

**Working Paper No. 34/10**

**Generalization of Age-Structured Bioeconomic  
Models in Theory and Practice**

**by**

**Stein Ivar Steinshamn**

SNF Project No. 5638  
Socio-economic effects of fisheries-induced evolution

The project is financed by the Research Council of Norway

INSTITUTE FOR RESEARCH IN ECONOMICS AND BUSINESS ADMINISTRATION  
BERGEN, AUGUST 2010  
ISSN 1503-2140

© Dette eksemplar er fremstilt etter avtale  
med KOPINOR, Stenergate 1, 0050 Oslo.  
Ytterligere eksemplarfremstilling uten avtale  
og i strid med åndsverkloven er straffbart  
og kan medføre erstatningsansvar.



## Generalization of Age-Structured Bioeconomic Models in Theory and Practice

**Abstract:** The harvesting functions and the stock dynamics in age-structured bioeconomic models are generalized in order to incorporate density dependence. Using this generalization anything from completely uniformly distributed fish to extreme schooling can be analyzed. The classical Beverton-Holt model comes out as a special case of the generalized model. Both the theoretical outline as well as practical numerical examples are provided, and the generalization can be applied both for simulation as well as optimization purposes given appropriate software.

Non-linear programming is applied to maximize the net present value with the new updating and harvesting functions are used as constraints. One practical result is that pulse fishing seems to become less and less economically profitable as we move from uniformly distributed fish to schooling species. The main reason why pulse fishing cease to be optimal in schooling fisheries, is that the economies of scale present in search fisheries gradually disappear when we move from search fisheries to schooling fisheries. This has important implications for how fish stocks ought to be managed in the future.

**Keywords:** Bioeconomic modelling, age-structured models, optimal harvesting, pulse fishing.

Age-structured models have been popular among fisheries scientists and fisheries managers for many years (Baranov 1918; Leslie 1945), and the Beverton and Holt (1957) model is the most commonly applied such model. Such models have been dominating among biologists for several decades (Hilborn and Walters 2001), whereas surplus growth models have retained a strong position in economics and in bioeconomic modelling (Scott and Munro 1985). This has changed recently, and more and more economists agree that age-structured models are necessary in order to cover the complexity of real world fisheries and fish stocks (Townsend 1986; Wilen 1985 & 2000). A large variety of bioeconomic age-structured models has now been developed (Tahvonen 2010). Such models can be used both for simulation as well as numerical optimization. Simulation models have traditionally been the most common ones as numerical optimization has been regarded as much more difficult, and analytical optimization as almost impossible (Clark 1990). The possibility to do numerical optimization with such models has become more realistic along with the occurrence of gradually more sophisticated software. Even the long held belief that analytical optimization is almost impossible has been challenged recently by, among others, Tahvonen (2010) who also presents a useful survey of age-structured optimization models.

Notwithstanding the large variety of age-structured bioeconomic models, the basic structure has more or less remained the same. Consequently these models have up to now primarily been representative of so-called search fisheries (e.g. bottom-trawl) where the fish is uniformly distributed in the water. Many of the most important fisheries in the world are, however, based on pelagic schooling species, for which existing age-structured models are not adequate. The purpose of the present article is to generalize the main relationships in an age-structured model in order to also include schooling species and, in general, be able to handle all kinds of density dependence in the stock. As a result the traditional

Beverton-Holt model comes out as just a special case of the generalized model. The model presented here can also be transformed to an aggregated surplus growth model if necessary by focusing on only one year-class.

## Background

Age-structured models can first and foremost be divided between simulation models and optimization models. Simulation models are used both for economic and biological purposes. In biology they are used mainly for forecasting and for simulating outcomes and strategies. In economics they are used to evaluate and compare scenarios and stylized harvesting patterns to see which ones yield the highest return. But as far as simulation models are concerned, the difference between biology and economics is not very significant.

Optimization models, on the other hand, are used to find the best strategies subject to given constraints and are mainly used in economics. The most common method is non-linear programming. Due to the complexity of age-structured models optimal solutions may not always be possible to attain even with well specified problems. In this article it is shown that even after conventional age-structured models have been generalized to deal with all kinds of density dependence, they can still be applied both for simulation and optimization purposes. The combination of the generalization outlined here and the fact that steadily more advanced software has become available recently, provide reasons to be quite optimistic about prospects for handling large and complex numerical optimization models.

A basic assumption in conventional age-structured models is that the so-called catchability coefficient is a constant. This again is equivalent to assuming that the density of fish is proportional to the abundance of fish. This assumption is more or less correct for most demersal species as these are usually uniformly

distributed, but may be way off the mark for pelagic schooling species. This fact is a major motivation for the present article, and in the following the implications of relaxing this assumption will be outlined.

Emphasis will be put on the effects of introducing catch and updating equations that not only represents uniformly distributed fish stocks but also any degree of schooling and unevenly distributed fish. This will be done along a continuum from completely uniformly distributed fish on one side to extreme schooling on the other side. First the mathematics will be outlined and at the end a numerical example will be given.

## Outline of the theory

The definitions of fishing mortality,  $F$ , and natural mortality,  $M$ , are fundamental in age-structured modeling. These are first and foremost related to the instantaneous change in the stock as follows:

$$\dot{N} = -(F + M)N(t) \tag{1}$$

where  $N(t)$  is the number of fish in a single cohort at a particular time,  $t$ .<sup>1</sup> For simplicity, and to avoid unnecessary subscripts, only a single age-class is investigated at the moment, and only the number of fish are concentrated upon. The corresponding biomass is easily found by multiplying the number of fish in each year-class by the weight at age for that year-class. The interpretation of (1) is that the instantaneous change in the stock is the sum of the change due to the harvesting activity,  $F \cdot N$ , and the instantaneous change in the stock due to natural mortality,  $M \cdot N$ . The change due to harvesting is defined as the instantaneous catch and can be written

---

<sup>1</sup>Dots are used to denote time derivatives. Time dependence in the variables are often ignored for notational convenience.

$$\dot{C} = F_t N(t). \quad (2)$$

The corresponding total values can be found as follows. The number of fish,  $N(t)$ , can be found by solving the differential equation given by (1). Total catch can be found by the time integral of (2) after the solution from (1) has been inserted. Each of these will be dealt with in separate sections. These tasks are straightforward under standard assumptions, but they are more demanding in the generalized model presented in the following. The standard assumptions are that  $F$  and  $M$  are constant in each time period. Remember that age-structured models usually are formulated as a combination of a continuous and discrete. The number of fish and the harvest can be found as continuous variables, but the model is usually updated using discrete time steps where the mortalities are constant over these time steps. This is no longer the case in the generalized model as the fishing mortality here can vary within a time period although the fishing effort, which is the control variable, is constant.

Another fundamental parameter, both in age-structured and aggregated bioeconomic modeling, is the catchability coefficient,  $q$  (Clark 1990). This can be defined as the relationship between fishing mortality and fishing effort:

$$F_t = qE_t \quad (3)$$

where  $E$  is the actual fishing effort exerted at time,  $t$ . Both equation (2) and (3) are hence correct by definition. The main and most important difference between the present approach and previous literature is that in the following  $q$  will no longer necessarily be just a constant as it usually is, and therefore neither will  $F$ . This comes as a result of modelling the density and distribution of fish explicitly.

Another way to formulate the instantaneous catch is by the expression

$$\dot{C} = k\rho E_t \quad (4)$$

where  $k$  is a selectivity parameter for this cohort and  $\rho$  is a density parameter. Equation (4), which includes density explicitly, may not be familiar to all but it is fairly obvious. The efficiency of the effort with respect to a particular cohort of fish depends on its selectivity towards that cohort and the density of fish in the cohort. The density of fish, however, is more complex than just the number of fish in a given area, and this is of particular interest here. Only in the case of uniformly distributed fish is the density proportional to the abundance of fish in a given area. In that case the density parameter can be written:

$$\rho = \frac{N}{V} \quad (5)$$

where  $V$  is the total volume of water screened and is supposed to be constant. Equations (2) – (5) imply that in this case the catchability coefficient is given by a constant

$$q = \frac{k}{V} \quad (6)$$

as  $k$  and  $V$  are both constants. The hypothesis about a constant catchability coefficient, as given by (6), hinges therefore on the assumption in (5) which is only valid for uniformly distributed fish. In other words, this represents the traditional case of age-structured modeling.

Fish, however, is not always uniformly distributed in the water, and for schooling fish stocks the catchability coefficient,  $q$ , is no longer constant. The reason for this is that in such fisheries density is not proportional to abundance as the fish continue to cluster no matter how small the stock is. In the case



of extreme schooling the density remains constant and not the catchability coefficient. When the stock abundance is reduced, it is simply the number of schools that is reduced and not the density as such because that remains the same within each school. In this situation it is either full density within schools or zero density between schools.

Total abundance of fish can in general be found by integrating over the density profile as follows:

$$N(\rho) = \int_0^{\rho} f(r) dr \quad (7)$$

where  $f(r)$  is the number of fish within a small concentration area,  $dr$ , and  $\rho$  is the maximum concentration. Equation (7) is valid both for any degree of schooling. The difference is that for non-schooling species  $\rho$  varies whereas for schooling species it does not. It follows from (7) that

$$N'(\rho) = f(\rho).$$

The inverse function  $\rho(N)$ , of which equation (5) is a special example, may be just as interesting. In the special case of (5) the density is simply proportional to the abundance and represents a uniformly distributed fishery. In the case of a pure schooling fishery, on the other hand, the density is constant,  $\rho = \bar{\rho}$ .

As interest is put particularly on the intermediate cases, a continuous function that goes from proportional to constant is needed. A straightforward function that fulfils this requirement is the following:

$$\rho(N) = hN^{\alpha} \quad (8)$$

where  $h$  is a constant. When  $\alpha = 0$  it is a pure schooling fishery as  $\rho$  then is constant and equal to  $h$ . When  $\alpha = 1$  it is a uniformly distributed fishery

as  $\rho$  then is proportional to  $N$ . In the latter case the constant  $h = \frac{1}{V}$ . The parameter  $\alpha$  will become the key parameter in the following analysis.

### Age-structured modelling

The next question is: how can this be incorporated in an age-structured model? For this purpose the equations for updating the number of fish in each cohort and the equations for calculating the catch from each cohort are needed in the general case where  $0 \leq \alpha \leq 1$ . The familiar Beverton-Holt model turns out to be a special case of the general model, namely the case when  $\alpha = 1$ .

### Stock dynamics

Updating the number of fish in a single cohort over time is considered first. This is found by solving the differential equation given in (1). Inserting the general expression for  $\rho$  from (8) into (4) yields:

$$\dot{C} = khN^\alpha E.$$

Combining this with (2) and (3) it is found that the catchability coefficient in the general case is no longer a constant but can be written as a function of  $N$  :

$$q(N) = khN^{\alpha-1}. \tag{9}$$

When  $\alpha = 1$  it is seen that equation (9) is consistent with equation (6), and  $q$  is just a constant independent of  $N$ , namely  $q = kh$ . By the definition in equation (3) and the expression for  $q(N)$  in (9) the fishing mortality can be rewritten

$$F = khN^{\alpha-1} E. \tag{10}$$

The stock in numbers at any point in time can now be derived by inserting (10) into (1) yielding the non-linear differential equation:

$$\dot{N} = -khN^\alpha E - MN. \quad (11)$$

Solving (11) the following general expression is found:

$$N(t) = \left\{ \left[ N_0^{1-\alpha} + \frac{khE}{M} \right] e^{-M(1-\alpha)t} - \frac{khE}{M} \right\}^{\frac{1}{1-\alpha}} \quad (12)$$

where  $N(0) = N_0$  is considered given. In this expression it is assumed that  $E$  does not change over time. In order to find the stock updated from one period to the next, which is often relevant in age-structured modelling, the expression becomes

$$N(t+1) = \left\{ \left[ N_t^{1-\alpha} + \frac{khE_t}{M} \right] e^{-M(1-\alpha)} - \frac{khE_t}{M} \right\}^{\frac{1}{1-\alpha}} \quad (13)$$

where  $N_t$  is given from the previous step, and  $E_t$  is the effort which, in this case, is assumed constant within each time period. It is readily seen that in the case of zero effort  $N(t+1) = N_t \cdot e^{-M}$  as expected.

In the special case where  $\alpha = 1$  it is found, by taking the limit, that equation (12) reduces to the familiar expressions from the Beverton-Holt model:

$$N(t) = N_0 e^{-(khE+M)t}$$

and

$$N(t+1) = N_t e^{-(khE_t+M)}.$$

Remember that  $F = khE$  in this case. On the other hand, in the special case of a extreme schooling, that is when  $\alpha = 0$ , the following expressions are found:

$$N(t) = \left( N_0 + \frac{khE}{M} \right) e^{-Mt} - \frac{khE}{M}$$

and

$$N(t+1) = \left( N_t + \frac{khE_t}{M} \right) e^{-M} - \frac{khE_t}{M}.$$

And in the intermediate case, where  $\alpha = \frac{1}{2}$ , the expressions become:

$$N(t) = \left[ \left( \sqrt{N_0} + \frac{khE}{M} \right) e^{-\frac{M}{2}t} - \frac{khE}{M} \right]^2$$

and

$$N(t+1) = \left[ \left( \sqrt{N_t} + \frac{khE_t}{M} \right) e^{-\frac{M}{2}} - \frac{khE_t}{M} \right]^2.$$

The latter one can be rewritten as a polynomial in effort<sup>2</sup>:

$$N(t+1) = aN_t - bN_t^{1/2}E_t + cE_t^2 \quad (14)$$

where

$$a = e^{-M}, \quad b = 2e^{-\frac{M}{2}} \cdot \frac{khg}{M}, \quad c = \left( \frac{khg}{M} \right)^2, \quad g = (1 - e^{-\frac{M}{2}}). \quad (15)$$

Notice that  $a$ ,  $b$ ,  $c$  and  $g$  are unique numbers after  $M$ ,  $k$  and  $h$  have been specified.

### Calculating the catch

Total catch can, by definition, be calculated by taking the integral of the instantaneous catch over time

---

<sup>2</sup>Also the more general  $N(t)$  can be written as polynomial in effort by substituting  $N_t$  by  $N_0$  and letting  $a$ ,  $b$ , and  $c$  be functions of time.

$$C = \int \dot{C} dt = \int FN dt. \quad (16)$$

The total catch from one particular cohort is then in the general case given by the function

$$C(t) = \int_0^t khE \left\{ \left[ N_0^{1-\alpha} + \frac{khE}{M} \right] e^{-M(1-\alpha)\tau} - \frac{khE}{M} \right\}^{\frac{\alpha}{1-\alpha}} d\tau \quad (17)$$

after  $F$  from (10) and  $N(t)$  from (12) have been inserted into (16) and assuming that effort is constant. Total catch during one time period can be written

$$C_t = \int_t^{t+1} khE_t \left\{ \left[ N_t^{1-\alpha} + \frac{khE_t}{M} \right] e^{-M(1-\alpha)\tau} - \frac{khE_t}{M} \right\}^{\frac{\alpha}{1-\alpha}} d\tau \quad (18)$$

where  $N(t) = N_t$  is given and the effort exerted in this period is  $E_t$ .

Again it is useful to look at special cases, and the catch within a single period is concentrated upon. It is easy to verify that in the case of uniformly distributed fish,  $\alpha = 1$ , the well-known expression for catch in numbers:

$$C_t = \frac{khE_t}{khE_t + M} N_t \left[ 1 - e^{-(khE_t + M)} \right],$$

familiar from the Beverton-Holt model, is found by taking the limit of (17). Again  $F = khE$ . In the case of extreme schooling fisheries,  $\alpha = 0$ , the expression for total catch is particularly simple:

$$C_t = khE_t$$

assuming that the effort is constant and equal to  $E_t$  within each period. This is in accordance with intuition, namely that catch is proportional to effort and independent of the stock in purely schooling fisheries.

As seen, the updating of number of fish per cohort is particularly simple in the  $\alpha = 1$  case whereas the total catch function is particularly simple in the  $\alpha = 0$  case. And both these extreme cases lend themselves easily to age-structured modelling also when the objective is to do optimization.

In the intermediate case, on the other hand, when  $0 < \alpha < 1$ , the integral in (17) seems almost unsolvable as a general expression to be valid for all  $\alpha$ . It is, however, possible to solve it for particular values of  $\alpha$  such as  $1/3$ ,  $1/2$  and  $2/3$  to mention a few. Many of these solutions consist of quite messy expression and are often too complicated to be used in age-structured optimization models using non-linear programming; at least very strong computational power is needed. For simulation purposes, on the other hand, it is straightforward to find the catch by numerical integration of (17) or (18) for any value of  $\alpha$ .

For optimization purposes, therefore, particular values of  $\alpha$  will be concentrated upon. The case when  $\alpha = 1/2$  seems to be the case with the least messy solution. It is also a useful case to analyze as it lies midway between zero and one and therefore is highly representative of the intermediate case. In this case total catch is given by a quadratic function of effort for a given stock:

$$C_t = \gamma\sqrt{N_t}E_t + \Gamma E_t^2 \quad (19)$$

where

$$\gamma = \frac{2khg}{M}, \quad \Gamma = \left(\frac{kh}{M}\right)^2 \cdot (2g - M) \quad (20)$$

and  $g$  defined as in (15). Both  $\gamma$  and  $\Gamma$  are unique numbers when  $k$ ,  $h$ , and  $M$  are specified. It is therefore quite straightforward to apply these expressions, (19) and (20), in an age-structured optimization model when adequate software is available.<sup>3</sup>

---

<sup>3</sup>Another value of  $\alpha$  that can be used for optimization is  $2/3$ . In this case  $C_t = aN_t^{2/3}E +$

It is also relatively easy to verify that these expressions, (14) and (19), fulfil the criterion that total catch plus total natural mortality during one time period is equal to the change in the stock during the same period. This is an obvious, but at the same time important, criterion for the model to make sense.

When applying this model in practice it is important to be aware of the fact that in the general case there is a restriction on effort. Except in the case when  $\alpha = 1$  the stock can be reduced to zero during one period if the effort is sufficiently high. Effort levels higher than the one that drives the stock to zero are therefore meaningless. Only when  $\alpha = 1$  the stock will approach zero asymptotically. In the general case,  $0 < \alpha < 1$ , there is a critical limit on effort,  $E_{\max,t}$ , that must be taken into account. This is given by

$$E_{\max,t} = \frac{N_t^{1-\alpha} M}{kh [e^{M(1-\alpha)} - 1]}. \quad (21)$$

In the special case that  $\alpha = 1/2$ , this reduces to

$$E_{\max,t}^{\alpha=1/2} = \frac{\sqrt{N_t} M}{kh(e^{M/2} - 1)},$$

and in the extreme case with  $\alpha = 0$  it becomes

$$E_{\max,t}^{\alpha=0} = \frac{N_t M}{kh(e^M - 1)}.$$

When  $\alpha = 1$ ,  $E_{\max}$  is infinity.

There is also another critical effort level beyond which the catch starts to decline. However, it is relatively easy to verify that this level is always higher than the one given by (21) and therefore the latter will be binding first and hence is the only one that needs to be considered.

---

$bN_t^{1/3}E^2 + cE^3$  where  $a$ ,  $b$  and  $c$  depend on  $k$ ,  $h$  and  $M$  only. In fact, when  $\alpha = \frac{n-1}{n}$  and  $n$  is an integer greater than zero, it can be shown that  $C_t$  is a polynomial of degree  $n$  in effort. This, however, is only useful for practical purposes when  $n$  is small.

## Numerical example

In this section a numerical example based on a hypothetical fish stock is presented. The purpose of this example is to illustrate one of the most important implications of changing  $\alpha$  when everything else is kept equal and simple. Therefore complicating aspects, such as a stock-recruitment relationship etc., although they could easily have been included, are avoided.

Three values of  $\alpha$  are investigated, namely 0, 1/2 and 1. It is an age-structured model with four year-classes and constant recruitment. The optimization is done over a period of 10 years. The objective function is to maximize the undiscounted net revenue over the whole period given by

$$\pi = \sum_{t=1}^{10} \sum_{a=1}^4 (pw_a C_{a,t} - \kappa E_t)$$

where  $p$  is the price per unit biomass harvested,  $w_a$  is weight at age  $a$  and  $\kappa$  is the cost per unit effort. The reason why discounting is ignored is that the effect of discounting is only a somewhat higher effort and that harvest is brought forward in time. It does not affect the qualitative implications of changing the value of  $\alpha$ . Net revenue is maximized subject to the dynamic equations (13) and the restrictions on effort given by (21). There is also an additional constraint guaranteeing that the the stock left behind after the optimization period is no smaller than the initial stock. The last constraint is introduced to guarantee sustainability such that the remaining stock is not mined out towards the end of the time horizon. More specifically, in this case it implies that the stock in period 11 is no smaller than the initial stock in period one. The model is solved in a GAMS environment using KNITRO ([www.ziena.com/knitro.html](http://www.ziena.com/knitro.html)) as solver.

The numerical specification of the model is given in appendix. All exogenous



parameters are the same for all values of  $\alpha$  except the cost of effort. The reason for this that the effort can no longer be interpreted the same way when  $\alpha$  is changed. When  $\alpha = 1$ , effort is proportional to fishing mortality,  $F$ , as defined in conventional age-structured models. When  $\alpha = 0$ , however, effort is simply proportional to harvest. For  $0 < \alpha < 1$  the interpretation of effort lies somewhere between these two extreme points. The way costs have been specified in the stylized model, therefore, is by calibrating the parameters such that the cost is the same for a typical harvest given a representative initial stock. Here it has been calibrated such that a harvest equal to 2.7 has the same cost for all three values of  $\alpha$  with the initial stock.

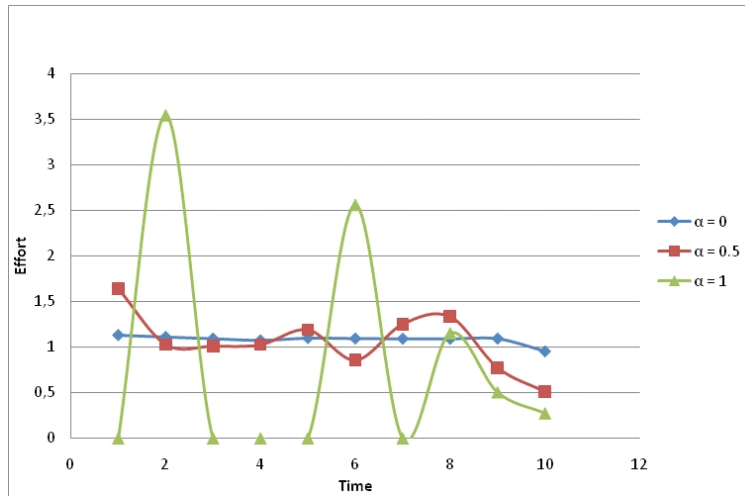


Figure 1. Optimal effort over time for various values of  $\alpha$ .

The results are shown in figure 1. The most interesting and intriguing aspect of this figure is that the harvesting pattern becomes gradually more even the lower the value of  $\alpha$  is. When  $\alpha = 1$ , that is with the classical Beverton-Holt model, typical pulse fishing patterns occur. That such patterns are optimal

within this setting was discovered already several decades ago (Hannesson 1975). And the main reason why pulse fishing is optimal is the lack of perfect selectivity combined with non-linear individual growth of the fish. Ideally the fish should be harvested at the age at which yield-per-recruit is maximized, but this is only possible with perfect selectivity. Lack of perfect selectivity therefore makes some extent of pulse fishing optimal due to economies of scale.

Economies of scale exist when  $\alpha > 0$  as the instantaneous catch function is homogenous of degree  $1 + \alpha$ . The strongest economies of scale occur when  $\alpha = 1$  as instantaneous catch then is homogenous of degree two. The higher  $\alpha$  is, the more it pays off to let the stock build up over a period of time and then take a substantial harvest instead of harvesting the same amount each period. As  $\alpha$  gets smaller this effect gradually disappears, and this is the explanation behind the pattern seen in figure 1.

This is reconfirmed by running the model for various combinations of growth and selectivity. The growth is divided between linear and kinked (piecewise linear), and the selectivity is divided between perfect (uniform) and knife-edge. The beneficial effects of pulse fishing compared to an even fishing pattern is expected to be higher when the individual growth of fish is non-linear than when it is linear. This is also confirmed by the results. The results are summarized in Table 1.

*Table 1. Degree of Optimal Pulse when  $\alpha = 1$ .*

	Selectivity	
growth	uniform	perfect
linear	strong pulse	weak pulse
kinked	strong pulse	even fishing pattern

The equivalent table when  $\alpha = 0$  consist of an even fishing pattern for all four combinations of growth and selectivity, and this table is therefore not shown.

## Summary

In this article the main relationships in age-structured modelling, in particular the expression for updating the number of fish in each year-class from one period to the next and the expression for calculating the catch from each year-class in each period, have been generalized compared to traditional age-structured models in order to take density dependence in the harvest function properly into account. This generalization makes it possible to include everything from fish stocks characterized by pure schooling to fish stocks characterized by perfect uniform distribution in the model. The new and generalized model has the property that the classical Beverton-Holt model comes out as a special case. The generalized model can be used both for simulation and optimization although optimization is clearly more demanding, at least computationally. Nevertheless, a numerical example has been provided. In this example the new model has been formulated as an optimization model using non-linear programming and applied on a hypothetical fish stock. It was found that the higher the degree of schooling behavior among the fish, the more even is the optimal fishing pattern. With completely uniform distribution of fish, the well-known result that pulse fishing is optimal is reconfirmed.

The finding that pulse fishing patterns cease to be optimal going from uniformly distributed fish stocks to schooling fisheries is a novel discovery. The explanation why pulse fishing cease to be optimal in schooling fisheries, is that the economies of scale present in search fisheries gradually disappear when we move from search fisheries to schooling fisheries. This ought to be looked much deeper into using models representing real world fisheries as it may have im-

portant implications for how such fisheries ought to be managed in the future. After all, the real value of  $\alpha$  for most fisheries is most likely somewhere between zero and one rather than exactly equal to one as assumed in traditional age-structured models like the Beverton-Holt model. Basing all management on Beverton-Holt like models may therefore not only cause foregone revenue but also cause more variations in quotas from year to year than necessary. And more stable quotas over time are desired by fishermen as it makes their activity more predictable and therefore correct decisions regarding investment, etc., easier to make. Applying this model to real world fish stocks is an obvious topic for future research.

## References

- T.I. Baranov, T.I., On the question of the biological basis of fisheries, Nauch. Issledov. Iktiolo. Inst. Izv. I (1) pp. 81–128 (Moscow) (1918).
- R.J.H. Beverton and S.J. Holt, On the dynamics of exploited fish populations. Fisheries Investigations Series 2(19). London: Ministry of Agriculture, Fisheries and Food, 1957.
- C.W. Clark, C.W., Mathematical Bioeconomics: The optimal management of renewable resources. New York, Wiley & Sons, 1990.
- R. Hilborn and C.J. Walters, Quantitative Fisheries Stock Assessment: Choice, Dynamics and Uncertainty, 3rd. edn. London, Chapman & Hall, 2001.
- R. Hannesson, Fishery dynamics: A North Atlantic cod fishery, Canadian Journal of Economics 8(1975):151-173.
- P.H. Leslie, On the use of matrices in certain population mathematics, Biometrika 33(1945): 183 - 212.
- A.D. Scott and G.R. Munro, The Economics of Fisheries Management, in Handbook of Natural Resource and Energy Economics. Volume 2, (A.V. Kneese

and J.L. Sweeney, Eds.). Amsterdam and Oxford, North-Holland, 1985.

O. Tahvonen, Age structured optimization models in fisheries bioeconomics: A survey, in "Optimal Control of Age-structured Populations in Economy, Demography, and the Environment", R. Boucekine, N. Hritonenko, Y. Yatsenko (editors). Abingdon UK, Routledge, 2010.

R.E. Townsend, A critique of models of the American Lobster fishery, *J. of Environ. Econom. and Management* 13(1986): 277 - 291.

J.E. Wilen, J.E., Bioeconomics of renewable resource use, in *Handbook of Natural Resource and Energy Economics. Volume 1*, (A.V. Kneese and J.L. Sweeney, Eds.) Amsterdam and Oxford, North-Holland, 1985.

J.E. Wilen, Renewable resource economists and policy: what differences have we made? *J. of Environ. Econom. and Management* 39(2000): 306 - 327.

## Appendix

In this appendix the exogenous parameters in the numerical model are given.

Age-specific parameters are given in Table A1.

<i>age</i>	$N_1$	$w$	$k$	$h$
1	4	1	0.4	0.8
2	3	3	0.6	0.8
3	2	5	0.8	0.8
4	1	7	1.0	0.8

Other parameters such as costs, price and natural mortality are given in Table

A2. Costs must be  $\alpha$ -specific in order for the model to be consistent.

$\alpha$	$p$	$\kappa$	$M$
0	2	0.81	0.2
0.5	2	1.0	0.2
1.0	2	1.35	0.2

Recruitment is constant and set to four in all periods. Specification of the

runs summarized in Table 1 above is given in Table A3.

*Table A3. Characterization of the Growth and Selectivity Patterns Used.*

	Growth		Selectivity	
	linear	kinked	uniform	perfect
age	$w$	$w$	$k$	$k$
1	1.0	1.0	1.0	0.0
2	3.0	3.0	1.0	0.0
3	5.0	5.0	1.0	1.0
4	7.0	5.0	1.0	1.0