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Optimal Transmission Switching under Demand Uncertainty

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Problem description

In our thesis we explore the optimization problem Optimal Transmission Switching (OTS). We provide a description of the Alternating Current (AC) and the Direct Current (DC) Optimal Power Flow (OPF) problem, the OTS problem, as well as a presentation and evaluation of two OTS heuristics. Further, we review the existing OTS literature, where consensus is that transmission switching can provide signifcant cost reductions for a system operator. Since these conclusions are made on the basis of inconclusive test cases, we review and improve the most used test case within the OTS research community and evaluate how transmission switching will affect a system operator for a number of power demand scenarios. Further, we extend our studies to include larger, newer test cases like the Great Britain and Polish networks. Finally, we assess the risk related to transmission switching by generating a number of demand scenarios and evaluate the effect on the networks studied.

Preface

This master's thesis is written within Manegerial Economics and Operations Research (AØO) at the Department of Industrial Economics and Technology Management (IØT), at the Norwegian University of Science and Technology (NTNU). The thesis explores the properties of the Optimal Transmission Switching problem and two heuristics used to solve it. A central part of the work investigates the potential economical benefits and risks of transmission switching under demand uncertainty.

Working on this thesis has been a long and exciting journey. We started the project with very limited knowledge of nonlinear optimization. Although the complexity of the problem has been overwhelming at times, we have been able to study it and familiarize ourselves with the existing literature since August 2014, when we wrote our project thesis also based on this topic. Gradually, we were able to understand the challenges revolving the topic and to make independent choices regarding the direction of this thesis. The process has been challenging with many setbacks, but we feel like we have come a long way. Now, we are excited to present our results.

Our sincerest gratitude goes to our main supervisor, postdoctoral researcher Alois Pichler, for his constant support, guidance and motivation. Without his encouragement and academic insights, this thesis would have looked very differently. We also wish to thank postdoctoral researcher Francesco Piu, who has been an invaluable help in the process of reaching an understanding of the OTS problem. Finally, we would like to thank Professor Asgeir Tomasgard for his help and valuable input.

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Abstract

This thesis studies the Optimal Transmission Switching (OTS) problem, a nonlinear, nonconvex combinatorial optimization problem, classified as NP-hard. The motivation for this work stems from our observations that the network test cases applied to evaluate the potential of OTS are inconclusive. We create new, more realistic versions of the existing test cases and evaluate the solutions given by an Alternating Current (AC) based OTS heuristic and a Direct Current (DC) approximated OTS heuristic. Our results show that in most cases, the DC heuristic returns unreliable results, and that the AC heuristic should be applied to solve the OTS problem.

We explore the literature revolving the OTS problem, and create load scenarios for each test case. We wish to answer whether transmission switching can lead to significant reductions in generator dispatch costs and to determine the risk involved when changing the topology of a network. We conclude that for the test cases studied in our thesis, there is little to no cost reduction potential related to transmission switching and that this conclusion holds for a number of load scenarios. Our results differ from the literature because we have extended our studies to include larger, newer networks, as well as modeling the networks to run in normal conditions. Further, we conclude that there is little risk involved with transmission switching leading to network blackout. Contrary to what has been reported in OTS literature, we show that a congested network rarely leads to significant cost reductions, but rather the opposite. Although our results look discouraging for the potential of OTS, further studies need to be made on modern power networks to be able to reach an ambiguous conclusion on the future of OTS. This advocates for a closer collaboration with the power industry in the years to come.

Sammendrag

Denne oppgaven studerer optimeringsproblemet, *Optimal Transmission Switching (OTS)*. Motivasjonen for oppgaven springer ut fra våre observasjoner om at testnettverkene som er brukt til å evaluere potensialet i denne problemstillingen er mangelfulle. Vi skaper nye, mer realistiske versjoner av de eksisterende testnettverkene og evaluerer løsningene som er gitt av en vekselstrømsbasert heuristikk og en likestrømstilnærmet heuristikk. Resultatene våre viser at i de fleste tilfeller, så vil likestrømsheuristikken gi upålitelige resultater, mens vekselstrømsheuristikken er foretrukket som løsningsmetode for OTS-problemet.

Vi gjennomgår litteraturen som omhandler OTS-problemet, og genererer lastscenarioer for hvert testnettverk. Vi ønsker å besvare hvorvidt linjetopologiendringer kan føre til signifikante besparelser i generatorkostnader og å kvantifisere risikoen knyttet til denne topologiendringen. Vi konkluderer at det er lite eller intet besparelsespotensiale for våre testnettverk, og at denne konklusjonen er holdbar for en rekke lastscenarioer. Våre resultater skiller seg ut fra de som er rapportert i litteraturen. Dette er fordi vi har utvidet våre studier til å inkludere større, nyere nettverk, i tillegg til at nettverkene er modellert til å kunne anses å operere i normal tilstand. Videre konkluderer vi med at det er lite risiko knyttet til at topologiendringene skal føre til overbelastning og strømbrudd i nettet. I motsetning til det som har blitt rapportert i litteraturen, viser vi at et høyt belastet nettverk sjeldent fører til signifikante kostnadsbesparelser, men heller det motsatte. Selv om våre resultater ser nedslående ut for potensialet til OTS, må vi presisere at problemet må utforskes videre på moderne kraftnett for å kunne nå en entydig konklusjon. Dette vil kreve et økt samarbeid med kraftindustrien i de kommende årene.

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List of Abbreviations

PF	Power Flow
OPF	Optimal Power Flow
ACOPF	Alternating Current Optimal Power Flow
DCOPF	Direct Current Optimal Power Flow
OTS	Optimal Transmission Switching
MINLP	Mixed Integer Nonlinear Program
MILP	Mixed Integer Linear Program
IPM	Interior Point Method
LMP	Locational Marginal Price

Chapter 1

Introduction

The power industry calls on technological research aiming to develop a smarter electrical network. Sissine (2007) argues that the development of Smart Grid should include technology to improve reliability and efficiency of a power network, as well as employing dynamic optimization of network operations and resources. However, new network infrastructure can be expensive, making optimization of the already existing network topology a highly relevant study case.

This thesis studies the Optimal Power Flow (OPF) and the Optimal Transmission Switching (OTS) problems through the widely applied power flow solver MATPOWER. The OPF problem seeks to determine the optimal power flow in a network consisting of generator and load nodes. Its objective is to minimize generator dispatch costs. The two main formulations for solving the OPF problem are the Alternating Current Optimal Power Flow (ACOPF) problem and the Direct Current Optimal Power Flow (DCOPF) problem, where the DCOPF problem represents a linearization of the nonlinear ACOPF problem. The ACOPF is the core of independent system operator power markets. It is applied in a long term perspective for system planning, and in a short term perspective for day-ahead and real-time procedures. The Optimal Transmission Switching (OTS) problem is an extension of the OPF problem, where the option of switching off lines in the network in a short term perspective is included. The objective of the OTS becomes to search for the optimal combination of lines to switch off in order to minimize generator dispatch costs while still satisfying the OPF restrictions. It still remains a theoretical concept, and is not yet applied by system operators.

The OTS problem is an NP-hard Mixed Integer Nonlinear Program (MINLP), and is solved using a heuristic based on the DCOPF problem, or in recent years, an ACOPF based heuristic. These will be referred to as DC and AC heuristics. During the last ten years, transmission switching has received attention for having a huge cost reduction potential for power system operators. Through reviewing the literature and studying the problem ourselves, we have made the following observations:

- The OTS problem is mainly studied on small test cases, where network data are outdated.
- The test cases also lack necessary network data, and cannot be considered to represent realistic networks.
- The test cases were designed to study the AC power flow problem, which is a feasibility task, and not designed for optimization.
- The conclusion that OTS will result in large cost reductions are based on two to three demand scenarios, where the total demand of the network is scaled up or down mostly around 10%.

These observations are our motivation to study the OTS problem further. We wish to extend OTS research to involve larger, more realistic test cases. Further, we will make an effort to improve the network data of the most frequently used test case to investigate whether we can arrive at the same conclusion; that OTS will greatly decrease generator dispatch costs. Before applying a heuristic to solve the OTS problem, we will examine the characteristics and reliability of the DC and AC heuristics.

When an initial set of lines are chosen by the heuristic to be switched off, we wish to investigate how these line switches will affect the system operator throughout the day, i.e., subject to continuous network load changes at each load bus. It will be done by discretizising the continuous load changes into stages, where the load at each bus is changed with a relatively high correlation between buses.

The main objectives of our work can be summarized as follows:

1. Improve the network data of the already studied test case in OTS literature.
2. Investigate OTS on larger, newer, more realistic networks.
3. Test the performance and quality of the AC and DC heuristics used to solve the OTS problem.
4. Explore whether or not transmission switching can reduce generator dispatch costs over time, when subject to demand uncertainty.
5. Quantify the risk related to transmission switching, both in an economical and operational contingency perspective.

The remainder of this thesis is structured in three parts. Part one presents relevant theory revolving the OPF and the OTS problem. Chapter 2 includes an introduction to basic power systems terminology, before we present the AC and DCOPF problem, the MATLAB solver and finally the OTS problem. Chapter 3 presents an overview of the OTS literature.

The second part is included to provide an overview of the methodology and choices we have made to arrive at our objectives. Chapter 4 will include a discussion of the weaknesses of the test cases existing today and what we have done to modify the ones we have chosen for our OTS study. In Chapter 5, we aim to explain the idea behind the AC and DC heuristics applied, and examine their performance on the modified test cases. Finally, in Chapter 6, the method of how we simulate various demand scenarios based on a log-normal distribution is presented.

In the third part, the results from our simulations will be presented and commented, before finally discussing them in the light of our objectives. Chapter 7 presents our results regarding the cost reduction potential of OTS, and important findings are commented on. In Chapter 8, we present our findings regarding operational risk. Our results are discussed in Chapter 9, before we arrive at our final conclusion in Chapter 10.

Part I

Background and Theory

This part will provide a presentation of the underlying physics governing the Optimal Transmission Switching Problem, as well as its mathematical properties. The literature revolving OTS is reviewed and our contribution is set into context.

Chapter 2

The Optimal Transmission Switching Problem

This chapter presents the necessary theoretical background in order to understand the deduction of the Optimal Transmission Switching (OTS) problem. We first present the electrical components in a power network, before we introduce the basis of the OTS problem, namely the Optimal Power Flow (OPF) problem. The solving method of this problem is then introduced, before we present the OTS problem and comment on its properties.

2.1 Power systems terminology

In a transmission network, various transmission lines are interconnected at network nodes. These nodes are referred to as *buses* in power systems theory. Some of these are generator buses, i.e. they are connected to power generators which generate real and reactive power at generator g, P_n^g and Q_n^g. Others are load buses which serve elecricity to consumers. These buses have a reactive and active power demand, or load, associated to them, represented by Q_n^d and P_n^d.

The power flowing on a transmission line can be divided in two parts. P_{knm} is the real power which flows on line k connecting buses n and m, and is what consumers pay for, minus power loss. Q_{knm} is the reactive power flowing on line k, which is a necessary physical property when describing AC systems power flows. Power is represented by a single complex number having P

as the real part and Q as the imaginary part. When considering an AC system, voltage levels vary sinusoidally at a constant given frequency. V_n and θ_n denote the voltage magnitude and voltage angle at bus n, which describe the position in the sine wave at time t=0 (Elgerd, 1977).

When determining the power flowing on a transmission line, there are a number of electrical elements that are taken into account. Electrical impedance describes a measure of opposition to a current when a voltage is applied, and is expressed in Cartesian form as:

$$Z = R + jX \tag{2.1}$$

The real part of impedance is the resistance, R, a measure of the opposition to a current in a conductor. X is the imaginary part of impedance, called reactance, which represents a measure of the opposition to a change in current or voltage due to a circuit element's inductance or capacitance.

Y is the electrical admittance assigned to each transmission line k in the network. It is a complex number, and represents how easily current flows through a circuit. It is the inverse of impedance, given by:

$$Y = Z^{-1} = \frac{1}{R + jX} = G + jB \tag{2.2}$$

The real part of admittance is the electrical conductance, G, which gives a measure of the ability of an element to pass electric current. It is defined as the inverse of the electrical resistance R. The imaginary part of the admittance is susceptance, B, a measure of the ease in which alternating current passes through the transmission line, and is expressed as the inverse of reactance, X.

An Optimal Power Flow (OPF) problem is a power system optimization problem concerned with finding an optimal operating point of a power system, which minimizes a given objective function subject to network constraints (Madani et al., 2013). One example of an objective function is to minimize the total power loss in the network, while another is to minimize generator dispatch costs. In this thesis we will consider the latter. The constraints of an OPF problem ususally consist of a Power Flow problem based on Kirchoff's laws, as well as bus voltage limits, line power flow limits and other network operation limits. In the Alternating Current Optimal Power Flow problem (ACOPF), P_{knm} and Q_{knm} are nonlinear functions of the voltage magnitude and angle. The ACOPF is an NP-hard problem, and is complex in many ways. The AC power flow introduces nonlinearities, and computationally the problem is difficult to solve as the optimization has nonconvexities (Cain et al., 2012). A simpler version of the OPF is the DCOPF problem. The name DCOPF can be misleading; It is not a power flow solution for a direct current network, but a linearized version of the ACOPF which is reminiscent of Ohm's law for DC current. We will apply the general term OPF to include both ACOPF and DCOPF.

2.2 Nomenclature

Indices and sets

n, m	bus indices
k	transmission line index
knm	transmission line k connecting buses n and m
G	set of generators
K	set of transmission lines
N	set of buses

Parameters

B_{nm}	susceptance on the transmission line connecting buses n and m
G_{nm}	conductance on the transmission line connecting buses n and m
Q_n^d, P_n^d	reactive and real power demand at bus n
Q_k^{loss}, P_k^{loss}	reactive and real power loss across line k
c_n	unit power cost from generator at bus n

V_n^{min}, V_n^{max}	minimum and maximum voltage magnitude at bus n
P_n^{min}, P_n^{max}	minimum and maximum real power generation at bus n
Q_n^{min}, Q_n^{max}	minimum and maximum reactive power generation at bus n
$\Delta\theta^{min}, \Delta\theta_n^{max}$	minimum and maximum voltage angle difference
S_k^{max}	maximum apparent power flow on line k
b_{nm}^P	shunt susceptance at transmission line k going from bus n to m

Variables

P_n^g, Q_n^g	real and reactive power generated at bus n
V_n	voltage magnitude at bus n
θ_n	voltage angle at bus n
θ_{nm}	voltage angle difference $(\theta_n - \theta_m)$
P_{knm}	real power flow on line k between buses n and m
Q_{knm}	reactive power flow on line k between buses n and m

2.3 The ACOPF problem

The ACOPF problem is considered to be the most realistic formulation of the OPF problem as it includes the full Power Flow problem with system and resource constraints. In this thesis, we will study test cases that are all AC power distribution networks. There are numerous formulations of the ACOPF problem, the first given by Carpentier (1962). Since then, several formulations have been used, with some of the most common being the polar power-voltage formulation, the rectangular power voltage formulation and the rectangular ACOPF-current-voltage formulation (Cain et al., 2012). We will present the ACOPF problem using the widely applied polar power-voltage formulation. This is also the formulation implemented in MATPOWER (Zimmerman et al., 2011). The problem formulation is presented below:

$$\underset{V,\theta,P,Q}{\text{minimize}} \sum_{n\in G} c_n P_n^g \tag{2.3}$$

$$\text{s.t.} \quad P_{knm}^2 + Q_{knm}^2 \leq (S_k^{Max})^2 \ \forall k \in K \tag{2.4}$$

$$h(V,\theta,P,Q) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.5}$$

$$V_n^{min} \leq V_n \leq V_n^{max} \ (n \in N) \tag{2.6}$$

$$\Delta\theta^{min} \leq \theta_n - \theta_m \leq \Delta\theta^{max} \ (n \in N) \tag{2.7}$$

$$P_n^{min} \leq P_n^g \leq P_n^{max} \ (n \in G) \tag{2.8}$$

$$Q_n^{min} \leq Q_n^g \leq Q_n^{max} \ (n \in G) \tag{2.9}$$

2.3.1 The objective function

Equation (2.3) refers to the objective function of the problem that minimizes the total cost of generating power at each generator bus. Some formulate it as a quadratic or a piecewise linear function. Within OTS literature, studies have only included the marginal cost of producing power [$/MWh], which we also do. This is multiplied with power generation [MW], to find the generator dispatch costs [$/h]. Note here that some models also include the costs of generating reactive power, but in most cases these are negligible.

2.3.2 Line flow constraints

Equation (2.4) represents the line flow constraints, which constitute the flow limit of apparent power in a given line in the network. The combination of reactive power and true power is called apparent power. It is the product of a circuit's voltage and current, without reference to phase angle. In other words, when a circuit is purely resistive, as in the case of a DC circuit, then apparent power is equal to real power. However, in inductive or capacitive circuits, as in the case of an AC circuit, apparent power is greater than real power.

S_k^{Max} is the maximum absolute value of the apparent power flowing on line k linking buses n and m, and is often reffered to as a *thermal limit*. P_{knm} and Q_{knm} represent the active and reactive power flow on line k between buses n and m, and can be rewritten as follows:

$$\begin{aligned} P_{knm}(\theta, V) &= V_n V_m (G_{nm} \cos\theta_{nm} + B_{nm} \sin\theta_{nm}) - G_{nm} V_n^2 && \forall k \in K \\ Q_{knm}(\theta, V) &= V_n V_m (G_{nm} \sin\theta_{nm} - B_{nm} \cos\theta_{nm}) + V_n^2 (B_{nm} - b_{nm}^p) && \forall k \in K \end{aligned} \tag{2.10}$$

The thermal limits of a network determines to a large extent the capacity of the network, in addition to network operating limits. If the power flowing on the line exceeds the thermal limit, it will cause the OPF problem to not converge, i.e. that the network blacks out. We distinguish between two network scenarios. The first is when the network is running in *normal* conditions. This is the case during most of the year in modern power networks, when lines and generators are not pushed to their peak capacity. However, it does happen for some networks that parts of the grid are *congested*. This is the case when transmission lines or generators are close to overloaded, operating at their maximum capacity. Mathematically, it can be interpreted as being close to the problem's feasibility limit. As power demand is fluctuating throughout the day, and the fact that a modern power network is able to supply power in most of these fluctuations, it would be reasonable to assume that a network will rarely experience severe congestion.

2.3.3 The Power Flow problem

Equation (2.5) represents a compact form of the Power Flow (PF) problem. It is a well known problem in the field of power system engineering with the aim of finding the steady-state point of an electric power system (Grainger and Stevenson, 1994). In the PF problem, one usually differentiates between PV buses and PQ buses. PQ buses are commonly known as *load buses* where the net real and reactive power demand, P_n^d and Q_n^d, are known quantities. PV buses are usually referred to as *generator buses* where the real power injection, P_n^g, and the voltage magnitude, V_n, are known. The PF problem consists of the real and reactive power balance equations for each PQ bus and only the real power balance equation for PV buses. As the net reactive power injection is assumed to be unknown, we would get an additional unknown variable by including the reactive power balance equation for PV buses. The problem is a set of nonlinear equations representing Kirchoff's laws and network operation limits. They are called balancing constraints as they ensure that the amount of power flowing into a node must equal the amount of power flowing out of the node. Readers are referred to Andersson (2004) for a detailed description of the underlying physics of the PF equations.

The power balance equations are formulated in the following way:

$$\sum_{m \in N} V_n V_m (G_{nm} \cos(\theta_{nm}) + B_{nm} \sin(\theta_{nm})) + P_n^d - P_n^g = 0 \quad (n \in N) \tag{2.11}$$

$$\sum_{m \in N} V_n V_m (G_{nm} \sin(\theta_{nm}) - B_{nm} \cos(\theta_{nm})) + Q_n^d - Q_n^g = 0 \quad (n \in N \backslash G) \tag{2.12}$$

Equation (2.11) represents the real power balance, while equation (2.12) represents the reactive power balance. For the two PF equations, we collect the unknown variables in a vector X:

$$X := ((V_n)_{n \in G}, (\theta_n)_{n \in N}, (P_n^g + jQ_n^g)_{n \in G}) \tag{2.13}$$

Equations (2.3) and (2.4) can then be expressed in vector notation:

$$h(X) := \begin{pmatrix} h_P(X) \\ h_Q(X) \end{pmatrix} = \begin{pmatrix} h_P(\theta, V) \\ h_Q(\theta, V) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.14}$$

2.3.4 Network operating limits

Equations (2.6)–(2.9) are inequality restrictions representing network operating limits. They ensure that the voltage magnitude, voltage angle and reactive and active power generated stay within the operating limits at each bus. These restrictions are rewritten as $X \in B$. We then get a compact formulation of the ACOPF problem, a nonlinear problem with both equality and inequality constraints:

$$\text{minimize} c(X) \tag{2.15}$$
$$\text{s.t.} \quad h(X) = 0 \tag{2.16}$$
$$g(X) \leq 0 \tag{2.17}$$
$$X \in B, \tag{2.18}$$

2.4 The DCOPF problem

The DCOPF is a linear approximation of the ACOPF problem, where voltage magnitudes are approximated to one, and reactive power variables are set to zero. This leaves us with the decision variables P_n^g and θ_n. The approximations of the ACOPF model, (2.3) - (2.9), result in the following model:

$$\underset{P,\theta}{\text{minimize}} \sum_{n \in G} c_n P_n^g \tag{2.19}$$
$$\text{s.t.} \quad P_k^{min} \leq P_{knm} \leq P_k^{max} \quad \forall k \in K \tag{2.20}$$
$$P_n^g - \sum_m B_{nm}(\theta_n - \theta_m) = P_n^d \quad \forall n \in N \tag{2.21}$$
$$P_{knm} = B_{nm}(\theta_n - \theta_m) \quad \forall k \in K \tag{2.22}$$
$$\Delta\theta^{min} \leq \theta_n - \theta_m \leq \Delta\theta^{max} \quad \forall n \in N \tag{2.23}$$
$$P_n^{min} \leq P_n^g \leq P_n^{max} \; (n \in G) \tag{2.24}$$

The objective function (2.19) remains the same as in the ACOPF model. The line flow constraints are simplified to (2.20), with upper and lower limits on active power flow. The approximated power flow for each line is calculated by (2.22). Constraints (2.20) and (2.22) could be combined into one constraint. The PF problem is described by (2.21). The inequality constraints, (2.23) and (2.24), describe the network operating limits for voltage angle differences and power generation limits, respectively.

2.5 Solving the ACOPF and DCOPF problems

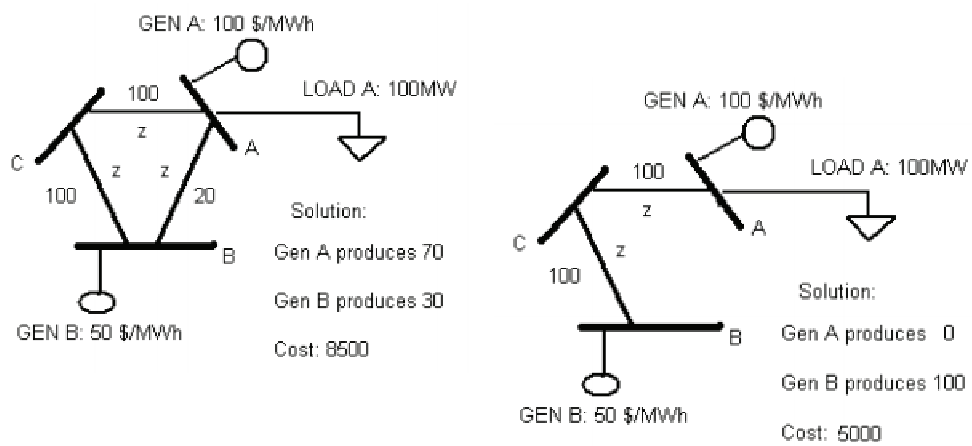
Early approaches to solving the OPF problem include gradient methods, sequential quadratic programming and sequential linear programming. These techniques fall short considering the slow convergence and limited field of application (Capitanescu et al., 2007). Accross the years, numerous approaches have been applied, as documented in the extensive survey by Cain et al. (2012).

Throughout this thesis we will refer to MATPOWER as an OPF solver. MATPOWER is a package of MATLAB M-files for solving PF and OPF problems (Zimmerman et al., 2011). It employs all of the standard steady-state models typically used for power flow analysis. An ACOPF is executed by calling *runopf* with a case file name as the first argument. The output is stored in a *results* file, including, amongst others, the objective function value, computation time, bus voltage magnitudes and angles, real and reactive generator power injections, real and reactive power loss and real and reactive power flowing on each transmission line. Similarly, the DCOPF is solved by calling *rundcopf*. By default, *runopf* and *rundcopf* solve an OPF problem using a primal-dual interior point solver included in MATPOWER called MATLAB Interior Point Solver (MIPS). Interior Point Methods (IPM) are a class of iterative algorithms applied to solve nonlinear and linear problems. The main characteristic of the IPM is that it reaches a solution by moving through the interior points of the solid defined by the optimization problem instead of moving through the surface of the solid. It is an appealing approach to the OPF problem mainly because of its ease of handling inequality constraints by logarithmic barrier functions, its speed of convergence and the fact that a strictly feasible initial point is not required (Capitanescu et al., 2007). A more detailed presentation of the principles governing IPM can be found in Appendix A.

2.6 Introducing the Optimal Transmission Switching problem

The OTS problem formulation is strongly linked with the OPF problem. It seeks to change the network topology by switching on or off one or more transmission lines in the network in order to relieve overload and voltage problems, improve security and reduce losses (Fuller et al., 2012). More recently, it has been suggested and examined as an important optimization model to reduce generator dispatch costs. The objective of the OTS problem is to find the transmission line switching actions that provide the highest cost reduction while still securing power supply. The physical characteristics of electricity imply that electrical power flows on each active line of the network, also lines that are economically inefficient. Removing or adding a line produces a global effect on the network, and switching off a selection of transmission lines can reduce the total production cost. The principal is illustrated with the simple three-bus systems in Figure (2.1).

Figure 2.1: Illustration of the OTS principle (Hedman et al., 2008b).



The OTS is a reformulation of the DCOPF or ACOPF problem, where a binary variable λ_k is introduced. λ_k returns 0 when line k is fully in service and 1 when line k is removed from the system. The solution provides the location of switches under a given operational scenario. When introducing the binary variable λ_k to the ACOPF problem presented in (2.3)-(2.9), we get the following formulation of the AC-OTS problem:

$$\underset{V,\theta,P,Q,\lambda_k}{\text{minimize}} \sum_{n\in G} c_n P_n^g \tag{2.25}$$

$$\text{s.t.} \quad (P_{knm}(V,\theta)\times(1-\lambda_k))^2 + (Q_{knm}(V,\theta)\times(1-\lambda_k))^2 \leq (S_k^{Max}\times(1-\lambda_k))^2 \quad \forall k\in K \tag{2.26}$$

$$P_n^g - \sum_{m\in N}(P_{knm}(V,\theta)\times(1-\lambda_k)) = P_n^d \quad \forall n\in N \tag{2.27}$$

$$Q_n^g - \sum_{m\in N}(Q_{knm}(V,\theta)\times(1-\lambda_k)) = Q_n^d \quad \forall n\in N\backslash G \tag{2.28}$$

$$V_n^{min} \leq V_n \leq V_n^{max}, \quad \forall n\in N \tag{2.29}$$

$$\Delta\theta^{min} \leq \theta_n - \theta_m \leq \Delta\theta^{max}, \quad \forall n\in N \tag{2.30}$$

$$P_n^{min} \leq P_n \leq P_n^{max}, \quad \forall n\in G \tag{2.31}$$

$$Q_n^{min} \leq Q_n \leq Q_n^{max}, \quad n\in G \tag{2.32}$$

$$\lambda_k \in \{0,1\} \quad \forall k\in K \tag{2.33}$$

Similarly, when introducing the binary variable λ_k to the DCOPF problem presented in (2.19)-(2.24), we get the following formulation of the DC-OTS problem:

$$\underset{P,\theta,\lambda_k}{\text{minimize}} \sum_{n\in G} c_n P_n^g \tag{2.34}$$

$$\text{s.t.} \quad P_k^{min}(1-\lambda_k) \leq P_{knm} \leq P_k^{max}(1-\lambda_k) \quad \forall k\in K \tag{2.35}$$

$$P_n^g - \sum_m B_{nm}(\theta_n-\theta_m) = P_n^d \quad \forall n\in N \tag{2.36}$$

$$P_{knm} = B_{nm}(\theta_n-\theta_m)(1-\lambda_k) \quad \forall k\in K \tag{2.37}$$

$$\Delta\theta^{min} \leq \theta_n - \theta_m \leq \Delta\theta^{max} \quad \forall n,m\in N \tag{2.38}$$

$$P_n^{min} \leq P_n \leq P_n^{max} \quad (n\in G) \tag{2.39}$$

$$\lambda_k \in \{0,1\} \quad \forall k\in K \tag{2.40}$$

The OTS based on the DCOPF model, returns a Mixed Integer Linear Program (MILP). Using an

ACOPF formulation to solve the OTS problem will return an NP-hard non-convex Mixed Integer Nonlinear Program (MINLP) which gives rise to further computational challenges. Based on either DCOPF or ACOPF, the main impediment to use OTS in practice is the long computation time needed to solve it. This aspect has made the development of heuristics for the OTS problem necessary.

Chapter 3

Literature survey

3.1 The IEEE118 case

The IEEE118 case, hereby called case118, is by far the most studied test case within OTS literature. Case118 is a MATPOWER benchmark test case found at the University of Washington online archive. The test case lacks some of the data necessary to study OPF, including line thermal limits and generator costs. Studies that report a significant cost reduction potential, have used the original case118 and added necessary data from Blumsack (2006) to the test case. In Blumsack's thesis, there is not given an explicit explanation of the methodology used to determine thermal limits or generator marginal costs. Through e-mail correspondance with him, he stated that the costs were generated as to exhibit a merit order dispatch curve with roughly the same shape that you would see in a thermal-based system in the US. The cost data were not necessarily meant to be realistic except for the relative cost of peak versus base load generators, and the relative share of peak versus base load generation. The thermal limits, or the upper bounds of the line flow constraints, were generated in order to congest portions of the network. There was no real basis for these limits other than wanting to have some lines fully loaded.

3.2 DCOPF based heuristics

OTS in an economic context was first examined by O'Neill et al. (2005). Their paper investigated OTS in a market context, where market-based payments for transmission was considered. However, for our thesis and the literature reviewed in this section, the market perspective is not considered. We assume a system operator that uses OPF calculations to determine power dispatch.

Fisher et al. (2008) were some of the first to examine OTS under this assumption in an economic context. They seperated the DC-OTS problem into a collection of subproblems and solved them using CPLEX. The test case studied was case118, where they modelled generator costs as linear based on the calculations of Blumsack (2006). Further, they assumed resistance and shunt capacities as zero and line thermal limits of 9900 MVA, which in practice means that the transmission lines have infinite capacity. They looked at the test case in a peak and off-peak load scenario, and their best result was measured to be a 25% cost reduction with 38 lines open. An N-1 contingency analysis was performed with the conclusion being that switching off transmission lines did not necessarily have a negative impact on network reliability. An extension of this paper was published by Hedman et al. (2008a), where they discussed computational issues and how OTS affected nodal prices, load payments and flowgate prices. A more detailed contingency analysis of OTS on case118 was later published by Hedman et. al. in two papers (Hedman et al., 2009, 2010).

Fuller et al. (2012) published a paper where they discussed the great computational challenges related to the OTS problem. The previous heuristics employed solved a sequence of MILPs, removing one line at a time, with each MILP having all binary variables, one for each line. Fuller et. al. suggested two new significantly faster heuristics with far fewer variables in each MILP. A similar heuristic was also presented by Ruiz et al. (2012). Similar to Fisher et al. (2008), they used a DC approximation and attempted to replicate the data used for case118. Together with the 662-bus model (Vaderbei), they explored the OTS heuristic using three different load scenarios: low, medium and high demand, where they multiplied the loads with 80%, 90% and 100%, respectively. Their highest cost reduction was found to be 14% in the medium demand scenario for case118. They concluded that their heuristic significantly decreased the computation time, but that the results could not be deemed reliable until the problem was examined in a nonlinear context. Shortly after, Potluri and Hedman (2012) found that the results from a DC approximation were generally poor compared to the estimated cost based on running the ACOPF before and after line removal. It is worth noticing, however, that studies on DC heuristics versus AC heuristics have only been explored on a few smaller test cases, mostly case118.

3.3 A shift towards ACOPF based heuristics

Following this, Soroush and Fuller (2014) compared the reliability of the DC heuristic with the actual cost reductions one would get with the ACOPF model. Tests were done with case118 and the IEEE300 case. They also created an AC heuristic, and tested it under three different load scenarios: low, high and very high demand. They concluded that the DC approximated estimates of cost reduction were poor, especially in the scenarios where the demand was high. However, their AC heuristic yielded high cost reductions for high demand scenarios (16%), but at the expense of a high computation time. The network data used for case118 is the same as before, taken from Blumsack.

Recently, Capitanescu and Wehenkel (2014) proposed an AC heuristic algorithm that aimed to break down the complexity of the original MINLP problem. The approach was applied on case118, where the generator costs and thermal limits were changed in accordance with the suggestions from Blumsack (2006). They considered three load scenarios: normal, high and very high. Their final conclusion was that their heuristic returned good quality suboptimal solutions with relatively small computational efforts. However, they emphasized that further studies were needed on larger, more realistic networks.

Coffrin et al. (2014b) very recently studied the properties of various formulations of the OTS problem and developed an AC heuristic. The test cases employed were the RTS96 case and case118, which is based on the data from Blumsack with a few of their own modifications. They concluded, as others have done before them, that a DC model was not appropriate for OTS studies as it "[...]exhibited significant AC-feasibility issues while both underestimating and overestimating the benefits of line switching in different contexts". A new observation was that they showed that line switching did not bring economic benefits to the standard MATPOWER benchmark test cases. However, they suggested that OTS may be beneficial on congested networks with reported cost reductions up to 29%. They concluded their paper by emphasizing that the network studied, case118, is a small network from the sixties and that it remains unclear whether or not these results will hold for modern power networks.

3.4 Summary and our contribution

OTS research in an economic perspective started in the middle of the '00s, where heuristics and solvers were based on a DC approximation. Later on, it was shown that a DC approximation returns poor and unreliable results, and therefore different AC heuristics were developed. The majority, whether with a DC or an AC heuristic, takes a positive outlook on OTS, arguing that the economic benefits are high, ranging up to a 29% cost reduction when switching off lines.

These results are mostly based on the small case118, a US network from the sixties, where thermal limits are modelled so that parts of the network is congested and with no real cost data. Some have also studied the RTS96 and IEEE300 cases. Many have extended their studies with three or four different load scenarios.

Our contribution is to explore the OTS problem on larger and newer networks, as the Polish and Great Britain networks, with 2–3000 buses each. We also explore a modified version of case118, for comparative reasons. The networks studied will include realistic network data, and OTS will be examined under a much larger set of realistic load scenarios.

Part II

Methodology

Our main objective is to investigate whether or not transmission switching has the potential of being economically beneficial for a system operator. In order to do this, we need to have access to realistic networks and solve the OTS problem using a reliable solving method. This part focuses on the choices we have made to create a basis for evaluating OTS.

Chapter 4

Modifications of MATPOWER test cases

As presented in Chapter 3, the OTS literature is characterized by testing with a version of case118 that represents a congested network with unrealistic generator costs. We aim to modify this network to a noncongested state to shed light on how OTS can affect dispatch costs for a power network that runs in *normal* conditions. Further, to extend studies with larger, newer networks, we study the Polish and Great Britain (GB) networks. This chapter presents the methods employed to create necessary network data for our chosen test cases, and explain the modifications done to each of them.

4.1 The lack of necessary network data

Due to the sensitive nature of power infrastructue, network data from real power networks are difficult to obtain. This has led to the frequent use of a collection of test cases where many are based on data that are more than thirty years old (University of Washington). Through studying these test cases, we discovered that most of them lack necessary network data. Another discovery was that it is unclear whether or not these test cases are suitable for optimization studies, as they were originally designed to test the AC power flow problem, which is a feasibility task.

Coffrin et al. (2014a) surveyed and collected the existing test cases used for OPF studies, and reached the same conclusion that crucial network data was either missing or unrealistic. They presented which test cases lack necessary data for optimization applications. The test cases we have chosen are displayed in table 4.1, where cells containing "–" indicate missing data.

Name	Original source	Generator capabilities	Generator costs	Thermal limits
case118	University of Washington	–	–	–
Polish	Zimmerman et al. (2011)	✓	✓	✓
Great Britain	Bukhsh and McKinnon	✓	–	–

Table 4.1: Survey of test case data (Coffrin et al., 2014a)

4.1.1 Thermal limits

In MATPOWER, thermal limits are represented by the upper bound of apparent power flow, S_k^{Max} of the line flow restriction in the ACOPF problem:

$$P_{knm}^2 + Q_{knm}^2 \leq (S_k^{Max})^2 \; \forall k \in K \tag{4.1}$$

As there exists little realistic data available today on line characteristics of a network, we need to make an approximation of the thermal limits. In most test cases, thermal limits are set unrealistically high, leading to lines with infinite capacity. This is clearly unfortunate in an optimization context, as it would remove an important restriction to the problem. The thermal limit of a transmission line is determined based on several aspects, such as the voltage level of the line, the maximum current flowing on the line, the line length, the conductor type, the diameter of the phase conductors and weather conditions. If data on these characteristics were available, we could quite easily calculate the thermal limit of each line. However, to the best of our knowledge, not a single test case exists with these characteristics enclosed. A normal approach to take when calculating thermal lines, is with a Surge Impedance Loading (SIL) (Nayak et al., 2006). This approach has some significant limitations if used in OPF calculations, as it only takes into consideration the voltage level and the surge impedance of a line. When the line length is not considered, the SIL will not return a measure of a transmission line's power transfer capability. It will return the same thermal limits for lines within the same voltage level, when in fact in real life, these limits can vary significantly (Kundur et al., 1994).

To create more realistic thermal limits that go beyond SIL, our initial idea was to randomly distribute thermal limits taken from a test case of the same size which had realistic data from a reliable source, namely the Polish network created by Roman Kolab with data from Poland's National Grid Operator (PSE S.A). In this way we could, contrary to before, obtain an upper bound to an important restriction. We did see, however, that this method had some significant limitations. It did not take into consideration the line length or conductor type. It was not until we came across the recently published report by Coffrin et al. (2014a), that we decided to change our method. This report presented a linear regression model focusing on the values of resistance, reactance and nominal voltage. These values can be found in MATPOWER test cases as the parameters *r, x* and *baseKV*, respectively. It returns reasonable upper bounds for the line thermal limits, when these line parameters are known. It was found that the ratio of resistance to impedance could provide insight to a line's conductor type, as "[...]this ratio should be independent of the line length, whereas the values taken individually would be proportional to the length". Another observation was that conductors of the same type could have different thermal limits, depending on their nominal voltage. The regression model presented is based on two test cases that have realistic thermal limits, the Polish network and the Irish network provided by EIRGRID, and shows that an approximation of the thermal limit could be derived using the following equation:

$$S_k^{Max} = (baseKV)e^{-5.0886}(\frac{x}{r})^{0.4772} \tag{4.2}$$

Equation (4.2) can be used when r and x are given in the data set for the line k in question, and when *baseKV* is the same in both connecting buses. When this is not the case, Coffrin et al. (2014a) presented an alternative way of calculating the thermal limits, by expressing them through available parameters, such as the voltage magnitude bounds and the line's voltage angle difference bounds:

$$(S_k^{Max})^2 = (V_n^{max})^2 y_{nm}^2((V_n^{max})^2 + (V_m^{max})^2 - 2V_n^{max}V_m^{max}\cos(\theta_{nm}) \tag{4.3}$$

θ_{nm} is assumed to be 15 °, as it is suggested as a reasonable value by, amongst others, Van Hertem et al. (2006). For more details on the derivation of these two equations, the reader is referred to Coffrin et al. (2014a).

4.1.2 Generator capabilities

A test case should include box constraints on a generator's real and reactive injections, specified in the ACOPF formulation as:

$$P_n^{min} \leq P_n^g \leq P_n^{max} \; (n \in G) \tag{4.4}$$

$$Q_n^{min} \leq Q_n^g \leq Q_n^{max} \; (n \in G) \tag{4.5}$$

A true capability curve should involve some tradeoff between real and reactive capability, so that it is not possible to produce P_n^{max} and Q_n^{max} simultaneously. In power system optimization, this curve is usually approximated by a specified upper and lower bound on active and reactive power injection.

For the Polish and the GB networks, these capabilities are included in the data set and are based on realistic values (table 4.1). In case118, however, these values are not included. Coffrin et al. (2014a) have resolved this by creating a generation fuel category classification model, which randomly assigns a fuel category to each generator in the network. By collecting data from the U.S. Energy Information Administration data set, they looked at the distribution of capacity of each generator fuel type, and from these statistical properties, they allocated a capacity to each of the generators in the network.

4.1.3 Generator costs

A common feature in many test cases, is that generators are assigned quadratic cost fuctions, often with the same coefficients. To be able to obtain reasonable output from an optimization problem, the cost data needs to be as realistic as possible. In order to model more realistic generation costs, we have made the assumption of a non-competitive environment with one system operator.

Coffrin et al. (2014a) have also identified the lack of realistic generator costs as an issue when exploring optimization problems. They randomly assigned a fuel type to the generators in the network with a corresponding cost of fuel. The fuels in the network are only fossil fuels. However, these costs do not give a proper representation of which generators and fuel types that are actually in the network. Further, it does not include other costs of generating power, such as CO_2 emission costs or operation and maintenance costs, which will naturally vary according to fuel type.

To assign costs to each generator in the network, we have categorized them according to fuel type. A joint report by the IEA and the OECD Nuclear Energy investigates electricity generator costs in 2010 (OECD, IEANEA, 2010). The costs were obtained by surveying 200 power plants in 21 countries. Based on this report, Comaty (2013) set out to determine marginal costs of producing 1 MWh of energy in Europe assigned to each technology. The marginal cost includes the fuel costs, the operation and maintenance cost and the CO_2 emission cost. Many marginal cost calculations are levelised costs that include investments cost and amortization (MacDonald, 2010; Tarjanne et al., 2008; Anderson, 2007). However, as we are not considering a long term investment perspective, we chose the marginal costs of Comaty (2013), displayed in Table 4.2. To be able to apply these costs to the test cases, each generator node and its fuel type must be identifiable.

Technology	Marginal cost [$/MWh]
Coal	47.4
Gas – CCGT	72.0
Gas – AGT	91.5
Landfill gas	32.2
Hydro	5.3
Pump storage	10.9
Wind	36.2
Biomass	9.3
Nuclear	22.0
Oil	158.9

Table 4.2: Marginal costs assigned to technology

4.2 Modifying the test cases

4.2.1 Case118

Case118 consists of 118 buses, 54 generators and 186 lines and was first introduced by Rich (1993). It represents a portion of the American Electrical Power System in the Midwestern USA as of December 1962. As previously mentioned, the test case lacks thermal limit data, generator capabilities and realistic cost data. When creating a more realistic version of case118 we have focused on these aspects. The major part of the generator bus names are taken from the real names of power plants. Our first approach to assigning better cost data was to track down the type of generator, indentify the fuel type and assigned the marginal cost of production from Table 4.2. This proved challenging, as the test case is outdated and many of these generators do not exist today. Very recently, Coffrin et al. (2014a) developed a new version of the test case, found in the NESTA archive. In this version, thermal limits are updated with the method explained in subsection 4.1.1. Cost data and generator capacities are modelled based on statistical properties of today's existing generators in the U.S., as explained in subsection 4.1.2. We use the test case provided by the NESTA archive in our studies, mainly for comparative reasons.

Further, the test case is scaled down to 80% of its original load to be able to explore load changes throughout a day.

4.2.2 The Great Britain network

The Great Britain (GB) network is a test case created by W. A. Bukhsh at the University of Edinburgh. It consists of 2224 buses, 394 generators and 3207 transmission lines, where the network data is obtained from the GB SQSS Review working group report [1]. Realistic generator capacities are already included in the data set, so no changes are done to these. As in the original case118, thermal limits are set unrealistically high, with each line having 9900 MVA as its limit. New thermal limits were calculated with equation (4.2).

By contacting Mr. W. A. Bukhsh, we were able to access information on the location of each generator plant in the network. With this information, we retrieved the number of generators and fuel type at each node, and assigned generator marginal costs to them based on the values presented in Table 4.1.

4.2.3 The Polish network

The Polish network is the MATPOWER test case 'case2736sp', which is created by Roman Korab at the Silesian University of Technology in Poland. It represents the Polish 400, 220 and 110 kV networks during summer 2004 conditions. It consists of 2736 buses, 420 generators and 3504 lines. Through contact with the creator of the test case, we were ensured that the generator costs and thermal limits are set within a realistic range, and we did not attempt to modify it. It is worth noticing however that only the centrally dispatched generators are assigned nonzero costs. The rest of the system is assumed constant with a fixed amount of energy produced at each node.

[1] Availabe online: http://www.nationalgrid.com/uk/Electricity/Codes/gbsqsscode/workinggroups/intgeneration/

Chapter 5

The OTS Heuristic

As many power systems have thousands of transmission lines, the OTS problem becomes computationally expensive. A system operator has limited time available, and an implementation of OTS in day-ahead and real-time procedures is not practical. The focus on transmission switching heuristics has therefore increased in order to obtain good suboptimal solutions within a reasonable computation time. In this thesis we apply the heuristic presented by Soroush and Fuller (2014), which has been implemented by Francesco Piu, postdoctoral researcher at the University of Bergamo. This chapter will present the idea and theory behind the heuristic based on both ACOPF and DCOPF, before we proceed to test the performance of the heuristic on the test cases presented in Chapter 4. Finally, we will evaluate which input parameters should be used in the heuristic in order to reveal the cost reducing potential of the test cases within a reasonable computation time.

5.1 The AC heuristic

This heuristic idea is based on the optimal primal and dual variables of the reformulated ACOPF problem (5.1)-(5.9) below.

$$\underset{V,\theta,P,Q}{\text{minimize}} \sum_{n\in G} c_n P_n^g \tag{5.1}$$

$$\text{s.t.} \quad (P_{knm}(V,\theta)\times(1-\lambda_k))^2 + (Q_{knm}(V,\theta)\times(1-\lambda_k))^2 \le (S_k^{Max}\times(1-\lambda_k))^2 \quad [\gamma_k], \quad \forall k\in K \tag{5.2}$$

$$P_n^g - \sum_{m\in N}(P_{knm}(V,\theta)\times(1-\lambda_k)) = P_n^d \quad [\pi_n^P], \quad \forall n\in N \tag{5.3}$$

$$Q_n^g - \sum_{m\in N}(Q_{knm}(V,\theta)\times(1-\lambda_k)) = Q_n^d \quad [\pi_n^Q], \quad \forall n\in N\backslash G \tag{5.4}$$

$$V_n^{min} \le V_n \le V_n^{max}, \quad \forall n\in N \tag{5.5}$$

$$\Delta\theta^{min} \le \theta_n - \theta_m \le \Delta\theta^{max}, \quad \forall n\in N \tag{5.6}$$

$$P_n^{min} \le P_n^g \le P_n^{max}, \quad \forall n\in G \tag{5.7}$$

$$Q_n^{min} \le Q_n^g \le Q_n^{max}, \quad n\in G \tag{5.8}$$

$$\lambda_k = 0 \quad [\alpha_k], \quad \forall k\in K \tag{5.9}$$

It is similar to the AC-OTS problem, with the exception of constraint (5.9) where all λ_ks are set to zero. The dual variables are presented in brackets behind the corresponding restriction.

5.1.1 Heuristic variables

π_n^P and π_n^Q are the dual variables or shadow prices corresponding to the nodal power balance equality constraints (5.3) and (5.4). These variables can be interpreted as the locational marginal price (LMP) at each node. The definition of the LMP at any given bus is the sensitivity of the objective function to the addition of an extra MW of demand at the bus. According to market rules, power should flow from a lower nodal price to a higher nodal price, but as the network follows Kirchoff's laws, this is not always the case. Sometimes power might flow from higher to lower nodal prices, which is not beneficial in an economic perspective. Hence, the values of these dual variables play an important role in selecting which lines to cut with a cost reducing objective.

The variable λ_k represents the fraction of line k removed from service, meaning that $(1-\lambda_k)$ represents the fraction left in service. Realistically, λ_k can only take the values of 1 or 0, but here it is treated as a continous variable in order to derive α_k. By setting λ_k=0 in (5.9), the problem is equivalent to the original ACOPF problem presented in equations (2.3) – (2.9). Therefore, an optimal solution to the ACOPF problem is also optimal in (5.1) – (5.9).

The dual variable α_k describes the sensitivity of the objective function to a marginal change in the status of a transmission line. As we wish to decrease the costs, we are interested in the lines that have a corresponding negative α_k. Once the α_k values are known, lines may be ranked based on the indicated effect on costs, and help us conclude whether it is profitable to cut the line in question. We refer to α_k as the line ranking parameter.

5.1.2 Deriving α_k

The line ranking parameter, α_k, is the key behind the heuristic idea, and is calculated from the solution of the ACOPF. In order to derive an expression for α_k, we evaluate the optimality conditions for the reformulated ACOPF. The Karush-Kuhn-Tucker conditions (KKT) of the reformulated ACOPF at its optimal solution are satisfied by the optimal primal and dual variables from the ACOPF and $\lambda_k = 0$. We derive the KKT conditions using the Lagrangian function for problem (5.1) - (5.9):

$$
\begin{aligned}
\mathscr{L} = \sum_{n \in G} c_n P_n^g &+ \pi_n^P \left[P_n^d - (P_n^g - (P_{knm}(V,\theta) \times (1-\lambda_k))) \right] \\
&+ \pi_n^Q \left[Q_n^d - (Q_n^g - (Q_{knm}(V,\theta) \times (1-\lambda_k))) \right] \\
&+ \gamma_k [(S_k^{Max} \times (1-\lambda_k))^2 - ((P_{knm}(V,\theta) \times (1-\lambda_k))^2 + (Q_{knm}(V,\theta) \times (1-\lambda_k))^2 \right] \\
&+ \alpha_k \left[\lambda_k \right]
\end{aligned}
\tag{5.10}
$$

A necessary condition for a point to be optimal is that it is a stationary point to the Lagragian function. A classic method for finding a stationary point to the Lagrangian function is the Lagrangian multiplier method, where we search for the stationary point by solving the set of equations constituting $\nabla\mathscr{L}$=0 (Lundgren et al., 2010). In order to derive an expression for α_k, we evaluate $\frac{\partial \mathscr{L}}{\partial \lambda_k} = 0$. Due to losses, the power flow is not constant across the line. It is therefore necessary to specify the flow for each end of the line, through P_{kmn} and P_{knm}.

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial \lambda_k} &= \pi_n^P(P_{knm}(V,\theta)) + \pi_m^P(P_{kmn}(V,\theta)) \\ &+ \pi_n^Q(Q_{knm}(V,\theta)) + \pi_m^Q(Q_{kmn}(V,\theta)) \\ &+ 2\gamma_k(-(P_{knm}^2(V,\theta) \times (1-\lambda_k)) - (Q_{knm}^2(V,\theta) \times (1-\lambda_k)) + ((S_k^{Max})^2 \times (1-\lambda_k))) \\ &+ \alpha_k = 0, \quad \forall k \in K\end{aligned} \tag{5.11}$$

We evaluate the problem at λ_k=0. This gives the following equation:

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial \lambda_k} &= \pi_n^P(P_{knm}(V,\theta)) + \pi_m^P(P_{kmn}(V,\theta)) \\ &+ \pi_n^Q(Q_{knm}(V,\theta)) + \pi_m^Q(Q_{kmn}(V,\theta)) \\ &+ 2\gamma_k(-P_{knm}^2(V,\theta) - Q_{knm}^2(V,\theta) + (S_k^{Max})^2) \\ &+ \alpha_k = 0, \quad \forall k \in K\end{aligned} \tag{5.12}$$

Applying the complimentarity slackness condition on the line limit constraint (5.2) at λ_k=0 gives that either the constraint is not active in the point and the dual variable $\gamma_k = 0$, or the constraint is active and $(S_k^{Max})^2 - P_{knm}^2(V,\theta) - Q_{knm}^2(V,\theta) = 0$.

$$\gamma_k((S_k^{Max})^2 - P_{knm}^2(V,\theta) - Q_{knm}^2(V,\theta)) = 0, \quad \forall k \in K \tag{5.13}$$

From (5.13) we can simplify equation (5.12) to:

$$\alpha_k = -\pi_n^P(P_{knm}(V,\theta)) - \pi_m^P(P_{kmn}(V,\theta)) - \pi_n^Q(Q_{knm}(V,\theta)) - \pi_m^Q(Q_{kmn}(V,\theta)), \quad \forall k \in K \tag{5.14}$$

As we can only retrieve values for P_{knm} and Q_{knm} from the solution of the ACOPF, we need to replace P_{kmn} and Q_{kmn} with the following relations:

$$P_{kmn}(V,\theta) = -P_{knm}(V,\theta) + P_k^{loss} \tag{5.15}$$

$$Q_{kmn}(V,\theta) = -Q_{knm}(V,\theta) + Q_k^{loss} \tag{5.16}$$

This gives the final expression for α_k applied in the heuristic.

$$\alpha_k = (\pi_m^P - \pi_n^P) \times P_{knm}(V,\theta) - P_k^{loss}\pi_m^P + (\pi_m^Q - \pi_n^Q) \times Q_{knm}(V,\theta) - Q_k^{loss}\pi_m^Q, \quad \forall k \in K \tag{5.17}$$

We use equation (5.17) to calculate α_k for each line in the network.

5.2 The DC heuristic

The reformulated DCOPF is presented in problem (5.18)-(5.24) below.

$$\underset{P_g,\theta,\lambda_k}{\text{minimize}} \sum_{n \in G} c_n P_n^g \tag{5.18}$$

$$\text{s.t.} \quad P_k^{min}(1-\lambda_k) \le P_{knm} \le P_k^{max}(1-\lambda_k) \quad [\gamma_k], \quad \forall k \in K \tag{5.19}$$

$$\sum_{n \in G} P_n^g - \sum_m B_{nm}(\theta_n - \theta_m) = P_n^{dem} \quad [\pi_n], \quad \forall n,m \in N \tag{5.20}$$

$$P_{knm} = B_{nm}(\theta_n - \theta_m)(1-\lambda_k) \quad [\sigma_k], \quad \forall k \in K \tag{5.21}$$

$$\Delta\theta^{min} \le \theta_n - \theta_m \le \Delta\theta^{max} \quad \forall n,m \in N \tag{5.22}$$

$$P_n^{min} \le P_n^g \le P_n^{max} \quad (n \in G) \tag{5.23}$$

$$\lambda_k = 0 \quad [\alpha_k], \quad \forall k \in K \tag{5.24}$$

An expression for α_k is derived from the KKT conditions of problem (5.18)–(5.24), and α_k can be calculated from the optimal primal and dual variables of the DCOPF problem:

$$\alpha_k = (\pi_m^P - \pi_n^P) P_{knm} \quad \forall k \in K \tag{5.25}$$

As reactive power is not accounted for in the DCOPF, equation (5.25) does not include any reactive power terms or losses. The first factor describes the difference between the LMP at the to and from node, and the second term is the power flow on the line.

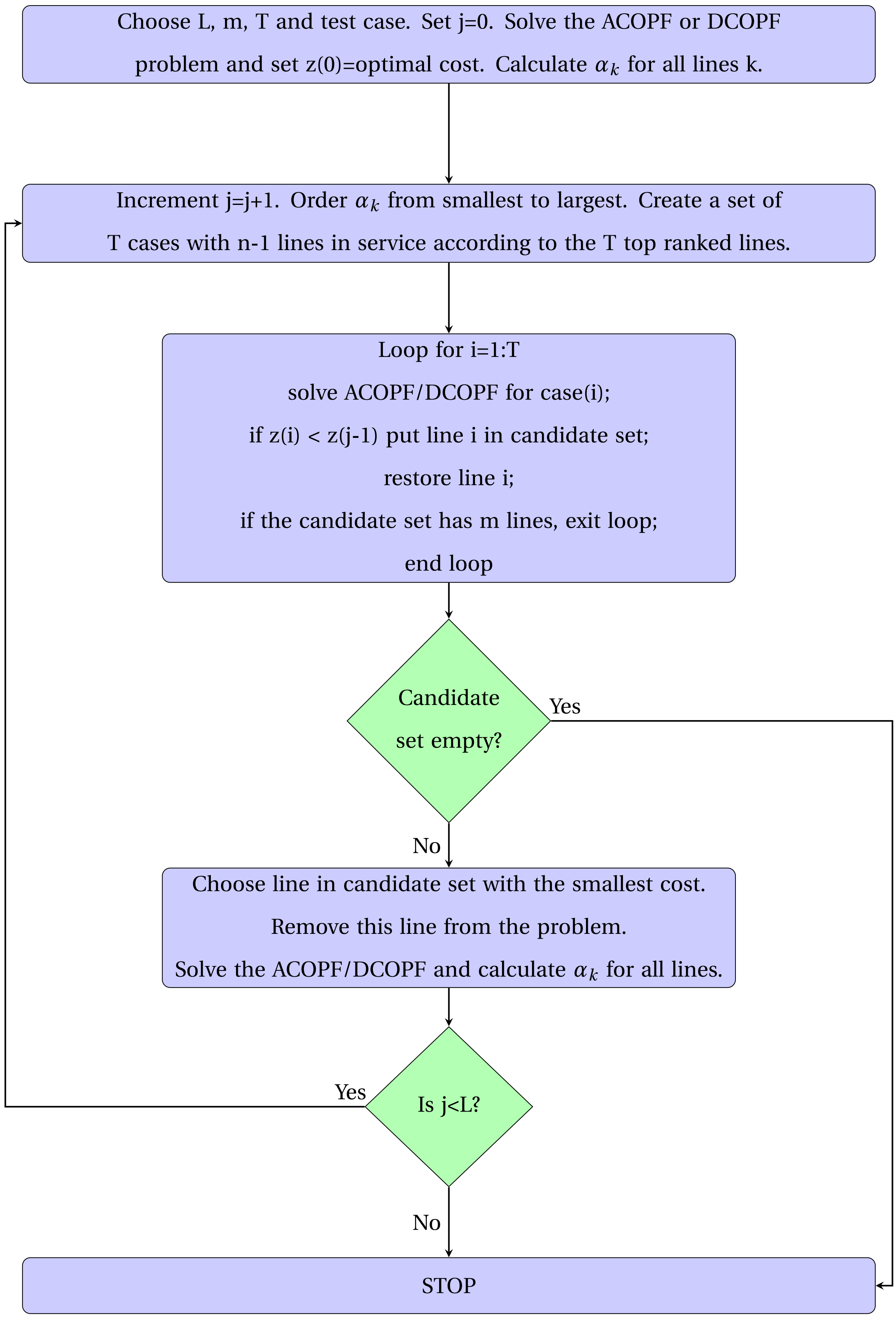
5.3 Heuristic steps

The AC and DC heuristics are constructed in the same way. They differ in the calculation of α_k and in the use of the ACOPF or DCOPF. The heuristic has four input parameters: L, m, T and a test case.

Heuristic nomenclature

L	Desired maximum number of line removals
m	Maximum number of cost-reducing line removals to be stored in candidate set
T	Maximum number of times the ACOPF or DCOPF can be solved in the inner loop of the heuristic
j	Counts the number of lines permanently removed from service
z	Optimal objective function value
n	Number of lines in service in test case

The flowchart on the next page describes the procedure of the heuristic algorithm. In brief terms, the heuristic calculates the α_k value for each line by solving the ACOPF or DCOPF problem and ranks the lines based on these values. It tests the top T ranked lines by switching them off one-by-one and reoptimizing to calculate the change in costs. The line switch contributing to the highest cost reduction is selected and switched off permanently before the algorithm proceeds to the next iteration.


Choose L, m, T and test case. Set j=0. Solve the ACOPF or DCOPF problem and set z(0)=optimal cost. Calculate α_k for all lines k.
Increment j=j+1. Order α_k from smallest to largest. Create a set of T cases with n-1 lines in service according to the T top ranked lines.
Loop for i=1:T
solve ACOPF/DCOPF for case(i);
if z(i) < z(j-1) put line i in candidate set;
restore line i;
if the candidate set has m lines, exit loop;
end loop
Candidate set empty?
Yes
No
Choose line in candidate set with the smallest cost.
Remove this line from the problem.
Solve the ACOPF/DCOPF and calculate α_k for all lines.
Is j<L?
Yes
No
STOP

We will make a few additional comments to the flowchart. L defines the number of iterations in the outer loop, while T defines the number of iterations in the inner loop. To reduce the computation time, the candidate set is filled with the m first cost reducing switches before the best switching action among these is chosen. As a result of this, some of the T line switches may not be explored. As α_k is not a perfect indicator, some of these unexplored lines might provide higher cost reductions than the switches in the candidate set. We have considered the possibility of alternating the heuristic algorithm to make cost reductions an increasing function of m by filling the candidate set with the best switching decisions after having solved all T ACOPF problems. Some heuristic tests with $m = T$ were done to force the heuristic to explore all T switching actions. This did not result in a significant improvement of the results, but, naturally, increased the computation time. Thus, the heuristic algorithm was not changed. Another aspect of the heuristic to keep in mind, is that each permanent line switch changes the topology and produces a global effect on the network. The switching line chosen in one iteration affects which lines that will be good switching candidates in the next iteration. Due to these characteristics, it is necessary to make some numerical experiments with different combinations of the input parameters to reveal the true cost reduction potential of each test case. Before we can arrive at that, we first evaluate the DC heuristic in order to determine its reliability, before choosing which heuristic to proceed with.

5.4 DC heuristic solutions tested on ACOPF

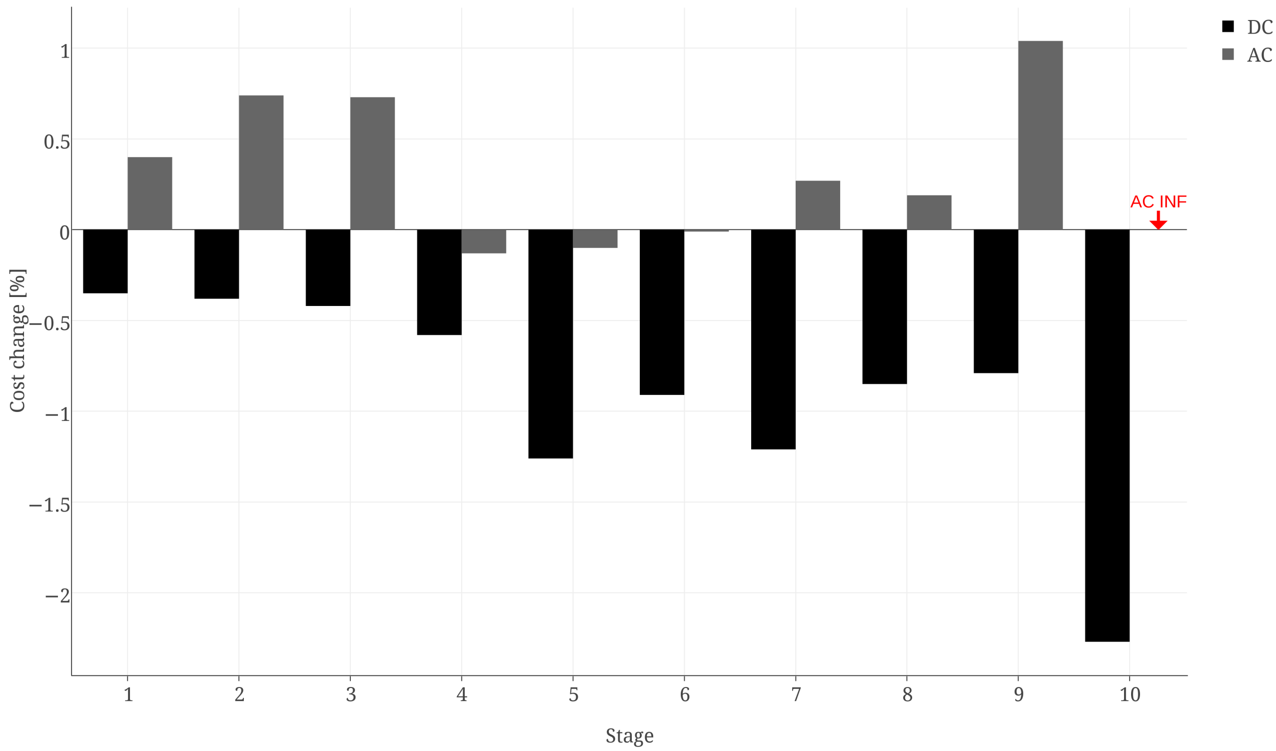
In real life market operations, DCOPF based applications are favored simply due to a significantly shorter computation time. As presented in the literature survey, recent studies show results indicating that the switching actions suggested by the DC heuristic are not always beneficial when applied to the ACOPF. The studies are performed on standard MATPOWER test cases. In this section, we extend these studies by applying the DC heuristic to our three modified test cases, and evaluate it for different load scenarios. The objective is to measure how well the solution from the DC heuristic performs under realistic system conditions modelled by the ACOPF. We rewrite the AC heuristic implementation of Francesco Piu to represent the DC heuristic.

Depending on the different DC-feasibility regions of the test cases, we select specific load levels for each test case to evaluate the DC heuristic on. For case118, we select 10 load stages ranging from 50% to 160% of the original load. For the Polish network, we have 9 load stages ranging from 90% to 130%. For the GB network, we select 12 load stages ranging from 90% to 110%. We apply the DC heuristic to each of the test cases subject to different load stages, and register the cost reductions and switching actions. We compare these cost reductions to the reductions achieved when applying the same switching actions to the ACOPF.

The results are presented in figure 5.1 - 5.3, with stages representing increasing loads. The text AC INF in the figures indicates when the switching actions suggested by the DC heuristic prove infeasible in the ACOPF.

5.4.1 Case118

Figure 5.1: DC switching actions applied to ACOPF – case118



In 6 out of 10 stages in figure 5.1, 1-3 and 7-9, the switching actions suggested by the DC heuristic result in increased costs when applied to the ACOPF. For stages 4-6 the switching actions are also ACOPF beneficial, but the benefits presented by the DC heuristic are overestimated. For stage 9, the DC heuristic presents a set of switching actions with a cost reduction of -2.3%. However, the set of switches causes ACOPF infeasibility.

5.4.2 The Polish network

Figure 5.2: DC switching actions applied to ACOPF – Polish network

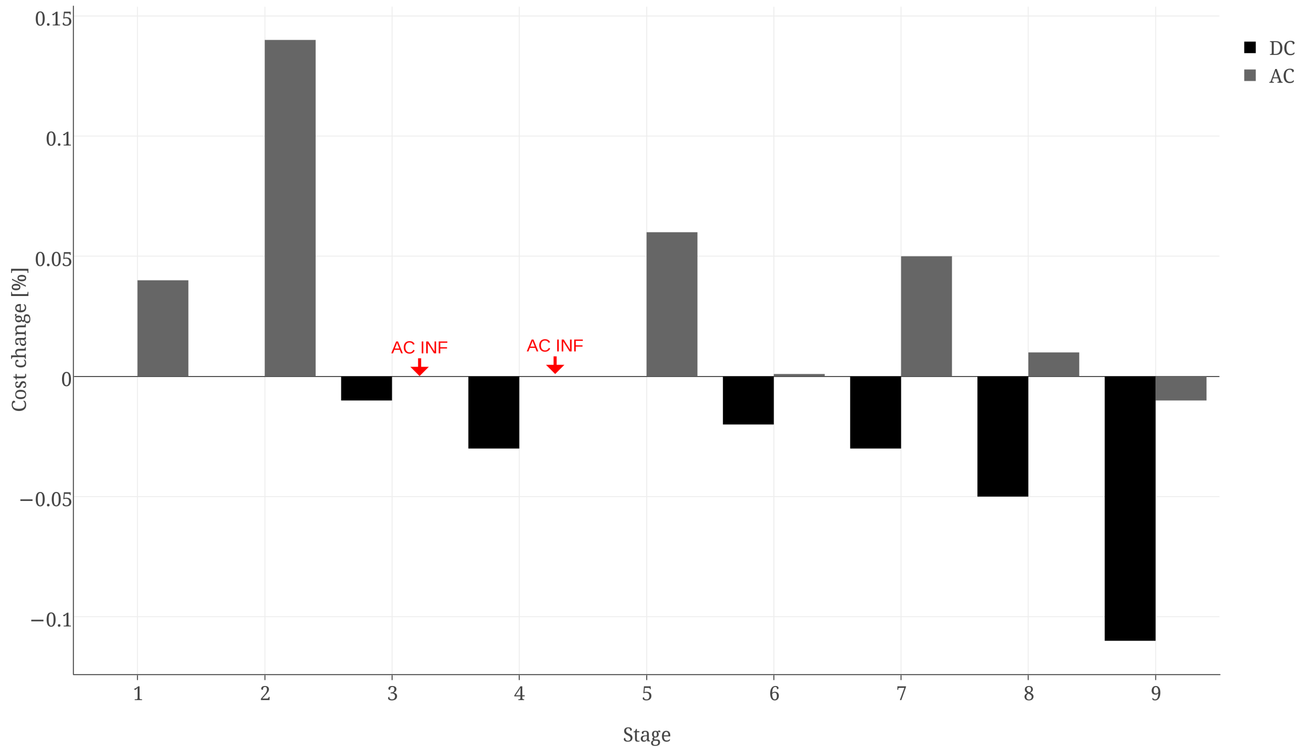
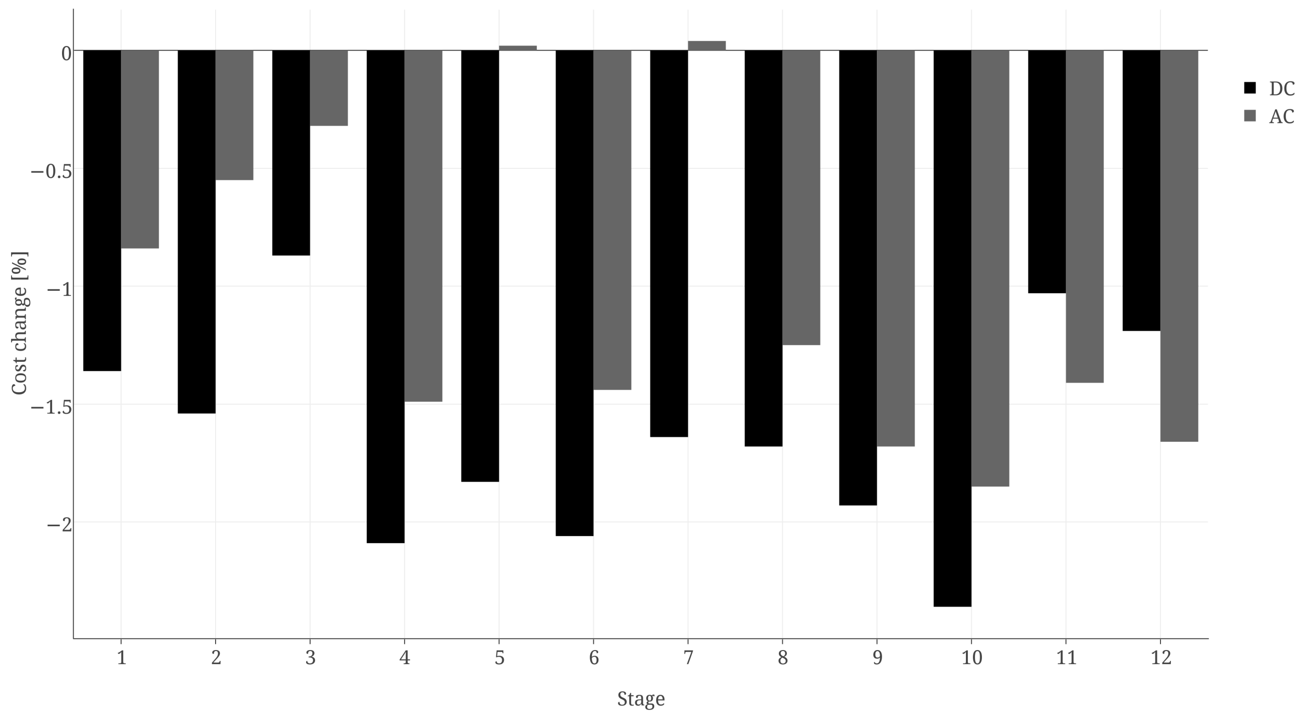


Figure 5.2 shows results from the tests done on the Polish network, where only the results from stage 9 indicate cost reductions in an ACOPF context. As we observed with case118, the cost reductions found with the DC heuristic are overestimated. For load stages 1, 2 and 5, the best switching actions found by the DC heuristic yield close to no cost reductions in the DCOPF, while the same switching actions applied to the ACOPF cause an increase in costs. In stages 3 and 4, the DC heuristic reports cost reductions, but the switching actions are ACOPF infeasible. For the remaining stages 6, 7 and 8, the switching actions result in increased costs when applied to the ACOPF.

5.4.3 The GB network

Figure 5.3: DC switching actions applied to ACOPF – GB network



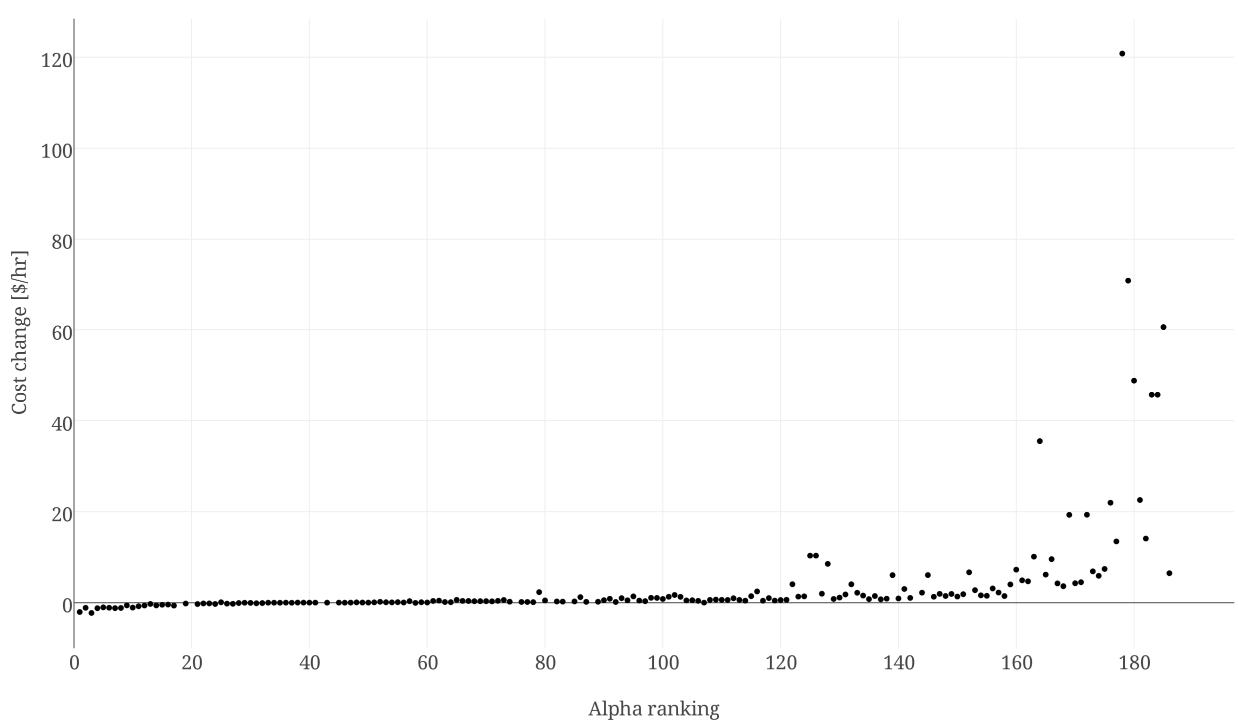
The results for the GB network differ from those of the two other test cases. Only two out of twelve stages return an increase in costs for the ACOPF. For the remaining stages, the switching actions suggested by the DC heuristic also provide cost reductions for the ACOPF. In stages 11 and 12, the cost reductions achieved are larger for the ACOPF than what is estimated by the DC heuristic.

For the Polish network and case118, the DC heuristic does not perform well, and many of the suggested switching actions lead to infeasibility or increased costs when applied to the ACOPF. For the GB network, the DC heuristic performs quite well. Still, the risk of an increase in costs for the ACOPF is present, and the cost reductions presented by the DC heuristic are often larger than what is actually achieved in an ACOPF context. As our primary focus is on quality of results, it is not crucial that computation time stays within real-life practical limits. We consequently choose to proceed with the AC heuristic.

5.5 Performance testing of the AC heuristic α_k ranking

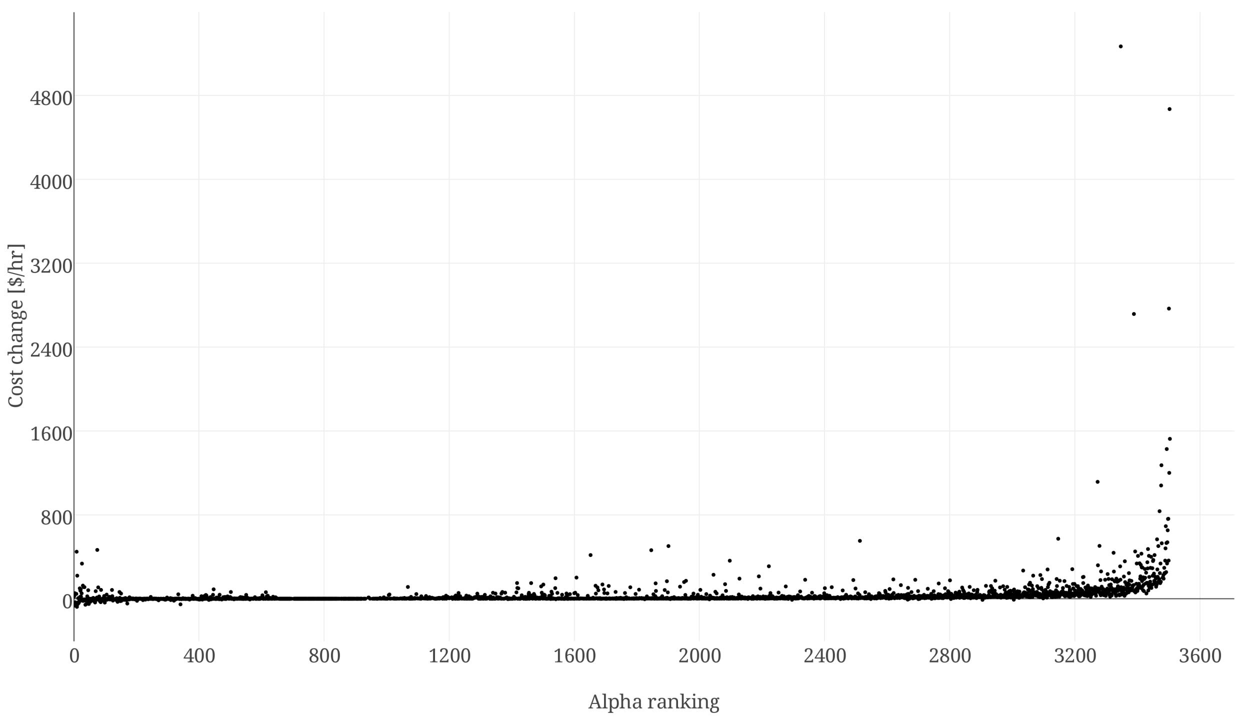
The implementation of the AC heuristic by Francesco Piu has, prior to our work, only been tested on smaller MATPOWER test cases and Blumsack's version of case118. To ensure that the heuristic delivers reasonable results also for larger and modified test cases, we measure its performance against the exact method of line switching. The heurstic will only work well if α_k is a good predictor of the cost reductions actually achieved when line k is put out of service. We measure this by removing one line at the time and computing the changes in cost, so that each line has a cost change associated with it. We compare the results to the ranking of the lines based on α_k. The line with the smallest α_k will be ranked first. The results are displayed in figures 5.4 - 5.6.

Figure 5.4: Cost changes when cutting one line versus α_k ranking of line – case118



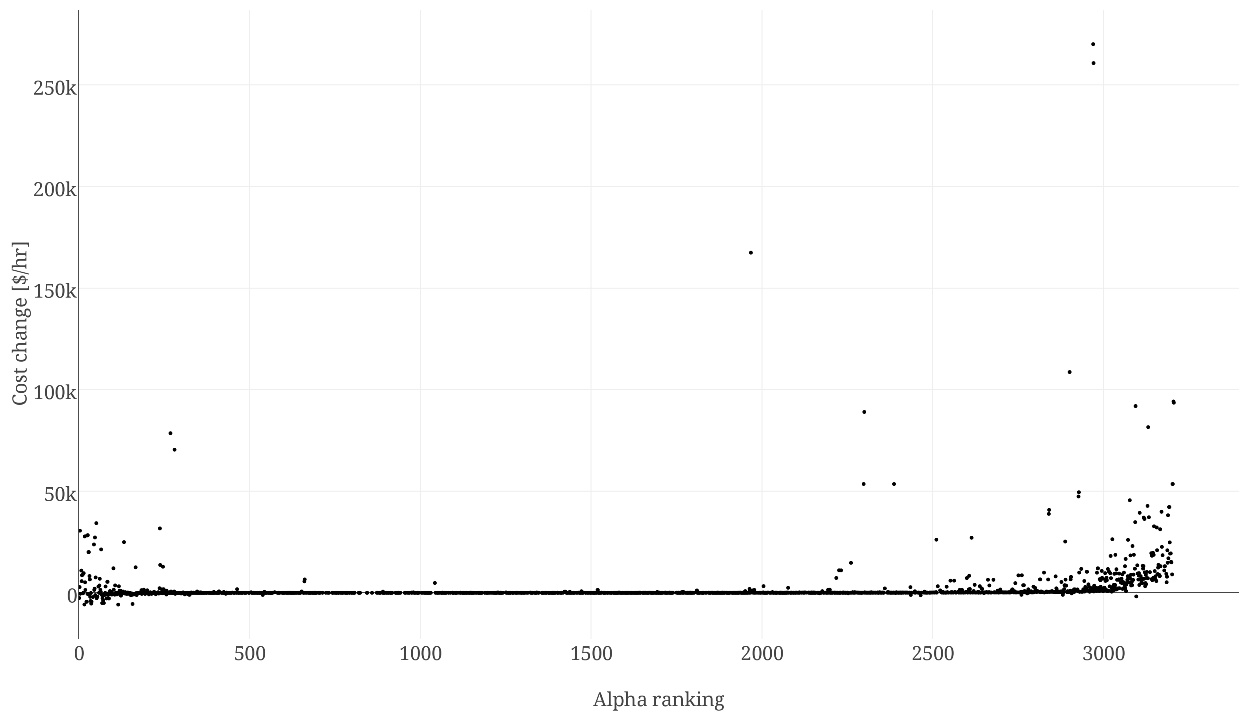
The results for case118 in figure 5.4 show that the lines resulting in cost reductions when cut are positioned to the left with a high α_k ranking, whereas the lines that increase costs when cut are positioned to the right with a low α_k ranking. This implies that α_k is a good predictor, and we can expect the heuristic to perform well.

Figure 5.5: Cost changes when cutting one line versus α_k ranking of line – Polish network



The results for the Polish network in figure 5.5 show the same general trend as case118, the best lines to cut are highly ranked, while the worst lines have a low α_k ranking. However, among the top ranked lines positioned to the left, we also find some lines that result in a cost increase when cut. α_k is still a good predictor, but not as accurate as seen for case118.

Figure 5.6: Cost changes when cutting one line versus α_k ranking of line – GB network



The results from the GB network in figure 5.6 are similar to those from the Polish network. The best lines to cut are found to the left with high α_k rankings, but we find more "bad" lines with a high α_k ranking as well.

For the larger test cases, α_k is less accurate. It manages to rank the lines that will provide the largest cost reductions well, but among these are also lines that increase costs when cut. This affects our decision of the input variables, especially T, the maximum desired line removals. To put it in perspective; if α_k were perfect, we would not need T as we could simply cut the line ranked highest by α_k in each iteration. For case118, α_k is quite accurate, and we can apply a low T as it is only nessecary to check the very top ranked lines to find the best candidate for switching. For the larger test cases we need to apply a higher T in order to obtain a larger scope of lines to test in each iteration.

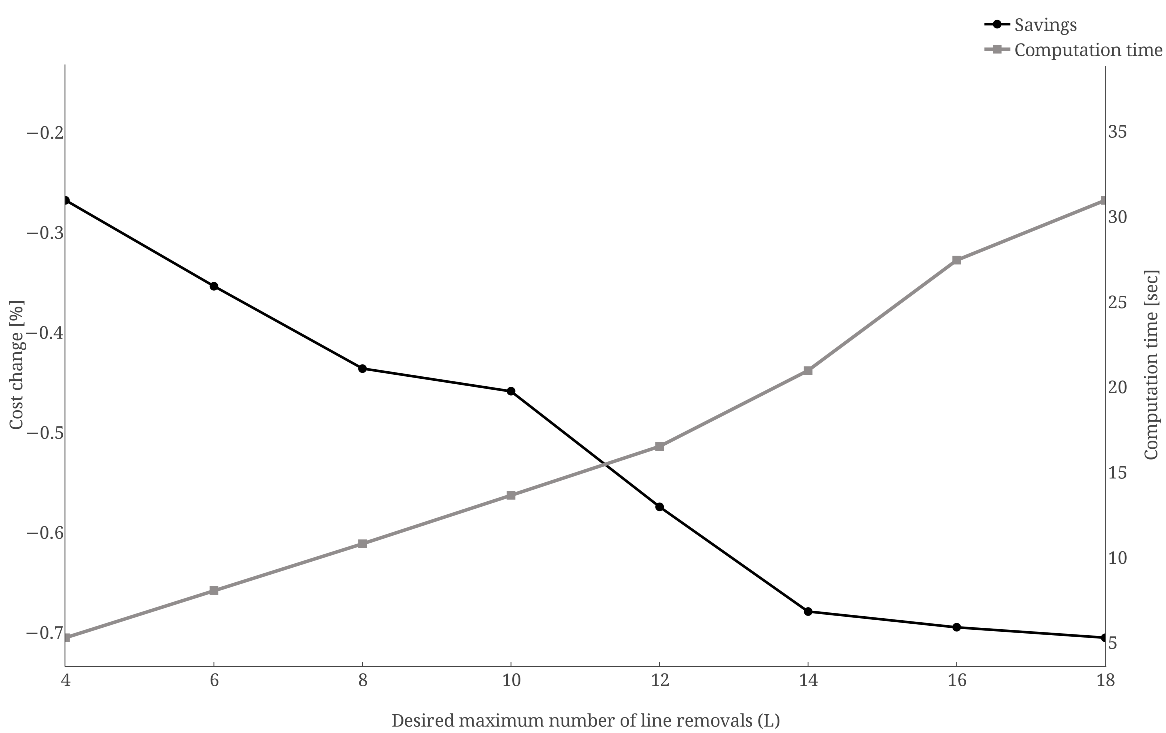
5.6 Choosing input parameters for the heuristic

As the heuristic results and execution time vary depending on the chosen input parameters, the choices of L, m and T must be made considering computation time, while ensuring that the true cost reduction potential of each test case is revealed. We have run the AC heuristic with different input values for the three chosen test cases. Based on the results, we have decided which L, m and T to apply later on.

5.6.1 Case118

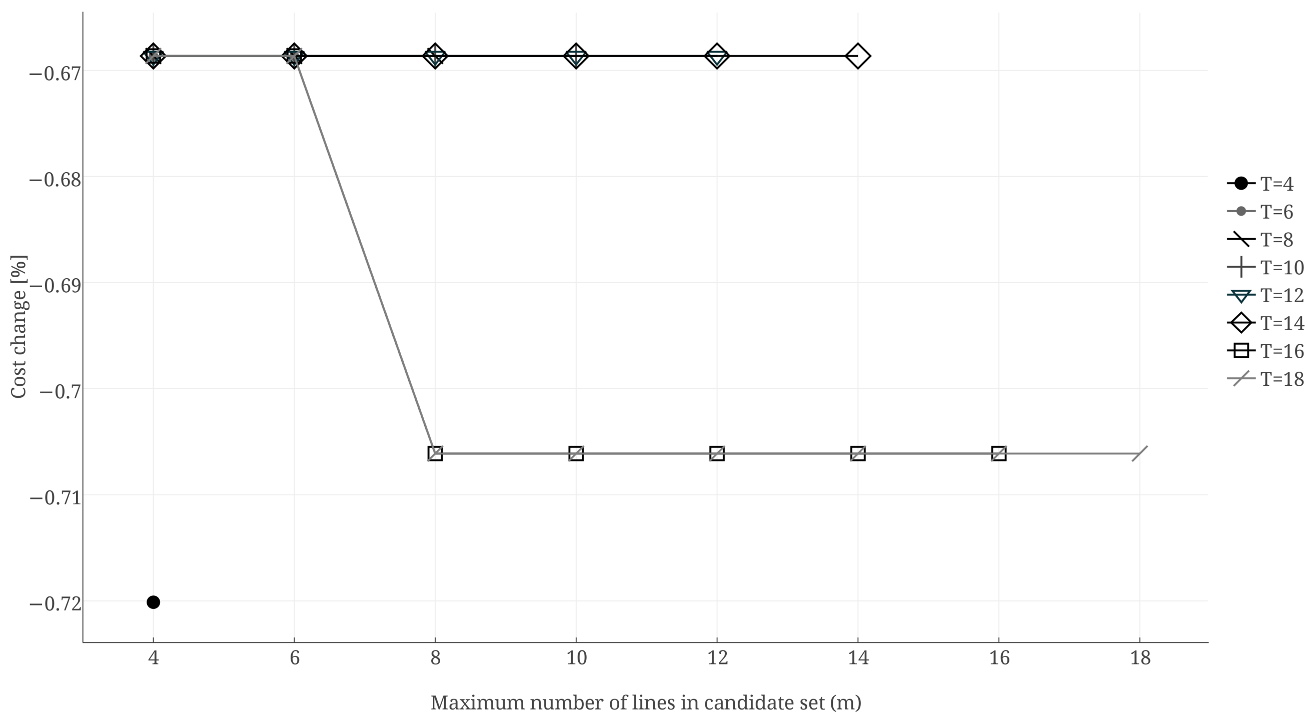
Due to the small size of case118 and the accuracy of α_k, the computation time for the heuristic is short and we are able to run many tests with different input values. We tested combinatorially for $L = 4-18$, $m = 4-18$ and for $T = 4-18$, for even numbers only and with the restriction $T \geq m$. This resulted in 288 different combinations and heuristic tests. We first have a look at the average cost reduction for increasing desired line removals, L, versus computation time, displayed in figure 5.7.

Figure 5.7: Choosing number of lines to cut (L) – case118



We observe that the computation time increases steadily, but is in general short. The cost reduction line flattens out at $L = 14$. After this, searching for additional lines to cut will return marginal cost reductions or infeasibility while increasing the computation time. We therefore choose $L = 14$. We then have a look at the values of m and T for $L = 14$, displayed in Figure 5.8.

Figure 5.8: Choosing values for m and T – case118



A general observation from the 228 heuristic tests performed on case118 is that increasing values of T and m, for each L, result in increasing or identical cost reductions. From figure 5.8, we observe that for $T = 6-14$ the cost reductions stay the same for increasing values of m. For $T = 16-18$, cost reductions increase from $m = 6$ to $m = 8$. However, the value of T resulting in the highest cost reduction is $T = 4$. This contradicts the general observations, but reflects the random aspect of the heuristic; a small T forces us to choose a different line switch than with a larger T. This choice might give lower cost reductions now, but the new topology can reveal higher possible cost reductions in later iterations. As a low value of T also reflects a shorter computation time, we choose $T = 4$ and $m = 4$ when applying the heuristic to case118 in later studies.

5.6.2 The GB and Polish networks

When choosing input values for the two larger test cases, we have chosen a slightly different approach than for case118. Due to the size of the GB and Polish networks, and the lower accuracy of α_k, we cannot conduct as many tests of the heuristic, considering the computation time. To get an impression of how large L should be, we have run the heuristic once for each test case with high values of L, m and T [50, 20, 70], with a computation time of 192 min for the GB network and 242 min for the Polish network.

Figure 5.9: Number of lines cut versus cost reductions – Polish network

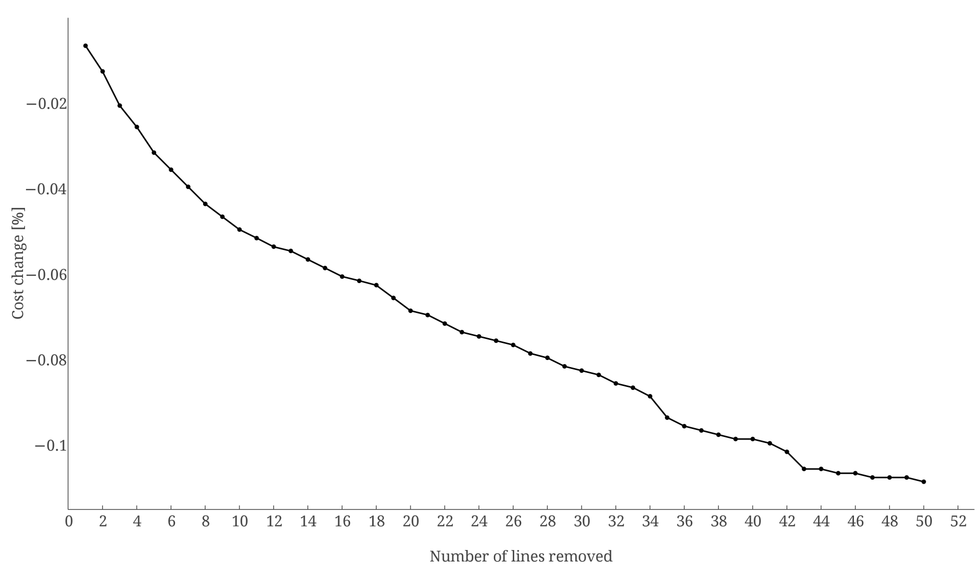
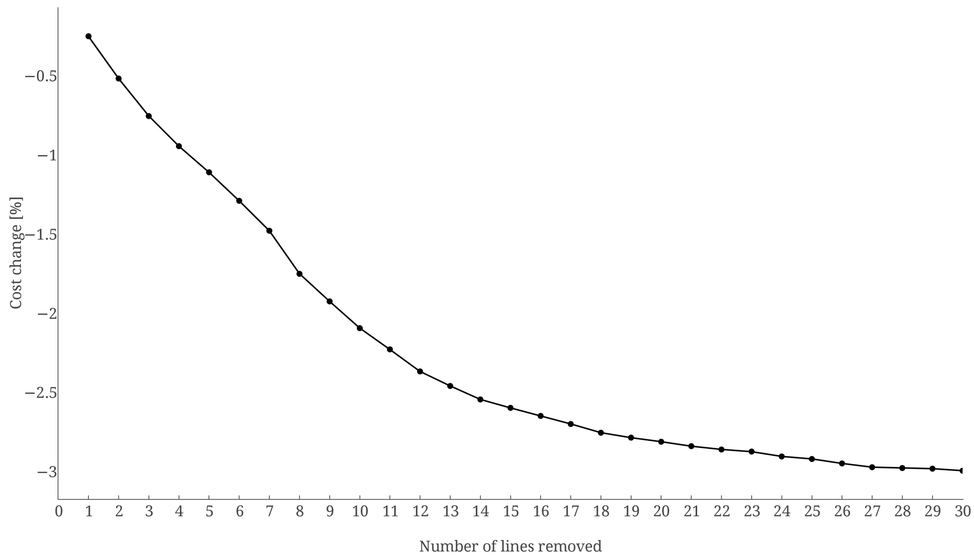
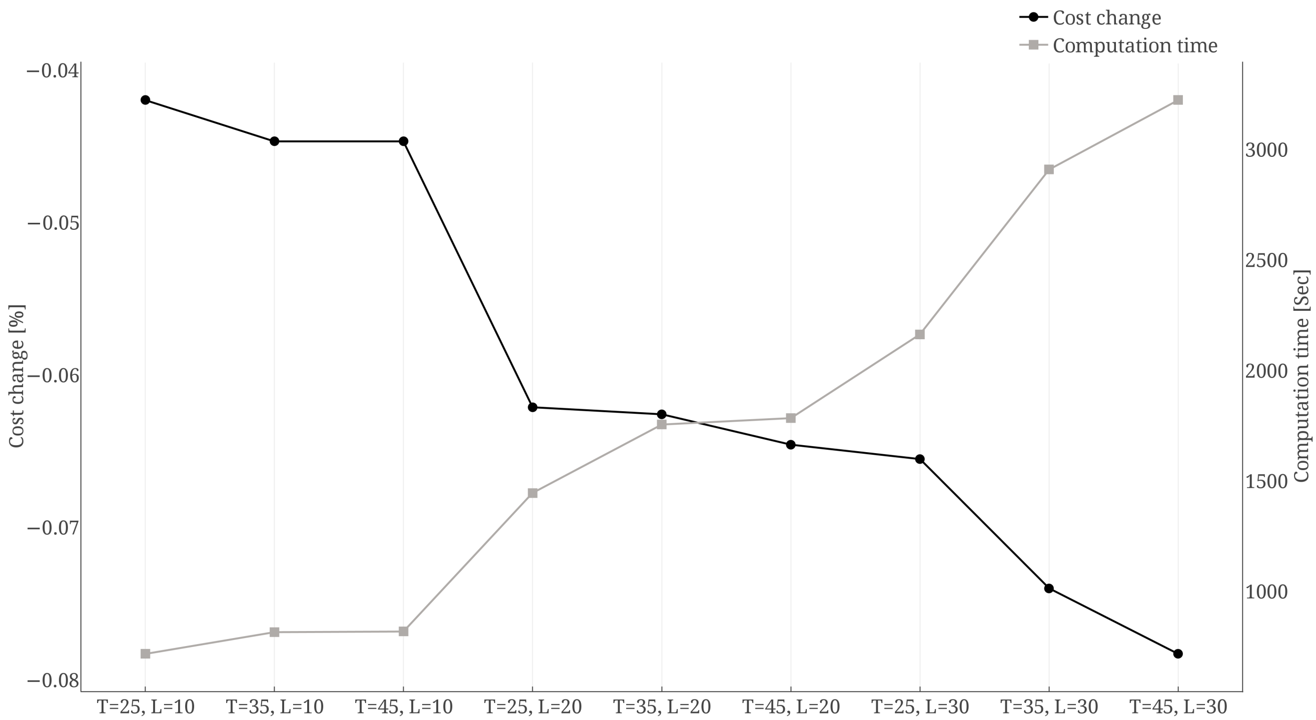


Figure 5.10: Number of lines cut versus cost reductions – GB network



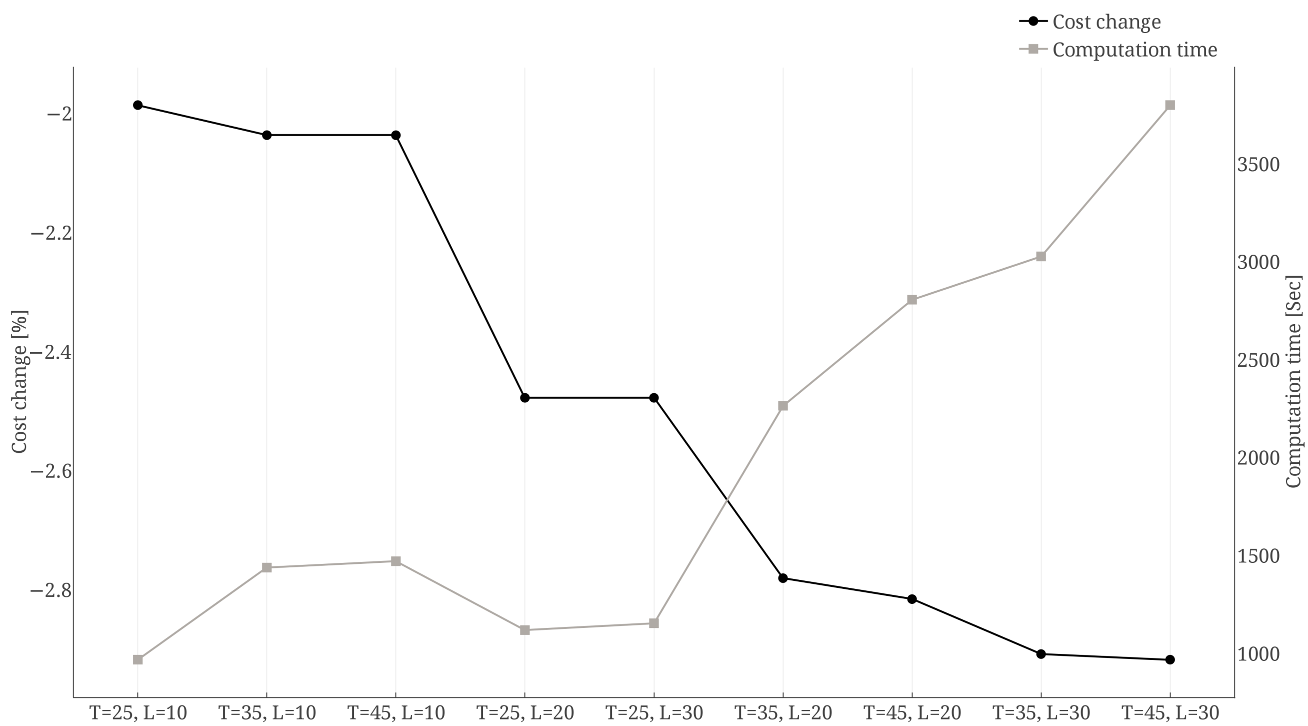
For both test cases, the first lines to be cut contribute to the largest share of the cost reductions. The additional value of switching off a line diminishes as the number of line switches increases, which is in accordance with findings from Pichler et al. (2014) and Fuller et al. (2012). Based on these results, we choose to evaluate the heuristic on the two test cases for L =10, 20 and 30. When studying the behaviour of the candidate set, we observe that it is not necessary to use a high m. Several of the T lines evaluated by the heuristic do not contribute to cost reductions, and will therefore not be stored in the candidate set, in accordance with the less accurate α_k ranking. m can therefore be set to a lower value without limiting the chance of finding good switching candidates. We set $m = 12$ for all further evaluations of the heuristic on the two large test cases. Another consequence of the less accurate α_k ranking is the need for a larger T, which allows us to evaluate a larger scope of switching candidates. We choose to evaluate the heuristic on the two test cases for T =25, 35 and 45. Hence we have nine combinations of input values T and L, with results presented in figures 5.11 and 5.12.

Figure 5.11: Cost reductions versus computation time, m=12 – Polish network



For the Polish network, we observe that both cost reduction and computation time have a flatter area in the middle of the graphs before they increase again. We choose the combination $T = 45$, $L = 20$. Although it is possible to achieve higher cost reductions, it comes with a cost of a rapidly increasing computation time.

Figure 5.12: Cost reduction versus computation time, m=12 – GB network



For the GB network we observe that the highest values of T and L, $T = 45$ and $L = 30$, yield the highest cost reductions, but at a very high computation time. We therefore choose $T = 45$, $L = 20$, which yields cost reductions close to this, but at a shorter computation time.

Table 5.1 presents the chosen input values of the AC heuristic for our following studies.

Name	L	m	T
Case118	14	4	4
Polish network	20	12	45
GB network	20	12	45

Table 5.1: Final input values for AC heuristic

Chapter 6

Generation of load scenarios

When solving the OTS problem, we look at a snapshot of a network at a given point in time with fixed loads at each bus. However, the total load in a network is subject to continuous change throughout the day, and is also season dependent. After solving the OTS with the heuristic, we get a set of lines that we choose to switch off. Many questions arise in relation to this. For how long can these switches be economically beneficial for the system operator? Is there any risk involved with performing these switches? Can the system operator be sure that the system can still supply each load bus after performing these switches?

In order to explore these questions, we evaluate the network subject to line switches under a set of different load scenarios, as system operators continuously face uncertainty through the randomness of demand or load levels (Ramos et al., 2006). Up to this point, this load uncertainty has either been ignored or slightly touched upon in OTS literature. The most commonly applied method is to create three load scenarios, where the total load of the network is either reduced or increased by a given percentage, normally ranging from 80–120%. We do see a need, however, to explore demand uncertainty further. As the randomness of electric consumption is characterized by its uncontrollability and its dependence on variables outside the electric network (Gómez-Expósito et al., 2008), we wish to generate load scenarios where the load at each load bus changes with an element of randomness. Once these scenarios are generated, we can explore how the system behaves and be more fit to answer the questions above.

This chapter will present the statistical method we use to simulate load scenarios, and a presentation of the data selection we have used to generate these scenarios.

6.1 Changing network loads

To change the load at each bus we construct a function that generates dependent random numbers from a log-normal distribution, which are multiplied with the original load at each load bus, referred to as $P^{d,0}$. Parameters and MATLAB functions used to construct this function are presented below:

σ	Standard deviation of the normal distribution
μ	Mean of the normal distribution
n	Number of buses in the test case
$ones(n)$	Returns an n-by-n matrix of ones.
$eye(n)$	Returns an n-by-n identity matrix with ones on the main diagonal and zeros elsewhere
c	Assumed correlation between bus loads
ρ	Correlation matrix between bus loads, expressed as $(c * ones(n) + (1-c) * eye(n))$
τ	Average growth in load as found from historical data
$mvrnd(\mu,\rho)$	Returns a 1-by-n matrix of random vectors chosen from a multivariate normal distribution with mean μ and correlation ρ

We start with an original load matrix, $P^{d,0}$=$[P_1^d,\ldots,P_n^d]$, where n represents the number of buses in the network. To change this, we generate a random matrix Y drawn from a normal distribution with mean μ and standard deviation σ, as shown in equation (6.1).

$$Y = \mu + \sigma \cdot mvrnd(\mu, \rho) - \frac{\sigma^2}{2} \tag{6.1}$$

Y returns a large vector of dependent random variables. The term $-\frac{\sigma^2}{2}$ represents the bias corrector (Cordeiro and McCullagh, 1991). There is a problem with equation (6.1), however, as it could possibly return negative values, that, when multiplied with the original load matrix, results in negative loads. To adjust for this, we use a log-normal distribution, as it can only take positive real values. Given the normally distributed random matrix, Y, a log-normally distributed random matrix can be calculated through equation (6.2):

$$X = e^Y = e^{(\mu + \sigma \cdot mvrnd(\mu, \rho) - \frac{\sigma^2}{2})} \tag{6.2}$$

To change the loads, we multiply the original load matrix with the average growth in load we want to explore, $(1 + \tau)$ and with the matrix X for given values of c, σ and μ. In this way, the load at each load bus will be changed dependently on changes in loads at other buses while varying with σ around the load level determined by $(1 + \tau)$.

6.2 Choosing input parameters

To simulate realistic load scenarios, we started with the British network. Historical demand data of the entire British network was found at National Grid (2014). It provides a complete set of historic demand data since the beginning of British Electricity Trading and Transmission Arrangements in April 2005. From this, we can compute the average growth in demand at each hourly interval (τ_t) and the corresponding standard deviation (σ_t). We have also assumed both case118 and the Polish network to have the same load behavior as Great Britain, as detailed demand data was not found for Poland or the U.S. area that case118 originally was based on.

When changing the loads of an entire network, an important question that arises is whether or not these loads are correlated. Temperature changes, time of day, week or month are factors that influence demand. Often, loads of the same type, e.g. residential or industrial consumers, show fairly similar behavior (Baran and McDermott, 2009), and will react similarly to outside factors. We have limited our load change method by assuming that the consumers in the network are of a residential type, where they will have a fairly similar demand behavior. When this assumption is made, the correlation will be high, but still have small variations (Hernández et al., 2012), and the correlation c is set to 95%. Further, the mean of the normal distribution, μ, is set to 0%, as we wish to examine the effects of varying the load around the average growth in load, $(1+\tau)$.

In order to look at how transmission switching will affect the total generator costs when subject to demand changes, we explore OTS in stages of demand changes within the feasibility area of each test case. The feasibility area of each test case is given in figure 6.1–6.3.

Figure 6.1: Feasibility area of case118

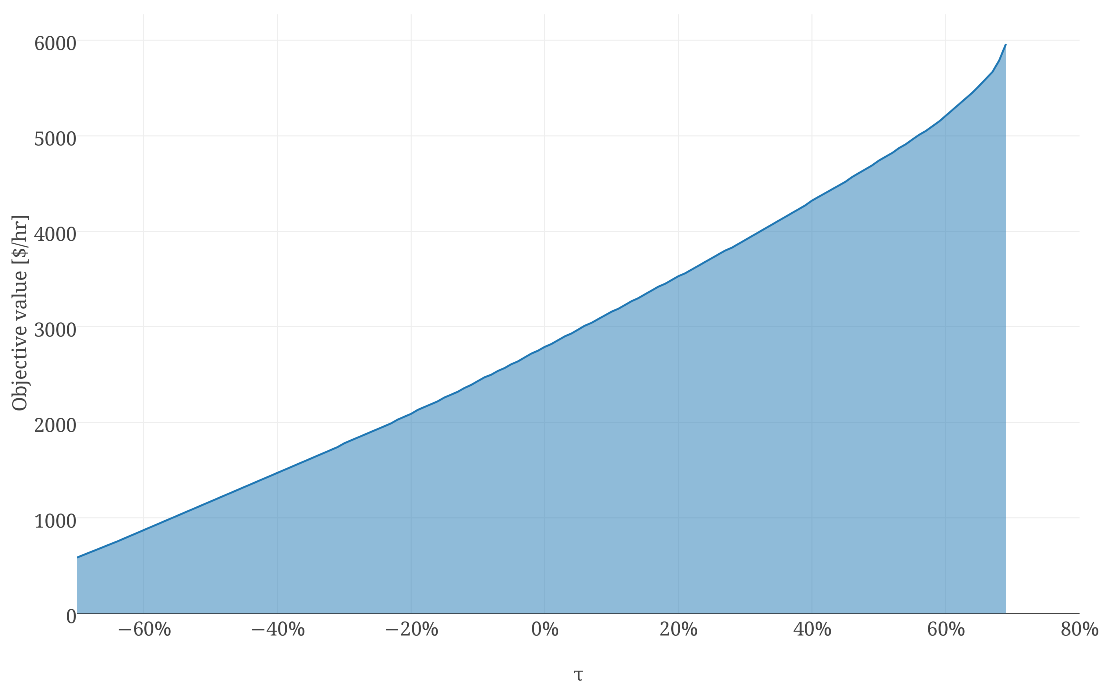


Figure 6.2: Feasibility area of the Polish network

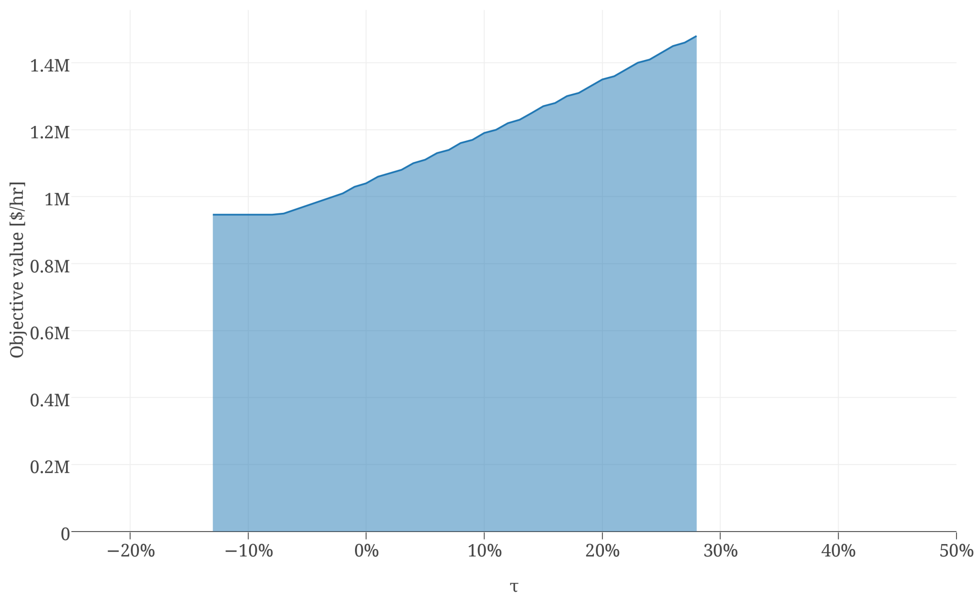
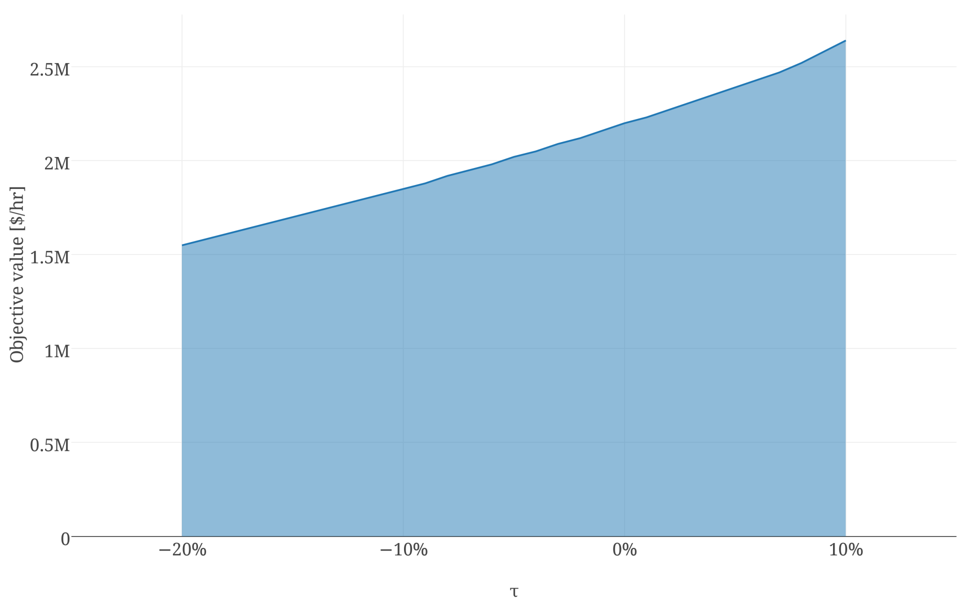


Figure 6.3: Feasibility area of the GB network



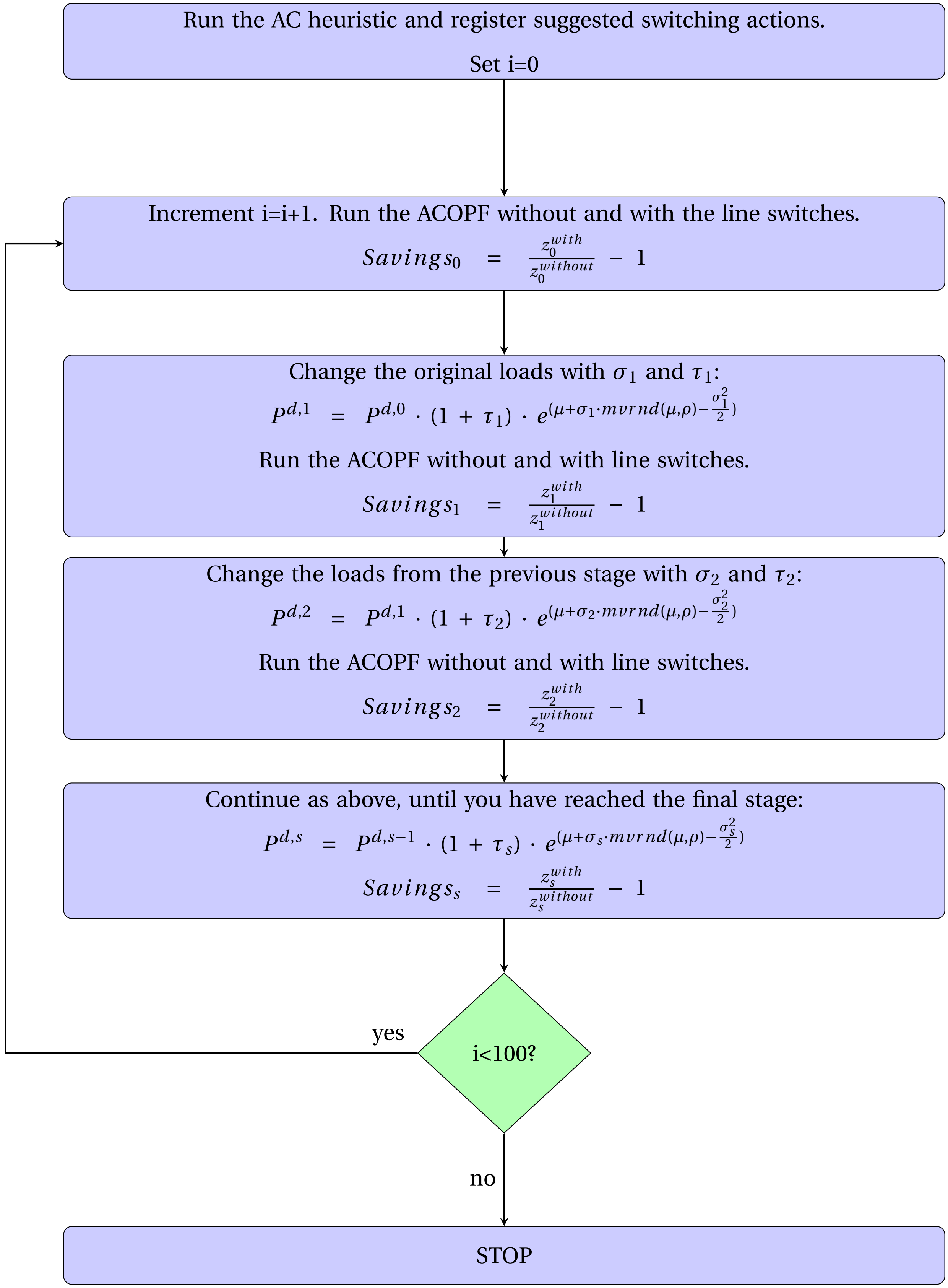
The feasibility area illustrates for which $(1+\tau)$ one can multiply the original loads with, and still have that the problem converges. What we observe is that the Polish and GB network have a small feasibility area, where the Polish network can handle a total load change between 87% and 127% , while the GB network can handle a total load change of 80–110% of the original load. In practice, this means that these networks are designed to study a point in time, and not to handle large load changes. From the data presented by National Grid (2014), we observe an increase of 170% from the lowest to the highest load during a day. Thus, only case118 has the sufficient feasibility region to handle the load change occuring throughout a day.

To be able to study the two large test cases for the same load variations, we would have to change the test case data to such an extent that it would not be prudent to do so without access to real network data. However, we will explore how the two large networks behave with a load change *up to* their convergence limits. We define stages where at each stage the load is increased with $\tau = 2\%$. From the National Grid (2014) data set, we calculate an average σ of 2% from hour to hour. We also define stages with decreasing loads of $\tau = -2\%$ for each stage, with the same standard deviation of $\sigma = 2\%$.

If the original load matrix in the test case is denoted $P^{d,0}$, then the load at stage 1 will be:

$$P^{d,1} = P^{d,0} \cdot (1+\tau) \cdot e^{(\mu + \sigma \cdot mvrnd(\mu,\rho) - \frac{\sigma^2}{2})} \tag{6.3}$$

We change the original loads at stage 1, and compare the objective value from the ACOPF with and without line switching. In the second stage, we change the loads from stage 1 and compare the new objective values with and without line switches. We do the same for the third stage, and continue until we have reached the network's feasibility limit. The load change procedure is iterated 100 times for each stage. Due to the random vector mvrnd(), the load scenario will be different for each iteration. We get an output of the cost reductions each time the load is changed in the 100 iterations. These will be presented in boxplots to investigate whether or not transmission switching will still be economically beneficial with varying loads. The results are presented in Chapter 7. A pseudocode of the procedure is given on the next page, where $z_s^{without}$ and z_s^{with} denote the objective function value from running the ACOPF without and with transmission switches at stage s.


Run the AC heuristic and register suggested switching actions.
Set i=0
Increment i=i+1. Run the ACOPF without and with the line switches.
$Savings_0 = \frac{z_0^{with}}{z_0^{without}} - 1$
Change the original loads with σ_1 and τ_1:
$P^{d,1} = P^{d,0} \cdot (1 + \tau_1) \cdot e^{(\mu + \sigma_1 \cdot mvrnd(\mu,\rho) - \frac{\sigma_1^2}{2})}$
Run the ACOPF without and with line switches.
$Savings_1 = \frac{z_1^{with}}{z_1^{without}} - 1$
Change the loads from the previous stage with σ_2 and τ_2:
$P^{d,2} = P^{d,1} \cdot (1 + \tau_2) \cdot e^{(\mu + \sigma_2 \cdot mvrnd(\mu,\rho) - \frac{\sigma_2^2}{2})}$
Run the ACOPF without and with line switches.
$Savings_2 = \frac{z_2^{with}}{z_2^{without}} - 1$
Continue as above, until you have reached the final stage:
$P^{d,s} = P^{d,s-1} \cdot (1 + \tau_s) \cdot e^{(\mu + \sigma_s \cdot mvrnd(\mu,\rho) - \frac{\sigma_s^2}{2})}$
$Savings_s = \frac{z_s^{with}}{z_s^{without}} - 1$
i<100?
yes
no
STOP

For case118, we have a much larger feasibility area which makes it possible to subject the test case to more realistic load scenarios. We follow the same procedure as described in the flow chart, and set the number of stages to 24. The first stage represents the load at 07:00 in the morning, the second stage at 08:00 in the morning an so on. We employ the hourly values for τ and σ that we have calculated from the National Grid (2014) database, when looking at Monday–Friday during the winter months. We make line switches based on the output of the heuristic at the original loads, which we assumed to be at 06:00 in the morning where demand is at its lowest. Following this, the loads at stage 1, will be:

$$L_{07:00} = L_{06:00} \cdot (1 + \tau_{07:00} \cdot e^{(\mu + \sigma_{07:00} \cdot mvrnd(\mu,\rho) - \frac{\sigma^2_{07:00}}{2}))} \tag{6.4}$$

Where $\tau_{07:00}$ is the average of the percentwise growth in load from 06:00–07:00 from the GB data set, and $\sigma_{07:00}$ is the standard deviation of the same data selection. At 08:00, in a similar way, the new loads will be:

$$L_{08:00} = L_{07:00} \cdot (1 + \tau_{08:00} \cdot e^{(\mu + \sigma_{08:00} \cdot mvrnd(\mu,\rho) - \frac{\sigma^2_{08:00}}{2}))} \tag{6.5}$$

The loads will not be reset for the next 24 hours, but will simply follow the statistical properties from each hourly interval. Once the loads have been changed for all 24 hours, the procedure is repeated to a total of 100 iterations.

Part III

Results

This part will present our most important findings, which will be discussed and compared with previous work within the OTS research community. Our objectives will be summarized, before we reach our final conclusions.

Chapter 7

Transmission switching and demand uncertainty

7.1 Overview

The heuristic from Chapter 5 is run on the modified test cases described in Chapter 4. The output of the heuristic is as follows:

Test case	Heuristic input parameters (L, m, T)	Number of lines switched off	Cost reductions [%]
Case118	14, 4, 4	13	-0.72%
Polish network	20, 12, 45	20	-0.065%
GB network	20, 12, 45	20	-2.8%

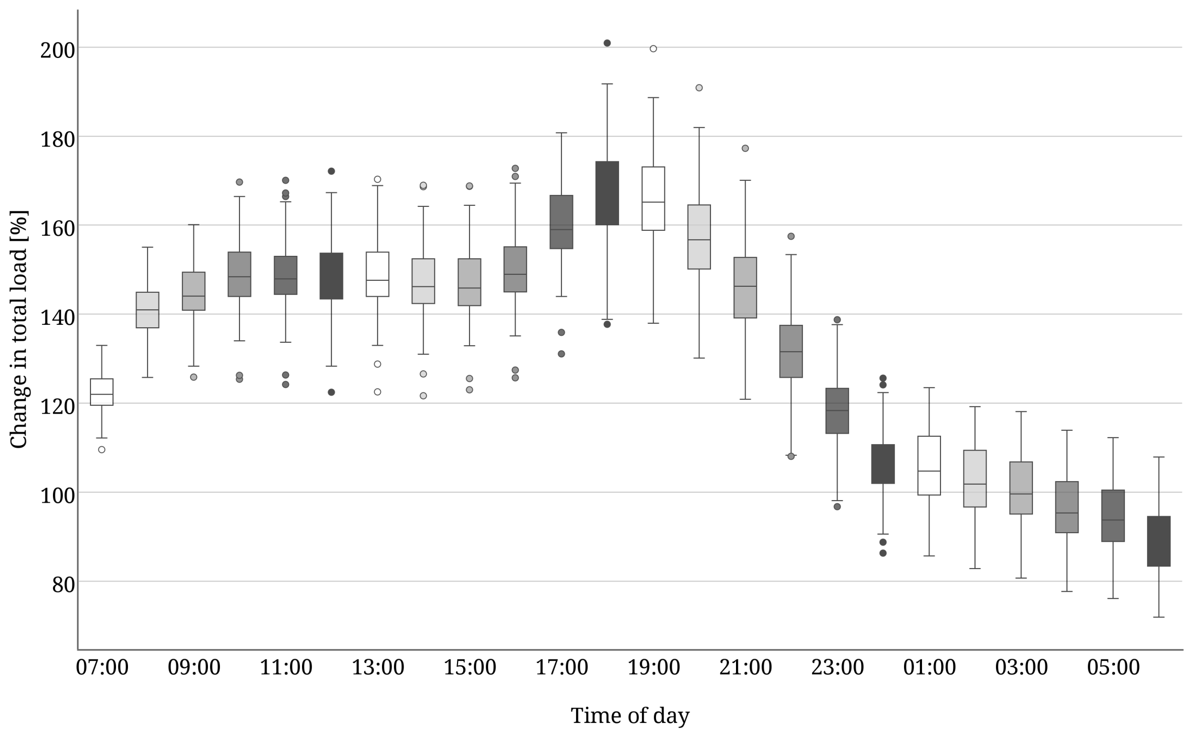
Table 7.1: Results from the OTS heuristic

These heuristic ouputs are based on the modified test cases with the original loads. The lines chosen here, are the lines that will be switched on and off when changing the loads of the network. What we observe is that even though the heuristic always returns a set of line switches that yield cost reductions for the system operator, these reductions are still quite small, especially for the larger test cases. The cost reductions from case118 are far below than what is reported in the OTS literature (6–30%).

7.2 Case118

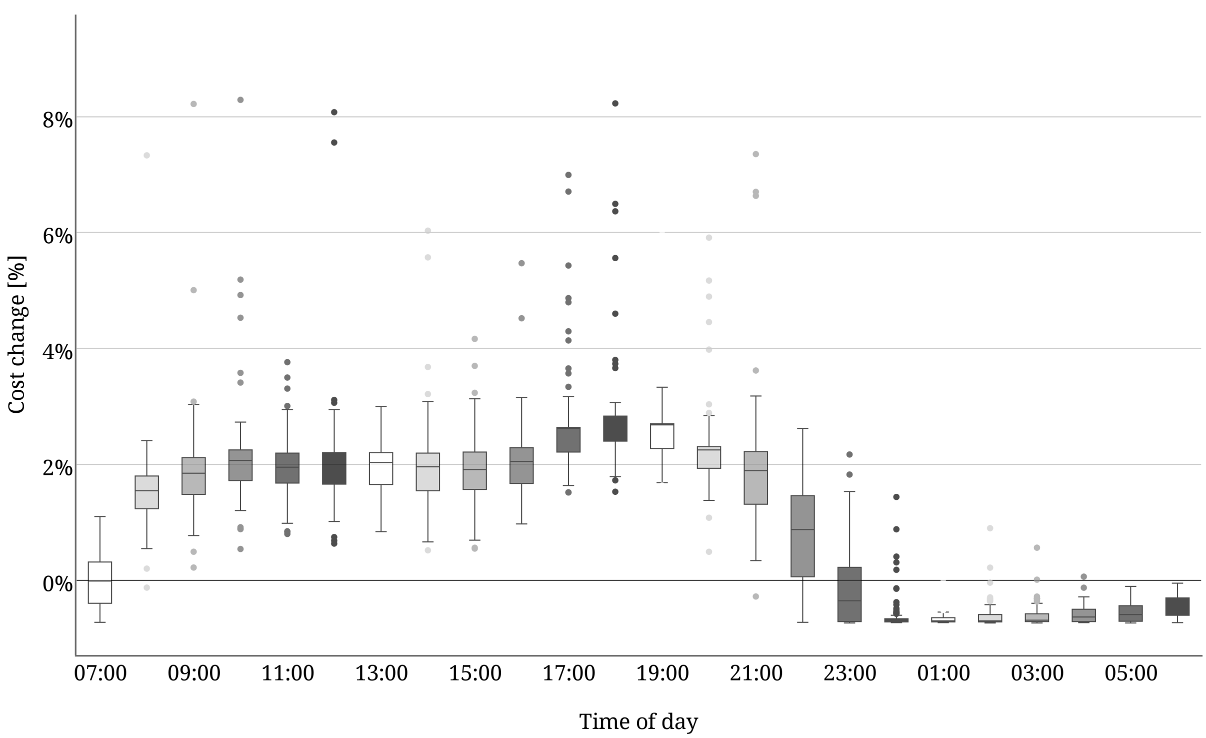
We create scenarios to simulate load changes throughout a day, as explained in Section 6. There will be 100 scenarios at each stage. The distribution of these scenarios is given in figure 7.1, where the y-axis represents the percentwise total change in load for each hour:

Figure 7.1: Total change in load from 06:00



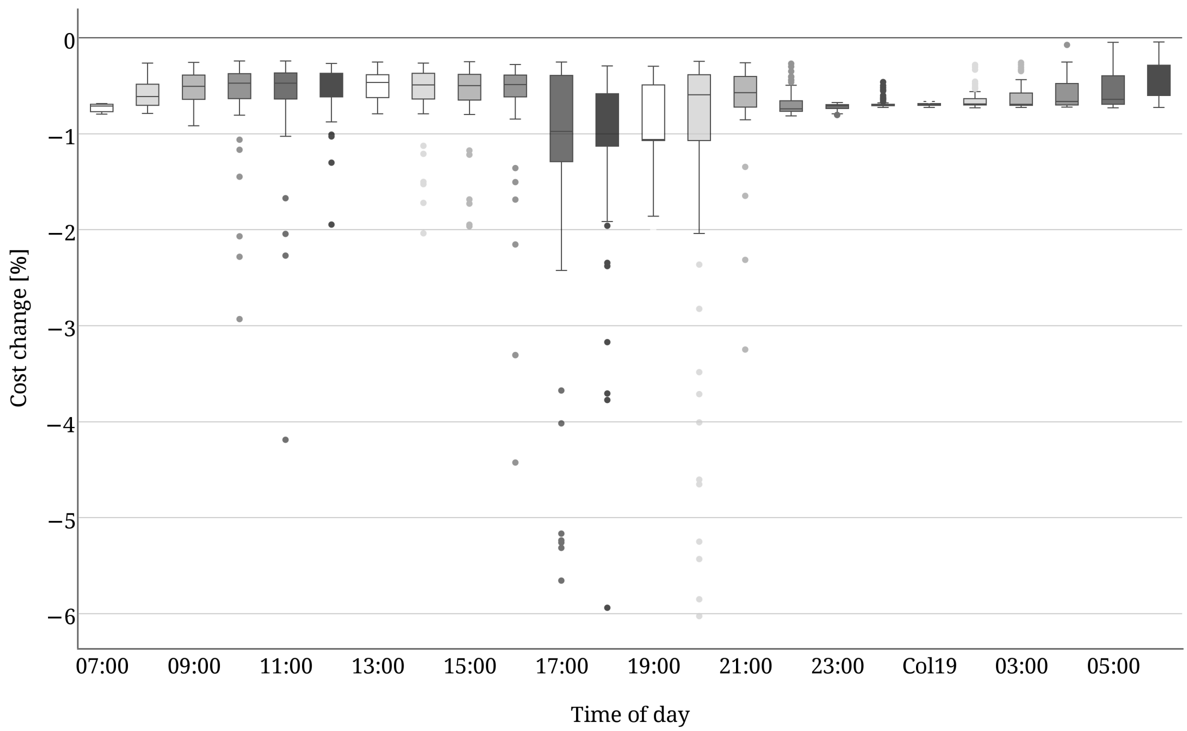
From the output of the heuristic, we get a set of lines that are switched off. With the original loads, these switches yield a cost reduction of 0.720%. These loads are changed from hour to hour throughout the day, and the corresponding cost reductions are shown in Figure 7.2. We can quickly see that the line switches suggested by the heuristic will not be good switching decisions throughout the day, as the line switches will increase costs for all hours with increasing load. When the load decreases again and returns to the original load level, as is the case for the network during the night, we can expect cost reductions around 0.5 to 0.7%. Another interesting observation is made by looking at what cost reductions we can expect when the network is considered to be congested. As was illustrated in figure 6.1, the OPF problem of the network will not converge with values of τ exceeding 70%. Figure 7.1 demonstrates that we will have values of τ around this feasibility limit at the times 09:00–13:00 and again at 16:00–21:00. This means that for these hours, many of the iterations do not converge, and of those who do converge, some will represent scenarios where parts or a majority of the network is congested, i.e. pushed to the maximum of its physical limits. For these times, we can see on the outlayers that the increase in costs due to line switching is as much as 8%. We have assumed that the original load is at 06:00 in the morning, meaning that the load is at its lowest of the day and will be subject to large increases. If the original load is assumed to be at the peak demand, at 19:00 in the night, loads will decrease throughout the next 24 hours. In this case, we have found that cost reductions are in general stable for each hour, ranging from 0.3 to 0.7%.

Figure 7.2: Case118 subjected to demand changes throughout 24 hours with 13 lines switched off



Initially, the heuristic suggested that 13 lines should be switched off, which was executed and presented in Figure 7.2. We already know that the heuristic simply looks at one given point in time, and cannot give any indications on whether or not these switches will be beneficial throughout the day. To explore if there is a set of line switches that would return more stable results over time, we simply switched the last line suggested by the heuristic on again. Figure 7.3 shows the cost reductions at each hour when the 12 first lines suggested by the heuristic are switched off. The difference between whether or not the 13th line is included makes an enormous difference. When the 13th line is not included, we see that the line switches will provide cost reductions for the system operator throughout the entire day, contrary to when all 13 lines are switched off. The cost reductions are in general stable around the original cost reductions of 0.7% or slightly above. The risk that the cost change will approach 0% is increasing slightly when the loads are decreasing, as can be seen in the time interval 01:00–05:00 in the night. In the time periods during the day when the network is partly congested, we observe cost reductions down to 6%. This is in stark contrast to when all 13 lines were switched off, where the congested scenarios increased costs by as much as 8%.

Figure 7.3: Case118 subjected to demand changes throughout 24 hours with 12 lines switched off



7.3 The Polish network

Figure 7.4 illustrates how the loads are increased in stages for the Polish network. We see that the total load change is up to 140% in the final stages, which exceeds the feasibility limit of 128% (Figure 6.2). When a set of line switches are made, we change the loads with a τ of 2% and a σ of 2% at each stage.

Figure 7.4: Total load change per stage ($\tau = 2\%, \sigma = 2\%$)

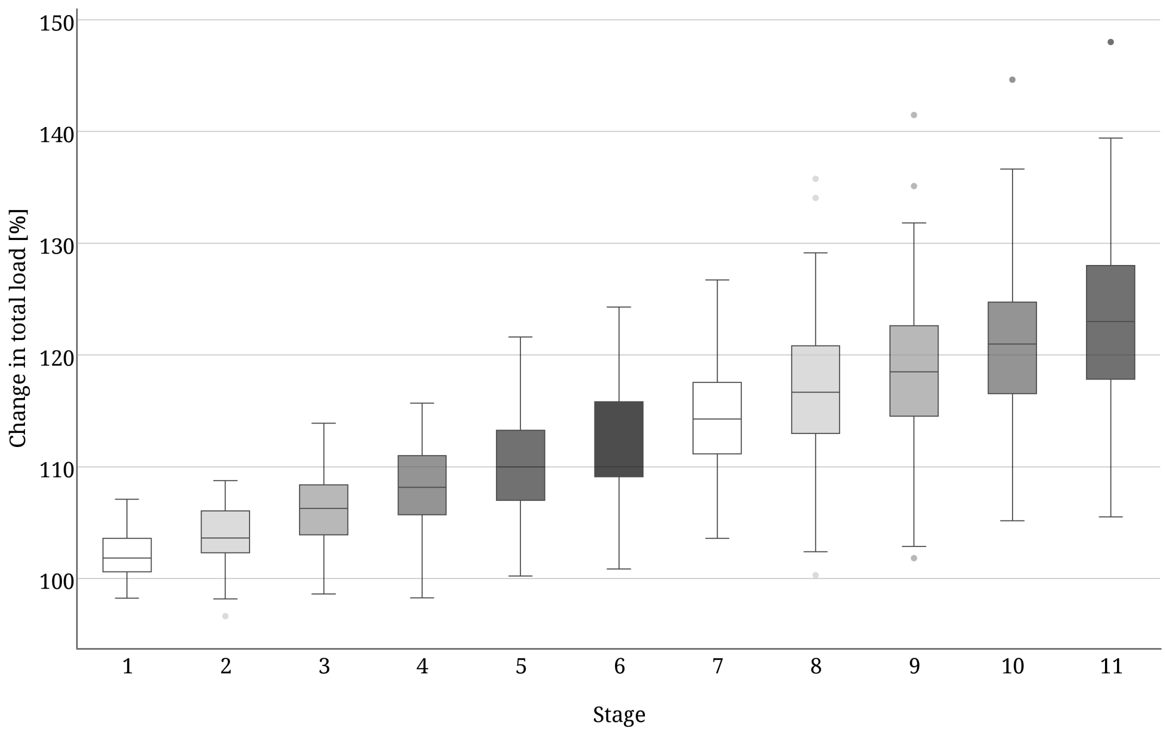
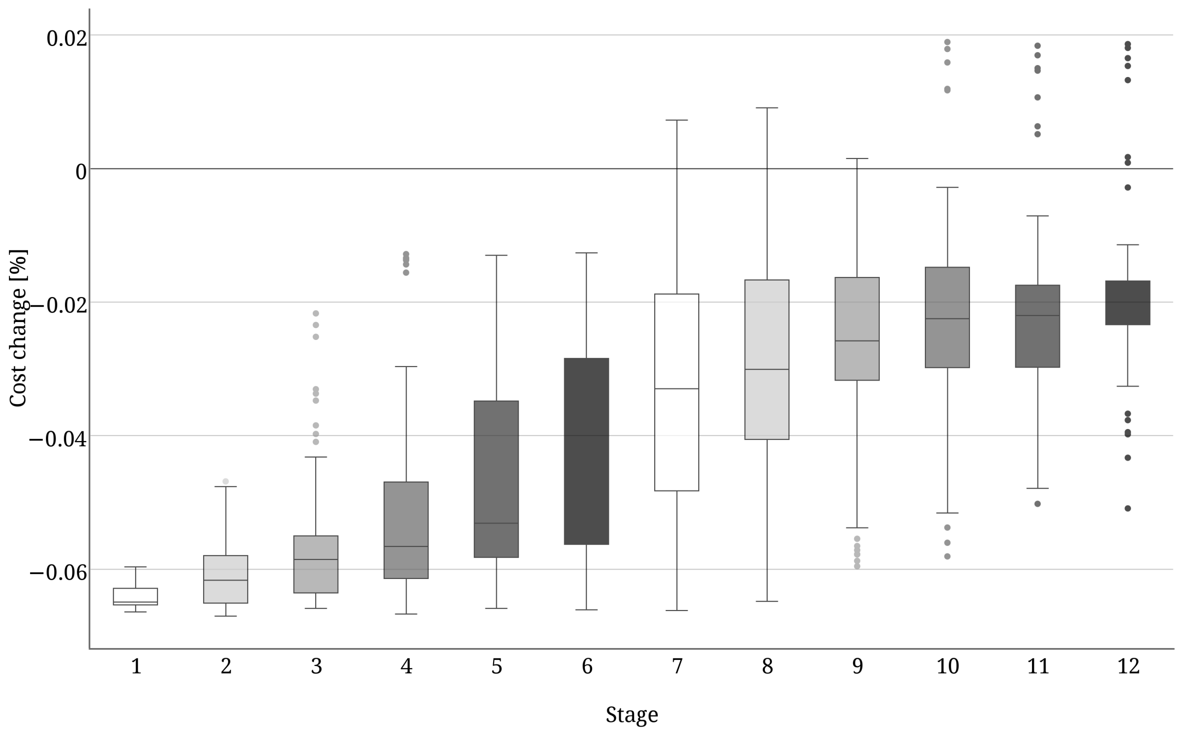


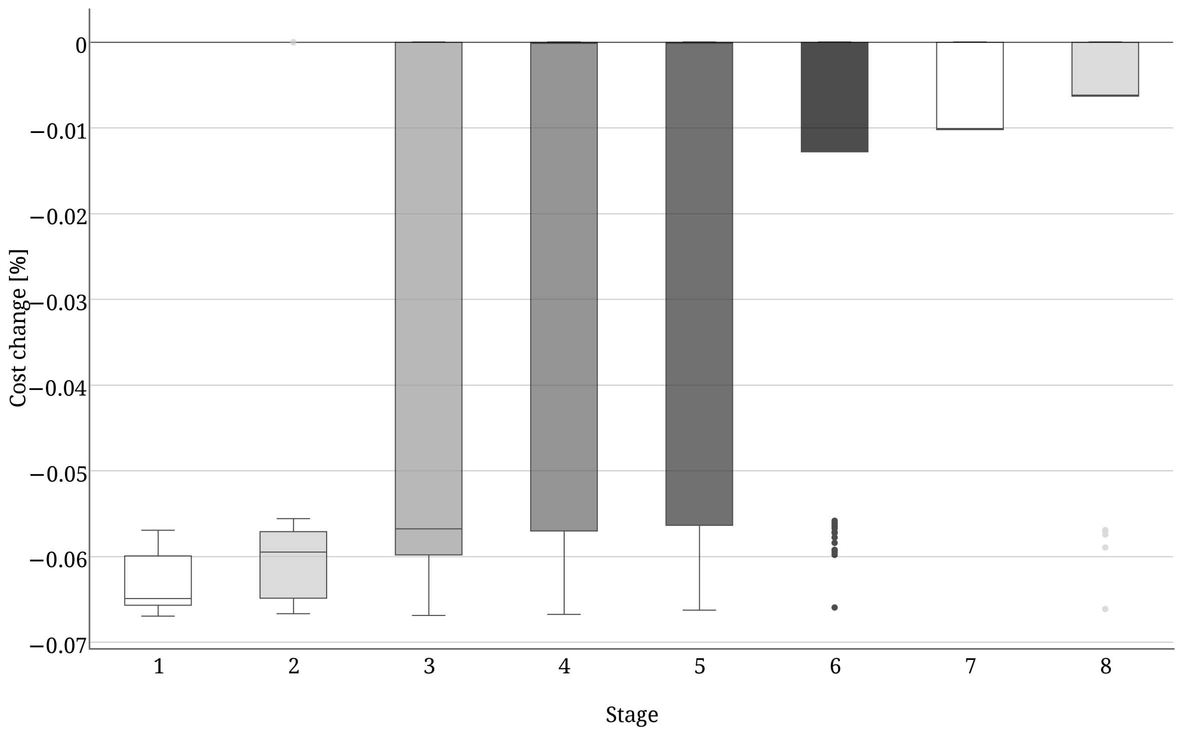
Figure 7.5 shows the reported cost reductions at each stage when switching off the lines suggested by the heuristic. We can see that for an increasing demand, transmission switching becomes less and less economically beneficial. In the first stages, we still have that the line switches provide cost reductions for the system operator, although these reductions are small. From stage 7 and onwards, we see an increasing risk that the line switches will increase costs. In the final three stages, the network is at its feasibility limit, meaning that some parts of the network will be congested. Contrary to that of case118 with 12 line switches, a congested state will not bring large cost reductions.

Figure 7.5: The Polish network with load change ($\tau = 2\%, \sigma = 2\%$)



The total load of the Polish network can only go down to 87% of the original load (Figure 6.2), so the number of stages where we can decrease the load is limited. Figure 7.6 shows the reported cost reductions at each stage when we change the loads with a τ of -2% and a σ of 2%. For the first stages we observe that cost reductions remain close to the original cost reductions of 0.065%. We also see that for decreasing loads, there is a higher risk that the line switches will result in a cost reduction close to 0%, with a few instances where the switches result in a minimal increase in costs.

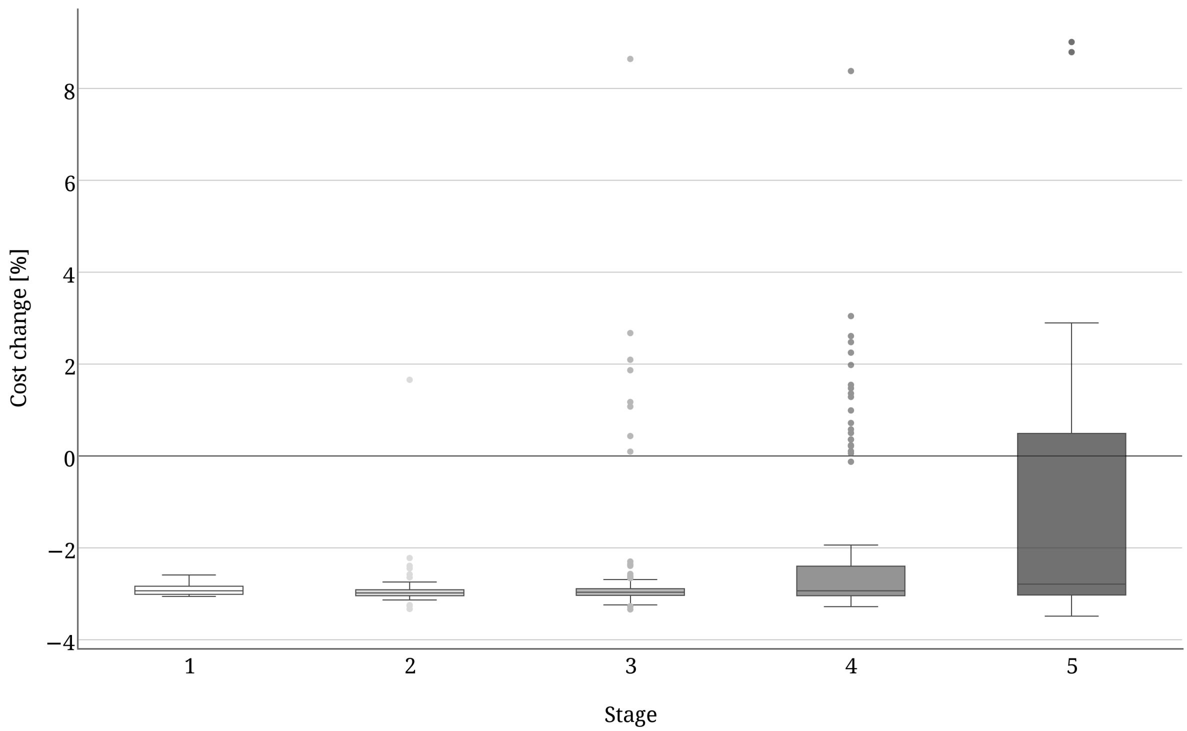
Figure 7.6: The Polish network with load change ($\tau = -2\%, \sigma = 2\%$)



7.4 The GB network

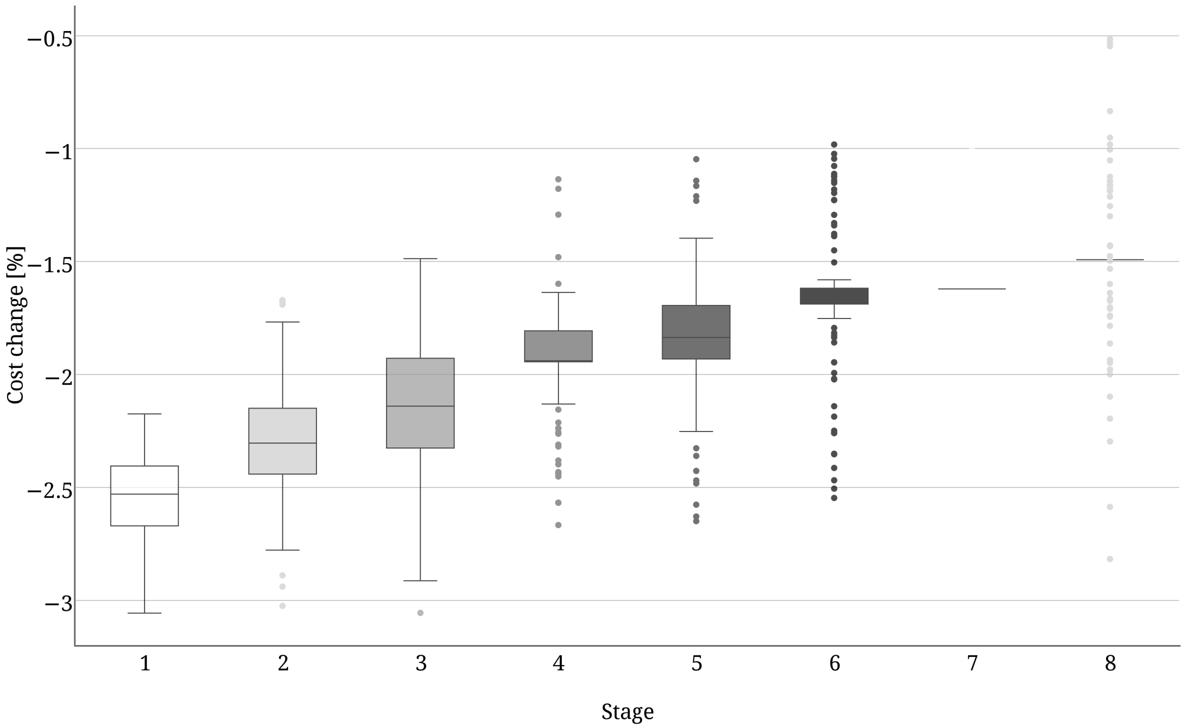
As the GB network can only handle a total τ of 10% (Figure 6.3), we increase the loads in five stages where at each stage, the loads are changed with $\tau = 2\%$ and $\sigma = 2\%$. The results are shown in Figure 7.7, where at stage 4 and 5 we have some scenarios that represent a congested network. The majority of the results are the same as with the previous networks; cost reductions are generally stable around the original cost reduction of 2.8%. However, in the stages where we consider the network to be congested, there are a few instances where the line switches result in an increase in costs up to 8.5%.

Figure 7.7: The GB network with load change ($\tau = 2\%, \sigma = 2\%$)



When the GB network is subjected to decreasing loads, as in Figure 7.8, the cost reductions when switching off lines decrease as well, although all scenarios return cost reductions.

Figure 7.8: The GB network with load change ($\tau = -2\%, \sigma = 2\%$)



Chapter 8

Operational risk related to transmission switching

This chapter presents our results from assessing the operational risk that a system operator is subjected to when considering transmission switching. We have measured risk using the following probabilities:

- P(1) – The probability that the problem converges both before and after transmission switching
- P(2) – The probability that the problem does not converge neither before, nor after transmission switching. I.e. that the loads are simply too high for the network to handle.
- P(3) – The probability that the problem does not converge due to line switching. The problem converges before transmission switching is executed, but not after.
- P(4) – The probability that the problem does not converge before line switching, but does converge after. I.e. that transmission switching prevents a network blackout.

8.1 Case118

Figure 8.1 illustrates the operational risk related to transmission switching for case118 under different load scenarios throughout a day when all 13 lines suggested by the heuristic are switched off. We observe a probability of 5–30% that the network will black out, i.e. that the problem will not converge, due to transmission switching (P(3)). This is the case during the hours of the day when demand is at its highest. From the peak hours 17:00–20:00, we observe a probability of 15–30% that the problem will not converge, neither before nor after transmission switching. When the network is not congested, we see that it can be operated normally for all load scenarios. However, as we illustrated in Figure 7.3, the probability that transmission switching will lead to cost increases is 100% for all hours when demand is increasing.

Figure 8.1: Risk related to switching off 13 lines – case118

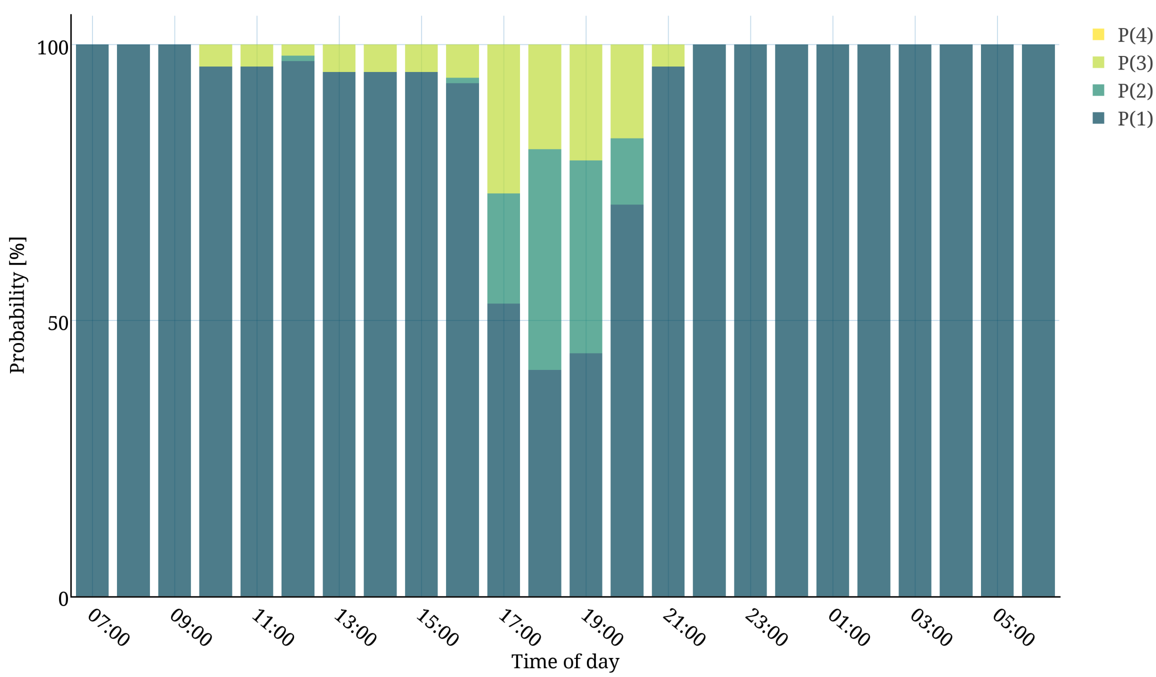
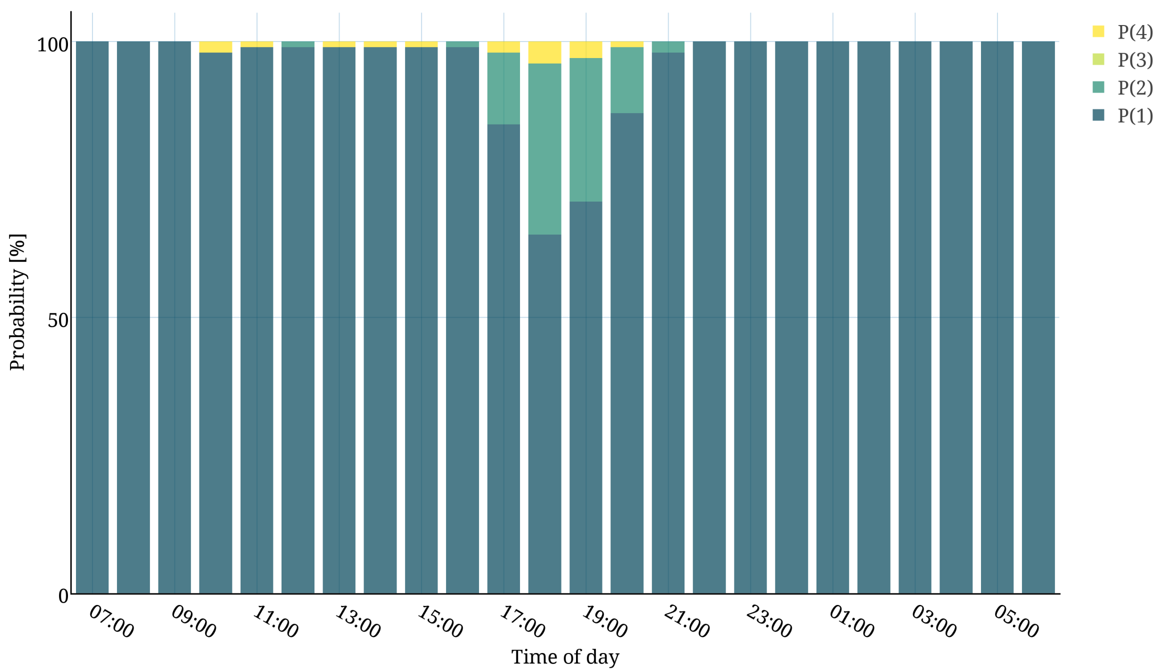


Figure 8.2 shows that the risk profile when switching off only 12 lines is in many ways opposite to when all 13 lines are switched off. When all lines are switched off, there is a relatively large presence of P(3). When only the first 12 lines are switched off, we observe no risk that the prob-

lem will black out due to transmission switching. The probability that transmission switching will actually help the problem to converge (P(4)) is present, although not much, in some of the congested scenarios.

Figure 8.2: Risk related to switching off 12 lines – case118



8.2 The Polish network

Figure 8.3 and 8.4 show the risk related to transmission switching with increasing and decreasing loads, respectively. The results are fairly similar to those of case118 with 12 lines switched off. We do not face any risk that the line switches will lead to a blackout in the network, as $P(3) = 0\%$ for all scenarios. Furthermore, there are a few instances in the congested scenarios where the line switches will prevent the network to black out (P(4)).

Figure 8.3: Risk related to transmission switching – Polish network ($\tau = 2\%, \sigma = 2\%$)

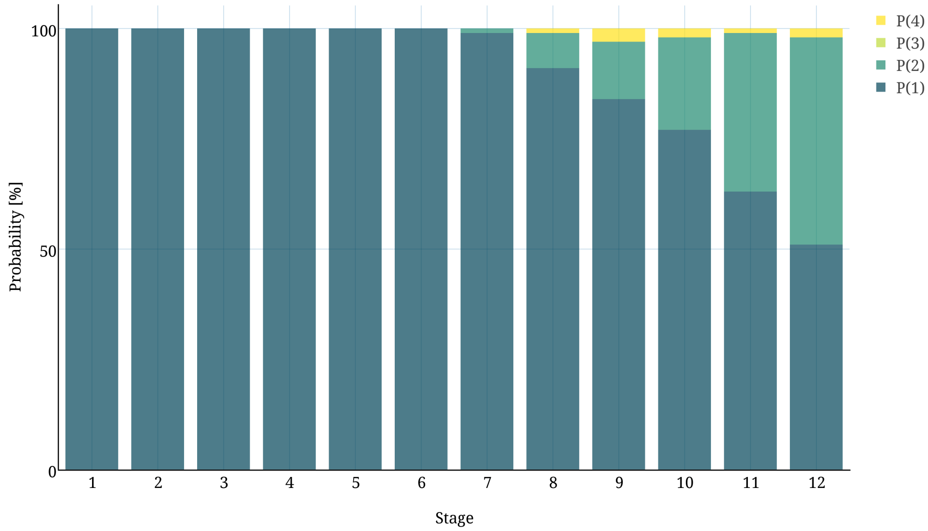
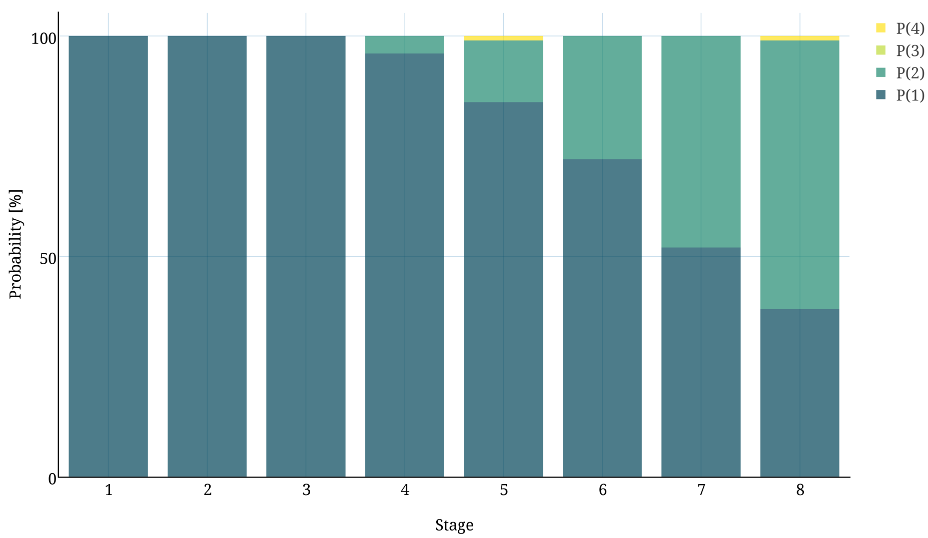


Figure 8.4: Risk related to transmission switching – Polish network ($\tau = -2\%, \sigma = 2\%$)



8.3 The GB network

Figure 8.5 shows the risk related to transmission switching for increasing loads. It shows that in most scenarios the network will run normally after transmission switching, as the problem usually converges, both before and after switching (P(1)), or does not (P(2)). The probability that transmission switching will help the problem to converge (P(4)) or make it not converge (P(3)), is roughly the same for all scenarios, except in stage 1.

Figure 8.5: Risk related to transmission switching – GB network ($\tau = 2\%, \sigma = 2\%$)

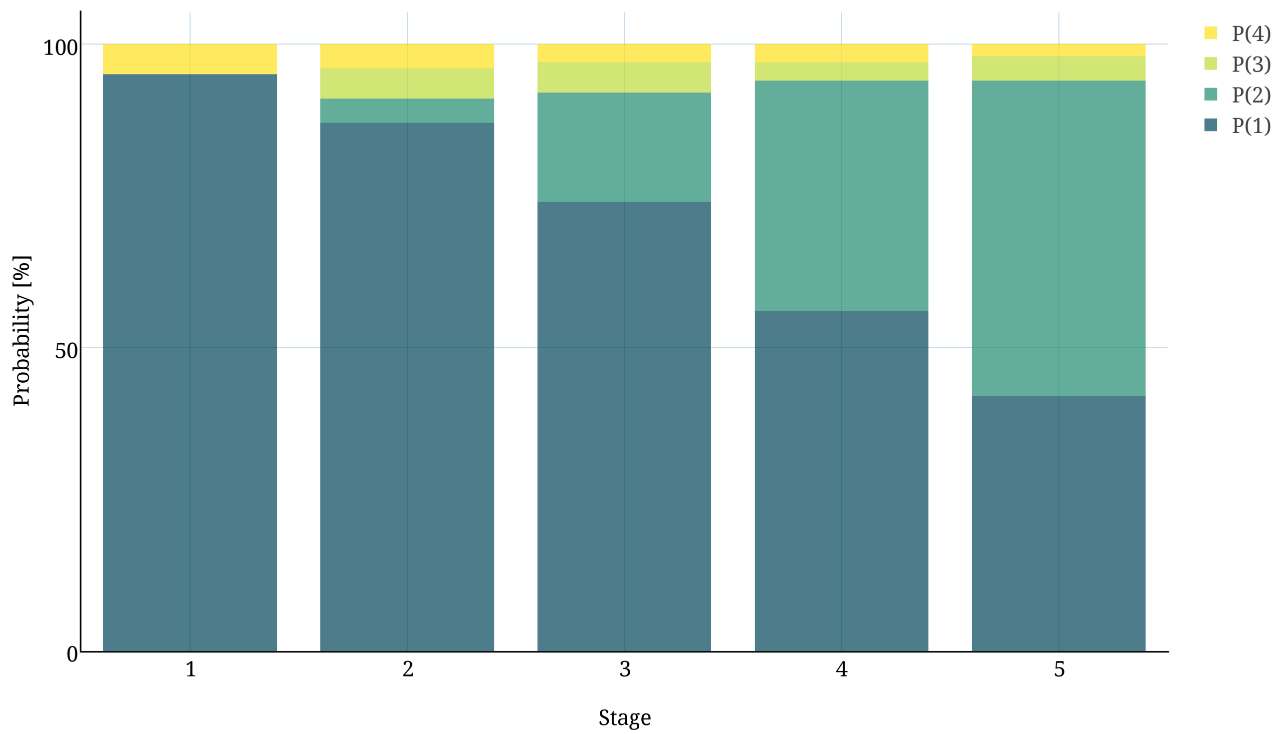
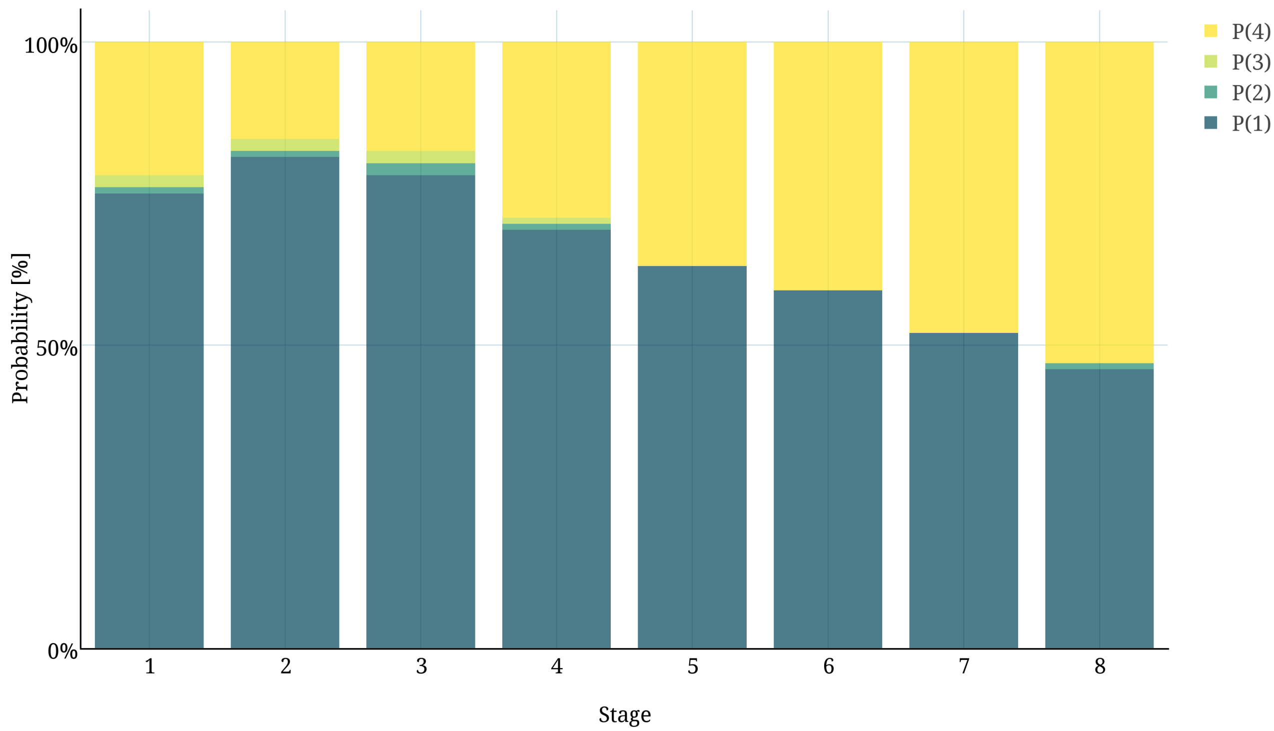


Figure 8.6 shows the same risks for decreasing loads. We see that the risk involved with transmission switching is different for decreasing and increasing loads. For decreasing loads, we have a high presence of the problem not converging when the lines are switched on, but with the help of transmission switching, it converges (P(4)). The probability that the problem will not converge due to transmission switching is only reported for a few instances in a few stages (P(3)).

Figure 8.6: Risk related to transmission switching – GB network ($\tau = -2\%, \sigma = 2\%$)



Chapter 9

Discussion

9.1 The AC and DC Heuristic

The discussion of the AC heuristic versus the DC heuristic depicts the classical tradeoff between computation time and quality of results. The AC heuristic will always return more accurate results, as it is based on the more realistic ACOPF problem. The only reason for basing the heuristic on the less accurate DCOPF is to avoid the computational difficulties of the ACOPF. Although our expectations to the DC heuristic performance are not as high as for the AC heuristic, the results must still be of a certain quality to have any value at all. The results from testing the DC heuristic on networks subject to load change were inconsistent. For both case118 and the Polish network, the suggested switching actions from the DC heuristic did not prove beneficial when applied to the ACOPF. Considering these results, a system operator relying on the DC heuristic to make switching decisions would expose himself to a huge risk of both higher costs and blackouts in the network. However, the results from the GB network differ. For the load changes, the DC heuristic switching actions generally performed well when applied to the ACOPF. In general, the results diplayed no pattern in the DC heuristic performance considering the size of the network or load level. We cannot conclude that the DC heuristic will perform badly for all networks, nor state the opposite. However, our results have demonstrated the unreliability of its switching recommendations, making the ACOPF a preferred basis for the heuristic.

Our study of the AC heuristic performance shows that the quality of the α_k ranking varies from network to network, and is in general poorer for the larger and more realistic networks. To obtain as accurate results as possible from the heuristic, it is necessary to perform some initial numerical experiments with the input parameters. We allow for quite long computation times in order to obtain good results, but as OTS is a short-term problem, the focus on computation time must be emphasized more if the AC heuristic should be applied in practice by system operators. With the way the heuristic is constructed today, it is still difficult to achieve good results within a reasonable computation time for large networks.

9.2 The congested case118

Since the start of OTS studies and up until now, the majority has drawn the same conclusion that there is a large potential for system operators to reduce generator dispatch costs through transmission switching. Contrary to most studies that report a cost reduction potential of up to 30% for case118, our results indicate the potential to be approximately -0.7%. These differences could only be explained through the modifications we have done to the original case118, as the heuristic principles we use to solve the OTS problem are the same as in many of the published studies.

The version of case118 that is mostly used in OTS literature represents a partly congested network operating up to its peak capacity with unrealistic generator costs. As we showed in Figure 7.2 and as Coffrin et al. (2014b) pointed out in their thesis, transmission switching can in some scenarios with the right line switches, lead to significant cost reductions when the network is congested. However, the same conclusion cannot be drawn in normal conditions.

This is why we believe that the common version of case118 is an unrealistic test case to base OTS studies on. We believe that our modified version of case118 is able to represent a more realistic snapshot of a network running at normal conditions, with the possibility of increasing loads to simulate a congested network. However, it is worth noticing that even this modified network is not perfectly representative of a realistic network existing today. Generator marginal costs do not include maintenance and operation costs nor CO_2 emission costs. It is based on an outdated network, and generator and line data are added based on statistical data from modern European networks, which are significantly larger than a small 118-bus network.

9.3 Can transmission switching be beneficial over time?

We loooked at two different sets of line switches for case118. In the first set, we switched off the 13 lines suggested by the heuristic and observed that when this network was subject to load changes throughout the day, transmission switching would not be beneficial to the system operator for even an hour. The switching actions resulted in increased costs in each scenario of increasing loads. In the hours during the day where the network was congested, costs actually increased by up to 8%, an observation contrary to what has been reported in the literature. However, we were able to obtain continual cost reductions throughout the day when we overrided the heuristic's suggestion, and only switched off the 12 first lines. With these switching actions, costs did not once during the day increase due to transmission switching and cost reductions even went as low as a 6% when the network was congested.

We learn two things from this. First, we observe that the heuristic's suggestion of switching actions does not necessarily imply that it is a good choice over time. A good choice would include that the switching actions are able to provide cost reductions over at least 4–5 hours during the day, which is a forecast that the heuristic cannot make. If the heuristic were extended to include this stochastic perspective, it would imply increasing the computation time to the extent where it would not make a useful tool for short-term operations. Second, the studies which report a huge cost reduction potential, have done so by studying an already congested test case with a limited amount of load scenarios close to the original load. We believe that the differences in our results are due to the fact that we study networks in normal condition with more realistic network data.

The Polish and GB networks show fairly similar results when subjected to load changes. When loads are increasing, line switching will in both networks yield less cost reduction, ultimately leading to cost increases for the highest demand scenarios. An interesting observation is that when the GB network is congested, cost increases of up to 8% occur, which again is contrary to what has been reported in the literature. For both test cases, no observations of line switching leading to higher cost reductions in congested scenarios were made. For decreasing loads, the line switches remain a good choice, although the cost reduction potential approaches 0%.

It may seem as though the positive outlook on OTS is based on one specific test case, in a congested state with the right set of line switches. In this case, we can achieve significant cost reductions. This conclusion does not hold for the remainder of our results, where transmission switching leads to small cost reductions, or cost increases, depending on the load scenario. It is worth noting, though, that we cannot draw an unambiguous conclusion that there is little potential in the future of OTS, although our results support that. The reason for this is that the problem itself is a highly complex nonlinear problem, where results and behavior differ largely between different networks. Furthermore, although we have extended our studies to newer, large networks with modifications done to simulate a real life power network, they can not be said to fully represent realistic networks. An indication of this is the feasibility limits of our test cases. The Polish network can only handle a total load change of -13 to 28%, while in the GB network the range is even smaller, ranging from -20% to 10%. This implies that the network cannot, in real time, handle the load change even during a normal day.

9.4 Operational risk related to transmission switching

When changing the topology of the network, we observed that in most scenarios transmission switching will not cause blackouts. However, for some load scenarios, the outputs differed between the test cases. For case118 with all 13 line switches, there was a high risk of blackouts due to transmission switching when demand was at its peak. However, when one of these lines was switched on again, transmission switching did not black out the network, but rather *helped* the network in a few instances to still operate. This again supports our previous observation that the choice of lines suggested by the heuristic is not necessarily a good choice, and that testing for different load scenarios is crucial if one actually were to switch off these lines.

For both increasing and decreasing loads in the Polish network, we observed that the line switches never resulted in the network blacking out, and a few instances were reported where transmission switching helped to keep the network operating. For increasing loads, the GB network showed instances where transmission switching both prevented and resulted in the network blacking out. For decreasing loads, however, transmission switching was a big contributor to that the network could operate for almost all load scenarios. These results follow the same pattern as the one described above, as they differ from case to case and even from scenario to scenario, which is natural given the many nonlinearities of the problem. Our results do not support, however, the OTS research community's claim that transmission switching does not have a negative impact on network reliability. It is worth mentioning that these claims are made on the basis of an N-1 contingency analysis, i.e., the system can survive the loss of any single component in the system, with the exception of radial lines (Hedman et al., 2010). This contingency analysis is not extended to include load scenarios where loads vary significantly from the original load level.

9.5 Summary

In Chapter 2, we presented relevant theory revolving the AC and DCOPF problems, as well as introducing the OTS problem, an NP-hard optimization problem that is studied throughout this thesis. In Chapter 3, the existing OTS literature is summarized. Our first objective was to improve the network data of case118, the test case that most results within OTS research are based on. The second of our objectives was to extend OTS studies by introducing larger, newer and more realistic networks. In Chapter 4, we identified the weaknesses of the version of case118 that has been used so far. It represents an outdated, partly congested network with unrealistic generator costs. Furthermore, we made necessary modifications to this network, as well as the Polish and GB networks. These modifications included changing thermal limits, generator capacities and generator marginal costs.

The third of our objectives, was to test the performance and quality of the AC and DC heuristics used to solve the OTS problem. In Chapter 5, the principles of the AC and DC heuristics were presented. The DC heuristic demonstrated poor behaviour for two out of our three test cases when its switching actions where applied in an ACOPF context. Thus, it cannot be said to be a reliable approach to solving the OTS problem. The AC heuristic must be applied if one wants to assure reliable results. We have also uncovered the varying accuracy of the α_k ranking depending on the network at hand, which affects the computation time necessary to ensure reliable results.

Based on the observations of the DC heuristic performance, we chose the AC heuristic to study OTS further. The fourth of our objectives was to explore whether or not transmission switching could reduce generator costs over time, when subject to uncertainty of demand. The method of how uncertainty of demand is simulated was explained in Chapter 6. We discretizised the changes of loads in stages, where at each stage the load at each bus was changed dependently on the load changes at other buses through a log-normal distribution. The results were presented in Chapter 7. They showed that when working with more realistic networks, the outlook of OTS in a cost reduction perspective is far more negative than what has been reported so far within the OTS research community. We have shown that large cost reductions can occur when case118, with the right set of switches, is modelled as a congested network. In general, cost reductions are small and transmission switching can only be slightly beneficial over a limited amount of hours. Based on our results, we do not consider transmission switching to be economically benefical over time.

Our final objective was to quantify the risk related to transmission switching, both in an economical and operational contingency perspective. This was presented in Chapter 7 and 8, where we showed that transmission switching very likely will lead to increases in costs over time, although the results varied between test cases and scenarios. In terms of operational risk, an overall conclusion would be that there is little operational risk involved with transmission switching. When the network is congested, we experienced cases where transmission switching caused the network to black out, but also cases where it helped the network not to black out. I.e. the operational risk involved for the system operator is on general low, but should be assessed in detail for each different network.

Chapter 10

Conclusion

Our results are, over all, discouraging for the future of OTS. Contrary to the OTS literature, we do not consider transmission switching to be beneficial over time. The networks have proven their individuality through the results, which makes it difficult to draw an unambiguous conlusion and proves yet another challenge related to transmission switching. We believe that we have created a better basis for studying transmission switching, through creating noncongested test cases with updated network data and testing them for a number of realistic demand scenarios. Nevertheless, despite our best efforts to evaluate OTS on a variety of test cases and scenarios, the networks studied in this thesis cannot be considered to fully represent modern power networks. Hopefully, more network data will be made available from the power industry in the future, which would provide a much better basis of examining the potential gains of OTS. Still, the complexity of the OTS presents a great challenge. The best available method for solving the OTS problem today is the AC heuristic, but the time needed for necessary pre-testing together with the computation time is still too long for practical use in day-ahead and real-time procedures.

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Appendices

Appendix A

The Interior Point Method

In this appendix, we will provide a theoretical presentation of the interior point method, which is the method employed by MATPOWER to solve OPF problems. The ACOPF problem can be written in compact form as:

$$\text{minimize} \quad c(X) \tag{A.1}$$

$$\text{subject to} \quad h(X) = 0 \tag{A.2}$$

$$g(X) \leq 0 \tag{A.3}$$

$$X \in B, \tag{A.4}$$

This is a nonlinear problem with both inequality and equality constraints. For a problem of this kind, interior point methods are often employed.

In dealing with a nonlinear optimization problem, interior point methods start with an interior point in the feasible region and then generate a sequence of interior points that converge to an optimal solution that may be on the boundary (Lundgren et al., 2010). In other words, we try to make a "shortcut" by searching along a *central path* in the feasible region instead of searching along the boundary of the feasible region. One way of doing this is to reformulate the problem using a barrier function.

A.1 Barrier function and Lagrangian

A barrier function can be called an interior penalty function, as it penalizes the deviation from the border of the feasible region (Lundgren et al., 2010). The most important barrier function is the logarithmic barrier function (Nocedal and Wright, 1999). If we have a general minimization problem with an inequality restriction $c_i(x) \geq 0$, we will have a strictly feasible region, here noted by $\mathscr{D}^0$. The logarithmic function can be defined thus:

$$-\sum_{i \in I} \ln c_i(X) \tag{A.5}$$

This barrier function forces the solver to search within the strictly feasible region as it is infinite outside $\mathscr{D}^0$, it is smooth inside $\mathscr{D}^0$ and the value of the function approaches infinity as it gets nearer to the boundary of $\mathscr{D}^0$. Introducing a barrier function to our problem results in the following formulation:

$$\text{minimize} \quad c(X) - \mu \sum_{k=1}^{K} \ln(Z_m) \tag{A.6}$$

$$\text{subject to} \quad h(X) = 0 \tag{A.7}$$

$$g(X) + Z = 0 \tag{A.8}$$

$$X \in B \tag{A.9}$$

$$Z > 0 \tag{A.10}$$

Note that we have now converted the inequality constraints into equality constraints by using a vector of slack variables Z, a logarithmic barrier function and the barrier parameter μ. Further we formulate the Langrangian of the problem, thus making it an unconstrained problem:

$$\mathscr{L}(X, \lambda, \gamma, \alpha, Z) = c(X) + \lambda^T h_P(X) + \gamma^T h_Q(X) + \alpha^T (g(X) + Z) - \mu \sum_{k=1}^{K} \ln Z_m \tag{A.11}$$

A.2 The Karush-Kuhn-Tucker (KKT) conditions

For a detailed deduction of the first order optimality conditions using the Lagrangian function, we refer to Lundgren et al. (2010). We will below focus on the MATPOWER creators Zimmerman et al. (2011), who state that the KKT conditions for our problem "are satisfied when the partial derivatives of the Lagrangian above are all set to zero." The partial derivatives for a fixed barrier parameter μ of $\mathscr{L}^{\mu}(X,\lambda,\gamma,\alpha,Z)$ are:

$$f(X,\lambda,\gamma,\alpha,Z) = \begin{bmatrix} \mathscr{L}_x^{\mu} \\ \mathscr{L}_\lambda^{\mu} \\ \mathscr{L}_\gamma^{\mu} \\ \mathscr{L}_\alpha^{\mu} \\ \mathscr{L}_Z^{\mu} \end{bmatrix} = \begin{bmatrix} c_x(X) + \lambda^T h_{PX} + \gamma^T h_{Qx}(X) + \alpha^T g_x(X) \\ h_P(X) \\ h_Q(X) \\ g(X) + Z \\ \alpha^T - \frac{\mu}{Z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A.12}$$

A.3 Newton's method

In interior point methods, Newton's method can be used to solve Equation (A.12). The Newton step is given in vector notation below:

$$\begin{bmatrix} f_x & f_\lambda & f_\gamma & f_\alpha & f_Z \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta\lambda \\ \Delta\gamma \\ \Delta\alpha \\ \Delta Z \end{bmatrix} = -f(X,\lambda,\gamma,\alpha,Z) \tag{A.13}$$

Where $\begin{bmatrix} f_X & f_\lambda & f_\gamma & f_\alpha & f_Z \end{bmatrix}$ denotes the Hessian of the Lagrangian of our problem. This can be rewritten as:

$$\begin{bmatrix} c_{xx}(X)+\lambda^T h_{Pxx}+\gamma^T h_{Qxx}(X)+\alpha^T g_{xx}(X) & h_{Px} & h_{Qx}(X) & g_x(X) & 0 \\ h_{Px}(X) & 0 & 0 & 0 & 0 \\ h_{Qx}(X) & 0 & 0 & 0 & 0 \\ g_x(X) & 0 & 0 & 0 & I \\ 0 & 0 & 0 & I & \frac{\mu}{Z^2} \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta\lambda \\ \Delta\gamma \\ \Delta\alpha \\ \Delta Z \end{bmatrix} = -f(X,\lambda,\gamma,\alpha,Z) \tag{A.14}$$

We observe that the Hessian can be considered to be a sparse matrix, which can be beneficial to the computation time as one can use specialized algorithms that take advantage of this structure (Golub and Van Loan, 1996). The search direction is used to compute a new point $x^{(k+1)} = x^{(k)} + t^{(k)} d^{(k)}$. This search is done along a central path starting with an interior point in the feasible region. Cui (2013) presents a simple illustration of a central path, given in Figure A.1. It shows that one starts the search at any given interior point in the feasible region at t=0, and by using the Newton method moves towards an optimal solution on the boundary of the feasible region. For a more in-depth description of the central path, readers are referred to Roos et al. (2004).

Figure A.1: Central path in interior point methods