Portugal where the data is annual.

bias in the literature, keeping in mind that most of those are written on equities or bonds in general. My model also builds on a tradition on how to model sovereign debt, so a short review of the sovereign debt literature is also included.

3.1 Home bias

Home bias in assets have been a hot topic for research since French and Poterba (1991) documented the inefficiently low levels of diversification and Tesar and Werner (1995) showed a significant home bias and high turnover in equity holdings. The fact that investors worldwide seem to prefer investments at home over better hedging opportunities abroad has fascinated economists for some time. In their widely cited paper The Six Major Puzzles in International Macroeconomics Obstfeld and Rogoff (2001) list home bias in portfolios as one of their six puzzles. In their paper Tesar and Werner pointed to tax wedges or transaction costs as possible explanations of the home bias before proceeding to exclude transaction costs as a possible explanation due to the high turnover rate. Their results have later been revisited, notably by Warnock (2002) who rejects their results that the turnover is higher on foreign assets than domestic assets. Warnock does, however, confirm the result that transaction costs are not the cause of the observed home bias.

Obstfeld and Rogoff (2001) offer the explanation that home bias is partly explained by trade costs, not in assets, but in consumption goods, a view later rejected by, among others, Coeurdacier (2009). Another explanation of the home bias puzzle is information asymmetries and investment advantages. When domestic investors have informational advantages in their home market, their portfolios should be skewed towards domestic assets. A quite recent contribution into this strand of the literature is Van Nieuwerburgh and Veldkamp (2009). The importance of international diversification for hedging against fluctuations in income from human capital was raised by Baxter et al. (1998). The idea being that returns on domestic assets are closely correlated with wages. This meant the home bias puzzle was even more severe than what was believed earlier. This argument was turned around by Heathcote and Perri (2013), building on empirical results from Julliard (2003). Heathcote and Perri argue that investing in domestic assets could be a good hedge against labor income fluctuations. The arguments focusing on diversification for consumption smoothing are the ones most in line with the argument in my thesis as I argue that government debt might be a good hedge against tax fluctuations at home for domestic investors.

3.2 Sovereign debt

Modeling sovereign debt as a contract with limited enforcement is a tradition that goes back to Debt with Potential Repudiation by Eaton and Gersovitz (1981). A central element in the modeling of sovereign debt is that governments cannot commit to repay their debt and a mechanism is needed to keep them from defaulting each period. In their paper Eaton and Gersovitz model a world where governments are excluded from bond markets if they default. The mechanism has been modeled in different ways, such as ad hoc default penalties, endogenous penalties and market exclusion. A recent review on the literature on sovereign debt can be found in Aguiar and Amador (2013). My model consists of only two periods, so I model the penalty of default as cash penalties more along the lines of Cole and Kehoe (2000) and Niepelt and Dellas (2013).

In a recent paper Niepelt and Dellas (2013) model a situation where the composition of creditors matters for the default choice. They arrive at a model where official lending by institutions like the IMF or governments discipline the indebted government because they have greater means to penalize a defaulting government. In this paper I model a similar environment where the composition of foreign versus domestic creditors matter for the default decision. I do not require asymmetries in the ability to penalize, but exploit the assumption that governments care about the welfare of the population. My model is simpler in many ways, but goes further in modeling the default decision. Rather than focusing only on the binary choice between full repayment and full default I allow for partial default and model the continuous choice of default rate.

4 The model

4.1 Model environment

The model consists of a home country and the rest of the world and ends after two periods. The home country is inhabited by many investors that consume in both periods and make investment decisions in the first period to save for the second. They choose to save in domestic government bonds or in the world market. The rest of the world is inhabited by a group of investors with access to the market for government debt in addition many other agents without access to the market for sovereign debt. Both the home investors and foreign investors with access to the sovereign debt market are small groups compared to the rest of the world. The preferences of all agents can be represented by a utility function satisfying Gorman's aggregation theorem (Gorman, 1953), so both investor groups can be represented by a representative agent. The size of the population is the same in both periods. The Home country also has a government that finances public expenditure by taxation and debt. The foreign government is passive and only supply government provisions by lump sum tax. There are international markets for both the consumption good and financial assets. Prices on consumption goods are assumed constant and all interest rates are denoted in terms of the consumption good.

4.1.1 Timing

In the first period agents, both home and foreign, make their consumption and investment decisions based on their expectations about the second period. The interest rate on sovereign bonds is determined in the market for sovereign debt. This market is supplied by an exogenous amount of zero coupon debt obligations B, which correspond to the home governments' financing needs. The investors can perfectly predict how the government will act in all possible scenarios in period two, but as the world is inhabited by many agents their individual impact on the decisions made are negligible. All income is exogenous and there are no stochastic elements in the first period. At the beginning of the second period the outcome of the stochastic endowment is drawn. The government observes the realized endowment and chooses its tax and default rate. Agents receive their income and debt repayment, pay their taxes, consume and die.

4.1.2 Income and budget constraints

At the beginning of the first period both investors have a given wealth. Investors total wealth consists of all assets held in addition to their period one income less taxes payed in period one. Wealth is denoted I_1 for the Home investor and I_1^* for the Foreign investor. This wealth is used for consumption in period one and investment in sovereign debt at interest rate r. Both investor groups are assumed to be wealthy enough to hold the entire amount of debt if they spend their entire wealth, $I_1 > B$ and $I_1^* > B$. Agents are also able to lend or borrow at the world market with an exogenous interest rate r_R. This market coordinates the supply and demand of the rest of the world's inhabitants. Government bonds bought by home investors are denoted a and their position in the risk free asset is denoted a_R, conversely foreign investors holdings are denoted a^* and a_R^*. Their period one budget constraint is thus given by (1) for the home investor and (2) for the foreign.

$$c_1 + a + a_R = I_1 \tag{1}$$

$$c_1^* + a^* + a_R^* = I_1^* \tag{2}$$

In the second period, consumption is determined by the return on what agents saved from the first period in addition to a new endowment y^i for the home investor and y^* for the foreign. The second period endowment, interpreted as GDP, at home is uncertain and has a discrete distribution with the probability density function (PDF) $f(y^i) = P(y = y^i)$, $\sum_{i=1}^{n} f(y^i) = 1$ for n possible realizations. The return on the government bond will also depend on the realization of y^i through the actions of the home government. The second period budget constraint for each state of the world is described in (3) for home and (4) for foreign. Consumption is denoted c with the subscript denoting period 1 and 2, asterisk superscript denotes foreign consumption. The default rate for state i is denoted d^i and τ^i is the tax rate. $L(d^i, y^i)$ denotes the loss incurred by the home investor if it's government defaults in state i and is explained below.

$$c_2^i = y^i - \tau^i + (1 - d^i)(1 + r)a + (1 + r_R)a_R - L(d^i, y^i) \tag{3}$$

$$c_2^{*,i} = y^* - \tau^* + (1 - d^i)(1 + r)a^* + (1 + r_R)a_R^* \tag{4}$$

4.1.3 Preferences

The representative agents home and abroad are assumed to be identical. They only care about consumption. Public provisions are assumed not to affect agents' utility and are left out of the model. The representative agents discount the second period by β and we assume $0 \leq \beta \leq 1$. In making decisions agents maximize expected utility over the two periods, solving problem (5) and (6).

$$\begin{aligned} \max_{c_1, a, a_R} \quad & U = E[u(c_1) + \beta u(c_2^i)] \\ st. \quad & (1) \quad and \quad (3) \end{aligned} \tag{5}$$

$$\begin{aligned} \max_{c_1^*, a^*, a_R^*} \quad & U^* = E[u(c_1^*) + \beta u(c_2^{*,i})] \\ st. \quad & (2) \quad and \quad (4) \end{aligned} \tag{6}$$

4.1.4 Markets

There are three markets in this economy but only two is determined within the model. The world market for assets has an interest rate determined outside the model and need no market clearing condition. In (7) we impose market clearing in the market for government bonds and the market for consumption goods will clear by Walras' law.

$$B = a + a^* \tag{7}$$

4.1.5 Loss in case of default

The model is partly built on the tradition of debt with potential repudiation following Eaton and Gersovitz's (1981) seminal paper. The government may default on all, or a part of, its debt. To keep the government from doing so consistently creditors must have some means to punish the government if they default. I assume that the creditors are able to inflict a cost on the home country if the government chooses default. In this model the loss in case of default is taken by the inhabitants of the home country, it hurts the government indirectly as they care about the welfare of their citizens. The cost represents any penalty or sanction imposed by the international community and is denoted by $L(d^i, y^i)$. The specific loss function used here is specified in (8) and capture some key elements in the model.

$$L(d^i, y^i) = \begin{cases} 0 & \text{if } d^i = 0 \\ k + hy^i(d^i)^2 & \text{if } d > 0 \qquad k > 0, h > 0 \end{cases} \tag{8}$$

First, there is a fixed cost, k, associated with default. This seems reasonable since the signal effect of even a slight move away from full repayment would imply a large cost. It would create big reactions in markets and also trigger credit default swaps and other contracts. The fixed cost also makes the model applicable in a situation where both the default rate and the binary choice of default versus no default can be endogenized. Second, the loss depends positively on the rate of default and the realization of GDP. The positive dependence on GDP is motivated by both the creditors' opportunity and the creditors' willingness to impose sanctions. Higher GDP potentially makes sanctions more painful because you have more to lose. Creditors might also go easier on a country forced into default by low GDP than a country choosing default even with high GDP. The positive dependence on the square of d^i enables the model to incorporate partial default.

4.2 Government decision and default mechanism

The only risk factor investors face in the asset markets is that the home government might not repay all of the debt. The government choose a default rate, d^i for each state i. This yields a repayment rate of $(1 - d^i)$, giving investors a gross return on the bond of $(1 + r)(1 - d^i)$. The outcome of d^i is chosen by the government after observing the realized endowment. We assume that the government cannot discriminate between home and foreign investors. This assumption can be viewed as an institutional constraint based on very large penalties for default discrimination. The home government finance the repayment by taxing their inhabitants a lump sum τ^i. The government's budget constraint in the second period is given by (9).

$$\tau^i = (1 - d^i)(1 + r)B \tag{9}$$

The government maximizes their inhabitants utility subject to this constraint by choosing the default rate. The choice of d affect agents in tree ways; first default will reduce the actual payoff to whoever holds the bonds, secondly default triggers a loss for the inhabitants of the home country and thirdly it will imply a reduction in the tax-burden of the home inhabitants.

In period two the agents consume their entire second period income. Since the government only affects their inhabitant's income directly in the second period this means that maximizing utility translates into maximizing second period total income. The government acts after y^i is realized so the government solution will specify a default rate corresponding to all possible outcomes of y^i. The governments optimization problem for each state i is specified in (10).

$$\begin{aligned} \max_{\tau^i, d^i} \quad & I_2^i = y^i - \tau^i + (1 - d^i)(1 + r)a + (1 + r_R)a_R - L(d^i, y^i) \\ st. \quad & \tau^i = (1 - d^i)(1 + r)B \\ & 0 \leq d^i \leq 1 \end{aligned} \tag{10}$$

The solution to the problem would be a first order condition defining the interior solution, $d^{i,d}$, given in (11), and endpoint conditions given in (12) and (13) where f denotes the foreign share of bond holdings, $f = \frac{a^*}{B}$.

$$d^{i,d} = \frac{f(1 + r)B}{2hy^i} \tag{11}$$

$$d^i = 0 \quad \text{if} \quad y^i \geq \frac{[f(1 + r)B]^2}{4kh} \tag{12}$$

$$d^i = 1 \quad \text{if} \quad d^{i,d} > 1 \tag{13}$$

For every possible realization of y^i this yields a default rate, d^i, and taxes, $\tau^i = (1 - d^i)(1 + r)B$.

4.3 Further specification of the model

To simplify my analysis I limit the possible outcomes of the second period endowment to two values, y^L or y^H, and let p denote the probability of the low outcome, $0 \leq p \leq 1$.

Superscript L denotes the bad realization and H the good, $y^L < y^H$. In this case the households problems are represented by (14) and (15).

$$\begin{aligned} \max_{c_1, a, a_R} \quad & U = u(c_1) + p\beta u(c_2^L) + (1-p)\beta u(c_2^H) \\ st. \quad & (1) \quad and \quad (3) \end{aligned} \tag{14}$$

$$\begin{aligned} \max_{c_1^*, a^*, a_R^*} \quad & U^* = u(c_1^*) + p\beta u(c_2^{*,L}) + (1-p)\beta u(c_2^{*,H}) \\ st. \quad & (2) \quad and \quad (4) \end{aligned} \tag{15}$$

The solutions are given by the first order conditions in (16) and (17) for the home investor and in (18) and (19) for the foreign investor in addition to their budget constraints.

$$u'(c_1) = (1-p)\beta(1+r)u'(c_2^H)(1-d^H) + p\beta(1+r)(1-d^L)u'(c_2^L) \tag{16}$$

$$u'(c_1) = (1-p)\beta(1+r_R)u'(c_2^H) + p\beta(1+r_R)u'(c_2^L) \tag{17}$$

$$u'(c_1^*) = (1-p)\beta(1+r)u'(c_2^{*,H})(1-d^H) + p\beta(1+r)(1-d^L)u'(c_2^{*,L}) \tag{18}$$

$$u'(c_1^*) = (1-p)\beta(1+r_R)u'(c_2^{*,H}) + p\beta(1+r_R)u'(c_2^{*,L}) \tag{19}$$

I also specify utility to be of the constant relative risk aversion (CRRA) form, $u(c) = \frac{c^{1-\theta}}{1-\theta}$ for $\theta \neq 1$ and $u(c) = log(c)$ for $\theta = 1$, where θ can be interpreted as the coefficient of relative risk aversion. First order conditions with CRRA utility are shown in (20) and (21) for the home investor and in (22) and (23) for the foreign investor.

$$\frac{1}{1+r} = \beta c_1 \left(\frac{(1-p)(1-d^H)}{(c_2^H)^\theta} + \frac{(1-p)(1-d^L)}{(c_2^L)^\theta} \right) \tag{20}$$

$$\frac{1}{1+r_R} = \beta c_1 \left(\frac{1-p}{(c_2^H)^\theta} + \frac{p}{(c_2^L)^\theta} \right) \tag{21}$$

$$\frac{1}{1+r} = \beta c_1^* \left(\frac{(1-p)(1-d^H)}{(c_2^{*,H})^\theta} + \frac{p(1-d^L)}{(c_2^{*,L})^\theta} \right) \tag{22}$$

$$\frac{1}{1+r_R} = \beta c_1^* \left(\frac{1-p}{(c_2^{*,H})^\theta} + \frac{p}{(c_2^{*,L})^\theta} \right) \tag{23}$$

4.4 Equilibria

An equilibrium in this model are values of c_1, c_1^*, a, a_R, a^*, a_R^*, r, c_2^H, c_2^L, $c_2^{H,*}$, $c_2^{L,*}$ and d^i that satisfy equations (1), (2), $(3)_{i=H}$, $(3)_{i=L}$, $(4)_{i=H}$, $(4)_{i=L}$, (7), (20), (21), (22) and (23) in addition to the government decision rules, (11), (12) and (13). (11), (12) and (13) determine d^H and d^L, so the system determines 13 variables. An equilibrium must also satisfy the condition that the outcome is consistent with rational agents. The interest rate resulting from a certain belief about the default rate must lead to the same default rate.

The model can contain multiple equilibria depending on the values of the endowment distribution and the debt. Different expectations about default can lead to different equilibria. Expectations can be self-fulfilling in the sense that expectations of default drive up interest rates leading to default being the optimal choice for the government. The model contains three possible equilibria; 1) no default equilibrium where agents expect full repayment, 2) low default equilibrium where agents expect default in the low realization of the endowment and 3) full default equilibrium where agents expect default in both realizations of the endowment. This is discussed in further detail in section 5.2.

There is one equilibrium that is not fully determined by the model, namely the equilibrium where investors rationally expects full repayment. Here the interest rate will equal the risk-free rate and we are unable to determine who buys the bond, a and a^*. When there is no risk of the government defaulting both assets are risk free and both investors are indifferent between investing in the sovereign bond or the asset on the world market. Since the debt obligations are in exogenous supply we know that everything will be sold, but not who buys them. See section 5.2 for further discussion.

4.5 Parameterization

When parameterizing the model I set the deep parameters in accordance with existing literature and new parameters to get sensible results for the default rates and interest rates. I set the coefficient of relative risk aversion, θ, to 1, so I use log utility. The risk-free interest rate is set to 3%, $r_R = 0.03$ and the discount rate for both agents is set in accordance with that rate, $\beta = \frac{1}{1+r_R}$. The parameters of the loss function are chosen so that the resulting default decisions are within reasonable limits, $h = 1$ and $k = 0.15$. All these primary parameters stay constant throughout the thesis. The rest of the variables are subject to change during the analysis, but unless other values are declared the following values are the ones used. The second period endowment is normalized to one, $y^H = 1$ for the high outcome and the low outcome is set to 0.8, $y^L = 0.8$. The probability of the low outcome is set to 20%, $p = 0.2$. The second period endowment of the foreign investor is certain and equal to one, $y^* = 1$. Government debt is set to 150% of GDP, $B = 1.5$. Period one wealth is set to two for both agents, $I_1 = I_1^* = 2$. The exogenous tax the foreign agent has to pay in period two is 0.5, $\tau^* = 0.5$.

5 Analysis

In the previous section I developed a model that contributes to the understanding of what happened to the ownership structure of European sovereign debt in the period from 2007 to 2011. In this section I display and explain the predictions of the model. One main result is that a reduction in the share of foreign investors is a rational response of expected utility maximizers when home investors must repay their government's debt through taxes. This is due to consumption smoothing over the different states in the second period. Default reduces the value of bond holdings for both the foreign and domestic investor. The domestic investor is also penalized for his government's actions, but is compensated by a reduction in taxes. The government will in fact only default if the net effect to its citizens is positive. The fact that tax expenses and bond earnings are positively correlated makes government bonds a good hedging opportunity for the home investor. He realize that a loss on the investment will be compensated by a reduction in taxes. This effect will affect portfolio choice as long as there is some default risk and become stronger when risk increases. This implies that when default risk increases the interest rate increases and the foreign share of bond holdings decreases.

Domestic investors increase their holdings of the sovereign debt when faced with default risk because the return on the debt is negatively correlated with their other income. The fact that default usually happen when the realization of y^i is low may complicate this result. If default occurs when output is very low the correlation may be reversed together with the predictions of the model. The model is not sensitive to this possible complication and the bad realization of the endowment would have to be *very* low for the results to change.

The analysis is conducted by changing the level of default risk. In equation (11), from the government's solution, we see that the default rate is decreasing in GDP and increasing in total debt issued. Thus I change default risk by changing the income distribution and the level of debt, that is values for y^H, y^L, p and B. Throughout the main part of my analysis the focus will be on equilibria where the government defaults only in the bad realization of the endowment, but for some combinations of parameters more than one equilibrium may exist. These cases will be discussed in section 5.2. In section 5.1.1 to 5.1.4 I consider cases where the high outcome of GDP always results in full repayment and the government defaults only in the low outcome. Increased default risk is thus captured by an increased default rate in the bad outcome of GDP.

5.1 Analysis on the foreign share for different levels of risk

In this section I focus only on the equilibrium where investors expect default in bad times and full repayment in good times. I model the increased default risk in different ways. First I analyze changes in the expectations about second period GDP, second I analyze changes in both expectations and the period one realization of income, third I analyze what the model predicts for different levels of government debt and finally I include scenarios where the high and low outcomes remains the same, but the probability of a low outcome increases.

5.1.1 Different expectations about GDP

In this section I consider the effects of variation in the expectations of GDP by changing y^H and keeping $y^L = 0.8y^H$. The results can be seen in figure 2, where the x-axis is the expectation of second period GDP, the dotted line is the foreign share of holdings and the solid line the interest rate on the bond. The x-axis here is reversed such that default risk increase when you move right in the figure. All figures in the analysis chapter display the outcome with the highest risk at the far right of the x-axis. With lower expected second period endowment interest rates rise and the share of foreign holders of the sovereign bond fall. The models explanation is that when expectations about GDP fall, the expected default rate in case of the bad scenario increase. The increased default rate makes the bond less attractive for both investors, so the interest rate increases. To understand why the foreign share decrease we must understand what happens to the investors' demand for the bond. The foreign investor observes a decrease in expected return on the bond so he demands higher interest rates. The domestic investor experience the same decrease in expected return, but he knows that in case of default taxes would decrease as well. Low repayment on the bond thus corresponds to higher values of his other disposable income. His loss of utility is thus lower than for the foreign investor. The home investors then take advantage of the higher interest rates to acquire more of the bond.

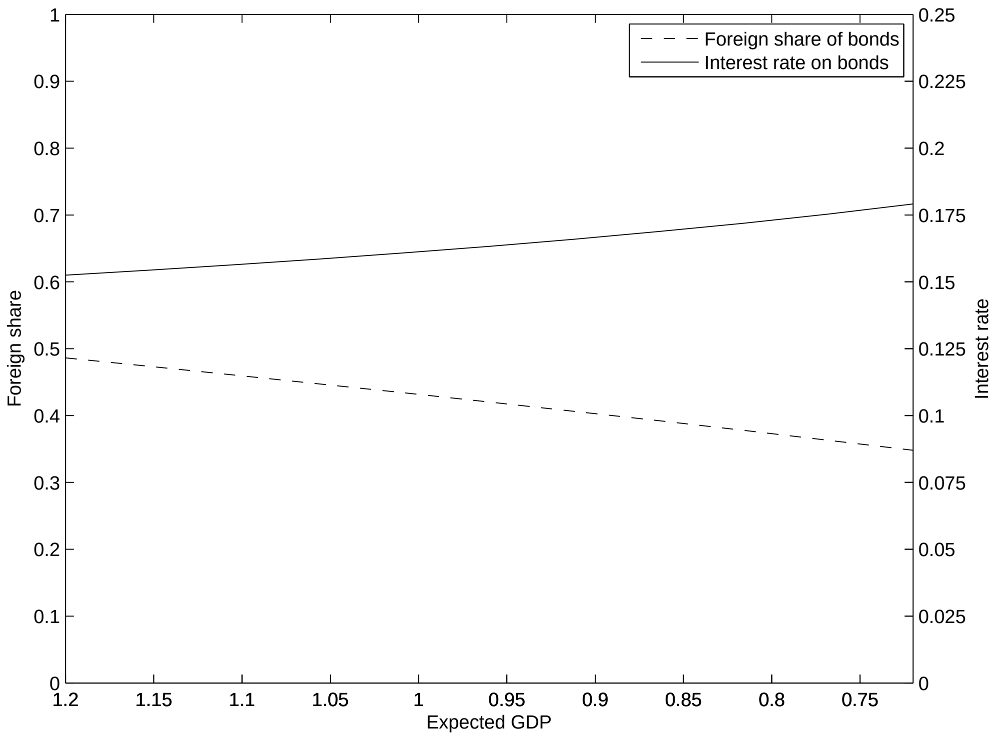


Figure 2: Interest rates and foreign shares for different values of expected GDP. High values of expected GDP leads to high shares of foreign bond holdings and low interest rates. With lower levels of expected GDP interest rates are higher and domestic investors buy more of the bond.

5.1.2 Different period one income and expectations about GDP

The above result shows what happens if only the second period distribution changes. In this section I let period one income for the home investor change together with the expectation of period two income I let $I_1 = y^H + 1$ and vary y^H in the same way I did in the previous section. This could be interpreted as a situation where the outcome tomorrow is correlated with the realization of income today. As seen in figure 3 the results are qualitatively the same, but the magnitudes are different. The effect on interest rates is stronger than before and the effect on foreign share of bond holdings is weaker. The key to understanding this result is related to effective demand. The home investor has a stronger incentive to invest in the home bond than the foreign investor when the default risk is higher. In this set-up, where income in period one and two are correlated, the strongest incentive for home investors to purchase home bonds coincide with a low income and less means to actually buy the bonds. So the interest rates increase more when income fall because of the negative effect lower period one home income has on the demand for bonds. The foreign share of bond holdings fall less because home investors increased demand for

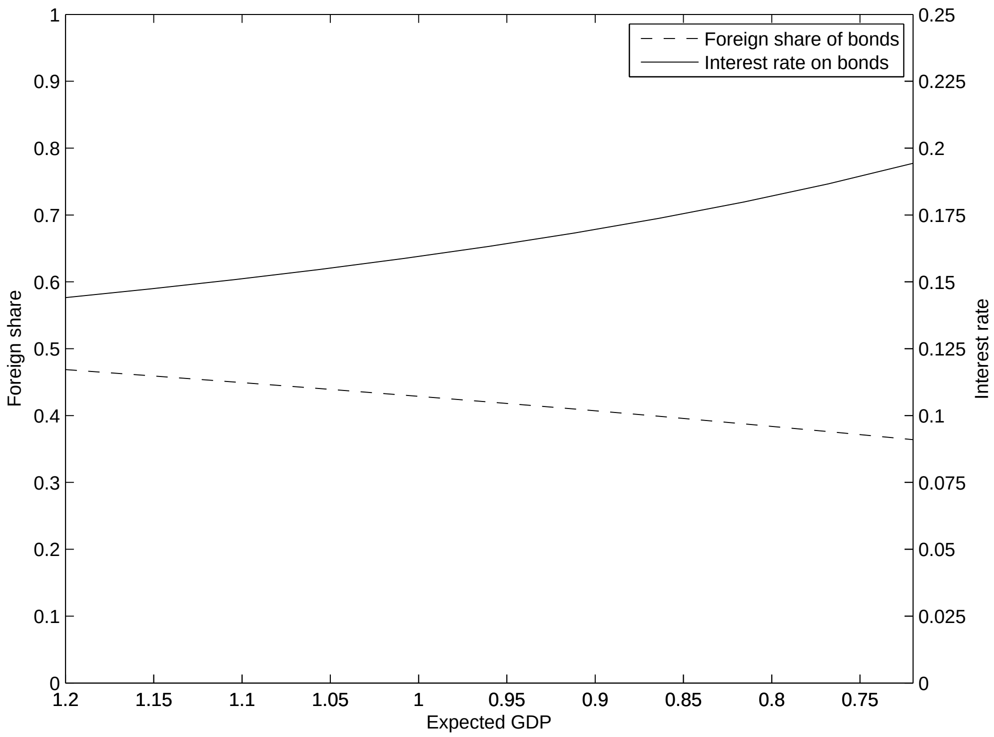


Figure 3: Interest rates and foreign shares for different values of period one income and expected GDP. Compared to figure 2 interest rates respond more, and the foreign share less, to different values of expected GDP. Lower period one income limits the home investor's ability to buy bonds, lowering effective demand for the bond and thus leads to higher interest rates.

home bonds in bad times are partly offset by the fact that they now have less wealth to spend on bonds.

5.1.3 Different values of total debt

The level of total government debt also affects the default decision and gross return on the government bond. Figure 4 shows that an increase in the home government's debt, B, will have the same effect on interest rates and the foreign share of bond holdings as lower expected GDP. Higher total debt increases default risk because the gains of default increase. High debt makes the government choose a higher default rate than with low debt. This has the same asymmetric effect on home and foreign investors as in the two previous cases. Both investors observe the increased risk and demand higher interest rates, but the home investor is inclined to buy more of the bond because his total period two income is less affected by default than the foreign investor.

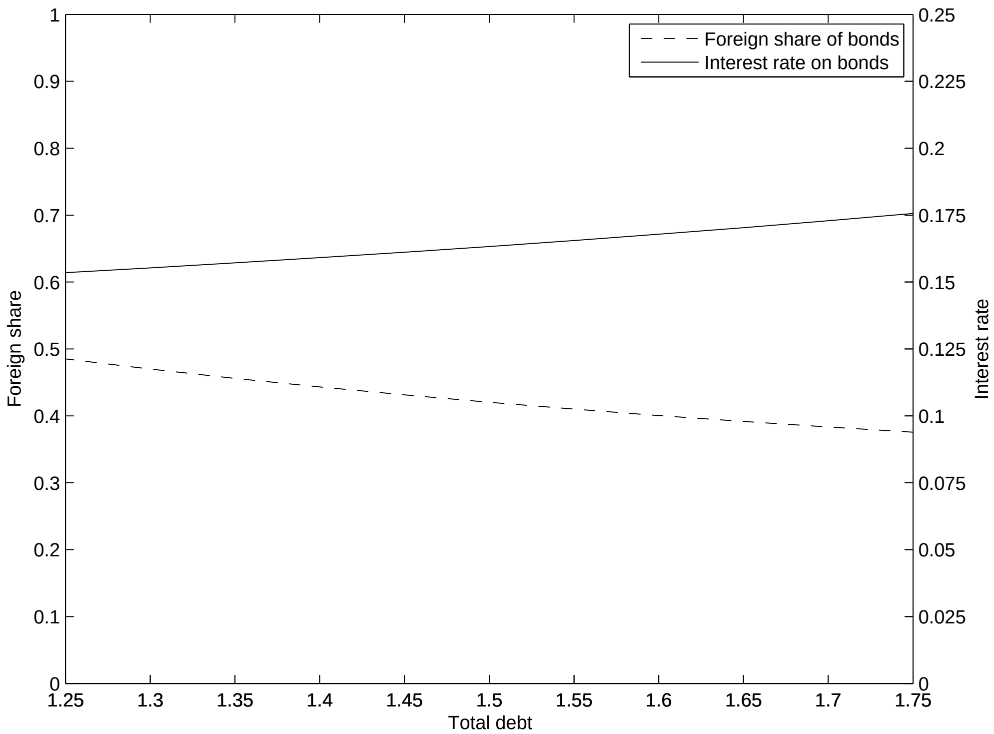


Figure 4: Interest rates and foreign shares for different values of total debt. High debt has the same effects as low expected GDP. Default risk increases, leading to a higher interest rate and a lower share of foreign holdings.

5.1.4 Different probabilities of low outcome

Another way to change the default risk is to change the probability of the low outcome occurring. Figure 5 shows how the interest rate and foreign share of bonds react to values of p from 10% to 40%. The qualitative results are the same, but in this case the interest rate moves a lot more than the share of foreign holdings. The reason for this is twofold. The foreign share moves less because the probability of default affects the marginal default rate less than a change in y^L. A reduction in y^L has a direct effect on the marginal default rate through the governments first order condition and indirectly through higher interest rates in the market. The default rate increase because of the lower endowment and because the interest rate is higher. A change in the probability of the low outcome only has this indirect effect on the marginal default rate. Less movement in the foreign share leads to a more responsive interest rate because an increase in the domestic share of bond holdings contributes to dampening the default risk.

The probability of default also moves the interest rate more because it has a much stronger effect on the expected default rate than a decrease in the low outcome. The

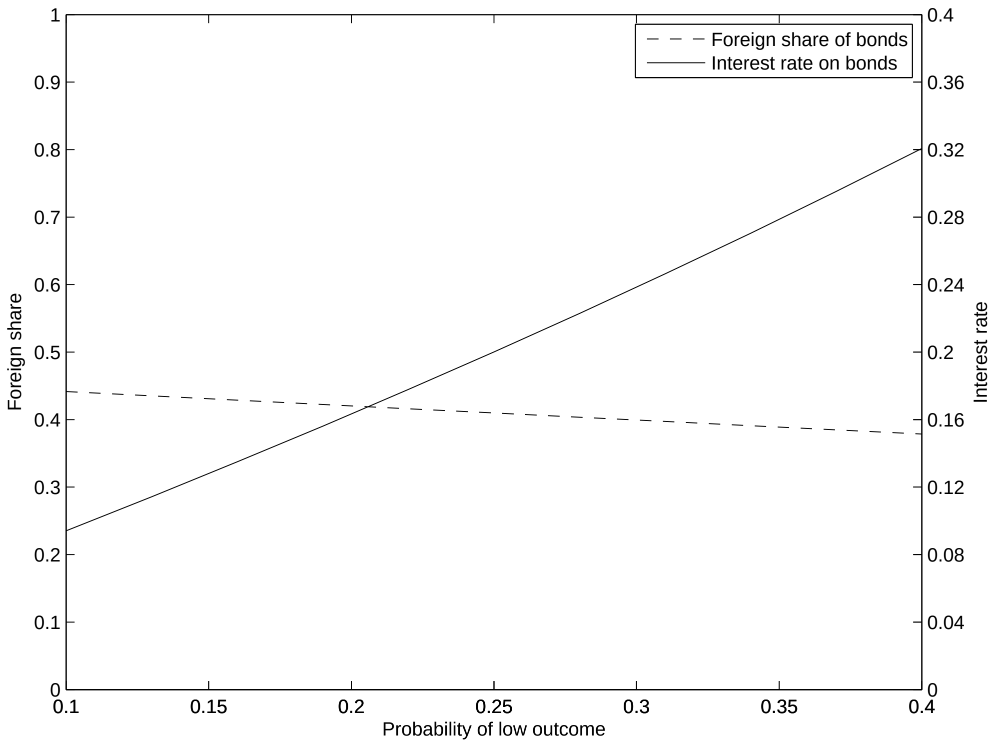


Figure 5: Interest rates and foreign shares for different probabilities of a bad outcome. The interest rates react very much in this case. The interest rate is much more responsive to this form of increased risk because increased probability of default has a much greater impact on expected default than a marginal increase in the default rate.

large expected default rate makes investors demand a very high interest rate. Consider two changes to the income distribution, both having the same effect on the expected endowment, one where we increase p and one where we decrease y^L. Remember that in this calibration of the model a high outcome always implies full repayment and a low outcome implies default. The expected default rate is thus pd^L. A lower y^L will increase the default rate in case of the low realization of the endowment. Increased probability of the low realization will both increase the probability of default and also increase the default rate because of the higher interest rate. With the parameterization used here a decrease in the expected second period endowment of about 2% by increasing p leads to an increase in the expected default rate of about 6.5 percentage points. An equal decrease in the expected endowment brought about by a lower bad outcome leads to an increase in the expected default rate of about 2 percentage points.

5.2 Analysis of multiple equilibria

The discontinuity that arises when the government goes from choosing full repayment to default leads to the possibility of multiple equilibria because of the interaction between interest rates and the default decision. In the low default equilibrium investors believe that the government will default if output is low. Here the interest rate is such that it is optimal for the government to default if output is low and not default if output is high. In the no default equilibrium investors believe in full repayment in both states. This leads to a low interest rate which makes it optimal for the government to repay all it's debt in both outcomes. In the full default equilibrium the high interest rates resulting from expectations of default in both realizations drive interest rates so high that the government defaults even if the income realization is high.

Specific combinations of B, y^H, y^L and p can all lead to multiple equilibria. In the first part of my analysis I focused on the equilibrium where investors expect default in bad times and full repayment in good times. In the following sections I discuss the existence of this equilibrium. I also examine for what values the other equilibria will exist. I do this by varying y^L and describe for what values the model has more than one equilibrium. There will be an upper threshold for y^L where expectations of default in bad times is no longer consistent with an equilibrium and a lower threshold where expectations of full repayment in good times is no longer consistent. This provides some answers to the uniqueness and existence of equilibria in the model in addition to being the basis for an interesting analysis on how a default situation might arise.

5.2.1 The possibility of certain repayment

In my earlier analysis I worked exclusively with the case where agents expect default in bad times and full repayment in good times. For my discussion of the different equilibria I include some additional notation to clarify what set of expectations led to that solution. I include subscripts for the interest rate and foreign share denoting the different sets of expectations; ND denotes expectations of no default, LD denotes expectations of default in the low realization and full repayment in the high realization of the endowment and FD denotes expectations of default in both the high and the low realization of the endowment. I also specify where r and f are dependent on y^L

Let's first consider the case where all agents expect full repayment even if the realization of the endowment is low. Interest rates will drop to the risk-free rate and the condition for the government to actually repay all of its debt in the low outcome is described in

(24). The condition on y^L is sufficient because the fact that $y^L < y^H$ ensures that full repayment in the low outcome implies full repayment in the high outcome.

$$y^L \geq \frac{[f(1+r_R)B]^2}{4kh} \tag{24}$$

With expectations of full repayment we have no way of determining a and a^* as both investors are indifferent between the government bond and the risk-free alternative. We are, however, sure that all of the debt will be bought, since we have an exogenous supply. Since the foreign share, f, is arbitrary we do not have *one* specific threshold value for where the equilibrium will exist. I can, however, specify a threshold value for the low outcome dependent on f, $\widehat{y}^L(f)$, given in (25).

$$\widehat{y}^L(f) = \frac{[f(1+r_R)B]^2}{4kh} \tag{25}$$

If, for a given foreign share f, the low outcome is lower than tis threshold, $y^L < \widehat{y}^L(f)$, the government will default in the low outcome and the equilibrium with expectations of full repayment even in bad times will not exist in this case. However, if the low outcome is higher than the threshold, $y^L > \widehat{y}^L(f)$ the government will not default at the low interest rate and the no default equilibrium will exist.

Assume then that investors expect that the government will default in the low outcome. Interest rates are determined in the market for sovereign debt and the condition for the government to repay its debt is described by the standard endpoint condition from the governments problem found in equation (12). From this condition we can find a threshold level, $\overline{y}^L$, that is given by (26). Contrary to $\widehat{y}^L(f)$, this value is fully determined in the model. When investors expect default we know who buys the debt and f is determined within the model.

$$\overline{y^L} = \frac{[f(y^L)_{LD}(1+r(y^L)_{LD})B]^2}{4kh} \tag{26}$$

If $y^L < \overline{y}^L$, an equilibrium where investors expect default under the low outcome and full repayment under the high outcome will exist. If the threshold, $\overline{y}^L$, is higher than $\underline{y}^L(f)$ the chosen parameterization allow for both the no default equilibrium and the low default equilibrium to exist. Both will be possible solutions to the model if we specify the low outcome to be in the region between the two threshold values, $\underline{y}^L(f) \leq y^L \leq \overline{y}^L$. Which one prevails depend on the expectations of the agents and is not discussed here.

5.2.2 The possibility of certain default

Let us now assume that investors believe the government will default in both realizations of the endowment. This leads to a very high interest rate and we can find threshold values for this to be a rational equilibrium in the same way as in the previous section. $\widetilde{y}^L$, defined by (27) is the threshold level for the full default equilibrium to exist.

$$y^H = \frac{[f(\widetilde{y}^L)_{FD}(1 + r(\widetilde{y}^L)_{FD})B]^2}{4kh} \tag{27}$$

If $y^L < \widetilde{y}^L$ the government will default in the high realization of the endowment. This also implies default in the bad realization since $y^L < y^H$ and the equilibrium will exist.

Consider again the case where investors expect default in the bad outcome and full repayment in the good outcome. It turns out that for sufficiently low y^L, the associated interest rate becomes so high that the government will default also in the high realization of the endowment. I can now find a lower threshold for y^L that means the government will default in the high realization of the endowment even if investors expect repayment. This will be the lower threshold for the low default equilibrium to exist. The intuition behind this is that when the endowment in the bad outcome becomes very low investors expect a high default rate in the state where the bad outcome is realized. This reduces the expected return on the bond and investors demand higher interest rates. This then changes the optimal behavior of the government. The threshold, $\underline{y}^L$, comes from the governments default condition and is defined by by (28).

$$y^H = \frac{[f(\underline{y}^L)_{LD}(1 + r(\underline{y}^L)_{LD})B]^2}{4kh} \tag{28}$$

If the low endowment is lower than this threshold, $y^L < \underline{y}^L$, expectations of default in the bad outcome and full repayment in the good outcome are no longer rational and the equilibrium will not exist.

5.2.3 Existence of the low default equilibrium

Figure 6 displays levels of y^L and f for which each of the different sets of expectations lead to a rational equilibrium. The figure is divided into six areas. Each area contains a box indicating which of the possible equilibria exists in that area. As before ND denotes the equilibrium where there is no default in either realization of the endowment, LD denotes the equilibria where the government defaults only in the bad realization and FD denotes the equilibrium where the government defaults in both realizations. The area below the $\widehat{f}(y^L)$-curve represents the area where the no default equilibrium exists. The area above the same curve represents the area where the government will choose to default in the bad outcome even when investors expect repayment and demand low interest rates. They default either because of a high share of foreign investors or a low value of the realized

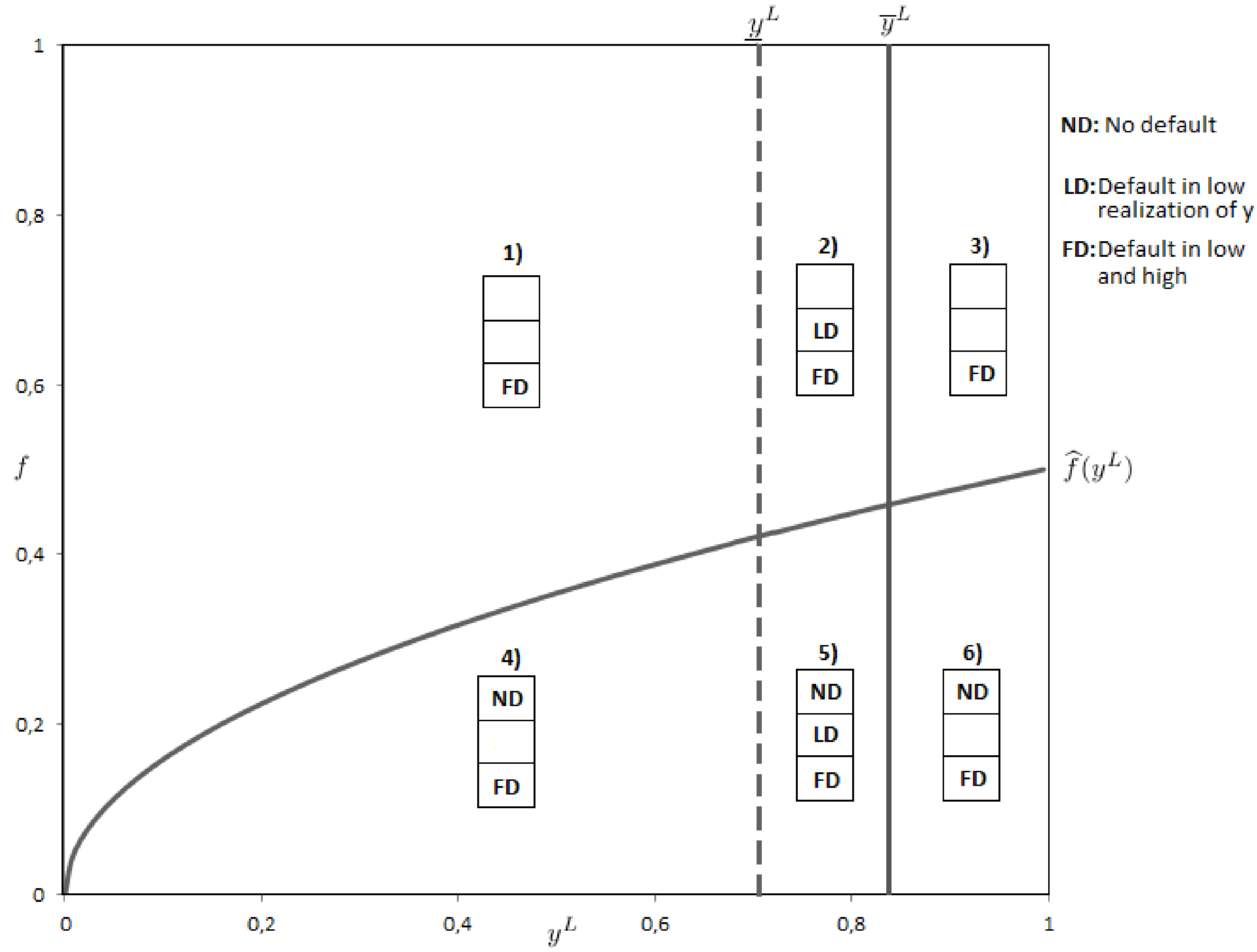


Figure 6: Existence of different equilibria for different values of y^L and the foreign share. The figure is divided into six areas where the notation in the boxes denotes which equilibria will be a possible solution for the corresponding values of the low realization of the endowment and the foreign share.

endowment. The vertical $\overline{y}^L$-line represents the upper threshold for which expectations of default in good times and repayment in bad times are rational. To the right of this line the value of the low realization of outcome is such that the government will repay all of its debt despite high interest rates resulting from expectations of default. The dotted $\underline{y}^L$-line represent the lower threshold for which the same low default equilibrium exists. For values of y^L to the left of this line the interest rates will become high enough for default to be the optimal choice for the government even if the realization of y^i is high. The low default equilibrium will exist in area 2) and 5).

To find values for the different threshold levels displayed in figure 6 I use the same parameterization as in the rest of the analysis. The upper threshold for the low default equilibrium, $\overline{y}^L$, is found by solving the model numerically assuming agents expect full repayment in the good outcome and default in the bad outcome and finding the value of y^L that prove their expectations wrong. I find that $\overline{y}^L = 0.84$, implying that if $y^L \geq 0.84$ the government will not default even if the interest rates on its bonds are high because of expectations of default. So expectations of full repayment in the good outcome and default in the bad outcome are only rational as long as $y^L < 0.84$.

By the same method I find the lower threshold, $\underline{y}^L$. I solve the model with expectations of full repayment in the good outcome and default in the bad outcome numerically and find the value of y^L that makes the government default even in the high realization of the endowment. I find that $\underline{y}^L = 0.73$. For $d^H = 0$, $d^L > 0$ to be a rational solution the low outcome must be larger than 0.73.

In the same way I find the threshold for the full default equilibrium, $\widetilde{y}^L$. Here I find that $\widetilde{y}^L > y^H$. y^L is assumed to be lower than y^H, so in this specification of the model, expectations of default in both realizations of the outcome always lead to default in both outcomes. The full default equilibrium will exist no matter what y^L is used.

The threshold value for the no default equilibrium, $\widehat{y}^L(f)$, will vary with the value of f. First I choose the value that yield the lowest possible threshold to capture all possible cases with multiple equilibria, $f = 0$. This yields $\underline{y}^L(0) = 0$, with all debt held by home investors the government would not default no matter how low the realized endowment is. Table 3 and 4 show some combinations of y^L and f where both the low default equilibrium and the no default equilibrium will exist. The values in table 4 corresponds to the $\widehat{f}(y^L)$-curve in figure 6. If we assume that the foreign share is 40% any y^L between 0.64 and 0.84 will yield a situation where all three equilibria exist. One where investors believe in full repayment in both realizations and the government does in fact repay the whole debt, one where investors believe the government will default in the bad realization and the government defaults because of the higher interest rates demanded by the investors and one were investors expect certain default and the government defaults in both realizations of the endowment.

The low default equilibrium, represented in area 2) and 5) in the figure, is stable to small changes in default level and the income distribution. The only thing that might move the solution from this equilibrium to another equilibrium is if all investors changed their expectations. For the same economic fundamentals a change in expectations could move the solution from the low default equilibrium to either the no default equilibrium or the full default equilibrium. If, for some reason, all agents suddenly stopped believing that the government would repay in the good scenario interest rates would rise a lot and

Foreign share f	**Threshold low outcome** $\underline{y}^L(f)$
0.00	0.00
0.05	0.01
0.10	0.04
0.15	0.09
0.20	0.16
0.25	0.25
0.30	0.36
0.35	0.49
0.40	0.64
0.45	0.81
0.50	0.99

Table 3: Threshold levels, $\underline{y}^L(f)$, for different values of f. For each level of foreign holdings, $\underline{y}^L(f)$ denotes the minimum level of the low outcome that makes full repayment possible. E.g. if the foreign share is 35% the low outcome must be at least 0.49 for the equilibrium with no default to exist.

Low outcome y^L	**Threshold foreign share** $\overline{f}(y^L)$
0.50	0.35
0.55	0.37
0.60	0.39
0.65	0.40
0.70	0.42
0.75	0.43
0.80	0.45
0.85	0.46
0.90	0.48
0.95	0.49
1.00	0.50

Table 4: Threshold levels, $\overline{f}(y^L)$, for different values of y^L. For each level of the low outcome, $\overline{f}(y^L)$ denotes the maximum level of the foreign share that ensures full repayment. E.g. if the low outcome is 0.7 the foreign share must be no higher than 42% for the equilibrium with no default to exist.

the government would default even in the high endowment state. The same is true for the other equilibria as well; changes in all investors' expectations could move the solution to one of the other existing equilibria.

These results give rise to an interesting analysis on what may cause a situation with default risk in the first place. If all agents assume that the government will repay all of its debt, the ownership structure of the debt is a factor in deciding if the government actually repays their debt. With a low outcome of 0.8, default will not happen as long as the foreign share is less than 45 %. If the foreign share exceeds 45% the government will default and the full repayment outcome is no longer an equilibrium. What happens exactly on the threshold level is not well described by my model, but with a foreign share below the threshold the economy will be in a stable low-interest equilibrium and above it will be in a stable high-interest equilibrium.

5.3 Correlation between the interest rate and the share of foreign holders

I also check what my model would predict about the correlation between bond yields and the foreign share of bond holders. I use the results from the different equilibria of my earlier analysis and find correlation coefficients between them. I do this for each of the cases in sections 5.1.1 - 5.1.4 and display the results in table 5. The correlation coefficients in each case are calculated on 11 observations of r and f resulting from different risk specifications.

Section	**Corr(r,f)**
GDP (5.1.1)	-0.9967
GDP and income (5.1.2)	-0.9957
Debt (5.1.3)	-0.9858
Probability (5.1.4)	-0.9991

Table 5: Predicted correlation between interest rates and foreign shares of debt holdings in the analysis from sections 5.1.1-5.1.4.

In table 5 we see the correlation coefficients between interest rates and the foreign share of bond holdings from my earlier analysis. They are calculated on the same data created for figures 2-5. In all these cases my model predicts correlation coefficients close to negative one. A high risk of default leads to a low share of foreign investors and high yields. Conversely low risk of default leads to a high share of foreign investors and low yields. There is no causation between r and f, both the foreign share of bond holdings and the interest rate are driven by a common factor, the default risk.

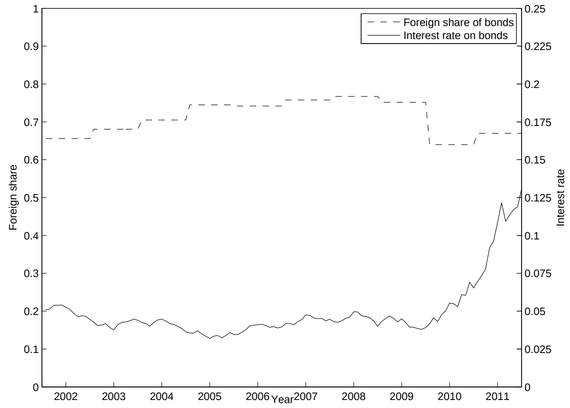
6 Discussion on the model's relevance for the situation in Europe

In the following sections I will attempt to draw parallels between the predictions of my model and the situation in Europe since the beginning of the millennium. All the scenarios analyzed in the section 5 could be consistent with different countries in Europe during and after the financial crisis. Almost all countries experienced expectations of negative growth in GDP and some countries, notably Greece, had major increases in debt and perceived probability of default.

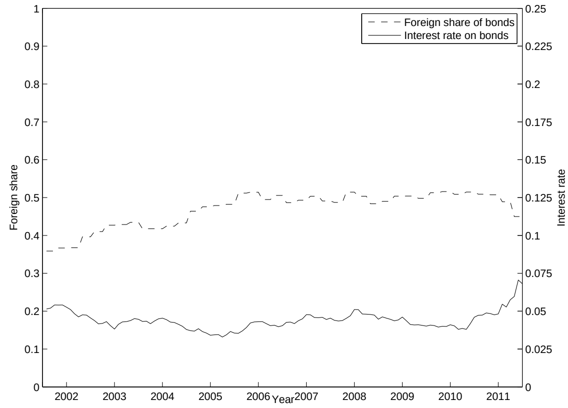
6.1 Declining foreign share of investors in PIIGS-countries

Figure 7a-7e display the foreign shares and interest rates for the PIIGS countries between 2002 and 2011. The graphs use the same data presented in section 2. The pre- and post-2007 trends established in section 2 are most apparent for Greece and Ireland, but visible for all countries. All the countries experienced increased foreign shares from 2002 to 2007 and a decline from 2007 to 2011. The low and stable interest rates in the period leading up to 2008 are argued to be caused, in part, by the introduction of the euro. The same can be said about the rising foreign share of debt holdings, as the introduction of the euro facilitated integration of financial markets and made diversification of portfolios across borders easier. The figures also show that the timing of the trend breaks happen about the same time as interest rates start to rise in 2008. With this in mind I argue that increased risk may have been a factor in causing the recent decline in the foreign share.

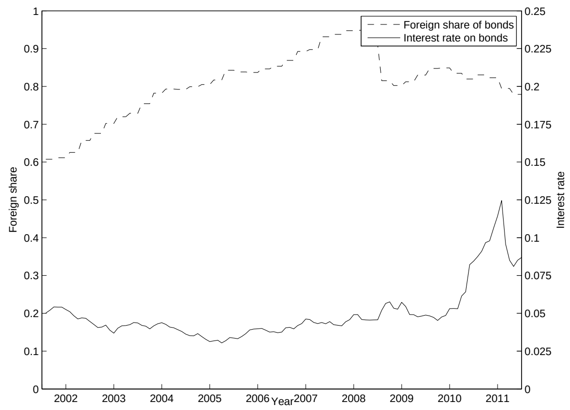
The case of Greece is shown in figure 7d. Around 2008 Greece had high debt and experienced expectations of low GDP which brought on concerns about default. When concerns about risk started to rise because of an increase in debt this can be compared to different situations in section 5.1. When debt rose my model would predict a higher interest rate and lower share of foreign holdings. This prediction is consistent with the data, after 2008 Greece experienced high interest rates and foreign investor sold off Greek debt. It could be the case that Greek investors were less afraid of buying the bond than foreign investors because they saw potential advantages like tax cuts, or rather absence of tax increase, if the government defaulted on its debt.



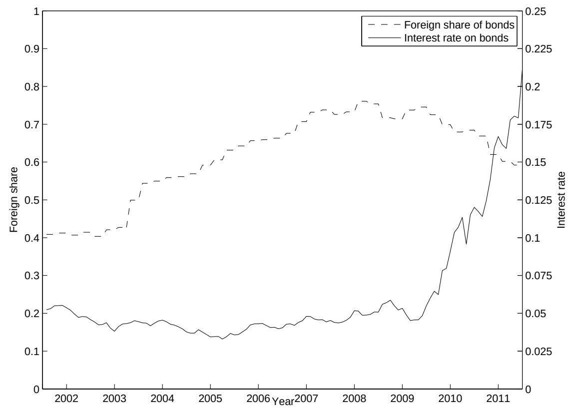
(a) Portugal



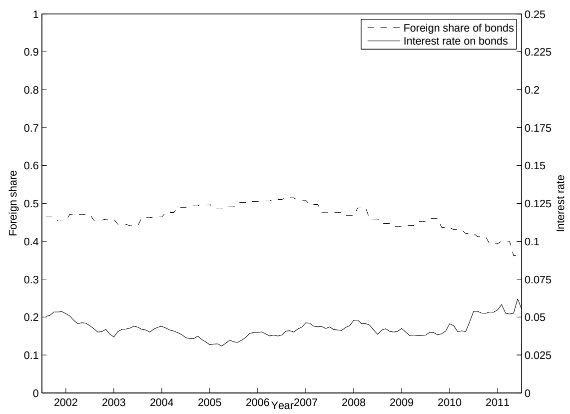
(b) Italy



(c) Ireland



(d) Greece



(e) Spain

Figure 7: Interest rates and foreign shares in the PIIGS-countries, 2002-2011

6.2 Correlation between yield and the foreign share

Andritzky (2012) concludes that the share of foreign investors in sovereign debt is negatively correlated with bond yields. That is, a high share of foreign investors is associated with low yields. He is unable to conclude on which way the causation goes. My model offers the possible explanation that there is no clear causation, but that both are affected by a common driver, the risk of default. The magnitude of the correlation coefficients from my model found in table 5 are way stronger than what we showed happened in the data in table 2. This reflects the fact that my model ignores many factors influencing both the interest rates and the foreign share. If we look at data from 2007-2011 the coefficients from the model fit much better. Table 6 displays the coefficients from the crisis years. In this time period we see much stronger negative correlation coefficients. The model gives a clear prediction about the correlation between bond yields and the foreign shares. Looking at the data, the model seems to explain the relationship better in times of crisis than in normal times. This is not surprising as the model is constructed to analyze situations with sovereign risk.

Country	**Correlation**
Portugal	-0.50
Italy	-0.74
Ireland	-0.55
Greece	-0.97
Spain	-0.70

Table 6: Correlation between interest rates and foreign shares of debt holdings in the PIIGS countries between 2007 and 2011.

6.3 Contribution to the origins of the sovereign debt crisis

There were many factors both inside and outside the troubled countries contributing to the crisis. In this section I explain how the mechanisms I propose in this thesis could have contributed to the situation.

My model shed light on some mechanisms that may have contributed to the situation in the European bond market. First it contains the unsurprising prediction that high debt-to-

GDP ratios and low expectations about future GDP results to higher risk of default. High debt and low GDP levels impede both the government's ability and willingness to make good on its debt. A second, and less explored, mechanism in the model is that the increase in foreign holdings of sovereign debt could have been a factor in increasing the default risk when the crisis hit. In my model an increase in the foreign share alone can cause default risk as the government only care about their own inhabitants. Third, an exogenous shock to expectations can create actual risk. An equilibrium where all investors expect full repayment from the Greek government may be fragile to both people's expectations and to the foreign share of debt holdings. These last two mechanisms were analyzed in section 5.2 and will be discussed in a European context in the following subsections.

6.3.1 A change in expectations

The model opens up the possibility of self-fulfilling expectations of default. It is possible that any one of the PIIGS-countries could be described in a similar manner as in the model with multiple equilibria. The situation in Greece before the crisis can be thought of as the good equilibrium where investors believed in full repayment and the Greek interest rate was equal to the secure German interest rate. When the financial crisis hit investors could for some reason start questioning the ability of the Greek government to repay if GDP took a turn for the worse. This change in expectations is a potential source of the move from a good to a bad equilibrium where interest rates rose in sovereign debt markets. The higher interest rates would in turn make default a credible outcome as GDP was falling during the crisis.

This is also an interesting environment to talk about contagion effects. The reason for shifting expectations could be default in other countries. As Greece requested the first bailout package in April 2010 the Portuguese and Irish interest rates started to rapidly increase. One interpretation is that the need for a bailout in Greece induced investors in Portuguese and Irish sovereign debt to expect default or similar measures in those countries. This change in expectations could have moved the market for those bonds from a good equilibrium to a bad one.

6.3.2 A high share of foreign investors could have contributed

Another implication of my model is that the increase in the foreign share of bond holdings, notably in Ireland and Greece as seen in figure 7c and 7d, could actually have contributed to the crisis. As discussed in section 5.2 the model contains multiple equilibria for certain parameter values. There is a threshold level for foreign holdings that would make the good equilibrium with full repayment become inconsistent with expectations. If foreign holdings of Greek sovereign debt at the beginning of the millennium was below this threshold, but increased as a result of increased diversification in the EU, it is possible that they reached the threshold level. The fact that more foreigners owned Greek debt in 2008 than in 2002 could have made the Greek government more willing to default on their debt and when the threshold was reached expectations about full repayment became irrational. This could have pushed Greece from a good equilibrium to a bad one.

7 Conclusion

This thesis has introduced a model of sovereign default and debt ownership with some features not covered to a large extent in the earlier literature. It highlights the importance of the nationality of bondholders for the default decision of a government, as the government is more willing to default on external debt than internal. The model also predicts changes in the ownership structure of sovereign bonds when economic fundamentals change. If the default risk increases, either by lower expected GDP, higher debt or increased probability of an economic downturn, the foreign bond holder will sell some of their bonds to domestic buyers who are more willing to accept the risk. Domestic holders are more willing to buy risky bonds because they realize that in the last instance they are the ones that must repay the debt through taxes and they understand that default leads to lower taxes. The model also shed light on the possibility of multiple equilibria and how shocks can move the government bond market from a good equilibrium to a bad one.

The mechanisms of the model offer possible explanations for some of the events in European sovereign debt markets in later years. It can be used to explain parts of both the increased interest rates and the decline in foreign bond holdings when the default risk increased. The same mechanism also offers an explanation to another, more general, finding in the literature, namely that the foreign share and yield differentials are negatively correlated. The model indicates that the sovereign bond crises could have been sparked by an exogenous shift in expectations or an increase in foreign holdings making the government more willing to default.

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A Matlab code

Appendix A.2-A.5 contains the Matlab codes i used to solve the model. Appendix A.1 contains a short explanation of the code.

A.1 Explanation of the code

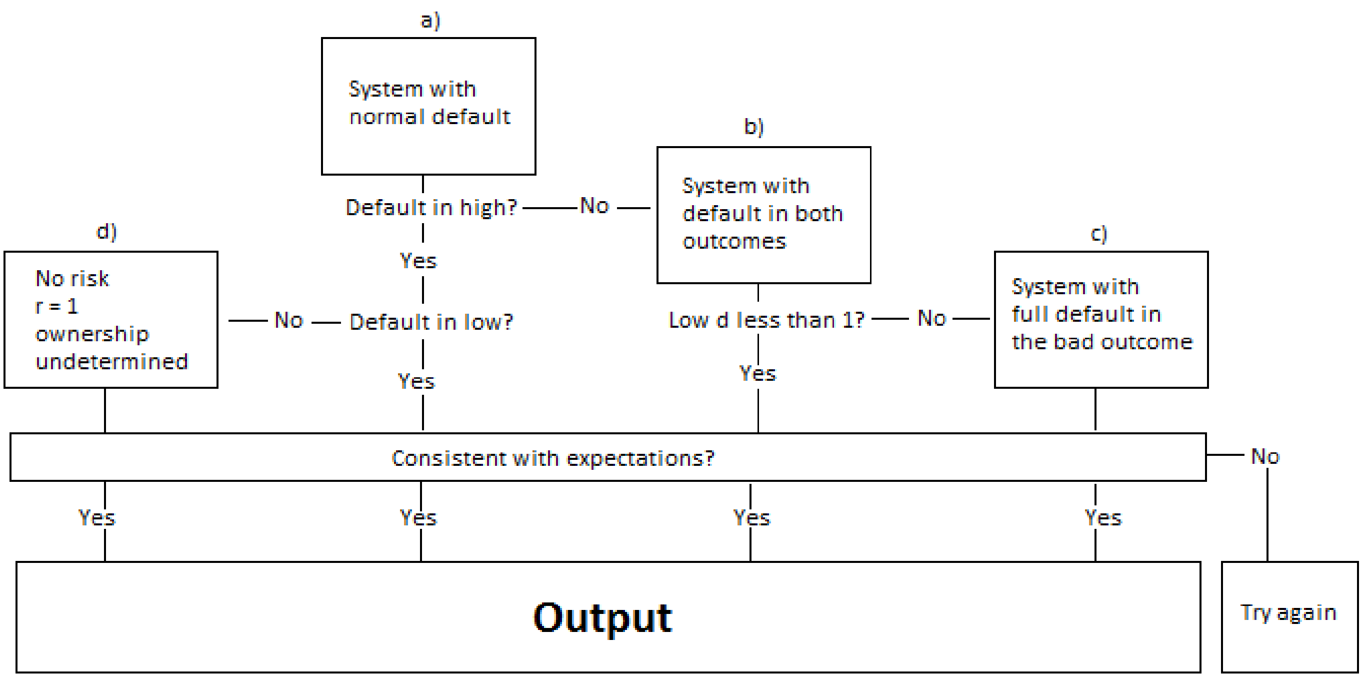

Figure 8: How the code runs

I use Matlab's *fsolve* command to numerically solve the model. *fsolve* cannot handle the "if"s from the governments optimality condition so I need solutions for all sets of expectations. The standard assumption is that the government will default in bad times and repay in good times. The system in this case is box a) in figure 8. If the government repays all its debt in both scenarios the solution is that the interest rate is equal to the risk free rate. If parameters or the income distribution is set so that the government will default even in bad times we must enter a new system with investor expecting default in both scenarios, box b). If this raises interest rates high enough to make it optimal for the government to default on all its debt in the bad scenario we need to update the expectations once more, box c). Once a consistent solution is found I display the output.

A.2 Main code

```
clear all
clc

%% Declare parameters
global pd B rr beta aa h in1 in1f yl yh yf tauf theta shock

B = 1.5;          % Level of debt
rr = 0.03;        % Risk-free interest rate
beta = 1/(1+rr);     % Discount factor
in1 = 2;          % Domestic period 1 wealth
in1f = 2;         % Foreign period 1 wealth
tauf = 0.5;       % Foreign tax rate
theta = 1;        % Coefficient of relative risk aversion
aa = 0.15;        % Fixed cost of default
h = 1;            % Variable cost of default parameter

%% Distribution of second period endowment
    % pd = probability of yl, (1-pd) = probability of yh

pd = 0.2;         % Probability of low endowment
shock = 0.8;          % Relative size of low endowment
yh = 1;                   % High outcome endowment
yl = shock*yh;            % Low outcome endowment
yf = 1;                   % Foreign endowment

%% Model - normal expectations
    % This version assumes that investors expect default in the bad outcome
    % and not in the good outcome.

x0 = [1 , 1 , 0.5 , 0.5 , 0.5 , 0.5 , 0.15 , 0.5 , 0.5 , 0.5 , 0.5 , 0.4, 0];
options = optimoptions('fsolve','display','off'); % Option to display output
[z,fval,exitflag] = fsolve(@normalsig,x0,options); % Call solver

        %% Output
c1 = z(1);        % Domestic period one consumption
c1f = z(2);           % Foreign period one consumption
a = z(3);         % Domestic bond holdings
ar = z(4);        % Domestic risk-free holdings
af = z(5);        % Foreign bond holdings
```

```
arf = z(6);       % Foreign risk-free holdings
r = z(7);         % Interest rate on bonds
c2h = z(8);       % Domestic period two consumption in high endowment
c2l = z(9);       % Domestic period two consumption in low endowment
c2fh = z(10);         % Foreign period two consumption in high endowment
c2fl = z(11);         % Foreign period two consumption in low endowment
d = z(12);        % Default rate in low endowment state
dh = z(13);       % Default rate in high endowment state

b = a/B;          % Domestic share of bond holding

%% Test section
    % Tests for the existence of equilibrium
        % To ensure the solution found is actually an equilibrium I need to
        % check it against the expectations assumed in the solver.

    % Test that yh leads to zero default
if yh > ((1-b)*(1+r)*B)^2/(4*aa*h)
    zerodefaultflag = 1;
else zerodefaultflag = 0;
end
    % Test if yl actually leads to default
if yl > ((1-b)*(1+r)*B)^2/(4*aa*h)
    lowdefaultflag = 0;
else lowdefaultflag = 1;
end
    % Test that d=1 is not the solution
if yl > ((1-b)*(1+r)*B)/(2*h)
    fulldefaultflag = 1;
else fulldefaultflag = 0;
end

if zerodefaultflag < 0.5     % If expectations of full repayment fails
                    %% Model - expectations of certain default

x0 = [1 , 1 , 0.5 , 0.5 , 0.5 , 0.5 , 0.15 , 0.5 , 0.5 , 0.5 , 0.5 , 0.4, 0];
options = optimoptions('fsolve','display','off'); % Option to display output
[z,fval,exitflag] = fsolve(@defdefsig,x0,options); % Call solver

c1 = z(1);
c1f = z(2);
a = z(3);
```

```
ar = z(4);
af = z(5);
arf = z(6);
r = z(7);
c2h = z(8);
c2l = z(9);
c2fh = z(10);
c2fl = z(11);
d = z(12);
dh = z(13);

b = a/B;

%% Test section
    % Tests for the existence of equilibrium
        % To ensure the solution found is actually an equilibrium I need to
        % check it against the expectations assumed in the solver.

    % Test that yh leads to zero default
if yh > ((1-b)*(1+r)*B)^2/(4*aa*h)
    zerodefaultflag = 1;
else zerodefaultflag = 0;
end
    % Test if yl actually leads to default
if yl > ((1-b)*(1+r)*B)^2/(4*aa*h)
    lowdefaultflag = 0;
else lowdefaultflag = 1;
end
    % Test that d=1 is not the solution
if yl > ((1-b)*(1+r)*B)/(2*h)
    fulldefaultflag = 1;
else fulldefaultflag = 0;
end

%%

    if fulldefaultflag < 0.5   % If partial default fails
        %% Model - expectations of certain default with full default in the
      % low outcome

x0 = [1 , 1 , 0.5 , 0.5 , 0.5 , 0.5 , 0.15 , 0.5 , 0.5 , 0.5 , 0.5 , 1, 0.4];
options = optimoptions('fsolve','display','off'); % Option to display output
```

```
[z,fval,exitflag] = fsolve(@def1defsig,x0,options); % Call solver

c1 = z(1);
c1f = z(2);
a = z(3);
ar = z(4);
af = z(5);
arf = z(6);
r = z(7);
c2h = z(8);
c2l = z(9);
c2fh = z(10);
c2fl = z(11);
d = z(12);
dh = z(13);

b = a/B;

%% Test section
    % Tests for the existence of equilibrium
        % To ensure the solution found is actually an equilibrium I need to
        % check it against the expectations assumed in the solver.

    % Test that yh leads to zero default
if yh > ((1-b)*(1+r)*B)^2/(4*aa*h)
    zerodefaultflag = 1;
else zerodefaultflag = 0;
end
    % Test if yl actually leads to default
if yl > ((1-b)*(1+r)*B)^2/(4*aa*h)
    lowdefaultflag = 0;
else lowdefaultflag = 1;
end
    % Test that d=1 is not the solution
if yl > ((1-b)*(1+r)*B)/(2*h)
    fulldefaultflag = 1;
else fulldefaultflag = 0;
end

%% Output in full default outcome

r
```

```
f = af/B
d
dh

    else
%% Output in certain default outcome
        r
        f = af/B
        d
        dh

end

        elseif lowdefaultflag < 0.5  % If the government will not default in
%% Output with certain repayment     % any state
            r = 0.03                 % the interest rate drop to the risk free rate
                                     % and ownership is undetermined.

            else
%% Output in normal default outcome

r
f = af/B
d
dh
end
```

A.3 System with normal default

```
%% System with normal default expectations
    % Filnename: normalsig.m

function G = twostate(x)
global pd B rr beta aa h in1 in1f yl yh yf tauf theta

G(1) = 1/(1+x(7))-beta*x(1)*(((1-pd)*(1-x(13))/x(8)^(theta))+(pd*(1-x(12))/x(9)^(theta)));
G(2) = 1/(1+rr) - beta*x(1)*(((1-pd)/x(8)^(theta))+((pd)/x(9)^(theta)));
G(3) = x(1) + x(3) + x(4) - in1;
G(4) = -x(8) + yh - (1+x(7))*(1-x(13))*B + (1+x(7))*(1-x(13))*x(3) + (1+rr)*x(4);
G(5) = -x(9)+yl-(1+x(7))*(1-x(12))*B+(1+x(7))*(1-x(12))*x(3)+(1+rr)*x(4)-aa-h*(x(12))^2*yl;
G(6) = 1/(1+x(7))-beta*x(2)*(((1-pd)*(1-x(13))/x(10)^(theta))+(pd*(1-x(12))/x(11)^(theta)));
G(7) = 1/(1+rr)-beta*x(2)*(((1-pd)/x(10)^(theta))+((pd)/x(11)^(theta)));
G(8) = x(2) + x(5) + x(6) - in1f;
G(9) = -x(10) + yf - tauf + (1+x(7))*(1-x(13))*x(5) + (1+rr)*x(6);
G(10) = -x(11) + yf - tauf + (1+x(7))*(1-x(12))*x(5) + (1+rr)*x(6);
G(11) = x(3) + x(5) - B;
G(12) = - x(12) + (((1-(x(3)/B))*(1+x(7))*B)/(2*h*yl));
G(13) = - x(13);



% x(1) = c1
% x(2) = c1f
% x(3) = a
% x(4) = ar
% x(5) = af
% x(6) = arf
% x(7) = r
% x(8) = c2h
% x(9) = c2l
% x(10) = c2fh
% x(11) = c2fl
% x(12) = dl
% x(13) = dh
```

A.4 System with certain default

```
%% System with expectations of default in both scenarios
    % Filename: defdefsig.m

function G = twostate(x)
global pd B rr beta aa h in1 in1f yl yh yf tauf theta

G(1) = 1/(1+x(7))-beta*x(1)*(((1-pd)*(1-x(13))/x(8)^(theta))+(pd*(1-x(12))/x(9)^(theta)));
G(2) = 1/(1+rr) - beta*x(1)*(((1-pd)/x(8)^(theta))+((pd)/x(9)^(theta)));
G(3) = x(1) + x(3) + x(4) - in1;
G(4) = -x(8)+yh-(1+x(7))*(1-x(13))*B+(1+x(7))*(1-x(13))*x(3)+(1+rr)*x(4)-aa-h*(x(13))^2*yh;
G(5) = -x(9)+yl-(1+x(7))*(1-x(12))*B+(1+x(7))*(1-x(12))*x(3)+(1+rr)*x(4)-aa-h*(x(12))^2*yl;
G(6) = 1/(1+x(7))-beta*x(2)*(((1-pd)*(1-x(13))/x(10)^(theta))+(pd*(1-x(12))/x(11)^(theta)));
G(7) = 1/(1+rr) - beta*x(2)*(((1-pd)/x(10)^(theta))+((pd)/x(11)^(theta)));
G(8) = x(2) + x(5) + x(6) - in1f;
G(9) = -x(10) + yf - tauf + (1+x(7))*(1-x(13))*x(5) + (1+rr)*x(6);
G(10) = -x(11) + yf - tauf + (1+x(7))*(1-x(12))*x(5) + (1+rr)*x(6);
G(11) = x(3) + x(5) - B;
G(12) = - x(12) + (((1-(x(3)/B))*(1+x(7))*B)/(2*h*yl));
G(13) = - x(13) + (((1-(x(3)/B))*(1+x(7))*B)/(2*h*yh));



% x(1) = c1
% x(2) = c1f
% x(3) = a
% x(4) = ar
% x(5) = af
% x(6) = arf
% x(7) = r
% x(8) = c2h
% x(9) = c2l
% x(10) = c2fh
% x(11) = c2fl
% x(12) = dl
% x(13) = dh
```

A.5 System with full default in bad times

```
%% System with expectation of full default in bad scenario
    % Filename: def1defsig.m

function G = twostate(x)
global pd B rr beta aa h in1 in1f yl yh yf tauf theta

G(1) = 1/(1+x(7))-beta*x(1)*(((1-pd)*(1-x(13))/x(8)^(theta))+(pd*(1-x(12))/x(9)^(theta)));
G(2) = 1/(1+rr) - beta*x(1)*(((1-pd)/x(8)^(theta))+((pd)/x(9)^(theta)));
G(3) = x(1) + x(3) + x(4) - in1;
G(4) = -x(8)+yh-(1+x(7))*(1-x(13))*B+(1+x(7))*(1-x(13))*x(3)+(1+rr)*x(4)-aa-h*(x(13))^2*yh;
G(5) = -x(9)+yl-(1+x(7))*(1-x(12))*B+(1+x(7))*(1-x(12))*x(3)+(1+rr)*x(4)-aa-h*(x(12))^2*yl;
G(6) = 1/(1+x(7))-beta*x(2)*(((1-pd)*(1-x(13))/x(10)^(theta))+(pd*(1-x(12))/x(11)^(theta)));
G(7) = 1/(1+rr) - beta*x(2)*(((1-pd)/x(10)^(theta))+((pd)/x(11)^(theta)));
G(8) = x(2) + x(5) + x(6) - in1f;
G(9) = -x(10) + yf - tauf + (1+x(7))*(1-x(13))*x(5) + (1+rr)*x(6);
G(10) = -x(11) + yf - tauf + (1+x(7))*(1-x(12))*x(5) + (1+rr)*x(6);
G(11) = x(3) + x(5) - B;
G(12) = - x(12) + 1;
G(13) = - x(13) + (((1-(x(3)/B))*(1+x(7))*B)/(2*h*yh));



% x(1) = c1
% x(2) = c1f
% x(3) = a
% x(4) = ar
% x(5) = af
% x(6) = arf
% x(7) = r
% x(8) = c2h
% x(9) = c2l
% x(10) = c2fh
% x(11) = c2fl
% x(12) = dl
% x(13) = dh
```