THERMOHALINE INSTABILITY IN ANISOTROPIC POROUS MEDIA

by

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Abstract

The onset of thermohaline convection in a horizontal porous layer is investigated theoretically. The layer is homogeneous, anisotropic and of infinite horizontal extent. Horizontal isotropy with respect to permeability, thermal diffusivity and solute diffusivity is assumed. For porous media with thermally insulating solid matrices, the stability diagram has the same shape as in the case of isotropy. The critical wave number is constant and equal to that of the one-component case. For thermally conducting matrices new features may occur: The locus of the direct mode in the stability diagram may not be a straight line, and the corresponding wave number may be non-constant. The initiation of salt fingers is studied by linear theory. It seems that the width of salt fingers is influenced by anisotropy in the diffusivities. Anisotropy may or may not favour salt fingers, depending on a dimensionless diffusion parameter D being greater than or less than 1.

INTRODUCTION

This paper is concerned with thermohaline convection in porous media (Nield, 1968; Taunton et.al., 1972). It is relevant in connection with groundwater pollution when the contaminant influences water density and temperature gradients are present. Temperature gradients may arise from geothermal heating, solar heating of saturated soil or discharge of warm waste water.

The case of warm, salt water overlying cold, fresh water is of special interest as it may cause a type of motion termed salt fingers (Taunton et.al. 1972; Turner, 1974). Salt fingers may develop if the fluid is unstable to convection but still statically stable. The convection is then driven by release of potential energy in the solute distribution. Salt fingers transport solute very efficiently vertically and may be responsible for contamination of groundwater reservoirs.

The literature on groundwater pollution often ignores buoyancy-driven convection (Fried, 1975). This may be justified as far as large-scale horizontal spreading of pollution is considered. Concerning the vertical transport, however, buoyancy effects must be taken into account.

The previous theories on thermohaline convection in porous media are concerned with isotropic materials. Natural porous media are in general anisotropic and inhomogeneous. In this paper anisotropic media are considered. Sediments usually have smallest permeability normally to the plane of sedimentation, see Davis (1969) and Bear (1972, p.124). Layered media may also be considered as anisotropic, with respect to motions having length scale considerably larger than the distance between the layers. See Bear (1972, p.156) and Moranville et.al. (1977).

A saturated porous medium with anisotropic permeability will also have anisotropic diffusion properties. The relation between these two types of anisotropy has been analyzed theoretically by Neale (1977). He showed that anisotropy in permeability is the more important type when the solid matrix is insulating, i.e. diffusion taking place in the fluid phase alone.

Convection in anisotropic porous media is a relatively new field of research, see Castinel & Combarnous (1975) and Kvernvold & Tyvand (1979). The present paper is concerned with the onset of thermohaline convection in a homogeneous, anisotropic porous layer. The effects of anisotropy on marginal stability and on the initiation of salt fingers are studied.

GOVERNING EQUATIONS

A fluid saturated porous layer of infinite horizontal extent is considered. It is bounded by two horizontal planes separated by a distance h. The differences in temperature and solute concentration between the lower and upper plane are ΔT and ΔS, respectively. The fluid density ρ is assumed to be a linear function of temperature T and solute concentration S :

$$\rho = \rho_0(1+\beta(S-S_0) - \gamma(T-T_0)) \qquad (1)$$

ρ_0, T_0 and S_0 are reference values of density, temperature and concentration. β and γ are expansion coefficients.

A cartesian frame of reference is chosen, with x- and y-axes at the lower boundary plane. The z-axis is directed upwards in the gravity field. The permeability, effective thermal diffusivity and solute diffusivity in horizontal directions are denoted by $K^{(1)}$, $\kappa_{mT}^{(1)}$ and $\kappa_{mS}^{(1)}$, respectively. Horizontal isotropy is hereby assumed. $K^{(3)}$, $\kappa_{mT}^{(3)}$ and $\kappa_{mS}^{(3)}$ denote the permeability and effective diffusivities in z-direction.

Dimensionless variables are introduced by taking

$$h,\ h^2/\kappa_{mT}^{(3)},\ \kappa_{mT}^{(3)}/h, \Delta T, \Delta S, \rho_0 \nu \kappa_{mT}^{(3)}/K^{(3)} \tag{2}$$

as units of length, time t, velocity $\vec{v}$ $(=(u,v,w))$, temperature, solute concentration and pressure p. According to Bear (1972, p.652), the governing equations may be written

$$\vec{v} + \mathcal{K} \cdot (\nabla p - Ra\ T\vec{k} + \tau Rs\ S\vec{k}) = 0 \tag{3}$$

$$\nabla \cdot \vec{v} = 0 \tag{4}$$

$$\frac{\partial S}{\partial t} + \vec{v} \cdot \nabla S = \tau \nabla \cdot (\mathcal{D}_S \cdot \nabla S) \tag{5}$$

$$c\ \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla \cdot (\mathcal{D}_T \cdot \nabla T) \tag{6}$$

Ra is the thermal Rayleigh number:

$$Ra = \frac{K^{(3)} g \alpha \Delta T h}{\nu \kappa_{mT}^{(3)}} \tag{7}$$

and Rs the solute Rayleigh number:

$$Rs = \frac{K^{(3)} g \alpha \Delta S h}{\nu \kappa_{mS}^{(3)}} \tag{8}$$

g is the gravitational acceleration, ν the kinematic viscosity and $c = (\rho c_p)_m/(\rho c_p)_f$ (Katto and Masuoka, 1967). c_p denotes the heat capacity at constant pressure. The subscripts m and f refer to the mixture of solid and fluid and to the fluid alone, respectively.

In (3) and (4) the Boussinesq approximation and Darcy's law are applied, and the small inertial terms $\frac{\partial \vec{v}}{\partial t}$ and $\vec{v} \cdot \nabla \vec{v}$ are neglected. In eq. (5) the solid matrix is assumed non-adsorbing, and the Soret effect is neglected (Bear, 1972; p.86).

Hydrodynamic dispersion is a quadratic function of the velocity at small velocities (Poreh, 1965). Accordingly it vanishes within our linear stability theory when the basic state is motionless. Dispersion effects on thermohaline convection have been studied by Rubin (1976).

The dimensionless tensors of permeability, solute diffusivity and thermal diffusivity will be written

$$\mathscr{K} = \xi(\vec{i}\vec{i} + \vec{j}\vec{j}) + \vec{k}\vec{k} \tag{9}$$

$$\mathscr{D}_S = \eta_S(\vec{i}\vec{i} + \vec{j}\vec{j}) + \vec{k}\vec{k} \tag{10}$$

$$\mathscr{D}_T = \eta_T(\vec{i}\vec{i} + \vec{j}\vec{j}) + \vec{k}\vec{k} \tag{11}$$

respectively. The following ratios have been introduced :

$$\xi = K^{(1)}/K^{(3)}, \quad \eta_S = \kappa_{mS}^{(1)}/\kappa_{mS}^{(3)}, \quad \eta_T = \kappa_{mT}^{(1)}/\kappa_{mT}^{(3)} \tag{12}$$

The ratio between the diffusivities of solute and heat in z-direction is denoted by :

$$\tau = \kappa_{mS}^{(3)}/\kappa_{mT}^{(3)} . \tag{13}$$

LINEAR STABILITY ANALYSIS

The static state of conduction is given by

$$T = S = -z \ , \quad \vec{v} = 0 \ , \quad p = p_0(z) \tag{14}$$

Small disturbances are added to this basic solution :

$$\begin{aligned} T &= -z + \theta(x,y,z,t) \\ S &= -z + S(x,y,z,t) \\ \vec{v} &= \vec{v}(x,y,z,t) \\ p &= p_0(z) + \pi(x,y,z,t) \end{aligned} \tag{15}$$

The boundary conditions are taken to be :

$$w = \theta = s = 0 \quad \text{at} \quad z = 0,1 \tag{16}$$

Both planes are thus assumed to be impermeable and perfect conductors of heat and solute. These conditions are chosen for mathematical simplicity, without qualitatively important physical effect being lost.

Eliminating the pressure from (3) yields :

$$Ra\nabla_1{}^2\theta - \tau Rs\nabla_1{}^2 s = (\nabla_1{}^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})w \tag{17}$$

Omitting nonlinear terms, (5) and (6) may be written

$$-w = (\tau\eta_s\nabla_1{}^2 + \tau\frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t})s \tag{18}$$

$$-w = (\eta_T\nabla_1{}^2 + \frac{\partial^2}{\partial z^2} - c\frac{\partial}{\partial t})\theta \tag{19}$$

s and θ are eliminated from the set of equations (17) - (19), and an equation in w is obtained :

$$\left[\left(\tau\eta_S\nabla_1^{\,2} + \tau\,\frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t}\right)\left(\eta_T\nabla_1^{\,2} + \frac{\partial^2}{\partial z^2} - c\frac{\partial}{\partial t}\right)\left(\nabla_1^{\,2} + \frac{1}{\xi}\,\frac{\partial^2}{\partial z^2}\right) + Ra\left(\tau\eta_S\nabla_1^{\,2} + \tau\,\frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t}\right)\nabla_1^{\,2} - \tau\,Rs\left(\eta_T\nabla_1^{\,2} + \frac{\partial^2}{\partial z^2} - c\frac{\partial}{\partial t}\right)\nabla_1^{\,2}\right]w = 0 \qquad (20)$$

The boundary conditions (16) can be expressed as :

$$\frac{\partial^4 w}{\partial z^4} = \frac{\partial^2 w}{\partial z^2} = w = 0 \qquad \text{at} \quad z = 0,1 \qquad (21)$$

A solution satisfying the boundary conditions may be written :

$$w = \sin n\pi z \;\; e^{i(kx+ly)+\sigma t} \qquad (22)$$

n is an integer, k and l are dimensionless wave numbers, and σ is the (complex) growth rate :

$$\sigma = \sigma_i + i\sigma_i \qquad (23)$$

The most unstable mode corresponds to $n = 1$. The growth rate is determined by the characteristic polynomial

$$\sigma^2 + B\sigma + C = 0 \qquad (24)$$

where

$$\left.\begin{aligned} B &= \tau(\eta_S\alpha^2 + \pi^2) + (\eta_T\alpha^2+\pi^2)/c + \alpha^2\,\frac{c\tau Rs - Ra}{c(\alpha^2+\pi^2/\xi)} \\ C &= \frac{\tau}{c}\left[(\eta_S\alpha^2+\pi^2)(\eta_T\alpha^2+\pi^2) + \alpha^2\,\frac{(\eta_T\alpha^2+\pi^2)Rs - (\eta_S\alpha^2+\pi^2)Ra}{\alpha^2+\pi^2/\xi}\right] \end{aligned}\right\} \qquad (25)$$

We have introduced the overall wave number (dimensionless)

$$\alpha = (k^2+l^2)^{\frac{1}{2}} \qquad (26)$$

Small disturbances of the static state will grow when $\sigma_r > 0$ and decay when $\sigma_r < 0$. We will first investigate marginal stability, defined by

$$\sigma_r = 0 \tag{27}$$

The real and imaginary part of Eq. (25) then give :

$$\sigma_i^2 = C \ , \qquad \sigma_i B = 0 \tag{28}$$

I) Direct (stationary) mode of marginal stability is defined by $\sigma_i = 0$. Eq. (28) then implies $C = 0$, which may be written as

$$Ra = \left(\frac{Rs}{\eta_S\alpha^2+\pi^2} + \frac{\pi^2}{\xi\alpha^2} + 1\right)\left(\eta_T\alpha^2+\pi^2\right) \tag{29}$$

The onset of convection sets in at a wave number which makes Ra an absolute minimum. The wave number corresponding to this minimum, critical value is determined by the polynomial

$$Q^4 + a_3Q^3 + a_2Q^2 + a_1Q + a_0 = 0 \tag{30}$$

where

$$\left.\begin{aligned} Q &= \alpha^2/\pi^2 \\ a_3 &= 2\eta_S^{-1} \\ a_2 &= \eta_S^{-2} + \frac{Rs}{\pi^2}\,\frac{\eta_T-\eta_S}{\eta_T\eta_S^2} - (\xi\eta_T)^{-1} \\ a_1 &= -\,2(\xi\eta_T\eta_S)^{-1} \\ a_0 &= -(\xi\eta_T\,\eta_S^2)^{-1} \end{aligned}\right\} \tag{31}$$

Only positive, real roots for Q have physical sense. In the appendix it is shown that there is one positive, real root. So there is always just one direct mode of marginal stability.

II) Oscillatory (overstable) mode of marginal stability is defined by $\sigma_1 \neq 0$. Eq. (21) then implies B = 0, which may be written as

$$Ra = c\tau Rs + \left(1 + \frac{\pi^2}{\xi\alpha^2}\right)\left[\eta_T\alpha^2+\pi^2+ c\tau(\eta_S\alpha^2+\pi^2)\right] \tag{32}$$

Ra attains its minimum at the wave number

$$\alpha = \pi\left[\frac{1 + c\tau}{\xi(\eta_T+c\tau\eta_S)}\right]^{\frac{1}{4}} \tag{33}$$

determining the critical thermal Rayleigh number for the oscillatory mode. The oscillatory frequency is given by

$$\sigma_1 = C^{\frac{1}{2}} \tag{34}$$

We have chosen to determine critical values for Ra when Rs is fixed. An opposite procedure would have given identical results.

PRESENTATION OF RESULTS

a) Media with thermally insulating matrices.

Figure 1 shows the stability diagram in the Rs, Ra plane with respect to porous media with thermally insulating solid matrices. This class of media is characterized by

$$\eta_T = \eta_S \tag{35}$$

because the diffusion processes of heat and solute differ solely by having different time scales. The subscripts of η will be dropped here. The fluid layer is unstable with respect to small disturbances in the unshaded part of the Rs, Ra plane (Fig.1). The coordinates have been modified in a way that allows arbitrary values of ξ and η.

The critical Rayleigh number for the direct mode is found from (29) :

$$Ra_c = Rs + \pi^2\left[\left(\eta/\xi\right)^{\frac{1}{2}} + 1\right]^2 \tag{36}$$

and for the oscillatory mode from (32) :

$$Ra_c = c\tau Rs + \pi^2\left[\left(\eta/\xi\right)^{\frac{1}{2}} + 1\right]^2 (1 + c\tau) \tag{37}$$

In both cases the critical wave number is given by

$$\alpha = \pi(\xi\eta)^{-1/4} \tag{38}$$

being the same as in one-component convection (Epherre,1975).

For solute Rayleigh number below a certain value Rs^*, a direct mode of disturbance is the most unstable one:

$$Rs^* = \frac{c\tau}{1-c\tau}\,\pi^2\left[\left(\eta/\xi\right)^{\frac{1}{2}} + 1\right]^2 \tag{39}$$

When $Rs > Rs^*$, an oscillatory mode is the most unstable mode of disturbance. The dotted curve in fig. 1 represents the supercritical transition from overstable to direct motion. Linear theory is assumed valid and the wave number is assumed to retain its critical value given by (33). This transition is given by

$$B^2 = 4C \tag{40}$$

These results are in accordance with the results of isotropy, see Weber (1975).

b) Media with thermally conducting matrices.

Equation (35) may still be satisfied if the solid matrix is a heat conductor. But usually this is not the case. Principally there is no correlation between η_T and ξ. The relation between η_T and ξ is not unique, either, but some kind of correlation does exist. The theory by Neale (1977) only deals with simple geometric models. We note that his results satisfy the restriction

$$(\eta_S - \xi)(\eta_S - 1) \leq 0 \qquad (41)$$

It is hereby stated that permeability is more strongly anisotropic than solute diffusivity. Unfortunately, we have no experimental support for this, because measurements of anisotropic diffusivity are lacking.

The critical wave number at the direct mode of marginal stability is given by a fourth degree algebraic equation (30). It is solved numerically by a Newton-Raphson iteration procedure. The corresponding critical Rayleigh number is found from (29).

Some results are displayed in figure 2(a) - (d). The unshaded part of the diagrams correspond to the fluid layer being unstable with respect to small disturbances. Fig. 3 shows the wave number of the most unstable mode of disturbance as a function of Rs, for the same choices of (ξ, η_S, η_T) as fig. 2. In fig. 2-3 we have chosen $c = 1$ and $\tau = 0.01$ for simplicity.

Fig. 2(a) represents a case which may be present in nature: $(\xi, \eta_S, \eta_T) = (10,5,1)$. $Rs^*/\pi^2 = 0.035$ where $Rs = Rs^*$ is the value above which the oscillatory mode takes over as the most unstable one.

Fig. 2 (b) is the "inverse" of the previous case: $(\xi, \eta_S, \eta_T) = (0.1, 0.2, 1)$. $Rs^*/\pi^2 = 0.068$.

Fig. 2 (c) and (d) represent experimental situations where metal threads are introduced into an isotropic medium. They constitute an anisotropic thermal diffusivity without influencing permeability or solute diffusivity. (Fig. 2 (c): $(\xi, \eta_S, \eta_T) = (1, 1, 10)$. $Rs^*/\pi^2 = 0.055$. Fig. (d) : $(\xi, \eta_S, \eta_T) = (1, 1, 0.1)$. $Rs^*/\pi^2 = 0.058$.

At $Rs = Rs^*$, there are small discontinuities in the wave numbers, displayed in Fig. 3. Rs^* is always positive, provided $c\tau \ll 1$.

When $\eta_S \neq \eta_T$, it turns out that the wave number of the direct mode of marginal stability is a function of Rs. The corresponding locus in the Rs, Ra plane is not a straight line. But the locus of the overstable mode of marginal stability is a straight line with a constant wave number. The dotted curve in Fig. 2 (a) - (d) represents the supercritical transition from oscillatory to direct motion as in Fig. 1. Its locus is a hyperbola.

Analytically, we will find asymptotic expressions for the slope $\frac{d(Ra)}{d(Rs)}$ of the direct mode curves and the corresponding wave number α, represented by $Q = \alpha^2/\pi^2$.

Case (i) : Assume $Q \ll 1$ when $|Rs| \gg \pi^2$. Then from (32) and (33)

$$Q = \left[\frac{Rs}{\pi^2}\, \xi\, (\eta_T - \eta_S)\right]^{-\frac{1}{2}} + O(Rs^{-1}) \tag{42}$$

The slope is given by

$$\frac{d(Ra)}{d(Rs)} = 1 + 0(Rs^{-\frac{1}{2}}) \tag{43}$$

This case is relevant when $Rs(\eta_T - \eta_S) > 0$.

Case (ii) : Assume $Q >> 1$ when $|Rs| >> \pi^2$. Then it follows that

$$Q = \left[\frac{Rs}{\pi^2} \frac{\eta_S - \eta_T}{\eta_T \eta_S{}^2}\right]^{-\frac{1}{2}} + 0(Rs^0) \tag{44}$$

with the corresponding slope

$$\frac{d(Ra)}{d(Rs)} = \frac{\eta_T}{\eta_S} + 0(Rs^{-\frac{1}{2}}) \tag{45}$$

This case is relevant when $Rs(\eta_T - \eta_S) < 0$.

Case (iii) : Assume that Q is of order one when $|Rs| >> \pi^2$. This implies $\eta_T = \eta_S$, a case which has been discussed separately.

We have obtained a general knowledge of the asymptotic behaviour of the direct mode curve. Its intersections with the coordinate axes are also known :

$$Rs = 0, \quad Ra = \pi^2\left[\left(\frac{\eta_T}{\xi}\right)^{\frac{1}{2}} + 1\right]^2, \quad \alpha = \pi(\xi\eta_T)^{-\frac{1}{4}} \tag{46}$$

$$Rs = -\pi^2\left[\left(\frac{\eta_S}{\xi}\right)^{\frac{1}{2}} + 1\right]^2, \quad Ra = 0, \quad \alpha = \pi(\xi\eta_S)^{-\frac{1}{4}} \tag{47}$$

Thereby the direct mode curve may be sketched in the general case.

INITIATION OF SALT FINGERS

Salt fingers may occur in the domain of the Rs, Ra plane where the fluid layer is unstable with respect to direct modes of disturbance but still statically stable (Turner, 1974). Then

$$Ra < 0, \qquad Rs < 0 , \qquad Ra - \tau Rs < 0 \tag{48}$$

Salt fingers are a highly nonlinear phenomenon which has not yet been adequately described theoretically. However, it is possible that linear theory may reveal the essential effects of anisotropy. We will study maximum growth rates at large absolute values of the Rayleigh numbers

$$|Ra| >> \pi^2 , \qquad |Rs| >> \pi^2 \tag{49}$$

The ratio between the Rayleigh numbers is denoted by

$$r = Ra/Rs$$

From the previous chapter we conclude that a necessary condition for instability is :

$$r < \max (D,1) \tag{50}$$

where we have introduced a dimensionless diffusion parameter

$$D = \frac{n_T}{n_S} = \frac{\kappa_{mT}^{(1)}}{\kappa_{mT}^{(3)}} \frac{\kappa_{mS}^{(3)}}{\kappa_{mS}^{(1)}} = \tau \frac{\kappa_{mT}^{(1)}}{\kappa_{mS}^{(1)}} \tag{51}$$

Salt fingers are characterized by a large wave number :

$$\alpha^2 >> \pi^2 \tag{52}$$

The fastestgrowing salt fingers indicated by linear theory are given by

$$\sigma = - \frac{dC}{d(\alpha^2)} \Big/ \frac{dB}{d(\alpha^2)} \tag{53}$$

from eq. (24). Equation (24), (25) and (54) subject to the conditions above determine the maximum growth rate

$$\sigma_{max} = \tau |Rs| \left[1 + (rD)^{\frac{1}{2}}\right]^2 \tag{54}$$

occurring at the wave number given by

$$\alpha = |Rs|^{\frac{1}{2}} \left[\left(\frac{r}{n_T n_S}\right)^{\frac{1}{2}} - \frac{r}{n_T}\right]^{\frac{1}{2}} \tag{55}$$

From the last equation it is indicated that the growth of salt fingers is possible only if

$$r < D \tag{56}$$

Compared with (50), it is seen that only a part of the unstable region in the third quadrant may give rise to salt fingers when $D < 1$. Compared with isotropy, the appearance of salt fingers is disfavoured.

When $D > 1$, however, the conditions (50) and (56) are coincident. This indicates a region of salt fingers which is larger than in the case of isotropy.

CONCLUSIONS

A theoretical investigation of thermohaline instability in anisotropic porous media has been performed. Important aspects of the problem is characterized by a dimensionless diffusion parameter D. $D = 1$ when the solid matrix is thermally insulating, and usually $D \neq 1$ when it is thermally conducting.

When $D = 1$ the stability diagram (Fig.1) is congruent to the case of isotropy. When $D \neq 1$, however, the direct mode of instability will show new features: A non-constant wave number and a locus of marginal stability in the Rs, Ra plane not in the form of a straight line. Linear theory suggests that salt fingers are disfavoured when $D < 1$ and favoured when $D > 1$, compared with isotropy ($D = 1$).

A layered medium represented as anisotropic will always have $\xi > 1$, see proof by Bear (1972, p.154). The most frequent situation in aquifers is actually $\xi > 1$ (Bear,1972, p.124) and probably $\eta_S > 1$ (Neale, 1977): η_T should be close t. 1 due to heat conduction in the solid phase.Accordingly $D < 1$,and salt fingers should be disfavoured compared with isotropy. This indicates that anisotropy and stratification tend to reduce the danger of pollution penetrating into aquifers from above.

APPENDIX

It will be shown that equation (32) has always just one positive, real root. The roots are denoted by Q_i (i =1,2,3,4).

Because ξ, η_T and η_S are positive and finite, the following inequalities are easily deduced from (32) and (33) :

$$Q_1 + Q_2 + Q_3 + Q_4 < 0 \qquad \text{(A 1)}$$

$$Q_2Q_3Q_4 + Q_1Q_3Q_4 + Q_1Q_2Q_4 + Q_1Q_2Q_3 > 0 \qquad \text{(A 2)}$$

$$Q_1Q_2Q_3Q_4 < 0 \qquad \text{(A 3)}$$

Because the coefficients in (32) are real, the roots are pairs of complex conjugates unless they are real. There are only two cases which satisfy (A 3) : One or three positive, real roots. In both cases there is one negative, real root (say Q_1) and at least one positive, real root (say Q_2). Then we derive from (A 1) and (A 2) :

$$Q_3 + Q_4 < 0 \qquad \text{(A 4)}$$

implying just one positive, real root.

NOTATION

a_i $(i = 1,2,3)$	coefficients defined in (31).
B	coefficient defined in (25).
c	$= (\rho c_p)_m/(\rho c_p)_f$.
C	coefficient defined in (25).
D	dimensionless diffusion parameter η_T/η_S.
$\mathcal{D}_S$	dimensionless tensor of solute diffusivity.
$\mathcal{D}_T$	dimensionless tensor of thermal diffusivity.
g	gravity acceleration.
h	depth of porous layer.
i	imaginary unit.
$\vec{i},\vec{j},\vec{k}$	unit vectors in x- y- and z- direction
k,l	dimensionless wave numbers.
$K^{(1)}$, $K^{(3)}$	permeabilities horizontally and vertically.
$\mathcal{K}$	dimensionless permeability tensor.
p	dimensionless pressure.
Q	$= \alpha^2/\pi^2$.
r	$= Ra/Rs$.
Ra	thermal Rayleigh number defined in (7).
Rs	solute Rayleigh number defined in (8).
S	dimensionless solute concentration.
t	dimensionless time.
T	dimensionless temperature.
$\vec{v}$	dimensionless velocity ($= u\vec{i} + v\vec{j} + w\vec{k}$)
α	dimensionless wave number.

β, γ	thermal and haline expansion coefficients.
ΔS	difference in solute concentration between lower and upper plane.
ΔT	difference in temperature between lower and upper plane.
η_S	haline anisotropy parameter ($= \kappa_{mS}^{(1)} / \kappa_{mS}^{(3)}$)
η_T	thermal anisotropy parameter ($= \kappa_{mT}^{(1)} / \kappa_{mT}^{(3)}$)
θ	dimensionless temperature perturbation.
$\kappa_{mS}^{(1)}, \kappa_{mT}^{(1)}$	molecular diffusivity of solute and heat horizontally.
$\kappa_{mS}^{(3)}, \kappa_{mT}^{(3)}$	molecular diffusivity of solute and heat vertically.
ν	kinematic viscosity.
ξ	permeability anisotropy parameter ($= K^{(1)} / K^{(3)}$).
π	dimensionless pressure perturbation.
ρ	density.
ρ_0	standard density.
σ	growth rate of disturbance.
τ	ratio between vertical diffusivities ($= \kappa_{mS}^{(3)} / \kappa_{mT}^{(3)}$).

ACKNOWLEDGEMENT

The author acknowledges Dr. J. E. Weber and Dr. E. Riis for their helpful comments.

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Figure legends:

Figure 1 Stability diagram in the (modified) Rs,Ra plane for media with thermally insulating matrices.
———, direct mode of marginal stability;
- - - -, overstable mode of marginal stability;
•••••••, transition from overstable to direct motion.

Figure 2 Stability diagrams in the Rs, Ra plane for some cases of media with thermally conducting matrices. The meaning of solid, broken and dotted curves is given in fig. 1.

(a) $\xi = 10$, $n_S = 5$, $n_T = 1$.

(b) $\xi = 0.1$, $n_S = 0.2$ $n_T = 1$.

(c) $\xi = 1$, $n_S = 1$, $n_T = 10$.

(d) $\xi = 1$, $n_S = 1$, $n_T = 0.1$.

Figure 3 The dimensionless wave number α of the most unstable mode as a function of Rs.

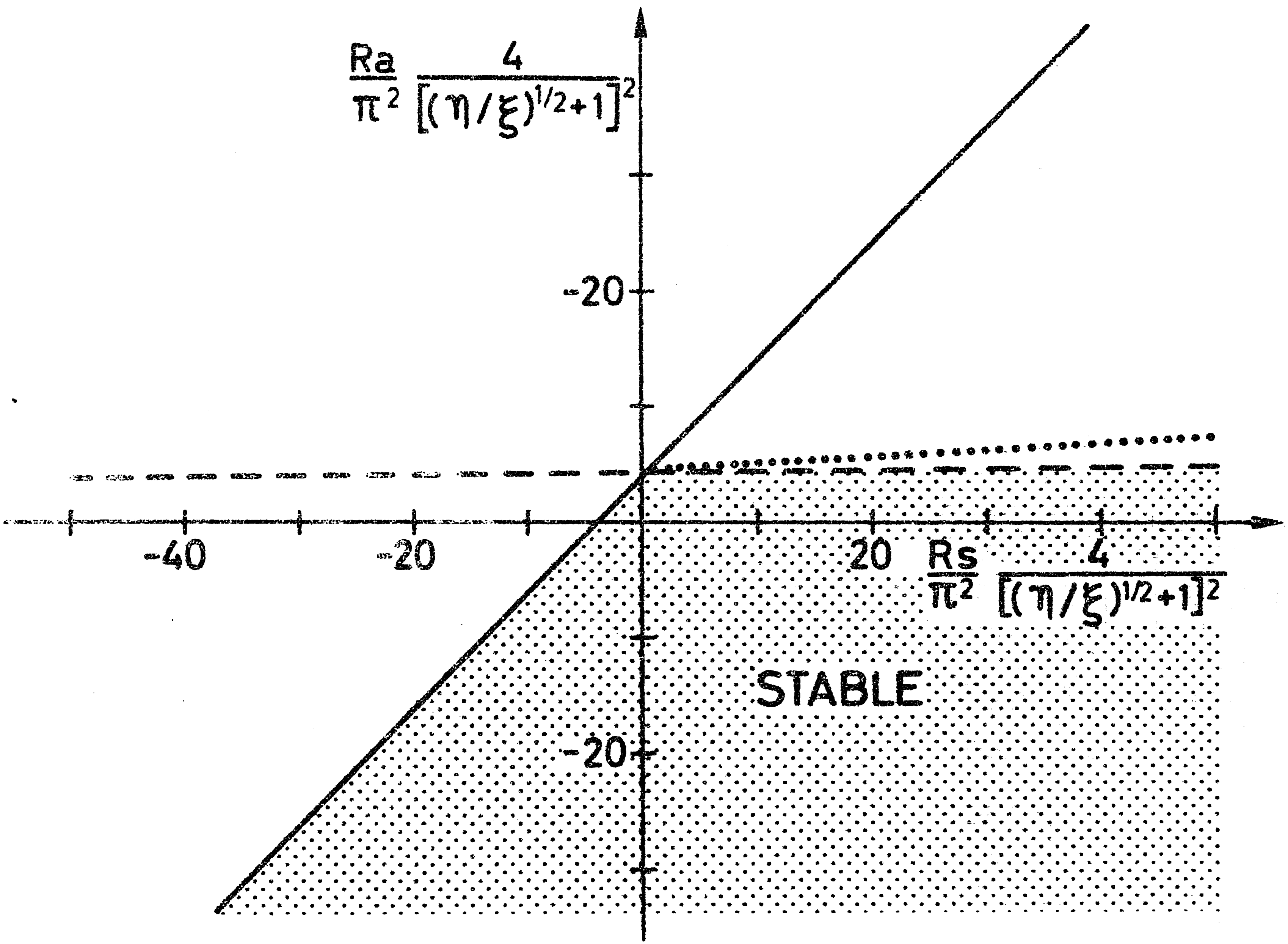
+-+-+-+ , $\xi = 1$, $n_S = 1$, $n_T = 1$,(isotropy);

------- , $\xi = 10$, $n_S = 5$, $n_T = 1$,(see fig.2(a));

-•-•-•- , $\xi = 0.1$, $n_S = 0.2$, $n_T = 1$,(see fig.2(b));

••••••• , $\xi = 1$, $n_S = 1$, $n_T = 10$,(see fig.2(c));

+++++++ , $\xi = 1$, $n_S = 1$ $n_T = 0.1$,(see fig.2(d)).



$\frac{Ra}{\pi^2} \frac{4}{[(\eta/\xi)^{1/2}+1]^2}$
-20
-40
-20
20
$\frac{Rs}{\pi^2} \frac{4}{[(\eta/\xi)^{1/2}+1]^2}$
STABLE
-20

FIG. 1

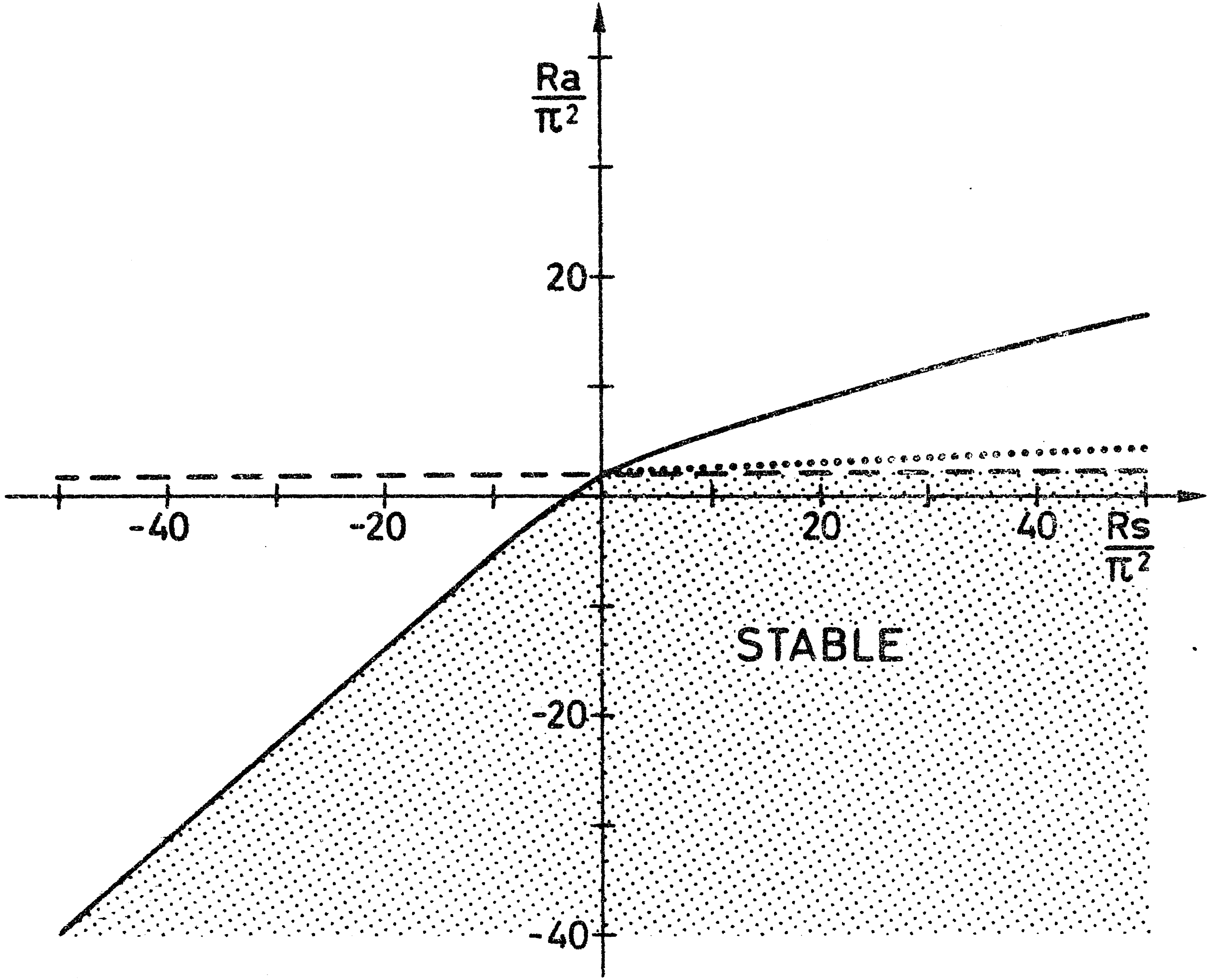


FIG. 2(a)

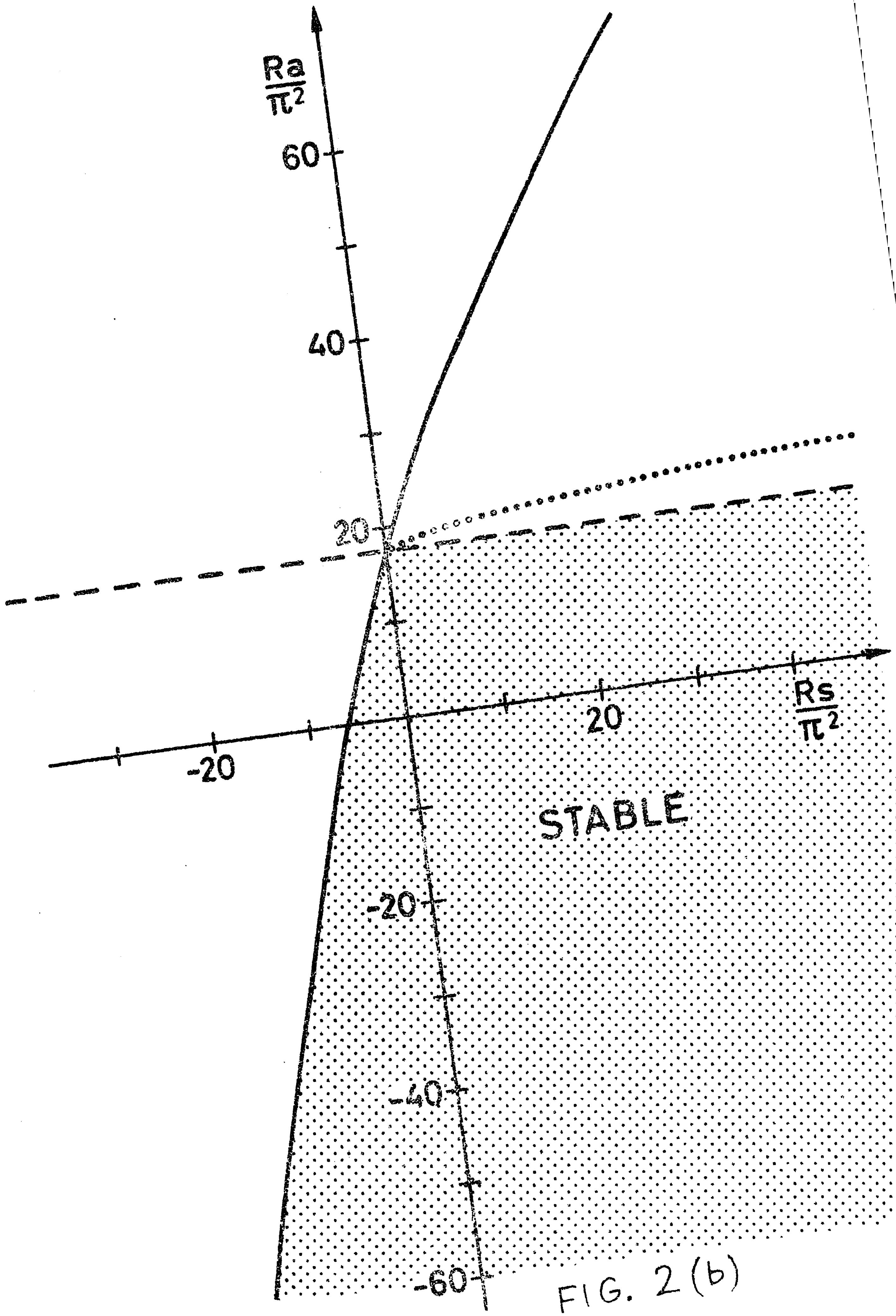


FIG. 2(b)

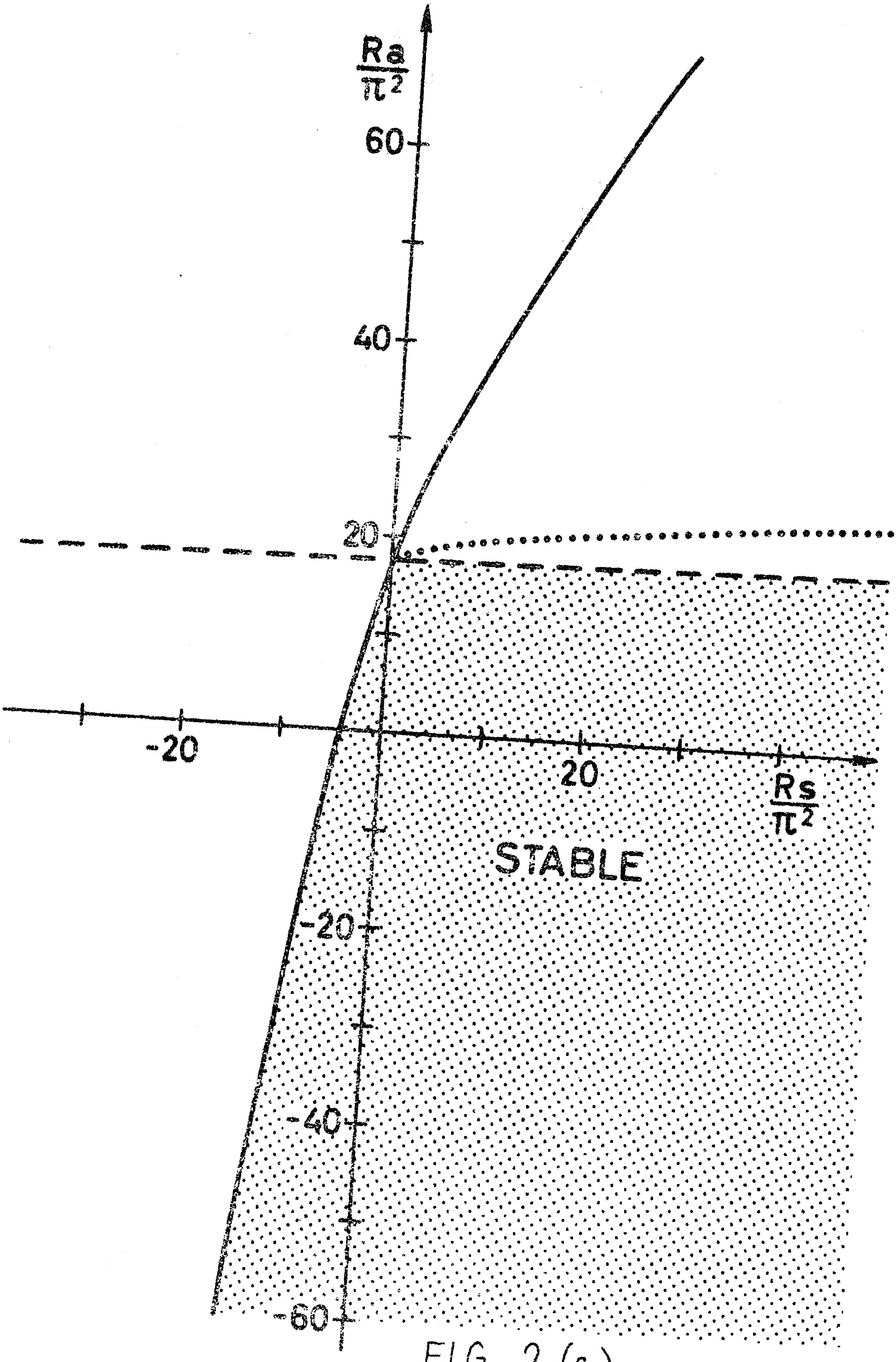


FIG. 2(c)

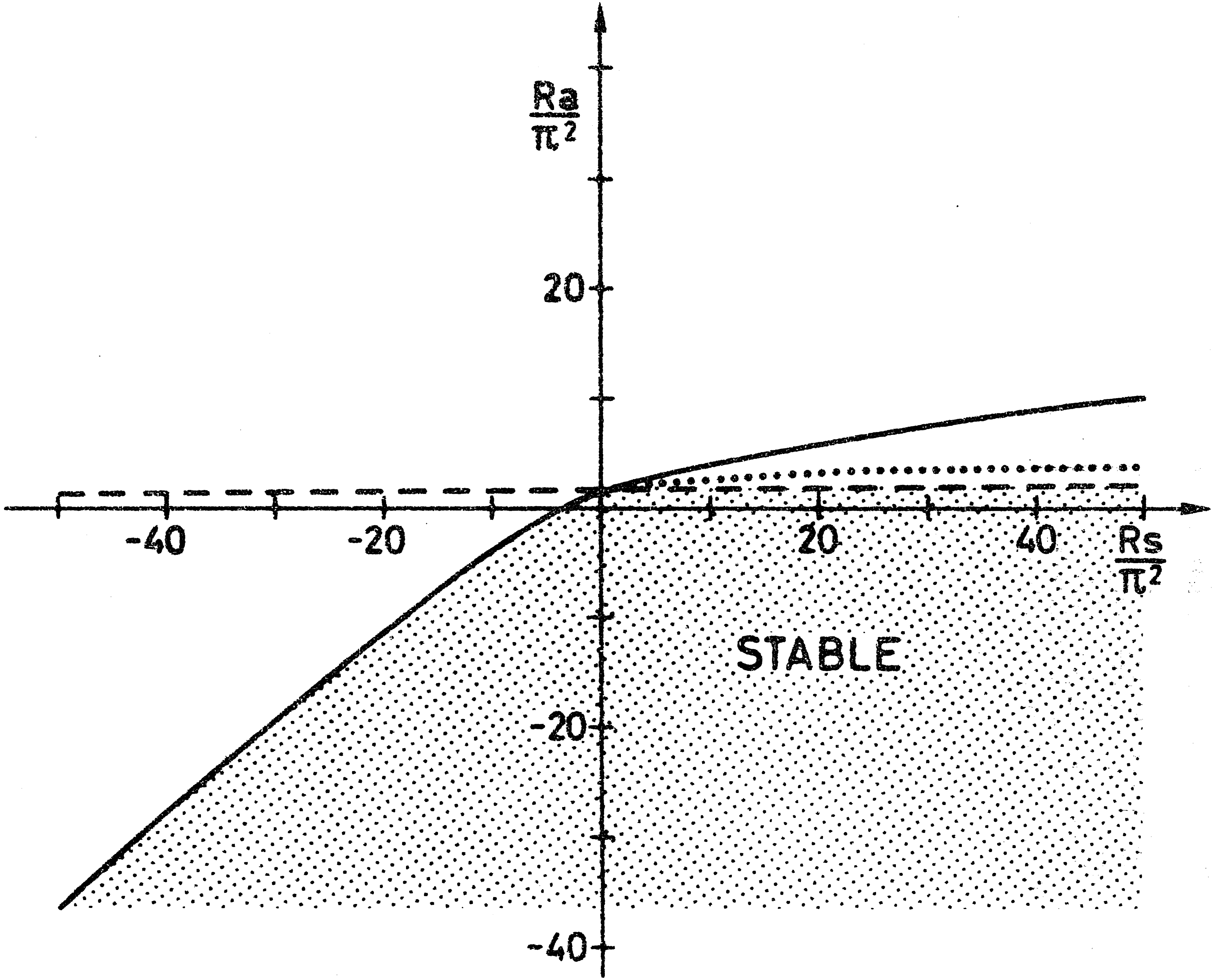


FIG. 2(d)

FIG. 3