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Teams and Tournaments in Relational Contracts*

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Abstract

This paper analyses and compares optimal relational contracts between a principal/firm and a set of agents when (a) only aggregate output can be observed, and (b) individual outputs can be observed. We show that the optimal contract under (a) is a team incentive scheme where each agent is paid a maximal bonus for aggregate output above a threshold and a minimal (no) bonus otherwise. The team's efficiency decreases with its size (number of agents) when outputs are non-negatively correlated, but may increase considerably with size if outputs are negatively correlated. In the case where individual output can be observed, we show that the optimal contract is a tournament scheme where the conditions for an agent to obtain the (single) bonus are stricter for negatively compared to positively correlated outputs. We finally show that if agents have bargaining power, firms may deliberately choose to organize production as a team where only aggregate output is observable. The team alternative is more likely to be superior under negatively correlated outputs.

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1 Introduction

An increasing number of firms tie compensation to their workers' performance, but the way firms arrange their incentive programs varies enormously (see Lemieux et al, 2009 and Bloom and Van Reenen, 2010). Some firms rely on team incentives in which bonuses are tied to the joint output of a team of workers. Other firms rely on tournament schemes in which workers compete against each other for bonuses or other rewards. And many firms combine both tournaments and team incentive schemes.

An important reason for this variation in how firms provide incentives to their employees may be attributed to technological differences. First, it is a matter of observability. Some firms only observe the aggregate output from teams of workers, while other firms may be able to get an exact measure of each individual's output. Second, it is a matter of technological or stochastic dependence between the workers. Some workers' outputs are positively correlated, such as sales agents who are exposed to the same business cycles. In other situations, workers' outputs are negatively correlated, for instance when specialists with different expertise meet different sets of demand from customers or superiors.

In this paper we study how these issues affect optimal incentive design. In contrast to previous literature, we focus on repeated game relational contracts. A relational contract includes variables that are hard to verify by a third party, such as the quality of a service or the value of a performance. As a result, the contract cannot be enforced by a court of law and needs to be self-enforcing. We study how observability and technological/stochastic dependence between workers affect the conditions for implementing self-enforcing relational contracts, and furthermore, what the optimal relational contract looks like in different situations.

In particular, we analyze and compare optimal relational contracts between a principal and a set of agents when (a) only aggregate output can be observed, and (b) individual outputs can be observed. We first show that the optimal contract under (a) is a team incentive scheme where each agent is paid a maximal bonus for aggregate output above a threshold and a minimal (no) bonus otherwise. This parallels Levin's (2003) characterization for the single-agent case. We show that the team's efficiency decreases with its size (number of agents, n) when outputs are non-negatively correlated, but that efficiency may increase considerably with size if outputs are negatively correlated. Negative correlation is beneficial for the team because it increases the marginal incentives for each team member to provide effort. This indicates that diversity and heterogeneity among team members can yield considerably efficiency improvements (see Horwitz and Horwitz, 2007, for a meta-analytic review documenting positive effects from team diversity).

We further allow agents to have ex post bargaining power over the values they have created. In such a setting, a team of agents can also create values in case the relational contract breaks down. Due to the well know free-rider problem, this outside value decreases in the number of agents. However, the weaker outside option strengthens the relational contract and thereby allows for a higher bonus and thus cet. par higher effort. In other words, the $1/n$ free-rider problem might be a blessing in relational contracts.

In case (b), where individual output is observable, Levin (2002) have shown that for independent outputs the optimal relational contract entails a stark RPE scheme (relative performance evaluation); a form of a tournament, where at most one agent is paid a (maximal) bonus. We point out that the efficiency of this tournament scheme increases with the number of agents, and hence becomes progressively better compared to a team when the number of independent agents increases.Then we extend the analysis to corelated variables, and show, for a parametric (normal) distribution, that the optimal contract is an RPE scheme where the conditions for an agent to obtain the (single) bonus are stricter for negatively compared to positively correlated outputs. The efficiency of this tournament contract is shown to improve with higher correlation (both positive and negative).

We finally point out that, if the firm can initially choose between organizations (technologies) that allow for either (a) only aggregate output or (b) individual outputs to be observed, and a subsequent reorganization is costly, the firm may choose (a), i.e. organize production as a team. Thus, even if alternatives (a) and (b) are equally costly to set up initially (e.g. in terms of output measurement investments), the team alternative may yield a higher subsequent surplus. This occurs because relational contract constraints may be affected in a way to favor the team alternative. We show that, although production efficiency in both alternatives increases with more negatively correlated outputs, the team alternative is more likely to be superior under such conditions.

Related literature: The closest related paper is the above mentioned Levin (2002). He considers a multilateral relational contract between a principal and n agents, and shows among other things that the stark RPE (tournament) scheme is optimal. Unlike Levin, we also consider the case where only aggregate output is observable. Moreover, we extend Levin's characterization to correlated outputs. Our paper is also related to the few papers considering team incentives in relational contracts, like Kvaløy and Olsen (2006, 2008), Rayo (2007) and Baldenius and Glover (2010). But in these papers individual outputs are observable, and so they do not consider how both observability and stochastic dependence between agents affect the optimal contract.[1]

Previous literature on incentive provision to multiple agents have mainly focused on risk sharing issues and the scope for cooperation. The informativeness principle (Holmström, 1979, 1982) states that an incentive contract should be based on all variables that provide information about the agents' actions. Stochastic and/or technological dependences between agents then typically call for "peer-dependent" incentive schemes such as teams or tournaments. By tying compensation to an agent's relative performance, the principal can filter out common noise and thereby exposing them to less risk (see Holmström, 1982; and Mookherjee, 1984).[2] And by tying compensa-

[1] Seminal contributions to the (formal) literature on relational contracting include Klein and Leffler (1981), Shapiro and Stiglitz (1984), Bull (1987) and MacLeod and Malcomson (1989).

[2] See also Lazear and Rosen (1981), Nalebuff and Stiglitz (1983) and Green and Stokey

tion to the joint performance of a team of agents, the principal can exploit complementarities between the agents' efforts and foster cooperation, see Holmström and Milgrom, 1990; Itoh (1991, 1992) and Macho-Stadler and Perez-Castrillo, 1993).[3]Our paper shows that stochastic dependence between agents is highly important for incentive design even in the absence of risk considerations, and that team incentives may be optimal even without classical team effects such as complementarities in production, peer pressure or peer monitoring.

Our paper is also related to a recent literature on endogenous formation of teams. While there is a vast agency literature that studies optimal incentives for teams[4], there is only a few papers that explore how and why firms may only hold a team of agents accountable for their joint output, even if individual accountability is technologically feasible. Mukherjee and Vasconcelos (2011) and Corts (2007) show that team production might help mitigate multitask problems, while Bar-Isaac (2007) show that teams consisting of juniors and seniors can restore the reputation concerns of seniors. We show that firms may use team production (team accountability) as a commitment device. By deliberately choose team assignment instead of individual assignment, the firm makes it more costly to breach the relational contract. But we also show that there is a limit to how many agents the firm should hold accountable. The optimal team size depends both on the agents' ex post bargaining power and on the type of dependence between the agents.

Finally, our paper is related to a literature on asset ownership and bargaining power in relational contracts, such as Baker, Gibbons and Murphy (2002), Halonen (2002) and Kvaløy and Olsen (2012). A central point here is that

(1983) for analyses of RPE's special form, rank-order tournaments.

[3]In addition, team incentives can provide implicit incentives not to shirk (or exert low effort), since shirking may have social costs (as in Kandel and Lazear, 1992), or induce other agents to shirk (as in Che and Yoo, 2001).

[4]Economists studying teams with unobservable individual ouputs, beginning with Alchian and Demsetz (1972), have mainly focused on the free-rider problem, in particular under what conditions the first-best outcome will be achieved, or what parameters affect the relative efficiency of teamwork. Influential papers include Holmstrom (1982) Rasmusen (1987), McAfee and McMillan (1991) and Legros and Matthews (1993).

agents' bargaining power may negatively affect the scope for relational contracting. In particular Halonen (2002) shows that agents may consider joint ownership of assets (similar to team production) in order to reduce outside options and thereby strengthen the relational contract between them. But Halonen does not consider principal-multiagent incentive problem, like we do.

The rest of the paper is organized as follows. Section 2 presents the model and analyses team incentives, given that only total output can be observed. Section 3 deals with the case where individual outputs can be observed, and Section 4 contains a comparative analysis of the two cases. The last section concludes.

2 Model

We analyze an ongoing economic relationship between a principal and n (symmetric) agents. All parties are risk neutral. Each period, each agent i exerts effort e_i incurring a private cost $c(e_i)$. Costs are strictly increasing and convex in effort, i.e., $c'(e_i) > 0$, $c''(e_i) > 0$ and $c(0) = c'(0) = 0$. Each agent's effort generates a stochastic output x_i, with marginal density $f(x_i, e_i)$. Expected outputs are given by $\bar{x}(e_i) = E(x_i | e_i) = \int x_i f(x_i, e_i) dx_i$ and total surplus per agent is $W(e_i) = \bar{x}(e_i) - c(e_i)$. First best is then achieved when $\bar{x}'(e_i^{FB}) - c'(e_i^{FB}) = 0$.

However, the parties cannot contract on effort provision. We assume that effort e_i is hidden and only observed by agent i. With respect to output, we consider two cases: Either individual outputs x_i are observable (IO), or only total output $y = \Sigma x_i$ is observable. In both cases, we assume that outputs are non-verifiable by a third party. Hence, the parties cannot write a legally enforceable contract on output provision, but has to rely on self-enforcing relational contracts.

2.1 Team: only total output observed

We first consider the case where individual output is unobservable, and hence the parties can only contract on total output provision. Each period, the principal and the agents then face the following contracting situation. First, the principal offers a contract saying that agent i receives a non-contingent fixed salary α_i plus a bonus $\beta_i(y)$, $i = 1...n$ conditional on total output $y = \Sigma x_i$ from the n agents[5]. Second, the agents simultaneously choose efforts, and value realization $y = \Sigma x_i$ is revealed. Third, the parties observe y and the fixed salary α_i is paid. Then the parties choose whether or not to honor the contingent bonus contract $\beta_i(y)$.

Conditional on efforts, agent i's expected wage in the contract is then $w_i = E(\beta_i(y)|\, e_1...e_n) + \alpha_i$, while the principal expects $\Pi = E(y|\, e_1...e_n) - \Sigma w_i = \Sigma_i E(x_i|\, e_i) - \Sigma w_i$. If the contract is expected to be honored, agent i chooses effort e_i to maximize his payoff, ie

$$e_i = \arg\max_{e_i'} \left(E(\beta_i(y)|\, e_i', e_{-i}) - c(e_i')\right) \tag{IC}$$

If the contract is not honored, the parties instead bargain over the realized values. Given a realization y, we assume that they agree on a spot price ηy, where $\eta < 1$ is the agents' share. More specifically, we assume that the spot price is determined by Nash bargaining. The agents are able to attain θy, $\theta \in [0, 1]$ in an alternative market. In Nash bargaining, the agents will then receive θy plus a share σ of the surplus from trade i.e. the spot price will be $S = \theta y + \sigma^i(y - \theta^i y) = \eta y$ where $\eta = \sigma + \theta(1 - \sigma)$. The parameter η can be interpreted as an index of the agents' total hold-up power.

In a one shot relationship, the parties have no incentives to honor the bonus contract, and so they have to rely on spot contracting. The expected spot price is then $\eta E(y|\, e_1...e_n) = \eta \Sigma \bar{x}(e_i)$. Agent i thus chooses spot effort e^s according to $\frac{1}{n}\eta \bar{x}'(e^s) - c'(e^s) = 0$, and so the expected spot price can be

[5] We thus assume stationary contracts, which have been shown to be optimal in settings like this (Levin 2002, 2003).

written $S = \eta\bar{y}(e^s)$, while the principal's expected spot profit is given by $\pi_s = (1-\eta)\bar{y}(e^s)$.

Now consider the repeated game. Like Levin (2002) we consider a multilateral punishment structure where any deviation by the principal triggers punishment from all agents. The principal honors the contract only if all agents honored the contract in the previous period. The agents honor the contract only if the principal honored the contract with all agents in the previous period. Thus, if the principal reneges on the relational contract, all agents insist on spot contracting forever after. And vice versa: if one (or all) of the agents renege, the principal insists on spot contracting forever after. A natural explanation for this is that the agents interpret a unilateral contract breach (i.e. the principal deviates from the contract with only one or some of the agents) as evidence that the principal is not trustworthy (see discussion in Bewley, 1999 Levin, 2002).

Now, (given that (IC) holds) the principal will honor the contract with all agents $i = 1, 2, ..., n$ if

$$-\Sigma_i\beta_i(y) + \frac{\delta}{1-\delta}\Pi \geq -\eta y + \frac{\delta}{1-\delta}\pi_s \tag{EP}$$

where δ is a common discount factor. The LHS of the inequality shows the principal's expected present value from honoring the contract, which involves paying out the promised bonuses and then receiving the expected value from relational contracting in all future periods. The RHS shows the expected present value from reneging, which involves spot trading of the realized outputs, and then receiving the expected value associated with spot trading in all future periods.

Agent i will honor the contract if

$$\beta_i(y) + \frac{\delta}{1-\delta}(w_i - c(e_i)) \geq \frac{1}{n}\eta y + \frac{\delta}{1-\delta}(\frac{1}{n}S - c(e^s)) \tag{EA}$$

where similarly the LHS shows the agent's expected present value from honoring the contract, while the RHS shows the expected present value from reneging.

Recall the definition $W(e_i) = E(x_i|\, e_i) - c(e_i)$ as the total surplus associated with agent i, and define 'modified' bonuses as follows:

$$b_i(y) = \beta_i(y) - \frac{1}{n}\eta y. \tag{1}$$

Following established procedures (e.g.Levin 2002) we obtain the following::

Lemma 1 *For given efforts* $e = (e_1...e_n)$ *there is a wage scheme that satisfies (IC,EP,EA) and hence implements* e*, iff there are bonuses* β *and fixed salaries* α *with* $b_i(y) = \beta_i(y) - \frac{1}{n}\eta y \geq 0$*, such that (IC) and condition (EC) below holds:*

$$\Sigma_i b_i(y) \leq \frac{\delta}{1-\delta}\Sigma_i(W(e_i) - W(e^s)), \tag{EC}$$

To see sufficiency, set the fixed wages α such that each agent's payoff in the contract equals his spot payoff, i.e. $\alpha_i + E(\beta_i(y)|\, e) - c(e_i) = \frac{1}{n}S - c(e^s) \equiv u^s$. Then EA holds since $\beta_i(y) - \frac{1}{n}\eta y \geq 0$. Moreover, the principal's payoff in the contract will be will be $\Pi = \Sigma_i(W(e_i) - u^s) = \Sigma_i(W(e_i) - W(e^s)) + \pi_s$, i.e. the surplus generated by the contract plus her spot profits. Then EC and (1) imply that EP holds. Necessity is verified in the appendix.

Following the standard assumption in the literature, we assume that the first order approach (FOA) is valid, and hence that each agent's optimal effort choice is given by the first-order condition (FOC):

$$\frac{\partial}{\partial e_i}E(\beta_i(y)|\, e_1...e_n) - c'(e_i) = 0$$

It is convenient to use the 'modified' (net) bonuses b_i when analyzing the contract. Since $Ey = \Sigma_j \bar{x}(e_j)$, the FOC can then be written

$$\frac{\partial}{\partial e_i}E(b_i(y)|\, e_1...e_n) + \frac{1}{n}\eta\bar{x}'(e_i) = c'(e_i) \tag{2}$$

Given that FOA is valid, the agents' optimal choices are characterized by the condition (2), which we will refer to as a 'modified' IC constraint. We will further assume that the 'monotone likelihood ratio property' (MLRP) holds for aggregate output y in the following sense: its density is assumed to be of the form $g(y; l(e_1...e_n))$ with $l_{e_i}(e_1...e_n) > 0$, and such that $\frac{g_l(y,l)}{g(y,l)}$ is increasing in y.

The optimal contract now maximizes total surplus $(\Sigma_i W(e_i) = \Sigma_i(E(x_i|\, e_i) - c(e_i)))$ subject to EC and the 'modified' IC constraint (2). Then we have the following:

Proposition 1 *The optimal symmetric scheme pays a maximal bonus to each agent for output above a threshold* $(y > y_0)$ *and no bonus otherwise. The threshold is given by* $\frac{g_l(y_0,l(e))}{g(y_0,l(e))} = 0$. *For* $l(e_1...e_n) = \Sigma_i e_i$ *no asymmetric scheme can be optimal.*

The maximal symmetric bonus is by EC $b_i(y) = b(y) = \frac{\delta}{1-\delta}(W(e) - W(e^s))$ when $e_i = e$ for all i. This result parallels that of Levin (2003) for the single agent case. The threshold property comes from the fact that incentives should be maximal (minimal) where the likelihood ratio is positive (negative). Since this ratio is monotone increasing, there is a threshold y_0 where it shifts from being negative to positive, and hence incentives should optimally shift from being minimal to maximal at that point.

2.2 Team size and efficiency

We will now study team size and efficiency. To see how size (i.e. number of agents in the team) affects efficiency, note from Proposition 1 that the IC constraint (2) can now be written

$$c'(e_i) = b\int_{y>y_0} g_i(y; e_1...e_n)dy + \frac{1}{n}\eta\bar{x}'(e_i)$$

where g_i denotes partial derivative of the density wrt e_i, and hence that the optimal solution $e_i = e$ (the maximal effort per agent that can be implemented) is given by

$$\frac{c'(e) - \frac{1}{n}\eta\bar{x}'(e)}{\int_{y>y_0} g_i(y; e...e)dy} = b = \frac{\delta}{1-\delta}\left(W(e) - W(e^s(n))\right) \quad (3)$$

The first equality shows the required bonus (per agent) to implement effort e (from the IC constraint). The second equality shows the feasible (maximal) bonus. When n increases, a single agent's marginal influence on his expected bonus payment (i.e. $b\int_{y>y_0} g_i(y; e...e)dy$) will be affected. If this marginal influence is reduced (as it typically will be for independent outputs), a larger bonus is required to maintain effort incentives (the first equality). A higher bonus is also required because the 'automatic incentive' ($\frac{1}{n}\eta\bar{x}'(e_i)$) is reduced when n increases. But a higher bonus is also feasible (the second equality) because the outside spot value $W(e^s(n))$ is decreasing in n. Which of these effects dominates will determine whether effort (per agent) will increase or decrease when the number of agents increases.

It is of particular interest to analyse teams with stochastic dependencies among the individual team members' contributions to total output. To make this analytically tractable we will *assume that outputs are (multi)normally distributed and correlated.* Given this assumption, and (by symmetry) each x_i being $N(e_i, s^2)$, then total output $y = \Sigma x_i$ is also normal with expectation $Ey = \Sigma e_i$ and variance

$$s_n^2 = var(y) = \Sigma_i var(x_i) + \Sigma_{i\neq j} cov(x_i, x_j) = ns^2 + s^2\Sigma_{i\neq j} corr(x_i, x_j)$$

It follows from the form of the normal density that the likelihood ratio is linear and given by $\frac{g_i(y,e_1...e_n)}{g(y,e_1...e_n)} = (y - \Sigma e_i)/s_n$. As shown above, the optimal bonus is maximal (minimal) for outcomes where the likelihood ratio is positive (negative), and hence has a threshold $y_0 = \Sigma e_i$. Applying the normal distribution, it then follows (as shown below, see (6)) that the marginal return to effort for each agent in equilibrium is given by

$$b \int_{y>y_0} g_i(y; e...e)dy = b/(Ms_n), \qquad M = \sqrt{2\pi} \tag{4}$$

Since by assumption now $\bar{x}(e_i) = Ex_i = e_i$, the IC condition (2) for each agent's (symmetric) equilibrium effort is therefore $c'(e_i) - \frac{1}{n}\eta = \frac{b}{s_n}\frac{1}{M}$. It then follows from (3) that the maximal effort per agent that can be sustained, is now given by

$$\left(c'(e) - \frac{1}{n}\eta\right) s_n M = b = \frac{\delta}{1-\delta}(W(e) - W(e^s(n)) \tag{5}$$

Consider now variation in team size. In line with the discussion above, a higher n has here three specific effects:

1. It reduces the outside spot value and thereby allows for a higher bonus, and thus cet par for higher effort.

2. It reduces the 'automatic incentive' $\frac{1}{n}\eta$ and thereby cet par the effort.

3. it affects the variance s_n^2 of the performance measure ($y = \Sigma x_i$)

If all agents' outputs are fully symmetric in the sense that all correlations as well as all variances are equal across agents, i.e. $var(x_i) = s^2$ and $corr(x_i, x_j) = \rho$ for all i, j, then the variance in total output will be

$$s_n^2 = ns^2 + s^2 \Sigma_{i \neq j} corr(x_i, x_j) = ns^2(1 + \rho(n-1))$$

If $\rho \geq 0$ the variance will increase with n and the third effect discussed above is detrimental for efficiency. Optimal n should therefore be smaller with larger ρ. Moreover, the standard deviation of total output (s_n) increases rapidly with n when $\rho \geq 0$ (at least of order $\sqrt{n}$), while all other terms in the relation (5) stay bounded, hence the effort per agent that can be sustained will then decrease rapidly with n. Large teams are therefore very inefficient if all agents' outputs are non-negatively correlated.

For negative correlations the situation is quite different. If $\rho < 0$ one can in principle reduce the variance to (almost) zero by including sufficiently many agents. The model then indicates that adding more and more agents to the team is beneficial, at least as long as $1 + \rho(n-1) > 0$ and the conditions for FOA to be valid is fulfilled. (We show below that for this to be the case, the variance of the performance measure, here s_n^2, cannot be too small.)

Note that assuming symmetric pairwise negative correlations among n stochastic variables only makes sense if the sum has nonnegative variance, and hence $1 + \rho(n-1) \geq 0$.[6] Given $\rho < 0$, there can thus only be a maximum number n of such variables (agents). And given $n > 2$, we must have $\rho > -\frac{1}{n-1}$.

Note also that for given negative $\rho > -\frac{1}{2}$, the variance is first increasing, then decreasing in n (it is maximal for $n = \frac{1}{2}(1 - \frac{1}{\rho})$). Hence the optimal team size in this setting is either very small ($n = 2$) or 'very large' (include all).

Proposition 2 *For symmetric agents, efficiency decreases rapidly with size if outputs are non-negatively correlated. For symmetric agents with negatively correlated outputs, efficiency first decreases (for $n > 2$) and then increases with increasing team size, hence efficiency is maximal either for a small or for a large team.*

The assumption of equal pairwise correlations among all involved agents is admittedly somewhat special, but illustrates in a simple way the forces at play when the team size varies. In reality there might be positive as well as negative correlations among agents. A procedure to pick agents for least variance would then be for each n, to pick those n that yield the smallest variance. Then compare across n, weighting the three effects discussed above.

[6] Indeed, as shown in the appendix, $1 + \rho(n-1) > 0$ is the condition for the covariance matrix to be positive definit, and hence for the multinormal model to be well specified.

2.3 When is the FOA valid?

We will now examine under what conditions the FOA is valid for the model analyzed in the previous section. Thus consider y normally distributed with expectation $Ey = \Sigma e_i$ and a variance that will be denoted by $s^2 = var(y)$ in this section (to simplify notation). As already noted, this distribution satisfies MLRP. Each agent is offered a 'gross' bonus $\beta(y) = b(y) + \mu y$, where $\mu = \eta/n < 1/2$, and $b(y)$ is the net 'bang-bang' bonus with threshold at y_0.

Given that the principal seeks to implement effort e_i^* from each agent this way, the optimal threshold is $y_0 = \Sigma e_i^*$. Agent i's expected payoff, given own effort e_i and efforts $e_j^* = e_i^*$ from the other agents, is then

$$
\begin{aligned}
& b \Pr(y > y_0 |\, e_i) + \mu E(y|\, e_i) - c(e_i) \\
& = b \Pr(y - \Sigma_{j \neq i} e_j^* - e_i > e_i^* - e_i) + \mu e_i + \gamma' - c(e_i) \\
& = b(1 - H(e_i^* - e_i)) + \mu e_i + \gamma' - c(e_i)
\end{aligned}
$$

where $H()$ is the CDF for a $N(0, s^2)$ distribution and $\gamma' = \mu \Sigma_{j \neq i} e_j^*$. The FOC for the agent's choice is

$$bh(e_i^* - e_i) + \mu - c'(e_i) = 0$$

where $h()$ is the density; $h() = H'()$. The FOA is valid if the agent's optimal choice is e_i^* and is given by this first-order condition, i.e. if

$$bh(0) + \mu - c'(e_i^*) = 0 \qquad (6)$$

and no other effort $e_i \neq e_i^*$ yields a higher payoff for the agent. We note in passing that $h(0) = 1/\sqrt{2\pi var(y)}$, verifying the formula (4) above.

Due to the shape of the normal density, the agent's payoff is generally not concave. The second derivative is $-bh'(e_i^* - e_i) - c''(e_i)$, where $h'(e_i^* - e_i) < 0$ for $e_i < e_i^*$. The payoff is locally concave at $e_i = e_i^*$ (since $h'(0) = 0$), hence e_i^* is a local maximum, but there may be other local maxima (other solutions to FOC) for $e_i < e_i^*$. The situation is illustrated in Figure 1, which depicts the agent's marginal revenue ($bh(e_i^* - e_i) + \mu$) and marginal cost for two values

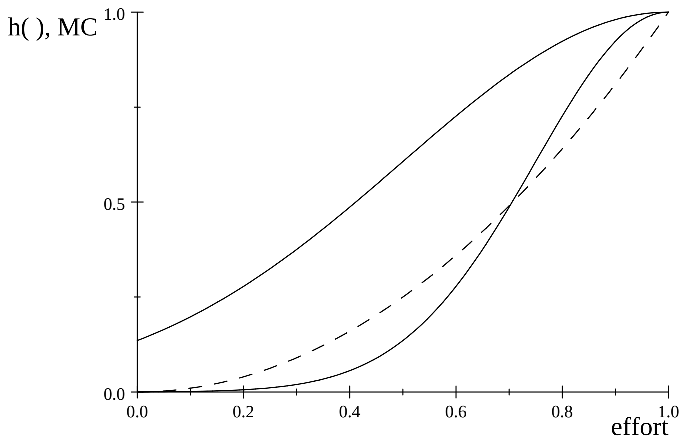

Figure 1: Illustration of FOC

of the variance $s^2 = var(y)$ (and for $\mu = 0$). If the variance is sufficiently small there is a local maximum at some $e_i < e_i^*$ (satisfying the FOC), and the figure indicates (comparing areas under MC and MR) that this local maximum dominates that at e_i^*. Indeed, since the density $h(0)$ is inversely proportional to the standard deviation s, we must have $b \to 0$ as $s \to 0$ (keeping e_i^* fixed). Since the probability of obtaining the bonus stays fixed in equilibrium (equal to $1 - H(0) = \frac{1}{2}$), independently of s, the expected bonus payment will go to zero as $s \to 0$, and the agent will surely deviate, because these expected payments will not cover the additional effort costs.

This shows that the FOA is valid here only if the variance of the performance measure (y) is not too small, and is confirmed in the following proposition, which also gives an estimate of which magnitude of s^2 is sufficient for the FOA to be valid for a parametric case (iso-elastic effort costs).

Proposition 3 *Given $e_i^* < e_i^{FB}$ (where $1 = c'(e_i^{FB})$) and $\mu \in \left[0, \frac{1}{2}\right)$, $\mu < c'(e_i^*)$, there is $s' > 0$ such that for $var(y) = s^2 > s'$ the FOA is valid (i.e. the agent's optimal choice is given by the FOC (6)). Moreover, it is then valid for any $\mu' \in (\mu, \frac{1}{2})$, $\mu < c'(e_i^*)$. There is also $s'' > 0$ such that FOA is not valid for $var(y) = s^2 < s''$*

For iso-elastic costs ($c(e) = ke^m$, $m \geq 2$) it further holds: For $\mu = 0$ FOA is *valid if* $\frac{e_i^*}{s} < K = K_0\sqrt{m-1}$*, where* $K_0 \approx 2.216$*. For* $m = 2$ *FOA is valid if* $\frac{e_i^* - \mu/2k}{s} < K' \approx 2.216$

Remarks. The conditions pertaining to iso-elastic costs are conditions that ensure a unique solution to the agent's FOC, and are hence sufficient, but not necessary conditions for FOA to be valid. Since e_i^* is expected output per agent in the model, the condition $\frac{e_i^*}{s} < K$ says that the standard deviation (SD) of total output relative to the individual mean should exceed the number $1/K = \frac{1}{K_0\sqrt{m-1}}$, which is decreasing in (the cost elasticity) m and is equal to $\frac{1}{2.216} = 0.451\,(45\ \%)$ for $m = 2$. This may seem like a relatively large fraction, but two remarks are relevant. First, since total expected output with n agents is ne_i^*, the condition says that the standard deviation of total output relative to the mean should exceed the number $1/Kn$, which amounts to 4.5 % for $n = 10$ agents when $m = 2$ Second, the model could have been specified with expected output per agent being $e_i + p$, with $p > 0$ being the expected output corresponding to "whistle as you work" effort, and hence the required ratio of s to expected output would be smaller.

2.4 A modified scheme

We saw in the previous section that for negatively correlated agents, the variance in the performance measure y could be made quite small by including many agents in the team. And we saw that this was beneficial for incentives and consequently for efficiency as long as the analysis building on FOA was valid. But for sufficiently small variance FOA is not valid, so this immediately raises the question of what a team can acheive under such circumstances. In the following we will consider a scheme that is at least approximately optimal for small variance, in the sense that it generates a surplus that converges to the optimal surplus when the variance goes to zero.

Mainly to simplify notation, consider here the case $\eta = 0$, so that the EC constraint is $0 \leq b(y) \leq \frac{\delta}{1-\delta}W(e)$.[7] To provide incentives, the bonus cannot be maximal for all outputs y, hence the expected bonus payment for an agent must be less than the maximal bonus, i.e. $E(b(y)|\, e) < \frac{\delta}{1-\delta}W(e)$. On the other hand, the agent's expected payoff from exerting effort must be non-negative; $E(b(y)|\, e) - c(e) \geq E(b(y)|\, e = 0) \geq 0$, so in any equilibrium we must have $c(e) < \frac{\delta}{1-\delta}W(e)$. It follows from this that the effort e_u^* and associated surplus $W(e_u^*)$ defined by

$$c(e_u^*) = \frac{\delta}{1-\delta}W(e_u^*) \tag{7}$$

constitute upper bounds for, respectively, the effort and surplus (per agent) that can be acheived in a relational contract. Note also that this upper bound can be acheived if there is no uncertainty, i.e. if (team) effort can be observed without noise; namely by paying the maximal bonus $b = c(e_u^*)$ to each agent conditional on total effort being at least ne_u^*.

We will now provide an incentive compatible and feasible scheme that converges to the upper bound as the variance in the performance measure goes to zero. The scheme is a simple modification of the threshold bonus scheme identified in Proposition 1, and consists of a relaxation of the threshold combined with an increase of the bonus relative to the latter scheme.

The problem we identified with the latter scheme was that for sufficiently small s the agent's payoff had two local maxima, at e^* and at $e^0 < e^*$, respectively, and that e^0 gave the highest payoff, so the agent would deviate from the supposed equilibrium effort e^*. The critical s is where the two local maxima yield the same payoff; i.e. $b(1 - H(0; s)) - c(e^*) = b(1 - H(e^* - e^0; s)) - c(e^0)$, where we as above have $\Pr(y > y_0|\, e) = 1 - H(e^* - e; s)$ and $H(\cdot; s)$ is the CDF for a $N(0, s^2)$ variable. In addition they both satisfy FOC, so $bh(e^* - e^0; s) = c'(e^0)$ and $bh(0; s) = c'(e^*)$.

For s below this critical level, the agent's payoff is higher at e^0. Now, this can be rectified by setting a lower threshold $y_0' < y_0 = ne^*$, i..e making it

[7] Also, to simplify notation, we drop subscripts, so e, b etc are scalars here.

easier to obtain the bonus, and at the same time increase the bonus level. For $y_0' = y_0 - \tau$ we have

$\Pr(y > y_0' | \, e_{j \neq i} = e^*, e_i) = \Pr(y - \Sigma_k e_k > ne^* - \tau - \Sigma_k e_k | \, e_{j \neq i} = e^*, e_i) = 1 - H(e^* - e_i - \tau; s)$

We can then choose τ and the bonus b such that e^* satisfies FOC and yields a payoff at least as high as the other local maximum e^0, i.e. such that we have

$$b(1 - H(-\tau; s)) - c(e^*) \geq b(1 - H(e^* - e^0 - \tau; s)) - c(e^0) \tag{8}$$

and

$$bh(-\tau; s) - c'(e^*) = 0 = bh(e^* - e^0 - \tau; s) - c'(e^0) \tag{9}$$

The smaller is τ, the smaller is the required bonus to satisfy FOC for e^*. The minimal such τ yields equality between the payoffs.

Now, this scheme can at most allow a bonus

$$b = \frac{\delta}{1 - \delta} W(e^*) \tag{10a}$$

Hence we see that the highest effort e that can be implemented by this scheme is the effort e^* defined by these conditions (8 - 10a); including equality in the first one.

For given s (below the critical level where FOA ceases to be valid), we cannot say whether the scheme is optimal. But we can show that the effort e^* it induces converges to the upper bound e_u^* identified above (see the appendix). Hence it induces an effort e^* and associated surplus $W(e^*)$ that are, for sufficiently small s, arbtrariliy close to the upper bounds for these entities that can be achieved in any relational contract. Thus we have

Proposition 4 *The modified threshold bonus scheme defined by (8 - 10a) is asymptotically optimal as the variance of output y goes to zero.*

It may be noted that for the set of variances $s^2 = var(y)$ sufficiently large to make FOA valid, the largest effort per agent that can be implemented must satsify $2c(e^*) \leq \frac{\delta}{1-\delta}W(e^*)$, and hence be considerably smaller than the upper bound e_u^* defined in (7). This is so because the agent obtains the bonus (b) with probability $\frac{1}{2}$ in equilibrium in the FOA scheme, hence we must have $b\frac{1}{2} \geq c(e^*)$ in that setting. This illustrates that a more precise performance measure can yield considerable benefits in relational contracting. The benefits are not associated with risk reduction (since all agents are risk neutral by assumption), neither with sharper competition, since in the team setting there is none. The benefits arise because a more precise measure strengthens individual incentives for effort, for a given bonus level. Since the bonuses in the relational contract are discretionary and hence must be kept within bounds, the added effort incentives coming from a more precise performance measure are valuable. And the value added may be considerable, as we have seen.

Thus far we have in this subsection taken the output variance ($s^2 = var(y)$) as an exogenous parameter. In Section 2.2 we pointed out that this variance can be substantially reduced if a team can be put together, consisting of several agents whose individual outputs are negatively correlated. As we now have illustrated, this may be of considerable value for the participants in the relational contract.

3 Individual outputs observed

Consider now the case where individual outputs are observable. The principal can then offer a bonus contract $\beta_i(x_1...x_n)$, to each agent $i = 1...n$, conditional on all individual outputs. Now, if the contract is expected to be honored, agent i's expected wage is then, for given efforts, $w_i = E(\beta_i(x_1...x_n)|\, e_1...e_n) + \alpha_i$, while the principal expects $\Sigma\bar{x}(e_i) - \Sigma w_i$. The agent then chooses effort

$$e_i = \arg\max_{e_i'} \left(E(\beta_i(x_1...x_n)|\, e_i', e_{-i}) - c(e_i')\right) \tag{11}$$

Like in the case where individual output is unobservable, we assume that if the contract is not honored, the parties instead bargain over the realized values. But now the principal agrees on a spot price ηx_i with each individual agent. In a one shot relationship, the parties still have no incentives to honor the bonus contract, and so they have to rely on spot contracting. Expected spot price is then $S = \eta \bar{x}(e_i^s)$. Agent i thus chooses spot effort e^s according to $\eta \bar{x}'(e_i^s) - c'(e_i^s) = 0$, while the principal's expected spot profit is given by $\pi_s = (1-\eta)\bar{x}(e_i^s)$. Note here that spot effort is higher than in the team case since the marginal revenue from effort $\eta \bar{x}'(e_i^s)$ is not divided by n.

In a repeated relationship, we still assume that the principal honors the contract only if all agents honored the contract in the previous period, and that the agents honor the contract only if the principal honored the contract with all agents in the previous period.

Now, (given that the IC condition (11) holds) the principal will honor the contract with all agents $i = 1, 2, ..., n$ if

$$-\Sigma_i \beta_i(x_1...x_n) + \frac{\delta}{1-\delta}\Pi \geq -\Sigma_i \eta x_i + \frac{\delta}{1-\delta}[n\pi_s] \tag{12}$$

Agent i will honor the contract if

$$\beta_i(x_1...x_n) + \frac{\delta}{1-\delta}(w_i - c(e_i)) \geq \eta x_i + \frac{\delta}{1-\delta}(S - c(e_i^s)) \tag{13}$$

These enforcement constraints are stricter than in the team case where individual output is not observable. The reason is that the spot surplus is higher, and so the long-term costs from deviating from the relational contract are lower. This in turn makes it possible to implement higher effort under team incentives, as will be discussed later.

Define 'modified' (net) bonuses: $b_i(x_1...x_n) = \beta_i(x_1...x_n) - \eta x_i$.It is then straightforward to show (as in the previous case where only $y = \Sigma_i x_i$ is observed) that we have:

Lemma 2 *For given efforts $e = (e_1...e_n)$ there is a wage scheme that satisfies (11),(13)-(12) and hence implements e, iff there are bonuses β and fixed salaries α with $b_i(x_1...x_n) = \beta_i(x_1...x_n) - \eta x_i \geq 0$, such that (11) and condition (14) below holds:*

$$\Sigma_i b_i(x_1...x_n) \leq \frac{\delta}{1-\delta}\left(\Sigma_i W(e_i) - nW(e^s)\right) \tag{14}$$

Here $W()$ denotes as before surplus per agent; $W(e_i) = E(x_i|\, e_i) - c(e_i)$. Assuming as before that FOA is valid, we can replace the IC constraint (11) with the first-order condition:

$$\frac{\partial}{\partial e_i}(E(\, b_i(x_1...x_n)|\, e_1...e_n) + \eta \bar{x}'(e_i) = c'(e_i) \tag{15}$$

The optimal contract then maximizes total surplus $(\Sigma_i W(e_i))$ subject to (14) and (15).

3.1 Independent outputs

Consider first independent outputs. This was analyzed by Levin (2002), who showed that the optimal contract is RPE with a bonus paid to at most one agent, namely the agent whose outcome yields the highest likelihood ratio. Moreover, the bonus is paid to this agent only if the likelihood ratio is positive. Given symmetric agents and strictly increasing likelihood ratios, this means that the agent with the largest output wins the bonus, but provided that his output exceeds some threshold x_0 (where the likelihood ratio $\frac{f_e(x_i;e_i)}{f(x_i;e_i)}$ is positive for $x_i > x_0$).

We will now use this result to analyze how the efficiency of this scheme varies with the number of agents (for independent ouputs). The next section considers correlated outputs.

With n agents, agent i's probability of winning the bonus b, given own output $x_i = x > x_0$, and given efforts $e_j = e$ from all others is now

$\Pr(\max_j x_j < x) = F(x;e)^{n-1}$. Hence the expected bonus payment to agent i is $b\int_{x_0}^{\infty} F(x_i;e)^{n-1} f(x_i;e_i)dx_i$, and the IC condition (15) takes the form:

$$b\int_{x_0}^{\infty} F(x_i;e)^{n-1} f_e(x_i;e_i)dx_i + \eta\bar{x}'(e_i) = c'(e_i) \tag{16}$$

In passing, it is worth noting that the integral here extends only over values of x_i where $f_e(x_i;e_i) > 0$. In a standard tournament, where agent i would obtain a bonus when he had the largest output, the integral would extend over all values of x_i. The payment scheme here, which we may call a modified tournament, thus provides stronger incentives (for a given bonus b) than a standard tournament scheme.

The optimal RPE bonus is maximal, i.e. $b = \frac{\delta}{1-\delta}(\Sigma_i W(e_i) - nW(e^s))$, where $W(e_i)$ is total surplus (for agent i) and $W(e^s)$ is the outside spot surplus per agent. Hence from (16) we have, in symmetric equilibrium

$$\frac{c'(e) - \eta\bar{x}'(e)}{\int_{x_0}^{\infty} F(x;e)^{n-1} f_e(x;e)dx} = b = \frac{\delta}{1-\delta} n(W(e) - W(e^s)) \tag{17}$$

It should be noted that the threshold x_0, which is defined by $f_e(x_0;e) = 0$, will generally depend on the equilibrium effort e.

Consider now variations in the number of agents. Higher n increases the competition to obtain the bonus (the probability of winning is reduced), so the bonus must be increased to maintain effort; this is captured by the first equality in (17). The second equality shows how much the bonus can be increased; namely by the increased total surplus. The question is then whether the latter is sufficient to compensate for the reduced probability of winning.

The answer is affirmative, and the reason is essentially that while the surplus on the RHS increases proportionally with n, the margnal probability (in the denominator) on the LHS decreases less rapidly, so that $n\int_{x_0}^{\infty} F(x;e)^{n-1} f_e(x;e)dx$ increases with n. This allows a higher effort (per agent) to be implemented, so we have:

Proposition 5 *For observable and independent individual outputs, effort per agent in the RPE scheme (the modified tournament) increases with the number of agents.*

When individual output measures are available, and these outputs are independent, we thus see that efficiency in the (modified) tournament is improved by including more agents. This is in sharp contrast to efficiency in a team for independent outputs: as we saw above the team efficiency rapidly decreases under such conditions.

3.2 Correlated outputs

Consider now correlated outputs. For tractability reasons we will then again consider normal distributions, and moreover limit attention to symmetric agents. A convenient feature of the multinormal distribution is that likelihood ratios are linear functions of the variables, and this simplifies comparisons of such ratios for these variables.

So assume now $x = (x_1...x_n)$ multinormal with $Ex_i = e_i$, $var(x_i) = s^2$ and (identical) correlations $corr(x_i, x_j) = \rho$. From the form of the multinormal distribution (see the appendix) the likelihood ratio for x_i is then

$$\frac{f_{e_i}(x|\, e_1...e_n)}{f(x|\, e_1...e_n)} = k_1(x_i - e_i) + k_2\Sigma_{i\neq j}(x_j - e_j) \tag{18}$$

with

$$k_1 = \tfrac{1+(n-2)\rho}{(1+(n-1)\rho)(1-\rho)s^2} > 0, \qquad k_2 = \tfrac{-\rho}{(1+(n-1)\rho)(1-\rho)s^2}$$

Note that $k_1 - k_2 = \frac{1}{(1-\rho)s^2} > 0$

As we show in the appendix, for symmetric agents the optimal symmetric scheme pays a maximal bonus to the agent with the highest likelihood ratio, provided this ratio is positive, and no bonus to the other agents. From symmetry (including symmetric efforts in equilibrium; $e_i = e^*$ all i) the agent with the highest output has the highest likelihood ratio, and this ratio is positive iff

$$x_i > e^* + \frac{\rho}{(n-2)\rho + 1} \Sigma_{j \neq i}(x_j - e^*) = E(x_i | x_{-i}) \tag{19}$$

This condition says that agent i's performance must exceed his expected performance, conditional on the performance of all other agents. Thus we have:

Proposition 6 *The optimal symmetric scheme pays a maximal bonus to the agent (say i) with the highest output, provided this output satisfies $x_i > E(x_i | x_{-i})$.*

Condition (19) can alternatively be interpreted as saying that the agent's deviation from the mean ($x_i - e^*$) must exceed some factor K ($K = \frac{(n-1)\rho}{(n-2)\rho+1}$) of the average performance deviation ($\frac{1}{n-1}\Sigma_{j \neq i}(x_j - e^*)$) for all the other agents. The factor K is increasing in ρ and varies from $-(n-1)$ to 1 over the permissible range for $\rho \in (-\frac{1}{n-1}, 1)$.

For positive correlation ($\rho > 0$) condition (19) is irrelevant if the other agents on average overperforms ($\frac{1}{n-1}\Sigma_{j \neq i} x_j > e^*$). To get the bonus agent i must then have the highest output; this must therefore exceed the average output from the others, and hence exceed the fraction $K < 1$ of this output. But if the other agents underperform, the required condition (19) says that agent i is allowed to underperform by at most a fraction K of their average underperformance.

For negative correlation ($\rho < 0$) the condition is again relevant only if the other agents on average underperforms ($\frac{1}{n-1}\Sigma_{j \neq i} x_j < e^*$). If that is the case, the condition then requires that agent i must overperform by at least a factor $|K| = \frac{(n-1)|\rho|}{(n-2)\rho+1}$ of their underperformance.[8]

[8] So if they on average underperform by 1% ($\frac{\frac{1}{n-1}\Sigma_{j \neq 1} x_j - e_1}{e_1} = -\frac{1}{100}$), agent i must

For $n = 2$ agents we now have that agent 1 gets the bonus if and only if he has the highest output ($x_1 > x_2$) and $x_1 - e^* > \rho(x_2 - e^*)$. This is illustrated in the figure below for $\rho = \frac{1}{2}$ (left) and $\rho = -\frac{1}{2}$(right). Agent 1 is to get the bonus for outcomes to the right of the broken line.

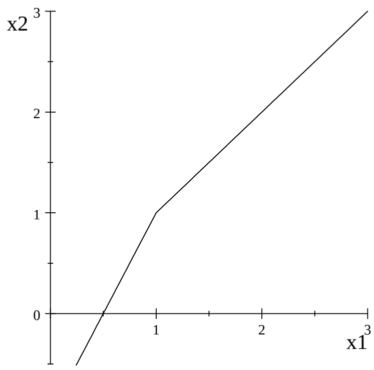

Figure 2a

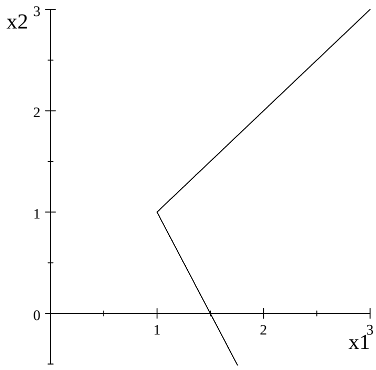

Figure 2b

In both cases the agent with the highest output gets the bonus if both of them have outputs that are above average ($x_1, x_2 > Ex_i = e^*$) . If agent 2 has below average output ($x_2 < Ex_i = e^*$) the requirement for agent 1 to get the bonus is less strict when there is positive correlation than when there is negative correlation. In the latter case, agent 1 must have an output well above average to obtain the bonus, and more so the worse is the output for agent 2. Under negative (positive) correlation, a bad performance by agent 2 raises (lowers) the expected *conditonal* performance of agent 1, and thus raises (lowers) the requirement –the hurdle– for agent1 to get the bonus.[9]

Having characterized the optimal scheme, we will now consider its incentive properties. To make the analysis tractable, we restrict attention to $n = 2$ agents. Consider then agent 1's incentives in this scheme, with 'reference

overperform by at least $|K|\%$ ($\frac{x_1-e_1}{e_1} > \frac{(n-1)\rho}{(n-2)\rho+1}(\frac{\frac{1}{n-1}\Sigma_{j\neq 1}x_j - e_1}{e_1}) = K(-\frac{1}{100}) = \frac{|K|}{100}$).

[9]To illustrate these points, if $\rho = .5$, and agent 2 has output 10% below expected ($x_2/e^* = .9$), agent 1 can only win if his output no more than 5% below expected. But if $\rho = -.5$, agent 1 must perform at least 5% *better* than expected in order to be eligible for the bonus (if in addition he wins).

point' (equilibrium) $e_1^* = e_2^*$. His probability of obtaining the bonus is

$$\Pr(x_1 > \max[x_2, e_1^* + \rho(x_2 - e_2^*)]) \equiv \Pr(B) = \int_{x \in B} f(x|\, e_1 e_2^*) \qquad (20)$$

So the marginal gain from effort is $\int_B f_{e_1}(x|\, e_1 e_2^*)$ and in symmetric equilibrium $e_1^* = e_2^* = e^*$ we will then have (given FOA valid)

$b \int_B f_{e_i}(x|\, e^*, e^*) + \eta - c'(e^*) = 0$

An interesting question is then: for given effort e^* to be implemented, how do marginal incentives vary with correlation ρ? E.g. do these marginal incentives get stronger when ρ increases, implying that a lower bonus is required to implement the same effort? We should bear in mind that this is a RPE scheme and that such schemes generally work well both for positive and negative correlations in other settings. Perhaps not surprisingly a similar property turns out to be true here.

Proposition 7 *For correlated variables and $n = 2$: Provided FOA is valid, the agent's FOC for (symmetric) equilibrium effort is*

$$b \frac{1}{\sqrt{2\pi} s} \frac{1}{2} \left(\frac{1}{\sqrt{1-\rho^2}} + \frac{1}{\sqrt{1-\rho}} \frac{1}{\sqrt{2}} \right) + \eta = c'(e^*) \qquad (21)$$

The marginal incentive in FOC (i.e. the expression on the LHS) is increasing in ρ for $\rho > \rho_0 \approx -0.236$ and decreasing in ρ for $\rho < \rho_0$. Hence, implementing a given effort requires a lower (higher) bonus when the correlation ρ increases for $\rho > \rho_0$ (for $\rho < \rho_0$).

This is illustrated in Figure 2, which depicts the marginal incentive as a function of ρ for the RPE scheme and for a standard tournament (dashed line).

As a function of ρ, the marginal incentive (MI) for effort is thus U-shaped in the optimal scheme, which again is a modified tournament. In comparison,

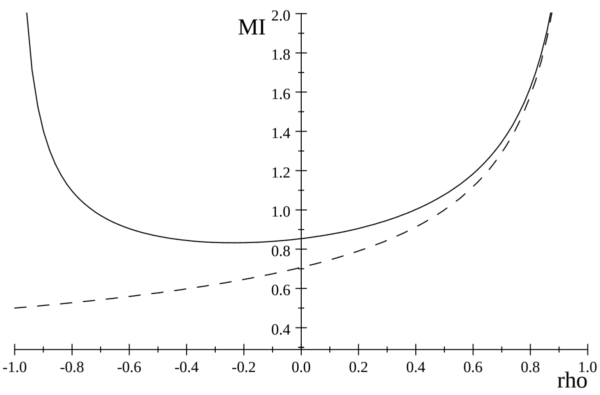

Figure 2: Marginal incentives as function of ρ

in a standard tournament the MI is monotone increasing in ρ (as shown by the dotted line; this MI is given by $\frac{d}{de_1}\Pr(x_1 > x_2) = \frac{1}{\sqrt{2\pi}s_d}$, where $s_d = \sqrt{2(1-\rho)}s$ is the SD of $x_1 - x_2$, and the formula follows from the normal distribution). In comparison the modified tournament yields higher MI for effort for every ρ (which allows a higher effort to be implemented with the same bonus), and the MI is high both for strongly positively correlated and for strongly negatively correlated outputs.

The latter property is caused by the specific criteria to obtain the bonus in the modified tournament, cfr the figures depicted above. In a standard tournament (ST) agent 1 wins and gets a bonus if $x_1 > x_2$, while in the modified tournament he gets a bonus only if $x_1 > x_2$ and $x_1 - e^* > \rho(x_2 - e^*)$. So the probability of obtaining the bonus is (all else equal) higher in ST, but the marginal effect of own effort on the probability (the marginal incentive MI), is higher in the modified tournament.

3.3 The validity of FOA

So far we have assumed FOA to be valid; this issue will now be examined more closely for the RPE scheme derived above. The question is then whether, for given symmetric efforts $e_1^* = e_2^*$ to be implemented by the modified tournament scheme, these efforts are indeed optimal choices for the respective agents.

In the appendix we show that the marginal gain to effort for agent 1 in the modified tournament scheme can be written as

$$\frac{b}{s}\Gamma(\frac{e_1 - e_1^*}{s}; \rho) + \eta - c'(e_1)$$

where $\Gamma(a; \rho)$ is a bell-shaped function defined as follows

$$\Gamma(a; \rho) = \frac{1}{\sqrt{1-\rho^2}}\frac{1}{2}\phi(\frac{a}{\sqrt{1-\rho^2}}) + \frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\phi(\frac{a}{\sqrt{2}\sqrt{1-\rho}})(1-\Phi(\frac{-a}{\sqrt{1+\rho}\sqrt{2}})), \tag{22}$$

where $\phi(z)$ is the standard normal density and $\Phi(z)$ its CDF. The FOC for $e_1 = e_1^*$ to be optimal (as stated in the Proposition above) can thus be written as $\frac{b}{s}\Gamma(0; \rho) + \frac{1}{2}\eta - c'(e_1^*) = 0$, and the local second order condition takes the form $\frac{b}{s}\Gamma_a(0; \rho)\frac{1}{s} - c''(e_1^*) \leq 0$ Since $\Gamma_a(0; \rho)$ turns out to be positive, these conditions imply that the standard deviation s cannot be too small. This is thus a necessary requirement for FOA to be valid in this setting. Moreover, we can also see that a 'large' s is sufficient for FOC to have a unique solution, and hence sufficient for FOA to be valid. More specifically we have the following result.

Proposition 8 *For given effort $e_i^* \leq e_i^{FB}$, a necessary condition for FOA to be valid is that*

$$\frac{e_i^*}{s} \leq \frac{m'}{1 - \eta/c'(e_i^*)}\sqrt{\pi}(\sqrt{2} + \sqrt{1+\rho})$$

where m' is the (local) elasticity of the marginal cost function; $m' = e_i^* \frac{c''(e_i^*)}{c'(e_i^*)}$. Moreover, there is $s' > 0$ such that FOA is valid for $s > s'$.

Since $\rho < 1$ then for $\eta = 0$ the necessary condition implies $\frac{e_i^*}{s} \leq m'K$, with $K = 2\sqrt{2\pi} \approx 5.01$, and hence that he standard deviation s of individual output must exceed a fraction $\frac{1}{m'K} \approx \frac{0.2}{m'}$ of its mean (e_i^*). Moreover, for ρ smaller this fraction must be larger, and in the limit ($\rho \to -1$) twice as large ($\frac{1}{m'K/2}$). Numerical calculations for iso-elastic costs with $m' = 1, 2$ indicate that s exceeding the latter fraction is also sufficient. For $\eta > 0$ the fraction can be smaller.

4 Teams or tournaments?

If individual outputs are observable, the principal may of course choose to base any discretionary bonuses only on aggregate output. Hence, if the relational contract constraints are unaffected by such a choice, the principal cannot do better with a scheme of the latter type. The RPE scheme based on individual outputs will then always be optimal. It follows that, if there is a choice between two equally costly technologies allowing for observation of, respectively, individual or aggregate output, the technology allowing individual output to be observed will be chosen. A (modified) tournament will then dominate a team.

However, if a chosen technology is costly to modify later on, the picture is no longer so clear. The reason is that relational contract constraints may be affected, in the sense that the respective outside options associated with spot trading will be different under the two technologies. Due to the free rider problem, spot trading is less efficient when only team output can be observed. Hence, if a team setup is chosen initially, then if the relational contract should break down, either a costly reorganization to individual output measurements and subsequent spot trading will take place, or (if reorganization costs are sufficiently high) spot trading based on team output will be the way the parties proceed. In any case the spot surplus will be smaller if the team organization was chosen initially. This implies in turn that the relational contract constraints are affected by the initial choice, and then it is no longer so clear that the team organization will be inferior. We will now examine this issue.

In the following we will assume that a reorganization of the team is so costly (relative to its benefits) that it will not take place if contract breakdown and subsequent spot trading should occur. The issue to be considered is then whether the surplus generated by the relational contract for the team (analyzed in Section 2) may dominate the surplus under the relational RPE contract based on individual outputs (analyzed in Section 3). Now, for each contract there will be a critical magnitude of the discount factor, say δ^{FB}, such that the contract generates the first-best surplus for $\delta \geq \delta^{FB}$, but not so for $\delta < \delta^{FB}$. A relatively simple way to compare the contracts is to compare their respective critical factors. The contract with the lower δ^{FB} will, for a range of $\delta' s$ exceeding the lower δ^{FB} strictly dominate the other.

For independent outputs, we know that the efficiency of the RPE tournament scheme improves with increasing number of agents, while the team's efficiency rapidly decreases with more agents. The team can thus only dominate if the number of agents is relatively small. In fact, for iso-elastic costs and independent outputs, the optimal team size (with respect to efficiency) turns typically out to be quite small ($n = 2$ or $n = 3$, depending on the magnitude of the elasticity and the magnitude of η). For quadratic costs (elasticity 2) we have the following.

Proposition 9 *For independent outputs and quadratic effort costs we have: the optimal team size (in the sense of having the lowest critical δ^{FB}) is $n = 2$ if $\eta < \eta_0 \approx 0.805$, and $n = 3$ if $\eta > \eta_0$. Moreover, for $n = 2$ agents, the team dominates the RPE tournament (in the same sense) if and only if $\eta > \eta_1 \approx 0.739$.*

Consider next correlated outputs. For negatively correlated outputs, we know that the optimal team size may be large (Section 2.2), and that the efficiency of the team may be quite high. It thus seems reasonable to conjecture that, under such conditions, a team may dominate the RPE tournament even for n large. The analysis of this issue is hindered, however, by the optimal RPE scheme being difficult to analyze for correlated outputs and arbitrary $n > 2$.

So we must at this stage confine the analysis to a comparison of the two schemes for $n = 2$ when outputs are correlated.

From the previous analysis we know that the RPE tournament has high efficiency both for strongly positive and strongly negative correlation. Since the team's efficiency is decreasing in ρ, it is thus to be expected that the tournament will tend to dominate for positive ρ. However, for negative ρ both schemes become more efficient with stronger (negative) correlation, hence it is not so clear what will happen there. It turns out that that the team's efficiency improves relatively more for strongly negative ρ, as shown in the following proposition.

Proposition 10 *For correlated multinormal outputs, quadratic costs and* $n = 2$ *we have: A Team dominates the RPE tournament in the sense of having a lower* δ^{FB} *iff* $\eta > \eta_0(\rho)$, *where* $\eta_0(\rho)$ *is increasing in* ρ *with* $\eta_0(\rho) \to 0$ *as* $\rho \to -1$, $\eta_0(\rho) \to 1$ *as* $\rho \to 1$, *and* $\rho_0(0) \approx 0.739$ *. This holds irrespective of the magnitude of* s *(the standard deviation for individual output), but for each* ρ, s *must be sufficiently large so that FOA is valid.*

The function $\eta_0(\rho)$ is depicted in Figure 3. The RPE tournament has highest critical δ^{FB} above the curve, and is hence dominated by the team there. For high ρ the RPE tournament does comparatively better in the sense that the the parameter set for which it is dominated is smaller. For strong negative correlations the opposite occurs: the team dominates there even for comparatively small parameter η.

5 Concluding remarks

Many businesses organize their employees in teams. According to Lawler (2001), 72 percent of Fortune 1000 companies make use of work teams, defined as groups of employees with shared goals or objectives. A large management literature has thus emerged investigating team composition, team

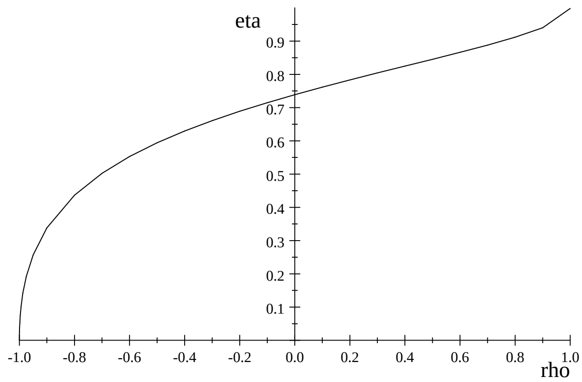

Figure 3: Illustration for Proposition 10

compensation, team leadership, and so forth. However, this literature is mainly empirical, and the theoretical literature is conceptual rather than formal.

The economics literature on teams is, in comparison, rather small. Theory has mainly focused on how the well-known free-rider problem can be solved or mitigated, while questions related to team size and team composition has remained unanswered, or not even asked. Moreover, endogenous formation of teams, in which firms deliberately choose to hold a team of workers accountable for their joint output, is not well understood.

Our paper contributes to the literature by deriving testable theoretical predictions on team incentives, team size, team composition and team formation. We've done so by analyzing optimal self-enforcing (relational) contracts between a principal and a set of agents where only aggregate output can be observed. We have then considered how the efficiency of the contract is affected by variations in the number of agents and in the correlations between the agents. Finally, we have compared with a situation where individual output is observable.

First, we showed that the optimal team contract entails an incentive scheme in which each agent is paid a maximal bonus for aggregate output above a threshold and a minimal bonus otherwise. We then considered optimal team size. To the extent this is studied in the formal literature, the standard result is that more agents increases the free-rider problem and thus weakens incentives and effort. In our model, this is not necessarily the case. More agents in a team have three effects: First, it reduces the marginal incentive effect of a given bonus, which is the standard 1/n free-rider problem. Second, it also reduces the teams' outside option. This strengthens the relational contract and thus allows for higher-powered incentives and thus higher effort. This positive effect of more agents is particularly strong if the agents' ex post bargaining power is high. Finally, it affects the variance of the performance measure. For positive correlations between the agents' outputs, the variance increases, while for negative correlations the variance is reduced. The latter is beneficial for the team because it increases the marginal incentives for each team member to provide effort.

Our model thus predicts that teamwork is more robust and more efficient when the team has high (ex post) bargaining power and when the team members' outputs are negatively correlated. The former implies that teamwork is more efficient (or prevalent) when the team is in a position to hold up values and sell their products in an alternative market. This is typically the case in human capital-intensive industries where groups of employees can potentially walk away with ideas, clients, innovations, etc

The latter - negative correlations - relates to questions concerning optimal team composition. In the management literature a central question is whether teams should be homogenous or heterogeneous with respect to both tasks (functional expertise, education, organizational tenure) and bio-demographic charachteresics (age, gender, ethnicity). One can conjecture that negative correlations are more associated with heterogeneous teams than homogenous teams, and also more associated with task-related diversity than with bio-demographic diversity. There is no reason to believe that e.g. men and women's outputs are negatively correlated. However, workers with different functional expertise may be differently exposed to common shocks, or meet different sets of demands from customers or superiors. This can create negative output correlations.

Interestingly, a comprehensive meta-study by Horwitz and Horwitz (2007), investigating 35 papers on the topic, finds no relationship between bio-demographic diversity and performance, but a strong positive relationship between team performance and task-related diversity. An explanation is that task-related diversity creates positive complementarity effects. We point to an alternative explanation, namely that diversity may create negative correlations that reduces variance and thereby increases marginal incentives for effort. The team members "must step forward when others fail". Diversity and heterogeneity among team members can thus yield considerably efficiency improvements.[10]

We have also compared with a situation where individual output is observable. For a parametric (normal) distribution, we have shown that the optimal contract is an RPE (relative performance evaluation) scheme; a form of a tournament, where the conditions for an agent to obtain the (single) bonus are stricter for negatively compared to positively correlated outputs. The efficiency of the RPE contract is shown to increase with the number of agents, and to improve with higher correlation (both positive and negative).

Now, if the firm can initially choose between organizations that allow for (a) only aggregate output or (b) individual outputs to be observed, we show that the firm may choose (a), i.e. to organize production as a team. Thus, even if alternatives (a) and (b) are equally costly to set up initially, the team alternative may yield a higher subsequent surplus.

There are two reasons for this. One is that teams create worse outside options. This is particularly the case under high ex post bargaining power. When individual outputs are observable, high bargaining power creates quite efficient spot contracts, while under team production the free-rider problem dampens the efficiency of the spot contract. Hence, since worse outside options strengthens the relational contract, higher bargaining power favor the team alternative.

[10] Hamilton et al (2003) provides one of a very few empirical studies on teams within the economics literature. They find that more heterogeneous teams (with respect to ability) were more productive (average ability held constant).

Second, negative correlations are even more beneficial for the relational team contract than for the relational RPE contract. That is, although efficiency in both alternatives increases with more negatively correlated outputs, the team alternative is more likely to be superior under such conditions. Hence, according to our model, team work is not only more robust and efficient under high bargaining power and negatively correlated outputs. The likelihood for firms to deliberately choose the team alternative, even if individual output is observable, is also higher under these conditions.

APPENDIX

Proof of Lemma 1:

It remains to verify that the conditions are necessary. Given a scheme $\tilde{\alpha}, \tilde{\beta}$ that satisfies (IC, EA, EP) and hence implements e. Let $\inf_y(\tilde{\beta}_i(y) - \frac{1}{n}\eta y) \equiv \xi_i$, and let $\beta_i(y) = \tilde{\beta}_i(y) - \xi_i$ and $\alpha_i = \tilde{\alpha}_i + \frac{1}{\delta}\xi_i$. Then IC holds for β. Moreover, $\inf_y(\beta_i(y) - \frac{1}{n}\eta y) = 0$ and EA holds for (α, β), since each agent's payoff is (by construction) unchanged for each realization y:

$$\beta_i(y) + \frac{\delta}{1-\delta}(\alpha_i + E(\beta_i(y)|\, e) - c(e_i)) = \tilde{\beta}_i(y) + \frac{\delta}{1-\delta}(\tilde{\alpha}_i + E(\tilde{\beta}_i(y)\Big|\, e) - c(e_i))$$

Then EP also holds, since the principal's payoff must be unchanged as well:

$$-\Sigma_i\beta_i(y) + \frac{\delta}{1-\delta}\Sigma_i(E(x_i|\, e) - E(\beta_i(y)|\, e) - \alpha_i) = -\Sigma_i\tilde{\beta}_i(y) + \frac{\delta}{1-\delta}\Sigma_i(E(x_i|\, e) - E\tilde{\beta}_i(y) - \tilde{\alpha}_i).$$

Taking inf and sup in EA and EP, respectively, and adding, we get

$$\sup_y\left(\Sigma_i\beta_i(y) - \eta y\right) - \Sigma_i \inf_y\left(\beta_i(y) - \frac{1}{n}\eta y\right) \leq \frac{\delta}{1-\delta}\Sigma_i(W(e_i) - W_n(e^s)$$

This shows that EC holds for β, since $\inf_y\left(\beta_i(y) - \frac{1}{n}\eta y\right) = 0$ for all i. This completes the proof.

Proof of Proposition 1.

The Lagrangian for the problem of maximizing total surplus ($\Sigma_i W(e_i) \equiv \Sigma_i(E(x_i|\, e_i) - c(e_i))$) subject to EC and the 'modified' IC constraint (2) is:

$$L = \Sigma_i W(e_i) + \int \left(\frac{\delta}{1-\delta}\left(\Sigma_i W(e_i) - nW_s\right) - \Sigma_i b_i(y)\right)\lambda(y)dy$$
$$+\Sigma_i\mu_i\left(\int b_i(y)g_l(y;l)l_{e_i}(e_1...e_n)dy + \frac{1}{n}\eta\bar{x}'(e_i) - c'(e_i)\right)$$

where we have used $\frac{\partial}{\partial e_i}E(b_i(y)|\, e_1...e_n) = \int b_i(y)g_l(y;l)l_{e_i}(e_1...e_n)dy$. This yields

$$\frac{\partial L}{\partial b_i(y)} = \mu_i g_l(y;l)l_{e_i}(e_1...e_n) - \lambda(y) \leq 0, \qquad b_i(y) \geq 0, \qquad \text{(compl slack)}$$

If $\mu_i = 0$, then $b_i(y) = 0$ for all y, hence $e_i = e_i^s$. For $b_i(y) > 0$ we have $\mu_i g_l(y,l)l_{e_i} = \lambda(y)$ and hence from (2)

$$\mu_i(c'(e_i) - \frac{1}{n}\eta\bar{x}'(e_i)) = \int b_i(y)\lambda(y)dy \geq 0$$

Thus for $e_i > e_i^s$ we cannot have $\mu_i < 0$. Hence we must have all $\mu_i > 0$.

For $y > y_0$ we have $g_l(y; l) > 0$ and hence $\lambda(y) > 0$, implying that EC is binding and at least one bonus is positive. In a symmetric solution the bonuses will thus all be equal and maximal for $y > y_0$.

On the other hand, for $y < y_0$ we have $g_l(y; l) < 0$ by MLRP and hence $b_i(y) = 0$ for all i.

Finally suppose $l(e_1...e_n) = \Sigma_i e_i$, and assume the solution is asymmetric; say that $e_i < e_j$. Let $b_0 = (b_i + b_j)/2$ and consider

$$\int b_0(y)g_l(y; l(e_1...e_n))dy = \frac{1}{2}\int b_i(y)g_l(y; l(e_1...e_n))dy + \frac{1}{2}\int b_j(y)g_l(y; l(e_1...e_n))dy$$

$$= \frac{1}{2}c'(e_i) + \frac{1}{2}c'(e_j) \geq c'(\frac{e_i+e_j}{2})$$

Hence the bonus $b_0(y)$ to each of i and j is feasible and would induce effort at least $\frac{e_i+e_j}{2} = e_0$ from each. Thus a slightly lower bonus to each is feasible and will induce effort e_0 from each. This yields higher value since the objective is concave. QED

Proof of Proposition 3.

To get a contradiction, suppose for given e_i^* that FOA is not valid for arbitrarily large s^2, and hence that, for any such s the agent's payoff is maximal for some $e_i = e_i(s) < e_i^*$. This optimum cannot be for $e_i(s) = 0$, since that would give payoff $b(1 - H(e_i^*)) + \gamma'$, where $bh(0) + \mu - c'(e_i^*) = 0$, and hence payoff (excluding γ') $\frac{c'(e_i^*)-\mu}{h(0)}(1 - H(e_i^*)) = (c'(e_i^*) - \mu)\int_{e_i^*}^{\infty} e^{-\frac{x^2}{2s^2}}dx$, which would become arbitrarily small for s sufficiently large. In comparison, the corresponding payoff for $e_i = e_i^*$ is $b(1-H(0))+\mu e_i^* - c(e_i^*) = \frac{c'(e_i^*)-\mu}{h(0)}\frac{1}{2}+\mu e_i^* - c(e_i^*)$, which is large for s large since $h(0) = \frac{1}{\sqrt{2\pi}s}$.

Now, for $e_i = e_i(s) > 0$ being an optimum, we must have $bh(e_i^* - e_i) + \mu - c'(e_i) = 0$, where $bh(0) + \mu - c'(e_i^*) = 0$ and hence $bh(0)(\frac{h(e_i^*-e_i)}{h(0)} - 1) = c'(e_i) - c'(e_i^*)$, where $bh(0) = c'(e_i^*) - \mu = const$. This implies (since $\frac{h(e_i^*-e_i)}{h(0)} = \exp(-\frac{(e_i^*-e_i)^2}{2s^2})$) that $e_i = e_i(s) \to e_i^*$, and hence that FOC has a solution

$e_i = e_i(s)$ arbitrarily close to e_i^* for s sufficiently large. But this is impossible, since the slope of the FOC expression at e_i^* is $bh'(0) - c''(e_i^*) = -c''(e_i^*)$, and hence there must be a left neighborhood where this expression is strictly positive for all s. This proves the first statement in the proposition.

The formulas for the agent's payoffs for $e_i = 0$ and $e_i = e_i^*$, i.e. $(c'(e_i^*) - \mu) \int_{e_i^*}^{\infty} e^{-\frac{x^2}{2s^2}} dx + \gamma'$ and $\frac{c'(e_i^*) - \mu}{h(0)} \frac{1}{2} + \mu e_i^* - c(e_i^*) + \gamma'$, respectively, show that the former will dominate for s sufficiently small (since $h(0) \sim \frac{1}{s}$) Hence FOA cannot be valid for s sufficiently small.

Now consider variations in μ. Suppose FOA is valid for $\mu \in \left[0, \frac{1}{2}\right)$ and hence that $b(1 - H(e_i^* - e_i)) + \mu e_i - c(e_i) < b(1 - H(0)) + \mu e_i^* - c(e_i^*)$ for all $e_i < e_i^*$ when $bh(0) + \mu - c'(e_i^*) = 0$. Consider $\mu' > \mu$ and let $b' < b$ be given by

$$b'h(0) + \mu' - c'(e_i^*) = 0, \quad \text{i.e.} \quad b'h(0) + \mu' = bh(0) + \mu$$

Then consider

$$\begin{aligned} & b'(H(e_i^* - e_i) - H(0)) + \mu'(e_i^* - e_i) \\ & = (b' - b)(H(e_i^* - e_i) - H(0)) + (\mu' - \mu)(e_i^* - e_i) \\ & \qquad + b(H(e_i^* - e_i) - H(0)) + \mu(e_i^* - e_i) \\ & > (b' - b)(H(e_i^* - e_i) - H(0) - h(0)(e_i^* - e_i)) + (c(e_i^*) - c(e_i)), \end{aligned}$$

where the inequality follows from FOA being valid for (μ, b). The CDF $H(x)$ is concave for $x > 0$ (since then $H''(x) = h'(x) < 0$), hence the term multiplying $(b' - b)$ is negative. Since $b' < b$ we then see that FOA is valid also for (b', μ').

Finally consider the case of iso-elastic costs. In general, a sufficient condition for FOA to be valid is that the agent's FOC has no solution for $e_i < e_i^*$. Due to the bell-shaped form of the normal density, this will occur if the variance s^2 exceeds a critical value that yields tangency between the agent's marginal revenue and marginal cost curves at some $e_i < e_i^*$ This critical s^2 is defined by the following conditions:

$$bh(0) + \mu - c'(e_i^*) = 0 = bh(e_i^* - e_i) + \mu - c'(e_i) \tag{23}$$

$$-bh'(e_i^* - e_i) = c''(e_i) \tag{24}$$

We have $h(x) = \phi(\frac{x}{s})\frac{1}{s}$, where $\phi(z) = e^{-z^2/2}/\sqrt{2\pi}$ is the standard normal density. For iso-elastic costs the above conditions (23 - 24) thus take the form

$$\tfrac{b}{s}\phi(0) + \mu - km(e_i^*)^{m-1} = 0 = \tfrac{b}{s}\phi(\tfrac{e_i^*-e_i}{s}) + \mu - km(e_i)^{m-1} \qquad \textit{and}$$

$$\tfrac{b}{s}\phi(\tfrac{e_i^*-e_i}{s})\tfrac{e_i^*-e_i}{s}\tfrac{1}{s} = km(m-1)(e_i)^{m-2}$$

Letting now

$$B = \tfrac{b}{s^m}\phi(0), \qquad \mu' = \tfrac{\mu}{s^{m-1}}, \qquad d^* = \tfrac{e_i^*}{s}, \qquad d = \tfrac{e_i}{s},$$

the above conditions, reflecting (23 - 24) can be written as

$$B + \mu' - km(d^*)^{m-1} = 0 = B\exp(-\tfrac{(d^*-d)^2}{2}) + \mu' - kmd^{m-1} \qquad \textit{and}$$

$$B\exp(-\tfrac{(d^*-d)^2}{2})(d^* - d) = km(m-1)d^{m-2}$$

For $\mu = 0$ this yields

$$\exp(\tfrac{(d^*-d)^2}{2}) = (\tfrac{d^*}{d})^{m-1} \qquad \textit{and} \qquad (d^* - d) = (m-1)d^{-1}$$

Letting $x = \frac{d^*}{d}$ this yields

$$d^2(x-1)^2/2 = (m-1)\ln x \qquad \textit{and} \qquad (x-1)d^2 = m-1$$

and hence x is the solution to $x - 1 = 2\ln x$, i.e. $x \approx 3.5129$. So we have $(d^*)^2 = d^2x^2 = \frac{m-1}{x-1}x^2$, and hence $\frac{e_i^*}{s} = d^* = K_0\sqrt{m-1}$, where $K_0 = \sqrt{\frac{x^2}{x-1}} \approx 2.216$. This proves the first assertion for iso-elastic costs.

To verify the second assertion, note that for $m = 2$ the conditions (23 - 24) defining the critical s take the form

$$B + \mu' - 2kd^* = 0 = B\exp(-\tfrac{(d^*-d)^2}{2}) + \mu' - 2kd \qquad and$$

$$B\exp(-\tfrac{(d^*-d)^2}{2})(d^* - d) = 2k$$

This yields

$$2k(d^*-d) = B\left(1 - \exp(-\tfrac{(d^*-d)^2}{2})\right) = 2k\exp(\tfrac{(d^*-d)^2}{2})(d^*-d)^{-1}\left(1 - \exp(-\tfrac{(d^*-d)^2}{2})\right)$$

and hence

$$(d^* - d)^2 = \left(\exp(\tfrac{(d^*-d)^2}{2}) - 1\right).$$

So $(d^* - d)^2 = z$ where $\ln(z+1) = z/2$, i.e. $z \approx 2.513$. This yields

$$B + \mu' - 2kd^* = 0 \qquad and \qquad B\exp(-\tfrac{z}{2})\sqrt{z} = 2k$$

where $\mu' = \frac{\mu}{s}$ and $d^* = \frac{e_i^*}{s}$. Hence we have

$$\tfrac{e_i^* - \mu/2k}{s} = d^* - \tfrac{\mu'}{2k} = \tfrac{B}{2k} = \exp(\tfrac{z}{2})/\sqrt{z} \approx 2.216$$

This completes the proof.

Proof of Proposition 4

We have $H(x;s) = \Phi(\frac{x}{s})$, and $h(x;s) = \phi(\frac{x}{s})\frac{1}{s}$ where $\Phi()$ is the N(0,1) CDF and $\phi()$ its density. The relations (8 - 10a) can then be written as

$$b(1 - \Phi(\frac{-\tau}{s})) - c(e^*) \geq b(1 - \Phi(\frac{e^* - e^0 - \tau}{s})) - c(e^0) \tag{25}$$

$$b\phi(\frac{-\tau}{s})\frac{1}{s} - c'(e^*) = 0 = b\phi(\frac{e^* - e^0 - \tau}{s})\frac{1}{s} - c'(e^0) \tag{26}$$

$$b \leq \frac{\delta}{1-\delta}W(e^*) \tag{27}$$

Note first that for the minimal $s = s_c$ for which the FOA is valid, all relations hold with equality, and $\tau = 0$. Denote the associated effort and bonus by $e^* = e_c^*$ and $b = b_c$, respectively.

We show below that for any $s < s_c$, the optimal threshold scheme of this type has all relations (25 - 27) binding, and implements some $e^* \in (e_c^*, e_u^*)$.

Given the latter property, it is straightforward to see that $e^* \to e_u^*$ as $s \to 0$. For suppose that (at least along a subsequence) $e^* \to e_l^* < e_u^*$ as $s \to 0$. Note that we then must have $\frac{\tau}{s} \to \infty$ as $s \to 0$. For if not, then $b \to 0$ by FOC for e^* in (26), which implies a negative payoff at e^*. For the same reason we must also have $\frac{e^*-e^0-\tau}{s} \to \infty$. Then we must have $e^0 \to e_l^0 = 0$ as $s \to 0$, for otherwise the payoff at e^0 would converge to $-c(e_l^0) < 0$. This is impossible, since the payoff at e^0 exceeds that at $e = 0$, and hence must be non-negative.

Taking limits in the first relation (25) with equality, we then get $\lim b \cdot 1 - c(e_l^*) = 0$, and hence from the last equation (for b) that $c(e_l^*) = \frac{\delta}{1-\delta} W(e_l^*)$. This cannot hold for $e_l^* < e_u^*$, hence we must have $e_l^* = e_u^*$.

It remains to prove the claim that for any $s < s_c$, the optimal threshold scheme of this type has all relations (25 - 27) binding, and implements some $e^* \in (e_c^*, e_u^*)$.

We first show that for any $s < s_c$, effort $e^* = e_c^*$ can be implemented with $b = b_c$, and a suitable choice of τ. Indeed, fix $e^* = e_c^*$ and $b = b_c$, and let $\tau(s)$ and $e^0(s)$ be defined by the FOCs (26) for e^* and e^0, respectively. For $s = s_c$ we have $\tau = 0$ and all relations hold with equality. We can show that the payoff difference will increase as s decreases; this follows from (as shown below) $\frac{d\Delta}{ds} < 0$, where Δ is the payoff difference;

$$\Delta = b(\Phi(\frac{e^* - e^0 - \tau}{s}) - \Phi(\frac{-\tau}{s})) - (c(e^*) - c(e^0)), \tag{28}$$

and $\tau = \tau(s)$ and $e^0 = e^0(s)$.

This shows that for any s less than the critical level s_c, it is feasible to implement the effort $e^* = e_c^*$ associated with s_c, and that this can be done by adjusting the threshold downwards, keeping the bonus fixed at $b = b_c$ (the FOA bonus associated with s_c). Moreover, the agent's payoff at e^* is then strictly larger ($\Delta > 0$) than his payoff at the other local maximum e^0. (And the payoff at e^0 is positive, since it exceeds the payoff at $e = 0$.) For fixed $s < s_c$ this leaves room for increasing the bonus and/or the threshold (reducing τ), thereby increasing the implemented effort.

Note that by increasing b, keeping τ fixed, effort e^* will increase (by FOC), and the payoff difference will also increase, since $\frac{d\Delta}{db} = \Phi(\frac{e^*-e^0-\tau}{s}) - \Phi(\frac{-\tau}{s}) > 0$ (by the envelope property). Hence b and e^* can be increased if the EC constraint $b \leq \frac{\delta}{1-\delta}W(e^*)$ is not binding.

By reducing τ, keeping b fixed, effort e^* will increase (by FOC), and the payoff difference will decrease, since $\frac{d\Delta}{d\tau} = (c'(e^*) - c'(e^0))\frac{1}{s} > 0$. Moreover, this relaxes the EC constraint $b \leq \frac{\delta}{1-\delta}W(e^*)$, and is hence feasible. Thus τ can be reduced and effort e^* increased if the payoff constraint $\Delta \geq 0$ is not binding.

These arguments show that, for given $s < s_c$, the optimal (feasible) threshold bonus has the EC constraint as well as the payoff constraint binding. Thus all the relations (25 - 27) will be binding, and the scheme implements an effort $e^* > e_c^*$. This verifies the claim.

It remains to prove $\frac{d\Delta}{ds} < 0$, where Δ is given by (28), $\tau = \tau(s)$ and $e^0 = e^0(s)$ are given by the FOCs in (26), and b and e^* are kept fixed ($e^* = e_c^*, b = b_c$). To this end, note that we have, for the payoff at e^0:

$$\frac{d}{ds}\left(b(1-\Phi(\tfrac{e^*-e^0-\tau}{s})) - c(e^0)\right) = -b\phi(\tfrac{e^*-e^0-\tau}{s})\frac{d}{ds}(\tfrac{e^*-e^0-\tau}{s}) - c'(e^0)\frac{de^0}{ds}$$

$$= -b\phi(\tfrac{e^*-e^0-\tau}{s})\frac{1}{s^2}\left(-\frac{d(e^0+\tau)}{ds}s - (e^* - e^0 - \tau)\right) - c'(e^0)\frac{de^0}{ds}$$

$$= c'(e^0)\frac{1}{s}\left(\frac{d(e^0+\tau)}{ds}s + (e^* - e^0 - \tau)\right) - c'(e^0)\frac{de^0}{ds}$$

$$= c'(e^0)\left(\frac{d\tau}{ds} + \frac{e^*-e^0-\tau}{s}\right)$$

Similarly, for the payoff at e^*:

$$\frac{d}{ds}\left(b(1-\Phi(\tfrac{-\tau}{s})) - c(e^*)\right) = -b\phi(\tfrac{-\tau}{s})\frac{d}{ds}(\tfrac{-\tau}{s})$$

$$= b\phi(\tfrac{-\tau}{s})\frac{1}{s^2}(s\frac{d\tau}{ds} - \tau) = c'(e^*)(\frac{d\tau}{ds} - \frac{\tau}{s})$$

Hence

$$\frac{d\Delta}{ds} = c'(e^*)(\frac{d\tau}{ds} - \frac{\tau}{s}) - c'(e^0)\left(\frac{d\tau}{ds} + \frac{e^*-e^0-\tau}{s}\right)$$

$$= (c'(e^*) - c'(e^0))(\frac{d\tau}{ds} - \frac{\tau}{s}) - c'(e^0)\frac{e^*-e^0}{s}$$

From the FOCs (26) and the fact that $\phi'(z) = -z\phi(z)$ we obtain

$\phi'(\frac{-\tau}{s})(-\frac{s\tau'-\tau}{s^2}) = \frac{1}{b}c'(e^*)$ *ie*

$-(\frac{-\tau}{s})\phi(\frac{-\tau}{s})(-\frac{s\tau'-\tau}{s^2}) = \frac{1}{b}c'(e^*)$ *ie*

$-\frac{\tau}{s}\frac{s}{b}c'(e^*)(\frac{s\tau'-\tau}{s^2}) = \frac{1}{b}c'(e^*)$ *ie*

$-\tau(\frac{s\tau'-\tau}{s^2}) = 1$ *ie* $\frac{\tau}{s}(\frac{d\tau}{ds} - \frac{\tau}{s}) = -1$

This yields

$\frac{d\Delta}{ds} = (c'(e^*) - c'(e^0))(-\frac{s}{\tau}) - c'(e^0)\frac{e^*-e^0}{s} < 0$

This completes the proof.

Proof of Proposition 5

From (17) we have

$$\frac{c'(e) - \eta\bar{x}'(e_i)}{n\int_{x_0}^{\infty} F(x;e)^{n-1} f_e(x;e)dx} = \frac{b(n)}{n} = \frac{\delta}{1-\delta}(W(e) - W(e_s))$$

Consider

$$s(n) = n\int_{x_0}^{\infty} F(x;e)^{n-1} f_e(x;e)dx = \int_{x_0}^{\infty} \frac{d}{dx}(F(x;e)^n)\frac{f_e(x;e)}{f(x;e)}dx$$

Letting $h(x) = \frac{f_e(x;e)}{f(x;e)}$ here denote the likelihood ratio, we have, integrating by parts

$$s(n) = \int_{x_0}^{\infty} \frac{d}{dx}(F(x;e)^n - 1)h(x)dx = h(x_0) + \int_{x_0}^{\infty} (1 - F(x;e)^n)h'(x)dx$$

where $h(x_0) = 0$ by definition of x_0. Given MLRP we have $h'(x) > 0$ and hence we see that $s(n)$ *is increasing in* n.

This implies that $\frac{c'(e)-\eta\bar{x}'(e_i)}{s(n,e)}$ shifts down with n, and hence that effort per agent (e) increases.

Proof of Proposition 6

Consider the problem of maximizing total surplus ($\Sigma_i W(e_i) = \Sigma_i(E(x_i|\,e_i) - c(e_i))$) subject to (14) and the 'modified' IC constraint (15). Letting $x = (x_1...x_n)$ and $e = (e_1...e_n)$, the Lagrangian for the problem is

$$L = \Sigma_i W(e_i) + \int \left(\tfrac{\delta}{1-\delta}\left(\Sigma_i W(e_i) - nW_s\right) - \Sigma_i b_i(y)\right)\lambda(x)$$
$$+\Sigma_i \mu_i \left(\int b_i(x) f_{e_i}(x;e) + \tfrac{1}{n}\eta \bar{x}'(e_i) - c'(e_i)\right),$$

where we have used $\frac{\partial}{\partial e_i} E(b_i(x)|\,e) = \int b_i(x) f_{e_i}(x;e)$ (and the integrals are multiple integrals over vector x). This yields

$$\frac{\partial L}{\partial b_i(x)} = \mu_i f_{e_i}(x;e) - \lambda(x) \leq 0, \qquad b_i(x) \geq 0, \qquad \text{(compl slack)}$$

If two agents are paid a positive bonus, then $\mu_i f_{e_i}(x;e) = \lambda(x) = \mu_j f_{e_j}(x;e)$, so their weighted likelihood ratios must be equal; $\mu_i \frac{f_{e_i}(x;e)}{f(x;e)} = \mu_j \frac{f_{e_j}(x;e)}{f(x;e)}$. But this can only occur for a set of measure zero, hence at most one agent is paid a bonus (almost surely).

If $f_{e_i}(x;e) < 0$ then $b_i(x) = 0$. If $f_{e_i}(x;e) > 0$ then $\lambda(x) > 0$, and agent i is paid the bonus ($b_i(x) > 0$) if and only if he has the largest weighted likelihood ratio. Also, the bonus is maximal since EC is binding.

In a symmetric solution the weights (multipliers) μ_i will be equal, and hence the agent with the largest likelihood ratio will get the bonus, provided this ratio is positive.

Now consider variables with identical variances and identical correlations ($corr(x_i, x_j) = \rho$ all $i \neq j$.). The multinormal density has the form

$$C \exp(-\tfrac{1}{2}(x-e)'\Sigma^{-1}(x-e))$$

where Σ is the covariance matrix. Under our assumptions we have $\Sigma = s^2 R$, where the correlation matrix R can be written as

$$R = (1-\rho)I + \rho J,$$

where each element of J is $J_{ik} = 1$. Note that $J^2 = nJ$. We will show below that

$$R^{-1} = \frac{1}{(1+(n-1)\rho)(1-\rho)} Q, \quad \text{where} \quad Q = (1+(n-1)\rho)I - \rho J \quad (29)$$

Note that the matrix Q has elements $(1+(n-2)\rho)$ on the diagonal, and $-\rho$ off the diagonal.

From the formula for R^{-1} and the definitions of k_1, k_2 in the text it follows that the quadratic form in the multinormal density can be written

$$-\tfrac{1}{2}(x-e)'\Sigma^{-1}(x-e) = -\tfrac{1}{2}\left(k_1\Sigma_i z_i^2 + k_2\Sigma_{i\neq j} z_i z_j\right), \qquad z_i = x_i - e_i$$

Differentiation of the density wrt e_i then yields the formula (18) for the likelihood ratio in the text

From the formula (18) it follows that agent $i's$ likelihood ratio is positive iff the inequality in (19) holds. We now verify the last equality in (19), i.e. the validity of the expression for $E(x_i|\, x_{-i})$. To this end note that for the normal distribution the conditional expectation of, say x_1 can be written

$$E(x_1|\, x_{-1}) = E(x_1) + \Sigma_{12}\Sigma_{22}^{-1}(x_{-1} - Ex_{-1}),$$

where $\Sigma_{12} = s^2(\rho, ..., \rho)$ is the $(n-1)-$dimensional vecor of covariances $cov(x_1, x_j)$, $j > 1$, and Σ_{22} is the covariance matrix for $x_{-1} = (x_2...x_n)'$. It follows from (29) that $s^2\Sigma_{22}^{-1}$ has the same form as R^{-1}, with n replaced by $n-1$. Hence $\Sigma_{12}\Sigma_{22}^{-1} = (\rho...\rho)R_{n-1}^{-1}$, and each element of this $(1\times n)$ matrix is, from (29):

$$((\rho...\rho)R_{n-1}^{-1})_i = \tfrac{\rho}{(1+(n-2)\rho)(1-\rho)}((1+(n-3)\rho) - (n-2)\rho) = \tfrac{\rho}{(1+(n-2)\rho)}$$

This verifies the last equality in (19).

It remains to verify the formula (29) for R^{-1}. To this end consider

$$RQ = ((1-\rho)I + \rho J)((1+(n-1)\rho)I - \rho J)$$

$$= (1-\rho)(1+(n-1)\rho)I + (1+(n-1)\rho)\rho J - (1-\rho)\rho J - \rho^2 nJ$$

$= (1-\rho)(1+(n-1)\rho)I + 0J$

This proves the formula for R^{-1}.

Finally we check positive definiteness, and verify that R is positive definit iff $1+(n-1)\rho > 0$. To verify this we will show that the determinant of R is

$$\Delta_n = (1-\rho)^{n-1}\left((n-1)\rho+1\right)$$

To see this, note that the element q_{ij} in the inverse matrix $Q = R^{-1}$ equals C_{ji}/Δ_n, where C_{ji} is the cofactor of element ji in matrix R. So for element nn we have $q_{nn} = C_{nn}/\Delta_n = \Delta_{n-1}/\Delta_n$, hence $\frac{1+(n-2)\rho}{(1+(n-1)\rho)(1-\rho)} = \Delta_{n-1}/\Delta_n$. The formula for Δ_n then follows by induction.

Proof of Proposition 7

We will show that for $e_1^* = e_2^*$ the marginal gain from effort is

$$\int_B f_{e_1}(x|\, e_1 e_2^*) = \tfrac{1}{s}\Gamma(\tfrac{e_1 - e_1^*}{s};\rho)$$

where $\Gamma(a;\rho)$ is the function defined in (22) in the text. The agent's FOC then takes the form $\frac{b}{s}\Gamma(0;\rho) - \eta/2 - c'(e_1^*) = 0$, which is precisely the formula (21) stated in the proposition.

The normal density depends on (vector) x via a quadratic form in $x - e$,, hence it satisfies $f_{e_i}(x;e)dx = -f_{x_i}(x;e)$. Taking account of the definition of the set B of outcomes (the set where agent 1 is paid a bonus) in (20) , we thus have

$$\int_B f_{e_1}(x|\, e_1 e_2^*) = -\left(\int_{-\infty}^{e_2^*} dx_2 \int_{e_1^*+\rho(x_2-e_2^*)}^{\infty} dx_1 + \int_{e_2^*}^{\infty} dx_2 \int_{x_2}^{\infty} dx_1\right) f_{x_1}(x|\, e_1 e_2^*)$$
$$= -\left(\int_{-\infty}^{e_2^*} dx_2 \left[f(x|\, e_1 e_2^*)\right]_{x_1=e_1^*+\rho(x_2-e_2^*)}^{x_1=\infty} + \int_{e_2^*}^{\infty} dx_2 \left[f(x|\, e_1 e_2^*)\right]_{x_1=x_2}^{x_1=\infty}\right)$$

where

$$f(x|\, e_1 e_2^*) = k\exp\left(-\frac{(x_1-e_1)^2+(x_2-e_2^*)^2-2\rho(x_1-e_1)(x_2-e_2^*)}{2(1-\rho^2)s^2}\right)$$
$$= k\exp\left(-\frac{((x_1-e_1)-\rho(x_2-e_2^*))^2+(1-\rho^2)(x_2-e_2^*)^2}{2(1-\rho^2)s^2}\right), \qquad k = \frac{1}{2\pi s^2\sqrt{1-\rho^2}}$$

This implies

$$[f(x|\,e_1e_2^*)]_{x_1=e_1^*+\rho(x_2-e_2^*)}^{x_1=\infty}$$

$$= k\exp\left(-\tfrac{(x_2-e_2^*)^2}{2s^2}\right)\left[0-\exp\left(-\tfrac{((x_1-e_1)-\rho(x_2-e_2^*))^2}{2(1-\rho^2)s^2}\right)\right]_{x_1=e_1^*+\rho(x_2-e_2^*)}$$

$$= -k\exp\left(-\tfrac{(x_2-e_2^*)^2}{2s^2}\right)\exp\left(-\tfrac{(e_1^*-e_1)^2}{2(1-\rho^2)s^2}\right)$$

and

$$[f(x|\,e_1e_2^*)]_{x_1=x_2}^{x_1=\infty} = k\exp\left(-\tfrac{(x_2-e_2^*)^2}{2s^2}\right)\left[0-\exp\left(-\tfrac{((x_2-e_1)-\rho(x_2-e_2^*))^2}{2(1-\rho^2)s^2}\right)\right]$$

$$= -k\exp\left(-\tfrac{(x_2-e_2^*)^2}{2s^2}\right)\exp\left(-\tfrac{((x_2-e_2^*)+(e_2^*-e_1)-\rho(x_2-e_2^*))^2}{2(1-\rho^2)s^2}\right)$$

So

$$\int_B f_{e_1}(x|\,e_1e_2^*) = k\int_{-\infty}^{e_2^*}\exp\left(-\tfrac{(x_2-e_2^*)^2}{2s^2}\right)dx_2\exp\left(-\tfrac{(e_1^*-e_1)^2}{2(1-\rho^2)s^2}\right)$$

$$+k\int_{e_2^*}^{\infty}\exp\left(-\tfrac{(x_2-e_2^*)^2+((x_2-e_2^*)(1-\rho)+(e_2^*-e_1))^2}{2(1-\rho^2)s^2}\right)dx_2 \qquad (z_2 = x_2-e_2^*)$$

$$= k\int_{-\infty}^{0}\exp\left(-\tfrac{(z_2)^2}{2s^2}\right)dz_2\exp\left(-\tfrac{(e_1^*-e_1)^2}{2(1-\rho^2)s^2}\right)$$

$$+k\int_0^{\infty}\exp\left(-\tfrac{z_2^2(1-\rho^2)+(z_2(1-\rho)+(e_2^*-e_1))^2}{2(1-\rho^2)s^2}\right)dz_2$$

We have

$$z_2^2(1-\rho^2)+(z_2(1-\rho)+(e_2^*-e_1))^2$$

$$= z_2^2(1-\rho^2+(1-\rho)^2)+2z_2(1-\rho)(e_2^*-e_1)+(e_2^*-e_1)^2$$

$$= 2(1-\rho)(z_2^2+z_2(e_2^*-e_1)+\tfrac{1}{4}(e_2^*-e_1)^2)+(e_2^*-e_1)^2(1-\tfrac{1}{2}(1-\rho))$$

$$= 2(1-\rho)(z_2+\tfrac{1}{2}(e_2^*-e_1))^2+(e_2^*-e_1)^2\tfrac{1}{2}(1+\rho)$$

Hence

$$\int_0^{\infty}\exp\left(-\tfrac{z_2^2(1-\rho^2)+(z_2(1-\rho)+(e_2^*-e_1))^2}{2(1-\rho^2)s^2}\right)dz_2$$

$$= \int_0^{\infty}\exp\left(-\tfrac{2(1-\rho)(z_2+\frac{1}{2}(e_2^*-e_1)^2}{2(1-\rho^2)s^2}\right)dz_2\exp\left(-\tfrac{(e_2^*-e_1)^2\frac{1}{2}(1+\rho)}{2(1-\rho^2)s^2}\right)$$

$$= \int_0^{\infty}\exp\left(-\tfrac{(z_2+\frac{1}{2}(e_2^*-e_1))^2}{(1+\rho)s^2}\right)dz_2\exp\left(-\tfrac{(e_2^*-e_1)^2}{4(1-\rho)s^2}\right), \qquad (z_2' = \tfrac{z_2+\frac{1}{2}(e_2^*-e_1)}{\sqrt{1+\rho}s}\sqrt{2})$$

$$= \int_{\frac{\frac{1}{2}(e_2^*-e_1)\sqrt{2}}{s\sqrt{1+\rho}}}^{\infty}\exp\left(-\tfrac{(z_2)^2}{2}\right)dz_2\tfrac{s\sqrt{1+\rho}}{\sqrt{2}}\exp\left(-\tfrac{(e_2^*-e_1)^2}{4(1-\rho)s^2}\right)$$

Hence we have

$$\int_B f_{e_1}(x|\, e_1 e_2^*) = k\tfrac{1}{2}\sqrt{2\pi}s\exp\left(-\tfrac{(e_1^*-e_1)^2}{2(1-\rho^2)s^2}\right)$$

$$+k\int_{\frac{e_2^*-e_1}{s\sqrt{1+\rho}\sqrt{2}}}^{\infty} e^{-\frac{z^2}{2}}dz\tfrac{s\sqrt{1+\rho}}{\sqrt{2}}\exp\left(-\tfrac{(e_2^*-e_1)^2}{4(1-\rho)s^2}\right), \qquad k=\tfrac{1}{2\pi s^2\sqrt{1-\rho^2}}$$

Setting $e_2^* = e_1^*$ and using $\phi(z) = \frac{1}{\sqrt{2\pi}}e^{-z^2/2}$ verifies the formula (21), and completes the proof.

Proof of Proposition 8

Given that the marginal gain to effort for agent 1 in the modified tournament scheme can be written as

$$\tfrac{b}{s}\Gamma(\tfrac{e_1-e_1^*}{s};\rho)+\eta-c'(e_1)$$

where $\Gamma(a;\rho)$ is given by (22), the FOC for $e_1 = e_1^*$ to be optimal is $\frac{b}{s}\Gamma(0;\rho)+\frac{1}{2}\eta-c'(e_1^*)=0$, and the local second order condition is $\frac{b}{s}\Gamma_a(0;\rho)\frac{1}{s}-c''(e_1^*)\leq 0$ Since $\phi'(0)=0$ we see that

$$\Gamma_a(0;\rho) = \tfrac{1}{\sqrt{1-\rho^2}}\tfrac{1}{2}\phi(0)^2 = \tfrac{1}{\sqrt{1-\rho^2}}\tfrac{1}{4\pi}$$

Since $\Phi(0) = \frac{1}{2}$ and $\phi(0) = 1/\sqrt{2\pi}$, this in turn implies

$$\tfrac{\Gamma_a(0;\rho)}{\Gamma(0;\rho)} = \tfrac{\frac{1}{\sqrt{1-\rho^2}}\frac{1}{2}\phi(0)}{\frac{1}{\sqrt{1-\rho^2}}\frac{1}{2}+\frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\frac{1}{2}} = \tfrac{\phi(0)}{1+\frac{\sqrt{1-\rho^2}}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}} = \tfrac{1/\sqrt{\pi}}{\sqrt{2}+\sqrt{1+\rho}}$$

From the FOC we have $\frac{b}{s}\Gamma(0;\rho) = c'(e_1^*)-\eta$, hence the SOC can be written as

$$c''(e_1^*) \geq \tfrac{b}{s}\Gamma_a(0;\rho)\tfrac{1}{s} = \tfrac{c'(e_1^*)-\eta}{\Gamma(0;\rho)}\Gamma_a(0;\rho)\tfrac{1}{s} = \tfrac{c'(e_1^*)-\eta}{s}\tfrac{1/\sqrt{\pi}}{\sqrt{2}+\sqrt{1+\rho}}$$

This can be rearranged to yield the formula stated in the proposition.

We will now show that for s sufficiently large, the FOC has a single solution ($e_1 = e_1^*$), which then must be a maximum, since the local SOC holds strictly for s large. To get a contradiction, suppose that, for every $s' > 0$ there is $s > s'$ such that FOC has a solution $e_1 = e_1(s) \neq e_1^*$, i.e. such that

$$\tfrac{b}{s}\Gamma(\tfrac{e_1-e_1^*}{s};\rho)+\eta-c'(e_1) = 0 = \tfrac{b}{s}\Gamma(0;\rho)+\eta-c'(e_1^*),$$

implying

$$\Gamma(\frac{e_1-e_1^*}{s};\rho)/\Gamma(0;\rho)=\frac{c'(e_1)-\eta}{c'(e_1^*)-\eta}$$

Then letting $s\to\infty$ (if necessary along a subsequence) we see that $e_1(s)\to e_1^*$. Hence $a(s)=\frac{e_1(s)\to e_1^*}{s}\to 0$, and the last equation above yields

$$\frac{1}{a(s)}(\frac{\Gamma(a(s);\rho)}{\Gamma(0;\rho)}-1)\frac{1}{s}=\frac{c'(e_1)-c'(e_1^*)}{e_1-e_1^*}\frac{1}{c'(e_1^*)-\eta}$$

Letting now $s\to\infty$, the LHS behaves like $\frac{\Gamma_a(0;\rho)}{\Gamma(0;\rho)}\frac{1}{s}$ and hence converges to zero, while the RHS converges to $\frac{c''(e_1^*)}{c'(e_1^*)-\eta}$. This yields a contradiction and thus completes the proof.

Proof of Proposition 9

It follows from (5) that the critical discount factor to implement first best effort e^{FB} is for a team with n independent agents given by

$$\left(c'(e^{FB})-\frac{1}{n}\eta\right)s_nM=\frac{\delta^{FB}}{1-\delta^{FB}}(W(e^{FB})-W_s(n))$$

i.e., since $c'(e^{FB})=1$ here:

$$\frac{\delta^{FB}}{1-\delta^{FB}}=\frac{\left(1-\frac{1}{n}\eta\right)M}{W(e^{FB})-W_s(n)}s_n$$

Consider now quadratic costs: $c(e)=\frac{k}{2}e^2$, with associated surplus per agent:

$$W(e)=e-\frac{k}{2}e^2$$

First-best effort and surplus is then

$$:\qquad 1=ke^{FB},\qquad W^{FB}=\left(\frac{1}{k}\right)-\frac{k}{2}(\frac{1}{k})^2=\frac{1}{2k}$$

Spot effort is given by $\eta/n)=c'(e_s)=ke_s$, and surplus is then:

$$W(e_s)=\left(\frac{\eta}{nk}\right)-\frac{k}{2}(\frac{\eta}{nk})^2=\frac{1}{2kn^2}\eta\,(2n-\eta)$$

Hence the critical discount rate is here given by

$$\frac{\delta^{FB}}{1-\delta^{FB}}=\frac{\left(1-\frac{1}{n}\eta\right)M}{W(e^{FB})-W_s(n)}s_n=\frac{(1-\eta/n)M}{\frac{1}{2k}-\frac{1}{2kn^2}\eta(2n-\eta)}s\sqrt{n}=\frac{(1-\eta/n)\sqrt{n}M}{1-\frac{1}{n^2}\eta(2n-\eta)}2ks$$

We have now

$$\frac{d}{dn}\left(\frac{(1-\eta/n)\sqrt{n}}{1-\frac{1}{n^2}\eta(2n-\eta)}\right)=\frac{1}{2}\frac{\sqrt{n}}{(n-\eta)^2}(n-3\eta)>0 \qquad iff \qquad n>3\eta$$

Hence, if $3\eta \leq 2$, i.e. $\eta \leq \frac{2}{3}$, then the critical discount rate δ^{FB} is increasing for $n \geq 2$, meaning that teams with $n > 2$ do worse than teams with $n = 2$ with respect to achieving FB.

The critical discount rate δ^{FB} is always increasing for $n > 3$ (since $\eta < 1$), hence teams with $n > 3$ will always do worse than teams with $n = 3$ regarding achieving FB.

Comparing $n = 2$ and $n = 3$:

$$\frac{(\frac{\delta}{1-\delta})^{FB}_{n=2}}{(\frac{\delta}{1-\delta})^{FB}_{n=3}}=\frac{\frac{(1-\eta/2)\sqrt{2}}{1-\frac{1}{2^2}\eta(2\cdot 2-\eta)}}{\frac{(1-\eta/3)\sqrt{3}}{1-\frac{1}{3^2}\eta(2\cdot 3-\eta)}}=\sqrt{6}\frac{2\eta-6}{9\eta-18}\geq 1 \text{ for } \eta\geq\eta_0=0.805\,42$$

Hence we have (for quadratic costs): *wrt achieving FB, the optimal team size is $n = 2$ if $\eta < \eta_0 = 0.805\,42$, and $n = 3$ if $\eta > \eta_0$.*

This proves the first part of the proposition. The second part (comparison with the RPE tournament) follows from the proof for Proposition10 below.

Proof of Proposition 10

For normal distributions, we have that effort in the modified tournament is for given bonus given by FOC, see (21):

$$b\frac{1}{\sqrt{2\pi}s}\frac{1}{2}\left(\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\right)+\eta=c'(e)$$

The EC conditon requires $b \leq \frac{\delta}{1-\delta}n(W(e)-W_s)$, hence effort is given by (when $s=\sqrt{v}$)

$$\frac{\delta}{1-\delta}(W(e)-W_s)=\frac{b}{n}=\frac{c'(e)-\eta}{\frac{1}{\sqrt{2\pi}s}\frac{1}{2}\left(\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}\frac{1}{\sqrt{2}}\right)2}=\frac{c'(e)-\eta}{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}\frac{1}{2}}\sqrt{\pi v}$$

Consider next the relational Team contract.

The maximal effort per agent that can be sustained in the team is given by (5), where now $M=\sqrt{2\pi}$ and $s_2^2=2v(1+\rho)$, and hence (5) is

$$\left(c'(e)-\frac{1}{n}\eta\right)\sqrt{2\pi}\sqrt{2v(1+\rho)}=b=\frac{\delta}{1-\delta}(W(e)-W_s(n))$$

Compare now critical $\delta's$ to implement FB. They are given by the following conditions, for the tournament and the team, respectively

$$\frac{\delta}{1-\delta}=\frac{c'(e^{FB})-\eta}{W(e^{FB})-W_s}\frac{\sqrt{v\pi}}{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}\frac{1}{2}}=\frac{1-\eta}{W(e^{FB})-W_s}\frac{2\sqrt{v\pi}}{\sqrt{2}\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}}$$

$$\frac{\delta}{1-\delta}=\frac{c'(e^{FB})-\eta/2}{W(e^{FB})-W_s(2)}\sqrt{2\pi}\sqrt{2v(1+\rho)}=\frac{1-\eta/2}{W(e^{FB})-W_s(2)}2\sqrt{v\pi}\sqrt{1+\rho}$$

For quadratic costs we have $W(e^{FB})=\frac{1}{2k}$, and

$$W(e_s(n=2))=\frac{1}{2kn^2}\eta\left(2n-\eta\right)_{n=2}=\frac{1}{8k}\eta\left(4-\eta\right), \qquad W_s=\frac{1}{2k}\eta\left(2-\eta\right)$$

Hence the tournament has highest critical δ^{FB} iff

$$1<\left(\frac{1-\eta}{W(e^{FB})-W_s}\frac{2\sqrt{v\pi}}{\sqrt{2}\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}}\right)/\left(\frac{1-\eta/2}{W(e^{FB})-W_s(2)}2\sqrt{v\pi}\sqrt{1+\rho}\right)$$

$$=\left(\frac{1-\eta}{\frac{1}{2k}-\frac{1}{2k}\eta(2-\eta)}\frac{1}{\sqrt{2}\frac{1}{\sqrt{1-\rho^2}}+\frac{1}{\sqrt{1-\rho}}}\right)/\left(\frac{1-\eta/2}{\frac{1}{2k}-\frac{1}{8k}\eta(4-\eta)}1\sqrt{1+\rho}\right)$$

$$=\frac{1}{2}\frac{2-\eta}{1-\eta}\frac{\sqrt{2}-\sqrt{\rho+1}}{\sqrt{1-\rho}}$$

where the last equality follows from straightforward algebraic calulations.

Consider then the critical combination $\frac{1}{2}\frac{2-\eta}{1-\eta}\frac{\sqrt{2}-\sqrt{\rho+1}}{\sqrt{1-\rho}}=1$, ie

$$\eta=\frac{2\left(\sqrt{-\rho+1}+\sqrt{\rho+1}-\sqrt{2}\right)}{2\sqrt{-\rho+1}+\sqrt{\rho+1}-\sqrt{2}}$$

The tournament has highest critcal δ^{FB} above the curve defined by $\eta(\rho)$. Differentiating, we obtain

$$\frac{d}{d\rho}\left(\frac{2\left(\sqrt{-\rho+1}+\sqrt{\rho+1}-\sqrt{2}\right)}{2\sqrt{-\rho+1}+\sqrt{\rho+1}-\sqrt{2}}\right)=\frac{\sqrt{2}(\sqrt{2}-\sqrt{\rho+1})}{\sqrt{1-\rho}\sqrt{\rho+1}\left(2\sqrt{1-\rho}+\sqrt{\rho+1}-\sqrt{2}\right)^2}>0$$

The increasing curve shows that the tournament gets relatively better vis-a-vis the team when ρ increases.

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