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Refinery production planning - model and solution method

by

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Abstract

In this paper we present a general model and solution approach for refinery process planning. The model is nonlinear and has a flexible description to account for different configurations at a refinery. The solution approach is based on solving the nonlinear model directly with a commercial solver. Since the model is highly nonlinear we apply a special procedure to find a good starting solution. We test standard commercial nonlinear solvers on a set of standard test examples.

Contents

1 Introduction

Production planning at a refinery is a complex task. There are many processes and qualities to consider in the planning. We consider a tactical planning problem where the planning period is one month. In this paper we describe model and solution methods developed and used in a testing platform. This platform is further described in detail in [6]. The purpose with the platform is to get an understanding of the refinery process and test different scenarios and assumptions. When developing a general model we need to allow any refinery to be modeled. This means that we need to be flexible and in Figure 1 we illustrate the main components in the refinery process considered. There is an availability of crude oils with specific characteristics. Typical characteristics are density and sulphur contents. These crude oils are then mixed together in Crude Distilling Units (CDU). The crude oils are heated and different fractions will condense at different temperature levels. After a CDU the fractions will go though some processes to improve some of the characteristics. The result is components and examples on components are light fuel oil, naphtha and jet fuel. These components are later blended together into specific products. These products have specific requirements on a set of characteristics.

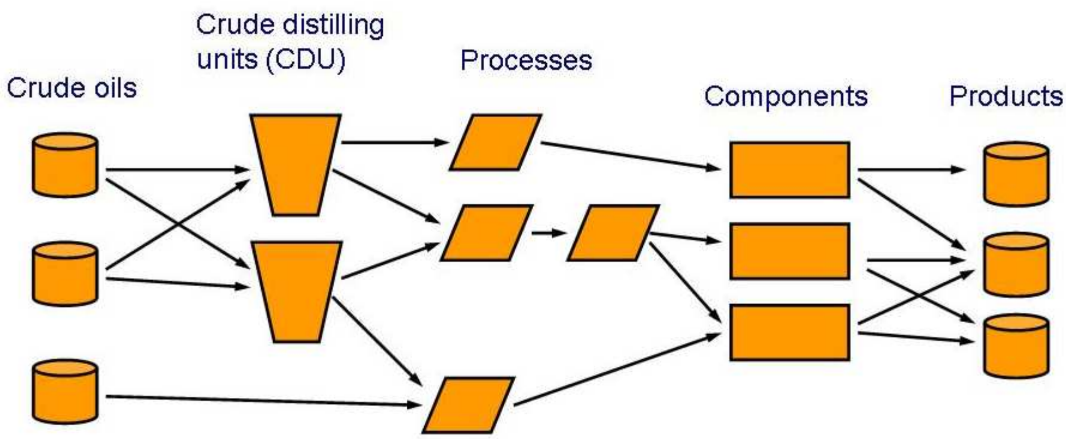

Figure 1: An illustration of a general refinery process.

The processes that appear in a refinery are generally very nonlinear. We have developed a general purpose model which allow the user to use very flexible functions for each of the underlying processes. The standard approach in refinery decision support systems to solve such a nonlinear model is to use a Sequential Linear Programming (SLP) method. Here the nonlinear problem is approximated with a linear formulation and solved. The result is then used to make another revised approximation. This process is repeated until some convergence criteria is satisfied. There are several drawbacks with using such an approach. One is a slow convergence and another that the sensitivity analysis is not correct with respect to the original nonlinear model. In this report we discuss and test different solution approaches. We also test the model and solution method on a set of standard test examples. We also compare a set of well known nonlinear solvers.

The platform keeps all data in an Excel sheet. The data in the Excel sheet is used as an input for a set of models implemented in the AMPL modeling language (see [2]). The models are solved using commercial solvers for nonlinear optimization models and the solution is inserted back into the Excel sheet. A feature of the platform is also the possibility to automatically print, for example, the system network or solutions. An example of a network model is given in Figure 2.

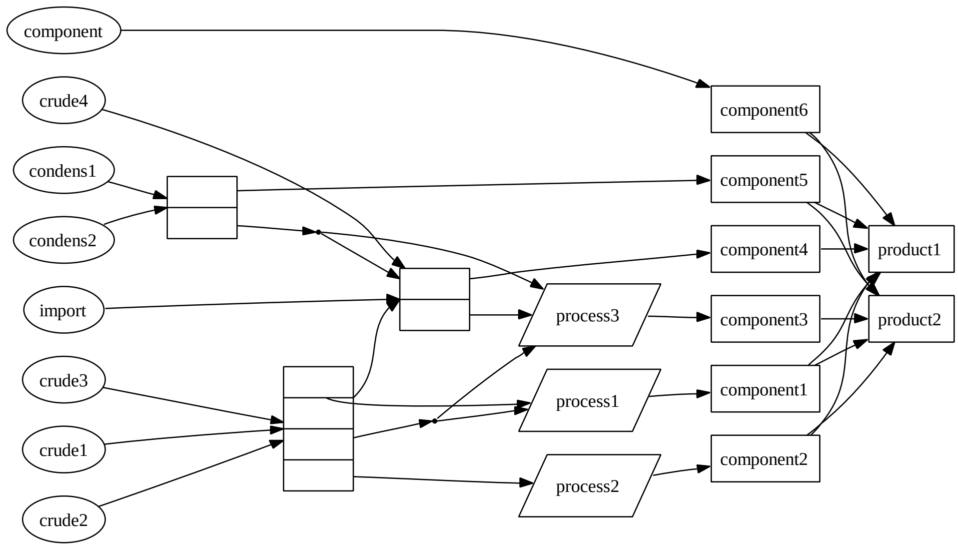

Figure 2: An example of a network defined in the platform.

In Section 2 we define the general nonlinear model used in the platform. In Section 3 we discuss the solution method, test examples and results. In 4 we make some concluding remarks.

2 Model

We will develop the model in a number of steps. These include the index sets, parameter data, decision variables, objective function and constraints.

Sets

We start by defining the index sets used in the model. These are general and can include any number of, for example, crude oils or components.

O_{Cru}	:	Set of Crude oils, index i
O_{Imp}	:	Set of Import oils, index i
O_{Com}	:	Set of Component oils, index i
O_{Con}	:	Set of Condens oils, index i
O	:	Set of all oil inputs to the planning process: $O_{Cru} \cup O_{Imp} \cup O_{Com} \cup O_{Con}$, index i
G	:	Set of processes with several inputs and several outputs, index c
F	:	Set of outputs from CDU or processes, index f.
G_c^{in}	:	Set of inputs from CDU c.
G_c^{out}	:	Set of outputs to CDU c.
P	:	Set of processes with only one output and one or several inputs, index p.
P_p^{in}	:	Inputs from process p.
P_p^{out}	:	Output from process p (singleton).
C	:	Set of components, index j.
C_j^{in}	:	Input to component j, index f.
K	:	Set of products, index k.
Q	:	Set of qualities, index q.
S	:	Set of outputs from all splits.
S_i	:	Set of outputs from splits of oil or output i, index s.
T	:	Set of time periods, index t $(1, 2, \ldots, N_t)$.

Parameter data

There is a large number of data required. Some are defining for example the available volume of crude oils or the demanded volumes of products. Other more special data is connected with the processes and define how the output is linked through the input. We define the data linked to each of the parts in the refinery process.

Crude oil:

$\overline{S}_{it}, \underline{S}_{it}$	:	Max and min supply of oil i during time period t.
$S0_i$	:	Initial storage of oil i.
$c_i^{O,Tend}$	:	Storage value at the end of the planning process for oil i.
c_{it}^{O}	:	Cost of oil i in time period t.
$c_i^{O,T0}$	:	Storage value at the beginning of the planning process for oil i.
$c_i^{O,stor}$	:	Storage cost for oil i.
S_{iqt}^{Q}	:	Quality of oil i for quality q bought in time period t.
$U0_{iq}^{Q,oil}$	:	Initial quality concentration for storage of oil i and quality q.

CDUs:

F_{cif}^{frac}	:	Fraction of output f given input i in CDU c.
S_{cifq}^{frac}	:	Quality fraction of quality q from CDU c from input i giving output f.
$\overline{C}_{ct}^{cdu}, \underline{C}_{ct}^{cdu}$	:	Max and min volume generated in CDU c in time period t.
c_{ct}^{cdu}	:	CDU operating costs at CDU c in time period t.

Processes:

Pol_{piz}^{vol}	:	Polynomial coefficients (volume) ($z = 0, 1, 2$) for input i in process p.
Pol_{pqiz}^{qual}	:	Polynomial coefficients (quality) ($z = 0, 1, 2$) for input i, quality q in process p.
$c_{pt}^{proc,vol}$	:	Process operating costs (volume based) at process p in time period t.
$c_{pt}^{proc,mass}$	:	Process operating costs (mass based) at process p in time period t.
$\underline{D}_{pt}^{in,proc}, \overline{D}_{pt}^{in,proc}$	:	Lower and upper process capacity in to process p in time period t.
$\underline{D}_{pt}^{out,proc}, \overline{D}_{pt}^{out,proc}$	:	Lower and upper process capacity out from process p in time period t.

Components:

$U0_{jq}^{Q,com}$	:	Initial quality concentration for storage of component j and quality q.
$c_j^{C,stor}$	:	Storage cost for component j.
$C0_j$	:	Initial storage of component j.
$c_j^{C,Tend}$	:	Storage value at the end of the planning process for component j.
$c_j^{C,T0}$	:	Storage value at the beginning of the planning process for component j.

Products:

$U0_{kq}^{Q,prod}$	:	Initial quality concentration for storage of product k and quality q.
$\overline{D}_{kqt}^{Q,prod}, \underline{D}_{kq}^{Q}$	:	Max and min Quality requirement of product k for quality q in time period t.
$c_{kt}^{K,vol}$	:	Volume value of product k in time period t.
$c_{kt}^{K,mass}$	:	Mass value of product k in time period t.
$\overline{D}_{kt}, \underline{D}_{kt}$	:	Max and min demand of product k during time period t.

Variables

There are four types of decision variables. The first set is associated with the various physical flows (or mass) in the refinery. The second type is associated with the measured quality characteristics. The third is associated with the volumes e.g. buying or storage, in each of the time periods. The fourth type is help variables making the model easier to understand or express.

Crude oil

$x_{it}^{oil,buy}$ = Volume of oil i bought in time period t.
$x_{it}^{oil,avail}$ = Volume of oil i available in time period t (help variable).
$x_{ict}^{oil,cdu}$ = Flow of oil i to CDU c in time period t.
$x_{ipt}^{oil,proc}$ = Flow of oil i to process p in time period t.
$x_{ijt}^{oil,com}$ = Flow of oil i to component j in time period t.
$u_{iqt}^{Q,oil}$ = Quality concentration of oil i for quality q in time period t.
$v_{it}^{oil,stor}$ = End storage of oil i in time period t.

CDUs and processes

y_{cft}^{cdu} = Volume generated in CDU c of output f in time period t.
$u_{cfqt}^{Q,cdu}$ = Quality concentration after CDU c for output f and quality q in time period t.
z_{pt}^{in} = Volume to process p in time period t.
z_{pft}^{out} = Volume from process p of output f in time period t.
w_{pt}^{in} = Mass to process p in time period t.
w_{pt}^{out} = Mass from process p in time period t.
$u_{cfqt}^{Q,proc}$ = Quality concentration after process p for output f and quality q in time period t.

Components and products

z_{jt}^{avail} = Volume of component j available in time period t (help variable).
$u_{jqt}^{Q,com}$ = Quality concentration in component j and quality q in time period t.
$v_{jt}^{com,stor}$ = End storage of component j in time period t.
z_{jkt}^{blend} = Volume blended from component j to product k in time period t.
$v_{kt}^{prod,vol}$ = Volume of product k in time period t.
w_{kt}^{prod} = Mass produced of product k in time period t (help variable).
$u_{kqt}^{Q,prod}$ = Quality concentration in product k and quality q in time period t.

Assumptions

In the model we make some general assumptions.

- Storage is only used for input oils and components.
- Only one output from a process.

- All functions used in the model are quadratic polynomials. There are three coefficients associated with the polynomials: a0 (constant), a1 (linear term), a2 (quadratic term).

Objective function

The objective function express the profit as the overall revenue (z_1) minus the total cost (z_2). The revenue is derived from selling of products (either measured in volume or in mass) and the value of having components and crude oils in storage after the finishing time period. The total cost consists of initial inventory of components and crude oils, cost of buying crude oils, CDU operating costs, process operating costs (either measured in volume or mass) and storage costs for crude oils and components.

$$\begin{aligned}
\max \quad z = \quad & z_1 - z_2 \\
z_1 = \quad & \sum_{k\in K}\sum_{t\in T}(c_{kt}^{K,vol} v_{kt}^{prod,vol} + c_{kt}^{K,mass} w_{kt}^{prod})+ \\
& \sum_{j\in C}(c_j^{C,Tend} v_{j,last(T)}^{com,stor} - c_j^{C,T0} v_{j,0}^{com,stor})+ \\
& \sum_{i\in O}(c_i^{O,Tend} v_{i,last(T)}^{oil,stor} - c_i^{O,T0} v_{i,0}^{oil,stor}) \\
z_2 = \quad & \sum_{i\in O}\sum_{t\in T} c_{it}^{O} x_{it}^{oil,buy} + \sum_{p\in P}\sum_{t\in T}(c_{pt}^{proc,vol} z_{pt}^{in} + c_{pt}^{proc,mass} w_{pt}^{in})+ \\
& \sum_{c\in G}\sum_{t\in T}(c_{ct}^{cdu} * \sum_{i\in O} x_{ict}^{oil,cdu})+ \\
& \sum_{i\in O}\sum_{t\in T} c_i^{O,stor} v_{it}^{oil,stor} + \sum_{j\in C}\sum_{t\in T} c_j^{C,stor} v_{jt}^{com,stor}
\end{aligned}$$

Constraints

The constraints provide limits on supply and demand, quality requirements, flow conservation, process balances and computations of quality values.

Crude oils

$$v_{i,0}^{oil,stor} - S0_o = 0, \quad \forall i \tag{1}$$

$$x_{it}^{oil,avail} - x_{it}^{oil,buy} - v_{i,t-1}^{oil,stor} = 0, \quad \forall i,t \tag{2}$$

$$v_{i,t-1}^{oil,stor} + x_{it}^{oil,buy} - v_{it}^{oil,stor} - \sum_{c \in G} x_{ict}^{oil,cdu} - \sum_{p \in P} x_{ipt}^{oil,proc} - \sum_{j \in C} \sum_{f \in C_j^{in} \cap S_i} x_{fjt}^{oil,com} - \sum_{j \in C} x_{ijt}^{oil,com} = 0, \quad \forall i,t \tag{3}$$

$$u_{i,q,0}^{Q,oil} - U0_{iq}^{Q,oil} = 0, \quad \forall i,q \tag{4}$$

$$u_{iqt}^{Q,oil} * x_{it}^{oil,avail} - S_{iqt}^{Q} x_{it}^{oil,buy} - u_{i,q,t-1}^{Q,oil} * v_{i,t-1}^{oil,stor} = 0, \quad \forall i,q,t \tag{5}$$

CDUs

$$y_{cft}^{cdu} - \sum_{i \in O} F_{cif}^{frac} x_{ict}^{oil,cdu} = 0, \quad \forall c,f,t \tag{6}$$

$$u_{cfqt}^{Q,cdu} * y_{cft}^{cdu} - \sum_{i \in G_c^{in} \cap (O \cup S)} S_{c,i,f,q}^{frac} * F_{c,i,f}^{frac} * x_{ict}^{oil,cdu} = 0, \quad \forall c,q,f \in G_c^{out},t \tag{7}$$

$$y_{cft}^{cdu} - \sum_{s \in S_f} y_{cst}^{cdu} = 0, \quad \forall c,f \in G_c^{out} \cap S,t \tag{8}$$

$$u_{csqt}^{Q,cdu} - u_{cfqt}^{Q,cdu} = 0, \quad \forall c,f \in G_c^{out} \cap S,q, \; s \in S_f,t \tag{9}$$

$$u_{sfqt}^{Q,split} = u_{cfqt}^{Q,cdu} \text{ or } u_{cfqt}^{Q,proc} \qquad \forall s,f,q,t \tag{10}$$

Process

$$z_{pt}^{in} - \sum_{f \in P_p^{in}} \Big(\sum_{c \in G} y_{cft}^{cdu} - \sum_{\overline{p} \in P} z_{\overline{p}ft}^{out} - \sum_{i \in O: i=f} x_{fpt}^{oil,proc}\Big) = 0, \quad \forall p,t \tag{11}$$

$$z_{pft}^{out} - \sum_{c \in G} \sum_{\underline{f} \in P_p^{in} \cap G_c^{out}} h_{c\underline{f}t}^{y}(y_{c\underline{f}t}^{cdu}) - \sum_{\overline{p} \in P} \sum_{\underline{f} \in P_p^{in} \cap P_{\overline{p}}^{out}} h_{\overline{p}\underline{f}t}^{z}(z_{\overline{p}\underline{f}t}^{out}) - \sum_{\underline{f} \in P_p^{in} \cap O} h_{\underline{f}pt}^{x}(x_{\underline{f}pt}^{oil,proc}) = 0, \quad \forall p,f \in P_p^{out},t \tag{12}$$

$$u_{pfqt}^{Q,proc} * z_{pft}^{out} - \sum_{\underline{f} \in P_p^{in}} \Big(\sum_{c \in G} h_{c\underline{f}qt}^{u1}(u_{c\underline{f}qt}^{Q,cdu}) + \sum_{\overline{p} \in P} h_{\overline{p}\underline{f}qt}^{u2}(u_{\overline{p}\underline{f}qt}^{Q,proc}) + \sum_{i \in O \cap \{\underline{f}\}} h_{iqt}^{u1}(u_{iqt}^{Q,oil})\Big) * \Big(\sum_{c \in G} h_{c\underline{f}t}^{y}(y_{c\underline{f}t}^{cdu}) + \sum_{\overline{p} \in P} h_{\overline{p}\underline{f}t}^{z}(z_{\overline{p}\underline{f}t}^{out}) + \sum_{i \in O \cap \{\underline{f}\}} h_{\underline{f}pt}^{x}(x_{\underline{f}pt}^{oil,proc})\Big) = 0, \quad \forall p,q,f \in P_p^{out},t \tag{13}$$

$$w_{pt}^{in} - \sum_{c \in G} \sum_{f \in P_p^{in} \cap G_c^{out}} u_{cfqt}^{Q,cdu} * y_{cft}^{cdu} - \sum_{\overline{p} \in P} \sum_{f \in P_p^{in} \cap P_{\overline{p}}^{out}} u_{\overline{p}fqt}^{Q,proc} * z_{\overline{p}ft}^{out} - \sum_{f \in P_p^{in} \cap O} u_{fqt}^{Q,oil} * x_{fpt}^{oil,proc} = 0, \quad \forall p,q='density',t \tag{14}$$

$$w_{pt}^{out} - \sum_{f \in P_p^{out}} z_{pft}^{out} * u_{pfqt}^{Q,proc} = 0, \quad \forall p,q='density',t \tag{15}$$

$$z_{pft}^{out} - \sum_{s \in S_f} z_{pst}^{out} = 0, \quad \forall p,f \in P_p^{out} \cap S,t \tag{16}$$

$$u_{psqt}^{Q,proc} - u_{pfqt}^{Q,proc} = 0, \quad \forall p,f \in P_p^{out} \cap S,q, \; s \in S_f,t \tag{17}$$

Components

$$
\begin{aligned}
v^{com,stor}_{j,0} - C0_j &= 0, && \forall j && (18)\\
z^{avail}_{jt} - \sum_{p\in P}\sum_{f\in P^{out}_p\cap C^{in}_j} z^{out}_{pft} - \sum_{i\in O}\sum_{f\in C^{in}_j} x^{oil,com}_{ijt} & && &&\\
- \sum_{c\in G}\sum_{f\in G^{out}_c\cap C^{in}_j} y^{cdu}_{cft} - v^{com,stor}_{j,t-1} &= 0, && \forall j,t && (19)\\
v^{com,stor}_{jt} - v^{com,stor}_{j,t-1} - \sum_{p\in P}\sum_{f\in P^{out}_p\cap C^{in}_j} z^{out}_{pft} & && &&\\
- \sum_{i\in O}\sum_{f\in C^{in}_j\cap S^{oil}_i} x^{oil,com}_{fjt} - \sum_{i\in O}\sum_{f\in C^{in}_j} x^{oil,com}_{ijt} & && &&\\
- \sum_{c\in G}\sum_{f\in G^{out}_c\cap C^{in}_j} y^{cdu}_{cft} - \sum_{k\in K} z^{blend}_{jkt} &= 0, && \forall j,t && (20)\\
u^{Q,com}_{j,q,0} - U0^{Q,com}_{jq} &= 0, && \forall j,q && (21)\\
u^{Q,com}_{jqt} * z^{avail}_{jt} - \sum_{p\in P}\sum_{f\in C^{in}_j\cap P^{out}_p} u^{Q,proc}_{pfqt} * z^{out}_{pft} - & && &&\\
\sum_{i\in O}\sum_{f\in C^{in}_j\cap S_i} u^{Q,oil}_{iqt} * x^{oil,com}_{fjt} - \sum_{i\in O}\sum_{f\in C^{in}_j} u^{Q,oil}_{iqt} * x^{oil,com}_{ijt} - & && &&\\
\sum_{c\in G}\sum_{f\in C^{in}_j\cap G^{out}_c} u^{Q,cdu}_{cfqt} * y^{cdu}_{cft} - u^{Q,com}_{j,q,t-1} * v^{com,stor}_{j,t-1} &= 0, && \forall j,q,t && (22)
\end{aligned}
$$

Products

$$
\begin{aligned}
v^{prod,vol}_{kt} - \sum_{j\in C} z^{blend}_{jkt} &= 0, && \forall k,t && (23)\\
w^{prod}_{kt} - v^{prod,vol}_{kt} * u^{Q,prod}_{kqt} &= 0, && \forall k, q =' density', t && (24)\\
u^{Q,prod}_{kqt} * v^{prod,vol}_{kt} - \sum_{j\in C} u^{Q,com}_{jqt} * z^{blend}_{jkt} &= 0, && \forall k,q,t && (25)
\end{aligned}
$$

Crude oils

$$
\begin{aligned}
\underline{D}^{oil,avail}_{it} \le x^{oil,avail}_{it} &\le \overline{D}^{oil,avail}_{it} && \forall i,t && (26)\\
\underline{S}_{ot} \le x^{oil,buy}_{it} &\le \overline{S}_{ot} && \forall i,t && (27)\\
\underline{D}^{oil,stor}_{it} \le v^{oil,stor}_{it} &\le \overline{D}^{oil,stor}_{it} && \forall i,t && (28)\\
\underline{D}^{Q,oil}_{iqt} \le u^{Q,oil}_{iqt} &\le \overline{D}^{Q,oil}_{iqt} && \forall i,q,t && (29)
\end{aligned}
$$

CDU

$$\underline{D}_{ict}^{oil,cdu} \leq x_{ict}^{oil,cdu} \leq \overline{D}_{ict}^{oil,cdu} \quad \forall i,c,t \quad (30)$$

$$\underline{D}_{cft}^{cdu,out} \leq y_{cft}^{cdu} \leq \overline{D}_{cft}^{cdu,out} \quad \forall c,f,t \quad (31)$$

$$\underline{C}_{ct}^{cdu} \leq \sum_{f \in G_c^{out}} y_{cft}^{cdu} \leq \overline{C}_{ct}^{cdu}, \quad \forall c,t \quad (32)$$

$$\underline{D}_{pfqt}^{Q,cdu} \leq u_{pfqt}^{Q,cdu} \leq \overline{D}_{pfqt}^{Q,cdu} \quad \forall p,f,q,t \quad (33)$$

Process

$$\underline{D}_{pt}^{in,proc} \leq z_{pt}^{in} \leq \overline{D}_{pt}^{in,proc}, \quad \forall p,t \quad (34)$$

$$\underline{D}_{ipt}^{oil,proc} \leq x_{ipt}^{oil,proc} \leq \overline{D}_{ipt}^{oil,proc} \quad \forall i,p,t \quad (35)$$

$$\underline{D}_{pft}^{out,proc} \leq \sum_{f \in P_p^{out}} z_{pft}^{out} \leq \overline{D}_{pt}^{out,proc}, \quad \forall p,t \quad (36)$$

$$\underline{D}_{pft}^{proc,out} \leq z_{pft}^{out} \leq \overline{D}_{pft}^{proc,out} \quad \forall p,f,t \quad (37)$$

$$\underline{D}_{pt}^{proc,Min} \leq w_{pt}^{in} \leq \overline{D}_{pt}^{proc,Min} \quad \forall p,t \quad (38)$$

$$\underline{D}_{pt}^{proc,Mout} \leq w_{pt}^{out} \leq \overline{D}_{pt}^{proc,Mout} \quad \forall p,t \quad (39)$$

$$\underline{D}_{pfqt}^{Q,proc} \leq u_{pfqt}^{Q,proc} \leq \overline{D}_{pfqt}^{Q,proc} \quad \forall p,f,q,t \quad (40)$$

Components

$$\underline{D}_{ijt}^{oil,com} \leq x_{ijt}^{oil,com} \leq \overline{D}_{ijt}^{oil,com} \quad \forall i,j,t \quad (41)$$

$$\underline{D}_{jt}^{com,avail} \leq z_{jt}^{avail} \leq \overline{D}_{jt}^{com,avail} \quad \forall j,t \quad (42)$$

$$\underline{D}_{jt}^{com,stor} \leq v_{jt}^{com,stor} \leq \overline{D}_{jt}^{com,stor} \quad \forall j,t \quad (43)$$

$$\underline{D}_{jqt}^{Q,com} \leq u_{jqt}^{Q,com} \leq \overline{D}_{jqt}^{Q,com} \quad \forall j,q,t \quad (44)$$

Products

$$\underline{D}_{jkt}^{blend,vol} \leq z_{jkt}^{blend} \leq \overline{D}_{jkt}^{blend,vol} \quad \forall j,k,t \quad (45)$$

$$\underline{D}_{kt}^{prod,vol} \leq v_{kt}^{prod,vol} \leq \overline{D}_{kt}^{prod,vol} \quad \forall k,t \quad (46)$$

$$\underline{D}_{kt}^{prod,mass} \leq w_{kt}^{prod} \leq \overline{D}_{kt}^{prod,mass} \quad \forall k,t \quad (47)$$

$$\underline{D}_{kqt}^{Q,prod} \leq u_{kqt}^{Q,prod} \leq \overline{D}_{kqt}^{Q,prod} \quad \forall k,q,t \quad (48)$$

all variables ≥ 0,

The function $h_*^+(x)$ is a second order polynomial of the form
$h_*^+(x) = P_*^{+,0} + P_*^{+,1} * x + P_*^{+,2} * x^2$
where $P_*^{+,0}$, $P_*^{+,1}$ and $P_*^{+,2}$ are the coefficients in the polynomials and where $*$ denotes the indexes and $+$ denotes the variable of the polynomial. The indexes $*$ consist of either three or four indexes. We use abbreviations for the variables $+$, where y, z, x, $u1$, $u2$, and $u3$ corresponds to y_{cft}^{cdu}, z_{pft}^{out}, $x_{fpt}^{oil,proc}$, $u_{iqt}^{Q,oil}$, $u_{cfqt}^{Q,cdu}$, and $u_{cfqt}^{Q,prod}$, respectively.

A summary and explanation of all constraints are found in Table 1.

Constraint set	Description
	Crude oils
(1)	sets the initial oil level.
(2)	available volume of oil each time period.
(3)	crude oil balance.
(4)	inial qualities of different oils.
(5)	quality balance of oil.
	CDUs
(6)	volume balance through the CDU.
(7)	quality balance in and out of the CDUs.
(8)	split of flows after a CDU
(9)	quality balance at a split after a CDU.
(10)	connect quality before and after a split.
	Processes
(11)	flow balance to the processes
(12)	balance flow through processes
(13)	quality balance through processes.
(14)	mass flow in to a process.
(15)	mass flow out of a process.
(16)	split of flows after a process.
(17)	quality balance at a split after a process.
	Components
(18)	initial storage of components.
(19)	total volumes available of components in each time period.
(20)	flow balance for the components.
(21)	sets the initial quality of components.
(22)	quality balance of components.
	Products
(23)	blending of products.
(24)	mass flow of products.
(25)	quality balance for products.
	Bounds
(26)	lower and upper limits on available oil.
(27)	lower and upper limits on volumes of oil to buy.
(28)	lower and upper limits on stored volumes of oil.
(29)	lower and upper limits on qualities of oils.
(30)	lower and upper limits on volumes in to CDUs.
(31)	lower and upper limits on volumes for each output from CDUs.
(32)	lower and upper limits on volume of all outputs generated in CDUs.
(33)	lower and upper limits on qualities after CDUs.
(34)	lower and upper limits on volumes in to processes.
(35)	lower and upper limits on oils to processes.
(36)	lower and upper limits on volume of all outputs from processes.
(37)	lower and upper limits on volume of output from processes.
(38)	lower and upper limits on mass to processes.
(39)	lower and upper limits on mass from processes.
(40)	lower and upper limits on qualities after processes.
(41)	lower and upper limits on volumes to components.
(42)	lower and upper limits on available components.
(43)	lower and upper limits on stored volumes of components.
(44)	lower and upper limits on qualities of components.
(45)	lower and upper limits on blending volumes.
(46)	lower and upper limits on produced volumes of products.
(47)	lower and upper limits on produced mass of products.
(48)	lower and upper limits on qualities of products.

Table 1: Description of the constraints in the model.

3 Computational experiments

3.1 Solution method

The model includes many equality constraints and the constraints are very nonlinear. Blending constraints where the quality is unknown are known to be very nonlinear. To use a SLP method if often coupled with problems as the linear approximations of the nonlinear functions often is of low quality. To overcome this in SLP methods different types of bounding procedures are used. Another difficulty is to find a good feasible solution. Without a good starting guess the nonlinear method may fail even to find a feasible solution.

In our approach we start by formulating an LP model where we pre-generate all possible paths for any flow. This means that we for example generate a path from a crude oil through the CDU through the processes and the components tanks to the products. In Figure 3 we illustrate one path-variable. With such variables we can explicitly compute all qualities on the path. This is a simplification of the general model but provides a much stronger description than using separate flows (as the variables in the general model). With this path-flow model, PathLP, we can generate a feasible flow while approximating the qualities for the entire model. Given a solution to the path-flow model, we can compute the real quality values in the processes and blending for this flow. We then solve a linearized problem of the nonlinear model. We perform two linearizations. The first one is done at the solution to the path-flow model with computed qualities. The second linearization is done at the solution to the first linearization after computing the qualities again. This will provide a high quality starting solution for the nonlinear solvers.

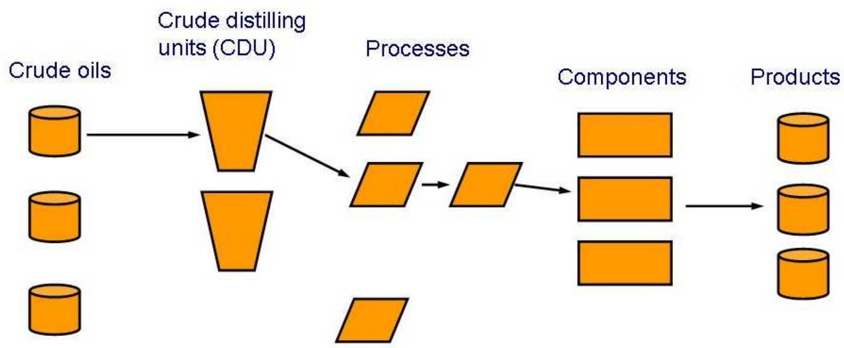

Figure 3: Illustration of one path-variable.

3.2 Computational experiments

We have studied three different cases named Aronofsky, AronofskyNLB and SmallNLBP. Case Aronofsky is the classic refinery model of Aronofsky *et al.* [1]. It consists of four processing units and produces five products over one time period. The refinery flow chart in Figure 4 illustrates the refinery process with all units, inputs, components and products. It is a linear programming problem. Case AronofskyNLB is a modification of Case Aronofsky where the number of components are reduced to give a nonlinear blending of products (the qualities). For a comprehensive description of the problem and the modifications see [6]. Case AronofskyNLB gives a problem which is a nonlinear programming problem.

Case-Aronofsky

The refinery process in the Aronofsky case is described in Figure 7. The platform generates a set of output files and the flowchart is one such. Another result is a flowchart with the optimal flows and this is shown in Figure 5. This case can be described as LP problem and when we solve the path-flow model we obtain the optimal solution directly. We are also interested in the qualities and to provide an example we present the octane values in Figure 6.

Case-AronofskyNLB

In this case we have nonlinear blending and hence a nonlinear problem. The refinery process is described in Figure 7 and the solution in Figure 8. The solution found is guaranteed a local optimal solution. However, as the problem is non-convex we can not guarantee a global optimal solution. The octane values are given in Figure 9.

Case-SmallNLBP

Case SmallNLBP has a process which is nonlinear as well as a nonlinear blending of products (the qualities) with one planning period. The refinery process is illustrated in Figure 10 and the optimal solution is given in Figure 11. For this case we present the density values in Figure 12.

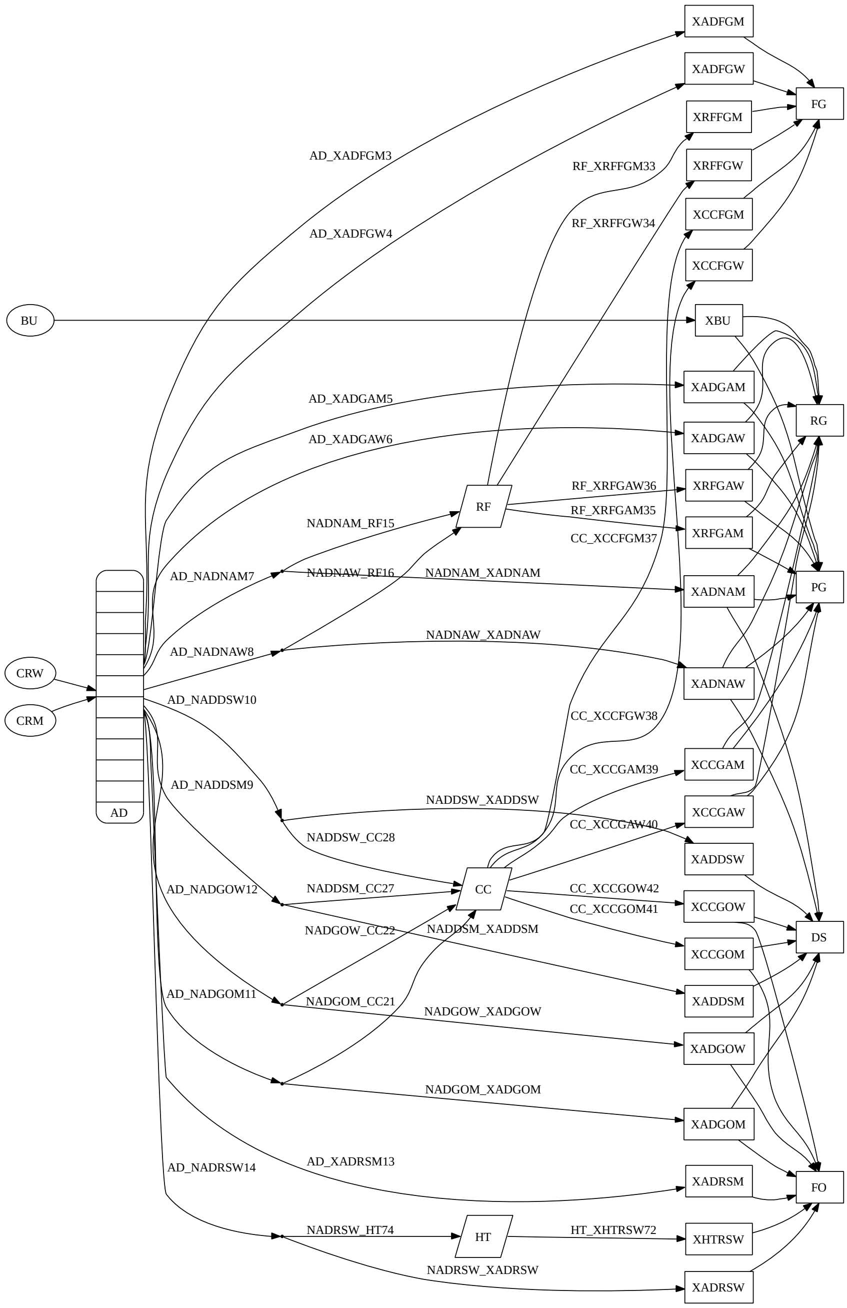

Figure 4: A refinery flow chart of Case Aronofsky.

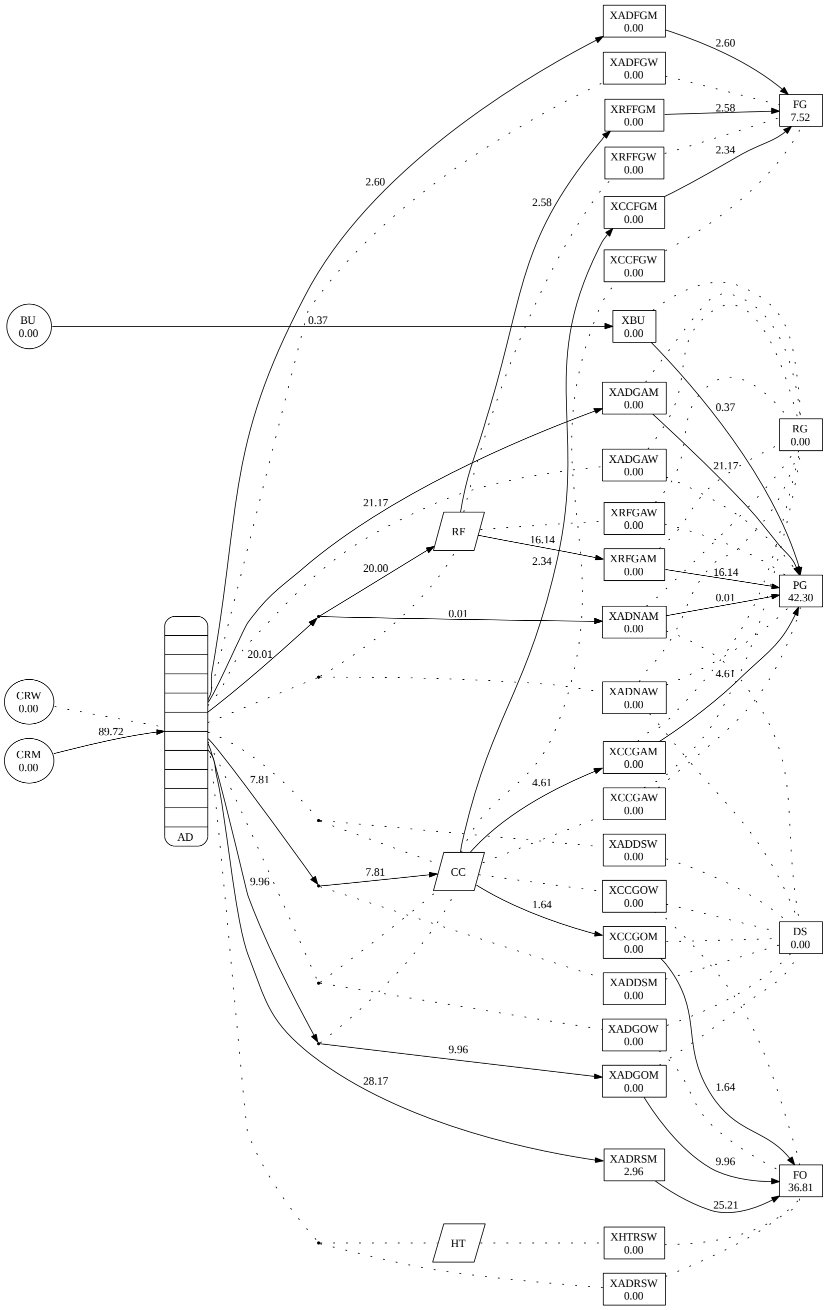

Figure 5: Volume flow for the solution to Case Aronofsky.

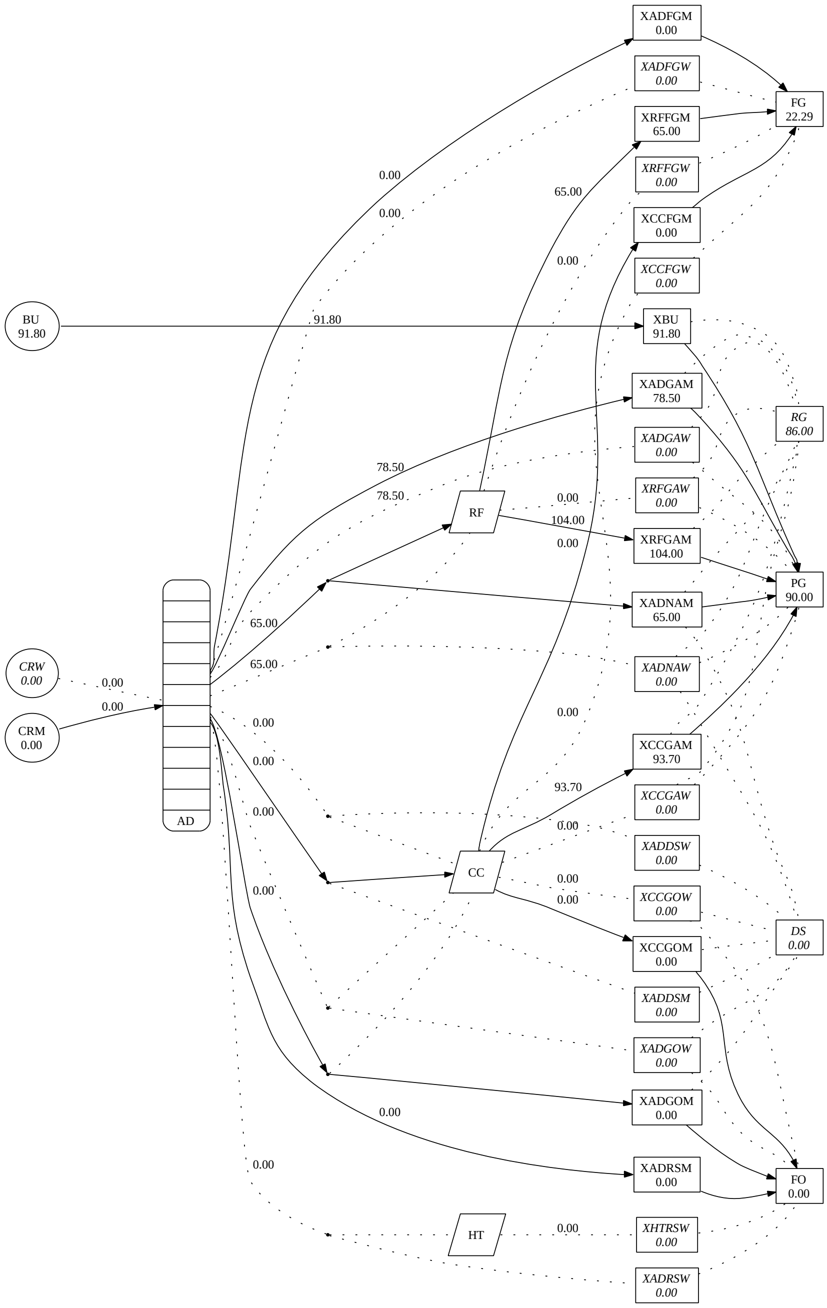

Figure 6: Octane levels for the solution to Case Aronofsky.

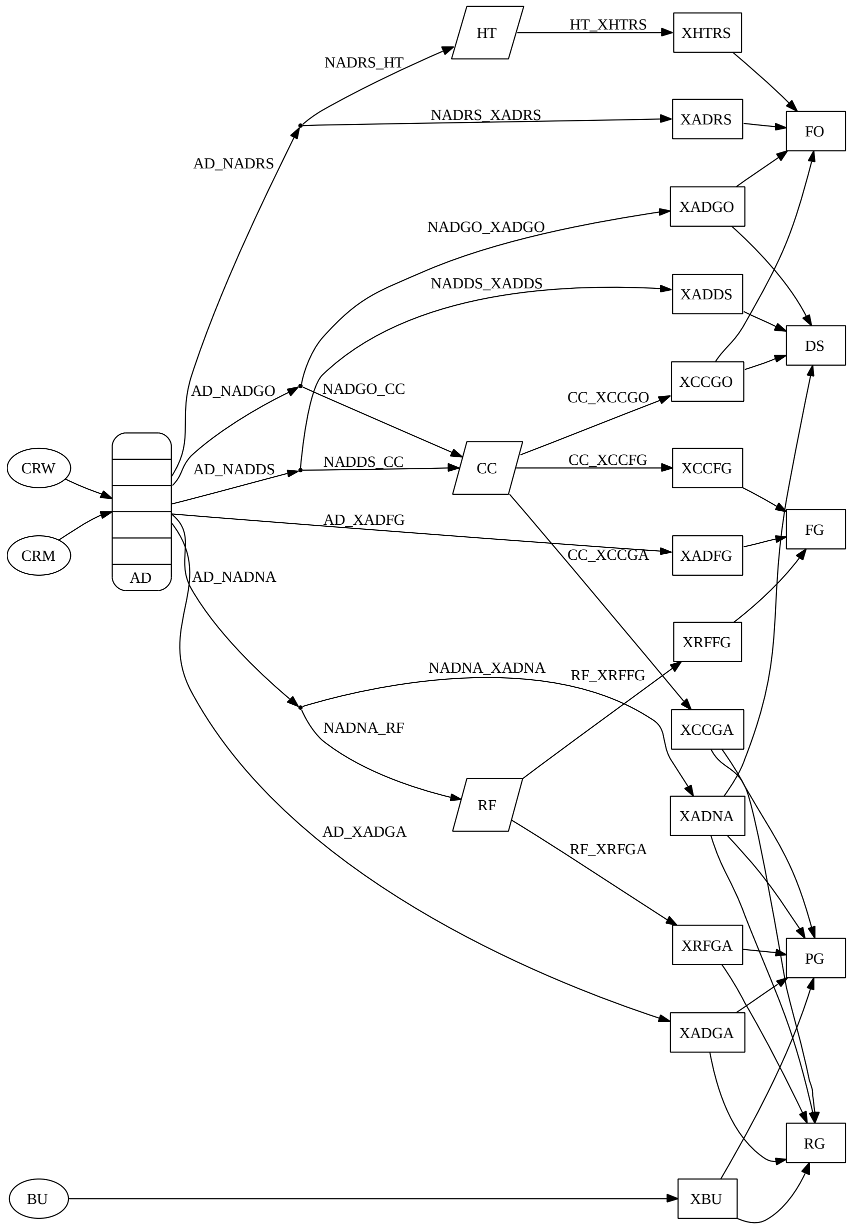

Figure 7: A refinery flow chart of Case AronofskyNLB (i.e with nonlinear blending).

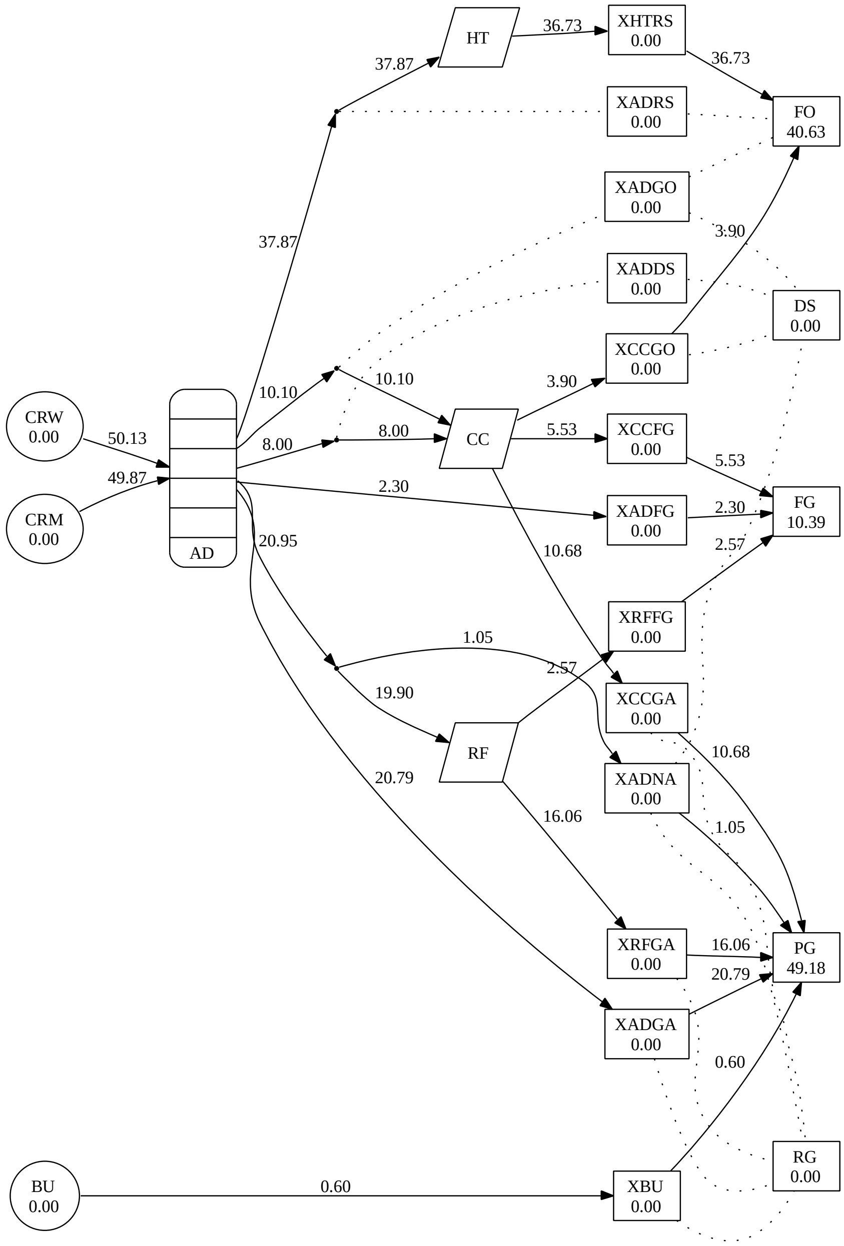

Figure 8: Volume flow for the solution to Case AronofskyNLB

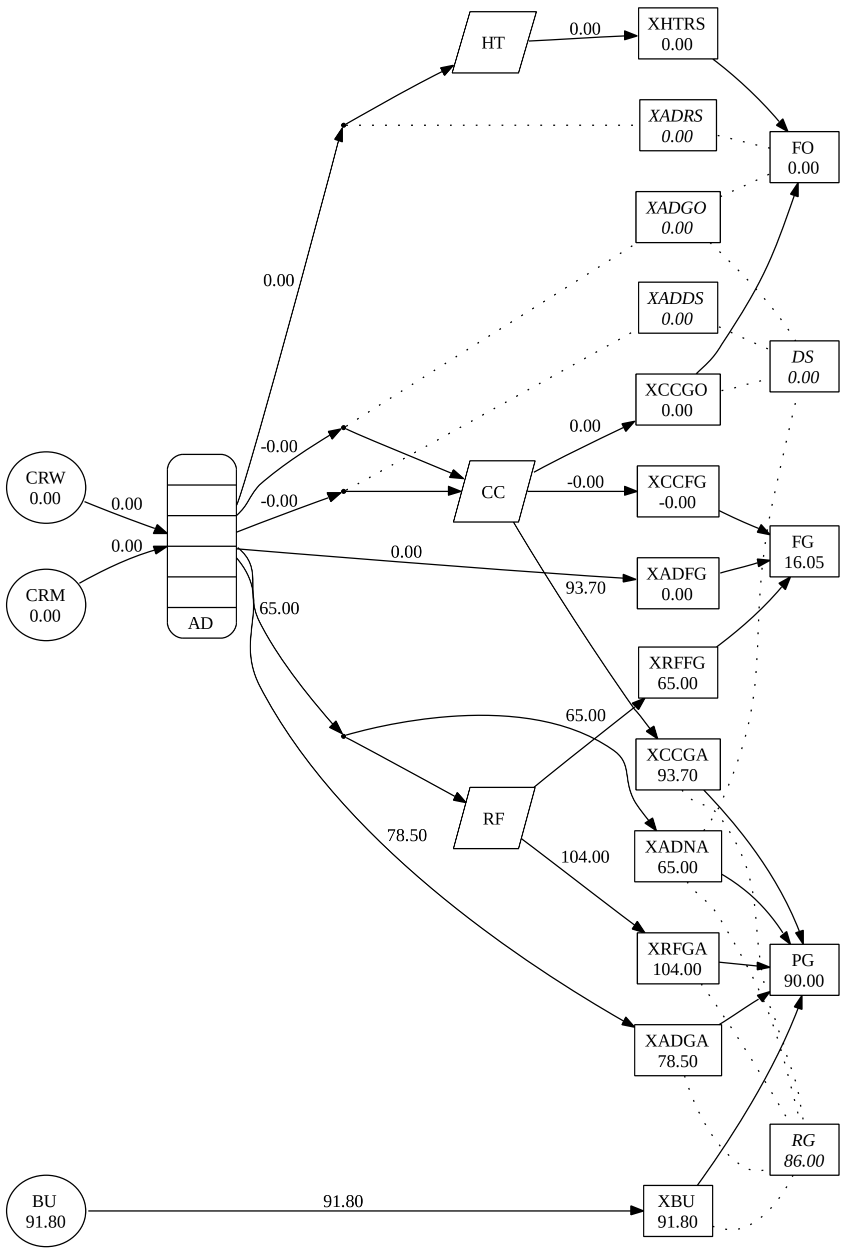

Figure 9: Octane levels for the solution to Case AronofskyNLB

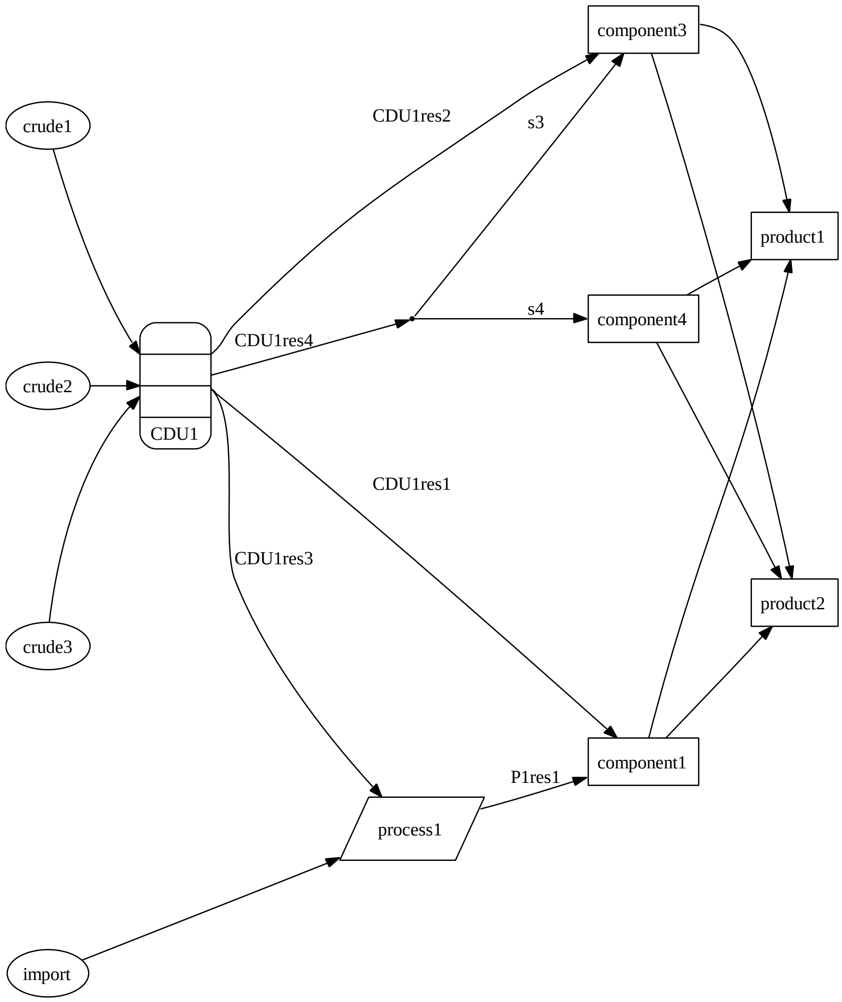

Figure 10: A refinery flow chart for Case SmallNLBP.

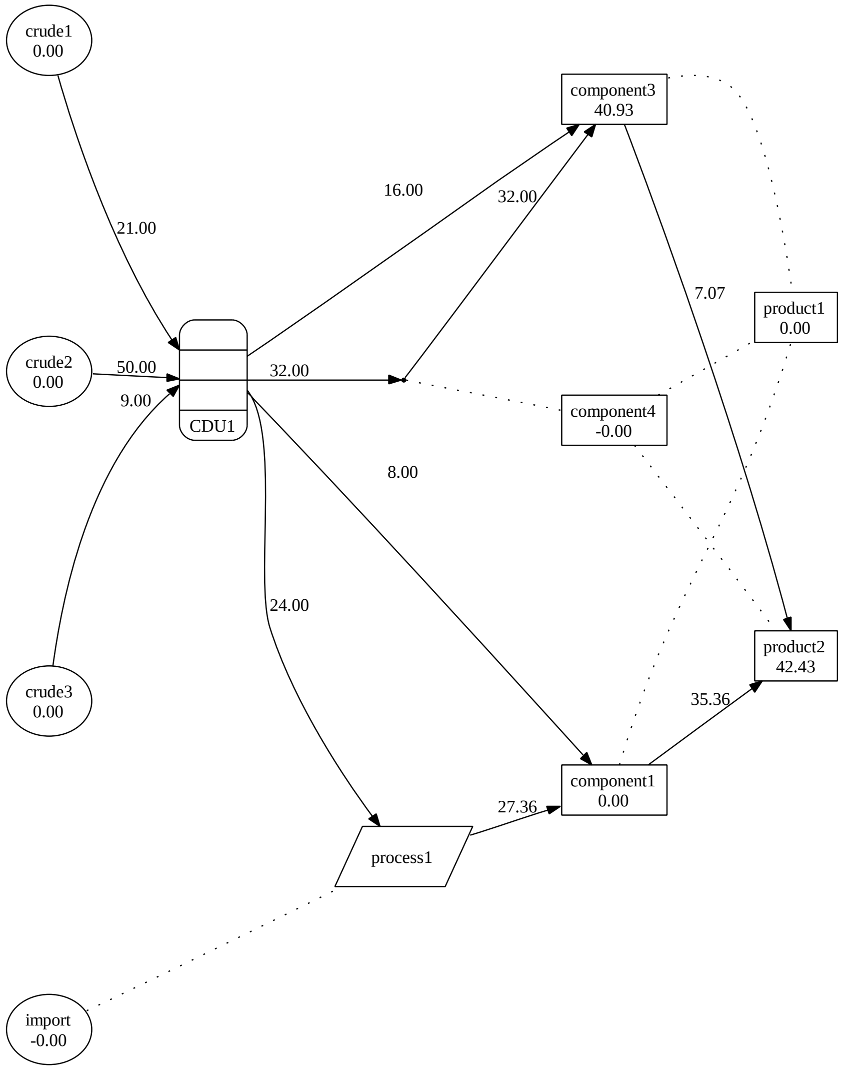

Figure 11: Volume flow for the solution to Case SmallNLBP.

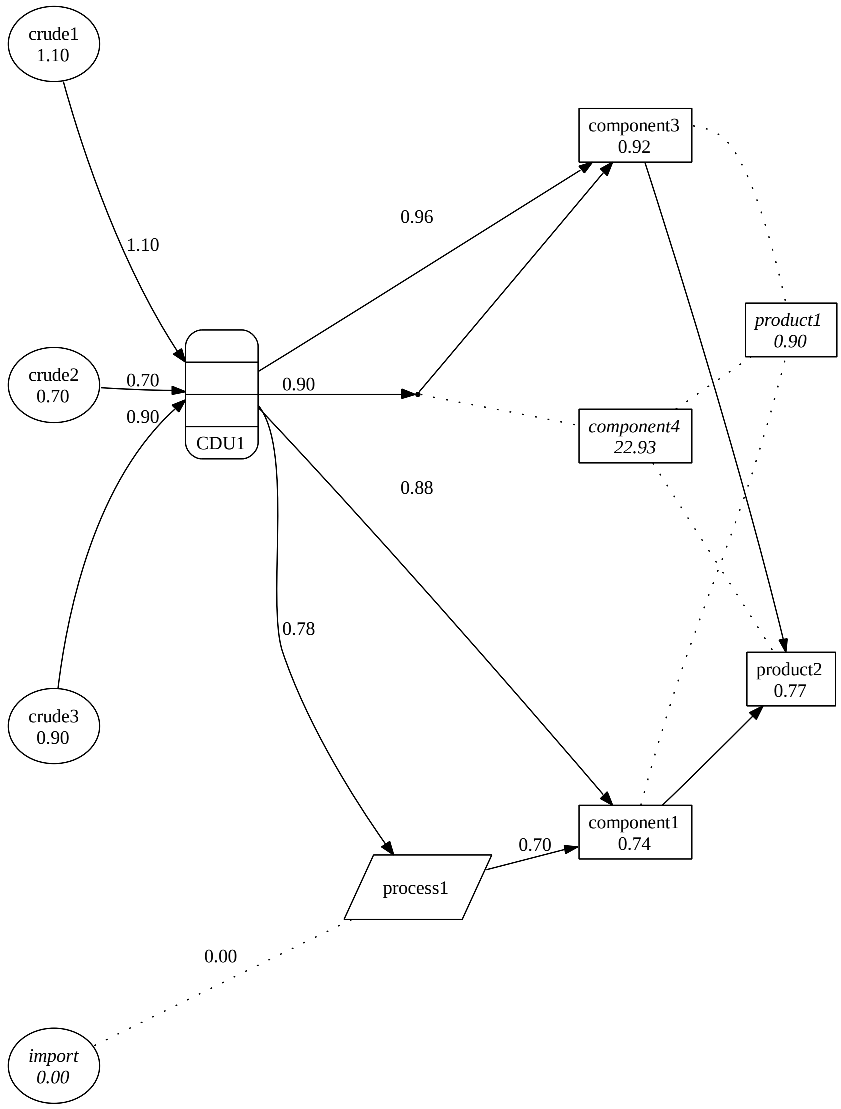

Figure 12: Density levels for the solution to Case SmallNLBP.

3.3 Function approximations

The functions $h^+_*(x)$ used in the nonlinear model are often unknown, i.e. the coefficients $P^{+,0}_*$, $P^{+,1}_*$ and $P^{+,2}_*$ are not known. Therefore, we have developed an application which can be used to approximate these coefficients. The user provides the data which should be approximated in a polynomial. We denote the data with $\hat{x}^{out}$ and the data that should be used to build up the approximation with $\hat{x}$. This is an identification problem and we formulate this as a minimum least-square optimization problem and use the coefficients $P^{+,0}_*$, $P^{+,1}_*$ and $P^{+,2}_*$ as decision variables. The optimization problem can be formulated as

$$\min \ \left\|P^{+,0}_* + P^{+,1}_* * \hat{x} + P^{+,2}_* * \hat{x}^2 - \hat{x}^{out}\right\|_2^2$$

In Figure 13 we illustrate the data and the found approximate function. The problem is nonlinear but convex, hence optimal solutions can be found. In the application, the input, output and properties are given in an Excel sheet. The solution, i.e. the coefficients $P^{+,0}_*$, $P^{+,1}_*$ and $P^{+,2}_*$ are written back to the same Excel sheet.

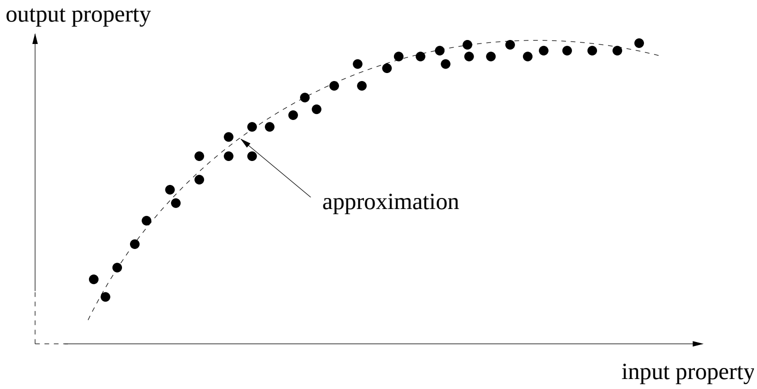

Figure 13: A set of measurement points of an input property (horizontal axis) and resulting output property (vertical axis) is given as well as the approximate function of the output property given the input property (broken line).

3.4 Nonlinear solvers

The nonlinear constraints for the blending makes the problem difficult to solve. Even with a good start solution determined by the proposed solution approach method, it is sometimes difficult to find a good solution to the nonlinear problem. We solved Case SmallNLBP using different start solutions with five different solvers. The solvers tested are NPSOL ([4]), IPOPT ([9]), DONLP2 ([8] and [7]), SNOPT ([3]) and MINOS ([5]). We tried three different start solutions. The first one was when no start solution was given to the solver (No). The second one was when only the PathLP problem was solved and the concentrations were determined from the flow (PathLP). The third one was when the whole described method was applied to generate the start solution (Full) The result from this test is given in Table 2. The method SNOPT did not find a feasible solution when the solution to PathLP was used as start solution. We indicate this with a '-' in the table.

Start solution	DONLP2	IPOPT	NPSOL	SNOPT	MINOS
No	109.56	0	0	0	0
PathLP	0	0.000013	0	-	915.14
Full	0	-2054.20	915.14	713.04	921.09

Table 2: Objective value (profit) when solving Case SmallNLBP with different start solutions with different solvers.

The solver statuses (provided back as a solution report) given in Table 3 indicate the difficulties the solvers had to find the optimal solution.

	Start solution		
Solver	No	PathLP	Full
DONLP2	no descent in QP	reached maxit steps	5 iter: no acceptable step size
IPOPT	optimum	optimum	maximal number of iterations exceeded
NPSOL	first-order optimal but not converged	serious ill-conditioning and stuck	optimum
SNOPT	optimum	the current point cannot be improved	the current point cannot be improved
MINOS	optimum	optimum	optimum

Table 3: Solver status when solving Case SmallNLBP with different start solutions with different solvers.

4 Concluding remarks

Refinery planning problem can be very nonlinear and hard to solve. In this report we describe a general purpose model and solution approach. This is used in a platform for refinery planning and can be used for different tests and scenario analysis. The model includes flexible process descriptions using second order polynomials. Tests show that the starting solution is important for the convergence behaviour. Moreover, the performance of general nonlinear solvers shows very different behaviour. he best performance is given by the solver MINOS and NPSOL.

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