WATER WITH POWER:
Market power and supply shortage in dry years[1]

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Abstract:
The purpose of this paper is to analyse how market power may affect the allocation of production between seasons (summer and winter) in a hydro power system with reservoirs and where inflow in winter is uncertain. We find that even without market power we expect lower average prices during summer than during winter. Furthermore, we find that market power may in some situations lead to more sales during summer and in other situations to less sales during summer. Thus market power is found to have an ambiguous effect on the supply shortage in years with low inflow.

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1 Introduction

In some countries the electricity production is dominated by hydro power.[3] The hydro power production system is often quite complex, especially in those cases where water can be stored in reservoirs. In particular, there is uncertainty concerning the inflow of water. Storage in one period may lead to spill of water if there is a large inflow in the next period. In such cases, how do we expect producers with market power to behave? For example, one may ask whether exertion of market power may lead to a more severe shortage. Could it be that they produce early on, to have less water available later on?[4] If so, exertion of market power is expected to worsen situations with supply shortage in dry years and may lead to dramatic price hikes. With a few notable exceptions, there are no studies raising this question.[5] The purpose of this paper is to analyse whether market power can lead to a more severe shortage in periods with a limited supply (dry year) compared to a situation with perfect competition.

Our study is motivated by the observations in the Nordic power market 2002-03. The spot price in January 2003 was more than 80 øre/kWh, while the average price in a year with normal inflow is approximately 20 øre/kWh. The producers claimed that the reason for the price hike was the low water inflow to the reservoirs in autumn 2002. Data seems to support this claim, saying that it was in fact an extraordinarily low rainfall in the late autumn

[3] In New Zealand 80% of production is from hydro, in Chile 70%, Brazil 97% and Norway close to 100%.

[4] A similar type of question has been analysed by Stiglitz (1976) with regard to exploitation of exhaustible natural resources. Assuming positive extraction cost and rate of interest and iso-elastic demand he finds that a monopolist is more conservation minded than what is socially optimal.

[5] There are some studies of the allocation of water between different time periods, see Førsund (1994), Bushnell (2003), Scott and Read (1996), Crampes and Moreaux (2001) and Skaar and Sørgard (2003). But none of these studies introduce uncertainty concerning water inflow.

The issue of water allocation between periods when there is uncertainty about inflow has been analysed by Mathiesen (1992). In this study however, producers are assumed to behave as price-takers only.

In a more recent study Garcia et al (2001) analyse strategic behaviour in an infinite horizon duopoly model where two hydro power producers can storage water and there is uncertainty concerning water inflow. The question of market power and storage when inflow is uncertain has also been analysed by Johnsen (2001). See below for more details concerning these two studies.

of 2002.[6]

But is this the whole picture? Could it be that strategic producer behaviour also contributed to the price hike?[7] Victor D. Norman, the Minister responsible for competition policy, said the following at the outset of the period with supply shortage:

'A situation with low prices during summer and high prices during winter may indicate that there has been an abuse of market power'. (Dagbladet *13.11.2002)*

The argument is that a producer with market power could benefit from producing a large amount during summer time, thereby limiting the supply in winter. By doing so the producer could earn a large profit from high prices during winter. In April 2003, when the supply shortage came to an end, the Norwegian Competition Authority suggested that one should consider to split the largest Norwegian hydro power producer, Statkraft, into several independent firms. One argument for doing so was the following:[8]

'.., *The Competition Authority will not rule out the possibility that lack of competition may have increased the difficulties [supply shortage - our remark] we have experienced*'.

We formulate a model where we are able to analyse how a producer with market power would distribute his sales between summer and winter. During autumn there will be either heavy rain or little rain. If there is heavy rain, the inflow is so large that some water may be spilled (reservoirs are full). Whether some water is spilled or not depends on the inflow and the size of the reservoirs. If there is little rain during autumn, all inflow can be stored in reservoirs and used for production in the winter season.

First, we show that even under perfect competition the average price during summer is lower than average price during winter. The reason is that a high inflow can lead to waste of water (reservoirs are full), and then it would have been better to sell a little more during summer at a low price than to wait and risk a spill of water if there is a large inflow. The implication is that one cannot conclude whether there has been an abuse of market power or not by just observing price differences between summer and winter. In contrast, when there is a zero probability of spill of water we find that absence of

[6]See, for example, Bye et al. (2003).

[7]We became recently aware that Førsund et al. (2003) has raised the same questions as we do, in a report for the Ministry of Oil and Energy.

[8]From a letter the Norwegian Competition Authority sent to the Ministry of Trade and Industry, the owner of Statkraft.

market power will lead to identical prices in summer and winter. In such a case a price difference between summer and winter would indicate exertion of market power.

Second, we find that exertion of market power has an ambiguous effect on the distribution of sales between summer and winter (storage). On the one hand, a producer with market power may sell a large quantity during summer in order to constrain his supply and obtain a high price during winter. Or he may choose to do the opposite, selling a low quantity during summer to achieve a higher summer price. In this latter case market power may lead to a more limited difference between prices summer and winter.

Our result contrasts with Garcia et al. (2001), who found that market power always leads to higher prices during summer. The driving force behind their result is the modeling of the demand side. They apply a rectangular demand function, where the price during winter time is exogenously given. Then a shift of production from summer to winter will have no effect the winter price. In contrast, in our model there is a trade off. A shift of production from summer to winter would lead to higher prices during summer and lower prices during winter. This explains why we found that market power in some instances can lead to a shift in production from summer to winter, and in other instances to a reallocation of production from winter to summer.

Johnsen (2001) analyses market power and storage in a situation with limited transmission capacity between two regions connected by a single radial transmission line. He applies a simple two-period model similar to the one used in our paper. A numerical example is provided to illustrate that a monopolist finds it profitable to increase production in the first period when inflow is certain. The monopolist does this to avoid the possibility of becoming export constrained in the second period if high inflow occurs. Thus, storage is concluded to be lower in the monopoly case than in the competitive case.

We abstract from the possibility of transmission constraints as we look only at allocation of water between periods within a single geographic area. Also, different from Johnsen (2001), we analyse situations where the size of the water reservoir may constrain production and situations where the energy constraint may not be binding. As mentioned above, we find that market power has an ambiguous effect on storage. This is in contrast to Johnsen (2001) who finds that storage is lower in the monopoly case.

This article is organised as follows. In the next section we present the model, and in Section 3 we analyse the equilibrium outcomes in four different regimes. In Section 4 we apply a linear demand function, while in Section 4 we provide some concluding remarks. Proofs are presented in Appendices A and B.

2 The model

We consider an industry where electricity is generated from water, and we will use the Norwegian market as an illustration.

With reference to the graphs in figure 1 a hydrological year may be described in terms of four periods (of somewhat unequal length). Starting in spring, (between week 16 and 21) there is a large inflow of water from snow melting. In summer there is little rain and typically consumption exceeds additional inflow in this period. The autumn is normally a rainy season where inflow again exceeds consumption, while in winter the precipitation comes mainly in the form of snow that is unavailable for electricity production until spring.[9] This seasonal pattern repeats. Inflow varies considerably over the year, while consumption changes less. Also, inflow in any period is highly uncertain, while consumption is considerably less uncertain. Thus we will treat inflow as uncertain and consumption as deterministic.

Although the graphs indicate four distinct periods, the analysis is conducted within a two-period model, interpreted as summer and winter. Our concern is the allocation of water between these two periods. In particular, how much water will be used for electricity production in summer and thereby, how much water will be stored for later use? This allocation is studied for two different modes of producer behaviour: A price taker and a price setter. Our interest lies with finding out who will use most water during summer and thus have less available for the winter season.

The model is illustrated in Figure 2. To simplify the analysis we disregard the multi-year dimension, i.e., the possibility to even out extreme inflows over several years. In our analysis, the hydrological year starts with the inventory U_1.[10] At the decision point in summer this inventory is known, while the autumn inflow U_2 is uncertain. Our analysis is one of decision making under

[9] 2002 was a dry year with little inflow during the autumn. This is illustrated in Figure 1, where the reservoir level during the autumn 2001 is considerably higher compared to the autumn 2002.

[10] U_1 may alternatively be thought of as an inflow of water at the beginning of period 1.

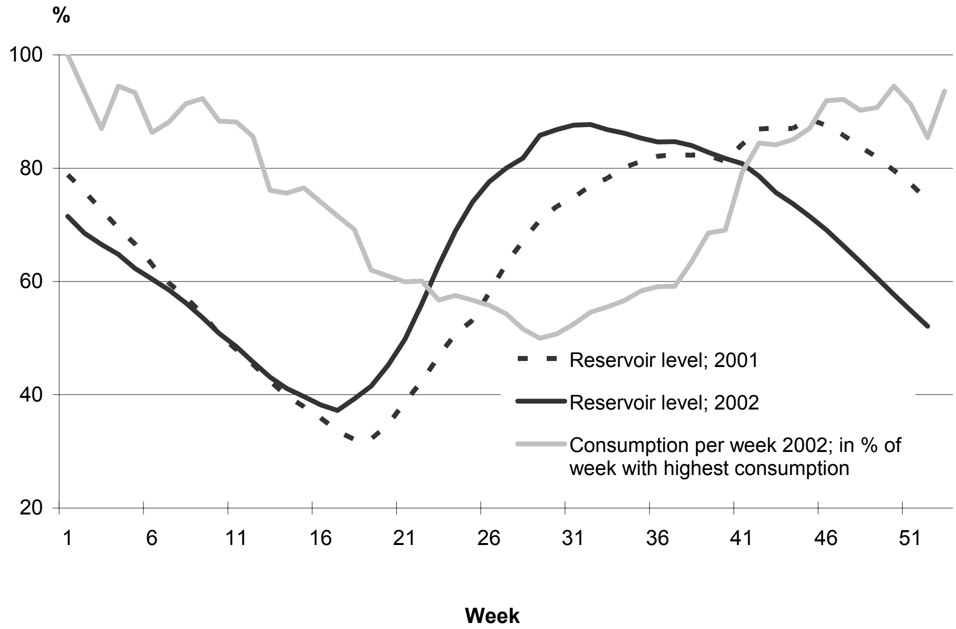

Figure 1: Seasonal pattern for reservoir level and consumption.

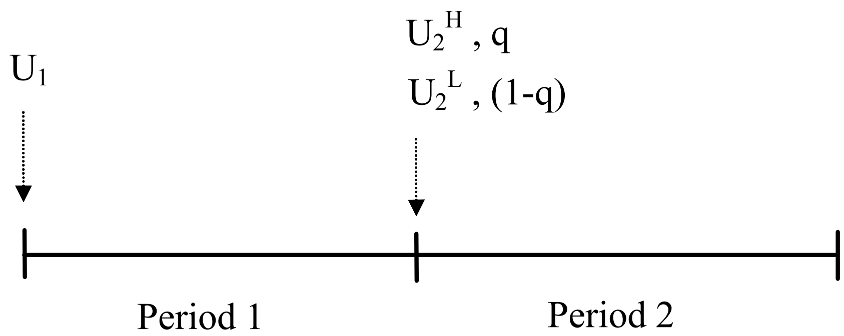

Figure 2: Illustration of the model.

uncertainty with respect to the autumn inflow. This inflow is thought of as materializing at the beginning of period 2. We assume that the inflow will be high (U_2^H) or low (U_2^L) with probabilities q and $(1-q)$ respectively, and that $U_2^H > U_2^L$. Furthermore, we will assume that U_2^L is so low that reservoir capacity (R) always is sufficient, while U_2^H is so high compared to R, that some water may be spilled.

We want to compare the outcome of a competitive equilibrium with that of an industry of producers with market power. To represent the latter we consider the collusive outcome. In the subsequent presentation and discussion of the model we simplify by talking about one producer that may behave as a price taker or as a monopolist when making his decisions, i.e., production (in the two periods): x_1 respectively x_2^i, $i = H, L$. Furthermore, let w^i denote spill of water in state i, $i = H, L$.

Both kinds of producers seek to maximize profits, the difference between them being how they regard the market prices. A monopolist assumes he can set prices, while a price taker by definition assumes that prices are given beyond his control. Let $p_t(.)$ and $D_t(.)$, $t = 1, 2$ denote inverse and direct demand respectively.

The objective for the monopolist can be formulated as[11]

$$\underset{x_1, x_2^L, x_2^H}{Max}\, p_1(x_1)x_1 + qp_2(x_2^H)x_2^H + (1-q)p_2(x_2^L)x_2^L.$$

The maximization of the sum of producers and consumers surpluses would generate the competitive equilibrium

$$\underset{x_1, x_2^L, x_2^H}{Max}\, \varphi_1(x_1)x_1 + q\varphi_2(x_2^H)x_2^H + (1-q)\varphi_2(x_2^L)x_2^L, \text{ where } \varphi = \int_0^x p(s)ds.$$

There are several constraints on decision variables x_1, x_2^H and x_2^L, some of which are obvious, like production has to be non-negative. In addition, we have the following restrictions:

(i) Period 1 production:

$$x_1 \leq min\{U_1, D_1(0)\}.$$

[11] Except for water values, variable production costs of electricity from hydro are small. In line with this, other variable costs than water values are by assumption set to zero in our analysis.

Production has to stay within the given amount of water (U_1), and the producer will never supply more than demand at a price of zero.

(ii) Available energy after autumn inflow:

$$Z_i \equiv U_1 - x_1 + U_2^i - w^i \leq R, \qquad i = H, L.$$

What is not consumed for production in period 1 ($U_1 - x_1$) will be stored. With the addition of autumn inflow (U_2^i), stored water has to be within the reservoir capacity (R). This restriction may imply that some water (w^i) has to be spilled. Thus Z_i denotes the available energy for production in period 2.

(iii) Period 2 production:

$$x_2^i \leq min\{Z_i, D_2(0)\}, \qquad i = H, L.$$

Production has to stay within the given amount of energy (Z_i).[12] Furthermore, the producer will never produce beyond demand at a price of zero.[13]

3 Equilibrium outcomes

Our concern is as mentioned above: Will a firm with market power store more or less water from summer to winter than a firm without market power? In order to highlight this issue, we make a few assumptions that further delineate the analysis.

For given demand, represented by $D_1(p_1)$ and $D_2(p_2^i)$ (or equivalently the willingness to pay $p_1(x_1)$ and $p_2(x_2^i)$), the model has four parameters R, U_1 and U_2^i, $i = H, L$. We now rule out parameter combinations that are of little relevance to our issue.

Assumption 1:

$$x_1 < U_1.$$

The initial inventory of water is sufficient for optimal production in any mode of producer behaviour in period 1. We are concerned with the consequences of shortage or surplus of energy in winter, not a shortage in summer.

[12] If demand, $D_2(0)$, is lower than available energy, the surplus is spilled. This spill comes in addition to the spill of water caused by limited reservoir capacity, w_i.

[13] We assume that demand during period 2 is independent of the state of world with respect to inflow at the beginning of the period.

Assumption 2:

$$U_1 - x_1 + U_2^L < R.$$

Inflow in a dry autumn (U_2^L) is so small that the reservoir never becomes binding. Hence, $w^L = 0$. Assumption 2 also applies to any mode of behaviour. It is the high inflow state that may cause spill of water.

Assumption 3:

$$x_2^i = Z_i \text{ whenever } p_2(Z_i) > 0, i = H, L.$$

Assumption 3 relates to period 2 production in both states. With regard to state L, we qualify Assumption 3 further by considering only combinations of inflow and reservoir capacity resulting in a positive price and marginal revenue in period 1 and positive expected price and marginal revenue in state L. The implications are that all the available energy is used both in the competitive and collusive equilibrium if there is low inflow and that the realised price in this state is positive by definition.

In state H on the other hand, there may be so much water available that the monopoly producer would like to produce less than what is available in order to ensure a positive marginal revenue from sales in state H. We make the assumption that authorities can enforce production as long as there is demand at a positive price. In particular, we assume that the authorities can detect and prevent water from being spilled (in excess of w^H) as long as there is positive demand. This assumption applies to the monopolist, who in some situations in state H would otherwise spill water in order to increase the price.[14] We relax this assumption in Section 4.3 below.

Through assumptions $1 - 3$ the producers' maximization problem is reduced to a question of finding production levels in period 1 and in the high inflow state of the world in period 2.

$$\underset{x_1, x_2^H}{Max} \quad \{p_1(x_1)x_1 + qp_2^H(x_2^H)x_2^H + (1-q)p_2^L(U_2^L + U_1 - x_1)[U_2^L + U_1 - x_1]\}.$$

Subject to

[14]The assumption that there is no spill of water is common in the literature, see for example Johnsen et al (1999) and Crampes and Moreaux (2001). Morover, The Norwegian Competition Authority also made such an assumption in an acquisition case in the Norwegian power market in 2001-02 (Statkraft acquiring Agder Energi).

$$x_2^H = \min\{Z^H, D_2(0)\}.$$

Next, we characterise different equilibrium outcomes (regimes) depending on the parameter values.

3.1 The four regimes

For a given demand, available energy and the reservoir capacity may constrain the solution. The various combinations of these parameters are illustrated in Table 1.

		Reservoir capacity, R	
		$R = x_2^H$	$R > x_2^H$
Energy, $U_1 + U_2^H$	$U_1 + U_2^H = x_1 + x_2^H$	R4: R constrains; energy constrains; $p_2(x_2^H) > 0$.	R3: Energy constrains; $p_2(x_2^H) > 0$.
	$U_1 + U_2^H > x_1 + x_2^H$	R2: R constrains; $p_2(x_2^H) > 0$; $w^H > 0$.	R1: $D_2(0)$ constrains; $p_2(x_2^H) = 0$. $w^H = 0$.

Table 1: The four regimes

Regime 1.

The inflow in state H is high. In this regime, reservoir capacity is also large and further production in period 2 is constrained by demand. Hence, we have that $p_2(x_2^H) = 0$ and any additional water is spilled. In the low inflow state, and by assumption 3, the energy constraint is binding and $p_2(x_2^L) > 0$.

When considering the optimal allocation of water between periods, a price taking firm, at the margin, either sells one unit in period 1 at the price p_1 or stores it; with probability $(1-q)$ the unit sells at price $p_2(x_2^L)$ or with probability q it sells at a price equal to zero. In a competitive equilibrium the price in period 1 equals the expected price of period 2,

$$p_1 = (1-q)p_2(x_2^L) + qp_2(x_2^H) = (1-q)p_2(x_2^L). \tag{1}$$

The monopoly firm considers its marginal revenue rather than the price it obtains when allocating water between periods. Thus the equilibrium in this regime has to satisfy

$$\begin{aligned} p_1[1-\frac{1}{|e_1|}] &= (1-q)p_2(x_2^L)[1-\frac{1}{|e_2|}] + qp_2(x_2^H)[1-\frac{1}{|e_2|}] \\ &= (1-q)p_2(x_2^L)[1-\frac{1}{|e_2|}], \end{aligned} \tag{2}$$

where e_1 and e_2 denote price elasticities of demand in period 1 and 2 respectively.

Regime 2.

In this regime, the reservoir capacity constrains the amount of available energy in state H. The producer is unable to satisfy all demand in period 2, also in a wet year. This implies that $p_2(x_2^H) > 0$. In summer the producer knows that a high inflow in autumn will lead to spill of water, which means that the marginal unit stored in summer does not make it to the winter.[15]

If there is low inflow, however, the marginal unit stored will be used for production in winter. The equilibrium condition in a situation with no market power is as follows:

$$p_1 = (1-q)p_2(x_2^L). \tag{3}$$

In the monopoly equilibrium the equivalent condition is

$$p_1[1-\frac{1}{|e_1|}] = (1-q)p_2(x_2^L)[1-\frac{1}{|e_2|}]. \tag{4}$$

Regime 3.

In this regime, inflow in state H is low compared to reservoir capacity and demand, whereby $p_2(x_2^H) > 0$. Furthermore, because the reservoir capacity is non-binding, there is no spill of water and the marginal unit stored in period 1 has a positive value in state H.

Because of the non-binding reservoir, water is optimally allocated between periods. Thus in a competitive equilibrium the price of period 1 has to equal the expected price of period 2

$$p_1 = (1-q)p_2(x_2^L) + qp_2(x_2^H). \tag{5}$$

Similarly, the equilibrium condition for the monopoly has to satisfy

[15] We might say that $p_2(x_2^H)$ is irrelevant because the marginal unit stored in summer never survives to period 2 state H.

$$p_1[1 - \frac{1}{|e_1|}] = (1-q)p_2(x_2^L)[1 - \frac{1}{|e_2|}] + qp_2(x_2^H)[1 - \frac{1}{|e_2|}]. \tag{6}$$

Regime 4.

In this regime, both energy and reservoir constrain production. By definition, the limited energy implies that prices are positive in both states as in regime 3. The reservoir constraint further implies that the producer cannot freely allocate the scarce energy resource between periods, as in regime 3, whereby production in period 1 will be higher than optimal, and the price in period 1 will be lower than the expected price of period 2.[16]

The competitive equilibrium of this regime is therefore characterized by

$$p_1 < (1-q)p_2(x_2^L) + qp_2(x_2^H) \tag{7}$$

Similarly, in the monopoly equilibrium, the marginal revenue of period 1 is lower than the expected marginal revenue of period 2

$$p_1[1 - \frac{1}{|e_1|}] < (1-q)p_2(x_2^L)[1 - \frac{1}{|e_2|}] + qp_2(x_2^H)[1 - \frac{1}{|e_2|}]. \tag{8}$$

With reference to the four regimes described above and by focusing on the competitive case, we can state the following proposition with respect to the expected price difference between period 1 and period 2:

Proposition 1 *Assume that* $0 < q < 1$*. If the reservoir constraint is binding in equilibrium, then the price in period* 1 *is lower than the expected price in period* 2*.*

Proof. When the reservoir constraint is binding in the high inflow state we have that $x_2^H = R$. The reservoir constraint is binding in regimes 2 and 4 and the competitive equilibrium in both these regimes is characterized by $p_1(x_1) < (1-q)p_2(x_2^L) + qp_2(x_2^H)$. ■

Proposition 1 tells us that even under perfect competition the price in period 1 may be lower than the expected price in period 2. Such price differences may very well be the result of an efficient allocation of resources between periods. The intuition here is that if we do not use the water to

[16] Assumption 1 and the infeasibility of moving water (storage) forward in time (from period 2 to period 1), rule out the possibility of having $p_1 > (1-q)p_2(x_2^L) + qp_2(x_2^H)$.

produce electricity in period 1, there is a probability that water may be lost due to the reservoir constraint. This result is important, because it means that when the price in summer proves to be lower than the price in winter year after year this is no proof of exertion of market power.

3.2 Equilibria in the same regime

Since we are unable to verify the existence of market power from observing price differences between periods, we proceed by comparing price differences between the competitive and the monopoly equilibrium. As an alternative to focusing directly on the price difference we look at period 1 production. If period 1 production is lower in one case than the other, this means that storage is higher and also that the price in period 2 will be lower.

Above we have characterized four regimes and conditions on equilibrium prices and marginal revenues with and without market power, respectively. For a given parameter set $(R, U_1, U_2^i, i = H, L)$ there is no guarantee that the competitive and monopoly equilibria fall into the same regime. In fact, numerical examples show that the competitive regimes do not overlap completely with the corresponding monopoly regimes. Moreover, regime 1 implies that marginal revenue will be negative. We deal with this more complex situation below.

In this section, we compare production (and storage) of the competitive and monopoly equilibria where by assumption both belong to the same regime. In addition, we assume that marginal revenue corresponding to competitive equilibrium production in state H is positive.[17] One implication is that regime 1 is ruled out of the analysis in this section.

Let $\widehat{x}_1^v$ and $\widetilde{x}_1^v$ represent equilibrium production in period 1 in regime v ($v = 1, 2, 3, 4$) under competition and monopoly, respectively. The equilibrium production levels are derived from the equilibrium conditions stated in equations (1) to (8). We consider the price elasticities in the competitive equilibrium as the benchmark. Let $|e_t|$ denote the absolute value of the price elasticity in the competitive equilibrium. We then ask whether the introduction of market power will lead to reallocation of production.

[17] The assumption that price and marginal revenue are positive holds for iso-elastic demand where $|e_t| > 1$. However, since we we are interested in whether storage in the monopoly case is higher or lower than in the competitive case we only need to assume that marginal revenue is positive in state H at the level of competitive equilibrium output.

Proposition 2 *Assume that* $|e_t| > 1$ *and that reservoir capacity and inflow are such that the same regime applies to both the competitive and the monopoly equilibrium. Then,*
i) in regime 2, $\widehat{x}_1^2 > \widetilde{x}_1^2$ *if* $|e_1| < |e_2|$.
ii) in regime 3, $\widehat{x}_1^3 > \widetilde{x}_1^3$ *if* $|e_1| < |e_2|$.
ii) while $\widehat{x}_1^4 = \widetilde{x}_1^4$ *in regime* 4.

Proof. See Appendix A. ■

Proposition 2 tells us that as long as the competitive and the monopoly equilibrium are in the same regime, the price elasticity of demand is decisive for whether market power leads to more or less production in period 1. If, for example, demand is less price elastic in period 1 than in period 2, monopoly production in period 1 is lower than the competitive production. The reason is that a firm with market power will exploit differences in market characteristics. Note that the result in Proposition 2 does not depend on the probability of high inflow.

3.3 Equilibria in different regimes

As mentioned, the borderlines that define cut-off values between the four regimes are not identical for the two equilibria. For certain values of reservoir capacity and inflow $(R, U_1, U_2^i, i = H, L)$, the competitive equilibrium may for instance be in regime 4, while the monopoly equilibrium belongs to regime 3. Then the simple criteria we reported in Proposition 2 may no longer apply.

When we compare equilibria in different regimes, we need to define the cut-off values between the different regimes in our two cases. Each regime is defined for a certain range of inflow and reservoir values as illustrated in Table 1. In order to be able to describe the cutoff values between all 4 regimes, we make use of period 1 equilibrium production determined by the first order conditions applying to the different regimes. After having defined each regime under perfect competition and monopoly, we then compare period 1 production (and storage) in situations where the equilibria are in different regimes. Thus, we confine the discussion to the range of inflow and reservoir values where the regime related to the competitive equilibrium is different from the regime associated with the monopoly equilibrium. We continue to assume that marginal revenue is non-negative as described in the previous section. The situations where regimes are identical are covered by proposition 2.

We start by defining the cut-off values between the different regimes. The cut-off values are illustrated in Table 2.

	Cut-off value	
Regime	Competitive	Collusive
2 and 4	$U_1 + U_2^H = \widehat{x}_1^2 + R$	$U_1 + U_2^H = \widetilde{x}_1^2 + R$
3 and 4	$U_1 + U_2^H = \widehat{x}_1^3 + R$	$U_1 + U_2^H = \widetilde{x}_1^3 + R$

Table 2: Regime borders

If we look at the equilibrium conditions defined for regime 2 (equations (3) and (4)) and regime 3 (equations (5) and (6)) we find that $\widehat{x}_1^2 > \widehat{x}_1^3$ and $\widetilde{x}_1^2 > \widetilde{x}_1^3$. A special situation arises when $|e_1| = |e_2|$. Then we have that period 1 production is the same in the competitive and the monopoly case both in regime 2 and regime 3, $\widehat{x}_1^2 = \widetilde{x}_1^2$ and $\widehat{x}_1^3 = \widetilde{x}_1^3$. Accordingly, the cut-off values defining the borderlines between the regimes are identical and there is no combination of inflow and reservoir capacity resulting in different equilibria. If $|e_1| \neq |e_2|$ however,then for some values of inflow and reservoir capacity the competitive regime is different from the monopoly equilibrium regime and we can state the following proposition. As in Proposition 2, let us consider the price elasticity in the competitive equilibrium as the benchmark:

Proposition 3 *Assume that $|e_t| > 1$ and that reservoir capacity and inflow $(R, U_1, U_2^i, i = H, L)$ are such that different regimes apply to the competitive and the monopoly equilibrium. Then,*
i) if $|e_1| < |e_2|$, period 1 production in the monopoly case is always lower (and storage higher) than in the competitive equilibrium.
ii) if $|e_1| > |e_2|$, period 1 production in the monopoly case is always higher (and storage lower) than in to the competitive equilibrium.

Proof. See Appendix B. ■

We see that period 1 production in the monopoly equilibrium is always lower or equal to period 1 equilibrium production in the competitive case if demand in period 1 is less price elastic than demand in period 2. This result holds irrespective of whether the competitive and the monopoly equilibrium belong to the same regime (Proposition 2) or not (Proposition 3). Put differently, we find that when $|e_1| < |e_2|$ in the competitive equilibrium, storage under perfect competition will never be higher than storage under monopoly.

This is in line with the results we found in Proposition 2, and it shows that our basic result is quite robust.

This result however, rests on the assumption that price and marginal revenue in state H are both non-negative. Sufficient inflow in state H may lead to a situation where the monopolist would want to spill some of the available water. Also, in the competitive case the price in state H may be driven to zero if inflow is sufficiently large. According to Assumption 3, the monopolist may be forced to produce a quantity that implies a negative marginal revenue in state H. The only situations where authorities allow spill of water are when the price in state H is zero or the reservoir is full. We analyze these situations in the next subsection through an example with linear demand.

4 An example: Linear demand

Let us now introduce a linear demand function. Such a demand function implies that the price will become zero for a large enough production quantity. In contrast, with iso-elastic demand the price will never equal zero and marginal revenue to a monopolist will be positive (when demand is elastic).[18] Linear demand and Assumption 3 implies that regime 1 equilibria are possible and that marginal revenue may be negative in equilibrium. Since the first order conditions are identical under regimes 1 and 2, we let $\widehat{x}_1$ and $\widetilde{x}_1$ represent period 1 production in these regimes under competition respectively monopoly.

We employ the following inverse demand functions[19]:

$$p_1 = \alpha_1 - \beta_1 x_1, \tag{9}$$

$$p_2 = \alpha_2 - \beta_2 x_2^i, \text{ where } i = H, L. \tag{10}$$

Note that the parameter β captures the market size, while the parameter α captures the willingness to pay. This is easily seen from the monopoly price and quantity; $p = \alpha/2$ and $x = \alpha/2\beta$.

[18] When demand is inelastic there is no monopoly equilibrium. Hence, analysing monopoly equilibrium with iso-elastic demand by assumption rules out the case we want to characterise.

[19] As mentioned above (see footnote 10), demand parameters may differ between periods, but not over the states of the world. In period 2, when $x_2^H \neq x_2^L$, it follows that $p_2(x_2^H) \neq p_2(x_2^L)$.

4.1 Equilibria in the same regime

Substituting the demand functions defined in (9) and (10) into the equilibrium conditions of regimes 1 to 4, we can state the following proposition:

Proposition 4 *Assume linear demand as specified in* (9) *and* (10)*, and that reservoir capacity and inflow are such that the same regime applies to both the competitive and the monopoly equilibrium. Then,*
i) in regimes 1 *and* 2*,* $\widehat{x}_1 > \widetilde{x}_1$ *if* $\alpha_1 > (1-q)\alpha_2$*.*
ii) in regime 3*,* $\widehat{x}_1^3 > \widetilde{x}_1^3$ *if* $\alpha_1 > \alpha_2$*.*
ii) in regime 4 $\widehat{x}_1^4 = \widetilde{x}_1^4$*.*

Proof. See Appendix A. ■

Proposition 4 shows that the parameter α is crucial for understanding how market power affects allocation of production. At equal prices in the two periods the absolute value of the price elasticity of demand is lower in the period with the highest α. A firm with market power would then find it optimal to reallocate production so that the price is highest in the period with the highest parameter α. If $\alpha_1 > \alpha_2$, reallocation leads to less production in period 1 in the monopoly equilibrium and higher storage $(U_1 + U_2^H - x_1)$. This result is in line with what we found in Proposition 2. In both cases production is increased in the period with the highest absolute value of price elasticity of demand.

We also note that in regime 1 and regime 2 equilibria the willingness to pay for electricity in period 1 only have to be higher than the expected willingness to pay in state L for storage to be higher in the monopoly equilibrium.

4.2 Equilibria in different regimes

The cut-off values between different regimes in the case of linear demand are reported in Table 3.

As indicated by Table 3 there are regime 1 equilibria in the competitive case if reservoir capacity and inflow are sufficiently high; $R > \frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H > \widehat{x}_1^3 + \frac{\alpha_2}{\beta_2}$. The two remaining cut-off values in the competitive case are identical to the values listed in Table 2 above.

In order to make a graphical representation of these regimes, we use a numerical example where we focus on variations in state H inflow (U_2^H) and

	Cut-off value	
Regime	Competitive	Monopoly*
1 and 2	$R = \frac{\alpha_2}{\beta_2}$	$R = \frac{\alpha_2}{\beta_2}$
1 and 3	$U_1 + U_2^H = \widehat{x}_1^3 + \frac{\alpha_2}{\beta_2}$	$U_2^H = y(U_1, U_2^L, R = \frac{\alpha_2}{\beta_2})$
2 and 4	$U_1 + U_2^H = \widehat{x}_1 + R$	$U_1 + U_2^H = \widetilde{x}_1 + R$
3 and 4	$U_1 + U_2^H = \widehat{x}_1^3 + R$	$U_1 + U_2^H = \widetilde{x}_1^3 + R$
2 and 3		$U_2^H = y(U_1, U_2^L, R)$

Table 3: Cut off values assuming linear demand. * The cut off value between regimes 2 and 3 is defined below in equation (11).

reservoir capacity (R). Assume that $q = 0.5$, $U_1 = 0.6$, $U_2^L = 0$, $\beta_1 = \beta_2 = 1$ and $\alpha_1 = \alpha_2 = 1$. The border lines between the different regimes in the case of a competitive equilibrium are illustrated in figure 3.

The cut-off values in the monopoly case listed in Table 3 depend on whether marginal revenue in state H is positive or negative. Marginal revenue in state H is non-negative as long as $x_2^H \leq \frac{1}{2}\frac{\alpha_2}{\beta_2}$. If $R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$ marginal revenue in state H is never negative because of the reservoir constraint. In addition, regime 1 is infeasible since regime 1 equilibria require that $R > \frac{\alpha_2}{\beta_2}$; $p_2(\frac{\alpha_2}{\beta_2}) = 0$. Thus, there is no cut-off value between regime 1 and 2 or 1 and 3 as long as marginal revenue in state H is positive.

Furthermore, when marginal revenue is non-negative in state H it follows from the equilibrium conditions that $\widetilde{x}_1^3 < \widetilde{x}_1$. For intermediate values of inflow and reservoir capacity, $\widetilde{x}_1 + R > U_1 + U_2^H > \widetilde{x}_1^3 + R$, regime 4 equilibria apply.

Let us turn to the combinations of inflow and reservoir capacity where marginal revenue in state H is negative. We observe from the equilibrium conditions that in such cases $\widetilde{x}_1^3 > \widetilde{x}_1$. Instead of a range of inflow/reservoir values where regime 4 applies, there is now a range of values where it seems that both regime 2 and regime 3 apply. Recall that the equilibrium solution should be in regime 3 if $U_1 + U_2^H < \widetilde{x}_1^3 + R$ and in regime 2 if $U_1 + U_2^H > \widetilde{x}_1 + R$. Because $\widetilde{x}_1^3 > \widetilde{x}_1$, we know that we are in regime 2 if $U_1 + U_2^H > \widetilde{x}_1^3 + R$ and in regime 3 if $U_1 + U_2^H < \widetilde{x}_1 + R$. In situations where $\widetilde{x}_1^3 + R > U_1 + U_2^H > \widetilde{x}_1 + R$, the monopoly equilibrium is either in regime 3 or in regime 2 depending on the level of profit associated with the relevant regime. This is illustrated in Figure 4, where $\Delta\Pi$ indicate the direction of increased profits.

We calculate profits $\Pi(.)$ using regime 2 and 3 equilibrium output. Then

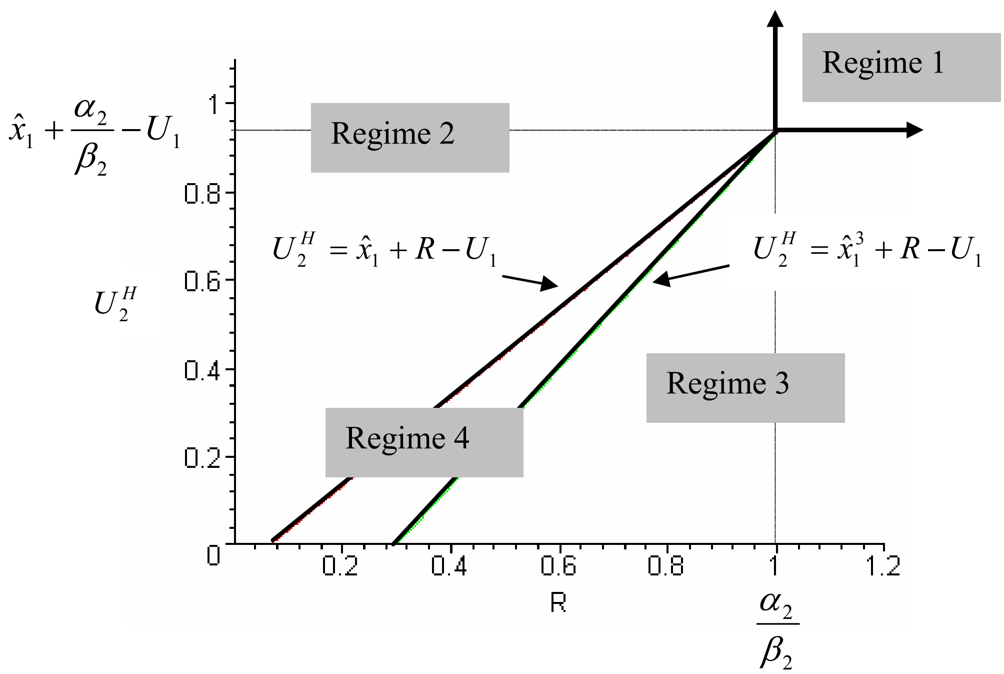

Figure 3: The border lines between the 4 different regimes in the competitive case.

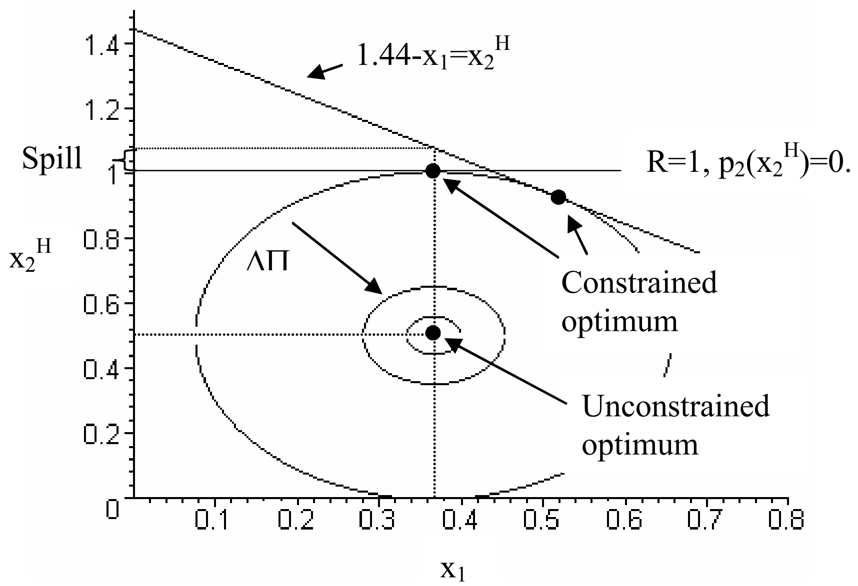

Figure 4: The two constrained optimum points illustrate situations where the monopoly producer is indifferent between a regime 2 and regime 3 equilibrium output in period 1. In regime 2 the reservoir constraint is satisfied, while the energy constraint is satisfied in regime 3. The unconstrained optimum is the monopoly choice of equilibrium production when there is no limit to his ability to spill water.

we set the difference $\Pi(\widetilde{x}_1) - \Pi(\widetilde{x}_1^3) = 0$ and solve for inflow in state H.

We find that a monopoly producer would be indifferent between a regime 2 and 3 strategy (when marginal revenue in state H is negative) if

$$\begin{aligned} U_2^H &= \frac{1}{2}\frac{\beta_2\alpha_1 - \beta_1\alpha_2 + 2(1-q)(\beta_2)^2 U_2^L - 2\beta_1\beta_2 U_1}{(\beta_1+\beta_2-q\beta_2)\beta_2} \\ &\quad + \frac{\sqrt{(2\beta_2 R - \alpha_2)^2(\beta_1+\beta_2)(\beta_1+\beta_2-q\beta_2)}}{(\beta_1+\beta_2-q\beta_2)\beta_2} \\ &= y(U_1, U_2^L, R). \end{aligned} \tag{11}$$

If $U_2^H > y(U_1, U_2^L, R)$, then profit is higher in regime 2. If so, the monopolist would choose regime 2 equilibrium output in period 1 .

The cut-off value between regimes 1 and 3 is found by inserting $R = \frac{\alpha_2}{\beta_2}$ into equation (11). The cut-off value between regimes 1 and 2 is identical to the competitive case; $R = \frac{\alpha_2}{\beta_2}$. The border lines between the different regimes under monopoly are illustrated in Figure 5.

Comparing the regime border lines in the competitive case (Figure 3) with the borders applying to the monopoly case (Figure 5), we observe that the border lines do not overlap perfectly.[20] Thus, as Figure 3 and 5 show, we have equilibria in different regimes for some values of inflow and reservoir capacity.

Using the border lines defined in Table 3 together with our knowledge of period 1 production and storage related to the different regimes we state the following proposition:

Proposition 5 *Assume linear demand as specified in* (9) *and* (10). *Furthermore, assume that reservoir capacity and inflow are such that different regimes apply to the competitive and the collusive equilibria. Then,*
i) if $\alpha_1 > \alpha_2$, *period* 1 *production in the monopoly equilibrium is always lower* (*and storage higher*) *than in the competitive equilibrium.*
ii) if $\alpha_1 < \alpha_2$ *and* $\alpha_1 > (1-q)\alpha_2$, *period* 1 *production in the monopoly equilibrium will be higher or lower than in the competitive equilibrium.*
iii) if $\alpha_1 < (1-q)\alpha_2$, *period* 1 *production in the monopoly equilibrium is always higher* (*and storage lower*) *than in the competitive equilibrium.*

[20] In our numerical example, the cut off value between regimes 3 and 4 however are identical in the competitive and collusive case.

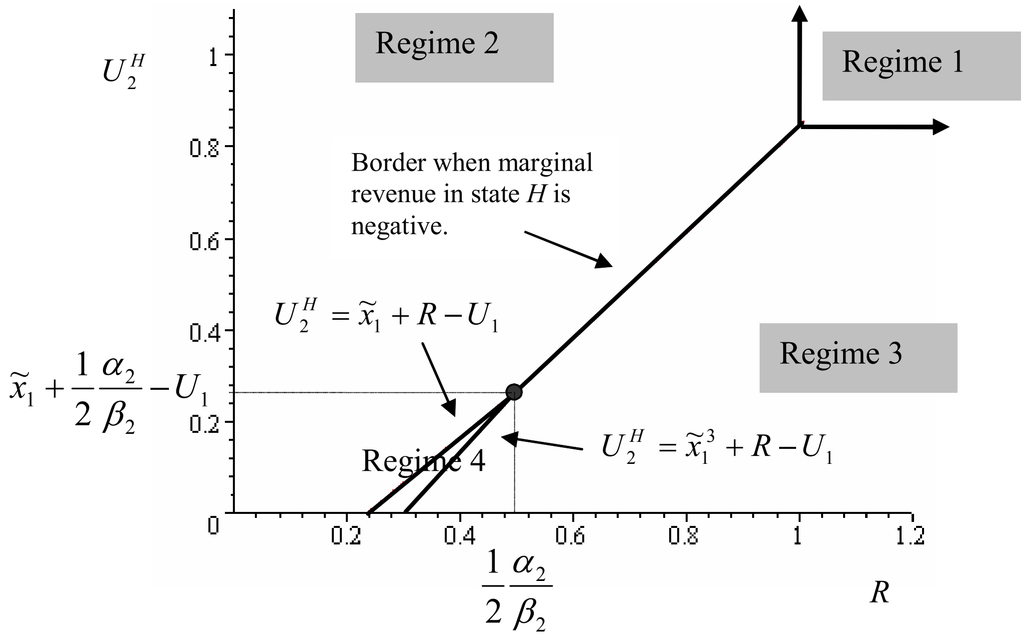

Figure 5: The border lines between the 4 regimes in the monopoly case.

Proof. See Appendix B. ■

Propositions 4 and 5 both show that storage in the monopoly case is always higher than storage in the competitive case if the willingness to pay for electricity in period 1 is higher than the willingness to pay for electricity in period 2; $\alpha_1 > \alpha_2$. This result holds irrespective of whether the monopoly and competitive equilibrium belong to the same regime or not. The intuition behind this result is simply that under monopoly, the producer want to reduce sales in the period with the lowest price elasticity. Thus, if we have the opposite situation where the price elasticity in period 1 is higher than in period 2 (also when there is overflow in state H) storage in the monopoly case will always be lower than storage in the competitive case.

From proposition 4 we know that if $(1-q)\alpha_2 < \alpha_1 < \alpha_2$, there would be higher storage under regimes 1 and 2 in the monopoly case and lower under regime 3. Thus, whether there is higher or lower storage in the monopoly case depends on the combination of inflow and reservoir values. This is also the case when we focus on combinations of different regimes. For some combinations of different regimes storage will be higher and other combinations will result in lower storage in the monopoly case.

4.3 Relaxing Assumption 3

Above we have assumed that the authorities will force producers to produce electricity in state H as long as the price is positive (Assumption 3). Now, we relax this assumption.

As long as marginal revenue in state H in the monopoly case is positive, relaxing Assumption 3 adds no new insight to the problem of determining whether storage is higher or lower in the monopoly case than in the competitive case. This situation is covered in Propositions 4 and 5.

However, if we look at reservoir and inflow values where $R > \frac{1}{2}\frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H > \widetilde{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$, the situation is different. When we relax Assumption 3 and inflow and reservoir capacity are sufficiently high, the monopolist would produce a fixed amount equal to $\frac{1}{2}\frac{\alpha_2}{\beta_2}$ in state H. At this production level marginal revenue in state H is zero.

When marginal revenue in state H is zero, allocation of water between period 1 and period 2 state L is determined by:

$$p_1[1 - \frac{1}{|e_1|}] = (1-q)p_2(x_1^L)[1 - \frac{1}{|e_2|}]. \tag{12}$$

Using the linear demand functions from (9) and (10) we get that period 1 production under monopoly is equal and fixed to $\widetilde{x}_1$ when $R > \frac{1}{2}\frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H > \widetilde{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$.

Now, we compare the monopoly equilibrium to the competitive equilibrium for different values of inflow and reservoir capacity. As long the competitive equilibrium is in regime 1 or 2, we have that period 1 production is equal to $\widehat{x}_1$. Thus, we find as shown in Prpositions 4 and 5 that storage is higher in the monopoly case ($\widetilde{x}_1 < \widehat{x}_1$) if $\alpha_1 > (1-q)\alpha_2$.

When we look at regimes 3 and 4 competitive equilibria the condition that $\alpha_1 > (1-q)\alpha_2$ is no longer sufficient to conclude that storage is higher in the monopoly case. As long as $\alpha_1 > (1-q)\alpha_2$, this implies that $\widetilde{x}_1 < \widehat{x}_1$. However, because regime 3 and 4 competitive equilibria imply less production in period 1 than in regimes 1 and 2 we also have that $\widehat{x}_1^3 < \widehat{x}_1$ and $\widehat{x}_1^4 < \widehat{x}_1$. Thus, we are unable to conclude that storage is higher in the monopoly case simply by looking at the condition $\alpha_1 > (1-q)\alpha_2$. If the opposite is true, $\alpha_1 < (1-q)\alpha_2$, then storage in the competitive case will always be higher than in the monopoly case.

5 Some concluding remarks

The main question of this paper is whether market power would lead to higher or lower storage from summer to winter. We compare the monopoly equilibrium to the competitive equilibrium in a simple two period model with uncertainty concerning water inflow. We analyze situations where storage may or may not be constrained by the existing reservoir capacity, and where inflow is so high that the energy constraint may or may no longer be binding. We find as a general result that market power, represented by the collusive equilibrium, would not lead to lower storage if demand is more price elastic in the winter period. If on the other hand, demand is less price elastic in winter compared to summer, storage would not be lower in the competitive case.

Whether demand is more or less price elastic in winter than in summer is an empirical question. To our knowledge there is no empirical evidence available at present. Thus, it is not possible to conclude that market power lead to less storage during summer and thereby increases the probability of a supply shortage in dry years.

A Equilibria in the same regime

Here we provide proof of proposition 2 and 4 where we have equilibria in the same regime.

A.1 Proof of Proposition 2

We have that period 1 production in competitive equilibrium ($\widehat{x}_1$) under regime 2 is determined by

$$p_1 = (1-q)p_2(x_1^L).$$

If we divide through by p_2 we find that $\frac{p_1}{p_2} = 1-q$.

Period 1 collusive production ($\widetilde{x}_1$) under regime 2 solves

$$p_1[1-\frac{1}{|e_1|}] = (1-q)p_2(x_1^L)[1-\frac{1}{|e_2|}].$$

We find that $\frac{p_1}{p_2} = (1-q)\frac{[1-\frac{1}{|e_2|}]}{[1-\frac{1}{|e_1|}]}$. Period 1 competitive production ($\widehat{x}_1$) is the higher if $\frac{[1-\frac{1}{|e_2|}]}{[1-\frac{1}{|e_1|}]} > 1$ or $|e_1| < |e_2|$.

Under regime 3, period 1 production in the competitive equilibrium ($\widehat{x}_1^3$) is determined by

$$p_1 = (1-q)p_2(x_1^L) + qp_2(x_1^H).$$

In the case of collusion we have that period 1 production under regime 3 ($\widetilde{x}_1^3$) solves

$$p_1[1-\frac{1}{|e_1|}] = (1-q)p_2(x_1^L)[1-\frac{1}{|e_2|}] + qp_2(x_1^H)[1-\frac{1}{|e_2|}].$$

We have that the price in competitive regime 3 equilibrium is lower and period 1 production ($\widehat{x}_1^3$) is higher compared to monopoly if

$$(1-q)p_2(x_1^L) + qp_2(x_1^H) < \left[(1-q)p_2(x_1^L) + qp_2(x_1^H)\right]\frac{[1-\frac{1}{|e_2|}]}{[1-\frac{1}{|e_1|}]},$$

or $|e_1| < |e_2|$.

Under regime 4 we have that $x_2^H = R$ and that the energy constraint is binding both under competitive and collusive equilibrium. Thus $\widehat{x}_1^4 = \widetilde{x}_1^4 = U_1 + U_2^H - R$ in both cases and they are identical.

A.2 Proof of Proposition 4.

We substitute the inverse linear demand functions defined in equation (9) and (10) into the equilibrium conditions applying to regimes 1 to 4. Under regime 1 and 2, period 1 production in competitive equilibrium is equal to

$$\widehat{x}_1 = \frac{\alpha_1 - (1-q)\alpha_2 + \beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2}$$

and period 1 collusive production under the same two regimes is given by

$$\widetilde{x}_1 = \frac{1}{2}\frac{\alpha_1 - (1-q)\alpha_2 + 2\beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2}.$$

Period 1 competitive production ($\widehat{x}_1$) is higher than the collusive equilibrium production ($\widetilde{x}_1$) if $\alpha_1 > (1-q)\alpha_2$.

Under regime 3, period 1 production in the competitive equilibrium is determined by

$$\widehat{x}_1^3 = \frac{\alpha_1 - \alpha_2 + \beta_2(U_1 + qU_2^H) + \beta_2(1-q)U_2^L}{\beta_1 + \beta_2}$$

while in the case of collusion we have that period 1 production is equal to

$$\widetilde{x}_1^3 = \frac{1}{2}\frac{\alpha_1 - \alpha_2 + 2\beta_2(U_1 + qU_2^H) + 2\beta_2(1-q)U_2^L}{\beta_1 + \beta_2}.$$

The difference $(\widehat{x}_1^3 - \widetilde{x}_1^3)$ is equal to $\frac{\frac{1}{2}(\alpha_1 - \alpha_2)}{\beta_1 + \beta_2}$, where competitive period 1 production ($\widehat{x}_1^3$) is higher than collusive period 1 production ($\widetilde{x}_1$) if $\alpha_1 > \alpha_2$.

For proof of the regime 4 condition see subsection $A.1$.

B Equilibria in different regimes

Here we provide proof of Proposition 3 and 5 where we assume that inflow and reservoir capacity is such that different equilibria apply to the competitive and monopoly equilibrium.

B.1 Proof of Proposition 3.

Because we assume that marginal revenue corresponding to the competitive equilibrium is positive, we can rule out any regime 1 from the analysis. The only regimes we have to consider is 2, 3 and 4.

The possible combinations of equilibria in different regimes are illustrated in Table 4.

		Competitive regime		
		2	3	4
Collusive regime	2		**2,3**	**2,4**
	3	**3,2**		**3,4**
	4	**4,2**	**4,3**	

Table 4: Combinations of equilibria in different regimes; possible combinations in bold.

With reference to table 4 consider first a combination of inflow and reservoir capacity where we are in regime 3 in the competitive case and either in regime 2 or 4 in the collusive case. In this situation we can not have less storage under the collusive outcome compared to the competitive case. Under regime 2 there is overflow if high inflow occurs and some of the available energy is lost before the second period. In regime 4 the reservoir constraint is met. In contrast,.under regime 3 in state H there is no overflow and the reservoir constraint is not met. Regime 3 implies more period 1 production and less storage than any of the two other regimes.

Second, we consider situations where we have a regime 2 competitive equilibrium. In addition to a regime 2 equilibrium we could now also have a regime 3 or 4 equilibrium in the collusive case. If so, period 1 production is higher in the latter case.

Finally, in a situation where we have a regime 4 solution in the competitive case, we can either have a regime 2 or a regime 3 solution in the collusive case. Period 1 production would be lower in the collusive case if we have a regime 2 equilibrium and higher if we have a regime 3 equilibrium.

This leaves us with the following combinations of different regimes, collusive and competitive respectively, where storage is higher in the collusive case: $\{(2,3),\ (2,4),\ (4,3)\}$. On the other hand, storage is higher in the competitive case for the following combinations of regimes: $\{(3,2),\ (4,2),\ (3,4)\}$. The combination of regimes resulting in higher storage under competition is exactly the opposite to the ones defined for the collusive case.

Now, can we observe a situation where we have the regime combination $(4,3)$ for some values of inflow and reservoir capacity and the combination $(3,4)$ for others? The cut-off value between regime 4 and 3 in the competitive case is given by $U_1+U_2^H = \widehat{x}_1^3+R$, while in the collusive case the corresponding cut-off value is defined by $U_1 + U_2^H = \widetilde{x}_1^3 + R$. If inflow $(U_1 + U_2^H)$ is higher in any of the two cases we have a regime 4 solution. We observe that if $\widehat{x}_1^3 > \widetilde{x}_1^3$, then for some combinations of inflow and reservoir capacity we can have the regime combination $(4,3)$. The difference between $\widehat{x}_1^3$ and $\widetilde{x}_1^3$ is not affected by inflow or reservoir capacity. We have that $\widehat{x}_1^3 > \widetilde{x}_1^3$ if $|e_1| < |e_2|$. Thus, if we observe the regime combination $(4,3)$ for some values of inflow and reservoir capacity we can not have the opposite situation $(3,4)$ for other values of inflow and reservoir capacity. The same result holds if we compare the other two remaining combinations of regimes where storage is higher in collusive equilibrium.

In the competitive case we are in regime 3 if

$$U_1 + U_2^H - R - \widehat{x}_1^3 < 0.$$

In the collusive case, we are in regime 3 if

$$U_1 + U_2^H - R - \widetilde{x}_1^3 < 0.$$

When $|e_1| < |e_2|$ we have that $U_1 + U_2^H - R - \widehat{x}_1^3 > U_1 + U_2^H - R - \widetilde{x}_1^3$. This implies that there are more combinations of inflow and reservoir capacity resulting in regime 3 equilibrium output in the competitive case. For these combinations of inflow and reservoir capacity we have either a regime 2 or 4 equilibrium in the collusive case.

Also, because $|e_1| < |e_2|$ we have that

$$U_1 + U_2^H - R - \widehat{x}_1^2 > U_1 + U_2^H - R - \widetilde{x}_1^2.$$

This implies that there are less combinations of inflow and reservoir capacity resulting in regime 2 outcomes in the competitive case. Recall that we we have a regime 2 equilibrium if $U_1 + U_2^H - R - \widehat{x}_1^2 > 0$. If inflow and reservoir capacity is such that we have a regime 2 equilibrium in the collusive case but not in the competitive case, then we either have a regime 3 or 4 equilibrium in the competitive case. Thus, when the equilibrium is in different regimes and $|e_1| < |e_2|$, storage is higher in the collusive case.

B.2 Proof of Proposition 5.

With linear demand as defined in equation (9) and (10), then if inflow and reservoir capacity is sufficiently large we may have regime 1 equilibria.

The possible combinations of equilibria in different regimes are illustrated in table 5.

		Competitive regime			
		1	2	3	4
Monopoly regime	1		1,2	**1,3**	1,4
	2	2,1		**2,3**	**2,4**
	3	**3,1**	**3,2**		**3,4**
	4	4,1	**4,2**	**4,3**	

Table 5: Combinations of equilibria in different regimes; possible combinations in bold.

If we have a regime 1 competitive or monopoly equilibrium we know that $R > \frac{\alpha_2}{\beta_2}$. This implies that is is not possible to have either a regime 2 or 4 equilibrium as this requires that $R < \frac{\alpha_2}{\beta_2}$. Thus, as indicated in table 5 the regime combinations $\{(1,2),(1,4)\}$ and $\{(2,1),(4,1)\}$ are not possible. For a discussion of the remaining regime combinations we refer to the proof of Proposition 3 above.

This leaves us with the following combinations of different regimes where storage is higher in the monopoly case: $\{(1,3),\ (2,3),\ (2,4),\ (4,3)\}$. On the other hand, storage is higher in the competitive case for the following combinations of regimes: $\{(3,1),\ (3,2),\ (4,2),\ (3,4)\}$. As illustrated in subsection $B.1$, if we observe the regime combinations $\{(1,3),\ (2,3),\ (2,4),\ (4,3)\}$ for some values of inflow and reservoir capacity, we can not observe the regime combinations $\{(3,1),\ (3,2),\ (4,2),\ (3,4)\}$ for other combinations of inflow and reservoir capacity values.

In the competitive case we are in regime 3 if

$$U_1 + U_2^H - R - \frac{\beta_2(U_1 + qU_2^H) + \beta_2(1-q)U_2^L}{\beta_1 + \beta_2} < \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$$

for values of $0 < R < \frac{\alpha_2}{\beta_2}$ and

$$U_1 + U_2^H - \frac{\alpha_2}{\beta_2} - \frac{\beta_2(U_1 + qU_2^H) + \beta_2(1-q)U_2^L}{\beta_1 + \beta_2} < \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$$

for values of $R > \frac{\alpha_2}{\beta_2}$.

In the monopoly case, we are in regime 3 if

$$U_1 + U_2^H - R - \frac{1}{2}\frac{2\beta_2(U_1 + qU_2^H) + 2\beta_2(1-q)U_2^L}{\beta_1 + \beta_2} < \frac{1}{2}\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$$

for values of $0 < R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$,

$$U_2^H - y(U_1, U_2^L, R) < 0$$

for values of $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$ and

$$U_2^H - y(U_1, U_2^L, R = \frac{\alpha_2}{\beta_2}) < 0$$

for values of $R > \frac{\alpha_2}{\beta_2}$.

When $\alpha_1 > \alpha_2$ we have that $\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} > \frac{1}{2}\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$. It implies that there are more combinations of inflow and reservoir capacity resulting in regime 3 equilibrium output in the competitive case for values of $0 < R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$. For these combinations of inflow and reservoir capacity we have either a regime 1, 2 or 4 equilibrium in the monopoly case.

Also, because $\alpha_1 > \alpha_2$ we have that $\frac{1}{2}\frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2} < \frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2}$. This implies that there are less combinations of inflow and reservoir capacity resulting in regime 2 outcomes in the competitive case when $0 < R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$. In the collusive case we are in regime 2 if

$$U_1 + U_2^H - R - \frac{1}{2}\frac{2\beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2} > \frac{1}{2}\frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2}$$

In the competitive case, we are in regime 2 if

$$U_1 + U_2^H - R - \frac{\beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2} > \frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2}.$$

When $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$ there is no regime 4 equilibria in the monopoly case because $\widetilde{x}_1^3 > \widetilde{x}_1$. We know that we are in regime 2 if $U_1 + U_2^H > \widetilde{x}_1^3 + R$ and in regime 3 if $U_1 + U_2^H < \widetilde{x}_1 + R$. In situations where $\widetilde{x}_1^3 + R - U_1 > U_2^H > \widetilde{x}_1 + R - U_1$, the monopoly equilibrium is either in regime 3 or in regime 2. The cut-off value defined in equation (11) imply that $\widetilde{x}_1^3 + R - U_1 > y(U_1, U_2^L, R) > \widetilde{x}_1 + R - U_1$. If $R = \frac{1}{2}\frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H = \widetilde{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$, we have that $\widetilde{x}_1 = \widetilde{x}_1^3$ and $\widetilde{x}_1^3 + R - U_1 = y(U_1, U_2^L, R) = \widetilde{x}_1 + R - U_1$ as illustrated in figure 4.

When $\alpha_1 > \alpha_2$ we have that $\widehat{x}_1^3 + R - U_1 > \widetilde{x}_1^3 + R - U_1 > y(U_1, U_2^L, R)$ implying there are more combinations of inflow and reservoir capacity resulting in regime 3 equilibrium output in the competitive case when $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$. Furthermore, we have that $\widehat{x}_1 + R - U_1 > \widehat{x}_1^3 + R - U_1$ implying there are less combinations of inflow and reservoir capacity resulting in regime 2 outcomes in the competitive case when $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$.

The proof related to values of $R > \frac{\alpha_2}{\beta_2}$ is similar. When $\alpha_1 > \alpha_2$ we have that $\widehat{x}_1^3 + \frac{\alpha_2}{\beta_2} - U_1 > \widetilde{x}_1^3 + \frac{\alpha_2}{\beta_2} - U_1 > y(U_1, U_2^L, R = \frac{\alpha_2}{\beta_2})$ and $\widehat{x}_1 + \frac{\alpha_2}{\beta_2} - U_1 = \widehat{x}_1^3 + \frac{\alpha_2}{\beta_2} - U_1$. If $R = \frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H = \widehat{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$, we have that $\widehat{x}_1 = \widehat{x}_1^3$ as illustrated in figure 2.

Thus, when $\alpha_1 > \alpha_2$, we can only observe combinations of different regimes where storage is higher in the monopoly case: $\{(1,3), (2,3), (2,4), (4,3)\}$.

The proof related to the situation where $\alpha_1 < (1-q)\alpha_2$ is similar to the one shown for the case where $\alpha_1 > \alpha_2$. When $\alpha_1 < (1-q)\alpha_2$ we have that $\widehat{x}_1 < \widetilde{x}_1$ and $\widehat{x}_1^3 < \widetilde{x}_1^3$.

For the intermediate situation where $\alpha_1 > (1-q)\alpha_2$ and $\alpha_1 < \alpha_2$ we have that $\widehat{x}_1 > \widetilde{x}_1$ and $\widehat{x}_1^3 < \widetilde{x}_1^3$. For simplicity, we restrict the discussion to values of $R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$. Because $\widehat{x}_1 + R > \widetilde{x}_1 + R$ we have some combinations of inflow and reservoir capacity resulting in regime 2 solutions in the monopoly case and either regime 4 or 3 in the competitive case. In these situations storage is higher in the monopoly case. However, because $\widehat{x}_1^3 + R < \widetilde{x}_1^3 + R$ there are some combinations of inflow and reservoir capacity resulting in regime 3 equilibrium in the monopoly case while we are either in regime 2 or 4 in the competitive case. Thus, when $\alpha_1 > (1-q)\alpha_2$ and $\alpha_1 < \alpha_2$ storage can both be higher and lower in the monopoly case depending on the actual combination of inflow and reservoir capacity.

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