



A Valuation Model of Deferred Callability in Defaultable Debt

Yvonne Silden Langlo

Supervisor: Svein-Arne Persson

Master thesis in Financial Economics

**NORWEGIAN SCHOOL OF ECONOMICS AND
BUSINESS ADMINISTRATION**

This thesis was written as a part of the Master of Science in Economics and Business Administration program - Major in Financial Economics. Neither the institution, nor the advisor is responsible for the theories and methods used, or the results and conclusions drawn, through the approval of this thesis.

Executive Summary

In this thesis a model is developed for valuing risky perpetual debt with an embedded American call option that can be exercised after a protection period. These features are relevant for a hybrid capital instrument typically issued by banks and other financial institutions, partly as an outcome of regulatory requirements. There exist a large market for this instrument, the outstanding amount of hybrid capital securities was \$ 376 billion in 2005 (Mjøs and Persson, 2007). The model is based on a model by Mjøs and Persson (2010) where similar debt is valued, but where as a simplification the option is assumed to be a European type of option. Market practice indicates that this hybrid capital instrument is issued with an American option and a step-up coupon rate, in this sense the model developed in this thesis is more realistic than Mjøs and Persson's model because it incorporates these characteristics. An important result from this thesis is that the value of risky perpetual debt with an embedded American call option differs from the value of similar debt with a European call option. This is interesting because considering market practice and the characteristics of the two options would imply otherwise.

Preface

Working with this master thesis has been a great experience. It has been challenging and I have learned a lot that I believe will be useful in my future both personally and professionally.

I want to thank my supervisor Svein-Arne Persson for helping me find such an interesting topic to study, and for all the support and useful discussion along the way. I would like to thank Pareto Securities AS for financial support. They believed in me and found my topic very interesting, for this they gave me an unconditional scholarship of NOK 30 000.

Bergen, 15 June 2010

Yvonne Silden Langlo

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1. Introduction

In this thesis a model is developed for valuing risky perpetual debt with an embedded American call option that can be exercised after a protection period. This debt product is a kind of hybrid capital instrument, where the characteristics of this debt instrument are partly an outcome of global bank regulation. There exist a large market for this product, the outstanding amount of hybrid capital securities was \$ 376 billion in 2005 (Mjøs and Persson, 2007). The valuation model developed in this thesis is an extension of model developed by Mjøs and Persson (2010) for valuing a similar type of debt, but where as a simplification the option is assumed to be a European type of option. Market practice indicates that this product is issued with an American type of option.

The importance of hybrid capital can be illustrated by the outstanding amount of securities. The participants in the market and the regulatory framework are other reasons why this is an interesting topic to study. Issuers of hybrid capital are typically large, international financial institutions of high importance to the stability of the financial market, and they are subject to comprehensive regulatory requirements.

The structure of this thesis is as follows: In chapter two I give an overview of regulatory requirements and the market of hybrid capital. In chapter three I review related literature. In chapter four I present the valuation model. In chapter five I analyse my model. In chapter six I conclude on my analysis.

2. Hybrid capital

Perpetual debt with a protection period and an embedded American call option is a form of hybrid capital. Hybrid capital is a combination of debt and equity. Banks and other financial institutions are the main issuers of hybrid capital. Along with equity and subordinated debt, hybrid capital is considered as risk capital and the amount held by banks is subject to regulatory requirements. In this section the regulatory requirements and the hybrid capital market are discussed.

First, some important terms will be explained to ensure proper understanding throughout the thesis. A protection period is a time period that starts at time 0 and usually lasts for 10 years, where there is no call option that can be exercised. A call option is a right but not an obligation to buy an underlying security on a predetermined date(s), the underlying security is here the debt. The call option is embedded which means that the equity holders are the holders of the option, a third-party could alternatively hold the option but in this case it would have no affect on the debt value. A European option can be exercised at one predetermined date while an American option can be exercised at all times (in this case after the protection period). The option in the hybrid capital instrument can be exercised at the coupon dates, an option that can be exercised at certain times is a Bermudian type of option (Mjøs and Persson, 2010). As a simplification the option is modelled as an American option in the model developed in this thesis.

2.1 Regulation

Financial authorities impose capital requirements on banks and other financial institutions. The issuance of hybrid capital is partly an outcome of regulatory requirements. To explain why this is the case the rationale for regulations will be reviewed an overview of the Basel Committee as an important regulator will be provided in the following sequence.

2.1.1 Rationale for regulating financial markets

Banks and other financial institutions are subject to stricter regulation than other companies. Amongst other things they are subject to stricter capital requirements which impacts their

holdings of hybrid capital. Below some of the arguments for regulations of banks, which also holds for other financial institutions, is discussed.

One argument for regulation of banks is that it may help protect the customers of the banks. This argument is based on the assumptions that the banking industry is an oligopolistic industry whereas there are many customers. This industry characteristic can lead the consumers to lack market power and thereby getting exploited by the industry (Matthews and Thompson, 2008).

Another argument for regulation of banks is that it may reduce the danger of bank runs. This argument is based on assumptions of lack of transparency in the industry and possible failure of monitoring, that makes bank reputations become very important. A rumour that weakens the trust of a bank is therefore likely to cause a bank run, a bank run might lead to solvency problems and bank failure. Contagion is the spread of one bank's problems to other banks. Contagion causes systemic risk, which is the likelihood that one bank's problems will spread to the entire industry. Both contagion and systemic risks are of big concerns in the banking industry (Heffernan, 2005).

A third argument for bank regulation is that it can help ensure the stability of the financial system. Systemic risk, risk that affects the system as a whole, is of major concern in the financial system. There are characteristics of the banking industry that makes it more important to the financial system than other industries. Three important characteristics are that banks store people's wealth, provide liquidity, as well as help implement government monetary policy. Therefore the proponents of regulation argue that the industry should be under stricter regulation than other industries (Freixas and Rochet, 2008).

To sum up the arguments for regulation of banks are based on the fact that the social costs of banking crises are higher than the private costs. The arguments explain why banks and other financial institution are subject to capital requirements.

2.1.2 The Basel Comittee

The Basel Committee on Banking Supervision ("the Basel Committee") is a standing committee of banking supervisory authorities established in 1975. The Basel Committee works to establish common global regulatory standards for international banks (Heffernan, 2005). Representatives of bank supervisory authorities and central banks from Belgium,

Canada, France, Germany, Italy, Japan, Luxembourg, the Netherlands, Spain, Sweden, Switzerland, the United Kingdom, and the United States are represented in the Committee. It has a permanent secretariat based at the Bank for International Settlements in Basel (BIS), Switzerland (Basel Committee, 2006).

In 1975 and 1983 the Committee made the first agreements regarding supervisory responsibilities. A more comprehensive agreement were reached in 1988, the Basel Accord “International Convergence of Capital Measurement and Capital Standards”, often referred to as Basel 1. The objective of Basel 1 was to improve international financial stability through effective supervision of international banking operations. Basel 1 established minimum capital requirements for all international banks. A Basel Amendment was introduced in 1996, and in 2004 a revised framework were published, known as Basel 2 (Heffernan, 2005). The minimum capital requirements from Basel 1 remain in place in Basel 2 as the first out of three pillars; I The minimum capital requirement, II Supervisory Review Process, and III Market Discipline. The amount and characteristics of hybrid capital issued by banks can be affected by these capital requirements.

The Basel agreements apply to international banks in the member countries. Many countries however, require all domestic banks to adopt the standards, i.e., all credit institutions in the member countries of the EU adhere to the rules. Many regulators from the countries that are members of the Bank for International Settlements¹ but not members of the Basel Committee also require their banks to adopt the Basel rules (Heffernan, 2005).

The minimum capital Basel requirement is a total capital ratio of 8 %, tier 2 capital can maximum be equal to tier 1 capital. The capital ratio equals (Heffernan, 2005)

$$Total\ capital\ ratio = \frac{Risk\ capital}{Risk\ weighted\ assets},$$

where risk weights are assigned to the assets depending on the credit type, higher risk gives a higher risk weight.

¹ 56 members of BIS; Algeria, Argentina, Australia, Austria, Belgium, Bosnia and Herzegovina, Brazil, Bulgaria, Canada, Chile, China, Croatia, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong SAR, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Macedonia, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Romania, Russia, Saudi Arabia, Serbia, Singapore, Slovakia,

The constituents of capital are divided in three tiers (Basel Committee, 1988).

Tier 1 (Core capital)

- 1) Paid-up share capital/Common stock
- 2) Disclosed reserves

Tier 2 (Supplementary capital)

- 1) Undisclosed reserves
- 2) Asset revaluation reserves
- 3) General provisions/general loan-loss reserves
- 4) Hybrid (debt/equity) capital instruments
- 5) Subordinated debt

Tier 3

- 1) Short-term subordinated debt

Hybrid capital instruments can gain acceptance as risk capital, either tier 1 or tier 2 capital (Mjøs and Persson, 2010). This is one reason why hybrid capital is an attractive form of capital for banks and other financial institutions. According to Mjøs and Persson the perpetual horizon and the deferral of the exercise date of the call option is crucial to gain acceptance. The regulators require an infinite horizon, while investors typically prefer a finite maturity, embedding a call option after a protection period is a way of reconciling these conflicting objectives.

2.2 Hybrid capital market

The global outstanding amount of hybrid capital was \$ 376 billion in 2005. 57 % of the issuances were made by banks, 8 % by insurance companies, the remaining of non-financial institutions (Mjøs and Persson, 2007). The dominance of highly regulated industries reflect that the product characteristics are tailor-made to regulative requirements, and that the

characteristics are suited to highly regulated companies. Out of all perpetual securities 50 % were accepted as tier 1 capital and 28 % as tier 2 capital. Issuances are spread among the main markets; 32 % in the US Dollar market, 41 % in the Euro market and the remaining 28 % in the Pound Sterling market (Mjøs and Persson, 2010).

A company that needs capital can instead of raising hybrid capital for example raise common equity or debt, the choice of financing will amongst other things depend on tax legislation, direct and indirect issuance costs and other capital structure considerations. Benston et al. (2003) analyse 105 issues of hybrid capital by US bank holding companies in the years 1995-1997 with regards to what characterises issuers versus non-issuers. They find that hybrid capital issuers typically are larger, have higher tax-rates, more uninsured funding, and lower equity ratios.

Mapondera and Bossert (2005) have examined security issuances by 50 large European banks. They found that out of the issued senior market debt in the years 2000 to 2005, 28 % had a perpetual horizon. All the perpetual securities issued in 2004 and 2005 had an embedded American option after a protection period of typically 10 years, i.e., hybrid capital. The coupon rates were typically stepped up by 75-150 basis points at the first call date.

Mjøs and Persson (2007) argue that the step-up rate is a result of different preferences between investor and regulators, regulators prefer long maturities, while investors prefer finite maturities. A step-up coupon increases the incentive to call the option, so the regulators limit the size of the step-up to support the permanence of the hybrid capital.

Ineke et al. (2003) indicate that in real-life an embedded American call option will be exercised at the first possibility, i.e., at the end-date of the protection period. In all but one of the cases they have studied this has been the case. The exception was a result of an administrative error and the option was exercised at the second opportunity.

The previous figures illustrate the importance of the hybrid capital market. It is a very large market and many of the participants are large banks of high importance to the stability in the financial market. The research shows that the characteristics of the hybrid capital instrument valued in the model in this thesis are realistic in real-life.

3. Literature review

The valuation model in this thesis is mainly based on a model developed by Mjøs and Persson (2010). They have developed a closed-form solution for risky perpetual debt with an embedded European option after an initial protection period. In this thesis a valuation model for a similar product with an embedded American option. This model is developed in line with theirs as far as possible but closed-form solutions are not provided. Mjøs and Persson have expressed the value of debt including the call option as a portfolio of perpetual debt and barrier options with a time dependent barrier. They analyse how the call option affects the coupon at issue-at-par and the issuer's optimal bankruptcy decision. This effect will also be analysed in the model developed in this thesis.

Research on perpetual debt and debt-based derivatives are relevant for this thesis. The literature on corporate debt pricing often use valuation tools from continuous-time asset pricing to study basic questions of corporate finance, i.e, how firms maximize equity value (Broadie and Kaya, 2007). Structural bond pricing models value debt as a contingent claim on firm's assets. Since all securities of a firm are treated as derivatives on the firm's asset the price information for one class of security, typically equity, can be used to infer the value of another, typically debt (Ericsson and Reneby, 2005). Contingent claim pricing offers potential for closed form solutions, precise answers and the opportunity to analyse dynamics. Most of the existing models in the literature attempt to derive analytical valuation formulas for debt and equity values by using simplifications to avoid time and path dependence. To work in a time-independent setting models usually price perpetual bonds (Broadie and Kaya, 2007).

Classic literature on debt valuation include Black and Scholes (1973), Merton (1974) and Black and Cox (1976). The Black and Scholes/Merton model is the first, simplest and best known of the structural models. The framework only considers zero-coupon debt and default at given time horizon, never prior to maturity. Black and Cox (1976) extended this work by considering perpetual debt with endogenous default, the default occur when the value of the firm's assets reaches a lower threshold. The model developed in this thesis follow this approach. All of these papers assume a constant risk-free interest rate and absolute priority as done in this thesis. The Black and Scholes/Merton approach has been implemented in a

number of papers, i.e., Geske (1977), Ingersoll (1977a, 1977b), Merton (1977), Smith and Warner (1979), and more recently in Delianedis and Geske (1999) and Eom et al. (2004).

Several models are developed along the lines of the Black and Scholes/Merton model, but do in addition incorporate more realistic features, i.e., violation of the absolute priority rule, taxes, costly financial distress, debt renegotiation or stochastic interest rates. Models include those of Kim et al. (1993), Nielsen et al. (1993), Leland (1994), Longstaff and Schwartz (1995), Anderson and Sundaresan (1996), Leland and Toft (1996), Briys and de Varenne (1997), Mella-Barral and Perraudion (1997), Ericsson and Reneby (1998), Mella-Barral (1999), Fan and Sundaresan (2000), Duffie and Lando (2001), and Collin-Dufresne et al. (2001).

Important papers on valuing bonds with embedded derivatives include Brennan and Schwartz (1978), Brennan and Schwartz (1984), Cox et al. (1985), Fischer et al. (1989), Mello and Parsons (1992), Kim et al. (1993). Ingersoll (1977b) and Brennan and Schwartz (1977). These papers do not include a protection period which will impact the valuation. All of these early models treat bankruptcy and liquidation as the same event and assume absolute priority as in this thesis. Most recent research attempts to treat bankruptcy and liquidation events separately, e.g. François and Morellec (2004).

In this thesis the same approach as Mjøs and Persson (2010), Black and Cox (1976) and Leland (1994) is followed, thus risk is included through the volatility of the EBIT-process. Mjøs and Persson (2010) argue that this is supported by market practice where issuers typically pay a credit margin on top of a market reference interest rate, thus they are not directly exposed to the nominal interest rate levels. Another possibility is to apply a stochastic interest rate process, i.e., as Acharya and Carpenter (2002) have done. Acharya and Carpenter (2002) analyse corporate bonds valuation (with fixed maturity) and optimal call and default rules when interest rates and asset value are stochastic. However, they do not develop exact valuation formulas for the bonds.

Emanuel (1983) develops a valuation of perpetual preferred stock, based on the option-methodology of Black-Scholes. Preferred stock can be viewed as perpetual debt for analytical purposes. Emanuel does not cover options on preferred stock. Sarkar (2001) focus on perpetual bonds with an American call option. In contrast to as done in this thesis Sakar assumes that there is no protection period, i.e. the calls are immediately exercisable. A main

part of the paper deals with the optimal exercise timing of the call, the paper does neither include analytical valuation of the options nor optimal coupon or bankruptcy levels. Sarkar and Hong (2004) extend Sarkar (2001) and analyse the impact from callability on the duration of perpetual bonds, they find that embedding a call option reduces the optimal bankruptcy level and extends the duration of a bond. Their reduced optimal bankruptcy level matches Mjøs and Persson's (2010) results. Bank (2004) values call options on debt but does not calibrate coupons or consider the fact that debt-values are not log-normally distributed.

Broadie and Kaya (2007) introduces a numerical method that can be useful to extend existing models or build more complex ones. They use a lattice method that is common in the option pricing literature but has rarely been used in corporate debt pricing. They model the evolution of the firm's assets on a discrete lattice, and then uses a backward solution procedure for the valuation of other securities. Other authors that use numerical techniques in the context of corporate debt pricing include Brennan and Schwartz (1978). They value a bond paying discrete coupons but their model is very restrictive with an exogenous bankruptcy boundary, and their valuation method does not give debt and equity values separately.

Kish and Livingston (1992) test for determinants of whether call options are included in corporate bond contracts. Their findings are that the interest rate level, agency costs and bond maturity significantly affect whether a bond comes with an embedded call option.

Johnson and Stulz (1987) defined the concept of vulnerable options, options where the counterparty may default on the contract. Hull and White (1995) categorise risky derivative contracts into classes by default risk of the counterparty and the credit-risk of underlying asset. Embedded options are in a class of vulnerable options with both default risk of counterparty and credit-risk of the underlying assets, where the two risks cannot be separated. These authors focus primarily on the risk at option maturity. Literature on barrier options however, like Bjork (2004), include the risk of bankruptcy before the option matures.

4. The valuation model

A model for valuing risky perpetual debt with a protection period and an embedded American call option is presented in this section. The model is developed in line with Mjøs and Persson (2010), who have developed a model to value risky perpetual debt with a protection period and an embedded European option. The following sequence start by presenting the main assumptions underlying the model. Thereafter the EBIT-based market value process is presented. Thirdly formulas for the debt value at the end of the protection period are presented. The last section presents how the present value of the debt can be estimated.

4.1 Basic assumptions

In line with Black and Scholes (1973) and Merton (1973) the following perfect market assumptions are taken;

- All assets are infinitely separable and continuously tradeable
- There are no taxes, transaction costs, bankruptcy costs, agency costs or short-sale restrictions
- All agents have costless and immediate access to all information
- There exists a known constant risk-free rate of return r .

4.2 The EBIT-based market value process

In line with Goldstein et al. (2001) it is assumed that the issuer is a limited liability company with financial assets that generate an EBIT (earnings before interest and tax) cash flow denoted δ_t . The cash flow is given by the following stochastic differential equation

$$(1) \quad d\delta_t = \mu\delta_t dt + \sigma\delta_t dW_t,$$

where μ and σ are constants representing drift and volatility, δ_0 is the fixed initial cash flow level, and W_t is a standard Brownian motion under a fixed equivalent martingale measure. The market value of the assets at time t of the assumed perpetual EBIT stream equals

$$\begin{aligned} \hat{A}_t &= E_t^Q \left[\int_t^\infty e^{-r(s-t)} \delta_s ds \right] \\ (2) \qquad &= \frac{\delta_t}{r - \mu}, \end{aligned}$$

where the discount rate equals the risk-free interest rate in a risk neutral set-up. The initial value of the process is denoted

$$(3) \qquad \hat{A}_0 = A.$$

The total market value of the perpetual EBIT stream is the solution to the following stochastic differential equation

$$\begin{aligned} (4) \qquad d\hat{A}_t &= (r\hat{A}_t - \delta_t)dt + \sigma\hat{A}_t dW_t \\ &= \mu\hat{A}_t dt + \sigma\hat{A}_t dW_t. \end{aligned}$$

4.3 Debt value at the end-date of the protection period

In this section the formulas for valuing debt at the end-date of the protection period denoted time T are presented. The section covers formulas for risk-free debt, risky debt, callable debt, and risky and callable debt.

4.3.1 Risk-free perpetual debt

The time T market value of risk-free perpetual debt equals

$$(5) \qquad D_T = \int_0^\infty e^{-rt} cD dt = \frac{cD}{r},$$

where c is the constant coupon rate after time T , D is the face value of the debt claim and cD is the continuous coupon payment.

4.3.2 Perpetual debt with bankruptcy risk

The time T market value of perpetual debt with bankruptcy risk is a standard Black and Cox (1976) result, the value is given by the following equation

$$(6) \quad D_T^R(A_t) = \frac{cD}{r} - \left(\frac{cD}{r} - \bar{A} \right) \left(\frac{A_T}{\bar{A}} \right)^{-\beta},$$

where R indicates that the debt is risky, β is a constant, A_T is the market value of assets at time T known from the asset process, and \bar{A} is the market value of assets where bankruptcy is optimal for the equity holders. Assuming absolute priority \bar{A} also represents the remaining assets given to the debt holders upon bankruptcy. The first term is the value of perpetual risk-free debt at time T . The second term represents the net loss upon bankruptcy. Upon bankruptcy the debt holders loses an infinite stream of coupon payments which at the time of bankruptcy has a market value of $\frac{cD}{r}$, they receive the remaining assets with a value equal to \bar{A} , multiplying with the term $\left(\frac{A_T}{\bar{A}}\right)^{-\beta}$ gives the value at time T . The term $\left(\frac{A_T}{\bar{A}}\right)^{-\beta}$ can be interpreted as the present value of one monetary unit paid upon bankruptcy, i.e., a time T discount factor. A value of 1 means no discounting, hence

$$(7) \quad \max \left(\frac{A_T}{\bar{A}} \right)^{-\beta} = 1.$$

The value of equity as the residual claim on the assets is given by

$$E_T^R(A_t) = A_T - D_T^R(A_t) = A_T - \frac{cD}{r} + \left(\frac{cD}{r} - \bar{A} \right) \left(\frac{A_T}{\bar{A}} \right)^{-\beta}.$$

β is given by the following expression

$$(8) \quad \beta = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2r}}{\sigma^2}.$$

Black and Cox (1976) determine the optimal bankruptcy level from the perspective of the equity holders. For a given capital structure (D and E) and with an infinite time horizon, the

bankruptcy level is constant. The level is found by differentiating the expression for the value of equity with respect to \bar{A} , for details see appendix, section 7.1. The optimal bankruptcy level is given by

$$(9) \quad \bar{A} = \frac{\beta}{\beta + 1} \frac{cD}{r}.$$

4.3.3 Perpetual risk-free debt with an embedded European call option

The time T market value of perpetual risk-free debt with an embedded European call option equals

$$(10) \quad D_T^{CE}(A_t) = \begin{cases} \frac{cD}{r} & \text{for } A_T < A_H, \\ H & \text{otherwise,} \end{cases}$$

where

$$(11) \quad H = \lambda D,$$

and

$$(12) \quad \lambda \geq 0.$$

where CE indicates callable debt with a European option, H is a call compensation paid to the debt holders upon exercising of the option, and A_H is a call barrier. The call barrier A_H is a market value of the assets where it is contractually decided that the call option can be exercised. If the market value of the assets reaches this barrier it is optimal for the equity holders to exercise the call option. The equity holders will only exercise the option if it is at their advantage, hence it represents a loss for the debt holders. H is a contractually decided call compensation paid to the debt holders upon exercising of the call option. A positive compensation as a function of the face value of the debt is assumed.

The first expression (in equation 10) represents the debt value if the option is not exercised, this is the case if the asset value is lower than the call barrier at time T . In this case the debt

value equals the value of perpetual risk-free debt. The second expression represents the debt value upon exercising of the option, in this case the debt value equals the call compensation.

4.3.4 Perpetual risk-free debt with an embedded American call option

The time T market value of perpetual risk-free debt with an embedded American call option equals

$$(13) \quad D_T^{CA}(A_t) = \frac{cD}{r} - \left(\frac{cD}{r} - H \right) \left(\frac{A_T}{A_H} \right)^\alpha,$$

where CA indicates callable debt with an American option, and α is a constant. The first term is the market value at time T of perpetual risk-free debt. The second term represents the market value at time T of the net loss upon bankruptcy for the debt holders. Upon exercising of the call option the debt holders loses an infinite stream of coupon payments which at the time of bankruptcy has market value of $\frac{cD}{r}$. The debt holders will on the other hand receive a call compensation H . The term $\left(\frac{A_T}{A_H} \right)^\alpha$ can be interpreted as the present value of one monetary unit paid upon bankruptcy, i.e., a time T discount factor. A value of 1 means no discounting, hence

$$(14) \quad \max \left(\frac{A_T}{A_H} \right)^\alpha = 1.$$

α is given by the following expression (McDonald, 2006)

$$(15) \quad \alpha = \frac{-\mu + \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2r}}{\sigma^2}.$$

The value of equity as the residual claim on the assets is given by

$$E_T^{CA}(A_t) = A_T - D_T^{CA}(A_t) = A_T - \frac{cD}{r} + \left(\frac{cD}{r} - H \right) \left(\frac{A_T}{A_H} \right)^\alpha$$

4.3.5 Perpetual debt with bankruptcy risk and an embedded European call option

The time T market value of risky perpetual debt with an embedded European call option equals

$$(16) \quad D_T^{RCE}(A_t) = \begin{cases} D_T^R(A_t) & \text{for } A_T < A_H, \\ H & \text{otherwise,} \end{cases}$$

where RCE indicates risky callable debt with a European option. The first expression represents the debt value if the call option is not exercised, which will be the case of the asset values lower than the call barrier at time T . In this case the debt value equals the value of risky debt (without an option). The second term represents the debt value upon exercising of the option, in this case the debt value equals the call compensation.

4.3.6 Perpetual debt with bankruptcy risk and an embedded American call option

The time T market value of perpetual debt with bankruptcy risk and an embedded American call option equals

$$(17) \quad D_T^{RCA}(A_t) = \frac{cD}{r} - \frac{cD}{r} E[e^{-r\tau}] + E[Xe^{-r\tau}],$$

where

$$X = \begin{cases} \bar{A} & \text{for } \tau = \tau(\bar{A}), \\ H & \text{for } \tau = \tau(A_H), \end{cases}$$

and where the stopping time τ is given by

$$(18) \quad \tau = \inf\{t \geq T, \hat{A}_t = \bar{A} \text{ or } A_H\},$$

where RCA indicates risky callable debt with an American option, τ can be interpreted as the time where either bankruptcy or exercising of the call option will occur. The expression $\inf\{\cdot\}$ represents a function that returns the value of the first time t after the end-date of the

protection period T where the condition is fulfilled, i.e., the first time the asset process hits the bankruptcy barrier or the call barrier. \hat{A}_t is given from the asset process in expression (2).

The first term (in equation 15) is the value at time T of perpetual risk-free debt. The second term represents the value at time T of the debt holders' loss upon bankruptcy or exercising of the option. The debt holders loses an infinite stream of coupon payments which at the time of the event has the market value of $\frac{cD}{r}$, the loss is discounted with the factor $E[e^{-r\tau}]$. The third term represents the market value at time T of the debt holders' gain from either bankruptcy or exercising of the call option at time τ . The debt holders receive the remaining assets \bar{A} upon bankruptcy, or the compensation H upon exercising of the call option.

The discount factor for the debt holders' loss can be expressed as follows

$$(19) \quad E[e^{-r\tau}] = P(A_T(\tau) = \bar{A})\left(\frac{A_T}{\bar{A}}\right)^{-\beta} + P(A_T(\tau) = A_H)\left(\frac{A_T}{A_H}\right)^{\alpha}.$$

The discount factor is calculated as the probability of reaching the bankruptcy barrier times the respective discount factor, plus the probability of reaching the call barrier times the respective discount factor.

The market value of the gain for the debt holders can be expressed as follows

$$(20) \quad E[Xe^{-r\tau}] = P(A_T(\tau) = \bar{A})\bar{A}\left(\frac{A_T}{\bar{A}}\right)^{-\beta} + P(A_T(\tau) = A_H)H\left(\frac{A_T}{A_H}\right)^{\alpha}.$$

The market value of the gain is calculated as the probability of reaching the bankruptcy barrier times the respective compensation and discount factor, plus the probability of reaching the call option barrier times the respective compensation and discount factor.

The probabilities of reaching the barrier bankruptcy or the call barrier are given by the following expressions from Karlin and Taylor (1975), for details see appendix, section 7.2,

$$(21) \quad P(A_T(\tau) = A_H) = \frac{e^{(-2\mu A_T)/\sigma^2} - e^{(-2\mu\bar{A})/\sigma^2}}{e^{(-2\mu A_H)/\sigma^2} - e^{(-2\mu\bar{A})/\sigma^2}},$$

and

$$(22) \quad P(A_T(\tau) = \bar{A}) = 1 - P(A_T(\tau) = A_H).$$

The probabilities are bounded by lower and upper boundaries

$$0 \leq P(A_T(\tau) = \bar{A}) \leq 1,$$

and

$$0 \leq P(A_T(\tau) = A_H) \leq 1.$$

In the special case of drift equal to zero the following formulas holds

$$P(A_T(\tau) = A_H) = \frac{A_T - \bar{A}}{A_H - \bar{A}},$$

and

$$P(A_T(\tau) = \bar{A}) = 1 - P(A_T(\tau) = A_H).$$

The value of equity as the residual claim on the assets is given by

$$E_T^{RCA}(A_t) = A_T - D_T^{RCA}(A_t) = A_T - \frac{cD}{r} + \frac{cD}{r} E[e^{-r\tau}] - E[Xe^{-r\tau}]$$

4.4 Debt value at time 0

This section explains how the debt value at time 0 can be calculated. It starts with an explanation of how bankruptcy risk can be taken into account in the protection period, thereafter formulas for the debt value at time 0 are derived, thirdly is a description of how Monte Carlo simulation can be used to make estimates of debt values.

4.4.1 The bankruptcy risk in the protection period

In the protection period there exists a level of the market value of the assets where it is optimal for the equity holders to default on the debt. This value represents a bankruptcy barrier, assuming absolute priority it also represents the value of the remaining assets given to the debt holders upon bankruptcy. The embedded call option has a non-negative value to

the equity holders, based on this it is likely that the bankruptcy level will decrease due to inclusion of a call option. The bankruptcy barrier depends on time to maturity, at time T it is reasonable to believe that it is close or equal to the bankruptcy barrier after time T (\bar{A}). It is therefore reasonable to assume that as time to maturity decreases the higher the bankruptcy level will be.

In line with Mjøs and Persson (2010) it is assumed an exogenous time dependent bankruptcy barrier

$$B_t = B e^{\gamma t},$$

where B is the initial bankruptcy level at time 0, and γ is a constant that determines the curvature of the time-dependent bankruptcy level. The time of bankruptcy in the protection period is given by the stopping time

$$\tau_p = \inf\{t \geq 0, \hat{A}_t = B_t\},$$

where p denotes that this is the stopping time in the protection period, as earlier in the model \hat{A}_t is given by the asset process in equation (4). By modifying the original asset process with a drift adjustment of γ the modified asset process can be expressed as

$$(23) \quad dA_t = (\mu - \gamma)A_t dt + \sigma A_t dW_t,$$

where the stopping time equals

$$(24) \quad \tau_p = \inf\{t \geq 0, A_t = B\}.$$

This transformation has no economic impact, it simplifies the analysis by formally letting the bankruptcy level be a constant level B . It is assumed a continuous bankruptcy barrier at time T , that the bankruptcy barrier in the protection period at time T equals the bankruptcy barrier which is constant after the protection period, this gives

$$B_T = B e^{\gamma T} = \bar{A}.$$

The bankruptcy barrier can therefore be expressed as

$$(25) \quad B = e^{-\gamma T} \bar{A}.$$

4.4.2 Debt value at time 0

The general valuation formula for debt at time 0 equals

$$D_0(A_t) = \begin{cases} E \left[\int_0^{\tau_p} c_p D e^{-rt} dt + e^{-r\tau_p} B \right] = E \left[\frac{c_p D}{r} (1 - e^{-r\tau_p}) + e^{-r\tau_p} B \right] & \text{for } \tau_p \leq T, \\ \int_0^T c_p D e^{-rt} dt + e^{-rT} E[D_T(A_T)] = \frac{c_p D}{r} (1 - e^{-rT}) + e^{-rT} E[D_T(A_T)] & \text{otherwise,} \end{cases}$$

where c_p is the constant continuously coupon rate in the protection period, and p indicates the protection period. If the coupon rate is stepped up at time T then c_p differs from c . The first expression represents the debt value upon bankruptcy in the protection period. The debt value equals the present value of the coupon payments from time 0 to the time of bankruptcy, plus the present value of the remaining assets. The second expression represents the debt value if there is no bankruptcy in the protection period. The debt value equals the present value of the coupon payments in the protection period, plus the present value of the debt value at time T for the respective debt type.

Debt value at time 0 for risk-free debt equals

$$(26) \quad D_0 = \int_0^T c_p D e^{-rt} dt + e^{-rT} D_T = \frac{c_p D}{r} (1 - e^{-rT}) + e^{-rT} D_T.$$

Debt value at time 0 for risky debt equals

$$(27) \quad D_0^R(A_t) = \begin{cases} E \left[\int_0^{\tau_p} c_p D e^{-rt} dt + e^{-r\tau_p} B \right] = E \left[\frac{c_p D}{r} (1 - e^{-r\tau_p}) + e^{-r\tau_p} B \right] & \text{for } \tau_p \leq T, \\ \int_0^T c_p D e^{-rt} dt + e^{-rT} E[D_T^R(A_t)] = \frac{c_p D}{r} (1 - e^{-rT}) + e^{-rT} E[D_T^R(A_t)] & \text{otherwise.} \end{cases}$$

Debt value at time 0 for risk-free debt with an embedded European call option equals

$$(28) \quad D_0^{CE}(A_t) = \int_0^T c_p D e^{-rt} dt + e^{-rT} E[D_T^{CE}(A_t)] \\ = \frac{c_p D}{r} (1 - e^{-rT}) + e^{-rT} E[D_T^{CE}(A_t)].$$

Debt value at time 0 for risk-free debt with an embedded American call option equals

$$\begin{aligned}
(29) \quad D_0^{CA}(A_t) &= \int_0^T c_p D e^{-rt} dt + e^{-rT} E[D_T^{CA}(A_t)] \\
&= \frac{c_p D}{r} (1 - e^{-rT}) + e^{-rT} E[D_T^{CA}(A_t)].
\end{aligned}$$

Debt value at time 0 for risky debt with an embedded European call option equals

$$\begin{aligned}
(30) \quad D_0^{RCE}(A_t) &= \begin{cases} E \left[\int_0^{\tau_p} c_p D e^{-rt} dt + e^{-r\tau_p} B \right] = E \left[\frac{c_p D}{r} (1 - e^{-r\tau_p}) + e^{-r\tau_p} B \right] & \text{for } \tau_p \leq T, \\ \int_0^T c_p D e^{-rt} dt + e^{-rT} E[D_T^{RCE}(A_t)] = \frac{c_p D}{r} (1 - e^{-rT}) + e^{-rT} E[D_T^{RCE}(A_t)] & \text{otherwise.} \end{cases}
\end{aligned}$$

Debt value at time 0 for risky debt with an embedded American call option equals

$$\begin{aligned}
(31) \quad D_0^{RCA}(A_t) &= \begin{cases} E \left[\int_0^{\tau_p} c_p D e^{-rt} dt + e^{-r\tau_p} B \right] = E \left[\frac{c_p D}{r} (1 - e^{-r\tau_p}) + e^{-r\tau_p} B \right] & \text{for } \tau_p \leq T, \\ \int_0^T c_p D e^{-rt} dt + e^{-rT} E[D_T^{RCA}(A_t)] = \frac{c_p D}{r} (1 - e^{-rT}) + e^{-rT} E[D_T^{RCA}(A_t)] & \text{otherwise.} \end{cases}
\end{aligned}$$

4.4.3 Monte Carlo simulation

Monte Carlo simulation in Excel can be used to simulate several asset processes and the respective debt values. This is a commonly used method when pricing path-dependent options. The debt value at time 0 depends on the development in the asset price from time 0 to time T. The asset price equals (McDonald, 2006)

$$A_{\Delta T} = A_0 e^{(\mu - \gamma - \frac{1}{2}\sigma^2)\Delta T + \sigma\sqrt{\Delta T}Z(1)},$$

$$A_{2\Delta T} = A_{\Delta T} e^{(\mu - \gamma - \frac{1}{2}\sigma^2)\Delta T + \sigma\sqrt{\Delta T}Z(2)},$$

and so on, up to

$$A_T = A_{n\Delta T} = A_{(n-1)\Delta T} e^{(\mu - \gamma - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z(n)}.$$

Where Z is a standard normally distributed random variable

$$Z \sim N(0,1),$$

where the length of the protection period T is divided into $n = T/\Delta t$ intervals, and each time step is denoted Δt . The estimate of the debt value at time 0 equals

$$\bar{D} = \frac{1}{N} \sum_{i=1}^N D_{0i}.$$

Where each simulation i of the asset process generates the debt value D_{0i} , and the number of simulations are denoted N . Increasing the number of intervals and number of drafts increases the accuracy of the debt value estimate.

5. Analysis of the valuation model

In this section market values are calculated using the valuation model developed in the previous chapter. First, a base case scenario is presented, thereafter debt values are analysed by looking at different values of the asset value at time T , thirdly the issue to par coupon rates are analysed, in the fourth section a step-up coupon rate is analysed, in the fifth section the results are compared with Mjøs and Persson's (2010) results, thereafter some assumptions underlying the model are discussed, and in the end the analysis is summarised.

5.1 Base case

In table 1 some base case parameters are presented. The values and reasoning behind them are in line with Mjøs and Persson (2010). The values of δ_0 , μ and r gives an initial market value of assets of $A = 100$. The value of the volatility parameter σ corresponds to a somewhat risky company. The value of the interest rate r is often used for illustrative purposes. The coupon rate is set at 300 basis points above the risk-free rate. A protection period of 10 years is common for hybrid capital. The rest of the parameters are only relevant to the model developed in this thesis. It is possible to include a step-up coupon but the coupon rate is assumed to be constant. The value of the call barrier is set at $A_H = 100$, and the compensation parameter $\lambda = 1$, so that the call compensation equals the face value of the debt $H = D$.

Parameter	Value	Explanation	Eq.
δ_0	3	Initial value of EBIT process	
μ	2 %	Drift of EBIT process	
σ	25 %	Volatility of EBIT process	
r	5 %	Risk-free interest rate	
c	8 %	Step-up coupon rate after the protection period	
c^p	8 %	Coupon rate in the protection period	
D	70	Face value of debt	
λ	1	Compensation parameter	12
H	70	Call compensation upon exercising of call option	11
A	100	Initial market value of assets at time 0	3
A_H	100	Call option barrier	
γ	0.002	Curvature of bankruptcy barrier in the protection period	
\bar{A}	68.61	Bankruptcy barrier after protection period	9
B	67.25	Bankruptcy barrier in protection period	25
T	10	Length of protection period	
Δt	1/252	Length of time step within a year	
N	20 000	Number of drafts in the simulation	

Table 1. Base case parameters. All rates are annualized.

For each draft the following results are calculated

Result	Explanation	Eq
$\left(\frac{A_T}{\bar{A}}\right)^{-\beta}$	Time T discount factor	7
$\left(\frac{A_T}{A_H}\right)^\alpha$	Time T discount factor	14
$P(A_T(\tau) = \bar{A})$	Probability of reaching the bankruptcy barrier first	22
$P(A_T(\tau) = A_H)$	Probability of reaching the call barrier first	21
$E[e^{-r\tau}]$	Time T discount factor	19
$E[Xe^{-r\tau}]$	Market value of compensation at time T	20
D_T	Value of risk-free debt at time T	5
$D_T^R(A_t)$	Value of risky debt at time T	6
$D_T^{CE}(A_t)$	Value of debt with European call option at time T	10
$D_T^{CA}(A_t)$	Value of debt with American call option at time T	13
$D_T^{RCE}(A_t)$	Value of risky debt with European call option at time T	16
$D_T^{RCA}(A_t)$	Value of risky debt with American call option at time T	17
D_0	Value of risk-free debt at time 0	26
$D_0^R(A_t)$	Value of risky debt at time 0	27
$D_0^{CE}(A_t)$	Value of debt with European call option at time 0	28
$D_0^{CA}(A_t)$	Value of debt with American call option at time 0	29
$D_0^{RCE}(A_t)$	Value of risky debt with European call option at time 0	30
$D_0^{RCA}(A_t)$	Value of risky debt with American call option at time 0	31

Table 2. Results for each draft

The simulation gives the following results

Result	Value	Explanation
D_0	112.00	Value of risk-free debt at time 0
$\bar{D}_0^R(A_t)$	87.74	Simulated value of risky debt at time 0
$\bar{D}_0^{CE}(A_t)$	99.66	Simulated value of debt with European call option at time 0
$\bar{D}_0^{CA}(A_t)$	92.86	Simulated value of debt with American call option at time 0
$^{CE}(A_t)$	80.75	Simulated value of risky debt with European call option at time
$^{CA}(A_t)$	80.55	Simulated value of risky debt with American call option at time

Table 3. Results base case scenario

Including risk and/or callability reduces the debt value at time 0. The value of risky debt with an embedded American call option is worth less than risky debt with an embedded European call option. These results seem plausible as risk is in general expected to negatively affect debt value, and a call option will never be exercised unless it is valuable to the equity holders, hence it has a non-positive value for the debt holders.

5.2 Debt value at time T for different asset values

Table 4 and Figure 1 illustrate how the debt value at the end-date of the protection period, time T will be affected by the asset value at the time, A_T . This provides a good basis for analysing why the debt values for risky debt with a European and an American option can differ, and illustrates the importance of the call barrier. The protection period is ignored here, upon bankruptcy in the protection period the debt value at time T is irrelevant.

A_T	60	70	80	90	100	110
$\left(\frac{A_T}{\bar{A}}\right)^{-\beta}$	1.00	0.97	0.78	0.65	0.55	0.47
$\left(\frac{A_T}{A_H}\right)^{\alpha}$	0.45	0.57	0.70	0.85	1.00	1.00
$P(A_T(\tau) = \bar{A})$	1.00	0.25	0.00	0.00	0.00	0.00
$P(A_T(\tau) = A_H)$	0.00	0.75	1.00	1.00	1.00	1.00
$E[e^{-r\tau}]$	1.00	0.67	0.70	0.85	1.00	1.00
$E[Xe^{-r\tau}]$	68.61	46.45	49.19	59.26	70.00	70.00
D_T	112.00	112.00	112.00	112.00	112.00	112.00
$D_T^R(A_t)$	68.61	69.96	77.97	83.75	88.08	91.43
$D_T^{CE}(A_t)$	112.00	112.00	112.00	112.00	70.00	70.00
$D_T^{CA}(A_t)$	93.27	88.10	82.49	76.45	70.00	70.00
$D_T^{RCE}(A_t)$	68.61	69.96	77.97	83.75	70.00	70.00
$D_T^{RCA}(A_t)$	68.61	83.59	82.49	76.45	70.00	70.00

Table 4. Debt values at time T for different values of A_T

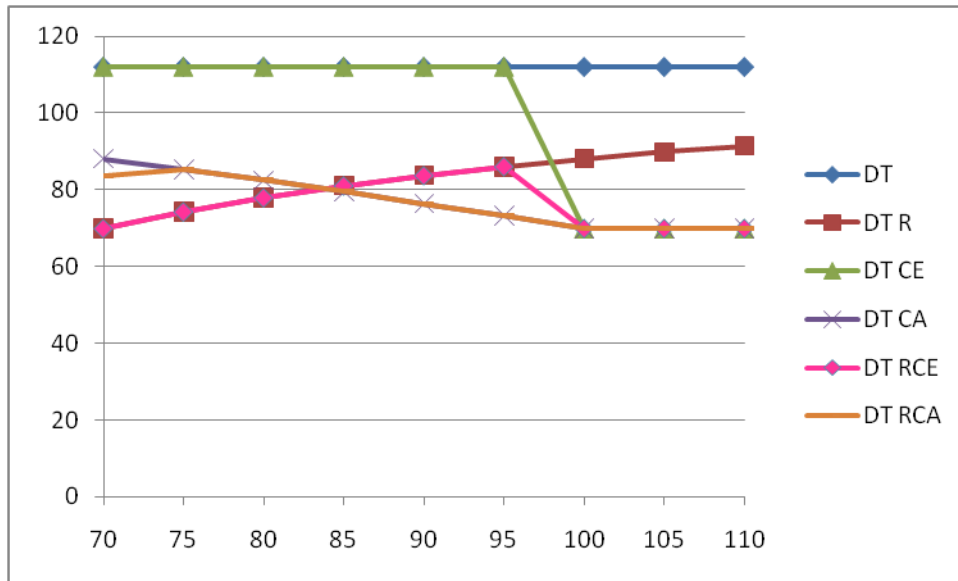


Figure 1. Debt value at time T for different values of A_T

The value of risk-free debt (DT) will not be affected by the asset value, and represents a constant, upper boundary for other debt values. As previously discussed including risk and/or callability will affect the debt value negatively.

The value of risky debt ($DT R$) increases as the asset value increases, as a higher asset value reduces the likelihood of bankruptcy. As A_T goes to infinity the value of risky debt equals the value of risk-free debt. For asset values lower than the bankruptcy barrier ($A_T < \bar{A} = 68.61$) bankruptcy will have occurred in the protection period, hence the debt value at time T is irrelevant.

The value of risk-free debt with a European option ($DT CE$) equals the call compensation for asset values higher than the call barrier ($A_T \geq A_H = 100$), since the option will be exercised in this case. For asset values lower than the call barrier, the debt value equals the value of risk-free debt (without an option).

The value of risk-free debt with an American option ($DT CA$) equals the call compensation for asset values higher than the call barrier ($A_T \geq A_H = 100$), since the option will be exercised in this case. For asset values lower than the call barrier, the debt value decreases as the asset value increases, as a higher asset value increases the likelihood of hitting the call barrier.

The value of risky debt with a European option (DT RCE) equals the call compensation for asset values higher than the call barrier ($A_T \geq A_H = 100$), since the option will be exercised in this case. For asset values lower than the call barrier, but higher than the bankruptcy barrier ($\bar{A} = 68.81 < A_T < A_H = 100$), the debt value equals the value of risky debt (without an option). For asset values lower than the bankruptcy barrier ($A_T < \bar{A} = 68.61$), bankruptcy will have occurred in the protection period, hence the debt value at time T is irrelevant.

The value of risky debt with an American option (DT RCA) equals the call compensation for asset values higher than the call barrier ($A_T \geq A_H = 100$), since the option will be exercised in this case. The probability of hitting the call barrier is 1 and the expected discount factor is 1 (no discounting) in this case. For asset values lower than the call barrier, but higher than the bankruptcy barrier ($\bar{A} = 68.61 < A_T < A_H = 100$), the debt value depends on the probabilities of hitting the barriers, the discount factor and the market value of the compensation (17). For example when $A_T = 70$ there is $\frac{1}{4}$ chance of hitting the bankruptcy barrier and $\frac{3}{4}$ chance of hitting the call barrier, hence the debt value is higher than the value of risky debt, and lower than the value of debt with an American option. For asset values lower than the bankruptcy barrier ($A_T < \bar{A} = 68.61$), bankruptcy will have occurred in the protection period, hence the debt value at time T is irrelevant.

From the previous discussion it is familiar that the value of the two types of risky and callable debt differs for asset values between the barriers ($\bar{A} = 68.61 < A_T < A_H = 100$). This is perhaps a surprising result based on observed market practice and the characteristics of the two option types. In the market options of American type are always exercised at the first possibility, based on this one might expect the debt values to be equal. By just considering the characteristics of the two option types, American versus European, we would expect debt with a European option to be most valuable. The value of a European call option represents the lower boundary of the value of an American option, since the American option can be exercised at the same time as the European and at all times thereafter. Embedding an option reduces the debt value, embedding an American option would therefore be expected to reduce the value by minimum the same as a European option. If this reasoning had been true the value of risky debt with a European option would be an upper boundary for debt with an American option. However, this is not the case. In Figure 2 the

values of the two types of risky and callable debt at time T for different asset values are illustrated (same as Figure 1 but more detailed).

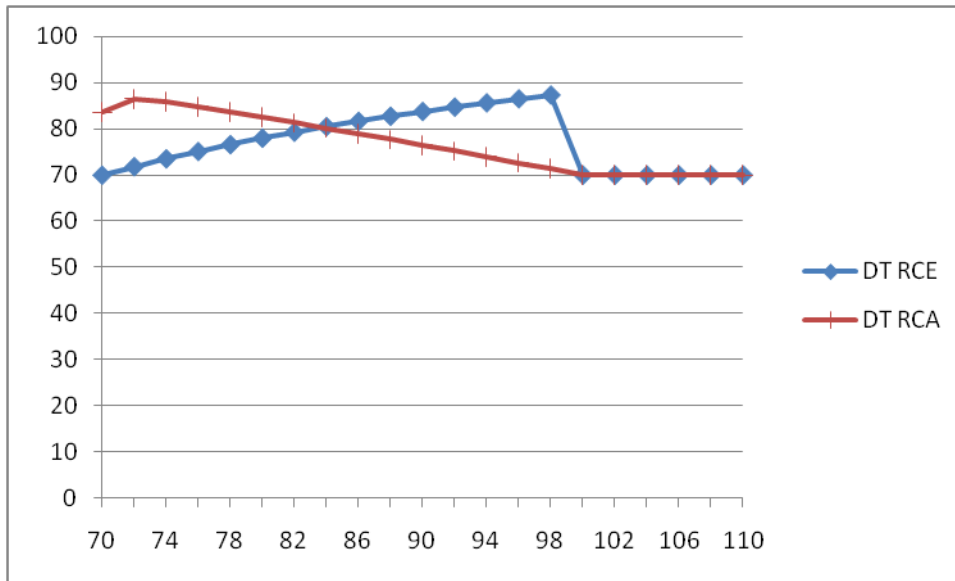


Figure 2. Debt value at time T for risky callable debt

Figure 2 illustrates the debt values at time T of risky debt with a European option ($DT RCE$) and an American option ($DT RCA$) for different asset values at time T . As previously explained, the risky debt with a European option equals the value of risky debt (without an option) when ($\bar{A} = 68.61 < A_T < A_H = 100$), while the value of risky debt with an American option depends on the probabilities of hitting the barriers, the discount factor and the market value of the compensation (17). For asset values close to the bankruptcy barrier ($\bar{A} = 68.61$) debt with an American option is most valuable, while for asset values close to the call barrier ($A_H = 100$) debt with a European option is most valuable. Recall that the initial value of the asset process is 100, so all of these asset values are relatively low.

The reason why risky debt with a European option is less valuable than with an American option for asset values close to the bankruptcy barrier, can be explained by looking at $A_T = 80$. In this case the value of risky debt with a European option equals the value of risky debt without callability (since the option cannot be exercised for $A_T < A_H$). The discount factor is 0.78, and the debt value is 77.97. The probability of hitting the call barrier first is 1, hence the value of risky debt with an American option equals the value of risk-free debt with an American option. The expected discount factor is 0.70, and the value of debt is 82.49. The earlier the process hits a barrier, the more negatively the debt value is affected

(less discounted). The compensation to the debt holders are higher upon exercising of the option than upon bankruptcy ($H > \bar{A}$). In this case the combination of less discounting and a lower compensation causes risky debt with a European option to be less valuable than with an American option.

The reason why risky debt with a European option is more valuable than with an American option for asset values close to the call barrier can be explained by looking at $A_T = 90$. In this case the value of risky debt with a European option equals the value of risky debt without callability (since the option cannot be exercised for $A_T < A_H$). The discount factor is 0.65, and the debt value is 83.75. The probability of hitting the call barrier first is 1, hence the value of risky debt with an American option equals the value of risk-free debt with an American option. The expected discount factor is 0.85, and the value of debt is 76.45. In this case the effect of higher discounting outweighs the lower compensation, the results is that risky debt with a European option is more valuable than with an American option.

The previous discussion illustrates the importance of the choice of call barrier, since the value of the two types of risky and callable debt differ for asset values between the bankruptcy barrier and the call barrier. Reducing the call barrier will increase the probability of exercising the option, and reduce the range of asset values where the values of the two types of risky debt differ.

5.3 Issue at par value

This section analyse the coupon rates that give issue at par value, i.e. market value of debt at time 0 equal to 70. The issue to par coupon rate equals the risk-free interest rate plus a credit margin. The credit margin reflects the value of the debt, debt holders will demand a higher credit margin for less valuable debt, both risk and callability reduces debt value. The issue to par coupon rates are useful because they are easily comparable for different debt types. The simulation method applied in the model makes it difficult to find an exact number, but coupon rates that give debt values approximately equal to 70 together with the corresponding bankruptcy levels are given in Table 5.

Result	$c_p = c$	\bar{A}	B
D_0	5 %	N/A	N/A
$D_0^R(A_t)$	5.74 %	49.23	48.25
$D_0^{CE}(A_t)$	5 %	N/A	N/A
$D_0^{CA}(A_t)$	5 %	N/A	N/A
$^{CE}(A_t)$	5.97 %	51.20	50.19
$^{CA}(A_t)$	5.73 %	49.14	48.17

Table 5. Issue to par value coupon rates and bankruptcy barriers

A coupon rate equal to the risk-free interest rate of 5 % gives issue to par for risk-free debt with or without callability. The issue to par coupon rate for risky debt without callability is 5.74 %, approximately the same as for risky debt with an American option. The issue to par coupon rate is around 25 basis points higher for risky debt with a European option relative to an American option. This explains that debt holders should demand around 0.25 % higher credit margin for risky debt with a European option than with an American option, and the same credit margin for risky debt with or without an American option. The value of risky debt with a European option is less valuable than with an American option, this is opposite to the base case scenario. This shows that the results are sensitive to the choice of coupon parameter.

5.4 Step-up coupon

A common characteristic of hybrid capital instruments is to include a step-up coupon rate at time T of 75-150 basis points (Mapondera and Bossert, 2005). The effect of a coupon rate that is increased by 100 basis points at time T is illustrated in Table 6.

Result	$c_p = 8\%$, $c = 9\%$	Issue to par coupon rates
$\bar{D}_0^{RCE}(A_t)$	84.56	5.79 %/6.79 %
$\bar{D}_0^{RCA}(A_t)$	84.37	5.72 %/6.72 %

Table 6. Step-up coupon

Compared to the base case scenario the debt values have increased, which is reasonable when increasing the coupon payments, the value of risky debt with a European option is still higher than with an American option. Including a step-up coupon has decreased the issue to

par coupon rate by around 20 basis points (from 5.97 %) for risky debt with a European option, it is approximately the same (5.73 %) for risky debt with an American option. Risky debt with an American option is more valuable than with a European option when issuing to par, similar to the case without a step-up coupon.

The decrease in the issue to par coupon rate for risky debt with a European option reflects the positive effect a higher coupon rate has on the debt value, which makes the debt holders demand a lower credit margin. The almost unaffected issue to par coupon rate for debt with an American option reflects that higher coupon payments increases the debt value, but it also increases the equity holders incentive to exercise the option which is negative for the debt holders.

5.5 Mjøs and Persson's model

Mjøs and Persson (2010) have developed a model for valuing risky perpetual debt with an embedded European call option after a protection period. The details of their solution are provided in the appendix, section 7.3. Although highly relevant, the results from their model are not directly comparable to the results derived in this thesis. The reason for this is that their exercise price is a debt value (the par value of the debt), while the model developed in this thesis' exercise price is an asset value (the exogenous call barrier A_H). The results from their model are provided Table 7, $D_0^{M\&P}$ denotes the debt value at time 0 from their model.

Result	Base case parameters	Issue to par
c	8 %	5.9657470148341 %
$D_0^{M\&P}$	80.27	70

Table 7. Results from Mjøs and Persson's model

Their results are close to the results derived in this thesis, although not directly comparable, the similarity indicates that the model derived in this thesis provides reasonable results and some conclusion from their analysis hold for this thesis' results as well. Mjøs and Persson claim that their coupon spreads are in a realistic size compared to observed market spreads, since the model developed in this thesis' coupon spreads are in the same magnitude this also holds for the derived results.

5.6 Discussion of assumptions underlying the model

This section will discuss some of the assumptions in the model. The assumptions of a known constant risk-free interest rate, absolute priority of the debt holders upon bankruptcy, that the underlying assets of the firm are traded, a constant bankruptcy level, and the exogenous call barrier. Papers who do the same (except the last assumption) include Black and Scholes (1973), Merton (1973), Black and Cox (1976) and Mjøs and Persson (2010).

The assumption of a constant risk-free interest rate is difficult to justify in a valuation model for risky fixed-income securities. The model developed in this follow the same approach as Mjøs and Persson (2010), Black and Cox (1976) and Leland (1994) and include risk through the volatility of the EBIT-process. Mjøs and Persson (2010) argue that this is supported by market practice where issuers typically pay a credit margin on top of a market reference interest rate, thus they are not directly exposed to the nominal interest rate levels. Briys and de Varenne (1997), Longstaff and Schwartz (1995) and Nielsen et al. (1993) extends the Black and Scholes/Merton model to amongst other things allow for stochastic interest rates.

Evidence shows that the assumption of absolute priority rarely holds, e.g. Franks and Torous (1989, 1994), Eberhart et al. (1990), LoPucki and Whitford (1990), Weiss (1990) and Betker (1991, 1992). The assumption of absolute priority affects the compensation paid to the debt holders upon bankruptcy. Deviations from absolute priority will reduce the compensation to the debt holders upon bankruptcy. The model can relatively easily be adjusted to incorporate deviations from absolute priority, i.e., by letting the compensation to the debt holders be a function of the remaining assets. Although deviations from the absolute priority will affect the debt values, the effect will be the same on both types of risky and callable, it is therefore probably not a critical assumption to this thesis' analysis.

It is assumed that the underlying assets of the firm is traded, however this is not the case. Ericsson and Reneby (2004) show that if any other contingent claim than the underlying asset is traded (e.g. equity) then this approach is ok for debt valuation. The issuers of hybrid capital instruments are typically large banks and other financial institutions where equity is traded, therefore this assumption is probably not of major concern to the valuation model.

Mjøs and Persson (2010) have examined the assumption of a constant bankruptcy level, they conclude that it produces the most correct coupon rates for alternative volatilities and option maturities. This assumption is therefore likely to be innocuous.

The call barrier (A_H) is assumed to be a contractually decided parameter value in the valuation model in this thesis. For asset values above this level it is optimal for the equity holders to exercise the option, however, it is not necessarily the optimal call barrier for the equity holders. Methods to find an optimal call barrier have been considered. One method considered was to find the barrier in the similar way as the bankruptcy barrier is derived, since the bankruptcy barrier is optimal for the equity holders. The bankruptcy barrier is found by differentiating the equity value with respect to the barrier, details are provided in the appendix, section 7.1. Unfortunately, it is not possible to solve for an optimal call barrier in the same manner. A second alternative considered was to do like Mjøs and Persson model (2010) and let debt value equal to par be the exercise price/barrier. This method is unfortunately not applicable in the model either, since the barrier is an asset value not a debt value. A third method considered was setting the debt value of risky debt with an embedded American option (17) equal to par, and solve for A_H . This equation depends on the asset value at time T so it cannot be solved analytically. Alternatively it can be found numerically in the similar manner as coupon rates that give issue to par were calibrated. However, this did not seem like a better solution. In lack of better ideas the call barrier is exogenous. As previously discussed the choice of the call barrier is important to the results from the model, since the debt values differ for different debt types for asset values below the call barrier. The assumption of an exogenous call barrier will therefore impact the results from the model.

5.7 Summary of analysis

The base case scenario illustrate that the debt value decreases when including risk and/or callability. An important result is that the value of risky debt with a European option differs from the value of risky debt with an American option. A more extensive analysis of different asset values at time T explains why the values differ. In section three I analyse the issue to par coupon rates, the credit margin should be around 25 basis points higher when issuing to par risky debt with a European option relative to risky debt with an American option. In the fourth section the effect of including a step-up coupon rate is analysed, this increases the

debt value and has a negative effect on the issue to par coupon rate for risky debt with a European option, the effect on debt with an American option is innocuous. In the fifth section my results are compared with the results from Mjøs and Persson's model, my model seems to behave properly and give reasonable results based on this comparison. In the sixth section some of the assumptions underlying of the model are discussed, the assumption of a constant risk-free interest rate is hard to justify and the choice of a call barrier parameter is important to the results from the model.

6. Conclusion

In this thesis a model is developed for valuing risky perpetual debt with an embedded European or American call option after a protection period. The two debt values differ which is interesting because considering market practice and the characteristics of the option types would imply otherwise. In the market options of American type are exercised at the first possibility, based on this one might expect debt values to be equal. The value of European option represents a lower boundary for an American option, based on this one might expect the value of debt with a European option to be an upper boundary for the value of debt with an American option.

The choice of parameter values in the analysis gives a 25 basis points higher credit margin when issuing to par debt with a European option relative to an American option, hence debt with an American option is most valuable. Including a step-up coupon rate of 100 basis points has no effect on the issue to par coupon rate for debt with an American option, but reduces the credit margin by around 20 basis points for debt with a European option.

The model developed is based on a model by Mjøs and Persson (2010) where similar debt with an embedded European option is valued. Although not directly comparable, the results derived in this thesis are close which indicates that the model behaves properly and give reasonable results. Market practice indicates that hybrid capital instruments are issued with an American option and include a step-up coupon rate, in this sense the model developed in this thesis is more realistic because it incorporates these characteristics.

Further research that would be interesting is to extend the model by for example introducing taxes, bankruptcy costs, or different bankruptcy priorities. Applying more advanced programming like for example C++ instead of Monte Carlo simulation could improve the accuracy of the results.

7. Appendix

7.1 Derivation of \bar{A}

$$\begin{aligned}
\frac{\partial E_T^R(A_t)}{\partial \bar{A}} &= \frac{\partial}{\partial \bar{A}} \left(A - \frac{cD}{r} + \left(\frac{cD}{r} - \bar{A} \right) \left(\frac{A}{\bar{A}} \right)^{-\beta} \right) = \frac{\partial}{\partial \bar{A}} \left(\left(\frac{cD}{r} - \bar{A} \right) \left(\frac{\bar{A}}{A} \right)^\beta \right) \\
&= -1 \left(\frac{\bar{A}}{A} \right)^\beta + \left(\frac{cD}{r} - \bar{A} \right) \beta \left(\frac{\bar{A}}{A} \right)^{\beta-1} \left(\frac{1}{A} \right) \\
&= -1 + \left(\frac{cD}{r} - \bar{A} \right) \beta \left(\frac{\bar{A}}{A} \right)^{-1} \left(\frac{1}{A} \right) = -1 + \left(\frac{cD}{r} - \bar{A} \right) \beta \left(\frac{1}{\bar{A}} \right) = 0 \\
\bar{A} &= \left(\frac{cD}{r} - \bar{A} \right) \beta = \frac{\beta}{\beta + 1} \frac{cD}{r}
\end{aligned}$$

7.2 Probabilities of reaching barriers

Let $\{A(\tau); \tau \geq 0\}$ be a Brownian motion process with drift $\mu \neq 0$ and variance σ^2 , and suppose $A(0) = A_T$. Let τ represent time after time T , and let τ be the first time the process reaches \bar{A} or A_H . Let \bar{A} and A_H be given and assume $\bar{A} < A_T < A_H$. The probability that the process reaches the barrier A_H before hitting the barrier \bar{A} is given by (Karlin and Taylor, 1975)

$$P(A(\tau) = A_H) = \frac{e^{(-2\mu A)/\sigma^2} - e^{(-2\mu \bar{A})/\sigma^2}}{e^{(-2\mu A_H)/\sigma^2} - e^{(-2\mu \bar{A})/\sigma^2}},$$

and

$$P(A(\tau) = \bar{A}) = 1 - P(A(\tau) = A_H).$$

If the drift equals zero ($\mu = 0$) the probabilities are given by

$$P(A(\tau) = A_H) = \frac{A - \bar{A}}{A_H - \bar{A}},$$

and

$$P(A(\tau) = \bar{A}) = 1 - P(A(\tau) = A_H).$$

7.3 Mjøs and Persson's model

The time 0 market value of debt with a European call option embedded equals

$$D_0(A)^{M\&P} = V_0(D_T^c) + L_0(A)$$

Where D_T^c denotes the time T payoff of perpetual debt including an embedded option to repay debt at par value D given no bankruptcy. $V_0(\cdot)$ represents the time 0 market value operator. L_0^A denotes the time 0 value of cash flows before time T, i.e. coupon and potential bankruptcy payments.

The time 0 market value of the debt payoff at time T equals

$$V_0(D_T^c) = V_k(A) + C_2^{do}(A, 0) - C_1^{do}(A, D)$$

Where $V_k(A)$ represent the part of the total value due to a possible discontinuity of the bankruptcy barrier at time T. In the case of continuity like I have assumed this term can be ignored. $C_2^{do}(A, 0)$ and $C_1^{do}(A, D)$ represents the time 0 market value of the debt payoff if the value of the company's assets is higher than the bankruptcy barrier both in the protection period and at time T. $C_2^{do}(A, 0)$ and $C_1^{do}(A, D)$ can be recognised as barrier options with exercise prices of 0 and D , respectively. The second term represents the time 0 market value of a call option on debt at time T with exercise price. The third term represents the short, embedded, call option on debt exercisable at time T with a strike equal to par value D .

Define

$$J = \frac{cD}{r} - \bar{A}$$

$$\bar{J} = \frac{cD}{r} - B$$

Where

$$C_2^{do}(A, 0) = C_0(A, 0)_\theta - \left(\frac{B}{A}\right)^{\left(\frac{2(\mu-\gamma)}{\sigma^2}-1\right)} C_0\left(\frac{B^2}{A}, 0\right)_\theta$$

$$C_0(A, 0)_\theta = \frac{cD}{r} e^{-rT} N(-f_2(A)) - J \left(\frac{A}{B} \right)^{-\beta} N(-f_1(A))$$

$$f_1(A) = \frac{\ln\left(\frac{\bar{A}}{A}\right) - \left(\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta\right)T}{\sigma\sqrt{T}}$$

$$f_2(A) = f_1(A) - \sigma\beta\sqrt{T}$$

$$C_0\left(\frac{B^2}{A}, 0\right)_\theta = \frac{cD}{r} e^{-rT} N\left(-f_2\left(\frac{B^2}{A}\right)\right) - J \left(\frac{\left(\frac{B^2}{A}\right)}{B} \right)^{-\beta} N\left(-f_1\left(\frac{B^2}{A}\right)\right)$$

$$f_1\left(\frac{B^2}{A}\right) = \frac{\ln\left(\frac{\bar{A}}{\left(\frac{B^2}{A}\right)}\right) - \left(\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta\right)T}{\sigma\sqrt{T}}$$

$$f_2\left(\frac{B^2}{A}\right) = f_1\left(\frac{B^2}{A}\right) - \sigma\beta\sqrt{T}$$

Where

$$C_1^{do}(A, 0) = C_0^D(A, D) - \left(\frac{B}{A}\right)^{\left(\frac{2(\mu-\gamma)}{\sigma^2}-1\right)} C_0^D\left(\frac{B^2}{A}, D\right)$$

$$C_0^D(A, D) = \left(\frac{cD}{r} - D\right) e^{-rT} N(-d_2(A)) - J \left(\frac{A}{\bar{A}} \right)^{-\beta} N(-d_1(A))$$

$$d_1(A) = \frac{\ln\left(\frac{\bar{A}}{A}\right) - \frac{1}{\beta}(\ln\left(\frac{cD}{r} - D\right) - \ln J) - \left(\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta\right)T}{\sigma\sqrt{T}}$$

$$d_2(A) = d_1(A) - \sigma\beta\sqrt{T}$$

$$C_0^D\left(\frac{B^2}{A}, D\right) = \left(\frac{cD}{r} - D\right) e^{-rT} N\left(-d_2\left(\frac{B^2}{A}\right)\right) - J \left(\frac{\left(\frac{B^2}{A}\right)}{\bar{A}} \right)^{-\beta} N\left(-d_1\left(\frac{B^2}{A}\right)\right)$$

$$d_1\left(\frac{B^2}{A}\right) = \frac{\ln\left(\frac{\bar{A}}{\left(\frac{B^2}{A}\right)}\right) - \frac{1}{\beta}(\ln(\frac{cD}{r} - D) - \ln J) - \left(\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta\right)T}{\sigma\sqrt{T}}$$

$$d_2\left(\frac{B^2}{A}\right) = d_1\left(\frac{B^2}{A}\right) - \sigma\beta\sqrt{T}$$

Where

$$L_0(A) = \frac{cD}{r} - \bar{J}\left(\frac{A}{B}\right)^{-\kappa} - C_2^{do}(A, 0)$$

The first term represents the value of the risk-free debt. The last two terms represents the time 0 market value of all cashflows before time T, modelled as the difference between immediately starting perpetual debt and a forward starting perpetual debt expressed as a barrier call option with exercise price 0 at time T.

$$\kappa = \frac{\mu - \gamma - \frac{1}{2}\sigma^2 + \sqrt{\left(\mu - \gamma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2r}}{\sigma^2}$$

$$C_2^{do}(A, 0) = C_0(A, 0)_\theta - \left(\frac{B}{A}\right)^{\left(\frac{2(\mu-\gamma)}{\sigma^2}-1\right)} C_0\left(\frac{B^2}{A}, 0\right)_\theta$$

$$C_0(A, 0)_\theta = \frac{cD}{r} e^{-rT} N(-h_2(A)) - \bar{J}\left(\frac{A}{B}\right)^{-\kappa} N(-h_1(A))$$

$$h_1(A) = \frac{\ln\left(\frac{B}{A}\right) - \left(\mu - \gamma - \frac{1}{2}\sigma^2 - \sigma^2\kappa\right)T}{\sigma\sqrt{T}}$$

$$h_2(A) = h_1(A) - \sigma\kappa\sqrt{T}$$

$$C_0\left(\frac{B^2}{A}, 0\right)_\theta = \frac{cD}{r} e^{-rT} N\left(-h_2\left(\frac{B^2}{A}\right)\right) - \bar{J}\left(\frac{\left(\frac{B^2}{A}\right)}{B}\right)^{-\kappa} N\left(-h_1\left(\frac{B^2}{A}\right)\right)$$

$$h_1\left(\frac{B^2}{A}\right) = \frac{\ln\left(\frac{B}{\left(\frac{B^2}{A}\right)}\right) - \left(\mu - \gamma - \frac{1}{2}\sigma^2 - \sigma^2\kappa\right)T}{\sigma\sqrt{T}}$$

$$h_2\left(\frac{B^2}{A}\right) = h_1\left(\frac{B^2}{A}\right) - \sigma\kappa\sqrt{T}$$

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