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Discussion paper

# Mixed contracts for the newsvendor problem with real options

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# Mixed contracts for the newsvendor problem with real options

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## Abstract

In this paper we consider the newsvendor model with real options. We consider a mixed contract where the retailer can order a combination of  $q$  units subject to the conditions in a classical newsvendor contract and  $Q$  real options on the same items. We provide a closed form solution to this mixed contract when the demand is discrete and study some of its properties. We also offer an explicit solution for the continuous case. In particular we demonstrate that a mixed contract may be superior to a real option contract when a manufacturer has a bound on how much variance she is willing to accept.

Keywords: newsvendor model, real options, discrete demand, mixed contract

## 1 Introduction

It is important for a firm to establish effective supply contracts with their suppliers/byers to enhance their performance in the supply chain. In capital-intensive industries improvements in the coordination of supply and demand may carry large economic benefits (Kleindorfer and Wu (2003)). Recently supply chain researchers have studied the performance of mixing two or more contracts, as the wholesale contract, the real option contracts and the usage of the spot market to improve the performance of one or more parties in the supply chain. The focus has been mainly on how the byers can establish effective supply contracts with their suppliers to achieve benefits as increased flexibility, reduce cost and adequate supply. Some authors have also focused on the pricing strategy of the suppliers.

In this paper we study the mix of a wholesale and a real option contract, and compare the performance of the mixed contract both with the single wholesale contract and with the real option contract. We model the negotiation process as a Stackelberg game, where the supplier is the leader and determines the wholesale price, and the option- and exercise price for the real option contract. Initially we assume that the agents are risk-neutral in the sense that they only care about expected profits. When two contracts have the same expected profit, however, the contract offering the lowest variance will be preferred.

The game is divided into two separate stages. At the first stage the supplier (leader) offers a wholesale contract, and chooses the wholesale price to maximize her expected profit. The buyer (follower) chooses the order quantity that maximizes his expected profit. We assume that both parties have full information on the demand distribution. The resulting contract is pareto optimal, and we will refer to this contract as the original contract.

At the second stage of the game the supplier is faced with the original contract, and wants to design a mixed contract to further advance profits. A new contract is feasible only if both parties have at least as much expected profit as in the original contract. The supplier hence search a feasible mixed contract to optimize profits.

If the supplier is risk-neutral, she can always extract all extra expected profit using a pure real option contract. A mixed contract cannot advance expected profits further since there is nothing more to take. If the supplier is risk-averse in the sense that she has a bound on how much variance she can tolerate, however, we show that expected profits can be enhanced considerably more by mixed contracts than by pure real option contracts. The explanation for this is quite simple. In the mixed contract the supplier has zero variance from the wholesale part of the contract, and as a consequence of this the variance of her profit falls much more rapidly (in comparison with the pure real option case) when the usage of the wholesale contract increases.

The paper is organized as follows: In section 2 we give a literature review to explain how our theory fits in the existing literature in the field. In Section 3 we review the basic properties of the newsvendor and real option contracts. The main result is Proposition 3.1 which offers

a closed form solution for the mixed contract in the discrete case. In Section 4 we examine some numerical examples to illustrate the some of the properties of the mixed contracts. In Section 5 we study the performance of mixed contracts for a risk-averse supplier. In particular we demonstrate that a mixed contract is superior to a real option contracts in enhancing profits for the supplier. In Section 7 we provide explicit formulas for the continuous case. Finally in Section 8 we offer some concluding remarks.

## 2 Literature review

A mixed option and wholesale contract has previously been addressed also by other authors in different type of settings. Cheng et al. (2003) considered a mixed wholesale and option contract for an exogenously given wholesale price. For the option contract to be effective, they suggest that the exercise price should be less than the wholesale price in the forward contract. Otherwise, they show that the supplier will take most of the profit improvement, leaving the buyer with little incentive to procure the options. To overcome this difficulty and achieve channel coordination, they propose a simple negotiation mechanism to share the profit improvement over the newsvendor model. Burnetas and Ritchken (2005) consider a mixed wholesale and option contract when the retailers demand distribution is influenced by pricing decisions (the retailer has an uncertain downward sloping demand curve). They show that the introduction of option contracts into the wholesale contract causes the wholesale price to increase and the volatility of the retail price to decrease. Conditions are derived under which the supplier is always better off with a mixed contract. They further find that the retailer will benefit from a mixed contract only if the demand uncertainty is low.

Barnes-Schuster et al (2002) consider a two-periodic mixed forward- and option- contract where the supplier has flexibility in choosing between a normal and a more expensive expedite production. They illustrate how options provide flexibility for the buyer to respond to market changes in the second period, but note that options not always coordinate the channel and may alleviate the individual rationality constraint. Barnes-Schuster et al (2002) show that contracts as the backup agreements analysed by Eppen and Iyer (1997), the quantity flexibility contract analysed

by Bassok and Anupindi 1998, Tsay and Lovejoy 1999, and the pay-to-delay capacity reservation contracts analyzed by Brown and Lee (1997), are all special cases of their proposed model. A buyer-supplier relationship with two ordering opportunities is also discussed/considered in Zhou and Wang (2009) and Weng (2004).

In the recent year there has been a focus on papers that combine the traditional long-term contracts, with the option of using spot market to sell the participants excess inventory or to buy additional inventory depending on the need. A literature survey that presents and discusses the literature that considers integrating long term contract as forward and options with short-term spot contracts in capital-intensive industries is given by Kleindorfer and Wu (2003). They illustrate the reviewed work with examples of goods and services currently being traded in both short-run and long-term contract markets and discuss the challenges of implementation. They conclude the survey by addressing unexplored research questions in the literature. A more recent survey that focuses on supply chain operation in the presence of a spot market, by Haksöz and Seshadri (2007), also reviews and discusses papers that consider the optimal mix of long term contracts and the usage of the spot market. They mention Akella et al (2001) and Seifert et al (2004) that mainly address the procurement problem for the buyer, and Wu et al (2002) and Golovachkina and Bradley (2002) that also consider the buyer-supplier coordination. More specific, Wu et al (2002) and Golovachkina and Bradley (2002) consider a real option capacity reservation contract where both parties have access to the spot market and the supplier has limited capacity while the spot market has unlimited supply. Golovachkina and Bradley (2002) focus on how access to the spot market affect buyer-seller coordination, while Wu et al (2002) study how to find the optimal balance between selling capacity using a forward contract and reserving capacity to sell in the spot market for a single supplier and multiple buyer supply chain. Both papers conclude that the optimal strategy for the supplier is to “set the exercise price sufficiently low to guarantee that the buyer will exercise the options and set the reservation price to achieve the trade-off between immediate and future revenues”, Golovachkina and Bradley (2002).

A buyer of commodity products has typically many different suppliers to procure from. By selecting the right mix of contracts from the long-term market (wholesale and option) and the short term (spot) market the buyer may increase the flexibility and enhance the profit. Martinez-

de-Albeniz and Simchi-Levi (2005) address the multi-periodic supplier selection problem for a buyer with access to forward contracts, real option contracts, and the spot market. They study how the buyer can find the portfolio of contracts that maximizes his expected profit, based on the flexibility-price trade-off of the potential contracts. This setting is particularly meaningful for commodity products where a large pool of suppliers is available. Through numerical examples, Martínez-de-Albeniz, and Simchi-Levi (2005), show that the “expected profitability of a portfolio contract dominates the long-term contract both in terms of the mean and the variance of profit”, while the real “option contracts may attain less profit variability compared to the portfolio contracts”. In order for the suppliers to get the buyer's attention they have to compete on price and flexibility. In Martínez-de-Albeniz and Simchi-Levi (2005) the suppliers' bids are exogenous, i.e., there is no competition among the suppliers. Martínez-de-Albeniz and Simchi-Levi (2009) analyze the behavior of the suppliers when they compete on the attention from the buyer. They present the optimal conditions for suppliers' bids and provided necessary conditions for equilibrium bids in a one period model. They find that the equilibria in pure strategies give rise to what they call cluster competition. Hazra and Mahadevan (2009) also address the supplier selection problem for a buyer with access to both the spot market and to long-term contracts through a supplier bidding process. They model the pricing behavior of the suppliers (offering capacity both through long term contracts and at the spot market) and derive expressions for the optimal contract mix for the buyer.

Serel (2009) discusses how to design a long term multi-period capacity reservation contract between a buyer and a long-term supplier when the buyer also has access to a spot market. The long-term contract gives the buyer access to a given volume in each period for a predetermined price. Serel (2009) derives an optimal inventory policy and presents numerical results that show that as uncertainty in the spot market increases, the usage of the long-term contract increases. Further, Serel (2009) note that the usage of the long-term contract also benefit the supplier through an increase in the utilization of the supplier capacity. Li et al (2009) consider a dynamic market where the buyer faces uncertainty in price and demand. Initially they discuss the buyer's trade-off between periodically purchasing from the spot market and signing a long-term contract with a single supplier. Then, they studied mixed strategies, purchasing commitments and contract cancellations. From computational results they find that increases in price (de-

mand) uncertainty favor long-term (short-term) suppliers.

Caldentey and Haugh (2009) study the benefits of using financial markets for a supply chain with a single supplier and a single budget constrained buyer. In their study they consider the standard wholesale contract (Cachon 2003), a flexible wholesale contract (the price and order quantity are contingent upon the history of the financial market up to the time for the physical transaction of goods) and a flexible wholesale contract where the retailer is able to hedge his budget constraint. They categorize scenarios in which the introduction of financial markets benefits both the buyer and the supplier and show that the supplier always will improve his performance if the buyer is able to hedge his budget constraint, while the buyer might actually be worse off when he can hedge his budget constraint.

Martinez-de-Albeniz, and Simchi-Levi (2006) study the trade-offs faced by a buyer signing a portfolio of forward and real option contracts with its suppliers (incurring inventory risk) and having access to a spot market (incurring spot price risk). They quantify the inventory risk and the spot price risk by studying the profit mean and variance for a given portfolio of contracts. Dong and Liu (2007) discuss the benefits of using a forward contract in addition to the spot market in a supply chain with two risk-averse participants who both have market powers. They report (find) that a firm's need for risk hedging is one potential fundamental driving force and show how risk can be reduced, shared, or shifted between the risk-averse participants in the supply chain. Managing risk in the supply chain is difficult, e.g., reducing (mitigating) one risk may increase another. Successful management of supply chain risks starts with an understanding of the various threats, individually and collectively (Sodhi and Lee (2007)). A variety of risks (strategic and operational) in the consumer electronics industry is studied in Sodhi and Lee (2007), some of them is accompanied with Samsung's response to the threat.

This work extends previous literature in that we study a mixed wholesale and real option contract with an endogenous determined wholesale price and provide closed form solution for both the suppliers pricing problem and the buyers procurement problem. Further, we provide new insight for the properties of the mixed contract. In particular we demonstrate that a mixed contract is superior for the manufacturer compared to a real option contract when the manu-

facturer is risk-averse (the variance of the expected profit is bounded above).

## 3 Theory

### 3.1 The classical newsvendor model

In the classical newsvendor model a retailer plans to sell a commodity in a market with uncertain demand. The retailer orders a number of units of the commodity from a manufacturer, and hopes to sell sufficiently many of these units to make a profit. We assume that the manufacturer faces a fixed manufacturing cost  $M$  and decides the wholesale price  $W$ , while the retailer can sell at a revenue price  $R$  and decides the order quantity  $q$ . We further assume that the retailer has complete knowledge of the distribution of market demand  $D$  (a random variable), and that the retailers selling price ( $R$ ) is exogenously determined and known. Unsold items can be salvaged at the specific price  $S$ . The mathematical solution to the classical newsvendor problem is very well known, and we list some properties below for easy reference.

Retailer's profit

The retailer's profit is denoted by  $\Pi_r(q)$ . Profits in the newsvendor model can be rewritten in several different ways. For the analysis we carry out in this paper it is convenient to express everything in terms of the random variable  $\min[D, q]$ . In that case

$$\Pi_r(q) = (R - S) \min[D, q] - (W - S)q \quad (1)$$

Manufacturer's profit

The manufacturer's profit is denoted by  $\Pi_m(q)$ . In the newsvendor model the manufacturer has a constant profit given by the expression

$$\Pi_m(q) = (W - M)q \quad (2)$$

Distribution of the demand

For simplicity we will assume that the demand  $D$  has a discrete distribution where the values  $d_1, d_2, \dots, d_n$  have probabilities  $p_1, p_2, \dots, p_n$ . In the classical newsvendor model the retailer



wants to choose an order  $q$  to maximize his expected profit. The optimal order quantity can then be found as follows: Let  $1 \leq k \leq n$  be the smallest integer s.t.

$$\sum_{i=1}^k p_i \geq \frac{R - W}{R - S}$$

The optimal order quantity is then  $q = d_k$ . In the degenerate case where  $\sum_{i=1}^k p_i = \frac{R-W}{R-S}$ , the expected profit is constant when  $q \in [d_k, d_{k+1}]$ . In all other cases the optimal order quantity is unique.

### 3.2 Real options

As an alternative to the contract in the newsvendor model the manufacturer can offer the retailer a contract with real options. The price for one option is  $c$ . Each option offers the right but not the obligation to buy one unit of goods at a fixed price  $x$ . Real option contracts of this type have been studied by many authors and the mathematical solution to the problem is very well known. For easy reference we list some of the main properties below.

If the retailer buys  $Q$  options, his profit  $\hat{\Pi}_r(Q, c, x)$  is given by

$$\hat{\Pi}_r(Q, c, x) = (R - x) \min[D, Q] - cQ \quad (3)$$

In this contract some of the risk is transferred to the manufacturer. The manufacturer's profit is now a random variable  $\hat{\Pi}_m(Q, c, x)$  given by

$$\hat{\Pi}_m(Q, c, x) = (x - S) \min[D, Q] - (M - S - c)Q \quad (4)$$

If the retailer wants to choose an order  $q$  to maximize his expected profit, the optimal order quantity can be found as follows: Let  $1 \leq k \leq n$  be the smallest integer s.t.

$$\sum_{i=1}^k p_i \geq \frac{R - c - x}{R - x}$$

The optimal order quantity is then  $q = d_k$ . In the degenerate case where  $\sum_{i=1}^k p_i = \frac{R-c-x}{R-x}$ , the expected profit is constant when  $q \in [d_k, d_{k+1}]$ . In all other cases the optimal order quantity is

unique.

### 3.3 The mixed contract

In this section we wish to define a mixed contract where the retailer can order a quantity  $q$  at the wholesale price  $W$ . In addition he can order a quantity  $Q$  of real options. As before the price of each option is  $c$  and each option gives the right but not an obligation to buy one unit of goods at the price  $x$ . The retailer's optimization problem is slightly more difficult as it now involves two variables;  $q$  and  $Q$ . Nevertheless it turns out that the optimal choices can be found by explicit formulas very similar to the one dimensional cases. The details are as follows:

If  $x \leq S$ , the retailer prefers to receive the full order  $q+Q$ , and the problem is in effect equivalent to a pure newsvendor contract with  $W' = \min[W, x + c]$ . We will hence assume that  $x > S$ . If the demand is  $D$ , the retailer makes a profit

$$\Pi_r[q, Q] = -Wq - cQ + \begin{cases} RD + S(q - D) & D \leq q \\ RD - x(D - q) & q \leq D \leq q + Q \\ R(q + Q) - xQ & q + Q \leq D \end{cases} \quad (5)$$

To simplify the analysis, we rewrite this as follows

$$\Pi_r[q, Q] = (R - W)q + (R - x - c)Q + \begin{cases} (R - S)D + (S - R)q + (x - R)Q & D \leq q \\ (R - x)D + (x - R)q + (x - R)Q & q \leq D \leq q + Q \\ 0 & q + Q \leq D \end{cases} \quad (6)$$

If  $d_k \leq q < d_{k+1}$ ,  $d_l \leq q + Q < d_{l+1}$ ,  $1 \leq k \leq l$ , the expected profit can be expressed on the form

$$\begin{aligned}
\mathbb{E}[\Pi_r[q, Q]] &= (R - W)q + (R - x - c)Q \\
&+ \sum_{i=1}^k ((R - S)d_i + (S - R)q + (x - R)Q)p_i \\
&+ \sum_{i=k+1}^l ((R - x)d_i + (x - R)q + (x - R)Q)p_i
\end{aligned} \tag{7}$$

After a rearrangement of terms, we get

$$\begin{aligned}
\mathbb{E}[\Pi_r[q, Q]] &= (x + c - W)q - \sum_{i=1}^k (x - S)(q - d_i)p_i \\
&+ (R - x - c)(q + Q) - \sum_{i=1}^l (R - x)(q + Q - d_i)p_i
\end{aligned} \tag{8}$$

If  $k = 0$ , i.e,  $0 \leq q < d_1$ ,  $d_l \leq q + Q < d_{l+1}$ ,  $l \geq 1$ , the first sum does not appear and

$$\mathbb{E}[\Pi_r[q, Q]] = (x + c - W)q + (R - x - c)(q + Q) - \sum_{i=1}^l (R - x)(q + Q - d_i)p_i \tag{9}$$

If  $0 \leq q < d_1$ ,  $0 \leq q + Q < d_1$ , both sums do not appear and

$$\mathbb{E}[\Pi_r[q, Q]] = (x + c - W)q + (R - x - c)(q + Q) \tag{10}$$

If  $W > x + c$ , it is clear that a global maximum for  $\mathbb{E}[\Pi_r[q, Q]]$  is obtained with  $q^* = 0$ , and if  $x + c > R$ , a global maximum must have  $Q^* = 0$ . In these two cases, the problem is reduced to a problem of pure contracts, and the solutions to these problems are well known. Moreover, if  $W < c + S$ , it is better to salvage overstocked items than paying  $c$  upfront, and  $Q^* = 0$  also in that case. To get a non-degenerate solution, we assume that  $W \leq x + c \leq R$ ,  $W \geq c + S$ , and  $x > S$ .

A key issue is to define the following pair of indices  $(k, l)$ :

- Let  $1 \leq k \leq n$  be the smallest integer s.t.

$$\sum_{i=1}^k p_i \geq \frac{x + c - W}{x - S} \quad (11)$$

- Let  $1 \leq l \leq n$  be the smallest integer s.t.

$$\sum_{i=1}^l p_i \geq \frac{R - x - c}{R - x} \quad (12)$$

A closed form solution for the mixed contract can then be formulated as follows:

**Proposition 3.1** *Let the indices  $(k, l)$  be defined by (11)–(12) and assume that  $W \leq x + c \leq R$ ,  $W \geq c + S$ , and  $x > S$ .*

- *If  $k \leq l$ , the expected profit  $E[\Pi_r[q, Q]]$  has a global maximum at  $q^* = d_k, Q^* = d_l - d_k$ .*
- *If  $k > l$ , a global maximum is obtained at a pure contract with  $Q^* = 0$ .*

*If any of the conditions  $W \leq x + c \leq R$ ,  $W \geq c + S$ , and  $x > S$  are violated, the problem is reduced to either a pure real option contract or a pure newsvendor contract:*

- *$W > x + c$ , leads to a pure real option contract, i.e.,  $q^* = 0$ .*
- *$x + c > R$ , leads to a pure newsvendor contract, i.e.,  $Q^* = 0$ .*
- *$W < c + S$ , leads to a pure newsvendor contract, i.e.,  $Q^* = 0$ .*
- *$x \leq S$  and  $W \leq x + c$ , leads to a pure newsvendor contract, i.e.,  $Q^* = 0$ .*
- *$x \leq S$  and  $W > x + c$ , is equivalent to a pure newsvendor contract with  $W' = x + c$ , and  $q^* = 0$ .*

**Proof** See the appendix.

### Some remarks

Proposition 3.1 gives a complete solution to the retailers optimization problem, and the key issues to the solution are the indices defined by (11) and (12). The retailer knows for sure that he will be able to sell the minimum demand  $D = d_1$  units. If  $W \leq x + c$ , the retailer will always order  $q \geq d_1$  in the newsvendor part of the contract. If  $W \leq x + c, W \approx x + c$ , there is risk of not selling more than  $d_1$  units and real options are a more favorable choice for additional orders. This is reflected in (11) as the right hand side is very small in this case, which in turn forces  $q$

down to the minimum  $d_1$  corresponding to the case  $k = 1$ .

If we gradually reduce  $W$  from the level  $W = x + c$ , the newsvendor part of the contract will be increasingly favorable, i.e., the retailer will order a larger fraction of the total order within the newsvendor part of the contract. When  $W$  has become sufficiently low, the real option part of the contract will be void because it has become too expensive compared with the alternative contract. This is exactly what happens when we get  $k > l$  in Proposition 3.1.

It is interesting to note that  $W$  does not appear in (12). Hence it appears that the total order level  $l$  is controlled by  $x$  and  $c$  alone. That is true in most cases, but when we get  $k > l$ , the total order level may in general be different from  $l$ . In that case the problem is transferred to a pure newsvendor contract, and in this contract the value of  $W$  controls the total order level. See Example 4.2.

### 3.4 Expected profits for the manufacturer

If the demand is  $D$  and  $x > S$ , the manufacturer makes a profit

$$\Pi_m[q, Q] = Wq + cQ - M(q + Q) + \begin{cases} SQ & D \leq q \\ x(D - q) + S(q + Q - D) & q \leq D \leq q + Q \\ xQ & q + Q \leq D \end{cases} \quad (13)$$

If we combine (13) with (5) we see that the profit for the supply chain is given by

$$\Pi_{\text{chain}}[q, Q] = \Pi_r[q, Q] + \Pi_m[q, Q] = (R - S) \min[D, q + Q] - (M - S)(q + Q) \quad (14)$$

Note that  $W, x$  and  $c$  do not appear in (14) as these prices only redistribute wealth between the retailer and the manufacturer. The optimal order quantity  $q_{\text{chain}}^*$  for the supply chain can be found solving a standard newsvendor problem, i.e., let  $1 \leq k \leq n$  be the smallest integer s.t.

$$\sum_{i=1}^k p_i \geq \frac{R - M}{R - S}$$

Then  $q_{\text{chain}}^* = d_k$ , and in the mixed contract optimal expected profit for the supply chain is

obtained at any combination of  $q, Q$  satisfying  $q + Q = q_{\text{chain}}^*$ .

### 3.5 Feasibility

We are assuming that the manufacturer decides the prices  $W, c$  and  $x$ , and tries to find combinations of these prices to maximize her expected profit. One particular scenario that we have in mind, however, is a setting where the retailer initially is offered a standard newsvendor contract (the original contract). In the original contract the manufacturer has chosen  $W$  to maximize her expected profit. When the manufacturer offers a new (mixed) contract she must keep in mind that the retailer will accept a new contract only if the expected profit from the new contract is at least as high as in the original contract. Any combination of  $W, c, x$  leading to a mixed contract where the expected profit is at least as high as in the original contract is called feasible. Note that  $W$  in the new contract need not be equal to the particular  $W$  used in the original contract.

When we search for feasible combinations of  $(W, c, x)$  to maximize expected profits for the manufacturer, the following principles are very useful: The manufacturer should seek feasible combinations where the expected profit for the supply chain is as large as possible while the expected profit for the retailer is as small as possible. If, in particular, we can find  $(W, c, x)$  such that

- $q + Q = q_{\text{chain}}^*$
- Expected profit for the retailer is equal to expected profit in the original contract

it is clear that expected profit for the manufacturer cannot be advanced further without breaking the feasibility constraints, i.e., such combinations must provide a global optimum for the manufacturer.

In all the numerical examples in Section 4 such special combinations can be found. The search can then be carried out very quickly as it suffices to search alternatives where  $q + Q = q_{\text{chain}}^*$ . Notice, however, that this principle fails in the example shown in Section 5. In that case we can no longer be sure that a global optimum is obtained when  $q + Q = q_{\text{chain}}^*$ , and it is

necessary to trace the whole domain of definition to be sure that a global maximum has been found. Some special care must then be taken as many standard optimization programs cannot handle discontinuous functions of this type. As the function is piecewise continuous with no strong gradients and the dimension of the search space is quite low, a simple grid search seems appropriate, and is what we used in Section 5 and 6.

## 4 Numerical examples

To illustrate the theory in Section 3, we consider a demand  $D$  with  $n = 10$  different values

$$(d_1, d_2, \dots, d_{10}) = (10, 20, 30, 40, 50, 60, 70, 80, 90, 100)$$

with uniform probabilities and where

$$R = 10, M = 4, S = 1$$

These values will be fixed throughout this section. The other variables  $W, c, x$  are chosen by the manufacturer and depends on the manufacturer's level of information.

### 4.1 Example

In this example we consider a version where the manufacturer has no or very little information on  $D$ , and offers the retailer a mixed contract with

$$W = 7, c = 3, x = 5$$

To find the optimal order quantities, we use (11) and (12). Since

$$\frac{x + c - W}{x - S} = 0.25$$

we get  $k = 3$  in (11). Correspondingly

$$\frac{R - x - c}{R - x} = 0.4$$

leads to  $l = 5$  in (12). Since  $k < l$ , it follows from Proposition 3.1 that the optimal order quantities are

$$q^* = d_3 = 30 \quad Q^* = d_5 - d_3 = 20$$

Expected profits using this contract is shown in Table 1.

Mixed contract: $W = 7, c = 3, x = 5, q^* = 30, Q^* = 20$		
	Manufacturer	Retailer
Expected profit	142.00	68.00
Standard deviation	36.00	94.11

Table 1: Expected profits in Example 4.1

## 4.2 Example

This example is similar to the previous example. The only difference is that  $W$  is smaller.

$$W = 5, c = 3, x = 5$$

To find the optimal order quantities, we use (11) and (12). Since

$$\frac{x + c - W}{x - S} = 0.75$$

we get  $k = 8$  in (11). Correspondingly

$$\frac{R - x - c}{R - x} = 0.4$$

leads to  $l = 5$  in (12). Since  $k > l$ , it follows from Proposition 3.1 that  $Q^* = 0$ . That leads to a pure newsvendor contract with  $W = 5$ , and the optimal order quantity in that contract is  $q^* = 60$ . Note that the solution is neither equal to  $d_k = 80$  nor  $d_l = 50$  in this case. Expected profits using this contract is shown in Table 2.



Mixed contract: $W = 5, c = 3, x = 5, q^* = 60, Q^* = 0$		
	Manufacturer	Retailer
Expected profit	60.00	165.00
Standard deviation	0.00	162.25

Table 2: Expected profits in Example 4.2

### 4.3 Example

In this example we assume that the manufacturer has full information on  $D$ , and chooses  $W, c, x$  to maximize her expected profit. We wish to compare the 3 different scenarios

- Classical newsvendor contracts
- Real option contracts (with or without feasibility constraints)
- Mixed contracts (with or without feasibility constraints)
- Classical newsvendor contract

In this case the manufacturer chooses  $W$  to maximize expected profit. Maximal expected profit is obtained with  $W^* = 7.30$ , and expected profits are shown in Table 3.

Newsvendor contract: $W^* = 7.30, q^* = 40$		
	Manufacturer	Retailer
Expected profit	132.00	54.00
Standard deviation	0	91.78

Table 3: Expected profits in a classical newsvendor model

- Unconstrained real option contract

In this case the manufacturer chooses  $c$  and  $x$  to maximize expected profit freely. Without feasibility constraints the solution is degenerate, i.e., is obtained in the limit where

$$c \rightarrow 0^+, x \rightarrow R^-$$

in such a way that the retailer orders the supply chain maximum order. In the limit the manufacturer takes all profit, leaving the retailer with an arbitrary small expected profit. An approximate solution is shown in Table 4.

Constrained real option contract: $c^* = 0.00001, x^* = 9.99997, Q^* = 70$		
	Manufacturer	Retailer
Expected profit	231.00	0.0008
Standard deviation	194.91	0.0006

Table 4: Expected profits in a unconstrained real option contract

- Constrained real option contract

In this case the manufacturer chooses  $c$  and  $x$  to maximize expected profit, but must take into account that the retailer must be offered a feasible contract, i.e., a contract where the retailer has at least as much expected profit as in the newsvendor contract above. Since  $D$  is discrete, maximum profit is obtained on a line segment, i.e., there are infinitely many solutions. Among these solutions we assume that the manufacturer chooses the pair implying minimum variance. This pair is unique, and is obtained at  $c = 1.03, x = 7.42$ , and expected profits are shown in Table 5.

Constrained real option contract: $c^* = 1.03, x^* = 7.425$		
	Manufacturer	Retailer
Expected profit	176.93	54.08
Standard deviation	139.14	55.77

Table 5: Expected profits in a constrained real option contract

- Unconstrained mixed contract

In this case the manufacturer chooses  $c, x$  and  $W$  to maximize expected profit freely. The solution to this problem is the same as for the unconstrained real option contract above. The manufacturer chooses  $W = 10$  to obtain what is in effect a pure real option contract. Without feasibility constraints the solution is degenerate, i.e., is obtained in the limit where

$$c \rightarrow 0^+, x \rightarrow R^- \quad W = 10$$

in such a way that the retailer orders the supply chain maximum order. In the limit the manufacturer takes all profit, leaving the retailer with an arbitrary small expected profit. An approximate solution is shown in Table 6.

Unconstrained mixed contract: $c^* = 0.00001, x^* = 9.99997, W^* = 10, q^* = 0, Q^* = 70$		
	Manufacturer	Retailer
Expected profit	231.00	0.0008
Standard deviation	194.91	0.0006

Table 6: Expected profits in a unconstrained mixed option contract

- Constrained mixed option contract

In this case the manufacturer chooses  $c$ ,  $x$  and  $W$  to maximize expected profit, but must take into account that the retailer must be offered a feasible contract, i.e., a contract where the retailer has at least as much expected profit as in the newsvendor contract above. The optimal choice for the manufacturer turns out to be a combination of a newsvendor contract that is equal to the original newsvendor contract plus a degenerate part where the retailer sells the real option part for a marginal profit. This contract is obtained in the limit where

$$c \rightarrow 0^+, x \rightarrow R^- \quad W = 7.30$$

in such a way that the retailer orders the supply chain maximum order. An approximate solution is shown in Table 7.

Constrained mixed contract: $c^* = 0.00001, x^* = 9.99997, W^* = 7.299, q^* = 40, Q^* = 30$		
	Manufacturer	Retailer
Expected profit	176.96	54.04
Standard deviation	122.41	91.78

Table 7: Expected profits in a constrained mixed option contract

If we compare this contract with the constrained real option contract above, we see that the manufacturer obtains the same expected profit in the two cases. That is not surprising as the real option contract can be designed to produce a contract where the feasibility constraint is binding, i.e., it is impossible to advance profits beyond that point. It is interesting to note, however, that the manufacturer can obtain a significantly smaller variance (122.41 compared to 139.14) via the mixed contract.

## 5 Mixed contracts under risk-aversion and variance constraints

As the manufacturer can always obtain maximal expected profit via real option contracts, mixed contracts cannot further advance expected profits. In the previous section we showed that a smaller variance of the expected profit for the manufacturer can sometimes be obtained, but our example only covers a very unrealistic case where the retailer agrees to sell goods for a marginally small profit. In this section, however, we will see that mixed contracts can enhance

manufacturers profits considerably if the manufacturer is risk-averse. The basic idea can be illustrated by the following example:

We return to the case considered in Table 7. In that case the manufacturer obtains an expected profit of 176.96 with a standard deviation of 122.41. In the original newsvendor contract the manufacturer has no risk, and a standard deviation of 122.41 imposes a considerable risk of losing money. If this implies a too high risk of bankruptcy, the manufacturer cannot offer this contract. Alternatively she will search for a new contract with a smaller risk, and clearly she must give up some profit to achieve that.

We will consider a new case where the manufacturer sets a limit to how much variance she can allow, and consider the case where

$$\text{sd}[\Pi_m[q^*, Q^*]] \leq 100 \quad (15)$$

That corresponds to a risk-averse manufacturer with a utility function

$$U(\text{expected profit, standard deviation}) = \begin{cases} \text{expected profit} & \text{if standard deviation} \leq 100 \\ -\infty & \text{otherwise} \end{cases} \quad (16)$$

At this stage it is important to remark that the retailer is risk-neutral, so the solution for the order quantities in Proposition 3.1 still applies.

If the extra condition in (15) is imposed, we must carry out a constrained search for alternatives satisfying (15). We will compare two cases: In the first case we use a real option contract, and in the second case we consider the same problem with a mixed contract. The final results are shown in the tables below.

Variance constrained real option contract: $c^* = 1.753, x^* = 5.617, Q^* = 70$		
	Manufacturer	Retailer
Expected profit	138.94	92.06
Standard deviation	99.99	94.92

Table 8: Variance constrained real option contract

Variance constrained mixed contract : $c^* = 0.66, x^* = 8.35, W^* = 6.80, q^* = 40, Q^* = 30$		
	Manufacturer	Retailer
Expected profit	152.05	78.95
Standard deviation	99.97	107.71

Table 9: Variance constrained mixed contract

Inspecting the results in Tables 8 and 9, we see that both problems have non-degenerate solutions, i.e., solutions obtained at inner points of the domains. We also see that a mixed contract offers a much more efficient way of reducing the variance for the manufacturer. With real option contracts, the manufacturer must give up 38.02 units of profit to obtain a variance that is sufficiently low, while in the mixed case it suffices to give up 24.91 units of profit to achieve this.

The explanation for this is quite simple. In the mixed contract the manufacturer has zero variance on the newsvendor part of the contract, and as a consequence of this the variance falls much more rapidly (in comparison to the real option case) when  $q$  increases.

#### *Alternative constraints*

In the previous example we considered a constraint on the form

$$\text{sd}[\Pi_m[q^*, Q^*]] \leq \text{sd}_{\max} \quad (17)$$

where  $\text{sd}_{\max}$  is a given constant. Alternatively one could consider constraints on the form

$$\frac{\text{sd}[\Pi_m[q^*, Q^*]]}{\text{E}[\Pi_m[q^*, Q^*]]} \leq C \quad (18)$$

where  $C$  is a given constant. Mathematically there is hardly any difference between the two constraints, but the unit free expression in (18) might be easier to interpret. If  $\Pi_m[q^*, Q^*]$  is close to a normal distribution, a value  $C = 2$  in (18) would imply that there is roughly 2.5% chance of negative profits, and generally  $C$  is the number of standard deviations that are needed before losses are incurred.

## 6 The continuous case

The main emphasis in this paper has been on the case when  $D$  is discrete. In our opinion the discrete case is more interesting as it features several issues that do not appear in the continuous case. In particular the discrete case is challenging due to degenerate cases with infinitely many solutions. These problems usually disappear in the continuous case.

As any continuous distribution can be approximated arbitrary well by a discrete distribution, however, the formulas for the continuous case comes more or less for free by passing to the limit. The basic result can be stated as follows:

**Corollary 6.1** *Assume that  $D$  has a continuous distribution with cumulative function  $F_D$  and that  $W \leq x + c \leq R$ ,  $W \geq c + S$ , and  $x > S$ . Find  $u, v$  and  $w$  such that*

$$u = F_D^{-1} \left( \frac{x + c - W}{x - S} \right) \quad v = F_D^{-1} \left( \frac{R - x - c}{R - x} \right) \quad w = F_D^{-1} \left( \frac{R - W}{R - S} \right) \quad (19)$$

- If  $u \leq v$ , the expected profit  $E[\Pi_r(q, Q)]$  has a unique global maximum at  $q^* = u, Q^* = v - u$ .
- If  $u > v$ , the unique global maximum is obtained at  $q^* = w, Q^* = 0$ .

*If any of the conditions  $W \leq x + c \leq R$ ,  $W \geq c + S$ , and  $x > S$  are violated, the problem is reduced to either a pure real option contract or a pure newsvendor contract:*

- If  $\min[W, x + c] \geq R$ , then  $q^* = 0, Q^* = 0$ .
- If  $W > x + c$  and  $x + c < R$ , then  $q^* = 0, Q^* = v$ .
- If  $x + c > R$  and  $W < R$ , then  $q^* = w, Q^* = 0$ .
- If  $W < c + S$ , then  $q^* = w, Q^* = 0$ .
- If  $x \leq S$  and  $W \leq x + c$ , then  $q^* = w, Q^* = 0$ .
- If  $x \leq S$  and  $W > x + c$ , then  $q^* = 0, Q^* = F_D^{-1} \left( \frac{R - x - c}{R - S} \right)$ .

### 6.1 Example

We now consider the case where  $D$  is  $N(\mu, \sigma^2)$ , with  $\mu = 160$  and  $\sigma = 40$ . To carry out the analysis, we first need to compute the reference values for the original newsvendor contract. As

before we use the values  $R = 10, M = 4, S = 1$ . Optimal profit for the manufacturer is then obtained using  $W^* = 8.95$ , and the results are shown in Table 10.

Newsvendor contract: $W^* = 8.95, q^* = 112.33$		
	Manufacturer	Retailer
Expected profit	556.00	97.44
Standard deviation	0	76.48

Table 10: Expected profits for the original newsvendor model

The optimal order quantity for the supply chain is obtained using  $q = 177.23$  with a total profit of 829.12, leaving considerable room for improvement. Like in Section 5, we assume that the manufacturer has a bound on how much variance she can tolerate, and we hence assume that standard deviation cannot exceed, e.g., 100. Profits under that constraint can be enhanced using a pure real option contract, and the optimal contract for the manufacturer is obtained using  $c = 1.98, x = 7.16$ . The results for this contract are shown in Table 11.

Real option contract: $c^* = 1.984, x^* = 7.160, Q^* = 139.2$		
	Manufacturer	Retailer
Expected profit	668.77	97.44
Standard deviation	100.00	46.10

Table 11: Variance constrained real option contract

As we already observed in the discrete case, a mixed contract can increase profits even further. In the mixed case, the optimal contract for the manufacturer is obtained using the values  $W = 8.961, c = 0.218, x = 9.674$ . The results for this contract are shown in Table 12.

Mixed contract : $c^* = 0.2176, x^* = 9.6739, W^* = 8.9612, q^* = 110.35, Q^* = 32.35$		
	Manufacturer	Retailer
Expected profit	679.80	97.44
Standard deviation	99.93	74.54

Table 12: Variance constrained mixed contract

If we compare Table 11 and 12, we see that the mixed contract is performing considerably better. The mixed contract enhances the supply chain profit from 653.44 in the newsvendor contract, to



777.23 for the mixed contract. There is still a some distance to the maximal supply chain profit, which is 829.12, but the manufacturer cannot advance the supply chain profit further unless she reduces her profit or violates the variance constraint.

## 7 Concluding remarks

This work extends previous literature in that we study a mixed wholesale and real option contract where the wholesale price is determined by the manufacturer together with the option and exercise price of the real option contract. We derive the optimal pricing strategy for the manufacturer, and the optimal procurement strategy for the retailer. We show that mixed contracts can enhance profits considerably if the manufacturer is risk-averse and that a mixed contract is superior to a real option contract in that it more efficiently reduces the variance of the profit.

To be able to apply the explicit formulas in Proposition 3.1 (discrete case) or Corollary 6.1 (continuous case), we need to assume that the retailer is risk-neutral. No such assumptions are needed for the manufacturer. In fact the explicit utility function used in (16) could have been replaced by an arbitrary function of expectation and variance. Such extensions are outside the scope of the present paper and are left for future research.

## 8 Appendix

In this appendix we provide the full details for the proof of Proposition 3.1.

**Lemma 8.1** *Let  $0 \leq C \leq d_n$  be a constant and define  $f_C : [0, C] \rightarrow \mathbb{R}$  by*

$$f_C[q] = E[\Pi_r[q, C - q]]$$

*If  $W \leq x + c$ ,  $f_C$  is non-decreasing on  $[0, \min[C, d_k]]$  and non-increasing on  $[\min[C, d_k], C]$ . Hence  $f_C$  has a global maximum at  $q = \min[C, d_k]$ .*

**Proof** To simplify the notation we define

$$\tilde{d}_i = \min[d_i, C] \quad i = 1, \dots, n$$

Since  $W \leq x + c$ , it follows from (9) or (10), that  $f_C$  is non-decreasing on  $[0, \tilde{d}_1]$ . Let  $k' \geq 1$ , and consider  $f_C$  on the interval  $[\tilde{d}_{k'}, \tilde{d}_{k'+1}]$ . By definition of  $k$  and (8) it is easy to see that  $q \mapsto f_C[q]$  is non-decreasing on  $[\tilde{d}_{k'}, \tilde{d}_{k'+1}]$  if  $k' < k$ , and  $q \mapsto f_C[q]$  is non-increasing on  $[\tilde{d}_{k'}, \tilde{d}_{k'+1}]$  if  $k' \geq k$ . Since  $f_C$  is continuous, this proves the lemma.

**Lemma 8.2** *Let  $0 \leq q \leq d_n$  be a constant and define  $g_q : [0, d_n - q] \rightarrow \mathbb{R}$  by*

$$g_q[Q] = \mathbb{E}[\Pi_r[q, Q]]$$

*If  $R \geq x + c$ ,  $g_q$  is non-decreasing on  $[0, \max[d_l - q, 0]]$  and non-increasing on  $[\max[d_l - q, 0], d_n - q]$ . Hence  $g_q$  has a global maximum at  $q = \max[d_l - q, 0]$ .*

**Proof** This lemma follows easily from the definition of  $l$  together with (8), (9) and (10).

### Proof of Proposition 3.1

**Proof** Assume that  $k \leq l$ , and let  $0 \leq q' \leq d_n$ ,  $0 \leq Q' \leq d_n$ . If  $q' \leq d_l$ , then by Lemma 8.1 and 8.2

$$\mathbb{E}[\Pi_r[q', Q']] \leq \mathbb{E}[\Pi_r[q', d_l - q']] \leq \mathbb{E}[\Pi_r[d_k, d_l - d_k]]$$

If  $q' \geq d_l$ , then

$$\mathbb{E}[\Pi_r[q', Q']] \leq \mathbb{E}[\Pi_r[q', 0]] \leq \mathbb{E}[\Pi_r[d_l, q' - d_l]] \leq \mathbb{E}[\Pi_r[d_l, 0]] \leq \mathbb{E}[\Pi_r[d_k, d_l - d_k]]$$

proving that  $q^* = d_k, Q^* = d_l - d_k$  is a global maximum for  $\mathbb{E}[\Pi_r[q, Q]]$ . That completes the proof in the case  $k \leq l$ .

We now consider the second case where  $l < k$ , and let  $0 \leq q' \leq d_n$ ,  $0 \leq Q' \leq d_n$ . If  $q' < d_l$ , then by Lemma 8.1 and 8.2

$$\mathbb{E}[\Pi_r[q', Q']] \leq \mathbb{E}[\Pi_r[q', d_l - q']] \leq \mathbb{E}[\Pi_r[d_l, 0]]$$

If  $q' \geq d_l$ , then by Lemma 8.2

$$\mathbb{E}[\Pi_r[q', Q']] \leq \mathbb{E}[\Pi_r[q', 0]]$$

This proves that a global maximum must be obtained at a point where  $Q = 0$ . If  $Q = 0$ , the problem is reduced to a classical newsvendor problem.

If any of the conditions  $W \leq x + c \leq R$ ,  $W \geq c + S$ , and  $x > S$  are violated, the problem is reduced to either a pure real option contract or a pure newsvendor contract. The solutions for these cases are straightforward and follows from the results in Section 3.1 and 3.2. Notice in particular that if  $x \leq S$ , then formula (5) does not apply. That case must hence be handled separately.

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