Public versus Private Health Care in a National Health Service[*]

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August 22, 2003

Discussion Paper 14/2003

Abstract

This paper studies the interplay between public and private health care in a National Health Service. We consider a two-stage game, where at stage one a Health Authority sets the public sector wage and a subsidy to (or tax on) private provision. At stage two the physicians decide how much to work in the public and the private sector. We characterise different equilibria depending on whether physicians coordinate labour supply or not, the physicians' job preferences, and the cost efficiency of private relative to public provision. We find that private provision tends to crowd out the NHS if physicians are sufficiently indifferent about where to work or the private sector is sufficiently cost efficient. Competition between physicians triggers a shift from public provision towards private provision, and an increase in the total amount of health care provided. The endogenous nature of labour supply may have counter-intuitive effects. For example, a cost reduction in the private sector is followed by a higher wage in the public sector.

JEL Classification: I11, I18, J42, L33

Keywords: Health care; Mixed oligopoly; Physicians

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[*]This article was partly written while Sørgard visited the University of California Santa Barbara, whose hospitality is gratefully acknowledged. Sørgard also thanks the Norwegian Research Council and the U.S.-Norway Fulbright Foundation for Educational Exchange for travel grants. We are indebted to Jan Erik Askildsen, John Cairns, Pau Olivella, Lise Rochaix-Ranson, Fred Schoyen, Odd Rune Straume and participants at the Royal Economic Society annual conference 2003 in Warwick, and the 21st NHESG meeting in Lund. The usual disclaimer applies.

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1 Introduction

Most health care systems involve a mixture of public and private provision. However, the role for private health care is different and more limited in health care systems characterised as a National Health Service (NHS) compared with private and mixed health care systems (see e.g. Besley and Gouveia, 1994). In a NHS, health care is mainly provided publicly and financed by general taxation. Still in most NHS systems there exists a parallell (and growing) private sector alongside the public one.[1] An important difference, though, is that patients receive public health care for free (or at a low cost), while they typically are charged the full cost of the medical treatment when seeking private health care.

In NHS systems that allow for private health care, physicians have the opportunity to work in the private sector. Interestingly, we typically observe that a substantial share of the physicians spend time in both sectors. For instance, in the UK most private medical services are provided by physicians whose main commitment is to the NHS. The UK Monopolies and Merger Commission (1994) estimated that about 61% of the NHS consultants had significant private work. Similar observations can be made in other countries with a NHS, like in France, Spain, and the Scandinavian countries (see Johnson, 1995, for an overview).

With a few notable exceptions, there are no studies that examine the interaction between public and private health care.[2] The purpose of this paper is to help fill this gap. We analyse the interaction between the public and the private sector in a NHS, emphasising the direct links between the two sectors both on the demand side and the supply side.[3]

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[1] In the UK, for instance, Propper (2000) reports that private health care expenditures have increased from 9% of total health care expenditures in 1979 to 15% in 1995. See e.g. Besley and Gouveia (1994) for an overview of public and private health care in different countries.

[2] Barros and Martinez-Giralt (2002) analyse the rivalry between preferred providers and out-of-plan providers under different reimbursement rules. Jofre-Bonet (2000) deals with the interaction between public and private providers when consumers differ in their income levels. Marchand and Schroyen (2001) analyse design of contracts to NHS doctors that induce an optimal mixture of public and private health care when the government takes distributional aspects into account.

[3] Rickman and McGuire (1999), (R&M) building on the model of Ellis and McGuire (1986), is closely related to our study. However, their approach is different from ours in many respects. First, we let a physician's utility be determined by total wage income in the public sector and the profits attained in the private sector. In contrast, in R&M a physician's utility from public sector work is determined by the performance of the public hospital as well as the satisfaction of her patients. Second, we assume an increasing marginal disutility of work. The reason for this is that each physician may face a soft time constraint, finding it more and more costly to supply an extra hour of labour. In contrast, R&M assume a constant marginal disutility.

The analysis will focus on the following questions: How does the option for NHS physicians to provide private health care affect the public provision? Does competition between physicians imply a larger or smaller scope for private health care? What role do job preferences and private sector costs play for the scope for public and private provision? How should the Health Authority set wages in the public sector, and should it tax or subsidise the private sector?

To analyse these questions, we consider a health care system characterised as a NHS. In this system there is a Health Authority (e.g. the Ministry of Health) responsible for providing health care to individuals in need for medical treatment. Public health care is free at the point of consumption, while patients are charged a (full cost) price if they visit a private clinic. The Health Authority's task is to implement the optimal mixture of public and private provision of health care. In this regard, it has two instruments; the public sector wage and a subsidy to (or tax on) private health care provision.[4]

The physicians' labour supply is important for the amount of public and private health care that will be provided. In the public sector physicians are on salary, while in the private sector they are self-employed and earn profits from their private practice. This means that the public sector wage as well as any private sector subsidies or taxes, affect their decision of how much to work in either sector. In addition, we find it reasonable that that physicians' job preferences play a role in determining their time spent in the two sectors. We assume that working in a public hospital and at a private clinic are imperfect substitutes from the physicians' perspective. As pointed out by Scott (2001), non-pecuniary job characteristics can be highly relevant in explaining the time physicians spend in the public and the private sector, as well as their preferences in this respect.[5]

In their setting, therefore, there are no direct links on the cost side between the two sectors. Third, we let the Health Authority act as a monopsonist in the labour market in the public sector, and the hospital then receives full-cost reimbursement. Although R&M have full-cost reimbursement in the public sector, they have no direct link between the costs associated with public health care and the physicians' revenue from such an activity. Finally, we assume strategic interaction between physicians, while R&M ignores the role of competition.

[4] We also observe that Health Authorities sometimes impose restrictions on the private earnings of the publicly employed physicians. In the UK, for instance, full-time NHS consultants are not allowed to earn more than 10% of their NHS salary on their private practice. This issue is analysed in detail by Gonzáles (2002).

[5] In the public sector physicians often have the opportunity for research and to specialise. Meeting colleagues and having access to medical facilities may also be important job characteristics for the public sector. In the private sector, on the other hand, important non-pecuniary job characteristics may be more autonomy or

On the demand side, we assume that public and private health care are homogeneous products with identical quality. Although this is not crucial for the analysis, the fact that it is the same physicians that provide health care in both sectors is an argument in favor of this assumption. Despite public and private health care being homogeneous products, patients prefer to be taken care of in the public sector because this means that they do not have to pay any out-of-pocket payments for the medical treatment. However, free public health care implies that some rationing takes place. Consequently, patients not served by the public sector have two options. They may either visit a private clinic or wait for public treatment.[6]

Allowing NHS physicians to establish private facilities outside the NHS system may have several potential e¤ects on the provision of public health care. In this paper, we focus on one particular e¤ect, which is the following: By restricting their labour supply in the public sector, physicians are able to increase the demand for private health care, and this may in turn increase the pro…tability of spending time in the private sector. This points to a potential instability in the NHS system in that the private sector tends to crowd out the public provision.[7] We therefore investigate in detail how the endogenous nature of physicians' labour supply a¤ects the Health Authority's wage setting and support to (or taxation of) the private sector. As it is an open question whether the physicians coordinate their labour supply or not, we contrast a competition case with a coordination case. Moreover, we also consider the situation where the two sectors are asymmetric in terms of the (cost) e¢ciency of providing health care.

There are some obvious informational asymmetries in the health care market. A large part of the literature is therefore concerned with the implications of such informational asymmetries for the amount of health care and the quality of it.[8] For example, several studies raise the issue of

professional freedom due to being self-employed.

[6] We do not explicitly model waiting time in the NHS system. However, in our model excess demand implies that some patients are neither served by a private nor a public provider. These patients can be considered as the waiting line, and the corresponding deadweigth loss may be interpreted as the waiting costs in the NHS system. There is, however, a reasonably large literature on waiting lists (see Cullis et al., 2000, for an overview), where some papers consider the interaction with the private sector, see e.g., Barros and Olivella (1999), Hoel and Sæther (2003), Iversen (1997) and Olivella (2002).

[7] This may explain the restrictions imposed on NHS physicians' private earings or time spent in the private sector we observe in some countries (see Johnson, 1995).

[8] See Frech (1996) for a discussion of issues in the health market in general, and Frech (1991) for a discussion of compensation schemes for physicians.

how the reimbursement scheme affects the provision of health care.[9] In this paper, though, we sidestep from some of the issues that have been investigated in detail in the literature.

The paper is organised as follows. In Section 2 we discuss various modelling issues, such as the formulation of demand and supply and the nature of the rivalry between the physicians. In Section 3 we derive the physicians' labour supply for given payments. In Section 4 we report results concerning the equilibrium outcomes in two separate cases. In the first one, the role of physicians' job preferences is analysed, while in the second case we consider asymmetric (cost) efficiencies in providing care. Finally, in Section 5, we summarise our findings.

2 Model

Consider a health care system characterised by a National Health Service (NHS). In this system there is a Health Authority (HA), hospital-based physicians, and individuals in need for medical treatment (patients). The HA is responsible for providing health care to patients. Public health care is assumed to be free of charge, while patients seeking private health care have to pay for the medical treatment as this is not included in the NHS. The demand for private health care is represented by the following inverse demand function:

$$p = 1 - Q_o - Q_p; \qquad (1)$$

where p is the marginal willingness to pay, Q_o is the quantity of health care provided by the public sector (o) and Q_p is the quantity of health care provided by the private sector (p). First, note from (1) that public and private health care are assumed to be perfect substitutes from the patients' perspective.[10] Second, note that we assume efficient rationing. As the public sector provision of health care increases, the marginal willingness to pay for private health care drops. Hence, the public sector has by assumption served those consumers with the highest willingness to pay for health care.[11] Those not served by the public sector will have

[9] Ellis and McGuire (1986, 1990) have considered how the reimbursement scheme affects the supply of health services, while Ma (1994) and Sharma (1998) have investigated how it affects quality as well as the incentives for reducing costs. For a survey of the literature, see Newhouse (1996) or Ennis (1998).

[10] McAvinchey and Yannopoulos (1994) find in an empirical study that private and public health care are substitutes. In some cases one could argue that private health care is of higher quality than public health care, and in other cases vice versa. Since we consider a situation where physicians operate in both the private and the public sector, we find it reasonable to assume public and private health care as perfect substitutes.

[11] This is consistent with a situation where the NHS is rationing patients according to severity of illness, leaving the easier (milder) cases for the private sector. For a

to seek private care to obtain treatment or wait for public treatment.

On the supply side, the important input to production is health personnel. Let us call them physicians. For ease of exposition, let us normalise input and output so that one unit of labour equals one unit of health care. Then Q_i denotes the units of labour used in sector i, where $i = o; p$. Since we focus on a specific health care product, it is plausible to assume that there is only a limited number of physicians qualified to supply the health care product in question in a specific area. In line with this, we simplify by assuming that there are only two physicians in the market, which may work in both the public and the private sector. Let q_i^k denote the labour supplied by physician k in sector i, where $k = 1; 2$: Total provision of health care in sector i is then given by $Q_i = \sum_k q_i^k$.

In the public sector, the physicians are on salary, earning the wage w per unit of labour. In the private sector physicians are self-employed and earns the profit from their private practice. Private sector revenues are equal to the price p, and possibly a transfer r from the HA per unit of health care (and thereby per unit of labour) provided. On the other hand, spending time providing health care generates disutility for the physicians. We find it plausible to assume a convex disutility function: the longer a physician initially works, the greater disutility from a marginal labour increase.[12] In line with this, we let the marginal disutility be influenced by a physician's total amount of labour in public and private sector.

However, it seems plausible as well to assume that a decision to work more in one of the sectors is influenced not only by total labour input, but also by how much the physicians works in that particular sector initially. The more she has worked in one sector, the higher marginal disutility in this particular sector. This implies that physicians are not indifferent about where to work, reflecting that they may have (intrinsic) preferences for working in a public hospital or at a private clinic.[13] A

similar approach, see e.g., Barros and Olivella (1999) and Olivella (2002).

[12] In our setting we consider physicians that work in both the public and private sector. The total amount of work can then be quite high, and each physician may face some restrictions on their labour supply: There are obviously physical limitations to how much each of them can work each day. Then it is natural to assume that each physician's total supply is approaching some kind of capacity constraint, and a convex disutility function captures such a case.

[13] This may be due to non-pecuniary factors like job characteristics. For instance, physicians may prefer to work in the public sector because of opportunities for research and specialising, meeting colleagues, access to medical facilities, etc. On the other hand, the private health care sector may be attractive because of, for instance, more autonomy due to being self-employed. See e.g. Scott (2001) for the importance of this.

disutility function that encompasses both elements is the following:

$$G^k = \left(q_o^k\right)^2 + \left(q_p^k\right)^2 + \delta q_o^k q_p^k; \tag{2}$$

where $0 < \delta < 2$. The parameter δ measures the degree of substitutability between working in the public and the private sector for each physician.[14] If $\delta \to 2$, the marginal disutility is determined by only the total amount of labour supplied. This corresponds to the case where physician k perceives working in a public hospital or at a private clinic as perfect substitutes. Contrary, if $\delta \to 0$, the marginal disutility is determined only by how much the physician works in either the private or public sector initially, implying that the allocation of labour supply between the two sectors matters. This refers to the case where physicians perceive public and private provision as imperfect substitutes.

We now have the following utility function for physician k:[15]

$$\pi^k = wq_o^k + (p + r - c)\, q_p^k - G^k; \tag{3}$$

where c denotes the marginal cost of providing health care in the private sector. The total marginal cost in private health care is the sum of c and the marginal disutility. With a slight abuse of terminology, in the following we refer to c as the marginal cost of private health care.

The HA is responsible for providing public health care. In line with this, we find it reasonable to assume that the HA has a monopsony role in the labour market for health care workers. In our model, we take this into account by allowing it to set w, the wage in the public sector. In addition, it can choose either to pay a per unit subsidy ($r > 0$) or impose a per unit tax ($r < 0$) on private health care.

The HA is in principle concerned about consumer surplus, profits as well as any possible distortions in the economy generated by taxes. From (1) we can derive the following utility function for persons demanding this particular health care service:

$$U = Q_o + Q_p - \frac{(Q_o + Q_p)^2}{2}. \tag{4}$$

[14] Alternatively, we can think of δ as the share of physicians that prefer to work in the public or the private sector relative to those that are indifferent between where to work.

[15] Note that we assume that physicians are not taking into account any patient benefit from health care when they maximize their utility. This non-altruistic approach is in contrast to some of the received literature, for example Rickman and McGuire (1999), where the patient's benefit enters the physician's utility function in a direct way. In principle, though, it should be simple to encompass altruism in our model. For example, it could be added as a downward shift in the disutility function.

Public health care is by assumption offered at a zero price to the patients. The cost of public health care, as well as any possible transfers to the private sector, are financed by distortionary taxes. Then we have the following social welfare function:[16]

$$W = U - pQ_p + \sum_k \pi^k - (1+\lambda)(wQ_o + rQ_p); \qquad (5)$$

where the parameter $\lambda \in (0;1)$ represents the marginal cost of public funds and captures the tax distortion. Rearranged, (5) can be written as

$$W = U - cQ_p - \sum_k G^k - \lambda (wQ_o + rQ_p)$$

Thus, the HA objective is to maximise patients' (gross) benefit of receiving medical treatment net of the physicians' costs of providing this health care as well as the social loss of financing health care associated with distortionary taxation.

Each physician determines her own labour supply in each sector. It is an open question whether physicians coordinate their decisions or not. For example, could it be that physicians coordinate their decisions in the private sector by establishing a joint private health care firm where both works? In theory, there are four possible situations. These are shown in Table 1 below.

In the situation called competition in Table 1, both physicians set their labour supply non-cooperatively. That would be the case where physician k maximises the utility function specified in (3), π^k, with respect to q_o^k and q_p^k. However, we know from theory that the players can jointly be better off in a collusive outcome. In such a case, the physicians would maximise joint utility, $\pi^1+\pi^2$, with respect to the physicians' labour supply in both sectors: $q_o^1; q_o^2; q_p^1$ and q_p^2. In this situation, denoted perfect coordination in Table 1, both physicians are expected to restrict their total supply of labour, thereby increasing the equilibrium price in the private sector. If each physician's discount factor is sufficiently high, we know that perfect coordination can be the equilibrium outcome in a repeated game.

[16] One could specify other objectives for the HA. We have checked whether excluding the physicians' utility in the welfare function, implying that the HA is concerned about patients' welfare and costly transfers only, matters for the analysis. However, it turns out that this does not change the results qualitatively. Details are available from the authors upon request.

Table 1: Coordination of labour supply?

		Private sector: Yes	Private sector: No
Public Sector	Yes	Perfect coordination	Public coordination
	No	Private coordination	Competition

In the two remaining situations, public coordination and private coordination, the physicians coordinate their labour supply in only one sector. However, we find neither of those two situations plausible. If the physicians have coordinated their labour supply in one sector, why should they not extend the cooperation to also include the other sector and thereby be better off? Therefore, we find it reasonable to contrast competition with perfect coordination. From now on we denote the latter simply coordination. We let superscript M and D denote coordination and competition, respectively. Whether coordination would be the equilibrium outcome is determined by exogenous factors such as period length and time preference rate. In addition, we may expect the structure of the private sector to be of importance. In particular, whether antitrust enforcement allows physicians to establish joint facilities can be decisive for whether a competitive outcome is attained or not in the labour market.

The rules of the game are the following:

- Stage 1: The Health Authority sets w and r.
- Stage 2: The physicians set q_i^k, where $i = o; p$ and $k = 1; 2$.

The model is, as usual, solved by backward induction.

3 Physicians' labour supply

Let us start by analysing the physicians' behaviour at stage 2 of the game. In particular, we are interested in how job preferences and the cost of private provision of health care affects the physicians' labour supply for a given wage and subsidy (or tax). This also enables us to see how a marginal change in the HA's policy instruments w and r affects the physicians' provision of public and private health care. In the competition game, physician k sets q_o^k and q_p^k to maximise (3), yielding the following first order conditions with respect to q_o^k and q_p^k:[17]

$$w - q_p^k = 2q_o^k + \delta q_p^k; \qquad (6)$$

[17] Second order conditions require that $\delta < -1 + 2\sqrt{2} \simeq 1.83$.

$$1 + r - \left(q_o^k + q_o^l\right) - 2q_p^k - q_p^l = c + 2q_p^k + \delta q_o^k; \qquad (7)$$

respectively, where $k; l = 1; 2$ and $k \neq l$. The left-hand side of (6) and (7) represent the marginal revenues of providing public and private health care, respectively, while the right-hand sides are the corresponding marginal costs. Notice the crowding-out effect each physician is facing when deciding her labour supply in the public and the private sector. When increasing the time spent at a public hospital, more patients are taken care of in the public sector, and this will, in turn, lower the demand for private health care. Thus, by restricting the labour supply in the public sector, the physician increases the profitability of working in the private sector.

Solving stage 2 of the game, yields the following equilibrium outcomes:

$$Q_o^D(w; r) = 2\frac{5w - (1 + r - c)(1 + \delta)}{8 - 3\delta - \delta^2} \qquad (8)$$

$$Q_p^D(w; r) = 2\frac{2(1 + r - c) - w(2 + \delta)}{8 - 3\delta - \delta^2}; \qquad (9)$$

where the superscript D denotes that we consider the competition game.

In the coordination game, the physicians set $q_o^1; q_o^2; q_p^1$ and q_p^2 to maximise joint profit, $\pi^1 + \pi^2$; yielding the following first order conditions with respect to q_o^k and q_p^k:[18]

$$w - \left(q_p^k + q_p^l\right) = 2q_o^k + \delta q_p^k; \qquad (10)$$

$$1 + r - c - \left(q_o^k + q_o^l\right) - 2\left(q_p^k + q_p^l\right) = 2q_p^k + \delta q_o^k; \qquad (11)$$

respectively. Again the left-hand side of (10) and (11) are the marginal revenues of providing public and private health care, respectively, and the right-hand sides are the corresponding marginal costs. While coordination eliminates the negative externality between physicians due to non-cooperatively labour supply present in the competition game, we see from the conditions that the crowding-out effect between public and private labour supply is also present in this case. Solving the first order conditions, yields the following outcomes at stage 2 of the game:

$$Q_o^M(w; r) = 2\frac{6w - (1 + r - c)(2 + \delta)}{8 - 4\delta - \delta^2} \qquad (12)$$

$$Q_p^M(w; r) = 2\frac{2(1 + r - c) - w(2 + \delta)}{8 - 4\delta - \delta^2}; \qquad (13)$$

where the superscript M denotes that we consider the coordination game.

[18] Second order conditions require $\delta < -2 + 2\sqrt{3} \simeq 1.46$.

As expected, physicians will work more in the public sector and less in the private sector when the wage (w) becomes higher, all else equal. The opposite is true when the HA subsidises private health care provision ($r > 0$). We also see that the marginal cost of private provision (c) affects the physicians' allocation of working time. A high c induces the physicians to spend less time in the private sector and more time in the public sector. The reason is that since physicians are self-employed in the private sector, they fully take into account the marginal cost in the private sector as this lowers the profit margin and therefore their private earnings. Notably, any potential production costs of public provision would not been taken into account by the physicians as these are covered by the HA and do not affect the physicians' wage income. These effects are present irrespective of whether physicians compete or coordinate their labour supply, though the magnitude can be different.

The effect of physician preferences, measured by δ, on private versus public labour supply (for given w and r) is, however, more complicated. Examination of the equilibrium outcomes enables the following statement.

Proposition 1 For given payments w and r, then

(i) $Q_o^j > 0$ if $w > \underline{w}^j$ and $Q_p^j > 0$ if $w < \overline{w}^j$, where $j = D; M$:

(ii) $Q_o^j > 0$ and $Q_p^j > 0$ if $\underline{w}^j < w < \overline{w}^j$, where $j = D; M$:

(iii) $\frac{\partial}{\partial \delta}\left(\overline{w}^j - \underline{w}^j\right) < 0$ and $\frac{\partial}{\partial c}\left(\overline{w}^j - \underline{w}^j\right) < 0$, where $j = D; M$:

A Proof is provided in Appendix B.

When physicians have the ability to decide their labour supply in the public and the private sector, and this affects the number of patients treated in either sector, the amount of public and private health care depends crucially on the public sector wage relative to potential profits in the private sector. From the Proposition it is clear that there is an upper and a lower bound on the wage that induces the physicians to work in both sectors.[19] If the wage becomes sufficiently low, physicians decide to spend time only in the private sector, while if the wage becomes sufficiently high they decide to only work in the public sector.

Less evident, though, is the effect of physicians' job preferences (measured by δ) on private versus public provision of health care. From part (ii) in the Proposition we see that as physicians become more indifferent about where to work, i.e. δ increases, the scope for a mixed health care system is reduced. The reason is that δ affects the physicians' division of labour between the two sectors. When δ is low job characteristics

[19] Alternatively, we could define this in terms of the support to, or taxation on, private health care supply (r).

matter relatively more than earnings in the two sectors for the physicians decision of how much time to spend in each sector. As the job characteristics differ in the public and private sector, physicians find it optimal to spend some time in both sectors in this case. On the other hand, when δ is high relative earnings in the two sectors are decisive for the physicians' labour supply, implying that they now tend to spend all their time in the sector that yield higher payments. This effect explains why the scope for a mixed system tends to be lower when δ is high than for a low δ.

A similar pattern is observed with respect to the private sector cost (c), but intuition is different. An increase in c, makes is less profitable to work in the private sector, inducing a shift in labour supply towards the public sector, all else equal. Consequently, the scope for a mixed system is decreasing in the marginal cost of private provision.

4 The Health Authority

Let us now turn to stage one of the game. At this stage the HA sets the public sector wage and the private sector subsidy (or tax). As pointed out in the previous section, both job preferences and the cost of private provision influence the physicians' allocation of labour between the public and the private sector. In the two next sections, we therefore examine in detail how these two factors affect the HA's wage and subsidy setting, and how this in turn determines the amount of public and private health care. We will also focus on how the nature of competition between physicians plays a role in this regard.

4.1 Physicians' job preferences

Let us start by examining the role of physicians' job preferences. For the moment, we assume that public and private provision are equally efficient, i.e. $c = 0$. At stage one of the game, the HA sets w and r to maximise (5), anticipating the physicians' labour supply responses. In the competition case, these are given by (8) and (9), and in the coordination case they are given by (12) and (13). The equilibrium outcomes in the competition and coordination game are presented in Table A in Appendix A.

Considering the scope for public and private health care, the following result can be established.

Proposition 2 Assume that $c = 0$. Then

(i) $Q_0^j > 0$ if $\delta < \bar{\delta}^j$, and $Q_p^j > 0$, where $j = D; M$:

(ii) $\bar{\delta}^M > \bar{\delta}^D$ and $\frac{\partial \bar{\delta}^j}{\partial \lambda} < 0$, where $j = D; M$:

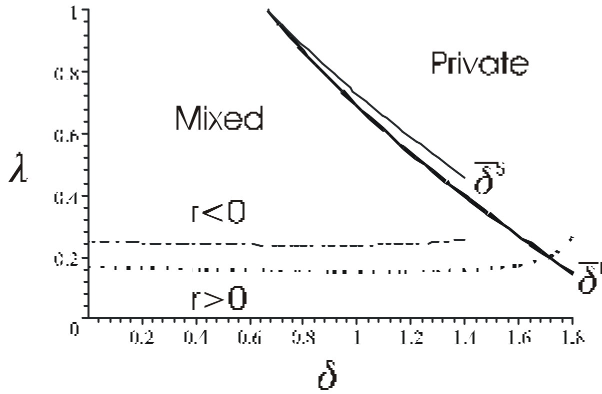

Figure 1: The scope for public and private health care depending on job preferences (±) and tax distortions (¸) when physicians compete (F) and coordinate (S).

A Proof is provided in Appendix B.

From the proposition it is clear that the stability of a NHS with a private sector alongside depends on physicians' job preferences and the cost of tax distortions. Figure 1 illustrates the two possible candidates for equilibrium of the game, where the thick lines represent the competition case (D) and the thin lines refer to the coordination case (M). We see that a mixed health care system is stable only if public and private sector work is perceived as su¢ciently di¤erentiated (i.e., a low ±) and the marginal cost of public funds (¸) is su¢ciently low. This is the case whether physicians compete or coordinate their labour supply. However, if both ± and ¸ becomes su¢ciently high then the equilibrium is characterised by a pure private health care system.

To understand the result, note that a reduction in labour supply in the public sector has distinctly di¤erent e¤ect on physicians' utility than a reduction in the private sector labour supply. By lowering the time spent at a public hospital, some patients are not served in the public sector but have to seek private health care to receive medical treatment. This increases the demand for private health care and the pro…t potential in the private sector, which in turn the same physicians bene…t from since they work in both sectors. As explained above, this e¤ect is stronger the more indi¤erent physicians are between working in the public and the private sector. When in addition the public provision is rather costly due to a high marginal cost of public funds, the HA faces a weaker incentive to mitigate physicians' shift toward private provision, potentially inducing the private sector to crowd out the public sector.

The result in Proposition 2 points to a fundamental problem within

NHS systems allowing for private sector provision alongside the public provision. When physicians have the opportunity to offer their services to the same patients in the private sector, the provision of free public health care can be seriously constrained. This is especially the case when relative earnings rather than job characteristics are decisive for the physicians labour supply and, in addition, costs of public funds is high due to tax distortions. Notably, this result emerges even though the HA has the opportunity to restrict private health care by means of public sector wage and private sector taxation. To fully understand the mechanisms at work let us therefore briefly consider the HA's optimal policy.

Proposition 3 Assume $c = 0$. Then

(i) $r^j > 0$ if $\lambda < \bar{\lambda}^j$, where $j = D; M$.

(ii) $\frac{\partial r^j}{\partial \lambda} < 0$ and $\frac{\partial w^j}{\partial \lambda} < 0$, where $j = D; M$.

(iii) $\frac{\partial r^j}{\partial \delta} < 0$ if $\delta < \hat{\delta}^j_r$, and $\frac{\partial w^j}{\partial \delta} > 0$, where $j = D; M$.

A Proof is provided in Appendix B.

From the Proposition we see that the HA subsidises the private sector when the cost of tax distortion is sufficiently small, while it imposes a tax on private health care when this cost becomes sufficiently large. This is illustrated in Figure 1. The rationale for subsidising private health care is related to the fact that public health care provision is fully financed through distortionary taxation. By allowing the private sector to take care of some patients, the HA is able to lower public spendings on health care and in turn the efficiency loss associated with tax distortion. However, if the marginal cost of public funds becomes sufficiently high, the HA finds it beneficial to instead impose a tax on private health care provision. This is partly because supporting private health care provision in itself becomes more costly, but also partly because a high λ lowers the wage and thus the public provision of health care, which is in line with part (ii) of the Proposition.

Perhaps more interestingly, we see from part (iii) of the Proposition that the HA responds to an increase in δ by raising the public sector wage and lowering the subsidy (or increasing the tax) on private provision. When δ is high relative earnings rather than job characteristics are decisive for physicians labour supply. In this case, the incentive for physicians to reshuffle patients from the public sector to the private sector is stronger. To mitigate this effect the HA therefore imposes a higher tax and offers a higher wage as a response to an increase in δ.

The exception to this rule is when δ is sufficiently high. Then the HA responds to an increase in δ by increasing the subsidy to (or lowering the tax on) private provision. As the HA still responds to a higher δ by increasing the public sector wage, this policy may seem inconsistent. However, the rationale for this policy is that total amount of health care provided becomes lower as δ increases. As it becomes more costly for the physicians to provide health care, the HA needs to stimulate the physicians' incentive to work more in both sectors.

Turning to the question of what role competition in the physicians market play, we have the following result.

Proposition 4 Assume $c = 0$ and $\delta < \bar{\delta}^M$. Then
(i) $w^M > w^D$ and $r^M > r^D$.
(ii) $Q_o^D < Q_o^M$ if $\delta < \tilde{\delta}_o$, and $Q_p^D > Q_p^M$.
(iii) $Q_o^D + Q_p^D > Q_o^M + Q_p^M$.

A Proof is provided in Appendix B.

Thus, in the coordination regime physicians face a higher public sector wage and a higher (lower) subsidy (tax) on private health care provision. The argument is that the HA finds private provision less desirable when physicians coordinate their labour supply as this leads to higher prices. In addition, the total amount of health care provided is less when physicians coordinate their activities than when they compete. This is because physicians find it profitable to restrict their labour supply in both sectors. Thus, in order to mitigate this negative effects of coordination, the HA sets a higher wage and a higher subsidy (or lower tax) in the coordination regime than in the competition regime.

From part (ii) of the Proposition we see that the scope for public health care is lower when physicians compete rather than coordinate, while the opposite is true for private health care. The reason is as follows. As explained above, the HA stimulates the physician to provide more health care under coordination by setting a higher wage and subsidy than under competition. The physicians respond by increasing their labour supply in both sectors, but more so in the public sector than in the private sector in order to keep high prices and profits.

However, if δ becomes sufficiently high, this pattern is changed. In this case, physicians spend more time in both sectors when they compete despite the fact that they receive a lower wage and a lower subsidy than under coordination. The reason is that under coordination, physicians take into account the effect of δ on total costs, while under competition only its own costs matter for a physician's labour supply.

Finally, we see that the total provision of health care is higher when physicians compete than when they coordinate their labour supply irrespective of the physicians' job preferences. This happens despite the fact that the HA attempts to induce more health care provision in the coordination regime.

4.2 Asymmetric cost efficiency

The results above were derived under the assumption that the public and the private sector were equally (cost) efficient in producing health care. In practice, this may not always be the case. In health care systems characterised as NHS, public hospitals may have access to inputs, like medical equipment, pharmaceuticals, etc., at lower prices than private clinics, for instance, because they are larger buyers. In this section we therefore assume a positive marginal cost in the private sector ($c > 0$). To focus on the effect of asymmetric cost efficiency, we abstract from the issue of physicians' job preferences by setting $\delta = 1$.

At stage one of the game, the HA sets w and r to maximise (5), anticipating the physicians' labour supply responses. In the competition case, these are given by (8) and (9), and in the coordination case they are given by (12) and (13). The equilibrium outcomes in the competition and coordination game are presented in Table B in Appendix A. Examining the scope for public and private health care, the following result can be established.

Proposition 5 Assume that $\delta = 1$. Then
(i) $Q_o^j > 0$ and $Q_p^j > 0$ if $\underline{c}^j < c < \bar{c}^j$, where $j = D, M$.
(ii) $\bar{c}^D > \bar{c}^M$, and $\underline{c}^D > \underline{c}^M$.
(iii) $\frac{\partial \bar{c}^j}{\partial \lambda} > 0$, and $\frac{\partial \underline{c}^j}{\partial \lambda} > 0$, where $j = D, M$.

A Proof is provided in Appendix B.

According to the proposition there is no interior solution if marginal costs in the private sector are either too high or too low. When c is sufficiently low we may have an equilibrium with private health care provision only, while the opposite occurs when c becomes sufficiently high. The reason is intuitive. Consider the case of a low c. On one hand, a low c makes private provision relatively more desirable since it is more efficient and yields lower prices, which both tend to increase the scope for private provision. On the other hand, a low c induces the physicians to work more in the private sector because they earn higher profits relative to the case of a high c. Thus, when c is low the HA must offer a high wage (or impose a substantial tax) to mitigate physicians' incentives to reallocate their labour supply towards the private sector.

Moreover, we see from (iii) that an increase in the marginal cost of public funds (λ) will reduce the scope for the public sector. The reason is obvious. By allowing the private sector to provide relatively more

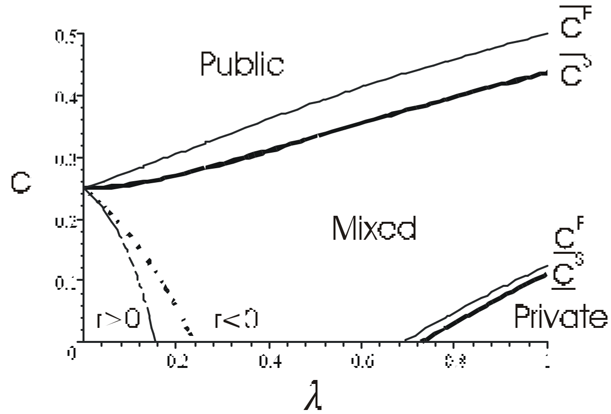

Figure 2: The scope for public and private health care depending on private sector costs (c) and tax distortions (λ) when physicians coordinate (S) or compete (F).

health care, and not subsidising private health care (see below), the HA can avoid serious distortions caused by taxation.

In Figure 2, we have shown the upper and lower bounds on c depending on the marginal cost of public fund in both the competition (D) and the coordination (M) case. From the figure (and result (ii) in the Proposition) we see that the critical values of c are higher when physicians compete rather than coordinate their labour supply. Thus, the scope for public provision is lower in the competition case. To understand this, note the trade-off the HA is facing. On one hand, free health care by the public sector typically leads to a smaller price distortion than what is the case with private health care. This is the case if the price-cost margin in the private sector is larger than the (negative) price-cost margin in the public sector. If so, public health care leads to a lower deadweight loss.

On the other hand, free public health care incur costs associated with distortionary taxation, a cost that is not present in a private sector. The higher the wage paid to physicians, and thereby the larger capacity in the public sector, the higher is the cost associated with distortionary taxation. If physicians compete, labour supply in the private sector will increase and the deadweight loss will be reduced. Thus, it is no surprise then that competition between physicians results in a greater scope for the private sector to provide health care. This result suggests that an increase in the number of physicians would lead to a greater scope for the private provision of health care. The intuition is that a larger number of physicians would result in a lower price-cost margin in the private sector, and therefore less concern for deadweight loss in the private sector.

Proposition 6 Assume that $\delta = 1$ and $\underline{c}^j < c < \bar{c}^j$, where $j = D; M$. Then

(i) $r^j > 0$ if $c < \hat{b}^j$, and $\frac{\partial \hat{b}^j}{\partial \lambda} < 0$.
(ii) $\frac{\partial r^M}{\partial c} < 0$ if $\lambda < \frac{\sqrt{241}-7}{12} \simeq 0.71$, and $\frac{\partial r^D}{\partial c} < 0$ if $\lambda < \frac{\sqrt{65}-5}{10} \simeq 0.31$.
(iii) $\frac{\partial w^j}{\partial c} < 0$.

A Proof is provided in Appendix B.

From the proposition we see that the HA subsidises (taxes) private health care if the marginal cost of private provision (c) is sufficiently low (high). The reason is that the HA is concerned about the total surplus in society, and encourages private health care only if it is sufficiently cost efficient relative to public provision. Since public support is raised through distortionary taxation, the decision of whether to subsidise (or tax) the private sector depends also on the size of the loss due to tax collection. This is illustrated in Figure 2, where the dashed lines represent the set of parameter values for which $r = 0$ in the competition case (thin line) and the coordination case (thick line), respectively.

Intuitively, we would expect that the HA responds to a cost reduction in the private sector by lowering the wage and increasing the support to private health care provision in order to reshuffle production from the public to the private sector. However, the picture is more complicated. All else equal, each physician responds to lower costs in the private sector by spending less time in the public sector and more time in the private sector. The HA must take this into account, and this may lead to some counter-intuitive results.

As we would expect a priori, the HA increases the support to private health care provision as a response to a cost reduction in the private sector. By doing so it re-enforces the physicians' incentive to work more in the private sector and in turn increases private health care production, which has become less costly. The exception to this rule is when tax distortions are sufficiently large. In this case the HA is concerned about the increase in private health care provision following a cost reduction, because this results in more costly public transfers. Therefore, it responds to a lower private sector cost by reducing its support to private health care if tax distortions are sufficiently high.

However, irrespective of the costs of taxation the HA increases the public sector wage following a cost reduction in the private sector. This is a response to the physicians' shift in labour supply towards the private sector when c becomes lower. By increasing the wage the HA mitigates the reduction in the public provision of health care. Then we have that in some cases the HA responds to a cost reduction in the private sector by increasing its support to both the private and the public sector. The intuition is that there is no one-to-one relationship between labour supply in the private and the public sector - a certain increase in labour supply in one sector would not lead to an identical reduction in labour supply in the other sector. The HA has one instrument tailored to the private sector (r) and one tailored to the public sector (w). It can therefore monitor the changes better by using both instruments than by using one of them, and in some cases it has to counterbalance the forces at work.

As these effects are present irrespective of whether physicians compete or coordinate their labour supply, let us now turn to the comparison of the two different regimes.

Proposition 7 Assume that $\delta = 1$ and $\underline{c}^D < c < \bar{c}^M$. Then

(i) $w^M > w^D$ if $c < \tilde{c}_w$, and $r^M > r^D$ if $c < \tilde{c}_r$:

(ii) $Q_o^M > Q_o^D$ and $Q_p^M < Q_p^D$.

(iii) $Q_o^M + Q_p^M < Q_o^D + Q_p^D$.

A Proof is provided in Appendix B.

We see from part (i) of the proposition that coordination between physicians involves a higher wage and a higher subsidy (or lower tax) than competition, given that the marginal cost of private provision is sufficiently low. The reason is that coordination enables the physicians to achieve higher profits in the private sector by restricting their labour supply in both the public and private sector. Thus, the HA has to set a high wage and subsidy to encourage the physicians to work more in both sectors, and by this increase the amount of public and private health care supplied in the coordination case.

However, for sufficiently high costs in the private sector this result may be reversed, and both wages and support to the private sector may be lower in the coordination regime than in the competitive regime. High costs of private provision will make it unattractive to encourage private provision. Since private provision is higher in the competition case than in the coordination case - all else equal - then support to the private provision is lower in the competition case than in the coordination case if c is sufficiently high. This has, in turn, implications for the wages setting. The HA can set a lower wage in the competition case than in the coordination case and still attract labour to the public sector, since now support for private provision is low in the competition case.

From part (ii) of the Proposition, we see that the scope for private health care provision is higher when physicians compete rather than coordinate their labour supply, while the scope for public health care is lower in this case. The reason for this is as follows. In the competition regime, physicians compete for market shares in the private sector, inducing each of them to provide a substantial amount of private health care. However, this has an indirect negative e¤ect on their labour supply in the public sector. Thus, in the competition regime each physician supplies more labour in the private sector than is the case under coordination, and compensates for this by supplying less labour in the public sector.

Finally, we see from part (iii) that a competitive labour market results in a larger total production of health care than is the case when the physicians' labour supply is coordinated. This implies that in our model the increase in production in the private sector due to a shift from coordination to competition is not o¤set by the indirect, negative e¤ect on public health care provision.

5 Concluding remarks

The purpose of this article has been to investigate the interaction between public and private health care in a NHS. We have emphasised the close links between the public and the private sector on both the demand and supply side, and focused on how endogenous labour supply, cost e¢ciency in the two sectors, and the nature of competition between physicians may a¤ect a HA's public policy. The paper provides the following three main …ndings.

First, we have pointed to a fundamental problem that may arise when physicians are allowed to earn revenues from private health care in addition to wage income from public health care. Physicians can increase the demand for private health care by restricting their supply of labour in the public health sector. The outcome in terms of health care system depends crucially on the physicians' job preferences. When physicians are close to being indi¤erent between work in the public and private sectors, the scope for a mixed health care system tends to be very limited.

Second, the endogenous nature of labour supply complicates public policy. In some cases results are in line with what we expect. For example, the HA supports the private sector if the cost of private health care is su¢ciently low. In other cases, though, it is not that straightforward. For example, consider the case of a more e¢cient private sector. This triggers a shift of labour supply from public to private health care. Then it is not obvious whether the government should respond by increasing or reducing the wage in the public sector. The latter may apparently be the right choice. But we …nd in our setting that in some cases it should respond to a cost reduction in the private sector by increasing the public wage, thereby dampening the shift of labour supply from the public to the private sector.

Third, we show that the nature of the rivalry between the physicians may be important for public policy. In our setting, physicians can either coordinate their labour supply or compete on labour supply. We find that competition between physicians results in an increase in private health care production and a reduction in public health care production. Then it is not obvious how competition affects the total production. In our model the first effect (increase in private health care) dominates, so that competition between physicians leads to an increase in total production of health care.

Before the paper is concluded, we would like to stress that our model is stylised and that many of the results are ambiguous. For example, (i) the public health care sector can either be driven out of the market or not, and (ii) private health care can be either taxed or subsidized. Ambiguity in a stylised model implies that we will also have to report ambiguous results in a generalised version of our model. This fact suggests that the model should not be used to predict some clear-cut policy recommendations, but rather to point out some mechanisms that may be of importance in mixed health care systems.

6 Appendix A: Equilibrium Outcomes

Table A: Equilibrium outcomes when $\delta > 0$ and $c = 0$:

	Competition (D)	Coordination (M)
W	$\frac{6+15\lambda-2\lambda^2-2\lambda\delta^2-3\lambda\delta-\delta^2-\delta}{12+44\lambda+31\lambda^2-\delta(1+2\lambda)(4+\delta+2\lambda(3+\delta))}$	$\frac{(2-\delta)(4+10\lambda+\delta+2\lambda\delta)}{4(3+12\lambda+8\lambda^2)-\delta(1+2\lambda)^2(4+\delta)}$
R	$\frac{2-9\lambda-17\lambda^2-\delta+\lambda\delta(3+5\lambda+\delta+2\lambda\delta)}{12+44\lambda+31\lambda^2-\delta(1+2\lambda)(4+\delta+2\lambda(3+\delta))}$	$\frac{4\left(1-3\lambda-4\lambda^2\right)-2\delta+\lambda\delta(4+\delta)(1+2\lambda)}{4(3+12\lambda+8\lambda^2)-\delta(1+2\lambda)^2(4+\delta)}$
Q_o	$2\frac{2+5\lambda-3\lambda^2-\delta(1+2\lambda)(1+\lambda)}{12+44\lambda+31\lambda^2-\delta(1+2\lambda)(4+\delta+2\lambda(3+\delta))}$	$2\frac{2+6\lambda-4\lambda^2-\delta(1+\lambda)(1+2\lambda)}{4(3+12\lambda+8\lambda^2)-\delta(1+2\lambda)^2(4+\delta)}$
Q_p	$2\frac{2+5\lambda+4\lambda^2-\delta(1+2\lambda)}{12+44\lambda+31\lambda^2-\delta(1+2\lambda)(4+\delta+2\lambda(3+\delta))}$	$2\frac{2+4\lambda+4\lambda^2-\delta(1+2\lambda)}{4(3+12\lambda+8\lambda^2)-\delta(1+2\lambda)^2(4+\delta)}$
P	$\frac{4+24\lambda+29\lambda^2-\delta(1+2\lambda)(2\lambda\delta+4\lambda+\delta)}{12+44\lambda+31\lambda^2-\delta(1+2\lambda)(4+\delta+2\lambda(3+\delta))}$	$\frac{4+28\lambda+32\lambda^2-\delta(1+2\lambda)(2\lambda\delta+6\lambda+\delta)}{4(3+12\lambda+8\lambda^2)-\delta(1+2\lambda)^2(4+\delta)}$

Table B: Equilibrium outcomes when $\delta = 1$ and $c > 0$.

	Competition (D)	Coordination (M)
W	$2\frac{2+5\lambda-\lambda^2-c\left(1-\lambda^2\right)}{(7+5\lambda)(1+3\lambda)}$	$\frac{5+12\lambda-6c(1+\lambda)}{7+28\lambda+12\lambda^2}$
R	$\frac{1-5\lambda(1+2\lambda)-2c(2-5\lambda(1+\lambda))}{(7+5\lambda)(1+3\lambda)}$	$\frac{2-8c-\lambda(1-c)(7+6\lambda)}{7+28\lambda+12\lambda^2}$
Q_o	$2\frac{1+3c+2\lambda(1+4c)-5\lambda^2(1-c)}{(7+5\lambda)(1+3\lambda)}$	$2\frac{1+3\lambda+3c(1+3\lambda)-6\lambda^2(1-c)}{7+28\lambda+12\lambda^2}$
Q_p	$2\frac{1+3\lambda-4c(1+2\lambda)+4\lambda^2(1-c)}{(7+5\lambda)(1+3\lambda)}$	$2\frac{1+2\lambda-4c(1+2\lambda)+4\lambda^2(1-c)}{7+28\lambda+12\lambda^2}$
P	$\frac{3+16\lambda+17\lambda^2+2c\left(1-\lambda^2\right)}{(7+5\lambda)(1+3\lambda)}$	$\frac{3+2\lambda(9+8\lambda)+2c(1+\lambda)(1-2\lambda)}{7+28\lambda+12\lambda^2}$

7 Appendix B: Proofs of Propositions

Proof of Proposition 1: Consider the competition game ($j = D$). Setting (8) and (9) equal to zero and solving for w, yield the following critical values for public and private provision, respectively,

$$\underline{w}^D = \frac{1+\delta}{5}(1+r-c);$$

$$\overline{w}^D = \frac{2}{2+\delta}(1+r-c):$$

Comparing these critical values, it is easily verified that

$$\overline{w}^D - \underline{w}^D = \frac{(8-3\delta-\delta^2)}{5(2+\delta)}(1+r-c) > 0:$$

Furthermore, we can prove that $Q_o^k(w; r) = 0$ for any $w < \underline{w}^D$ and $Q_p^k(w; r) = 0$ for any $w > \overline{w}^D$. Hence part (i) and (ii) of the Proposition must be true. Part (iii) is established by checking the following partial derivatives

$$\frac{\partial}{\partial\delta}\left(\overline{w}^D - \underline{w}^D\right) = -\frac{14+4\delta+\delta^2}{5(2+\delta)^2}(1+r-c) < 0$$

$$\frac{\partial}{\partial c}\left(\overline{w}^D - \underline{w}^D\right) = -\frac{(8-3\delta-\delta^2)}{5(2+\delta)} < 0$$

In a similar way the results for the coordination game ($j = M$) can be proved. QED.

Proof of Proposition 2: Setting Q_o^D and Q_o^M, reported in Table A in Appendix A, equal to zero and solve with respect to δ, we get the following critical values

$$\bar{\delta}^D = \frac{(2 - \lambda)(1 + 3\lambda)}{(1 + \lambda)(1 + 2\lambda)},$$

$$\bar{\delta}^M = 2\frac{1 + 3\lambda - 2\lambda^2}{1 + 3\lambda + 2\lambda^2}.$$

These critical values are plotted in Figure 1. Comparing the critical values, we have that

$$\bar{\delta}^M - \bar{\delta}^D = \frac{\lambda(1 - \lambda)}{(1 + \lambda)(1 + 2\lambda)} > 0.$$

Then taking the partial derivatives of the critical values, we find that

$$\frac{\partial \bar{\delta}^D}{\partial \lambda} = -\frac{19\lambda^2 + 14\lambda + 1}{(\lambda + 1)^2 (2\lambda + 1)^2} < 0,$$

$$\frac{\partial \bar{\delta}^M}{\partial \lambda} = -8\lambda \frac{2 + 3\lambda}{(1 + 3\lambda + 2\lambda^2)^2} < 0.$$

Finally, it can be shown that Q_p^D and Q_p^M are strictly positive for any relevant values of δ and λ. QED.

Proof of Proposition 3: Setting r^D and r^M, reported in Table A in Appendix A, equal to zero and solve with respect to λ, we get the following critical values

$$\hat{\lambda}_r^D = \frac{9 - 3\delta - \delta^2 - \sqrt{217 - 162\delta - 5\delta^2 + 14\delta^3 + \delta^4}}{2(2\delta^2 + 5\delta - 17)},$$

$$\hat{\lambda}_r^M = \frac{\sqrt{(2 - \delta)(200 - 76\delta - 26\delta^2 - \delta^3)} - (2 - \delta)(6 + \delta)}{4(8 - 4\delta - \delta^2)}.$$

These critical values are plotted in Figure 1. Then we can show that $r^j > 0$ for any $\lambda < \hat{\lambda}_r^j$ and $r^j < 0$ for any $\lambda > \hat{\lambda}_r^j$, which proves part (i) of the Proposition.

Part (ii) and (iii) are established by checking the partial derivatives of w^j and r^j with respect to λ and δ. It can be shown that $\frac{\partial w^j}{\partial \lambda} < 0$,

$\frac{\partial r^j}{\partial \lambda} < 0$ and $\frac{\partial w^j}{\partial \delta} < 0$ for any relevant values of λ and δ. However, in the competition case (D) we find that

$$\frac{\partial r^D}{\partial \delta} = -\frac{(2-\delta)^2+\lambda\left(16+5\delta^2-22\delta\right)+9\lambda^2\left(1+\delta^2\right)+\lambda^2\left(49\lambda^2+33\lambda-46\delta\right)-2\lambda^3\delta(15-4\delta)+4\lambda^4\delta(3+\delta)}{(12+44\lambda+31\lambda^2-\delta(1+2\lambda)(4+\delta+6\lambda+2\delta\lambda))^2}$$

Solving this for δ, we get the following critical value

$$\hat{\delta}_r^D = \frac{2+7\lambda+9\lambda^2-3\lambda^3-2\sqrt{\lambda\left(1+6\lambda+2\lambda^2\right)\left(2+2\lambda-2\lambda^2-5\lambda^3\right)}}{\left(1+\lambda+\lambda^2\right)(1+2\lambda)}.$$

It can be shown that $\hat{\delta}_r^D < \bar{\delta}^D$ for some values of $\lambda \in (0,1)$. Then by numerical computation we find that $\frac{\partial r^D}{\partial \delta} < 0$ for any $\delta < \hat{\delta}_r^D$, and $\frac{\partial r^D}{\partial \delta} > 0$ for any $\delta > \hat{\delta}_r^D$. In a similar way, we can prove this to hold for the coordination case (D) as well. QED.

Proof of Proposition 4: Setting $Q_o^D = Q_o^M$ and solve with respect to, for instance, δ, we get the following critical value

$$\tilde{\delta}_o = \frac{4+13\lambda+\lambda^2-\sqrt{64+240\lambda+145\lambda^2-334\lambda^3-111\lambda^4}}{2(3+\lambda)(1+2\lambda)}.$$

It can be shown that $\tilde{\delta}_o \in \left(0, \bar{\delta}^M\right)$ for $\lambda \in (0.64, 1)$. Then by numerical computation we find that $Q_o^D > Q_o^M$ if $\delta < \tilde{\delta}_o$ and $Q_o^D < Q_o^M$ if $\delta > \tilde{\delta}_o$. In a similar way, the rest of the Proposition can be proven. QED.

Proof of Proposition 5: (i) $\bar{c}^M$ (or $\underline{c}^M$) is found by setting the equilibrium value of q_p^M (or q_o^M) in the coordination regime (see Table B) equal to zero and solve with respect to c, yielding the following critical values

$$\bar{c}^M \equiv 2\frac{\lambda}{2\lambda+3} \text{ and } \underline{c}^M \equiv \frac{1}{3}\frac{-2+3\lambda^2}{1+2\lambda+\lambda^2}$$

In a similar way, we find

$$\bar{c}^D \equiv \frac{1}{2}\frac{4\lambda+1}{2\lambda+3} \text{ and } \underline{c}^D \equiv \frac{1}{5}\frac{5\lambda^2-3}{\lambda^2+2\lambda+1},$$

in the competition game.

(ii) It is easy to check that $\bar{c}^j > \underline{c}^j$, where $j = D, M$. From these expressions, we have that

$$\bar{c}^D - \bar{c}^M = \frac{1}{2(2\lambda+3)} > 0 \text{ and } \underline{c}^D - \underline{c}^M = \frac{1}{15\left(1+2\lambda+\lambda^2\right)} > 0$$

(iii) A marginal change in λ has the following effect on the critical values:

$$\frac{\partial \bar{c}^M}{\partial \lambda} = \frac{3\lambda}{2(1+\lambda)^3} > 0; \quad \frac{\partial \bar{c}^D}{\partial \lambda} = \frac{5}{(2\lambda+3)^2} > 0;$$

$$\frac{\partial \underline{c}^M}{\partial \lambda} = \frac{(2+3\lambda)8\lambda}{3(1+\lambda)^2(1+2\lambda)^2} > 0; \frac{\partial \underline{c}^D}{\partial \lambda} = \frac{2}{5}\frac{5\lambda+3}{(\lambda+1)(1+2\lambda+\lambda^2)} > 0.$$

Proof of Proposition 6: Setting r^M and r^D (reported in Table B in Appendix A) equal to zero, respectively, and then solve the expressions with respect to c, we find

$$\hat{c}^M = \frac{2-7\lambda-6\lambda^2}{8-7\lambda-6\lambda^2} \text{ and } \hat{c}^D = \frac{-1-2\lambda+5\lambda^2}{3+8\lambda+5\lambda^2};$$

which yields result (i).

From the equilibrium values reported in Table B in Appendix A, we have the following effects of a marginal change in c on the subsidy (or tax):

$$\frac{\partial r^M}{\partial c} = \frac{-8+7\lambda+6\lambda^2}{7+28\lambda+12\lambda^2} \text{ and } \frac{\partial r^D}{\partial c} = \frac{3+8\lambda+5\lambda^2}{(5+7\lambda)(1+3\lambda)}.$$

and on the wage:

$$\frac{\partial w^M}{\partial c} = -\frac{6(1+\lambda)}{7+28\lambda+12\lambda^2} \text{ and } \frac{\partial w^D}{\partial c} = -\frac{2(1-\lambda^2)}{(5+7\lambda)(1+3\lambda)};$$

Then we can easily verify the results reported in the Proposition. QED.

Proof of Proposition 7: Result (i) is found by setting $w^M = w^D$, reported in Table B in Appendix A, and then solve for c, yielding

$$\tilde{c}_w = \frac{7+32\lambda+73\lambda^2+116\lambda^3+24\lambda^4}{2(14+71\lambda-118\lambda^2+73\lambda^3+12\lambda^4)}.$$

It can be shown that $\frac{\partial(w^M - w^D)}{\partial c} < 0$, which implies that $w^M > w^D$ for any $c < \tilde{c}_w$. Furthermore, it can be shown that $\tilde{c}_w < \bar{c}^M$. Hence, $w^M > w^D$ for all relevant values of c. In a similar way, we can prove the rest of result (i).

Result (ii) is found by setting $Q_0^M = Q_0^D$ and solve with respect to c, yielding

$$\tilde{c}_0 = \frac{7+32\lambda+73\lambda^2+116\lambda^3+24\lambda^4}{2(14+71\lambda-118\lambda^2+73\lambda^3+12\lambda^4)}.$$

Then it can be shown that $Q_0^M > Q_0^D$ for any $c > \tilde{c}_0$. Furthermore, it can be shown that $\tilde{c}_0 < \underline{c}^M$. Hence, $Q_0^M > Q_0^D$ for all relevant values of c. In a similar way we can prove the rest of part (ii) and part (iii) of the proposition.

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