Low Energy Effects of a Left-Right Symmetric Model extended by a heavy Quark.

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Thesis submitted for the degree of
Master of Physics

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February 2010

ACKNOWLEDGMENTS

As I prepare to submit the product of my endeavours, there are certain groups and individuals who I must thank and acknowledge as providing necessary and/or complementary support.
Firstly, thanks are due to my supervisor, Professor Jan Olav Eeg for being available to field an increasing number of questions, for aiding me in unearthing source material and otherwise providing support and reassurance.
Thanks to Ivica Picek for rapid and helpful responses to email correspondence.
I would also like to thank Phil Minns for language support and corrections.
Thanks to the people who accompanied me on coffee breaks for entertaining conversation and for providing a welcome diversion from the insularity of thesis-writing.
Finally, to my entire family for listening ears, encouraging words and financial support.

Hannah Bos
Oslo, January 2010.

CONTENTS

INTRODUCTION

In this thesis I set out to explore the methods of and the space available when adding new quarks to the particle content of the Standard Model (SM) and the Left-Right Symmetric Model (LRSM).

Most observations of interactions mediated by the electromagnetic, weak and strong forces can be adequately explained by the Standard Model. It is known, however, that due to unexplained unnaturalnesses such as the hierarchy problem and the requirement of additional sources of CP-violation in order to explain baryogenesis, the Standard Model must be the effective theory of an extended model which is possibly broken down at a higher energy scale. The rare deviations of measurements from the Standard Model predictions make it difficult to grasp and constrain new physics. There are also a number of models which suffer under a large number of parameters and therefore reveal a difficulty which occurs during model-building, namely that there exists a fine line between a model which can predict the outcome of an experiment after inserting a reasonable number of parameters and a model which loses its predictive capability since the large number of parameters can be arranged in such a way that almost every possible outcome can be produced.

There is thus a wide range of models based on similar principles to the Standard Model which have been proposed as possible extensions, including Spontaneous Symmetry Breaking, an extended Higgs sector and an extended gauge symmetry at a higher energy scale. One aim of this thesis is to list and describe some representatives of those models, specifically the Left-Right Symmetric Model, the Little Higgs Model and the Standard Model with an extended quark sector.

A method of introducing new quarks to a Lagrangian and the quarks' mass-gaining process is discussed in detail. Subsequently the Left-Right Symmetric Model is extended by a vector-like quark isosinglet. In order to constrain new parameters, two types of interactions are analysed within the framework of the LRSM with the extra heavy quark, namely interactions which are correctly predicted by the Standard Model (such as Kaon and B-meson mass-mixing and CP-violation in meson decay) and interactions which have not yet been observed due to their minuteness (such as meson to lepton pair decay). Hence the parameters are constrained in such a way as to produce the same outcome for the correctly predicted interactions and a larger outcome for the interactions yet to be discovered in order to be distinguishable from the Standard Model.

While calculating these interactions, the opportunity to display more detailed derivations of the loop integral functions and more Feynman rules within the LRSM than are usually used in the literature is taken.

The thesis is roughly divided into three parts. The first chapter opens with a description of the Standard Model ingredients, including the Lagrangian, the concept of an effective Lagrangian, the gauge and discrete symmetries, the Higgs mechanism, the CKM matrix and renormalisability. Subsequently parameters, interactions and concepts involved in the analysis of the extent of new physics beyond the Standard Model in contemporary observations, such as meson mass-mixing, the oblique T- and R_b-parameters and the decoupling theorem are discussed.
The second chapter gives an overview of extensions of the Standard Model, including the Standard Model with a fourth generation of quarks, the LRSM and the Little Higgs Model. Within this, recently obtained constraints on the new parameters such as new particle masses and new phases are discussed.
The third part, consisting of chapters three and four, addresses the Standard Model and the LRSM extended by a vector-like heavy Top-quark. While the concept of introducing a vector-like quark to the Standard Model has been discussed in the literature, it has to the best of my knowledge not been discussed in the case of an LRSM. Meson mass-mixing and decay are subsequently discussed in order to constrain the new parameters.
The Appendix deals with the derivation of loop integral functions, Feynman rules within the LRSM and hadron matrix elements.

1. THE STANDARD MODEL

1.1 The Langrangian of the Standard Model

All equations in this section are taken from [1] unless cited otherwise. The Standard Model has an SU(3)$_c\times$SU(2)$_L\times$U(1)$_Y$ gauge symmetry which is broken down to SU(3)$_c\times$U(1)$_Q$ by Spontaneous Symmetry Breaking (SSB), which in this case is called the Higgs mechanism (chapter 1.2). SU(3) gauges the strong interaction such that fields transform according to their colour charge c which is only possessed by quarks and gluons. SU(2) gauges the weak interaction, under which left-handed fermions transform as doublets and right-handed fermions as singlets. The electromagnetic interaction is gauged by U(1), under which fermion fields transform according to their electrical charge Q. After the breaking mechanism the weak and the electromagnetic interaction are combined to give the electroweak interaction. The fermion fields transform under the U(1)$_Y$ gauge symmetry according to their weak hypercharge

$$Y = Q - T^3, \tag{1.1}$$

where T^3 denotes the third component of the weak isospin.
Before the SSB the Lagrangian describing the Standard Model is given by

$$L_{SM} = L_1 + L_2 + L_{QCD}, \tag{1.2}$$

where L_1 describes a Lagrangian symmetric under an U(1)-symmetry and L_2 a Lagrangian symmetric under an SU(2)-symmetry. After SSB L_1 and L_2 will combine to the electroweak Lagrangian L_{EW} which in turn can be split into the electromagnetic Lagrangian L_{EM} or into the QED Lagrangian and the weak Lagrangian L_W. The term L_{QCD} describes the strong interacting part.
The U(1)-symmetric part is given by

$$L_1 = \sum_f \bar{f}(i\not{D}_1 - m)f - \frac{1}{4}F_1^{\mu\nu}F_{1\mu\nu} \tag{1.3}$$

with one Dirac spinor for every fermion $f \in \{\nu_e, e, \nu_\mu, \mu, \nu_\tau, \tau, u, d, c, s, t, b\}$, the field tensor

$$F_1^{\mu\nu} = \partial_\mu B_\nu - \partial\nu B_\mu$$

and the covariant derivative

$$D_1^\mu = \partial^\mu + ig'B^\mu.$$

The SU(2)-symmetric Lagrangian describes three gauge fields A_μ^a and writes the following:

$$L_2 = \sum_{u,d} \left(\bar{Q}_L(i\not{D}_2)Q_L\right) + \sum_{e,\nu_e} \left(\bar{E}_L(i\not{D}_2)E_L\right) + \sum_{f} \left(\bar{f}_R(i\not{D}_2)f_R\right) - \frac{1}{4}F_{2a}^{\mu\nu}F_{2\mu\nu}^a$$

where $u \in \{u, c, t\}$, $d \in \{d, s, b\}$, $e \in \{e, \mu, \tau\}$ and $\nu_e \in \{\nu_e, \nu_\mu, \nu_\tau\}$.
The left-handed fields are arranged in doublets:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \text{and} \quad E_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}. \tag{1.4}$$

The field tensor for a SU(2) symmetry which describes the coupling of the gauge bosons to each other is given by

$$F_{2\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{ijk}A_\mu^j A_\nu^k$$

with the Levi-Civita symbol ϵ.
The covariant derivative writes

$$D_2^\mu = \partial^\mu - \frac{ig}{2}\sigma_i A_i^\mu$$

with the three Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{and} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Considering SSB, the U(1)- and the SU(2)-symmetric Lagrangian combine, while the QCD Lagrangian remains unchanged. The Lagrangian is also enhanced by a potential term of the Higgs doublet Φ (1.20) (introduced in chapter 1.2.2) and the Yukawa coupling, which couples the Higgs field to the fermions.

$$L_{SM} = L_{EW} + L_{QCD} + L_\phi + L_{Yukawa}$$

The Lagrangian L_ϕ couples the Higgs field to the gauge bosons of the electroweak interaction and to itself:

$$L_\phi = D_\mu\phi D^\mu\phi - V(\phi) \quad \text{with} \quad V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

where the parameters λ and μ are discussed in chapter 1.2.
The covariant derivative for the electroweak interaction is given by

$$D^\mu = \partial_\mu - \frac{ig}{2}\sigma^i A^{i\mu} - \frac{ig'}{2}B^\mu.$$

The physical gauge bosons are combinations of the four gauge fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2), \quad Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 - g'B_\mu) \quad \text{and} \quad A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A_\mu^3 + gB_\mu). \tag{1.5}$$

The electrical charge e and the weak mixing angle θ_W are defined in the following way:

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} \quad \text{and} \quad cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}.$$

Inserting these physical fields and rewriting the generators of the gauge groups alters the covariant derivative in the following way:

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{cos\theta_W} Z_\mu (T^3 - sin^2\theta_W Q) - ieA_\mu Q \tag{1.6}$$

with

$$T^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2), \qquad T^3 = \frac{1}{2} \quad \text{for} \quad u_L, c_L, t_L \qquad \text{and} \qquad T^3 = -\frac{1}{2} \quad \text{for} \quad d_L, s_L, b_L.$$

The Yukawa coupling writes

$$L_{Yukawa} = -\sum_{e,\nu_e} \lambda_e \bar{E}_L \phi e_R - \sum_{u,d} \left(\lambda_d \bar{Q}_L \phi d_R + \lambda_u \epsilon^{ab} \bar{Q}_{La} \phi_b^+ u_R \right) + h.c.$$

There are no right-handed neutrinos in the Standard Model, since they are left massless. The parameters $\lambda_{e,u,d}$ are discussed in chapter 1.4.
The electroweak Lagrangian writes

$$L_{EW} = \sum_{u,d} (\bar{Q}_L (i\not{D}) Q_L) + \sum_{e,\nu_e} (\bar{E}_L (i\not{D}) E_L) + \sum_f (\bar{f}_R (i\not{D}) f_R) - \frac{1}{4} F_{2a}^{\mu\nu} F_{2\mu\nu}^a - \frac{1}{4} F_1^{\mu\nu} F_{1\mu\nu}. \tag{1.7}$$

The QCD Lagrangian is given by [2]

$$L_{QCD} = \sum_q \bar{\psi}_q (i\not{D}_3 - m_q) \psi_q - \frac{1}{4} G_{\mu\nu}^j G_j^{\mu\nu}, \tag{1.8}$$

with

$$D_3^\mu = \partial^\mu + \frac{ig_3}{2} \lambda_l A^{l\mu}, \qquad l \in \{1, 2, 3, ..., 8\}$$

with the eight generators of SU(3) λ_l, the eight massless gluon fields A_μ^l and the field tensor

$$G_{\mu\nu}^j = \partial_\mu A_\nu^j - \partial_\nu A_\mu^j - g f_{ikl} A_\mu^k A_\nu^l.$$

There is one colour triplet for every quark q

$$\psi_q = \begin{pmatrix} q_{red} \\ q_{blue} \\ q_{green} \end{pmatrix}.$$

One can choose the Gell-Mann matrices as SU(3) generators

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$
$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

In this representation the structure constants f_{ikl} write

$$f_{123} = 1, \quad f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}, \quad f_{156} = f_{367} = -\frac{1}{2} \quad \text{and} \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}.$$

1.2 Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking (SSB) describes a symmetry which is obeyed by the Lagrangian but not by the vacuum. This means that if the Lagrangian is transformed under the symmetry it stays invariant but the vacuum transforms non-trivially (non-invariantly).
SSB can be used in models which have a higher degree of symmetry at a higher energy scale than at a lower energy scale, where one could regard the symmetry to be partly hidden. Examples include supersymmetry (SUSY) and Left-Right Symmetric Models (LRSM) (chapter 2.2).
Another important application of SSB is the Higgs mechanism, which is used in many models in order to give mass to the otherwise massless gauge bosons.
One simple approach is the ϕ^4-theory. The corresponding Lagrangian can be written as a kinetic term minus a potential term [3]

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) \qquad \text{with} \qquad V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4. \tag{1.9}$$

If SSB is used in a gauge symmetry, V is usually referred to as the Higgs potential. The value of ϕ which minimises the Higgs potential is called the vacuum expectation value (vev) $\phi_0 = |\langle 0|\, \phi\, |0\rangle|$. The symmetry transformation which leaves the Lagrangian invariant is the reflection of the field ϕ on the horizontal axis $\phi \to -\phi$.
A non-vanishing vev ($\phi_0 = v$) would not inherit the axial symmetry of the potential, nor would the Lagrangian described by a shifted field around this vev ($\phi' = \phi - v$) since it includes a term proportional to ϕ'^3.

$$L = \frac{1}{2}(\partial_\mu \phi')^2 + \mu^2\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

In this case a non-vanishing vev and therefore SSB is only possible if $\mu^2 > 0$. The symmetry was broken by choosing one vev and expanding the field around it.
If the Lagrangian of a scalar field is invariant under a continuous symmetry, the Goldstone theorem (which states that for every spontaneously broken continuous symmetry there appears one massless Goldstone boson in the Lagrangian) holds. It can be shown that the Goldstone bosons are massless even after including loop corrections. Those massless bosons provide the one physical degree of freedom the gauge bosons are missing in order to be able to acquire a mass.
On the other hand, if a symmetry remains unbroken the degree of freedom transfers to the scalar field, which thus becomes a physical massive Higgs boson. The scalar fields are in general integrated in a gauge theory in the following way [1]:

$$L = \frac{1}{2}(D_\mu \phi_i)^2 - V(\phi_i) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}. \tag{1.10}$$

The integration of the Higgs doublet in the Standard Model will be discussed in chapter 1.2.2. The fields ϕ_i are taken to be real-valued, which is only a matter of convention.

$F^a_{\mu\nu}$ is the gauge-tensor discussed in chapter 1.1. The covariant derivative which couples the gauge to the scalar fields is given by [3]

$$D_\mu \phi_i = \partial_\mu \phi_i + g A^a_\mu T^a_{ij} \phi_j$$

where T^a are real, antisymmetric representation matrices of the underlying gauge group. Every component of ϕ which acquires a vev can be shifted and the symmetry in this direction is broken

$$\phi_i(x) = \phi_{0i} + \chi_i(x). \tag{1.11}$$

Reinserting the shifted components (1.11) into the Lagrangian (1.10) gives masses to as many gauge bosons as broken symmetries with the following mass matrix [1]:

$$m^2_{ab} = g^2 (T^a \phi_0)_i (T^b \phi_0)_i.$$

The couplings of the Goldstone and the Higgs bosons to the fermions as well as the mass terms of the fermions due to the vevs of the scalar fields are introduced via the Yukawa coupling, which is chosen to be the most general renormalisable gauge invariant term that could be added to the Lagrangian [1]

$$L_Y = -\lambda_{dij} \bar{Q}_{Li} \phi d_{Rj} - \lambda_{uij} \epsilon^{ab} \bar{Q}_{Lai} \phi^+_b u_{Rj} + h.c.$$

where λ_d and λ_u are general complex matrices.

1.2.1 The R_ξ-gauge

Considering the same situation in the path integral formalism using the Faddeev-Popov gauge-fixing procedure shows that a gauge-fixing function needs to be chosen. In the path integral formalism the generating functional Z, from which correlation functions can be deduced, can be obtained by integrating over all possible fields. In this case the relevant fields are the gauge fields A and the scalar fields χ.

$$Z = \int DAD\chi e^{i \int d^4x L(A,\chi)} \tag{1.12}$$

Since the underlying gauge group assures that the Lagrangian stays invariant under the gauge field shift

$$A^a_\mu \to A'^a_\mu = A^a_\mu + \delta A^a_\mu \qquad \text{with} \quad \delta A^a_\mu = \frac{1}{g} \partial_\mu \alpha^a - f^{abc} \alpha^b A^c_\mu = \frac{1}{g} (D_\mu \alpha)^a, \tag{1.13}$$

the infinite number of gauge fields which are linked by this transformation are physically equivalent. However, the formula used in (1.12) integrates over all those unphysical degrees of freedom and therefore produces unphysical divergences. The problem is solved by introducing a gauge-fixing condition (G) in the form of a δ-function which ensures that only one of the gauge fields linked by (1.13) is counted. Introducing the gauge-fixing condition to the equation of the generating function gives [1]

$$Z = C \int DAD\chi e^{i\int d^4x L(A,\chi)} \delta(G(A,\chi)) det\left(\frac{\delta G(A',\chi)}{\delta\alpha}\right) \qquad \text{with} \quad C = \int D\alpha,$$

where C is an infinite constant. The gauge-fixing is arbitrary and can therefore include an additive arbitrary scalar function of x such as $\omega(x)$. Instead of adding it to the gauge-fixing condition it could be added in the delta function such as $\delta(G(A,\chi) - \omega(x))$. Since $\omega(x)$ is arbitrary, one could add or integrate over an infinite number of ω-functions. Using this and including a Gaussian weighting function centred at $\omega = 0$ the δ function drops out of the integral and a new term is added to the Lagrangian

$$Z = C' \int DAD\chi e^{i\int d^4x (L(A,\chi) - \frac{1}{2}G^2)} det\left(\frac{\delta G}{\delta\alpha}\right) \qquad \text{with} \quad C' = CN, \tag{1.14}$$

where N is a normalisation constant due to the Gaussian weighting function.
In the R_ξ-gauges the gauge-fixing condition is chosen in such a way that it cancels the terms in the original Lagrangian which mix gauge and scalar fields. It also includes a free parameter ξ which can chosen to be any number.

$$G^a = \frac{1}{\sqrt{\xi}}(\partial_\mu A^{a\mu} - \xi g F_i^a \chi_i) \qquad \text{with} \quad F_i^a = T_{ij}^a \phi_{0j}$$

Inserting this condition into the equation for the generating function (1.14) gives an effective Lagrangian from which one can read off the mass matrices of the gauge bosons

$$(m_A^2)^{ab} = g^2 F_i^a F_i^b = g^2 (FF^T)^{ab}, \tag{1.15}$$

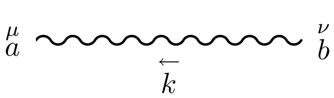
the Goldstone bosons

$$(m_G^2)_{ij} = \xi g_a^2 F_i^a F_j^a = \xi g_a^2 (F^T F)_{ij} \tag{1.16}$$

and the physical Higgs bosons

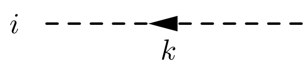
$$M_{ij} = \frac{\partial^2}{\partial\phi_i \partial\phi_j} V(\phi)|_{\phi_0}. \tag{1.17}$$

The dependence on the free parameter ξ of the mass matrix of the Goldstone bosons shows the unphysicality of these particles.
Calculating the propagator gives for the gauge bosons [1]



$$= \left(\frac{-i}{k^2 - m_A^2}\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 - \xi m_A^2}(1-\xi)\right]\right)^{ab} \tag{1.18}$$

and for the scalar fields



$$= \left(\frac{i}{k^2 - \xi m_G^2 - M^2}\right)_{ij} . \tag{1.19}$$

Here the mass matrices need not necessarily be diagonal, which means that the propagators above do not automatically describe the physical mass eigenstates of the gauge boson but a mixture between them.

The evaluation of the determinant in equation (1.14) results in another additional part to the Lagrangian. This part describes unphysical fields, called ghost fields, which couple exclusively to gauge and Higgs fields. Their mass matrix also depends on ξ and therefore they cannot be observed as external particles.

As previously noted, the parameter ξ can be chosen freely. It has been shown [1] that the theory remains renormalisable for every finite value of ξ as well as for $\xi \to \infty$. The gauge character of ξ is also reflected in the fact that it never shows up in any physical quantity such as an S-matrix element. However, it can be useful to choose ξ in a manner appropriate to the type of calculation. Common choices of the gauge parameter will be discussed in the next chapter after discussing the propagators (1.18,1.19) for the Standard Model.

1.2.2 The Glashow-Weinberg-Salam theory

The theory introduced by Glashow, Weinberg and Salam (GWS) combines the electromagnetic with the weak interaction and is therefore referred to as a theory of electroweak interaction. Considering Lagrangian (1.2) the GWS theory solves the problem of absent mass terms by Spontaneous Symmetry Breaking.

The scalar fields are chosen to be arranged in a complex doublet and therefore contain four degrees of freedom. A conventional parametrisation of the Higgs doublet containing four real scalars is given by [3]

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v+(h+i\phi^0)}{\sqrt{2}} \end{pmatrix} \tag{1.20}$$

with the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.21}$$

and the potential

$$V(\Phi) = -\mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2 .$$

It transpires that this vev value remains invariant under the gauge transformations of SU(2)xU(1) if two of the angles are chosen specifically and the last two are chosen to depend on each other. Thus one degree of freedom remains, producing a massive scalar field (the Higgs boson h) and leaves one of the gauge bosons (the photon) massless. The three broken symmetries lead to three massless Goldstone bosons $\phi^{\pm}$ and ϕ^0, where $\phi^{\pm}$ generate the masses of the $W^{\pm}$ bosons and ϕ^0 the mass of the neutral Z-boson.
Calculating the mass matrices for the gauge and Goldstone bosons (1.15, 1.16) in GWS theory in the R$_\xi$-gauge results in [1]

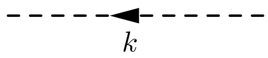
$$m_A^2 = \frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}, \qquad m_G^2 = \xi\frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 \\ 0 & g^2 & 0 \\ 0 & 0 & g^2+g'^2 \end{pmatrix}$$

where g and g' are the coupling constants of the gauge groups SU(2) and U(1) respectively.
The mass matrix of the gauge bosons shows that the gauge fields given by the gauge groups SU(2) and U(1), A_μ^a $(a = 1, 2, 3)$ and B_μ, are not the mass eigenstates. Rewriting the boson fields in the manner of (1.5) results in the fields for the physical bosons.
Through these transformations the mass matrix is diagonalised and the propagators (1.18, 1.19) decouple, which means that the field-mixing indices (a, b) and (i, j) can be dropped and there is exactly one propagator for each gauge and Goldstone boson.
The propagator for the gauge bosons is given by

$$\mu \;\underset{\overleftarrow{k}}{\sim\!\sim\!\sim\!\sim\!\sim\!\sim}\; \nu = \frac{-i}{k^2 - m^2}\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 - \xi m^2}(1-\xi)\right] \tag{1.22}$$

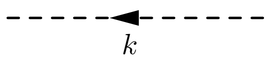
for m being either the mass of the W-bosons ($m_W = g\frac{v}{2}$), the Z-boson ($m_Z = \sqrt{g^2 + g'^2}\frac{v}{2}$) or the photon ($m_A = 0$).
After the symmetry breaking, the mass matrix of the physical Higgs fields (1.17) leave those elements which describe directions in which the symmetry has been broken as zero. Thus there is no contribution to the propagators of the Goldstone bosons (1.23). The only non-zero values in (1.17) are those elements relating to directions in which the symmetry remains. Hence (1.17) contributes to the propagator of the physical Higgs field (1.24).

The propagator for the Goldstone bosons is given by



$$= \frac{i}{k^2 - \xi m^2}, \tag{1.23}$$

with $m = m_W$ for $\phi^{\pm}$ and $m = m_Z$ for ϕ^0.
The Higgs boson propagates via

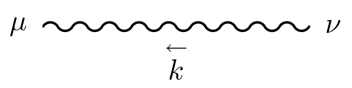


$$= \frac{i}{k^2 - 2\mu^2} \tag{1.24}$$

with $\mu^2 > 0$ since the term in the Higgs potential (1.9) proportional to μ^2 was chosen to be negative.
It is noticeable that different choices of ξ lead to different propagators of the gauge and Goldstone bosons. The three most common choices will be discussed below.

- *Lorentz gauge* ($\xi = 0$): This choice leaves the Goldstone bosons massless and is therefore useful for theoretical considerations.
- *Feynman-t'Hooft gauge* ($\xi = 1$): Here the Goldstone bosons are treated as scalars with the mass of the corresponding gauge boson. The second term of the gauge boson propagator is generally likely to cause divergences in loop integrals which are difficult to remove. Since this term is cancelled if $\xi = 1$ this choice is favoured in calculations including loops such as the box diagrams in meson-mixing processes (chapter 1.6.3). One disadvantage of this choice is that the Goldstone boson propagators must be considered additionally to the gauge bosons, which in turn leads to the fact that more diagrams must be calculated. In this context the Goldstone bosons are referred to as ghosts (which are not to be mistaken for the Faddeev-Popov ghosts).

- *Unitarity gauge* ($\xi \to \infty$): In this gauge the propagators of the Goldstone bosons vanish and the propagator of the gauge bosons converges to



$$= \frac{-i}{k^2 - m^2}\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 - m^2}\right].$$

Hence the Goldstone bosons are ignored and the number of diagrams to be calculated is reduced. It has been shown [4] that when making use of this choice while calculating box diagrams the calculation is not more intricate than in the Feynman-t'Hooft gauge. This combination can be especially useful if additional W gauge bosons are considered, such as in the Little Higgs Model (chapter 2.3).

1.3 The discrete symmetries

Beside the gauge group symmetries there are three discrete symmetries a Lagrangian can obey. These symmetries are parity (P) (also referred to as left-right symmetry or space inversion), time reversal (T) and charge conjugation (C). In classical three-space the transformations under these symmetries are intuitive. Parity mirrors the spatial coordinate (x→ -x), time inversion inverts the time coordinate (t→ -t) and charge conjugation transforms every particle into its antiparticle, this transformation only being applicable to relativistic quantum mechanics, since otherwise antiparticles would not exist. The transformations can be performed by defining P, T and C as operators, where P and C are unitary and T an anti-unitary operator.
In quantum field theory where fields are linear combinations of the creation- and annihilation-operators the situation is a little more complex. The transformation rules of the fields can be derived by the transformation rules of the creation- and annihilation-operators under the discrete transformations. Translating the classical transformation rules into the operator language gives [1]:

$$\begin{aligned}
P a^s_{\vec{p}} P &= e^{i\gamma_P} a^s_{-\vec{p}}, & P b^s_{\vec{p}} P &= e^{i\gamma'_P} b^s_{-\vec{p}} \\
C a^s_{\vec{p}} C &= e^{i\gamma_C} b^s_{\vec{p}}, & C b^s_{\vec{p}} C &= e^{i\gamma'_C} a^s_{\vec{p}} \\
T a^s_{\vec{p}} T &= e^{i\gamma_T} a^{-s}_{-\vec{p}}, & T b^s_{\vec{p}} T &= e^{i\gamma'_T} b^{-s}_{-\vec{p}}
\end{aligned}$$

where γ_i and γ'_i are random phases due to the unobservability of global phases. Inserting these transformation rules into the quantised fields (in quantum field theory the fields are linear combinations of creation- and annihilation-operators) the transformation rules for a scalar field, the electromagnetic field and a Dirac spinor can be derived (table 1.1) [1, 5].

	$\phi(t,\vec{x})$	$A_\mu(t,\vec{x})$	$\psi(t,\vec{x})$
P	$e^{i\alpha_P}\phi(t,-\vec{x})$	$A^\mu(t,-\vec{x})$	$e^{i\beta_P}\gamma^0\psi(t,-\vec{x})$
C	$e^{i\alpha_C}\phi^+(t,\vec{x})$	$-A^\mu(t,\vec{x})$	$-ie^{i\beta_C}\gamma^{2^T}\psi^{+^T}(t,\vec{x})$
T	$e^{i\alpha_T}\phi(-t,\vec{x})$	$A^\mu(-t,\vec{x})$	$e^{i\beta_T}\gamma^1\gamma^3\psi(-t,\vec{x})$
CP	$e^{i\alpha_{CP}}\phi^+(t,-\vec{x})$	$-A^\mu(t,-\vec{x})$	$-ie^{i\beta_{CP}}\gamma^0\gamma^{2^T}\psi^{+^T}(t,-\vec{x})$
CPT	$e^{i\alpha_{CPT}}\phi^+(-t,-\vec{x})$	$-A^\mu(-t,-\vec{x})$	$e^{i\beta_{CPT}}\gamma^5\psi^{+^T}(-t,-\vec{x})$

Tab. 1.1: Transformation rules of a scalar field, the electromagnetic field and a Dirac spinor under the discrete transformations P, C and T and the combined transformations CP and CPT.

The fact that the transformation rules of the vector field do not include a random phase is due to the composition of a vector field. A Dirac field is a linear combination of creation- and annihilation-operators of both particles and antiparticles which produce different phases under a discrete transformation. Due to the knowledge of the outcome of a discrete transformation of a field (with the exception of the phase), restrictions on the different phases can be made and those phases can be combined to one. However, a vector field is a linear combination of only particles' creation- and annihilation-operators. If the same strategy is applied to a vector field transformation the restriction (with the phase γ_i which is produced by creation- and annihilation-operators under a discrete transformation) $e^{i\gamma_i} = e^{-i\gamma_i}$ appears, meaning that $e^{i\gamma_i} = 1$. The other phases are arbitrary and can be used (in a manner analogous to the freedom of global rotation of the quark fields) to cancel phases which appear without physical meaning. One example is mentioned in chapter 2.2.2, in which these phases are used in order to cancel all phases but one of the Higgs potential in the LRSM.

The CPT theorem (Pauli-Lüders theorem) states that under the assumptions of a local field theory, Lorentz-invariance, the spin-statistic theorem and hermitian operators, the theory should be invariant under CPT-transformation. The Standard Model fulfils all these assumptions and is therefore CPT-invariant. A violation of CPT would directly imply the violation of Lorentz-invariance [6]. Due to the CPT theorem the violation of CP-invariance also implies the violation of T-invariance and vice versa. In the explored range of energies no CPT-violation has yet been found.

1.3.1 Requirements for CP-violation

A T- and therefore CP-transformation essentially transforms a quantity into its complex conjugate. This means that a Lagrangian or an amplitude only have the potential to violate CP-symmetry if they include complex numbers.

In the literature, three different kinds of complex phases can be distinguished between [5]:

- Weak phases, which occur in coupling constants and result, for example, from the rotation of the quark fields to their mass eigenstates (1.28) or a complex vev of a Higgs field, which on the condition that the phase cannot be removed by other symmetries is called spontaneous CP-violation. Weak phases have the property of being CP-odd, which means that they appear with an opposite sign in the CP-conjugate process.

- Strong phases, which are CP-even and result, for example, from Dirac matrix products with more than four matrices or on-shell scattering processes whose discussion goes beyond the scope of this thesis.

- Spurious phases, which characterise the phase difference between an amplitude and its CP-conjugate amplitude. One can, for example, choose to leave the original amplitude real and assign the whole phase difference to the CP-conjugate amplitude.

In this thesis only weak phases are of interest.
An amplitude receiving contributions from one or more of the phases mentioned above does not necessarily produce CP-symmetry violation, since the bra- and ket-vectors of the initial and final states can be arbitrarily rephased. Thus an amplitude must consist of at least two diagrams in order to violate CP-symmetry.

1.3.2 The discrete symmetries of the Standard Model Lagrangian

Using the transformation laws of table (1.1) it can be derived that the Lagrangian describing QED is P-, T- and C- and therefore also CP- and CPT-invariant without restrictions on the free phases α_i and β_i.
Almost the same is true for the Langrangian describing QCD (1.8), which is invariant under the discrete symmetries with the exception of one term. In order to solve another symmetry problem ($U(1)_A$-problem) an additive term is introduced [5]

$$L_\theta = \theta_{QCD} \frac{g_s^2}{32\pi^2} F^{a\mu\nu} \tilde{F}^a_{\mu\nu}$$

where θ_{QCD} is an optional parameter. This term violates P and T and therefore also CP. The new parameter is not invariant against a change of basis of the quark fields. In fact the transformation in order to achieve diagonal mass matrices (1.28) creates an additional parameter [5]

$$\theta_{QCD} \to \bar{\theta} \equiv \theta_{QCD} + \theta_{QFD}$$

with

$$\theta_{QFD} = ArgDet(M_u M_d)$$

and the non-diagonal mass matrices $M_u = \lambda v/\sqrt{2}$ and $M_d = \tilde{\lambda} v/\sqrt{2}$ (1.27) resulting from the Spontaneous Symmetry Breaking with the vacuum expectation values of the neutral component of the Higgs field v. The new parameter $\bar{\theta}$ is invariant against basis transformations of the quark field and can be interpreted in order to measure the amount of P- and CP-violation in QCD. Various measurements show that θ_{QCD} and θ_{QFD} almost cancel each other out, therefore the diminutiveness of $\bar{\theta}$ is regarded as a fine-tuning problem better known as the strong CP problem.

The situation is different in the Lagrangian describing electroweak interaction (1.7) which violates C-, P- and CP-symmetry, but not enough in order to explain the baryon-antibaryon asymmetry of the Universe. Even taking effects of temperature into account the prediction of the Standard Model is about the factor 10^{-12} times smaller than it should be in order to explain the amount of matter observed today [7]. The author of [7] also states that this missing CP-violation could be introduced via new imaginary phases due to the top- or heavier quarks. Since CP-violation is therefore one reason to extend the Standard Model, only this symmetry is considered here in detail.

According to [5] CP-violation cannot arise in pure gauge terms or in the non-diagonalised fermion mass terms, since there are always enough free global phases remaining to cancel out potential phases of the Standard Model Higgs vev. To show this, one can assume to have a complex Higgs vev:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ ve^{i\varphi} \end{pmatrix}$$

A transformation under SU(2)×U(1) would take the following from [1]:

$$\langle\phi\rangle \rightarrow e^{i\alpha^a \frac{\sigma^a}{2}} e^{i\frac{\beta}{2}} \langle\phi\rangle$$

with the three Pauli matrices $\sigma^a \in \{\sigma^1, \sigma^2, \sigma^3\}$. The following choice of phases then removes the phase of the vev:

$$\alpha^1 = \alpha^2 = \alpha^3 = 0, \qquad \text{and} \qquad \beta = -2\varphi,$$

showing that the phase φ has no physical significance. Since the vev of the Higgs field can be rotated to be real, there is no spontaneous CP-violation. Even if the vev were complex, there would be no spontaneous CP-violation, since a general CP-transformation produces a free phase [5]

$$(CP)\phi(t,\vec{r})(CP)^+ = e^{i\theta}\phi^{+^T}(t,-\vec{r})$$

which can be chosen as $\theta = 2\varphi$ in order to leave the vev invariant.

It is notable that these arguments are only valid in a theory with only one Higgs doublet. In a theory with two Higgs doublets two phases of two vevs could not be removed or kept invariant simultaneously and there would therefore be one physical phase. One example is the Lee Model [5] which is one of the simplest extensions of the Standard Model in order to achieve spontaneous CP-violation.

All these considerations show that the potentially CP-violating terms in the Standard Model must result from the transformation of the quark fields.

The electromagnetic part and the part including couplings of the quarks with the Z-boson are CP-invariant without the necessity of any rephasing due to the reasons which follow. The photon (A_μ) and the Z-boson field (Z_μ) both only produce a minus sign under CP-transformation and no global phases. There are also no CKM matrix elements (chapter 1.4) present because the photon and the Z-boson are neutral and therefore do not mix between differently charged quarks from different generations. The global phases resulting from the transformation of the quark fields cancel each other out since they belong to the same quark fields.
The CP-transformation of the charged W-bosons ($W^{\pm\mu}$) and the charged Higgs bosons ($\varphi^{\pm}$) produces a global phase [5].

$$\begin{aligned} (CP)W^{+\mu}(t,\vec{x})(CP)^+ &= -e^{i\xi_W}W^-_\mu(t,-\vec{x}), \qquad (CP)\varphi^+(t,\vec{x})(CP)^+ = e^{i\xi_W}\varphi^-(t,-\vec{x}) \\ (CP)W^{-\mu}(t,\vec{x})(CP)^+ &= -e^{-i\xi_W}W^+_\mu(t,-\vec{x}), \quad (CP)\varphi^-(t,\vec{x})(CP)^+ = e^{-i\xi_W}\varphi^+(t,-\vec{x}) \end{aligned}$$

In addition, the global phases resulting from the transformation of the quark fields do not cancel each other out since they belong to quarks of different families. The following condition for CP-invariance can be read from the Yukawa interaction Lagrangian
($L_Y - L_{mass}$: (1.26)-(1.27)) [5]:

$$V^*_{ak} = e^{i(\xi_W+\xi_k-\xi_a)}V_{ak} \tag{1.25}$$

where ξ_a and ξ_k are the phases resulting from the transformation of the quark fields u_a and d_k and V_{ak} the CKM matrix elements defined in chapter 1.4. Since ξ_W, ξ_k and ξ_a are free parameters, condition (1.25) (which belongs to a tree-level process) is always fulfilled. In the case of four multiplied CKM matrix elements condition (1.25) forces them to be real, which is known not be the case for all CKM matrix elements (chapter 1.4). Therefore processes including four or more vertices which mix families such as box diagrams (chapter 1.6.3) potentially violate the CP-symmetry.

1.3.3 Detection of CP- and T-violation

Since CPT-invariance is assumed, the violation of CP-symmetry is also a violation of T-symmetry. Due to the fact that T-symmetry violation is difficult to detect, only CP-symmetry-violating processes are listed here. All the data are taken from the Particle Data Group [8].
CP-violation is mostly detected in neutral meson decay. The two main asymmetries in Kaon decays are

$$|\eta_{+-}| = \frac{|A(K^0_L \to \pi^+\pi^-)|}{|A(K^0_S \to \pi^+\pi^-)|} = (2,233 \pm 0,012) \times 10^{-3}$$

and

$$\frac{[\Gamma(K^0_L \to \pi^- e^+\nu) - \Gamma(K^0_L \to \pi^- e^-\bar{\nu})]}{[\Gamma(K^0_L \to \pi^- e^+\nu) + \Gamma(K^0_L \to \pi^- e^-\bar{\nu})]} = (0,334 \pm 0,007)\%.$$

The indices L and S denote different mass eigenstates of the neutral Kaon, which will be discussed in chapter 1.6.
A larger value for the parameter describing CP-violation in neutral B-meson decay has been measured

$$sin(2\beta) = 0,678 \pm 0,025$$

with β [5] defined in terms of the CKM matrix elements introduced in chapter 1.4 .

$$\beta \equiv Arg\left(-\frac{V_{cd}V_{cb}^*}{Vj_{td}V_{tb}^*}\right)$$

Another CP-violating process is the decay of a neutral B-meson to a charged Kaon and a charged pion.

$$\frac{\left[\Gamma(\bar{B}_L^0 \to K^-\pi^+) - \Gamma(B_L^0 \to K^+\pi^-)\right]}{\left[\Gamma(\bar{B}_L^0 \to K^-\pi^+) + \Gamma(B_L^0 \to K^+\pi^-)\right]} = -0,101 \pm 0,015$$

The amount of CP-violation predicted for D^0-decays by the Standard Model is very small and there has not been any evidence for CP-violation in measurements.
Measurements of the electric dipole moment (EDM) of the neutron (chapter 2.2.4) constrain the amount of CP- and P-violation. The dipole moment has been measured to be smaller than 2,9$\times 10^{-26}$e cm.

1.4 The CKM matrix

The existence of the CKM matrix (named after Nicola Cabibbo, Makoto Kobayashi and Toshihide Maskawa) comes down to the fact that the coupling of the original quark fields (flavour eigenstates) to the vev of the Higgs field does not result in a diagonal mass matrix of the quarks. Thus the flavour eigenstates are not the physical mass eigenstates. The CKM matrix is the matrix which results from the multiplication of the rotation matrices which transform the quark fields of different weak isospins into their mass eigenstates.
The Yukawa coupling for the quarks in the Standard Model writes

$$L_Y = -\bar{Q}_{Li}\lambda_{ij}\Phi d_{Rj} - \epsilon^{ab}\bar{Q}_{Lia}\tilde{\lambda}_{ij}\Phi_b^+ u_{Ri} + h.c. \tag{1.26}$$

where Φ is the Higgs doublet (1.20). $\bar{Q}_{Li}$ and u_{Ri}, d_{Ri} are the three left-handed quark doublets (1.4) and the six right-handed quark singlets. The 3x3 matrices λ and $\tilde{\lambda}$ are arbitrary, since the requirement of gauge-invariance does not give any restrictions.
Inserting the vev of the Higgs field (1.21) in the Lagrangian of the Yukawa coupling gives the quark mass terms:

$$L_{mass} = -\frac{v}{\sqrt{2}}\bar{d}_{Li}\lambda_{ij}d_{Rj} - \frac{v}{\sqrt{2}}\bar{u}_{Li}\tilde{\lambda}_{ij}u_{Ri} + h.c. \tag{1.27}$$

Since λ and $\tilde{\lambda}$ are generally not diagonal, the quark flavour eigenstates used above are generally not the physical mass eigenstates. According to the singular value decomposition a random matrix can be diagonalised by two unitary matrices. The following transformation [1] leads to the physical mass eigenstates u' and d':

$$u_{Li} = U_{uij} u'_{Lj}, \qquad d_{Li} = U_{dij} d'_{Lj}. \tag{1.28}$$

Inserting the transformation of the quark fields in the flavour changing currents (for example the W-boson currents) reveals the nature of the CKM matrix

$$J_W^{\mu +} = \frac{1}{\sqrt{2}} \bar{u}_{Li} \gamma^\mu d_{Li} = \frac{1}{\sqrt{2}} \bar{u}'_{Li} \gamma^\mu V_{ij} d'_{Lj} \qquad \text{with} \qquad (U_u^+ U_d)_{ij} = V_{ij}. \tag{1.29}$$

The only constraint on the CKM matrix given by these theoretical considerations is unitarity, since it is a product of two unitary matrices. Thus the CKM matrix introduces new parameters to the Standard Model which must be determined by experiment.

Inserting the same quark transformation into the currents mixing with the Z-boson and the photon, the transformation matrices of the quark and antiquark are inverse to each other ($U_{u,d}^+ U_{u,d} = 1$), meaning that the Z-boson and the photon only couple to a quark and its antiquark and do not induce any flavour-changing processes. This is one of the conclusions of the GIM mechanism (named after Sheldon Lee Glashow, John Iliopoulos and Luciano Maiani, who proposed it in 1970) which states that there are no flavour-changing neutral currents (FCNC) at tree-level.

The original formulation of the GIM mechanism [9] involves the prediction of the c-quark, since at the time only the u-, d- and s-quarks had been discovered. One reason for the introduction of the fourth quark was the smallness of the branching ratio $\Gamma(K^0 \to \mu^+ \mu^-)/\Gamma_{total}(K^0)$ which is unexplained if only considering the u-quark as an intermediate particle. If one introduces the c-quark and uses the unitarity relation of the 2x2 CKM matrix the contributions with intermediate u- and c-quark have opposite signs and largely cancel each other out, which explains the smallness of the decay rate.

Generally the CKM matrix elements are labelled by the quark transition in which they are involved.

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Using unitarity gives twelve conditions

$$\begin{aligned} |V_{ui}|^2 + |V_{ci}|^2 + |V_{ti}|^2 = 1 \qquad &\text{for} \quad i \in \{d, s, b\} \\ |V_{id}|^2 + |V_{is}|^2 + |V_{ib}|^2 = 1 \qquad &\text{for} \quad i \in \{u, c, t\} \end{aligned} \tag{1.30}$$

and

$$\begin{aligned} V_{ui}^* V_{uj} + V_{ci}^* V_{cj} + V_{ti}^* V_{tj} = 0 \qquad &\text{for} \quad i, j \in \{d, s, b\} \quad \text{with} \quad i \neq j \\ V_{id} V_{jd}^* + V_{is} V_{js}^* + V_{ib} V_{jb}^* = 0 \qquad &\text{for} \quad i, j \in \{u, c, t\} \quad \text{with} \quad i \neq j. \end{aligned} \tag{1.31}$$

In the literature the last six conditions are primarily displayed in six unitarity triangles with the side length $\left|V_{ik}V_{jk}^*\right|$ or $\left|V_{ki}V_{kj}^*\right|$.
The following abbreviation will be used throughout this thesis:

$$\lambda_i = \begin{cases} V_{is}^* V_{id} & \text{for Kaon decays} \\ V_{is}^* V_{ib} & \text{for } B_s\text{-meson decays} \\ V_{id}^* V_{ib} & \text{for } B_d\text{-meson decays} \end{cases}$$

Therefore the third condition can be written as

$$\lambda_u + \lambda_c + \lambda_t = 0 \tag{1.32}$$

which when inserted while calculating meson decays is often referred to as the GIM mechanism.
The analysis of the Particle Data Group [8] based on experimental data using unitarity is displayed in the following CKM matrix:

$$V_{CKM} = \begin{pmatrix} 0,97419 \pm 0,00022 & 0,2257 \pm 0,0010 & 0,00359 \pm 0,00016 \\ 0,2256 \pm 0,0010 & 0,97334 \pm 0,00023 & 0,0415\,^{+0,0010}_{-0,0011} \\ 0,00874\,^{+0,00026}_{-0,00037} & 0,0407 \pm 0,0010 & 0,999133\,^{+0,000044}_{-0,000043} \end{pmatrix}$$

A general unitary NxN matrix can be expressed by N^2 parameters. There is at most one phase for every matrix element, which is denoted by α_{ij}. The Lagrangian should be invariant under global rotation of the quark fields $q_i \to e^{i\alpha_{u_i}} q_i$, where q is any quark and the quark phases can therefore be chosen freely. After rotating the quark fields by the CKM matrix, which happens in the flavour-changing currents induced by the W-boson (1.29) there are N^2 phases of the form $\alpha_{ij} + \alpha_{u_i} + \alpha_{d_j}$ for $i, j \in \{1, 2, 3, ..., N\}$.
If $\alpha_{u_i} = -\alpha_{i1} - \alpha_{d_1}$ is chosen, N phases are removed and the $N^2 - N$ remaining phases have the form $\alpha_{ij} - \alpha_{i1} - \alpha_{d_1} - \alpha_{d_j}$. $\alpha_{d_j} = -\alpha_{ij} + \alpha_{i1} + \alpha_{d_1}$ can then be chosen and therefore N-1 additional phases can be removed. Thus, due to the invariance of the Lagrangian under the global rotation of quarks, 2N-1 phases can be removed since they have no physical significance. Therefore $N^2 - (2N-1) = (N-1)^2$ parameters remain.
According to [5] N(N-1)/2 parameters should be rotation angles, which can be justified by the fact that a unitary matrix is a complex orthogonal matrix. Therefore (N-1)(N-2)/2 phases remain.
Thus in the case of three generations, the CKM matrix can be parameterised by three angles and one phase.
Several suggestions concerning the parameterisation have been made in the past. The most relevant ones will be discussed below.
The first attempt to parameterise the matrix V for the two generation case was made by Nicola Cabibbo. In this case all matrix elements can be rephased to be real valued. There is thus no CP-violation in the Standard Model with two generations.

The extension to three generations was made by Makoto Kobayashi and Toshihide Maskawa in 1973 [10]:

$$V = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix},$$

where $c_i = cos\theta_i$ and $s_i = sin\theta_i$. The angle θ_1 is called the Cabibbo angle.
The standard parameterisation was introduced by Chau and Keung in 1984 in order to better reflect the experimentally-observed small size of the CP-violating coefficients [11]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix},$$

where the new angles and phase are related to the Kobayashi-Maskawa (KM) parameterisation via

$$s_{12} = s_1 + O(10^{-4}), \qquad s_{23} = \sqrt{s_2^2 + s_3^2 + 2s_2s_3s_\delta} + O(10^{-4})$$

$$s_{13} = s_1s_3, \qquad s_{23}s_{\delta_{13}} = s_2s_\delta + O(10^{-3}).$$

Around the same time in 1984 another parameterisation was introduced by Wolfenstein [12]. Compared to the standard parameterisation it is not exact, but an expansion in one of the matrix elements, i.e. unitarity always holds up to a certain order of this element.
One of the best determined CKM matrix elements in this period was V_{us}, which was measured to be around 0,22. According to the Particle Data Group [8] the value used today does not vary much from the value assumed in 1983 ($|V_{us}| = 0,2255 \pm 0,0019$). From experimental results the range of most of the other matrix elements were known and expanded in $V_{us} = \lambda$. It transpires that the CKM matrix is close to unity. The diagonal is of order 1, V_{cd} is of order λ and V_{ts} and V_{cb} are of order λ^2. The last two elements V_{td} and V_{ub} were assumed to be of order λ^3, whereas it is known today that V_{ub} is actually of order λ^4 [13]. Similar to the standard parameterisation, the CP-violating imaginary part of the matrix elements is of the order λ^3. In addition to λ, three other parameters are required, which in Wolfenstein's parameterisation are called A, ρ and η.
In most cases it is sufficient to use this expansion up to the order of λ^3. When calculating CP-violating processes the imaginary part should be expanded to λ^5 while the real parts can be left unchanged.

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta)[+iA\eta\frac{1}{2}\lambda^5] \\ -\lambda & 1 - \frac{1}{2}\lambda^2[-i\eta A^2\lambda^4] & A\lambda^2[+iA\eta\lambda^4] \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \tag{1.33}$$

Note that the terms in brackets are not meant as products, but as the expansion of the imaginary parts to order λ^5.

The Wolfenstein parameters relate to the KM parameters, assuming that s_2 and s_3 are of order λ^2, by

$$\lambda = s_1, \qquad A\lambda^2 = \sqrt{s_2^2 + s_3^2 + 2s_2 s_3 cos\delta}$$
$$A^2\lambda^4\eta = s_2 s_3 sin\delta, \qquad A\lambda^2\sqrt{\rho^2+\eta^2} = s_3.$$

It was shown by Buras et al. [14] in 1994 that the Wolfenstein parameterisation can be achieved using the standard transformation

$$\rho = \frac{s_{13}}{s_{12}s_{23}}cos\delta, \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}sin\delta$$

and is therefore valid to all orders of λ. In their analysis they found two extra contributions which should be taken into account when discussing CP-violation:

$$\Delta V_{cd} = -iA^2\lambda^5\eta, \qquad \Delta V_{ts} = -iA\lambda^4\eta$$

and one which contributes to unitarity of the CKM matrix:

$$V_{td} = A\lambda^3(1-\tilde{\rho}-i\tilde{\eta}) \qquad \text{with} \quad \tilde{\rho} = \rho(1-\frac{\lambda^2}{2}) \quad \text{and} \quad \tilde{\eta} = \eta(1-\frac{\lambda^2}{2})$$

This parameterisation is often referred to as Buras-Wolfenstein parameterisation.

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta)[+iA\eta\frac{1}{2}\lambda^5] \\ -\lambda[-iA^2\lambda^5\eta] & 1-\frac{1}{2}\lambda^2[-i\eta A^2\lambda^4] & A\lambda^2[+iA\eta\lambda^4] \\ A\lambda^3(1-\tilde{\rho}-i\tilde{\eta}) & -A\lambda^2[-iA\lambda^4\eta] & 1 \end{pmatrix} + O(\lambda^4)$$

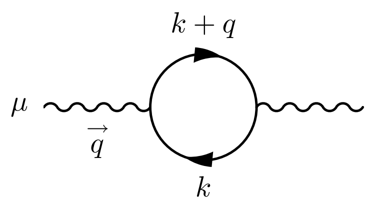
A fitting approach of the Particle Data Group [8] leads to the following Buras-Wolfenstein parameter:

$$\lambda = 0,2257\,^{+0{,}0009}_{-0{,}0010}, \quad A = 0,814\,^{+0{,}021}_{-0{,}022}, \quad \tilde{\rho} = 0,135\,^{+0{,}031}_{-0{,}016}, \quad \tilde{\eta} = 0,349\,^{+0{,}015}_{-0{,}017}$$

1.5 Renormalisation and renormalisation group equations

1.5.1 Renormalisation

A quantum field theory generally includes divergences when it comes to the calculation of diagrams accounting for more than the first order in α (simply another formulation for including loops with intermediate particles like virtual gauge bosons, fermions or scalar particles).
There are four categories of diagram in which these divergences appear: corrections to the gauge boson propagator, the fermion propagator, the vertex and the field strength. These four cases will be discussed briefly and methods of extracting (regularisation) and removing (renormalisation) will be given by means of the QED.
A free field theory is only valid in the non-interacting vacuum. Field strength renormalisation ensures the validity of an interacting theory, which takes into account the possibility of a state being created from the vacuum (unlike the free theory, in which the state is assumed to exist, so the probability of its creation is one). Considering this probability (often denoted by Z), it transpires [1] that Z is multiplied by the fermion propagator. When the fermion propagator is discussed below, the probability Z is included within the scale factor Z_2.
One example of the first order contribution to a gauge boson propagator is the photon propagator with two intermediate fermions [1]



$$= i\Pi_2^{\mu\nu}(q) \qquad (1.34)$$

where the index two denotes that the contribution concerns the second order of perturbation theory. The divergence enters through the integration (due to the intermediate loop fermion) over the four-momentum k. It transpires that this divergence is unphysical, since a manipulation of the Lagrangian in order to cancel it out is possible. This is achieved in the following manner.
The first step is to isolate the divergences. To achieve this a number of methods have been developed. These methods introduce a new parameter, which is taken to the limit of either 0 or $\pm\infty$ and which is supposed to be cancelled out by additional terms to the Lagrangian (counterterms). The most convenient regularisation method today is dimensional regularisation since it preserves the underlying gauge symmetry. The dimension of spacetime is in this case assumed to be d, where d can be taken as 4-ϵ.
The change of the spacetime dimension enables the calculation of the divergent integrals

and the isolation of the singularities by means of ϵ. In the case of the aforementioned example dimensional regularisation leads to

$$\Pi_2(q^2) \xrightarrow[\epsilon<<1]{} -\frac{2\alpha}{\pi}\int_0^1 dx x(1-x)\left(\frac{2}{\epsilon} - log\Delta - \gamma + log(4\pi) + O(\epsilon)\right)$$

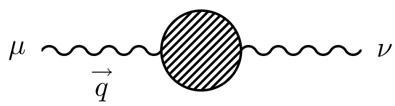
with $\Delta = m^2 - x(1-x)q^2$. The ϵ^{-1} divergence can be expressed as a logarithmic divergence of a cutoff scale Λ by using Pauli-Villars regularisation. In this regularisation method one introduces an extra fictitious intermediate particle with a very large mass Λ and therefore substitutes the propagator of the original particle by [1]

$$\frac{1}{(k+q)^2 - m^2 + i\epsilon} \rightarrow \frac{1}{(k+q)^2 - m^2 + i\epsilon} - \frac{1}{(k+q)^2 - \Lambda^2 + i\epsilon}.$$

When integrating from $k = 0$ to $k = \infty$ only the terms for $k = 0$ survive and the terms with diverging k cancel each other out. Because of this behaviour (allowing only small momenta k to contribute) the mass Λ is also known as the cutoff scale. The contribution to the photon propagator (1.34) writes in this regularisation method [1]

$$\Pi_2(q^2) \xrightarrow[\epsilon<<1]{} -\frac{2\alpha}{\pi}\int_0^1 dx x(1-x)\left(ln\left(\frac{x\Lambda^2}{\Delta}\right) + O(\Lambda^{-1})\right).$$

Taking the limit $\Lambda \rightarrow \infty$ would restore the real physical situation. Since the divergences which appear when taking this or the $\epsilon \rightarrow 0$ limit are not physical, ϵ and Λ vanish when physical quantities such as cross-sections are calculated.
It transpires that when taking all orders of perturbation theory into account they can be written as a geometric sequence of contributions and the gauge propagator is the following [1]:



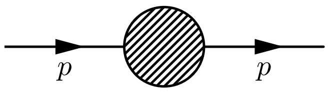
$$= \frac{-ig_{\mu\nu}}{q^2(1-\Pi(q^2))}. \qquad (1.35)$$

One defines the parameter

$$Z_3 = \frac{1}{1-\Pi(0)} \qquad (1.36)$$

which will be used later to renormalise the electron charge.

The procedure is similar when considering the fermion propagator, which can be written as [1]

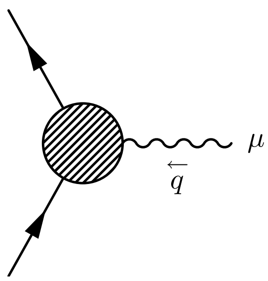


$$= \frac{i}{\not{p} - m_B - \Sigma(\not{p})}.$$

The rescaling factor which is defined describing these corrections and which is used later in order to cancel the vertex rescaling factor Z_1 is given by

$$Z_2 = \left(1 - \frac{d\Sigma}{d\not{p}}|_{\not{p}=m}\right)^{-1}. \tag{1.37}$$

Taking all loop contributions to a vertex into account can be summarised in the vertex function $\Gamma^\mu(q)$ [1]



$$= -ie\Gamma^\mu(q).$$

The rescaling factor due to these corrections is given by

$$Z_1 = \gamma^\mu(\Gamma^\mu(q=0))^{-1}. \tag{1.38}$$

In order to achieve a divergence-free theory, counterterms must be introduced which nullify the divergences due to loop integrals in each order of perturbation theory. If this is possible while conserving the underlying gauge symmetry, the Lagrangian is labelled renormalisable.

A more elegant method is a reinterpretation of the physical parameters of the Lagrangian such as the masses, charges and fields. Before adding the counterterms the Lagrangian is called the bare Lagrangian, depending on bare parameters.

Taking the example of the electromagnetic theory (neglecting the indices of the field tensor and the sum over the fermion fields), the bare Lagrangian writes [1]

$$L = \bar{f}(i\not{\partial} - m_0)f - e_0\bar{\psi}\gamma_\mu\psi B^\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

where m_0 and g_0 are not the parameters measured experimentally. It transpires that rescaling the fields and the electron charge by the rescaling factors (1.36, 1.37, 1.38) in the following way [1]

$$f_P = Z_2^{-1/2} f, \qquad B_P^\mu = Z_3^{-1/2} B^\mu, \qquad e = e_0 Z_2 Z_1^{-1} Z_3^{1/2} = e_0 Z_3^{1/2}$$

results in the aforementioned counterterms which cancel the unphysical divergences out of the diagrams shown above. Here the index P denotes the physical quantities.
The rewritten Lagrangian is given by

$$L = \bar{f}_P(i\partial\!\!\!/ - m_f) f_P - e\bar{f}_P \gamma_\mu f_P B_P^\mu - \frac{1}{4} F_P^{\mu\nu} F_{P\mu\nu} \tag{1.39}$$

$$+ \bar{f}_P(i\delta_2 \partial\!\!\!/ - \delta_m) f_P - e\delta_1 \bar{f}_P \gamma_\mu f_P B_P^\mu - \frac{1}{4}\delta_3 F_P^{\mu\nu} F_{P\mu\nu} \tag{1.40}$$

with

$$\delta_3 = Z_3 - 1, \quad \delta_2 = Z_2 - 1, \quad \delta_m = Z_2 m_0 - m, \quad \text{and} \quad \delta_1 = Z_1 - 1.$$

The same form of the Langrangian can be derived from the path integral formalism. The fields are assumed not to contribute to low energy physics if their momentum is much higher than the scale of the initial process, which means that they are only taken to be unequal to zero if their momentum is under a certain cutoff scale Λ. The counterterms are achieved by integrating out the fields (degrees of freedom) with a momentum close to the cutoff scale.
It is important to remark that the constant shift of the electron charge is only valid in the Lagrangian, i.e. at tree-level. Actual calculations introduce a q-dependence of the charge due to the use of the q-dependent propagator (1.35). Therefore the physical charge depends on the energy or the distance at which the process is considered and is given by

$$e_{eff} = \frac{e}{\sqrt{1 - [\Pi_2(q^2) - \Pi_2(0)]}}.$$

The same can be applied to the physical coupling constants which are therefore also energy dependent. It transpires that the coupling constants for QED and weak interaction increase with increasing energy whereas the coupling constant for QCD decreases for increasing energy. This phenomenon is also known as asymptotic freedom of QCD.

1.5.2 Renormalisation group equations

The Lagrangian (1.40) can also be derived using path integral methods to integrate out higher degrees of freedom. Following this method the Lagrangian can be iterated down to the considered scale. At every scale the Lagrangian has the form (1.40) but the parameters change. Since perturbation theory is valid, the parameters differ only slightly by a small change of scale. These transformations of the Lagrangian from one scale to another are called the renormalisation group. Strictly speaking, however, they do not belong to a

group, since the inverse element is missing as the process of integrating out degrees of freedom is not reversible.
The so-called flow of the Lagrangian parameters is represented by scale-independent functions (β- and γ-functions), where the β-function represents the rate at which the renormalised coupling changes given the bare coupling. A positive β-function shows that the coupling constant increases for increasing momenta and decreases for decreasing momenta. Writing down the change of arbitrary renormalised Green functions through changing scale leads to differential equations depending on the scale M and the parameters. In the case of ϕ^4-theory the β- and γ-functions are defined as [1]

$$\beta \equiv \frac{M}{\delta M}\delta\lambda, \qquad \gamma \equiv -\frac{M}{\delta M}\delta\eta \tag{1.41}$$

where $\delta\lambda$ represents the change in the coupling due to a scale transformation and $\delta\eta$ the change in the field. Generally there is one β-function for each coupling and one γ-function for each field. These functions are scale-independent since the Green functions are cutoff-independent and therefore the rescaling functions β and γ cannot depend on the scale due to dimensional analysis.
The resulting differential equation is called the Callan-Symanzik equation and is in the case of QED given by [1]

$$\left[M\frac{\partial}{\partial M} + \beta(e)\frac{\partial}{\partial e} + n\gamma_2(e) + m\gamma_3(e)\right] G^{(n,m)}(x_i; M, e) = 0$$

where n is the number of electron fields and m the number of photon fields in the Green function.
The rescaling functions can be obtained order by order in perturbation theory by inserting possible Green functions. It transpires that the β- and γ-functions are related to the counterterms of the Lagrangian.

1.6 Neutral meson-mixing

Neutral meson-mixing processes are often used to discuss the introduction of new particles beyond the Standard Model for the following reason.
New particles are generally too heavy to be detected directly, therefore it is more efficient to analyse their indirect contributions to observable physics. Heavy fermions or gauge bosons can be produced as intermediate particles in loop processes such as neutral meson-mixing. These new particles produce new Feynman diagrams and hence their contributing amplitudes are added to those of the Standard Model. If the contribution of new particles is non-negligible, the case is referred to as non-decoupling of heavy particles.
Despite the fact that the observed amount of neutral meson mass-mixing and the amount of observed CP-violation in meson decay (chapter 1.3.3) can be explained by the Standard

Model, these processes can be considered, not in order to explain data that cannot be explained by the Standard Model, but in order to constrain new physics. Heavy particles would be allowed only if they either give very small contributions or some cancellations have to take place in order to leave the outcome of the experiment unchanged.
A meson is a particle composed of a quark and an antiquark and is therefore a sub-category of a hadron. Mesons are also bosons. Every meson P^0 has an anti-partner $\bar{P}^0$. The two particles are linked by C-transformation. The discrete symmetries transform the neutral mesons in the following way [15], neglecting the free phases discussed in chapter 1.3:

$$C\,|P^0\rangle = -\,|\bar{P}^0\rangle\,, \qquad P\,|P^0\rangle = -\,|P^0\rangle\,, \qquad T\,|P^0\rangle = |P^0\rangle$$

Since the strong and the electromagnetic interactions do not induce any charged or flavour-changing currents, the neutral mesons cannot decay or mix via those forces. Considering the weak interaction, the $W^{\pm}$ bosons induce charged flavour-changing currents and therefore enable the neutral mesons and anti-mesons to decay or to oscillate between one another.

Due to the oscillation of P^0 and $\bar{P}^0$ a general state is given by the mixture of those two states [5]

$$|\psi(t)\rangle = \psi_1(t)\,|P^0\rangle + \psi_2(t)\,|\bar{P}^0\rangle\,.$$

The Schrödinger equation gives the time development of this state

$$\frac{\partial}{\partial t}\Psi = \mathbf{H}\Psi \qquad \text{with} \quad \Psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

where the 2x2 matrix **H** is given by $\mathbf{H}$=$\mathbf{M}$-$\frac{i}{2}\Gamma$, with

$$M_{ij} = m_0\delta_{ij} + \langle i|\,H_W\,|j\rangle + \sum_n P\frac{\langle i|\,H_W\,|n\rangle\,\langle n|\,H_W\,|j\rangle}{m_0 - E_n} \tag{1.42}$$

to second order in perturbation theory [5]. The states i and j describe the states P^0 and $\bar{P}^0$ where the states n are virtual intermediate states. H_W denotes the Hamiltonian of the weak interaction.
For interactions only including QCD and QED, **M** would be a diagonal mass matrix and Γ would not exist. The non-diagonal elements in **M** represent the mixing of the states P^0 and $\bar{P}^0$ and the existence of Γ represents the possibility of the neutral mesons decaying into other particles.
Requiring CP-invariance would restrict the matrix **H** in the following way [5]:

$$|H_{12}| = |H_{21}|$$

Thus a parameter measuring CP- and T-violation is given by

$$\delta \equiv \frac{|H_{12}| - |H_{21}|}{|H_{12}| + |H_{21}|}. \tag{1.43}$$

Other constraints deduced by CPT-invariance lead to the definition of the CPT- and CP-violating parameter θ.

$$\theta \equiv \frac{H_{22} - H_{11}}{\sqrt{4H_{12}H_{21} + (H_{22} - H_{11})^2}} \tag{1.44}$$

Due to their mixing, the neutral mesons P^0 and $\bar{P}^0$ are not the mass eigenstates if the weak interaction is considered. Requiring CPT-invariance, the states of the physical particles are given by the following linear combination of the original mesons [5]:

$$|P_H\rangle = p\,|P^0\rangle + q\,|\bar{P}^0\rangle \tag{1.45}$$

$$|P_L\rangle = p\,|P^0\rangle - q\,|\bar{P}^0\rangle \tag{1.46}$$

with $|p|^2 + |q|^2 = 1$.
Diagonalising **H** gives the following relation between p and q:

$$\frac{p}{q} = \sqrt{\frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}}$$

and the CP-violating parameter (1.43) can be written as

$$\delta = |p|^2 - |q|^2 = \langle P_L|P_H\rangle\,.$$

If CP-invariance were required δ would be equal to zero and P_H and P_L would become CP eigenstates with the eigenvalues ±1 and ∓1 respectively. The eigenstate with the eigenvalue +1 is called CP-even and that with the eigenvalue -1 CP-odd.
Assuming that P_H and P_L decay to a final state $|f\rangle$ which is CP-even, CP-invariance would require (here P_H is taken to be CP-even)

$$\langle f|\,H_W\,|P_L\rangle = \langle f|\,(CP)^+H_W(CP)\,|P_L\rangle = -\,\langle f|\,H_W\,|P_L\rangle \qquad => \qquad \langle f|\,H_W\,|P_L\rangle = 0$$

This means that if both P_L and P_H are observed to decay into a final state which is a CP eigenstate, CP-symmetry is violated. Therefore another CP-violating parameter can be defined as

$$\epsilon_f = \frac{\langle f|\,H_W\,|P_H\rangle}{\langle f|\,H_W\,|P_L\rangle} \tag{1.47}$$

where $|f\rangle$ is a CP eigenstate with the same eigenvalue as P_H.
The eigenvalues of **H** are given by

$$\mu_{H,L} = m_{H,L} - \frac{i}{2}\Gamma_{H,L}$$

where m is the mass and Γ the decay width of the corresponding meson. Thus the mass and decay width difference between the heavier and the lighter meson is given by

$$\Delta m \equiv m_H - m_L \qquad \text{and} \qquad \Delta\Gamma \equiv \Gamma_H - \Gamma_L.$$

Since the theoretical determination of the CP-violating parameters is always only approximate they must be derived for each system of neutral mesons individually. CP-violation has been observed in Kaon decay as well as in neutral and charged B-meson decay.

1.6.1 The neutral Kaon

The first signs of CP-violation were found in neutral Kaon decay in 1964 [8]. The CP-violation becomes apparent due to the fact that the physical mass state with the longer life-time K_L decays more often to $\pi^- e^+ \bar{\nu}_e$ than to $\pi^+ e^- \nu_e$. If CP were a symmetry K_L would be a CP eigenstate and the following equality would be true:

$$\langle \pi^- e^+ \bar{\nu}_e | H_W | K_L \rangle = \langle \pi^- e^+ \bar{\nu}_e | (CP)^+ H_W (CP) | K_L \rangle = \langle \pi^+ e^- \nu_e | H_W | K_L \rangle$$

This means the decay rates to the two final states would need to be equal.
The mass eigenstates are defined analogous to (1.46)

$$\begin{aligned} |K_L\rangle &= p\,|K^0\rangle + q\,|\bar{K}^0\rangle \\ |P_L\rangle &= p\,|K^0\rangle - q\,|\bar{K}^0\rangle \end{aligned}$$

where the indices stand for longer and shorter lifetimes. It transpires that K_L is the heavier meson. The observed difference in mass assuming CPT-invariance is given by [8]

$$\Delta m = (3,483 \pm 0,006) \times 10^{-12} MeV.$$

It is also observable that K_L is mostly CP-odd and K_S mostly CP-even.
From experimentally-achieved data it can be concluded that neutral Kaons mostly decay into the CP-even two-pion state with the isospin zero [5].

$$|2\pi, 0\rangle = |2\pi, I = 0\rangle = \sqrt{\frac{2}{3}}\,|\pi^+\pi^-\rangle - \sqrt{\frac{1}{3}}\,|\pi^0\pi^0\rangle$$

Thus the associated CP-violating parameter according to (1.47) is given by

$$\epsilon \equiv \frac{\langle 2\pi, 0| H_W |K_L\rangle}{\langle 2\pi, 0| H_W |K_S\rangle}. \tag{1.48}$$

This parameter can be linked to the CP-violating parameter δ (1.43) over

$$\frac{2\mathrm{Re}\epsilon}{1 + |\epsilon|^2} = \delta.$$

The assumption that the decay channel to $|2\pi, 0\rangle$ is the only one leads to a linear correlation of those two parameters [5]

$$\epsilon \approx \frac{\delta}{\sqrt{2}} e^{i\frac{\pi}{4}} \tag{1.49}$$

and an approximation for δ

$$\delta \approx -\frac{\mathrm{Im}(M_{12} A_0 \bar{A}_0^*)}{|M_{12} A_0 \bar{A}_0|} \tag{1.50}$$

with

$$A_0 \equiv C\lambda_u \langle 2\pi, 0| (\bar{s}\gamma^\mu \gamma_L u)(\bar{u}\gamma_\mu \gamma_L d) |K^0\rangle \quad \text{and} \quad \bar{A}_0 \equiv C\lambda_u^* \langle 2\pi, 0| (\bar{u}\gamma_\mu \gamma_L s)(\bar{d}\gamma^\mu \gamma_L u) |\bar{K}^0\rangle$$

where C is a factor of the Wilson coefficient obtained using operator product expansion (chapter 1.6.2) and λ_u is determined by the CKM matrix elements via $\lambda_u \equiv V_{us}^* V_{ud}$. The constant C is real, since the only potentially CP-violating phases in the Standard Model enter via the CKM matrix elements. If a model with other sources of CP-violation is considered, C might include CP-violating phases.
This approximation still holds for extensions of the Standard Model as long as the decay channel to $|0\rangle$ remains dominant.
The current value for ϵ obtained through measurements of the decay rate $K \to 2\pi$ is given by [8]

$$\epsilon = (2,229 \pm 0,010) \times 10^{-3} e^{i\frac{\pi}{4}}.$$

Carrying out a CP-transformation of the bra, the ket and the quark fields of A_0 by inserting $(CP)(CP)^+ = 1$ into the transition amplitude and using the transformation rules listed in chapter 1.3 gives

$$A_0 = C\lambda_u \langle 2\pi, 0| (\bar{s}\gamma^\mu \gamma_L u)(\bar{u}\gamma_\mu \gamma_L d) |K^0\rangle = C\lambda_u \langle 2\pi, 0| (-\bar{u}\gamma_\mu \gamma_L s)(-\bar{d}\gamma^\mu \gamma_L u) |\bar{K}^0\rangle = \lambda_u \lambda_u^{*-1} \bar{A}_0$$

where the sum of the free phases of the CP-transformation was taken to be 0 or π for simplicity.
Inserting this relation into (1.50) shows that δ and ϵ only depend on the matrix element M_{12} which can be determined in K^0-$\bar{K}^0$-mixing processes

$$\delta \approx -\frac{\mathrm{Im} M_{12}}{\Delta m} \qquad \text{and} \qquad \epsilon \approx -\frac{\mathrm{Im} M_{12}}{\sqrt{2}\Delta m} e^{i\frac{\pi}{4}}. \tag{1.51}$$

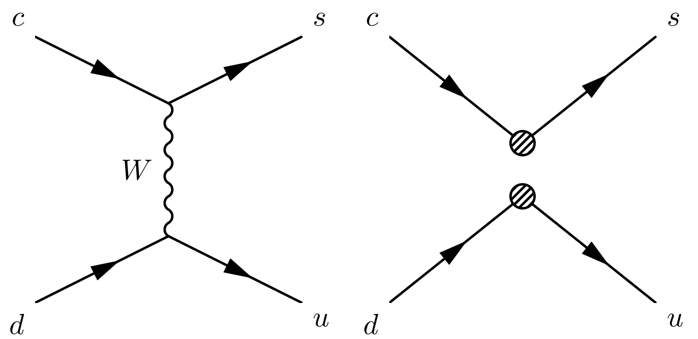
1.6.2 Operator product expansion and the effective Lagrangian

Before the amplitude for K^0-$\bar{K}^0$-mixing is derived explicitly, some general ideas on how to extend the tree-level-describing Lagrangian in order to express higher order interactions are displayed. The listed examples follow the lecture of Buras [16].
One possible way of imagining different types of particles is as degrees of freedom. The higher the considered energy scale, the more particles could be created and the more degrees of freedom exist, where the considered energy scale is referred to as the mass scale of the initial particle. In low energy processes it can be very intricate to include all types of particles or all degrees of freedom even though their mass scale is much higher than the scale of the initial particle. In this case it is useful to use an effective theory in which only particles at and under the mass scale of the low energy process are considered. Low energy processes are usually referred to as long distance (LD) processes and high energy processes as short distance (SD) processes. The effective Lagrangian is arrived at by integrating out higher degrees of freedom, which can literally be achieved by performing the integral in the path formalism over all particle fields with higher rest mass.
The procedure can be demonstrated on the decay $c \to su\bar{d}$ which is mediated by a charged W-boson. Since the mass of the W-boson m_W is much greater than the mass of the c-quarks,

the ratio between the momentum transfer and the W-mass must be very small $\frac{k^2}{m_W^2} << 1$. Evaluating the corresponding diagram with the common Feynman rules in the Feynman-t'Hooft gauge ($\xi = 1$) gives a prefactor dependent on the CKM matrix elements, the operator of the in- and outgoing currents and the contribution of the intermediate W-boson which can be expanded in the following way:

$$\frac{-ig_{\mu\nu}}{k^2 - m_W^2} = \frac{i}{m_W^2}\left(1 + O\left(\frac{k^2}{m_W^2}\right)\right)$$

Including only the leading order which is independent of k is analogous to integrating out the W-boson in the path formalism. The process is shown diagrammatically in figure 1.1.



(a) Shortdistance perspective (b) Multiplication of two currents with effective coupling constants including the shortdistance effects of the W boson

Fig. 1.1: The decay $c \to su\bar{d}$ to lowest order in perturbation theory [16].

The resulting diagram does not include the short distance contribution of the W-boson as an intermediate state but as part of the prefactor, and can be interpreted as a multiplication of the external currents with an effective coupling constant. It is therefore reminiscent of the description of decays in Fermi theory.
The general idea to expand local currents by means of local operators and effective coupling constants was introduced by Wilson in 1969 [17] who proposed that if A(x) and B(y) are currents located at x and y respectively and the distance between x and y is small the product can be expanded by means of the Wilson coefficients $C_n(x-y)$ and local operators $O_n(x)$.

$$A(x)B(y) = \sum_n C_n(x-y)O_n(x)$$

The Wilson coefficients are generally singular if $x = y$ and the local operators are of the form $(\bar{s}\gamma^\mu\gamma_L c)(\bar{u}\gamma_\mu\gamma_L d)$ which is taken from the above example.
The Wilson coefficient was derived to be proportional to [1]

$$C_n(x-y) \propto \left(\frac{1}{|x-y|}\right)^{d_A+d_B-d_n} \left(\frac{ln\left(\frac{1}{|x-y|^2\Lambda^2}\right)}{ln\left(\frac{M^2}{\Lambda^2}\right)}\right)^{\frac{a_n-a_A-a_B}{2b_0}} .$$

d_i denote the dimension of the operators, which is 1 for a bosonic field and 3/2 for a fermionic field and M the renormalisation scale. In the example given above, the dimensions would be $d_A = d_B = 3$ and $d_n = 6$ so that the first fraction is removed. The coefficients a_i and b_0 result from the γ-function and β-function respectively (1.41) [1]

$$\gamma_i = -a_i \frac{g^2}{4\pi} \qquad \text{and} \qquad b_0 = 11 - \frac{2}{3} n_f$$

at a fixed point g and with the number of fermions n_f. The mass scale Λ is chosen to fulfil the following relation:

$$1 = g^2 \frac{b_0}{8\pi} ln \left(\frac{M}{\Lambda} \right)$$

The local difference $|x - y|$ is inversely proportional to the mass of the intermediate boson. In the case of a W-boson one can insert

$$|x - y|^2 = \frac{1}{m_W^2}.$$

Calculating the transition amplitude from an initial state $|i\rangle$ to a final state $|f\rangle$ the Wilson coefficient can be factored out of the amplitude.

$$\langle f| A(x)B(y) |i\rangle = \sum_n C_n(x - y) \langle f| O_n(x) |i\rangle$$

Considering a process taking place at the scale M it transpires that the Wilson coefficient includes the contributions of particles with a rest mass higher than M. The expectation value of the local operators (referred to as the hadron matrix element) on the other hand includes all contributions due to low energy processes smaller than M. Since QCD is non-perturbative for low energies, advanced methods like lattice QCD are necessary to calculate the hadron matrix element and are not performed in this thesis. The corresponding values of the hadron matrix elements are taken from the literature.
The contributions to the Wilson coefficient can all be calculated in perturbation theory and are therefore discussed explicitly. Taking the above example and choosing M to be around the scale of a few GeV would mean that the W-boson and the top-quark need to be integrated out. If theories extending the Standard Model are considered, all contributing higher mass particles must be integrated out. One example of this is the heavy quarks in the four-generation case.
Another short distance effect which needs to be included is the gluon exchange between the initial quarks and, if it is a non- or semileptonic decay, also between the final quarks. Including gluon exchange can introduce new local operators which sum over colour cross-terms between the connected quark. The first order QCD corrections to the Wilson coefficients are shown in figure 1.2.

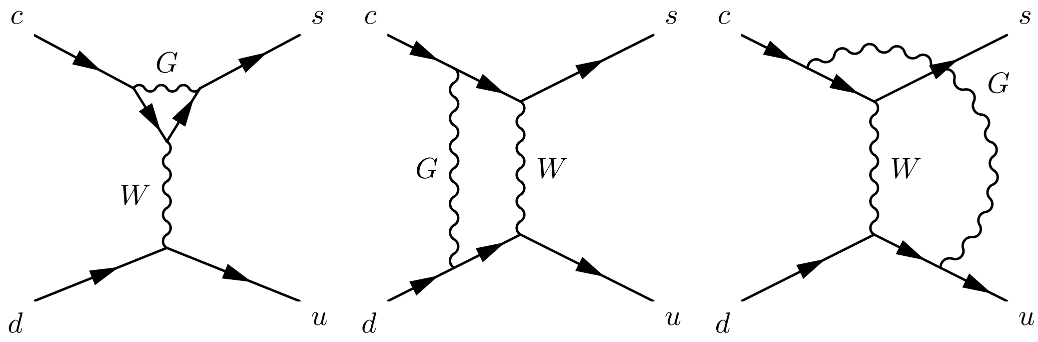
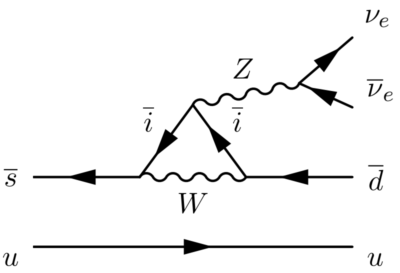


Fig. 1.2: Short distance corrections due to gluon exchange [16].

When calculating the first order QCD correction directly, unphysical divergences appear which can be removed by quark field renormalisation. Having carried out this renormalisation it transpires [1] that the divergences produced in the first diagram are completely cancelled out by field-strength renormalisation where the other two diagrams give (after renormalisation) contributions of the order $(\alpha/4\pi)ln(m_W^2/M^2)$[16].
Generally the contributions to the Wilson coefficients can be derived from the renormalisation group and therefore depend on the scale M. Thus the Wilson coefficients depend on the masses of the contributing particles with $m_i > M$ and on the scale M.
Altogether, using the operator product expansion separates the factors in the effective Hamiltonian in the Wilson coefficient (which describes the short distance contributions) and the hadron matrix element (which describes the long distance contributions) by choosing a scale M. Since a physical quantity such as the transition amplitude should not rely on an arbitrary scale, the M-dependence of the Wilson coefficient and the hadron matrix element cancel each other out.

An effective Lagrangian contains processes at higher-loop level as effective coupling constants and operators using OPE. In this thesis the effective Lagrangian is referred to as the Lagrangian describing interactions to one-loop corrections. Since FCNCs are forbidden at tree-level due to the coupling of the gauge bosons or the GIM mechanism, those processes are especially interesting in the study of CP-violation. The loop contributions due to the weak interaction are box and triangular diagrams. The example of the decaying charged Kaon shows how the diagram can be extended to one-loop level (1.52). The effective Hamiltonian would include all these contributions.

$K^+ \to \pi^+ \nu_e \bar{\nu}_e =$



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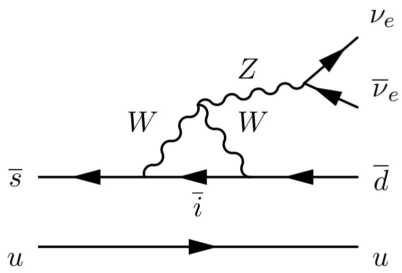

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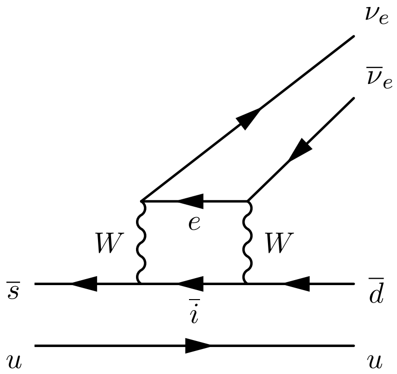


(1.52)

1.6.3 K^0-$\bar{K}^0$-mixing

The following calculation will be performed in the Feynman-t'Hooft gauge, which results in a charged massive Goldstone boson in addition to the W-boson in the loops. Therefore to the first order of perturbation theory K^0-$\bar{K}^0$-mixing can be described by the following eight diagrams:

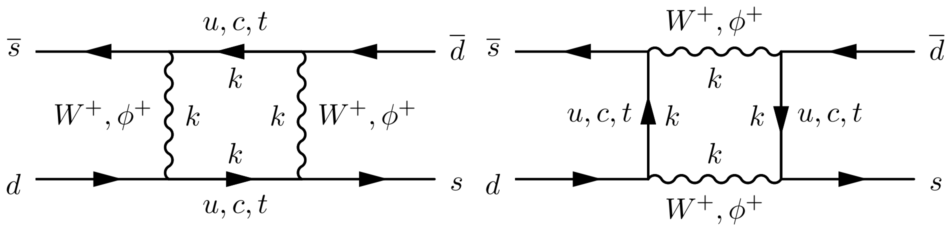


Fig. 1.3: Box diagrams contributing to K^0-$\bar{K}^0$-mixing. K^0 is defined as $\bar{s}d$ and $\bar{K}^0$ as $s\bar{d}$.

In unitary gauge only the first diagram would need to be considered. Since it has been shown that the change of the amplitude due to non-zero external momenta would be small compared to the uncertainty in the hadron matrix element in K^0-$\bar{K}^0$-mixing [18], the external momenta are neglected. Taking the external momenta in B^0-$\bar{B}^0$-mixing into account would give an even smaller contribution. Setting the external momenta to zero results in the fact that all internal particles carry the same momentum k.

Using the Feynman rules from Appendix A.1 and summing over all possible quarks in the loop gives the following amplitude for the box including two W-bosons:

$$A_{WW} = \frac{g^4}{4} \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j \int \frac{d^4k}{(2\pi)^4} \frac{k_\alpha k_\beta}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - m_W^2)^2} (\bar{s}\gamma^\nu \gamma^\alpha \gamma^\mu \gamma_L d)(\bar{s}\gamma_\tau \gamma^\beta \gamma_\sigma \gamma_L d)$$

with

$$\gamma_L = \frac{1-\gamma^5}{2}, \qquad \gamma_R = \frac{1+\gamma^5}{2}.$$

Introducing the identity $1 = \frac{1}{4} g^{\alpha\beta} g_{\alpha\beta}$ raises the index of k_β and lowers the index of γ^β. Using the Dirac matrix identity

$$\gamma^\mu \gamma^\nu \gamma^\sigma = g^{\mu\nu}\gamma^\sigma - g^{\mu\sigma}\gamma^\nu + g^{\nu\sigma}\gamma^\mu + i\epsilon^{\mu\nu\sigma\alpha}\gamma_\alpha \gamma_5,$$

the operator product can be simplified to

$$(\bar{s}\gamma^\nu \gamma^\alpha \gamma^\mu \gamma_L d)(\bar{s}\gamma_\tau \gamma_\beta \gamma_\sigma \gamma_L d) = 4(\bar{s}\Gamma^\mu d)(\bar{s}\Gamma_\mu d) = 4O_{sd} \qquad \text{with} \quad \Gamma^\mu = \gamma^\mu \gamma_L.$$

The internal momentum is substituted by $k \to \frac{k}{m_W}$ which leads to the following final expression:

$$A_{WW} = \frac{g^4}{4m_W^2} \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j F(x_i, x_j) O_{sd}$$

with

$$F(x_i, x_j) = \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - x_i)(k^2 - x_j)(k^2 - 1)^2} \qquad \text{and} \qquad x_i = \frac{m_i^2}{m_W^2}.$$

Defining a general integral helps in expressing the other three contributing amplitudes.

$$F_n(x_a, x_b, x_c, x_d) = -\frac{16\pi^2}{i} \int \frac{d^4k}{(2\pi)^4} \frac{k^n}{(k^2 - x_a)(k^2 - x_b)(k^2 - x_c)(k^2 - x_d)} \tag{1.53}$$

The prefactor is chosen in such a way as to cancel out the prefactor which occurs while solving the integral. The number of normalised mass terms in the argument equates to the number of mass-distinguishable particles in the loop. For example, the W-bosons and the massive Goldstone bosons count as one particle in the momentum integral since they have the same masses in the Feynman-t'Hooft gauge. An integral function with four arguments can only be realised in a theory with different types of W or single-charged gauge bosons with non-degenerate mass, such in the Little Higgs (chapter 2.3) or the Left-Right Symmetric

Model (chapter 2.2). The solution of F_n for $n = 0, 2$ for all possible numbers of different arguments is given in Appendix A.1. F_1 is always zero since the function under the integral is odd and the integral extends over the whole four-space.
The derivation of the contributions including one or two Goldstone bosons is analogous to the two-W-boson case. Thus the four contributions to $K^0 - \bar{K}^0$-mixing are given by

$$\begin{aligned}
A_{WW} &= -\frac{ig^4}{64\pi^2 m_W^2} \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j F_2(x_i, x_j; 1) O_{sd} \\
A_{W\phi} &= \frac{ig^4}{64\pi^2 m_W^2} \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j x_i x_j F_0(x_i, x_j; 1) O_{sd} \\
A_{\phi W} &= \frac{ig^4}{64\pi^2 m_W^2} \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j x_i x_j F_0(x_i, x_j; 1) O_{sd} \\
A_{\phi\phi} &= -\frac{ig^4}{64\pi^2 m_W^2 4} \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j x_i x_j F_2(x_i, x_j; 1) O_{sd}.
\end{aligned}$$

The whole amplitude is given by

$$\begin{aligned}
A &= -\frac{ig^4}{64\pi^2 m_W^2} \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j \left(F_2(x_i, x_j; 1) - 2x_i x_j F_0(x_i, x_j; 1) + \frac{1}{4} x_i x_j F_2(x_i, x_j; 1) \right) O_{sd} \\
&= -\frac{ig^4}{64\pi^2 m_W^2} C_0 O_{sd}
\end{aligned}$$

where C_0 is the Wilson coefficient depending on the masses of the internal particles with

$$C_0 = \sum_{i=u,c,t} \sum_{j=u,c,t} \lambda_i \lambda_j c_0(x_i, x_j; 1).$$

The exact coefficients are given in Appendix A.1.
Using unitarity of the CKM matrix and setting the quark mass to zero (Appendix A.1) results in the following Wilson coefficient:

$$C_0 = \lambda_c^2 c_0'(x_c; 1) + 2\lambda_c \lambda_t c_0'(x_c, x_t; 1) + \lambda_t^2 c_0'(x_t; 1)$$

Also including short distance QCD effects in the Wilson coefficients, it transpires that they give an additional factor for every possible loop, factoring out of the c_0'-functions (figure 1.4).
The prefactors η are evaluated at the mass scale corresponding to the quark. The values are given by [16]

$$\eta_{cc} = 1,38 \pm 0,20, \qquad \eta_{tt} = 0,57 \pm 0,01, \qquad \eta_{ct} = 0,47 \pm 0,04. \tag{1.54}$$

The updated Wilson coefficient writes

$$C_0 = \lambda_c^2 \eta_c c_0'(x_c; 1) + 2\lambda_c \lambda_t \eta_{ct} c_0'(x_c, x_t; 1) + \lambda_t \eta_t^2 c_0'(x_t; 1).$$

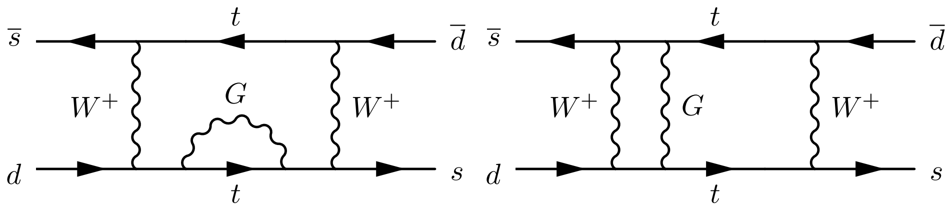


Fig. 1.4: Diagrams contributing to the short distance QCD corrections leading to η_t [16].

The operator O_{sd} obtains long distance corrections depending on the scale $M < M_c$ [16]. Therefore the new operator reads

$$[\alpha_s(M)]^{-2/9}\left[1+\frac{\alpha_s^3(M)}{4\pi}J_3\right]O_{sd}$$

where α_s is the scale-dependent coupling constant divided by 4π given by the renormalisation group equations and J_3 depends on the renormalisation scheme. The index three notes that since $M < M_c$ only the three lightest quarks must be taken into account in the loop calculation for the renormalisation group equations.

Since A includes the product of two identical operators, the Feynman rule for converting the vertex into the effective Lagrangian is vertex=-2iH$_{eff}$ [5] and therefore together with the definition of the Fermi constant $G_F = \frac{g^2}{4\sqrt{2}m_W^2}$ the effective Hamiltonian writes

$$H_{eff} = \frac{G_F^2 m_W^2}{4\pi^2}C_0 \times [\alpha_s(M)]^{-2/9}\left[1+\frac{\alpha_s^3(M)}{4\pi}J_3\right]O_{sd} + h.c..$$

Now the matrix elements M_{12} (1.42) simplify significantly. Since H_{eff} includes all virtual intermediate states, the non-diagonal element of M writes to second-order in perturbation theory

$$M_{12} = \langle K^0|\, H_{eff}^{+}\,|\bar{K}^0\rangle = \frac{G_F^2 m_W^2}{4\pi^2}C_0^* \times [\alpha_s(M)]^{-2/9}\left[1+\frac{\alpha_s^3(M)}{4\pi}J_3\right]\langle K^0|\, O_{sd}^{+}\,|\bar{K}^0\rangle\,.$$

The calculation of the hadron matrix element is difficult, since the operator O_{sd} corresponds to free quarks whereas the initial and final states K^0 and $\bar{K}^0$ consists of bounded quarks. The matrix elements are usually estimated by the vacuum insertion approximation, which splits the amplitude into several by inserting the vacuum state in all possible ways and multiplying by a correction factor B_K [5]

$$\langle K^0|\, O_{sd}^{+}\,|\bar{K}^0\rangle = -\frac{1}{3}B_K(M)f_K^2 m_K$$

with

$$\hat{B}_K = B_K(M)[\alpha_s(M)]^{-2/9}\left[1+\frac{\alpha_s^3(M)}{4\pi}J_3\right]$$

and the K-meson mass m_K and the K-meson decay constant f_K given in Appendix C.

The matrix element M_{12} now reads

$$M_{12} = -\frac{G_F^2 m_W^2}{12\pi^2}C_0^*\hat{B}_K(M)f_K^2 m_K.$$

1.7 Parameters used to approximate the size of physics beyond the Standard Model

1.7.1 The oblique T parameter

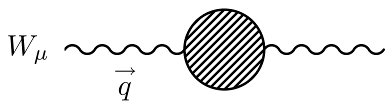
In general there are three oblique parameters (T, U and S) which describe the finite parts of the vacuum polarisation diagrams of the gauge bosons which have not been included in the renormalisation of the coupling constants, the masses or the fields. Here only the T-parameter is described in detail which is defined over [19]

$$T = \frac{4\pi}{m_Z^2 sin^2\theta_W cos^2\theta_W} \left[\Pi_{11}(0) - \Pi_{33}(0)\right] \tag{1.55}$$

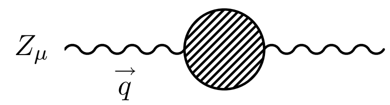
with

$$\begin{aligned}
\Pi_{11}(q) &= \frac{sin^2\theta_W}{e^2}\Pi_{WW}(q) \\
\Pi_{33}(q) &= \frac{sin^2\theta_W cos^2\theta_W}{e^2}\left(\Pi_{ZZ}(q) + 2sin^2\theta_W\Pi_{3Q}(q) + sin^4\theta_W\Pi_{QQ}(q)\right),
\end{aligned}$$

where Q denotes the charge of the intermediate particles. The contributions $\Pi_{WW}(q)$ and $\Pi_{ZZ}(q)$ to lowest order in perturbation theory are shown below.



$= \quad i\Pi_{WW}(q)g^{\mu\nu}$



$= \quad i\Pi_{ZZ}(q)g^{\mu\nu}$

The contributions $\Pi_{3Q}(q)$ and $\Pi_{QQ}(q)$ emerge from self-energy diagrams of the photon or the Z-boson and the photon but give zero in the limit of $q = 0$. This can be explained using the Ward identity and the assumption that the contributions do not diverge at $p^2 = 0$ [1]. The calculation of the T-parameter has been carried out in [20] for the Standard Model extended by a random number of vector-like singlets and quark doublets. The result for one extra u-type quark isosinglet (the Top-quark) is given by [21]

$$T = \frac{3}{16\pi sin^2\theta_W cos^2\theta_W}\left[\left|V_{tb}^{SM}\right| sin^2\theta\left(\theta_+(y_T, y_b) - \theta_+(y_t, y_b)\right) - sin^2\theta cos^2\theta\ \theta_+(y_T, y_t)\right] \tag{1.56}$$

with $y_i = m_i^2/m_Z^2$ and

$$\theta_+(y_i, y_j) = y_i + y_j - \frac{2y_i y_j}{y_i - y_j} ln \frac{y_i}{y_j}.$$

The measured value is given by $T = -0,03 \pm 0,09$ [21]. The high uncertainty in the experimental and theoretical value of T (due to the uncertainty in the Higgs mass) makes the T-parameter difficult to use to derive constraints on physics beyond the Standard Model.

1.7.2 The R_b parameter

This parameter describes the branching ratio of the Z-boson decaying to a b- and an anti b-quark [21].

$$R_b = \frac{\Gamma_b}{\Gamma_{had}}$$

The lowest order correction to the tree-level decay $Z \to b\bar{b}$ is dominated by the loops including the top-quark (1.5) [22] and would therefore be sensitive to other heavier t-quark type quarks.

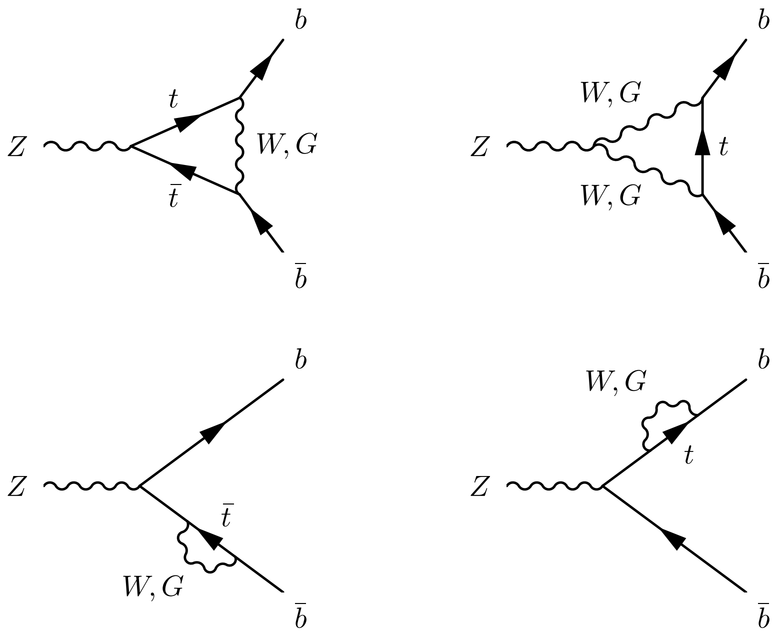


Fig. 1.5: The main lowest order contributions to the R_b-parameter.

The experimental and the Standard Model value of the R_b-parameter is given by [21]

$$R_b^{exp} = 0.21629(66) \qquad \text{and} \qquad R_b^{SM} = 0.21578(10).$$

1.8 The decoupling theorem

The decoupling theorem was established by Thomas Appelquist and James Carazzone in 1975 and is therefore also known as the Appelquist-Carazzone theorem [23]. The theorem

deals with the influence of heavy particles on renormalisable theories describing low energy interactions, such as a renormalisable effective Lagrangian. The Wilson description (chapter 1.6.2) showed that the effective Lagrangian can be split into two factors (one describing the low and one describing the high energy effects) in such a way that the heavy particles only appear explicitly in the Wilson coefficients. The decoupling theorem states that the contributions of the heavy particles must be suppressed by the heavy particle masses. Thus one can conclude that there are non-decoupling effects for finite particle masses, but that the heavy particles must decouple in the limit of infinite masses. The decoupling theorem also allows effects of heavy particles to appear in charge renormalisation [23, 24].
This theorem offers a simple way of controlling results obtained from calculations of loop corrections of physical quantities including heavy particles arising from a renormalisable extension of the Standard Model. However, one must exercise caution when taking the mass limit. Examples of possible misleading conclusions are diverging loop diagram corrections when taking the limit $m_t \rightarrow \infty$ in the Standard Model. This is not an exception to the decoupling theorem, since the non-decoupling arises from the fact that the renormalisability of the Standard Model is destroyed by integrating out the top- but not the bottom-quark [24]. The question also arises of whether it is physically justifiable to let the top-mass tend to infinity without letting all other particle masses of the Standard Model do so, since they all depend on the same Higgs vacuum expectation value v. The same situation occurs in the Left-Right Symmetric Model when calculating the B-meson to two muons decay (chapter 4.1). It will be explicitly shown (chapter 4.3) that one cannot let the single-charged Higgs scalar mass tend to infinity without doing the same with the right-handed W-boson mass, since both depend on the breaking scale (v_R) of the gauge group $SU(2)_R$.

2. EXTENSIONS OF THE STANDARD MODEL

2.1 A fourth generation in the Standard Model

Adding new generations of chiral quarks and leptons is a straightforward extension of the Standard Model, which was proposed even before the third generation had been discovered. There are many arguments against a fifth or higher family [25, 26, 27, 28], but the existence of a fourth generation cannot yet be excluded by existing data [25, 13].
In this section only the possibility of the existence of a new quark generation is discussed.

2.1.1 Advantages of a fourth family

Adding a fourth generation could remove some unnaturalnesses of the Standard Model.
It has been proposed that due to non-negligible loop corrections of the fourth generation quarks and leptons, the gauge couplings could be unified at a scale around $3,5 \times 10^{15}$ GeV [29], under the assumption that the Higgs boson is heavier than 174 GeV.

New heavy quarks could also contribute to an explanation of the strong CP problem (chapter 1.3.2) [30]. This is realised by introducing a new symmetry U(1)$_{new}$ under which only the new generation quarks transform non-trivially. In addition to the complex Higgs doublet, two complex Higgs singlets must be introduced, which also transform non-trivially under the new symmetry. Under certain assumptions about the masses of the new particles and the breaking scale of U(1)$_{new}$, the breaking of the new symmetry gives a value for $\bar{\theta}$ which is small enough to be consistent with recent measurements of the CP-violating parameters. Depending on the nature of U(1)$_{new}$ a new gauge boson is predicted. One well-known model is the Aspon Model, in which the Aspon is the massive gauge boson, which acquires its mass over the Higgs mechanism [31].

Another problem which could be solved is the missing sources of CP-violation predicted by the Standard Model. Introducing a new generation adds three new CP-violating phases to the CKM matrix (discussed in the next chapter) which then enter the effective couplings of mixing quark flavours. Since the imaginary parts of the effective couplings determine the CP-asymmetry of the reaction, the amount of CP-violation can be tuned according to the value of these phases. W.S. Hou states [32] that the amount of CP-violation can be enhanced by a factor of 10^{13}, which has been shown by means of the generalised form of the parameter J which was introduced by Jarlskog in order to estimate the amount of CP-violation in the Standard Model [33]. The enhancement is stated to leave the prediction of measured CP-violating processes such as the EDM (chapter 2.2.4) unchanged. The increase of the CP-violation is due to the large masses of the new quarks and the resulting large Yukawa couplings rather than the largeness of the new phases. Since the J-parameter depends on the mass-differences of quarks of different families, it is suppressed by the small quark masses or the small quark mass-differences.

The CP-violating phase in the B_s^0 system predicted by the Standard Model ($\Phi_s^{SM} \approx -2,3°$) [34] is noticeably smaller than the measured value ($\Phi_s = 2 * \beta_s = -40°$) [8]. The authors of [13] even claim that the value of the CP-violating phases could lie around $\Phi_s \approx -45°$. However, an enhancement of the value of the phase can be achieved by introducing new quarks [13].

2.1.2 Realisation of a fourth generation

Adding a new generation means to add a new quark doublet Q_4 which transforms with the same quantum numbers as the first three doublets.

$$Q_i \in \left\{ \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} T \\ B \end{pmatrix} \right\}.$$

The only restriction on the CKM matrix given by the Standard Model (chapter 1.4) is unitarity. Choosing a parametrisation with angles and imaginary phases gives six independent angles and ten independent phases [35]. Since the Lagrangian should be invariant under the global rotation of each quark field

$$q_i \rightarrow e^{i\alpha_i} q_i \qquad i \in \{1, ..., 8\}$$

seven of the ten phases can be removed by choosing α_i as linear combinations of those ten phases and three independent phases remain. Thus there are in total nine independent parameters in the 4x4 CKM matrix, five more than in the 3x3 case.
In recent analysis the parametrisation of Fritzsch and Plankl [36] was used. They show that a parametrisation of a unitary nxn matrix can be derived by the multiplication of several rotation matrices (where some of the matrix elements are treated as perturbations).

The 4x4 matrix is given by

$$V = \begin{pmatrix} c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ \begin{array}{c} -c_{23}c_{24}s_{12}-c_{12}c_{24}s_{13}s_{23}e^{i\lambda_{13}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} & \begin{array}{c} c_{12}c_{23}c_{24}-c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} & \begin{array}{c} c_{13}c_{24}c_{23} \\ -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} \end{array} & c_{14}s_{24}e^{-i\delta_{14}} \\ \begin{array}{c} -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}}+c_{34}s_{12}s_{23} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} -c_{12}c_{34}s_{23}-c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} \\ -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} \\ +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} c_{13}c_{23}c_{34} \\ -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} \\ \end{array} & c_{14}c_{24}s_{34} \\ \begin{array}{c} -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}}-s_{12}s_{23}s_{34} \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}}+c_{12}s_{23}s_{34} \\ -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{13}s_{34}ei\delta_{13} \\ +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} -c_{13}c_{23}s_{34} \\ -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} \\ -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} \\ \end{array} & c_{14}c_{24}c_{34} \end{pmatrix}$$

with $c_{ij} = cos\theta_{ij}$ and $s_{ij} = sin\theta_{ij}$.
Setting $c_{14} = c_{24} = c_{34} = 1$ and $\delta_{13} = \delta$ gives the parametrisation of the CKM matrix in the Standard Model, with the Cabibbo angle $\theta_C = \theta_{12}$.

2.1.3 Constraints on a fourth generation

Constraints on the new heavy quark masses and the mass-mixing angles can be derived from theoretical arguments as well as from experimental boundaries.
From Tevatron it is known that $m_T > 258GeV$. Considering the situation in which all B-quarks decay via the decay channel $B \to Zb$ the boundary $m_B > 268GeV$ is achieved [37].
A constraint on the splitting of the masses in the doublet comes from the contribution of a fourth generation to the oblique electroweak parameters S and T. Both parameters depend on the unknown Higgs mass m_H. In [37] experimental data for S and T is analysed and combined with the claim of minimal contribution due to the new particles.
The relation found is given by

$$m_T - m_B \cong \left(1 + \frac{1}{5} ln \frac{m_H}{115GeV}\right) \times 50GeV.$$

According to [32] the possibility of treating the Yukawa coupling by perturbation theory breaks down if $m_T, m_B > 600GeV$.
Summarising all these boundaries for the fourth generation quark masses and assuming $m_H = 115GeV$ gives

$$318GeV < m_T \leq 600GeV \qquad \text{and} \qquad 268GeV < m_B \leq 550GeV. \tag{2.1}$$

Information about the breakdown of perturbation theory can be acquired by analysing the behaviour of the Yukawa coupling of the new quarks for different energies or the renormalisation group equations. It transpires that since the new quarks acquire their mass through the same vev as the other three-generation quarks their Yukawa coupling is larger than 1,5

and the authors of [37] state that thus the use of perturbation theory is lost for energies higher than the TeV scale. It must be remembered that these predictions also rely on the mass of the Higgs boson and are therefore not definite. It will be discussed later (chapter 3.1) that these restrictions do not hold true for quarks which acquire their mass from a Higgs field other than the Standard Model one.
One theory in which the introduction of a fourth family does not lead to a negative Higgs quartic coupling producing large quantum corrections is supersymmetry [37].

Bobrowski, Lenz, Riedl and Rohrwild [13] discuss the boundaries of the new and the yet to be precisely assigned CKM matrix elements. Since $|V_{td}|$ and $|V_{ts}|$ can only be measured in meson mass-mixing and the fact that the PDG analysis [8] obtains its results for the matrix elements by assuming unitarity of the CKM matrix, these results cannot be used for the four-generation case. The values of the first two row elements and $|V_{tb}|$ are measured independently in tree-level decays and therefore the PDG results do not depend on the dimension of the CKM matrix.

$$\begin{array}{lll} |V_{ud}| = 0,97418 \pm 0,00027 & |V_{us}| = 0,2255 \pm 0,0019 & |V_{ub}| = 0,00393 \pm 0,00036 \\ |V_{cd}| = 0,230 \pm 0,011 & |V_{cs}| = 1,04 \pm 0,06 & |V_{cb}| = 0,0412 \pm 0,0011 \\ |V_{tb}| > 0,74 & & \end{array}$$

The analysis of [13] for D^0-mixing gives for two of the new CKM matrix elements

$$|V_{uB}V_{cB}| \leq \begin{cases} 0,00395 & \text{for} \quad m_B = 200 GeV \\ 0,00290 & \text{for} \quad m_B = 300 GeV \\ 0,00193 & \text{for} \quad m_B = 500 GeV. \end{cases}$$

The authors of [13] investigate possible values of the nine CKM parameters by evaluating tree-level and FCNC reactions for a large number of random parameters and discard all parameter regions which do not give results which agree with the experiment. How much the contribution due to the Standard Model with four generations departs from the original value predicted by the Standard Model is expressed by the parameter

$$\Delta = \frac{M_{12}^{SM4}}{M_{12}^{SM3}} = |\Delta| \, e^{i\Phi^{\Delta}}$$

which can be calculated for meson-mixing processes or tree-level decays (for details see the table on page 5 in [13]). The absolute values for these parameters are still under discussion. The calculated reactions do not contain contributions from the heavy B-quark. Therefore only the value of the heavy T-quark is analysed, which is taken to be between 300 and 650 GeV due to (2.1). The QCD corrections due to the fourth generations are assumed to be equal to those due to the third generation.

$$\eta_{TT} = \eta_{tT} = \eta_{tt}, \qquad \eta_{cT} = \eta_{ct}$$

The new CP-violating phases δ_{13}, δ_{14} and δ_{24} are taken to be unconstrained, i.e. between 0 and 2π.

With the boundaries of the new matrix elements from the above analysis, the new CKM matrix can be obtained by expanding every element in $V_{us} = \lambda$ up to the order of λ^4 in the manner of Wolfenstein's ansatz and setting

$$\begin{array}{lll} V_{ub} = s_{13} e^{-i\delta_{13}} =: A\lambda^4(r\tilde{h}o + i\tilde{\eta}) & V_{us} = s_{12} =: \lambda & V_{cb} = s_{23} =: A\lambda^2 \\ V_{uB} = s_{14} e^{-i\delta_{14}} =: (x_{14} - iy_{14})\lambda^2 & V_{cB} = c_{14} s_{24} e^{-i\delta_{24}} = (x_{24} - iy_{24})\lambda & \end{array}$$

where x_{14}, y_{14}, x_{24} and y_{24} are the parameters for s_{14}, δ_{14}, s_{24} and δ_{24} with the advantage that the new parameters are known to be smaller than one. Since the scale of V_{tB} is not known c_{34} and s_{34} are left unchanged.

The other four parameters A, λ, $\tilde{\eta}$ and $\tilde{\rho}$, with $\rho = \lambda\tilde{\rho}$ and $\eta = \lambda\tilde{\eta}$, are the well-known Wolfenstein parameters.

$$V = \begin{pmatrix} \begin{matrix} 1-\frac{\lambda^2}{2}-\frac{1}{8}(4x_{14}^2+4y_{14}^2+1)\lambda^4 \end{matrix} & \lambda & A(\tilde{\rho}-i\tilde{\eta})\lambda^4 & (x_{14}-iy_{14})\lambda^2 \\[2ex] \begin{matrix} -\lambda+\frac{1}{2}(x_{24}-iy_{24}) \\ (-2x_{14}+x_{24}-2iy_{14}+iy_{24})\lambda^3 \end{matrix} & \begin{matrix} 1-\frac{1}{2}(x_{24}^2+y_{24}^2+1)\lambda^2 \\ +\frac{1}{8}\big(-x_{24}^4-2(y_{24}^2-1)x_{24}^2 \\ -8iy_{14}x_{24}-y_{24}^4)-(4A^2+2y_{24}^2 \\ -8x_{14}(x_{24}-iy_{24})-8y_{14}y_{24}-1)\lambda^4 \end{matrix} & A\lambda^2 & (x_{24}-iy_{24})\lambda \\[2ex] \begin{matrix} s_{34}(-x_{14}+x_{24}-i(y_{14}-y_{24}))\lambda^2 \\ +Ac_{34}\lambda^3-\frac{1}{2}(A(-2i\tilde{\eta}-2\tilde{\rho})c_{34} \\ +s_{34}(x_{14}+iy_{14})(x_{24}^2+y_{24}^2+1)\big)\lambda^4 \end{matrix} & \begin{matrix} -s_{34}(x_{24}+iy_{24})\lambda-Ac_{34}\lambda^2 \\ +\frac{1}{2}s_{34}(-2x_{14}+x_{24}-2iy_{14}+iy_{24})\lambda^3 \\ -\frac{1}{2}(Ac_{34}(x_{24}^2+y_{24}^2-1))\lambda^4 \end{matrix} & \begin{matrix} c_{34} \\ -As_{34}(x_{24}+iy_{24})\lambda^3 \\ -\frac{1}{2}A^2c_{34}\lambda^4 \end{matrix} & \begin{matrix} s_{34} \\ -\frac{1}{2}(s_{34}(x_{24}^2+y_{24}^2))\lambda^2 \\ -\frac{1}{8}(s_{34}\big(x_{24}^4+2y_{24}^2x_{24}^2 \\ +y_{24}^4+4x_{14}^2 4y_{14})\big)\lambda^4 \end{matrix} \\[2ex] \begin{matrix} c_{34}(-x_{14}+x_{24}-i(y_{14}-y_{24}))\lambda^2 \\ -As_{34}\lambda^3+\frac{1}{2}(2A(i\tilde{\eta}+\tilde{\rho})s_{34} \\ +c_{34}(x_{14}+iy_{14})(x_{24}^2+y_{24}^2+1)\big)\lambda^4 \end{matrix} & \begin{matrix} -c_{34}(x_{24}+iy_{24})\lambda \\ +As_{34}\lambda^2+\frac{1}{2}c_{34}(-2x_{14} \\ +x_{24}-2iy_{14}+iy_{24})\lambda^3 \\ +\frac{1}{2}As_{34}(x_{24}^2+y_{24}^2-1)\lambda^4 \end{matrix} & \begin{matrix} -s_{34} \\ -Ac_{34}(x_{24}+iy_{24})\lambda^3 \\ +\frac{1}{2}A^2s_{34}\lambda^4 \end{matrix} & \begin{matrix} c_{34} \\ -\frac{1}{2}(c_{34}(x_{24}^2+y_{24}^2))\lambda^2 \\ -\frac{1}{8}c_{34}\big(x_{24}^4+2y_{24}^2x_{24}^2 \\ +y_{24}^4+4x_{14}^2+4y_{14}^2\big) \end{matrix} \end{pmatrix} \tag{2.2}$$

By choosing the nine parameters of the CKM matrix and the mass of the heavy T-quark in an appropriate way the problems mentioned above can be solved, although it should be noted that the new phenomenology is very sensitive to the choice of parameters [13]. It also transpires that the 3x3 CKM matrix could look different than previously thought.

The elements V_{td} and V_{ts} appear in the calculations of [13] along with other leading orders than previously believed. These new contributions are allowed since when calculating the parameter Δ the extra contributions due to the updated 3x3 CKM matrix cancel out the extra contributions arising from the new quarks.

2.2 The Left-Right Symmetric Model

2.2.1 Historical development

The assumption of a discrete left-right (parity) symmetric Langrangian at a high energy scale was first made by J. C. Pati and Abdus Salam [38]. They considered the gauge group $SU(4)_L \times SU(4)_R \times SU(4')$ which under the circumstances of non-vanishing coupling constants g_L and g_R reduces to the subgroup $SU(2)_L \times SU(2)_R \times SU(4')_{L+R}$. One aim of this model was to unite the leptonic with the baryonic number to form the fermion number $F = B + L$. The Pati-Salam gauge group can be embedded in the gauge group SO(10), which is one of the GUTs [39].

Recent attempts have been made by W.-M. Yang and H.-H. Liu [40] to extend the Pati-Salam Model to a $SU(2)_L \times SU(2)_R \times SU(4)_C \times SU(2)_G \times SO(3)_F \times D_p$ symmetry. This model generates super-heavy fermions as mirror particles of the Standard Model particles with masses just under the GUT scale ($\approx 10^{15}$GeV) and right-handed Majorana neutrinos. The lightest one has a mass around 1 TeV and serves as a dark matter candidate.

A further analysis of the subgroup $SU(2)_L \times SU(2)_R \times G$, where G is an arbitrary group commuting with $SU(2)_L \times SU(2)_R$ and obeying the left-right symmetry, has been published by R. N. Mohapatra and J.C. Pati [41]. They state that the left-right symmetry of the Lagrangian is a 'natural' symmetry which is broken at a scale higher than those explored experimentally until now. Since the breaking process leads to parity non-conserving terms, this mechanism introduces CP-violation naturally.

A follow-up paper by R. N. Mohapatra and G. Senjanovic [42] addresses the Higgs mechanism (described in the next chapter) and the realisation of such a model directly. It transpires that the prefactors belonging to the left- and right-handed part of the Higgs potential need to be of different scales in order to give different masses to the W_L- and W_R-boson. In order to introduce CP-violation other than doing so through the CKM matrix at least one of the vacuum expectation values of the Higgs fields needs to be imaginary, otherwise the Lagrangian would be CP-invariant. The minimal choice of fields in the Higgs sector is pointed out to be one bidoublet and two triplets. Models containing these Higgs fields are called Minimal Left-Right Symmetric Models (MLRSM).
Despite the boundaries on the parameters of the Left-Right Symmetric Models having changed, recently acquired data still allows an extension of the Standard Model by an LRSM. In [43] modern data is used to restrict the parameters such as the masses of the heavy W- and Z-boson. The symmetry referred to in more recent papers is $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, where P means parity-invariance.

There are several ways in which a left-right symmetric Lagrangian can result from the breaking of an SO(10)-symmetry. Depending on the Higgs content and the choice of whether SO(10) is combined with another U(1)-symmetry and thus embedded in E_6, SO(10) can be broken down to the Pati-Salam-/ the left-right symmetric- Lagrangian [44] or to the gauge group SO(4)×SO(6) [45] (which in turn can be broken down to the two aforementioned Lagrangians). The breaking process of the left-right symmetric Lagrangian to the Standard Model Lagrangian is described in chapter 2.2.2. Another possibility is to break the SO(10) symmetry down to an SU(5)-symmetry, but from there it is only possible to break directly to the symmetry of the Standard Model without the linking step of the Left-Right Symmetric Model [45].

Advantages of the LRSM which are often highlighted include the explanation of the smallness of the neutrino masses due to the seesaw relation which follows from the minimum condition of the Higgs potential and the solution of the strong CP problem (chapter 1.3.2) due to the parity-invariance of the Lagrangian. The strong CP-violating parameter transforms under parity transformation according to [29] $\Theta \leftrightarrow -\Theta$. Since the Lagrangian is required to be invariant under left-right transformation it follows that $\Theta = 0$.
A theory with an $SU(2)_L \times SU(2)_R$ symmetry would have the advantage of unifying the coupling constants of the Standard Model which has been shown in [46] for $M_{W_R} > 1$ TeV and a unification scale from 10^5 GeV up to $10^{17,7}$ GeV. In [47] the unification of the coupling constant was ensured by the introduction of extra fermions and scalars with exotic quantum numbers.

2.2.2 Higgs mechanism

In order to give mass to the fermions (including neutrinos) and bosons, several Higgs fields are required. A diagram often used in the literature (figure 2.1) shows the way in which the form of the Higgs fields and the parameters in the Higgs potential determine the outcome of the model. It emerges that three different outcomes are possible. Since one of the three outcomes is unlikely and one is hardly observable, just the third outcome is favoured by theorists.
In order to give mass to the neutrinos two of the Higgs fields must be triplets

$$\Delta_L = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & \frac{-\delta_L^+}{\sqrt{2}} \end{pmatrix}, \qquad \Delta_R = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & \frac{-\delta_R^+}{\sqrt{2}} \end{pmatrix} \tag{2.3}$$

where the vev of Δ_R breaks $SU(2)_R$ and therefore determines the mass scale of the new heavy bosons W_R and Z_R.
Another recent attempt made by Wu and Zhou suggests an LRSM with two Higgs bidoublets [48].

The electroweak symmetry breaking is enabled by the vev of the bidoublet

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}. \tag{2.4}$$

Since $SU(2)_R$ is required to be broken at a much higher energy scale than the electroweak symmetry breaking the vevs have to be of the order $v_R >> \kappa, \kappa'$, where v_R describes the vev acquired by the neutral component of the right-handed triplet and κ and κ' are the vevs acquired by the neutral components of the bidoublet. The vevs acquired by the Higgs fields are explicitly written down in equation (2.5).

Hence the symmetry of the left-right symmetric Lagrangian is broken in the following two steps:

$$SU(2)_R \times SU(2)_L \times SU(3)_c \times U(1)_Y \overset{\langle\Delta_R\rangle}{\rightarrow} SU(2)_L \times SU(3)_c \times U(1)_Y \overset{\langle\phi\rangle}{\rightarrow} SU(3)_c \times U(1)_Q$$

If one considered an additional U(1)-symmetry which ensures the parity-invariance of the Lagrangian, one would need an additional Higgs scalar in order to break that symmetry. Such an extension of the symmetry would also guarantee the equality of the coupling constants of SU(2)$_L$ and SU(2)$_R$ ($g_L = g_R$) at tree-level. The explicit U(1)-symmetry and the Higgs scalar are not considered in this thesis since they would not add much to the phenomenology of the LRSM, but the equality of the coupling constants is still assumed. Taking higher order corrections into account, the values of g_L and g_R differ but their deviation $g_L - g_R$ is always finite [41].

As seen above, the vev of the triplet Δ_L does not need to break any symmetry in order to achieve the right phenomenology, but it still needs to be included to preserve left-right symmetry of the Lagrangian. For this and other reasons (outlined below) it is possible to assume $v_L = 0$ or at least $\kappa, \kappa' >> v_L$.

Therefore the vevs follow the hierarchy

$$v_R >> \kappa, \kappa' >> v_L.$$

The vevs of the Higgs fields are given by [43]

$$\langle\phi\rangle = \begin{pmatrix} \kappa e^{i\alpha_1} & 0 \\ 0 & \kappa' e^{i\alpha_2} \end{pmatrix}, \qquad \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_1} & 0 \end{pmatrix}, \qquad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\theta_2} & 0 \end{pmatrix}.$$

Under $SU(2)_{L,R}$ those Higgs fields transform as [49]

$$\phi \rightarrow U_L \phi U_R^+, \qquad \Delta_L \rightarrow U_L \Delta_L U_L^+, \qquad \Delta_R \rightarrow U_R \Delta_R U_R^+$$

with $U_{L,R} \in SU(2)_{L,R}$.

Under parity they transform as

$$\phi \leftrightarrow \phi^+ \qquad \text{and} \qquad \Delta_L \leftrightarrow \Delta_R.$$

Using the degrees of freedom of the transformation laws under $SU(2)_L \times SU(2)_R$ two of the phases in the vevs can be eliminated.
Conventionally the phases of κ and v_R are chosen to be zero

$$\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix}, \qquad \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0 \end{pmatrix}, \qquad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \tag{2.5}$$

The Higgs fields are included in the Lagrangian over the traces of covariant derivatives [43]

$$L^{Higgs} = Tr[(D_\mu\Delta_L)^+(D^\mu\Delta_L)] + Tr[(D_\mu\Delta_R)^+(D^\mu\Delta_R)] + Tr[(D_\mu\phi)^+(D^\mu\phi)]$$

with

$$D_\mu\phi = \partial_\mu\phi + i\frac{g_L}{2}\vec{W}_{L\mu}\cdot\vec{\tau}\phi - i\frac{g_R}{2}\phi\vec{W}_{R\mu}\cdot\vec{\tau} \tag{2.6}$$

$$D_\mu\Delta_L = \partial_\mu\Delta_L + i\frac{g_L}{2}\left[\vec{W}_L\cdot\vec{\tau},\Delta_L\right] + ig'B_\mu\Delta_L \tag{2.7}$$

$$D_\mu\Delta_R = \partial_\mu\Delta_R + i\frac{g_R}{2}\left[\vec{W}_R\cdot\vec{\tau},\Delta_R\right] + ig'B_\mu\Delta_R. \tag{2.8}$$

The most general Higgs potential for the given fields which is invariant under parity is given by [43]

$$\begin{aligned}
V =& -\mu_1^2\left(Tr(\phi^+\phi)\right) - \mu_2^2\left(Tr(\tilde{\phi}\phi^+) + Tr(\tilde{\phi}^+\phi)\right) - \mu_3^2\left(Tr(\Delta_L\Delta_L^+) + Tr(\Delta_R\Delta_R^+)\right) \\
&+ \lambda_1\left(Tr(\phi\phi^+)\right)^2 + \lambda_2\left([Tr(\tilde{\phi}\phi^+)]^2 + [Tr(\tilde{\phi}^+\phi)]^2\right) + \lambda_3\left(Tr(\tilde{\phi}\phi^+)Tr(\tilde{\phi}^+\phi)\right) \\
&+ \lambda_4\left(Tr(\phi\phi^+)\left[Tr(\phi\tilde{\phi}^+) + Tr(\tilde{\phi}^+\phi)\right]\right) + \rho_1\left([Tr(\Delta_L\Delta_L^+)]^2 + [Tr(\Delta_R\Delta_R^+)]^2\right) \\
&+ \rho_2\left(Tr(\Delta_L\Delta_L)Tr(\Delta_L^+\Delta_L^+) + Tr(\Delta_R^+\Delta_R^+)Tr(\Delta_R\Delta_R)\right) \\
&+ \rho_3\left(Tr(\Delta_L\Delta_L^+)Tr(\Delta_R\Delta_R^+)\right) + \rho_4\left(Tr(\Delta_L\Delta_L)Tr(\Delta_R^+\Delta_R^+) + Tr(\Delta_L^+\Delta_L^+)Tr(\Delta_R\Delta_R)\right) \\
&+ \alpha_1\left(Tr(\phi\phi^+)\left[Tr(\Delta_L\Delta_L^+) + Tr(\Delta_R\Delta_R^+)\right]\right) \\
&+ \alpha_2\left(e^{i\delta_2}\left[Tr(\phi\tilde{\phi}^+)Tr(\Delta_R\Delta_R^+) + Tr(\phi^+\tilde{\phi})Tr(\Delta_L\Delta_L^+)\right] + h.c.\right) \\
&+ \alpha_3\left(Tr(\phi\phi^+\Delta_L\Delta_L^+) + Tr(\phi^+\phi\Delta_R\Delta_R^+)\right) + \beta_1\left(Tr(\phi\Delta_R\phi^+\Delta_L^+) + Tr(\phi^+\Delta_L\phi\Delta_R^+)\right) \\
&+ \beta_2\left(Tr(\tilde{\phi}\Delta_R\phi^+\Delta_L^+) + Tr(\tilde{\phi}^+\Delta_L\phi\Delta_R^+)\right) + \beta_3\left(Tr(\phi\Delta_R\tilde{\phi}^+\Delta_L^+) + Tr(\phi^+\Delta_L\tilde{\phi}\Delta_R^+)\right).
\end{aligned} \tag{2.9}$$

The only imaginary and therefore CP-violating prefactor in the potential is parameterised by the phase δ_2. This means that the Higgs potential is a source of explicit CP-violation in addition to the imaginary vev of the Higgs bidoublet.
The Higgs potential must be at its minimum with respect to all vev parameters. Using the minimum condition for the vev v_L gives the so-called seesaw relation [50]

$$(2\rho_1 - \rho_3)v_R v_L = \beta_1\kappa\kappa' cos(\theta_L - \alpha) + \beta_2\kappa^2 cos\theta_L + \beta_3\kappa'^2 cos(\theta_L - 2\alpha).$$

In the literature it is often assumed that due to an underlying symmetry (given for example by a GUT), all β parameters are zero. Other solutions would lead to unrealistic fine-tuning

under the requirement that v_R is at TeV scale. This requirement is necessary in order to create right-handed W- and Z-bosons with an observable mass (1-10 TeV).
It therefore follows that the left-hand side of the above equation must also be zero. Since v_R is breaking $SU(2)_R \times P$ and since $(2\rho_1 - \rho_3)$ can due to phenomenological reasons not be zero, v_L is set to zero.
In the last thirty years two different version of MLRSM have been popular. The manifest LRSM assumes a real Higgs potential ($\delta_2 = 0$) and no spontaneous CP-violations. This means no CP-violation due to Spontaneous Symmetry Breaking, which means that exclusively real Higgs vevs ($\alpha = \theta_L = 0$) are considered. This version was favourably used in the early development of the LRSM. These assumptions lead to almost equal right- and left-handed CKM matrices, distinguishable only by the quark mass signs (derived in chapter 2.2.5). The trade-off for the simplifying assumptions is the necessity of fine-tuning the Higgs potential parameters in order to result in the right energy scales for the Higgs vevs.
The pseudo-manifest LRSM assumes a real Higgs potential, but allows spontaneous CP-violation. This version does not predict as much CP-asymmetry as was found in B-meson decay and is therefore ruled out experimentally.
Recent papers therefore consider both explicit CP-violation due to an imaginary Higgs potential and spontaneous CP-violation. Ref. [43] states the following relation between the discrete CP-violating phase and the spontaneous CP-violating phase:

$$\alpha \propto sin^{-1}\left(\frac{2\,|\alpha_2|\,sin\delta_2}{\alpha_3\xi}\right)$$

where ξ is of the order $\frac{m_b}{m_t}$, where m_b and m_t are the masses of the bottom- and the top-quark.

Before SSB the Higgs fields contain 20 degrees of freedom. Eight belong to the bidoublet ϕ and six to each triplet Δ_L and Δ_R. After SSB six degrees of freedom are assigned to the longitudinal model of $W_L^{\pm}$, $W_R^{\pm}$, Z_L and Z_R. The other fourteen degrees form physical Higgs bosons. Hence there are four neutral scalars, two neutral pseudoscalars, two single-charged bosons and two double-charged Higgs [51]. All Higgs particles but one acquire mass around the scale of v_R. The lighter Higgs behaves similar to the Standard Model Higgs with a mass around the electroweak scale.

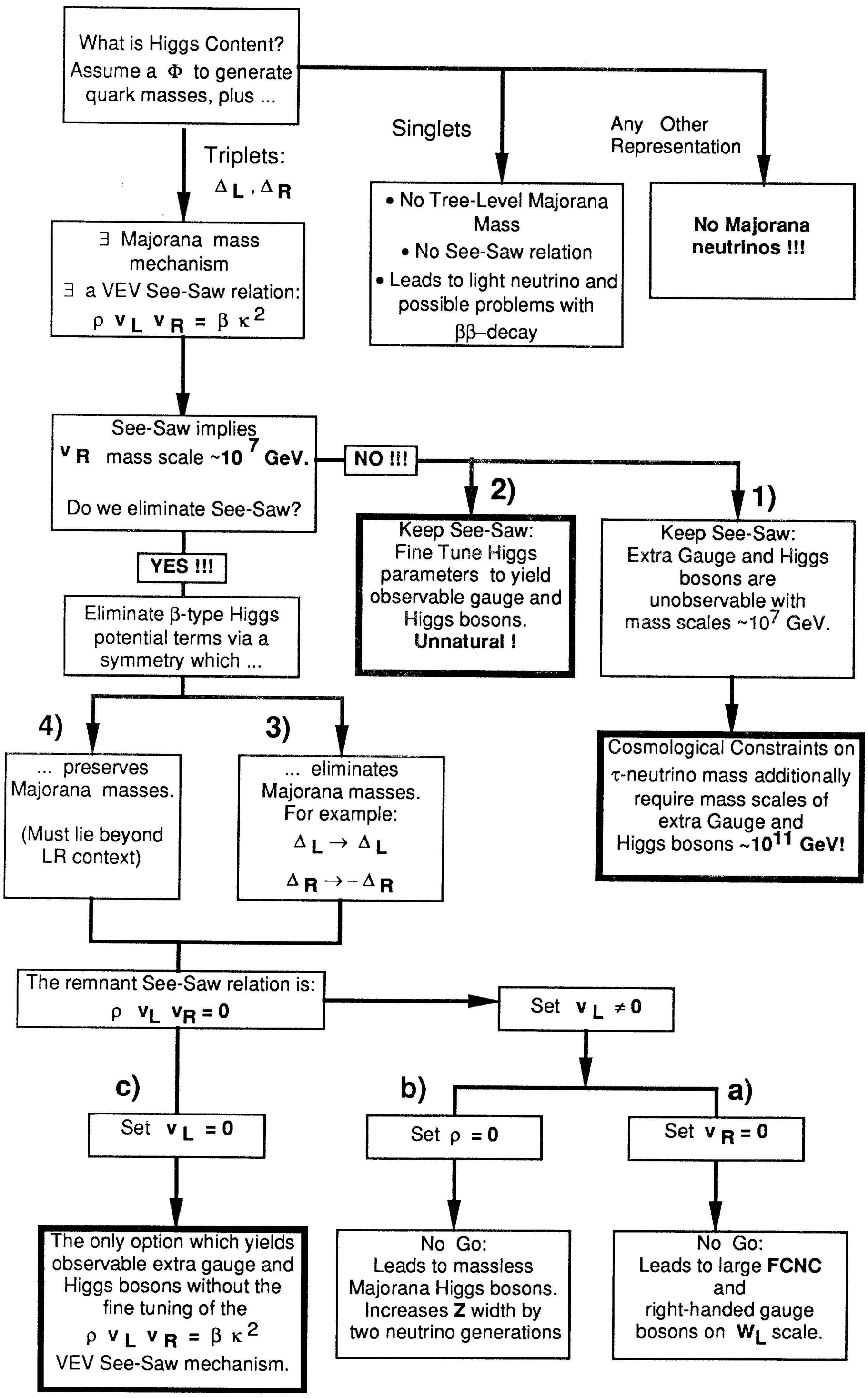


Fig. 2.1: Possibilities of the Higgs content of Left-Right Symmetric Models and the scenarios they result in. The bold framed scenarios are realistic.[50]

2.2.3 Field content

The left- and right-handed quark fields are given by

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

and transform as $Q_L \rightarrow U_L Q_L$ and $Q_R \rightarrow U_R Q_R$ under $SU(2)_{L,R}$ and as $Q_L \leftrightarrow Q_R$ under parity transformation.
The covariant derivative applying to the fermion fields is given by [5]

$$D_\mu = \partial_\mu - ig\sum_{k=1}^{3}(W_{Lk\mu}T_{Lk} + W_{Rk\mu}T_{Rk}) - ig' B_\mu Y \tag{2.10}$$

with the weak hypercharge Y and the generators of SU(2)$_L$ and SU(2)$_R$, T$_L$ and T$_R$.
In the Standard Model the third component of the electroweak field W^3 and B mix to give the physical Z-boson and the photon. In the MLRSM the situation is similar. Here the two third component W_L^3 and W_R^3 mix with B to form the Z-bosons Z_L and Z_R and the massless photon. It will be shown below that Z_L and Z_R are just the approximate physical mass eigenstates.
Evaluating the traces of the covariant derivatives of the vevs of the Higgs fields gives among others the mass term of the W-bosons W_L and W_R

$$L^{W-mass} = \begin{pmatrix} W_{L\mu}^- & W_{R\mu}^- \end{pmatrix} \begin{pmatrix} \frac{g^2}{2}(\kappa^2 + \kappa'^2 + 2v_L^2) & -g^2\kappa\kappa' e^{-i\alpha} \\ -g^2\kappa\kappa' e^{i\alpha} & g^2 v_R^2 \end{pmatrix} \begin{pmatrix} W_L^{+\mu} \\ W_R^{+\mu} \end{pmatrix}$$

where the element for W_R^+ and W_R^- is accurately given by $\frac{g^2}{2}(\kappa^2 + \kappa'^2 + 2v_R^2)$. From considerations of the scale of the Higgs vevs it is known that $v_R >> \kappa, \kappa'$ and therefore $\kappa^2 + \kappa'^2$ can be neglected. Since the mass matrix is not diagonal, W_L and W_R are not the physical mass eigenstates.
Calculating the eigenvalues of the mass matrix gives:

$$\begin{aligned} M_{W_1}^2 &= \frac{g^2}{2}\left(\kappa^2 + \kappa'^2 + v_R^2 - \sqrt{v_R^4 + 4\kappa^2\kappa'^2}\right) \approx \frac{g^2}{2}(\kappa^2 + \kappa'^2) = M_{W_L}^2 \\ M_{W_2}^2 &= \frac{g^2}{2}\left(\kappa^2 + \kappa'^2 + v_R^2 + \sqrt{v_R^4 + 4\kappa^2\kappa'^2}\right) \approx g^2 v_R^2 = M_{W_R}^2 \end{aligned} \tag{2.11}$$

Here W_1 and W_2 are the physical gauge bosons. Due to the scale difference of the vevs both κ and κ' can be neglected if in a sum with v_R and the masses of the W_1 and W_2 are approximately equal to those of W_L and W_R.
The transformation between these states is given by

$$\begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} = \begin{pmatrix} cos\zeta & -sin\zeta e^{-i\alpha} \\ sin\zeta e^{i\alpha} & cos\zeta \end{pmatrix} \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}$$

with [43]

$$tan\zeta = -\frac{\kappa\kappa'}{v_R^2} \approx -2\xi\left(\frac{M_{W_L}}{M_{W_R}}\right)^2 \tag{2.12}$$

which shows that the mixing between these states is negligibly small. In practice it is easier to calculate with the approximate mass eigenstates W_L and W_R since they only couple to left- or right-handed fermions. Hence the Lagrangian describing the W-current writes

$$\begin{aligned} L^{W-current} &= -\frac{g_L}{\sqrt{2}}\bar{u}_{Li}\gamma^\mu W^+_{L\mu}d_{Li} - \frac{g_R}{\sqrt{2}}\bar{u}_{Ri}\gamma^\mu W^+_{R\mu}d_{Ri} + h.c. \\ &= -\frac{g_L}{\sqrt{2}}\bar{u}_{Li}\gamma^\mu(cos\zeta W^+_{1\mu} - sin\zeta e^{-i\alpha}W^+_{2\mu})d_{Li} \\ &\quad -\frac{g_R}{\sqrt{2}}\bar{u}_{Ri}\gamma^\mu(sin\zeta e^{i\alpha}W^+_{1\mu} + cos\zeta W^+_{2\mu})d_{Ri} + h.c.. \end{aligned}$$

The mass term for the remaining gauge bosons is given by

$$L = \begin{pmatrix} W_{L3\mu} & W_{R3\mu} & B_\mu \end{pmatrix} M \begin{pmatrix} W_{L3\mu} \\ W_{R3\mu} \\ B_\mu \end{pmatrix}$$

with

$$M = \begin{pmatrix} \frac{g^2}{4}(\kappa^2+\kappa'^2+4v_L^2) & -\frac{g^2}{4}(\kappa^2+\kappa'^2) & -gg'v_L^2 \\ -\frac{g^2}{4}(\kappa^2+\kappa'^2) & \frac{g^2}{4}(\kappa^2+\kappa'^2+4v_R^2) & -gg'v_R^2 \\ -gg'v_L^2 & -gg'v_R^2 & -g'^2(v_L^2+v_R^2) \end{pmatrix}. \tag{2.13}$$

Calculating the eigenvalues for $v_L = 0$ gives the masses of the physical bosons Z_1, Z_2 and A

$$\begin{aligned} m_{Z_{1,2}} &= \frac{g^2}{4}(\kappa^2+\kappa'^2) + \frac{v_R^2}{2}(g^2+g'^2) \mp \frac{1}{4}\sqrt{g^4(\kappa^2+\kappa'^2)^2 - 4g^2g'^2v_R^2(\kappa^2+\kappa'^2) + 4(g^2+g'^2)^2v^4} \\ m_A &= 0 \end{aligned}$$

Under the requirement of $m_{W_L} = cos\theta_W m_{Z_1}$ the coupling constants are related over

$$g' = \frac{sin\theta_W}{\sqrt{cos\theta_W^2 - sin\theta_W^2}}g$$

which varies from the relation of the Standard Model. From now on the abbreviations $cos\theta_W = c_W$ and $sin\theta_W = s_W$ are going to be used.
The covariant derivative (2.10) only using the neutral boson part writes [5]

$$D_\mu = \partial_\mu - ig(W_{L3\mu}T_{L3} + W_{R3\mu}T_{R3}) - ig'B_\mu Y,$$

where it transpires that the weak hypercharge Y can be simply generalised to [43]

$$Y = \frac{B-L}{2} = Q - T_{L3} - T_{R3}$$

with the electrical charge Q.
Inserting the weak hypercharge yields

$$D_\mu = \partial_\mu - ig \left[T_{L3} \left(W_{L3\mu} - \frac{s_W}{\sqrt{c_W^2 - s_W^2}} B_\mu \right) + T_{R3} \left(W_{R3\mu} - \frac{s_W}{\sqrt{c_W^2 - s_W^2}} B_\mu \right) \right.$$
$$\left. + Q \frac{s_W}{\sqrt{c_W^2 - s_W^2}} B_\mu \right] .$$

In order to rotate the fields W_{L3}, W_{R3} and B to the states Z_L, Z_R and A such that Z_L is the Z-boson used in the Standard Model and A the photon one can read off some matrix elements of the rotation matrix by comparing the covariant derivative above to the covariant derivative of the Standard Model (1.6).

$$\begin{pmatrix} W_{L3\mu} \\ W_{R3\mu} \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & 0 & s_W \\ -\frac{s_W^2}{c_W} & -\frac{\sqrt{c_W^2 - s_W^2}}{c_W} & s_W \\ -\frac{s_w}{c_W}\sqrt{c_W^2 - s_W^2} & \frac{s_W}{c_W} & \sqrt{c_W^2 - s_W^2} \end{pmatrix} \begin{pmatrix} Z_{L\mu} \\ Z_{R\mu} \\ A_\mu \end{pmatrix} \tag{2.14}$$

Denoting the 3x3 matrix above as H, the matrix elements H_{11}, H_{13}, H_{23}, H_{31} and H_{33} were given by the previously mentioned requirement. The other elements were obtained by the requirement of unitarity of H. The same matrix but with opposite signs of the elements H_{13}, H_{23}, H_{31} and H_{22} was obtained in [5].
The mass matrix in the new basis is given by

$$M' = \frac{1}{4c_W^2} \begin{pmatrix} g^2(\kappa^2 + \kappa'^2) & g^2(\kappa^2 + \kappa'^2)\sqrt{c_W^2 - s_W^2} & 0 \\ g^2(\kappa^2 + \kappa'^2)\sqrt{c_W^2 - s_W^2} & g^2 \frac{4c_W^4 v_R^2 + (\kappa^2 + \kappa'^2)(1 - 4c_W^2 s_W^2)}{c_W^2 - s_W^2} & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

Since the upper part of the mass matrix is not diagonal, the states Z_L and Z_R are not the mass eigenstates. Analogous to the W-boson case the Z-bosons can be transformed to their mass eigenstates by the following rotation:

$$\begin{pmatrix} Z_L \\ Z_R \end{pmatrix} = \begin{pmatrix} cos\psi & -sin\psi \\ sin\psi & cos\psi \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$$

with the mixing angle

$$tan\psi \approx -\frac{(c_W^2 - s_W^2)^{\frac{3}{2}}}{4c_W^4} \frac{(\kappa^2 + \kappa'^2)}{v_R^2} \propto \frac{m_{W_L}^2}{m_{W_R}^2} .$$

Thus the mixing angles between the Z-bosons is about the factor χ^{-1} larger than the W-boson mixing angle but is still negligible. Therefore the bosons Z_L and Z_R are approximately the mass eigenstates with the following masses:

$$m_{Z_L}^2 \approx \frac{g^2}{2c_W^2}(\kappa^2 + \kappa'^2) \tag{2.15}$$

$$m_{Z_R}^2 \approx \frac{g^2 c_W^2}{2(c_W^2 - s_W^2)} v_R^2. \tag{2.16}$$

The results I achieved were actually only half as large as the quantities given above. Since these results did not agree with the results repeatedly listed in the literature [5, 52, 53] there must be a factor of two missing in my calculation or in the mass matrix (2.13).
Inserting the rotated gauge fields gives the covariant derivative for the neutral and charged gauge bosons

$$\begin{aligned} D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} \left(W_{L\mu}^+ T_L^+ + W_{L\mu}^- T_L^- + W_{R\mu}^+ T_R^+ + W_{R\mu}^- T_R^- \right) \\ - ieA_\mu Q - i\frac{g}{c_W} Z_{L\mu} \left(T_{L3} - s_W^2 Q \right) - ig\frac{\sqrt{c_W^2 - s_W^2}}{c_W} Z_{R\mu} \left(Y \frac{s_W^2}{c_W^2 - s_W^2} - T_{R3} \right) \end{aligned} \tag{2.17}$$

with the gauge fields [1, 5]

$$W_{L,R\mu}^{\pm} = \frac{1}{\sqrt{2}}(A_{L,R\mu}^1 \mp iA_{L,R\mu}^2)$$

and generators rewritten in the following way:

$$T_L^{\pm} = \begin{cases} \frac{1}{2}(\sigma^1 \pm i\sigma^2) & \text{for left-handed fermions,} \\ 0 & \text{for right-handed fermions} \end{cases}$$

and

$$T_R^{\pm} = \begin{cases} 0 & \text{for left-handed fermions,} \\ \frac{1}{2}(\sigma^1 \pm i\sigma^2) & \text{for right-handed fermions.} \end{cases}$$

This covariant derivative is equal to the covariant derivative derived in [5] apart from the opposite sign in front of the Z_R coupling and the photon coupling. The sign in front of the photon coupling was chosen to agree with the sign in the Standard Model covariant derivative in [1]. The quantum numbers of the quark fields are displayed in table (2.1).

	u_L	d_L	u_R	d_R
T_{L3}	1/2	-1/2	0	0
T_{R3}	0	0	1/2	-1/2
Q	2/3	-1/3	2/3	-1/3
Y	1/6	1/6	1/6	1/6

Tab. 2.1: Quantum numbers for left-and right-handed quark fields with $u \in \{u, c, t\}$ and $d \in \{d, s, b\}$.

2.2.4 Constraints on the MLRSM

Since the new particles lie in mass ranges which cannot be explored directly, constraints must be derived from indirect measurement. The heavy gauge and Higgs bosons contribute to loop diagrams within meson mass-mixing and meson decay. The authors of [43] carry out a whole analysis for an explicit and spontaneous CP-violating LRSM and derive constraints for the masses of the right-handed gauge boson W_R and some of the Higgs bosons. They also constrain the spontaneous CP-violating phase α.
The lowest boundary for the mass of W_R (2,5 TeV) is obtained from the $K_L - K_S$ mass-difference. The dominant new contribution is stated to be the amplitude with two intermediate c-quarks and one W_L- and one W_R-boson in the loop. Other boundaries come from B-$\bar{B}$-mixing and the calculation of CP-violation. These approximations generally give higher boundaries. Therefore the authors state that the lower boundary for M_{W_R} should lie around 4 TeV.
Two of the neutral Higgs scalars (H_1^0,A_1^0) cause flavour-changing currents (FCNH) and therefore contribute to the meson mass-mixing amplitudes. The two FCNH scalars are a linear combination of the neutral components of the Higgs bidoublet and their mass is given by [43]

$$m_{H_1^0,A_1^0} = \alpha_3 v_R^2$$

with the parameter from the Higgs potential α_3. From Kaon-mixing a lower mass boundary of 15 TeV is derived. The boundary increases to 25 TeV for $B_s - B_d$ mass-difference in order to give a low enough boundary to the right-handed W-boson. The single-charged Higgs H_2^+ is almost degenerate with the two FCNH since the electroweak breaking scale is much smaller than v_R ($m_{H_2} = \alpha_3(v_R^2 + \kappa^2/2) \approx \alpha_3 v_R^2$). Thus the same lower mass boundaries should be true for the single-charged Higgs boson whose properties are further discussed in Appendix B.
Another tool that can be used to constrain CP-violating quantities is the Neutron EDM (Electric Dipole Moment). The neutron EDM has not yet been measured, thus there is a lower boundary for its strength $d < 0,54 \times 10^{-23}$ ecm [8]. Assuming the neutron would have an electric dipole moment and considering it together with its measured magnetic moment, T- and P-symmetry and thus CP-symmetry would be violated. The upper boundary on the neutron EDM therefore gives an upper boundary to CP-violating interactions between the quarks within the neutron. The authors of [43] state that the main CP-violating contribution due to the LRSM is given by the four-quark operator $ud(s) \to ud(s)$ with an intermediate left-handed W-boson that couples to an intermediate right-handed W-boson. Since the coupling between these two bosons has been shown to be small (2.12) this contribution is also very small but still usable in order to constrain the right-handed W-boson mass.
A combination of the consideration of the CP-violating parameter ϵ (obtained from $K^0 - \bar{K}^0$-mixing) and the neutron EDM shows that small values of the phase α are preferred and the

authors of [43] fix the product $|rsin\alpha|$ to 0,05 with

$$r \equiv \frac{m_t}{m_b}\frac{\kappa'}{\kappa}$$

which transpires to be of the order 1.

2.2.5 Introducing a fourth family to the MLRSM

The new family is introduced in the same way as it was introduced to the Standard Model (chapter 2.1.2).

The most general Yukawa coupling in this model is given by [43]

$$L_Y = \bar{Q}_{Li}(h_{ij}\phi + \tilde{h}_{ij}\tilde{\phi})Q_{Rj} + h.c. \tag{2.18}$$

with $\tilde{\phi} = \tau_2\phi^*\tau_2$ and h, $\tilde{\text{h}}$ being 4x4 matrices.
The quark field indices now run from one to four and $u \in \{u, c, t, T\}$ and $d \in \{d, s, b, B\}$ where T is the new heavy top- and B the new heavy bottom-quark.
Transforming the Lagrangian under parity and requiring invariance shows that h and $\tilde{\text{h}}$ are hermitian. Inserting the vevs of the Higgs fields gives the following mass matrices for the quarks:

$$M_U = \kappa h + \kappa' e^{-i\alpha}\tilde{h}, \tag{2.19}$$

$$M_D = \kappa' e^{i\alpha}h + \kappa\tilde{h}. \tag{2.20}$$

If one assumes $M_U - M_D >> 0$ since the quark masses in one family are generally not of the same order it transpires that κ and κ' and h and $\tilde{\text{h}}$ should not be of the same order. The authors of [43] therefore assume $\kappa' << \kappa$ and h<< $\tilde{\text{h}}$. Hence the mass matrices reduce to

$$\begin{aligned} M_U &= \kappa h, \\ M_D &= \kappa' e^{i\alpha}h + \kappa\tilde{h}. \end{aligned}$$

Since the eigenvalues of a matrix are not changed through a similarity transformation, the flavour independence of the gauge couplings and the hermicity of h, it is possible to work in the basis where M_U is diagonal.

$$\bar{u}_L \to \bar{u}_L U, \qquad \bar{d}_L \to \bar{d}_L U, \qquad u_R \to U^+ u_R, \qquad d_R \to U^+ d_R \tag{2.21}$$

such that

$$UM_U U^+ = S_U \hat{M}_U$$

with $S_U = diag\{s_u, s_c, s_t, s_T\}$, where s_i is the sign of the quark masses and
$\hat{M}_U = diag\{m_u, m_c, m_t, m_T\}$.

Since h is diagonalised by the matrices U and U$^+$ but in general not $\tilde{h}$, M_D is in general not diagonal in this basis. On observing that in the basis used no rotation of u_L is required to achieve the physical mass eigenstates, the left-handed CKM matrix is given by

$$\bar{d}'_{Li} = \bar{d}_{Lj} V^{CKM+}_{Lji}$$

where d'_{Li} are the physical eigenstates.
The same holds true for the right-handed eigenstates and CKM matrix.

$$d'_{Ri} = S_U^{-1} V^{CKM}_{Rij} d_{Rj}$$

The singular value decomposition states that a random imaginary matrix can be transformed into diagonal form with the help of two unitary matrices. Therefore there are unitary matrices V_L^{CKM} and V_R^{CKM} such that

$$\hat{M}_D = V_L^{CKM+} M'_D S_U^{-1} V_R^{CKM} \tag{2.22}$$

is diagonal and has real non-negative entries which are interpreted to be the quark masses, i.e. $\hat{M}_D = diag\{m_d, m_s, m_b, m_B\}$. M'_D is the non-diagonal mass matrix in the new basis, i.e. $M'_D = UM_DU^+$.
Note that if $\alpha = 0$ the right- and left-handed CKM matrices are equal except for the quark mass signs.
Inserting equation (2.22) into (2.20) and using the hermicity of $\tilde{h}$ gives a relationship between the right- and the left-handed CKM matrices

$$\hat{M}_D \hat{V}_R^+ - \hat{V}_R \hat{M}_D = 2i\chi sin\alpha V_L^+ \hat{M}_U S_U V_L \tag{2.23}$$

with $V_R = S_U V_L \hat{V}_R$ and $\chi = \frac{\kappa'}{\kappa} \approx \frac{m_b}{m_t} \propto \lambda^3$.
Since $m_d << m_s << m_b << m_B$ all terms in a sum proportional to quarks of lower mass can be neglected. The left-hand side of (2.23) is therefore given by

$$K = \begin{pmatrix} -2im_d Im\hat{V}_{R11} & -m_s\hat{V}_{R12} & -m_b\hat{V}_{R13} & -m_B\hat{V}_{R14} \\ m_s\hat{V}^*_{R12} & -2im_s Im\hat{V}_{R22} & -m_b\hat{V}_{R23} & -m_B\hat{V}_{R24} \\ m_b\hat{V}^*_{R13} & m_b\hat{V}^*_{R23} & -2im_b Im\hat{V}_{R33} & -m_B\hat{V}_{R34} \\ m_B\hat{V}^*_{R14} & m_B\hat{V}^*_{R24} & m_B\hat{V}^*_{R34} & -2im_B Im\hat{V}_{R44} \end{pmatrix}.$$

According to chapter 2.1.3 the mass of the heavy B-quark is not allowed to be bigger than 600 GeV, so it is questionable whether the approximation above holds in this case. Therefore terms proportional to m_b should not be neglected when compared to terms proportional to m_B.
For the left-handed CKM matrix V_L the derived expression from chapter 1.4 can be used and contributions up to the order λ^4 should be included. Together with the right-hand side of (2.23) this gives 10 independent equations for the matrix elements of $\hat{V}_R$.

Using unitarity of $\hat{V}_R$ gives the missing elements [43]

$$
\begin{aligned}
\hat{V}_{Rii} &= S_{D_{ii}} e^{i\gamma_i} \\
\hat{V}_{R21} &= -s_d s_s \hat{V}^*_{R12} e^{i(\gamma_1+\gamma_2)} \\
\hat{V}_{R31} &= -s_d s_b \hat{V}^*_{R13} e^{i(\gamma_1+\gamma_3)} \\
\hat{V}_{R41} &= -s_d s_B \hat{V}^*_{R14} e^{i(\gamma_1+\gamma_4)} \\
\hat{V}_{R32} &= -s_s s_b \hat{V}^*_{R23} e^{i(\gamma_2+\gamma_3)} \\
\hat{V}_{R41} &= -s_s s_B \hat{V}^*_{R24} e^{i(\gamma_2+\gamma_4)} \\
\hat{V}_{R43} &= -s_b s_B \hat{V}^*_{R34} e^{i(\gamma_3+\gamma_4)}
\end{aligned}
$$

with the new parameters $sin\gamma_i = S_{D_{ii}} ImV_{Rii}$.
The right-handed CKM matrix can be obtained by using the relation $V_R = V_L S_U \hat{V}_R$.
According to [43] the right-handed CKM matrix in the three-family case has the same structure in λ as the left-handed CKM matrix enhanced with three more phases. All three phases are determined by the phase of the vev of the Higgs bidoublet α, so effectively V_L and V_R are distinguished from one another by only one phase.
Thus the right-handed CKM matrix which will be used in later analysis is given by

$$
V_R^{CKM} = \begin{pmatrix} s_u s_d e^{i\gamma_1}\left(1-\frac{\lambda^2}{2}\right) & s_u s_s e^{-i\gamma_2}\lambda & s_u s_b e^{-i\gamma_3} A\lambda^3(\rho - i\eta) \\ -s_c s_d e^{i(\gamma_1+2\gamma_2)}\lambda & s_c s_s e^{i\gamma_2}\left(1-\frac{\lambda^2}{2}\right) & s_c s_b e^{-i\gamma_3} A\lambda^2 \\ s_t s_d e^{i(\gamma_1+2\gamma_3)} A\lambda^3(1-\rho-i\eta) & -s_t s_s e^{i(\gamma_2+2\gamma_3)} A\lambda^2 & s_t s_b e^{i\gamma_3} \end{pmatrix} \tag{2.24}
$$

with the phases $\gamma_{1,2,3}$ as functions of $rsin\alpha$ and the mass signs

$$
\begin{aligned}
\gamma_1 &= -\, sin^{-1}[0,31(s_d s_c + 0,18 s_d s_t) rsin\alpha] \\
\gamma_2 &= -\, sin^{-1}[0,32(s_s s_c + 0,25 s_s s_t) rsin\alpha] \\
\gamma_3 &= -\, sin^{-1}[s_b s_t rsin\alpha].
\end{aligned} \tag{2.25}
$$

In the four-family case the situation would be far more complex.

2.3 The Little Higgs Model

A Little Higgs Model is a model extending the Standard Model describing the Higgs particle as a pseudo-Goldstone boson (PGB). PGBs are particles which are massless at tree-level but acquire mass via loop corrections. They can occur in theories in which the Higgs potential has a larger symmetry than the overall gauge symmetry of the complete Lagrangian [54]. Even though the theoretical background has been known for over 30 years [54] the embedding of the Higgs particle as a PGB in a theory extending the Standard Model was

not formulated until the last ten years [55]. Considering the Higgs as a PGB within the framework of the Standard Model would lead to constraints on the Higgs and top-quark mass (m_h <10 GeV and m_t <40 GeV) which are known to be incorrect [56]. The smallest extension of the Standard Model which makes use of the described method was named the Littlest Higgs Model [57] which is the Model described in this chapter.
The Little Higgs Model is built in such a way that it explains elements of unnaturalness which are contained in the Standard Model. One of these is the hierarchy problem (the fact that the one-loop contribution to the Higgs mass is quadratically dependent on the cutoff scale, which can be removed by renormalisation albeit only under the application of fine-tuning) which is solved by the introduction of an extra heavy Top-quark whose one-loop contributions to the Higgs mass cancel the one-loop contributions of the Standard Model top-quark. In [58] the cancellation of the contributions due to the t- and the T-quark is shown explicitly for the limit of a vanishing Standard Model Higgs vev (v=0). The contributing loops are shown in figure 2.2.

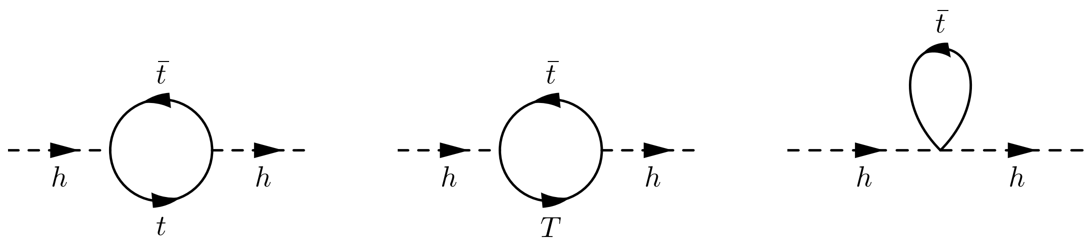


Fig. 2.2: Contributions to the one-loop correction of the Higgs mass square due to intermediate top- and Top-quarks in the limit of v=0. The graphic is taken from [58].

Adding the three contributions shows that the first two diagrams produce a contribution proportional to the square of the cutoff and breaking scale Λ^2 with the same sign and the third diagram a contribution proportional to Λ^2 with the opposite sign, which cancels the first two contributions. The remaining contribution to the Higgs mass square is proportional to the physical logarithm $log\Lambda^2/m_T^2$ with the Top-mass m_T. This topic will be discussed for the Standard Model extended by a vector-like quark in chapter 3.1.1.

2.3.1 Symmetry breaking

The gauge symmetry of the Littlest Higgs Model is described by the approximate global SU(5) gauge group, which is broken down by a vev f at a scale $\Lambda = 4\pi f$ to the gauge group SO(5), resulting in 14 Goldstone bosons which are grouped into a real singlet, a real triplet, a complex doublet and a complex triplet [59]. The first two fields serve as the longitudinal modes of the new gauge bosons, the complex Higgs doublet develops a non-vanishing vev and breaks the Lagrangian down to the electroweak Lagrangian and the complex triplet produces a physical neutral, a single- and a double-charged Higgs boson

similar to the complex triplet in the LRSM. In addition, there are two global subgroups $[SU(2) \times U(1)]^2 = [SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$ which break the global symmetry such that the neutral Higgs boson (which arises from the complex Higgs doublet) is only allowed mass terms at the two-loop level and bosons which arise from the complex Higgs triplet are allowed masses at one-loop level [57]. The subgroup $[SU(2) \times U(1)]^2$ is broken down at the same energy scale Λ to the electroweak gauge group $SU(2)_L \times U(1)_Y$.
Requiring a theory which solves the hierarchy problem without fine-tuning allows the breaking scale Λ to be at most 10 TeV, which equates to a vev of f≈1 TeV [60].

2.3.2 Particle content

The Littlest Higgs Model includes additional Higgs bosons, gauge bosons and one additional quark compared to the Standard Model particle content. The gauge group $[SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$ produces eight gauge bosons $W^1_{1,2,3}$, B^1, $W^2_{1,2,3}$ and B^2 which mix to give the physical mass eigenstates, which are the Standard Model gauge bosons $W^\pm_L$, Z_L, A_L and their heavy counterparts $W^\pm_H$, Z_H, A_H. The index L here denotes the light gauge boson and not the left-handed gauge boson as in the LRSM. The heavy gauge bosons acquire their masses from the real Higgs singlet and triplet, and are therefore of the order of f. According to [4] the following constraints can be derived:

$$W^\pm_H, Z_H > 1\,TeV \qquad \text{and} \qquad A_H > 500\,GeV.$$

An interesting property of the heavy Z- and W-gauge bosons is that if their coupling to the fermions is expanded in terms of their coupling to the light gauge bosons, the heavy Z- and W-boson couple to leading order only to left-handed fermions [59].
In order to cancel the quadratic divergences of the Higgs mass corrections (produced by the top-quark as discussed previously) one must introduce a heavy fermion which couples to the Higgs. The new particle (usually referred to as the Top-quark) is chosen to be introduced as a Weyl spinor with the following Yukawa coupling [59]:

$$L_Y = \frac{1}{2}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\chi_i\Sigma_{jx}\Sigma_{ky}u_3'^c + \lambda_2 f\tilde{t}\tilde{t}^c + h.c.$$

with $\chi_i = (b, t, \tilde{t})$ and the 5x5 matrices Σ which include the Higgs fields. One can read from this Lagrangian that the new heavy Top mixes with the Standard Model top but not with the lighter quarks. Coupling with the lighter quarks is not necessary, since the light quarks do not produce any quadratic divergences of the Higgs mass. Therefore the rotation of the quarks to their Standard Model eigenstates leaves only the t- and the T-quark in their non-physical form, which will also be the case when introducing the vector-like fermion to the Standard Model (chapter 3.1). Therefore it has the same impact on the CKM matrix elements. The CKM matrix alters from a 3x3 to a 4x3 matrix with the following elements [4]:

$$V^{LH}_{uj} = V^{SM}_{uj}, \quad V^{LH}_{cj} = V^{SM}_{cj}, \quad V^{LH}_{tj} = V^{SM}_{tj}(1 - \frac{x_L^2}{2}\frac{v^2}{f^2}), \quad V^{LH}_{Tj} = V^{SM}_{tj}x_L\frac{v}{f}$$

with $i \in \{d, s, b\}$, $x_L = \lambda_1^2/(\lambda_1^2 + \lambda_2^2)$ and the vev of the complex Higgs doublet v=246 GeV. If one would set $\theta = x_L \frac{v}{f}$ one would obtain the same parametrisation of the CKM matrix as that in chapter 3.1. The authors of [4] point out that the measured values of all CKM matrix elements not including t stay unaltered. Only the values measured for elements describing transitions with the top-quark would have to be reinterpreted.
There are several boundaries on the Top-mass between $m_T < 2$ TeV (in order to avoid fine-tuning for the cancellation of the Higgs mass contribution assuming that $m_h < 220$ GeV) and $m_T > 8 - 10$ TeV (due to electroweak precision measurements) [58].

The Higgs content consists of the unphysical longitudinal modes of the gauge bosons, the Standard Model neutral Higgs boson h, an additional neutral scalar Φ^0, a neutral pseudoscalar Φ^P and a charged and a double-charged Higgs boson Φ^+ and Φ^{++}.
The Feynman rules for the Littlest Higgs Model have been worked out in [59] and [61]. It is observable that the W_L-coupling to fermions equals the Standard Model coupling only to leading order but receives additional contributions at the order v^2/f^2. A similar situation would have occurred if W_1 were used in the LRSM instead of the left-handed W-boson which would have led to a higher order correction of the coupling, such that there would be a very small coupling to right-handed quarks. The coupling of the Z-boson is also altered such that it produces flavour-changing currents at the order of v/f.

2.3.3 Constraints on the Littlest Higgs Model

The parameters of the LHM can be constrained by analysing meson physics in the framework of the new model. Contributions to meson mass-mixing and the CP-violating parameter ϵ_K have been discussed in [4] and B-meson as well as Kaon decay in [61]. In both papers the contributions have been derived in unitary gauge (not in the commonly used Feynman-t'Hooft gauge) in order to avoid the calculation of a large number of diagrams due to the unphysical charged Higgs bosons describing the longitudinal modes of the gauge bosons. The QCD corrections have been chosen to be equal to those of the Standard Model, meaning that the QCD corrections relating to the Top-quark were taken to be equal to the corrections of the top-quark. This choice was justified by the fact that the QCD corrections are the smallest of many uncertainties. The same assumptions have been made in the calculations in this thesis.
The discussion of the box diagram contribution to meson mass-mixing is analogous to the discussion of the LRSM extended by a vector-like quark (chapter 3.2), which also points out that the inclusion of an extra heavy Top results in the need to include more diagrams, since the mass of the Top cancels the suppression factor v/f. The number of discussed parameters can be reduced to the masses of the new particles, the expansion factor v/f, the mixing angle between the light and the heavy gauge bosons s and the parameter x_L. The main contribution in addition to those existing in the Standard Model is the box with one intermediate

light gauge boson W_L and one intermediate heavy gauge boson W_H. The boxes including two heavy gauge bosons or the charged Higgs boson can be neglected, particularly considering that the coupling of the charged Higgs boson to the quarks is suppressed with v/f, contrary to the single-charged Higgs-quarks coupling in the LRSM (B.10). It transpires that the new contributions to $\Delta M_{d,s}$ and ϵ_K can enhance their previous (SM) values by at most 20% for $x_L \leq 0,8$ and by 56%/15% for $x_L \approx 1$ and f/v=5/10. The contribution to the mass-mixing of the Kaon can be neglected.

The decay rates of the meson have been derived to be enhanceble by at most 15%. In the calculations of the contributions mediated by the longitudinal mode of the heavy gauge boson and the single-charged Higgs, divergences appear which do not cancel out in the final result. The divergences are stated to be cancellable by charge renormalisation.

The Appelquist-Carazzone theorem (chapter 1.8) does not generally apply to the Little Higgs Model, since it is not renormalisable [62, 63]. Consulting the Feynman rules and one-loop calculations in [4, 61], the coupling to the new heavy particles is still suppressed by powers of the breaking scale f. The non-decoupling term (which was assigned to the charge renormalisation) is also in accordance with the decoupling theorem.

3. EXTENSIONS BY A VECTOR-LIKE QUARK ISOSINGLET

This chapter deals with the method of adding a massive vector-like up-type quark singlet to the Standard Model and to the LRSM. After describing how the new particle acquires mass and mixes with the Standard Model quarks, the B_d-mass-mixing and the CP-violating parameter ϵ_K will be calculated in the two extended models and in the LRSM. The experimentally-observed values of the mass-mixing and ϵ_K will then be used to constrain the new parameters.

3.1 Extending the Standard Model by a quark isosinglet

In chapter 2.1 it has been shown how a fourth generation of quarks can be added to the Standard Model. Since those two extra quarks acquire their mass through the same vev of a Higgs field as the other three-generation quarks, their masses must lie within a very narrow range (2.1). In order to create heavier quarks one must introduce additional Higgs fields whose vevs contribute to the masses of the new particles.
In order to keep the extension minimal, only a singlet up-type quark (a heavy Top) is added to the Standard Model. The quark is introduced as a vector-like isosinglet, where 'vector-like' means that the left- and right-handed projections of the fermion transform in the same way under the underlying gauge symmetry. The new Higgs field which gives a natural mass term to the new quark in the simplest way is a Higgs singlet φ which acquires a vev at an arbitrary energy scale w.
The most general Yukawa coupling which couples the quark fields of the Standard Model, the new quark field and the new Higgs field is given by

$$L_Y = -\bar{Q}_{Li}\lambda_{ij}\Phi d_{Rj} - \epsilon^{ab}\bar{Q}_{Lai}\tilde{\lambda}_{ik}\Phi_b^+ u_{Rk} - \bar{u}_{L4}\varphi\Gamma_k u_{Rk} \tag{3.1}$$

with $i = 1,2,3$, $j = 1,2,3$, $k = 1,2,3,4$ and the arbitrary complex numbers Γ_i. Thus λ describes a 3x3 matrix, $\tilde{\lambda}$ a 3x4 matrix and Γ a 1x4 matrix.
The Lagrangian can be rewritten in a part describing the Standard Model and a part describing the extension.

$$\begin{aligned} L_Y &= L_{YSM} + L_{Yextend} \\ &= -\bar{Q}_{Li}\lambda_{ij}\Phi d_{Rj} - \epsilon^{ab}\bar{Q}_{Lai}\tilde{\lambda}_{ij}\Phi_b^+ u_{Rj} - \epsilon^{ab}\bar{Q}_{Lai}\tilde{\lambda}_{i4}\Phi_b^+ u_{R4} - \bar{u}_{L4}\varphi\Gamma_k u_{Rk} \end{aligned}$$

Inserting the Standard Model vev of Φ (1.21) and $\langle\varphi\rangle = w$ (where w is generally complex, since the freedom of the gauge symmetry is already used to negate the phase of the vev of the Standard Model Higgs doublet) results in

$$L_Y = -\frac{v}{\sqrt{2}}\bar{d}_{Li}\lambda_{ij}d_{Rj} - \frac{v}{\sqrt{2}}\bar{u}_{Li}\tilde{\lambda}_{ij}u_{Rj} - \frac{v}{\sqrt{2}}\bar{u}_{Li}\tilde{\lambda}_{i4}u_{R4} - \bar{u}_{L4}w\Gamma_k u_{Rk}.$$

Introducing one Higgs scalar to the Standard Model is known as the truly minimal extension of the Standard Model and is the minimal extension which provides spontaneous CP-violation due to the complex phase of the vev of the scalar. However, the spontaneous CP-violating phase is not considered here for reasons discussed in the next chapter.

The last term gives exactly the missing column for the 4x4 mass matrix for the four quarks with charge 2/3

$$L_Y = -\frac{v}{\sqrt{2}}\bar{d}_{Li}\lambda_{ij}d_{Rj} - \bar{u}_{Lk}\tilde{\Lambda}_{kl}u_{Rl}$$

with

$$\tilde{\Lambda}_{kl} = \begin{pmatrix} & & & \frac{v}{\sqrt{2}}\tilde{\lambda}_{14} \\ & \frac{v}{\sqrt{2}}\tilde{\lambda}_{ij} & & \frac{v}{\sqrt{2}}\tilde{\lambda}_{24} \\ & & & \frac{v}{\sqrt{2}}\tilde{\lambda}_{34} \\ w\Gamma_1 & w\Gamma_2 & w\Gamma_3 & w\Gamma_4 + \frac{v}{\sqrt{2}}\tilde{\lambda}_{44} \end{pmatrix}.$$

Defining the matrices U_d and U_u which diagonalise λ and $\tilde{\Lambda}$ and inserting them into the flavour-changing current gives

$$J_W^{\mu+} = \frac{1}{\sqrt{2}}\bar{u}_{Li}\gamma^\mu d_{Li} = \frac{1}{\sqrt{2}}\bar{u}'_{Lk}\gamma^\mu V_{kj}d_{Lj} \qquad \text{with} \qquad V_{kj} = U^+_{uki}U_{dij}.$$

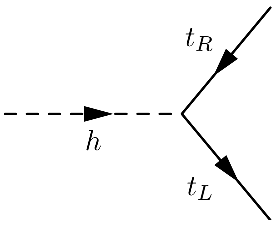
Thus the CKM matrix is a 4x3 matrix and has to be parameterised according to [5] by three CP-violating phases. One can therefore take the parameterisation of the CKM matrix in the four-generation case (2.2) without the fourth row.

$$V = \begin{pmatrix} c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} \\ \begin{array}{c} -c_{23}c_{24}s_{12} - c_{12}c_{24}s_{13}s_{23}e^{i\lambda_{13}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} & \begin{array}{c} c_{12}c_{23}c_{24} - c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} & \begin{array}{c} c_{13}c_{24}c_{23} \\ -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} \end{array} \\ \begin{array}{c} -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} + c_{34}s_{12}s_{23} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} -c_{12}c_{34}s_{23} - c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} \\ -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} \\ +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} c_{13}c_{23}c_{34} \\ -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} \end{array} \\ \begin{array}{c} -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} - s_{12}s_{23}s_{34} \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} + c_{12}s_{23}s_{34} \\ -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} & \begin{array}{c} -c_{13}c_{23}s_{34} \\ -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} \\ -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} \end{array} \end{pmatrix}$$

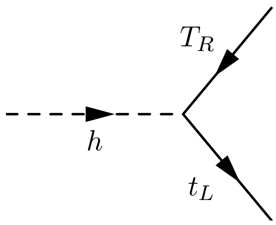
The extension of the Standard Model with a quark isosinglet has been extensively discussed in the papers by M.I. Vysotsky [64] and Picek and Radovicic [21].

3.1.1 Contributions to the Higgs mass square

The hierarchy problem of the Standard Model has been discussed within the chapter on the Little Higgs Model (chapter 2.3). In that specific case the hierarchy problem is solved by introducing a vector-like up-type quark. The cancellation (in the limit of a vanishing Higgs vev v=0) then takes place between the two contributions which have two intermediate t- or one t- and one T-quark and the contribution which has only one intermediate T-quark (figure 2.2) caused by the hhTT-vertex. Inserting the Standard Model Higgs doublet into Lagrangian (3.2) reveals that such a quartic coupling cannot be produced by a Yukawa coupling of that type, since the Higgs doublet is not multiplied by another Higgs field including the physical Higgs scalar. Thus the one-loop contribution to the Higgs mass square (in the limit of a vanishing vev) due to the top- and the Top-quark reduces to the first two diagrams in figure (2.2) with the following vertices:



$$= i\frac{\tilde{\lambda}_{tt}}{\sqrt{2}}$$



$$= i\frac{\tilde{\lambda}_{tT}}{\sqrt{2}}$$

Following [58] the one-loop contribution to the Higgs mass square is given by

$$\begin{aligned}\delta m_h^2 &= -6\tilde{\lambda}_{tt}^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2} - 6\tilde{\lambda}_{tT}^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_T^2} \\ &= -\frac{3}{8\pi^2}\left[\Lambda^2\left(\tilde{\lambda}_{tt}^2 + \tilde{\lambda}_{tT}^2\right) + \tilde{\lambda}_{tT}^2 m_T^2 ln\left(\frac{\Lambda^2 + m_T^2}{m_T^2}\right)\right]\end{aligned}$$

which shows that the hierarchy problem still exists in this extension of the Standard Model. A cancellation of the quadratic dependence on the cutoff scale Λ is generally possible since the factors $\tilde{\lambda}_{tt}$ and $\tilde{\lambda}_{tT}$ are undetermined and allowed to be complex. However, the requirement $\tilde{\lambda}_{tt} = \pm i\tilde{\lambda}_{tT}$ would cause a fine-tuning problem.

In order to solve the hierarchy problem in this model one could embed the theory in a larger symmetry group such as the Little Higgs Model (chapter 2.3) and could thus possibly justify a Yukawa coupling which multiplies Higgs fields, as in the Lagrangian (3.1) .

An analogous analysis with the same result is true for the following chapter, in which the vector-like Top-quark is introduced to the LRSM.

3.2 Extending the Left-Right Symmetric Model by a quark isosinglet

Introducing a vector-like fermion singlet to the LRSM Lagrangian does not result in any mass terms for the new fermion, since the Higgs bidoublet does not couple to singlets.

The problem can be solved by introducing a Higgs scalar which acquires a vev at an arbitrary scale analogous to the introduction of a vector-like fermion to the Standard Model.

$$L_Y = \bar{Q}_{Li}(h_{ij}\Phi + \tilde{h_{ij}}\tilde{\Phi})Q_{Rj} + \bar{u}_{L4}\varphi\Gamma u_{R4} + h.c. \tag{3.2}$$

Inserting the vev of the Higgs scalar shows that u_{L4} is already the mass eigenstate. Thus the heavy top-quark would not mix with the Standard Model quarks and would therefore not take part in flavour-changing processes such as meson-mixing. The detection of such a particle would therefore be difficult.

Another Ansatz is the introduction of a new Higgs doublet χ. Hence the original fermion doublets would couple to the new singlets.

$$L_Y = \bar{Q}_{Li}(h_{ij}\Phi + \tilde{h_{ij}}\tilde{\Phi})Q_{Rj} + \epsilon^{ab}\bar{Q}_{Lai}\Gamma_i\chi_b^+ u_{R4} + h.c.$$

The magnitude of the mass-mixing between the new heavy Top and the Standard Model quarks is determined by the Higgs vacuum expectation value of the Higgs doublet which is chosen to have the form

$$\langle\chi\rangle = \begin{pmatrix} 0 \\ w'e^{i\beta'} \end{pmatrix}$$

where β' is generally non-zero since all the free phases of the underlying gauge symmetry have already been used to cancel the imaginary parts of one component of the Higgs bidoublet Φ and Higgs triplet Δ_L. In order to avoid additional spontaneous CP-violation the angle β' is taken to be zero in the following calculation. The phenomenology of an extra CP-violating phase might be interesting when added to the Standard Model but would increase the number of parameters when added to the LRSM such that the predictability of the model would be compromised. One reason for this is that a spontaneous CP-violating phase (α) already exists in the LRSM.

Another choice of $\langle\chi\rangle$ such as assigning the first component a vev would yield unphysical (charge non-conserving) vertices. Further, $\langle\chi\rangle$ is assumed to break the symmetry at a scale much higher than the electroweak scale ($\omega' >> \kappa$).
Inserting the vev in the Lagrangian gives the following:

$$L_Y = L_{YLRSM} + w'\bar{u}_{Li}\Gamma_i u_{R4} + h.c. \qquad \text{with} \qquad L_{YLRSM} = \bar{u}_{Li}M_{Uij}u_{Ri} + \bar{d}_{Li}M_{Dij}d_{Ri}$$

where Γ_i are any complex numbers which are also assumed to be real.
Assuming the first three-generation quark fields are presented as LRSM mass eigenstates,

the down-type quark matrix M_D would already be diagonal and the new mass matrix for the up-type quarks would have the following form:

$$\tilde{M}_U = \begin{pmatrix} \kappa h_{uu} & 0 & 0 & \omega'\Gamma_1 \\ 0 & \kappa h_{cc} & 0 & \omega'\Gamma_2 \\ 0 & 0 & \kappa h_{tt} & \omega'\Gamma_3 \\ \omega'\Gamma_1 & \omega'\Gamma_2 & \omega'\Gamma_3 & 0 \end{pmatrix}$$

where κ is the upper component of the vacuum expectation value of the Higgs bidoublet Φ whose magnitude lies around the electroweak scale.

Assuming (following [21]) that the new heavy Top-quark only couples to the Standard Model top-quark and not to the u- and c-quark (justified by the hierarchy of the CKM matrix), Γ_1 and Γ_2 have to be zero. Hence the up-type quark mass matrix reduces to

$$\tilde{M}_U = \begin{pmatrix} \kappa h_{uu} & 0 & 0 & 0 \\ 0 & \kappa h_{cc} & 0 & 0 \\ 0 & 0 & \kappa h_{tt} & \Omega' \\ 0 & 0 & \Omega' & 0 \end{pmatrix} \qquad \text{with} \qquad \Omega' = \omega'\Gamma_3.$$

Identifying the eigenvalues of $\tilde{M}_U$ results in the masses of the up-type quarks

$$m_u = h_{uu}\kappa, \qquad m_c = h_{cc}\kappa, \qquad m_{t,T} = \frac{h_{tt}\kappa}{2} \mp \frac{1}{2}\sqrt{h_{tt}^2\kappa^2 + 4\Omega'^2}.$$

Hence the parameters h_{uu} and h_{cc} have the same values in this model as they do in the LRSM. They translate to the parameters given in the Standard Model over $h_{ii} = \frac{1}{\sqrt{2}}\lambda_i$ with $i = u, c$. As in the Standard Model, these parameters are free and have to be constrained by experiment. Since the parameters h_{tt} and Γ_3 can be of completely different orders, the sum in the square root determining the masses of the t- and the T-quark cannot be approximated without further assumptions. It transpires that any assumption concerning these parameters would lead to an unphysical situation with negative top-mass since $\sqrt{h_{tt}^2\kappa^2 + 4\Omega'^2} > h_{tt}\kappa$ for $\Gamma_3 > 0$.

Introducing both a Higgs doublet and a Higgs singlet introduces a new parameter which prevents the problem of a negative top-mass.

The new Lagrangian writes

$$L_Y = \bar{Q}_{Li}(h_{ij}\Phi + \tilde{h_{ij}}\tilde{\Phi})Q_{Rj} + \epsilon^{ab}\bar{Q}_{La3}\Gamma_3\chi_b^+ u_{R4} + \bar{u}_{L4}\varphi\Pi u_{R4} + h.c.$$

with the free parameter Π which is here taken to be real.

Choosing the vevs of the new scalar field as $\langle\varphi\rangle = \omega e^{i\beta}$ (with $\beta = 0$ and $\omega >> \kappa$) and inserting the vevs of the previously-discussed doublet, the LRSM Higgs bidoublet results in

$$L_Y = L_{YLRSM} + \Omega'\bar{u}_{L3}u_{R4} + \Omega\bar{u}_{L4}u_{R4} + h.c. \qquad \text{with} \qquad L_{YLRSM} = \bar{u}_{Li}M_{Uij}u_{Ri} + \bar{d}_{Li}M_{Dij}d_{Ri}$$

where $\Omega = \Pi\omega$.
Rewriting the matrices gives

$$L_Y = \bar{u}_{Lk}\tilde{M}_{Ukl}u_{Rl} + \bar{d}_{Li}M_{Dij}d_{Ri} + h.c.$$

with

$$\tilde{M}_U = \begin{pmatrix} \kappa h_{uu} & 0 & 0 & 0 \\ 0 & \kappa h_{cc} & 0 & 0 \\ 0 & 0 & \kappa h_{tt} & \Omega' \\ 0 & 0 & \Omega' & \Omega \end{pmatrix}.$$

The eigenvalues are

$$m_u = h_{uu}\kappa, \qquad m_c = h_{cc}\kappa, \qquad m_{t,T} = \frac{\Omega + h_{tt}\kappa}{2} \mp \frac{1}{2}\sqrt{(\Omega - h_{tt}\kappa)^2 + 4\Omega'^2}.$$

Requiring only positive top-masses produces the following relation between the parameters:

$$\Omega'^2 < \Omega h_{tt}\kappa$$

The physical mass fields result from the rotation $\bar{u}_L \to \bar{u}_L\tilde{U}^+$ and $u_R \to \tilde{U}u_R$ with the unitary matrix

$$\tilde{U} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\Omega'}{\sqrt{\Omega'^2+(m_t-h_{tt}\kappa)^2}} & \frac{\Omega'}{\sqrt{\Omega'^2+(m_T-h_{tt}\kappa)^2}} \\ 0 & 0 & \frac{m_t-h_{tt}\kappa}{\sqrt{\Omega'^2+(m_t-h_{tt}\kappa)^2}} & \frac{m_T-h_{tt}\kappa}{\sqrt{\Omega'^2+(m_T-h_{tt}\kappa)^2}} \end{pmatrix}.$$

The flavour-changing currents (B.1) can now be rewritten in two steps. The flavour eigenstates are first rotated to their LRSM mass eigenstates by the left- and right-handed CKM matrices V_L^{SM} (chapter 1.4) and V_R^{LRSM} (2.24). Following this rotation, the top-quark field is no longer the mass eigenstate of the top but a mixture between the top- and the new heavy Top-quark, as seen above. Hence the up-type quarks have to be rotated again which is formally done by the matrix $\tilde{U}$. Therefore the new Top-quark enters the flavour-changing currents in the following way:

$$J^{\mu+}_{W_L} = \frac{1}{\sqrt{2}}\bar{u}_{Li}\gamma^\mu d_{Li} = \frac{1}{\sqrt{2}}\bar{u'}_{Li}\gamma^\mu V^{CKM}_{Lij}d'_{Lj} \qquad \text{with} \qquad V^{CKM}_{Lij} = (\tilde{U}^+ V_L^{SM})_{ij} \tag{3.3}$$

$$J^{\mu+}_{W_R} = \frac{1}{\sqrt{2}}\bar{u}_{Ri}\gamma^\mu d_{Ri} = \frac{1}{\sqrt{2}}\bar{u'}_{Ri}\gamma^\mu V^{CKM}_{Rij}d'_{Rj} \qquad \text{with} \qquad V^{CKM}_{Rij} = (\tilde{U}^+ \tilde{V}_R^{LRSM})_{ij} \tag{3.4}$$

It transpires that the first two rows of the new 4x3 matrices V_L and V_R are the same as in the LRSM. The third and fourth row are given by

$$V_{Lti} = V^{SM}_{Lti}x \qquad \text{and} \qquad V_{LTi} = V^{SM}_{Lti}y \tag{3.5}$$

$$V_{Rti} = V^{LRSM}_{Rti}x \qquad \text{and} \qquad V_{RTi} = V^{LRSM}_{Rti}y$$

with

$$x = \frac{\Omega'}{\sqrt{\Omega'^2 + (m_t - h_{tt}\kappa)^2}} \qquad \text{and} \qquad y = \frac{\Omega'}{\sqrt{\Omega'^2 + (m_T - h_{tt}\kappa)^2}}$$

and $i = d, s, b$.

The GIM relation derived in the Standard Model (1.32) alters to

$$\lambda_u + \lambda_c + \lambda_t + \lambda_T = 0.$$

Inserting the new CKM matrix elements (3.5) and requiring the GIM relation to be true, gives the following constraint on the new parameters:

$$x^2 + y^2 = 1 \tag{3.6}$$

Hence the following parameterisation can be chosen:

$$x = cos\theta \qquad \text{and} \qquad y = sin\theta.$$

In paper [21] the same parameterisation has been used but with completely free parameters x and y. In this thesis the explicit dependence on the Higgs parameters is discussed.
The condition (3.6) puts the following constraint on the Higgs parameter:

$$\Omega'^2 = (m_t - h_{tt}\kappa)(m_T - h_{tt}\kappa)$$

The values of m_t and m_T could generally be taken from observation.
The new CKM matrix is not quite unitary and the unitary relations (1.30, 1.31) of the Standard Model alter to

$$\begin{aligned}
|V_{ui}|^2 + |V_{ci}|^2 + |V_{ti}|^2 + |V_{Ti}|^2 &= 1 && \text{for} \quad i \in \{d, s, b\}, \\
|V_{id}|^2 + |V_{is}|^2 + |V_{ib}|^2 &= 1 && \text{for} \quad i \in \{u, c\}, \\
|V_{id}|^2 + |V_{is}|^2 + |V_{ib}|^2 &= cos^2\theta && \text{for} \quad i = t, \\
|V_{id}|^2 + |V_{is}|^2 + |V_{ib}|^2 &= sin^2\theta && \text{for} \quad i = T
\end{aligned}$$

and

$$\begin{aligned}
V_{ui}^* V_{uj} + V_{ci}^* V_{cj} + V_{ti}^* V_{tj} + V_{Ti}^* V_{Tj} &= 0 && \text{for} \quad i, j \in \{d, s, b\} && \text{with} \quad i \neq j, \\
V_{id} V_{jd}^* + V_{is} V_{js}^* + V_{ib} V_{jb}^* &= 0 && \text{for} \quad i, j \in \{u, c, t, T\} && \text{with} \quad i \neq j \\
& && && \text{and} \quad i, j \notin \{t, T\}, \\
V_{id} V_{jd}^* + V_{is} V_{js}^* + V_{ib} V_{jb}^* &= cos\theta sin\theta && \text{for} \quad i, j \in \{t, T\} && \text{with} \quad i \neq j.
\end{aligned}$$

3.2.1 B_d-$\bar{B}_d$-mixing in the LRSM extended by a Top-isosinglet

Prior to beginning the analysis of the B-meson mass-mixing, the applicability of constraints on masses obtained in the framework of the LRSM is discussed. The lower boundary on the W_R-mass ($m_{W_R} > 2,5$ TeV) (which has been obtained from K^0-$\bar{K}^0$-mixing (chapter 2.2.4) [43]) is also valid in the LRSM extended by an extra heavy Top-quark, since the main contribution to Kaon mass-mixing is the box diagram containing two intermediate c-quarks. The new Top-quark does not mix with the c-quark and thus the new box diagram produced by the Top has little impact on the lower mass boundary of the right-handed W-boson.
The lower boundary on the single-charged Higgs boson H_2^+ is also taken into account ($m_{H_2} > 15$ TeV).

The main contributions to B_d-$\bar{B}_d$-mixing are the Standard Model contribution with two left-handed W-bosons and the contribution with a left- and a right-handed W-boson as intermediate particles as discussed in Appendix C.
Thus the mass-mixing is given by

$$\begin{aligned}\Delta m_{B_d} = \frac{G_F^2 M_{W_L}^2}{2\pi^2} \sum_{i,j=u,c,t,T} \Bigg| & 2V^*_{Liq_2} V_{Riq_1} V^*_{Riq_2} V_{Liq_1} \eta_{LRij} \sqrt{x_i x_j} \left[\left(\frac{x_i x_j}{x_{W_R}} + 4 \right) F_0(x_i, x_j, x_{W_R}, 1) \right. \\ & \left. - \left(1 + \frac{1}{x_{W_R}} \right) F_2(x_i, x_j, x_{W_R}, 1) \right] \langle B_d | O_{dbLR} | \bar{B}_d \rangle \\ & + C_0(x_i, x_j) \langle B_d | O_{dbLL} | \bar{B}_d \rangle \Bigg| \\ = \frac{G_F^2 M_{W_L}^2}{2\pi^2} \sum_{i,j=u,c,t,T} & \left| M_{ij}^{LR} + M_{ij}^{LL} \right| \end{aligned}$$

with

$$C_0(x_i, x_j) = \eta_{ij} V^*_{Lib} V_{Ljd} c_0'(x_i, x_j; 1) \qquad \text{and} \qquad \langle B_d | O_{dbLL} | \bar{B}_d \rangle = -\frac{1}{3} B_4^d(\mu) f_{B_d}^2 m_{B_d}$$

where the hadron matrix element is taken from [5].
The box diagram mediated by a left-handed W-boson and the single-charged Higgs boson H_2^+ would add a contribution proportional to

$$x_i x_j F_0(x_i, x_j, x_{H_2}, 1) - F_2(x_i, x_j, x_{H_2}, 1)$$

to the mass-mixing. This contribution would dominate for small scalar masses (since there is no suppression of $x_{W_R}^{-1}$) but can be neglected for masses larger than 15 TeV.
The contribution of the flavour-changing neutral Higgs scalars H_1^0 and A_1^0 which can already produce meson mass-mixing at tree-level is also not considered since the new isosinglet does not alter the value of this contribution.
In the case without the extra heavy Top the dominant contribution in the Standard Model and in the LRSM is given by the box diagram including two top-quarks. All other diagrams

are negligible. Since the new Top only couples to the Standard Model top the other contributions do not alter and remain negligible. Therefore the mass-mixing reduces to

$$\Delta m_{B_d} = \frac{G_F^2 M_{W_L}^2}{2\pi^2} \sum_{i,j=t,T} \left| M_{ij}^{LR} + M_{ij}^{LL} \right| .$$

Since it is known that the Standard Model contribution can explain the measured value of Δm_{B_d} new physics only finds space in this amplitude either if cancellations between the contributions take place or if the new contributions are used to constrain the CKM matrix elements differently than in the Standard Model.

In the following the outcome of the box diagram is discussed for various values of the mass of new heavy Top m_T and the mass of the right-handed W-boson m_{W_R}. The mass signs (s_c,s_t,s_d and s_b) which enter the mass-mixing over the right-handed CKM matrix are initially set to 1. Since the mass signs always appear in pairs, one of them can be fixed. Here s_c is chosen to be positive. It will be explicitly stated if any other choice of sign of s_t,s_d ands_b would lead to a phenomenologically different result. The choices of quark mass signs are grouped into the following regions:

s_t	s_d	s_b	Region
+	+	+	I
-	+	+	IV
+	-	+	III
+	+	-	III
-	-	+	II
-	+	-	II
-	-	-	IV

Tab. 3.1: Regions grouping the quark mass signs depending on their resulting mass-mixing value.

The first case to be briefly discussed $\theta = 0$ belongs to the pure LRSM case without any extra quark. The B_d-meson mass-difference in this case writes

$$\begin{aligned}\Delta m_{B_d} &= \frac{G_F^2 M_{W_L}^2}{2\pi^2} \left| M_{tt}^{LL} + M_{tt}^{LR} \right| \\ &= \frac{G_F^2 M_{W_L}^2}{2\pi^2} \left| V_{Ltb}^* V_{Ltd} \right|^2 \left| s_d s_b e^{i(\gamma_2(\alpha)+\gamma_3(\alpha))} x_{LR}(x_t, x_{W_R}) + x_{LL}(x_t) \right|\end{aligned}$$

with

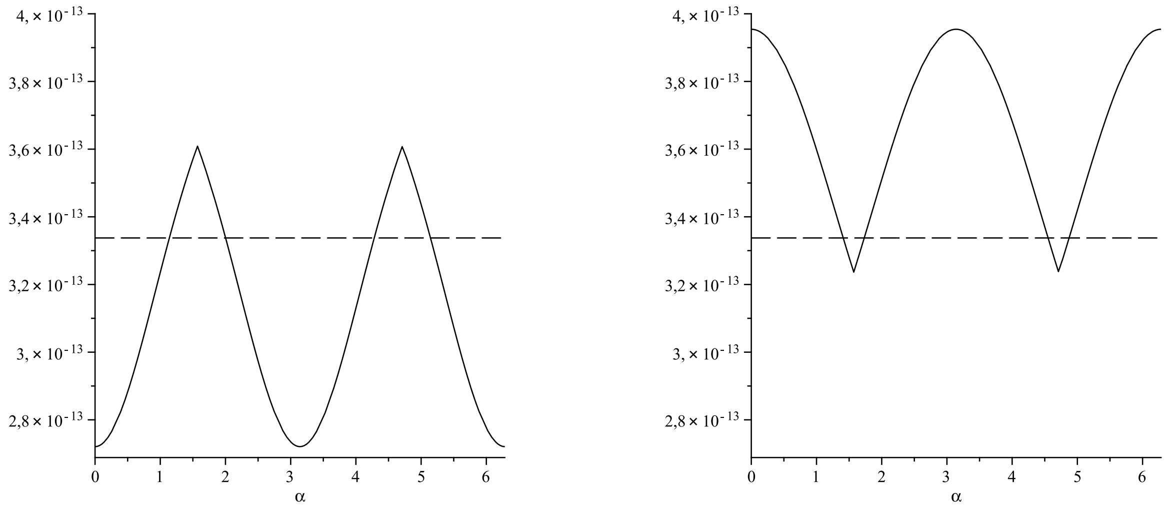
$$\begin{aligned}x_{LR}(x_i, x_j, x_{W_R}) =& 2\eta_{LRtt}\sqrt{x_i x_j} \left[\left(\frac{x_i x_j}{x_{W_R}} + 4 \right) F_0(x_i, x_j, x_{W_R}, 1) \right. \\ & \left. - \left(1 + \frac{1}{x_{W_R}} \right) F_2(x_i, x_j, x_{W_R}, 1) \right] \langle B_d | O_{dbLR} | \bar{B}_d \rangle \end{aligned} \tag{3.7}$$

and

$$x_{LL}(x_i, x_j) = \eta_{tt} c_0'(x_i, x_j; 1) \langle B_d | O_{dbLL} | \bar{B}_d \rangle . \tag{3.8}$$

Setting $\alpha = 0$ in addition describes the manifest LRSM. In this case the imaginary part of the amplitude is factored out of the sum of diagrams and therefore no cancellation between M^{LL} and M^{LR} arising from taking the norm takes place. There are cancellations since x_{LL} is negative and x_{LR} positive. For $m_{W_R} < 825 GeV$ the mass-mixing is dominated by x_{LR} but since such a small value for the mass of the right-handed W-boson has been excluded this case will not be discussed any further. For $m_{W_R} \approx 825$ GeV, M^{LR} cancels M^{LL} completely and approaches zero with raising m_{W_R}. If one allows the CKM matrix element $|V_{Ltd}|$ to alter by a maximum of 10 % from the value given by the Particle Data Group one obtains the constraint $m_{W_R} > 2,6$ TeV on the right-handed W-boson mass.
No quark mass sign choice would leave any space for the LRSM contribution. Giving the CKM matrix element $|V_{Ltd}|$ the freedom of ± 10% sets the following limit on the right-handed W-boson mass: $m_{W_R} > 2,5$ TeV (for region I and IV) and $m_{W_R} > 2,2$ TeV (for region II and III).
For $\alpha \neq 0$, solutions including the LRSM contribution without altering the CKM matrix elements are possible. The mass-mixing Δm_{B_d} oscillates with α where the difference between maximum and minimum becomes smaller with increasing m_{W_R}. The choices of the quark mass signs split into four regions (3.1). The resulting mass-mixing derived using a mass sign choice from region III/IV is larger/smaller than the experimentally observed value. Choices from region I or II lead to a mass-mixing value oscillating around the observed value (3.1). One can read from figure 3.1a that the predicted mass-mixing value derived using a quark mass sign choice of region I meets the experimentally observed value at $\alpha = 1,14(+\pi)$ or $\alpha = 2,01(+\pi)$. A quark mass sign choice of region II predicts the correct amount of mass-mixing for $\alpha = 1,41(+\pi)$ or $\alpha = 1,73(+\pi)$.
The value of θ is now taken to be non-zero and the new heavy Top-quark now contributes to the mass-difference between the B_d and the $\bar{B}_d$ meson. First the limit $m_{W_R} \longrightarrow \infty$ is considered where the LRSM contributions are zero and only the Standard Model case plus the new heavy Top-quark are considered. Thus the mass-difference writes

$$\begin{aligned}
\Delta m_{B_d} &= \frac{G_F^2 M_{W_L}^2}{2\pi^2} \left| M_{tt}^{LL} + M_{tT}^{LL} + M_{Tt}^{LL} + M_{TT}^{LL} \right| \\
&= \frac{G_F^2 M_{W_L}^2}{2\pi^2} \left| V_{Ltb}^* V_{Ltd} \right|^2 \eta_{tt} \left| cos^4(\theta) c_0(x_t) + 2cos^2(\theta) sin^2(\theta) c_0(x_t, x_T) \right. \\
&\qquad \left. + sin^4(\theta) c_0(x_T) \right| \langle B_d | O_{dbLL} | \bar{B}_d \rangle \\
&= \frac{G_F^2 M_{W_L}^2}{2\pi^2} \left| V_{Ltb}^* V_{Ltd} \right|^2 \eta_{tt} \left| c_0(x_t) + 2sin^2(\theta)(cos^2(\theta) c_0(x_t, x_T) - c_0(x_t)) \right. \\
&\qquad \left. + sin^4(\theta)(c_0(x_T) + c_0(x_t)) \right| \langle B_d | O_{dbLL} | \bar{B}_d \rangle .
\end{aligned} \tag{3.9}$$



(a) Region I

(b) Region II

Fig. 3.1: Oscillation of the mass-difference Δm_{B_d} for $m_{W_R} = 2,5$ TeV and mass signs chosen from region I and II. The dashed line represents the experimentally observed value.

All contributions in equation 3.9 are positive and the imaginary parts factor out, therefore no cancellations occur. Some constraints for the angle θ can be derived from the requirement of decoupling of the heavy Top for very large m_T (chapter 1.8). For very large heavy Top-masses, $c_0(x_T)$ is directly proportional to m_T whereas $c_0(x_t, x_T)$ is only proportional to $ln(m_T)$. Therefore if only large Top-masses are considered, the two contributions M_{tT}^{LL} and M_{Tt}^{LL} can be neglected and the product $sin^4\theta x_T$ must approach zero. This means $sin^4\theta < x_T^{-1}$ for very large masses m_T.

The first summand in equation (3.9) describes the measured mass-mixing, therefore the rest would need to be zero. This solution does not exist and therefore there is no space for an extra quark as long as the input parameters stay unchanged. If the CKM matrix element $|V_{Ltd}|$ is allowed to alter by a maximum of 10%, various combinations of θ and m_T are possible. Table (3.2) shows some possible values of the heavy Top-mass and the mixing angle if the CKM matrix element is allowed to deviate 10% from its value given by the Particle Data Group. All these values would be excluded when taking the constraint due to the T- and R_b- parameters (chapter 1.7) derived in [21] into account. The values obtained by allowing 5% deviation (3.3) would still be excluded by the T-parameter constraint but not by the R_b-parameter, which is the more reliable.

m_T	θ
1000	0,246
2000	0,190
3000	0,163
5000	0,134
10000	0,100

Tab. 3.2: Possible combinations of m_T and θ if the CKM matrix element $|V_{Ltd}|$ is allowed to deviate by 10%.

m_T	θ
1000	0,175
2000	0,139
3000	0,122
5000	0,103
10000	0,0791

Tab. 3.3: Possible combinations of m_T and θ if the CKM matrix element $|V_{Ltd}|$ is allowed to deviate by 5%.

The right-handed W-boson is now allowed to acquire a finite mass, meaning that the contributions due to the LRSM need to be taken into account. The mass-difference then writes

$$\begin{aligned}\Delta m_{B_d} &= \frac{G_F^2 M_{W_L}^2}{2\pi^2}\left|M_{tt}^{LL}+M_{tT}^{LL}+M_{Tt}^{LL}+M_{TT}^{LL}+M_{tt}^{LR}+M_{tT}^{LR}+M_{Tt}^{LR}+M_{TT}^{LR}\right| \\ &= \frac{G_F^2 M_{W_L}^2}{2\pi^2}\left|V_{Ltb}^* V_{Ltd}\right|^2 \Big| s_d s_b e^{i(\gamma_3(\alpha)+\gamma_2(\alpha))}\left(x_{LR}(x_t,x_{W_R}) + 2sin^2\theta\left(cos^2\theta x_{LR}(x_t,x_T,x_{W_R})\right.\right. \\ &\qquad \left.\left. -x_{LR}(x_t,x_{W_R})\right) + sin^4\theta(x_{LR}(x_T,x_{W_R}) + x_{LR}(x_t,x_{W_R}))\right) \\ &\qquad +x_{LL}(x_t) + 2sin^2\theta\left(cos^2\theta x_{LL}(x_t,x_T) - x_{LL}(x_t)\right) \\ &\qquad +sin^4\theta(x_{LL}(x_T)+x_{LL}(x_t))\Big| .\end{aligned}$$

For $\alpha = 0$ and $s_b s_d = 1$ the mass-mixing simplifies to

$$\begin{aligned}\Delta m_{B_d} = \frac{G_F^2 M_{W_L}^2}{2\pi^2}\left|V_{Ltb}^* V_{Ltd}\right|^2 \Big| & x_{LL}(x_t) + x_{LR}(x_t,x_{W_R}) + 2sin^2\theta\left(cos^2\theta\left(x_{LL}(x_t,x_T)\right.\right. \\ & \left.\left. +x_{LR}(x_t,x_T,x_{W_R})\right) - x_{LL}(x_t) - x_{LR}(x_t,x_{W_R})\right) \\ & +sin^4\theta\left(x_{LL}(x_T) + x_{LR}(x_T,x_{W_R}) + x_{LL}(x_t) + x_{LR}(x_t,x_{W_R})\right)\Big| .\end{aligned}$$

As in the case without the LRSM no solution for θ exists if the input parameters are not altered.

The functions used above have the following behaviour for very large heavy Top-masses and a fixed right-handed W-boson mass:

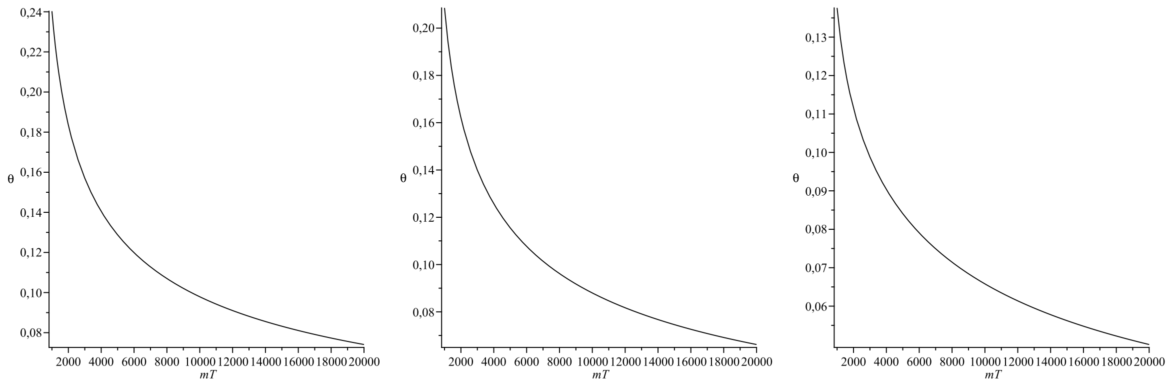
$$\begin{aligned} x_{LR}(x_t,x_T,x_{W_R}) &\propto \sqrt{x_T}, & x_{LL}(x_t,x_T) &\propto -ln(x_T) \\ x_{LR}(x_T,x_{W_R}) &\propto x_T ln(x_T), & x_{LL}(x_T) &\propto -x_T. \end{aligned}$$

Inserting these limits gives the following value for θ under the requirement of decoupling of large Top-masses:

$$sin^2\theta < \frac{1}{\sqrt{x_T ln(x_T)}} \tag{3.10}$$

In the following step first the right-handed W-boson mass and then the heavy Top-mass will be fixed in order to analyse whether there is space for both new particles to exist.

Keeping the right-handed W-boson mass fixed with $m_{W_R} > 1$ TeV leaves space for certain combinations of the heavy Top-mass and the mixing angle θ. It transpires that the smaller the mass of the right-handed W taken, the narrower the allowed mass range for the heavy Top. The mixing angles also increase with falling W-mass. For fixed W-mass there is always only one suitable Top-mass and mixing angle combination. Plotting the mixing angle dependent on the heavy Top-mass shows that the graph behaves in a parabola-like manner with the maximum angle for $m_T = 1$ TeV for $m_{W_R} > 2$ TeV. In figure 3.2 the suitable combinations for θ and m_T are plotted for the fixed W-masses 2,5 , 3 and 5 TeV.



(a) $m_{W_R} = 2{,}5$ TeV (b) $m_{W_R} = 3$ TeV (c) $m_{W_R} = 5$ TeV

Fig. 3.2: Mixing angle dependent on the heavy Top-mass for a fixed right-handed W-boson mass.

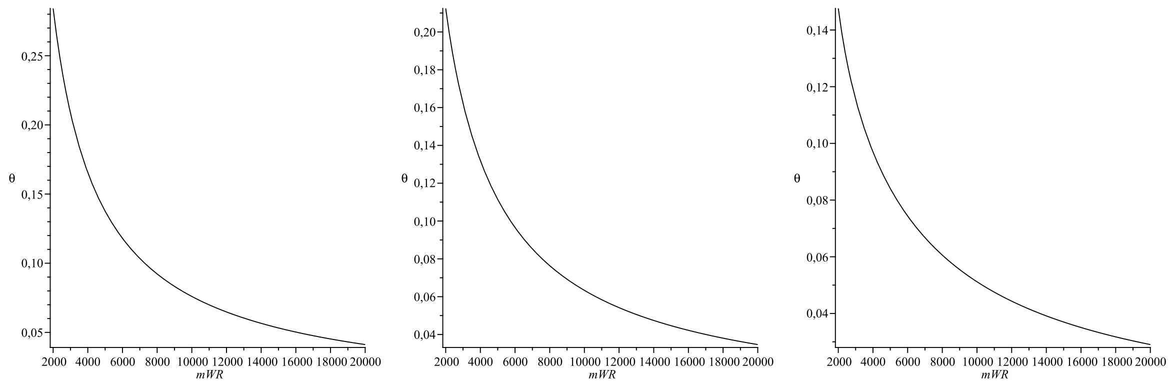
Assuming that both m_{W_R} and m_T are larger than 1 TeV, the allowed mass ranges of the heavy Top dependent on the fixed value of the boson are listed in table 3.4.

m_{W_R}/TeV	maximum of m_T/TeV
1	3,63
1,5	21,5
2,5	1500
3	2,92*10^4
5	2,41*10^9
10	$> \Lambda_{GUT}$

Tab. 3.4: Allowed mass range of the heavy Top-mass for various fixed right-handed W-boson masses.

This behaviour shows that the space for the extra heavy Top is provided by the LRSM contributions due to cancellations between the two. The smaller the LRSM contributions (which means the higher the W-mass), the smaller the contributions due to the heavy Top which is represented by the smaller mixing angle.
The correlation between the mixing angle and the right-handed W-mass for a fixed heavy Top-mass is shown in figure 3.3. There is no maximum limit for the W-mass, but a minimum. This minimum only lies over 1 TeV if the heavy Top-mass is fixed above 10 TeV and is

therefore not particularly relevant. The maximum mixing angle becomes smaller for a rising heavy Top-mass and converges on zero if $m_{W_R} \to \infty$ for every fixed heavy Top-mass.



(a) $m_T = 1$ TeV (b) $m_T = 2$ TeV (c) $m_T = 5$ TeV

Fig. 3.3: The mixing angle dependent on the right-handed W-boson mass for a fixed heavy Top-mass.

Using constraint (3.10) one can read a lower boundary of the right-handed W-mass from figure 3.3 (table 3.5). These lower boundaries should, however, only be seen as tendencies, since the θ-constraint is only valid for very large Top-masses and can be shifted for smaller Top-masses via an additional factor or a prefactor.

m_T/TeV	θ_{max}	lower boundary on m_{W_R}/TeV
1	0,193	3,2
1,5	0,149	4,6
2	0,126	5,6
3	0,100	7,3
5	0,0749	10,2

Tab. 3.5: Lower boundary on the W_R-mass derived from the requirement of decoupling for large Top-masses and a fixed angle $\alpha = 0$.

Allowing α to be unequal to zero extends the possible combination of parameters which are possible in this model in order to acquire the right mass-difference. Only values for α between 0 and $\frac{\pi}{2}$ are considered. Generally one finds a suitable combination of α and θ for all fixed values of m_{W_R} and m_T so that no masses can be excluded. It transpires that for each fixed pair of m_{W_R} and m_T there is a maximum value for θ when $\alpha = 0$ and a maximum value for α when $\theta = 0$. The maximum value of θ decreases with increasing heavy Top or right-handed W-boson mass. With the requirement that $m_{W_R} > 1$ TeV and $m_T > 1$ TeV the following constraint on θ is achieved:

$$\theta < 0.484$$

Allowing the input parameter to be corrected as in the CKM matrix elements this boundary should lie higher.
This constraint is looser than the one obtained from the first condition (3.10) when inserting $m_T = 1$ TeV.

$$\theta < 0.190$$

The maximum value of α is $\frac{\pi}{2}$ since the LRSM contribution decouples for very large W-boson masses.
Constraints could also be obtained using the T- and R_b-parameter. However, extending those calculations to the Left-Right Symmetric Model is a lengthy process since the new gauge bosons (the additional two neutral, two single-charged and two double-charged Higgs bosons) need to be considered in the loops. Even though the mixing between the physical Z-bosons Z_1 and Z_2 and W-bosons W_1 and W_2 has been neglected, the calculation goes beyond the scope of this thesis. Since the Standard Model prediction of the R_b-parameter needs a small positive shift in order to agree with the measured value, new contributions giving negative shifts can be used to constrain the particles involved in order to keep those contributions small. On the other hand, positive shifts must cancel negative contributions of the Standard Model and therefore yield to constraints relating parameters of the Standard Model to those of the extended model. In [65] detached contributions are used in order to constrain the masses of the new particles introduced in a Left-Right Symmetric Model.

3.2.2 CP-violation in neutral Kaon decay in the LRSM extended by a Top-isosinglet

The CP-violation in K^0- and $\bar{K}^0$-decay is calculated via the CP-violating parameter ϵ (1.51). The contribution M_{12} can be achieved from the contribution calculated in the last section by substituting all indices referring to a b-quark for indices referring to an s-quark. The experimental value of the CP-violating parameter has been measured [8] and can be explained by means of only the Standard Model

$$|\epsilon| = (2,228 \pm 0,011) \times 10^{-3}.$$

In the Standard Model the diagram with two intermediate top-quarks dominates the CP-violation since the CKM matrix elements for the transition from the t- to the s- and d-quark contain the CP-violating phase δ multiplied by the lowest order of λ compared to other CKM matrix elements.
When introducing an extra isosinglet Top-quark to the Standard Model, the updated value for ϵ leaves very little space for the new quark unless the parameters of the Standard Model can be altered. Considering only the tt-contributions and taking the constraints on θ and m_T into account [21] allows the CKM matrix element V_{td} to alter by at most 4% for a Top-mass of 1 TeV.
In the previous contributions the u-quark contribution was implicitly taken into account due to the application of the unitarity relations ($\lambda_u = -\lambda_c - \lambda_t - \lambda_T$). Considering the

contribution with one right-handed W-boson the application of the unitarity relations would be too complicated to be useful. The contributions including one or two intermediate u-quarks can be neglected because of their smallness compared to the other contributions. It is assumed that the QCD correction η_{ij} with i,j $\in$ $\{c, t, T\}$ are equal to the corrections in the case with two left-handed intermediate W-bosons (1.54). After these simplifications the CP-violating parameter ϵ depends on the masses of the two new particles m_T and m_{W_R}, the angles α and θ and the quark mass signs s_c, s_t, s_d, s_s and s_b. Due to cancellations between all the contributions, this scenario explains the value of the CP-violating parameter without the necessity of altering any input parameter.
Evaluating ϵ reveals that the signs always appear in pairs and therefore that one sign can be fixed. s_c is chosen to be equal to one and thus 16 different choices of the signs remain. It transpires that those 16 choices can be roughly attached to two regions.
Region Ia contains the four choices $s_d = -1$, $s_t = s_s = -1$, $s_s = s_b = -1$ and $s_t = s_s = s_b = -1$ where unmentioned signs are chosen to be plus one. Characteristic of region Ia is the graph shown in figure 3.4 which only alters slightly, changing m_{W_R}, θ and m_T. Thus the choices in region Ia give the following constraint on the angle of the Higgs bidoublet vev: $|rsin\alpha| > 0,5$.

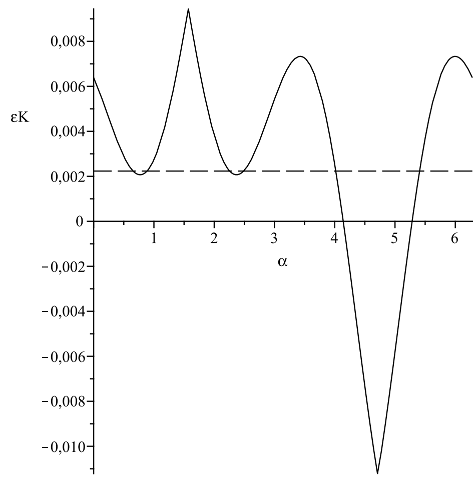


Fig. 3.4: The CP-violating parameter ϵ depending on α for fixed mixing angle $\theta = 0,15$, a fixed Top-mass $m_T = 1$ TeV, a fixed W-mass $m_{W_R} = 2,5$ TeV and the sign choices $s_d = -1$ and $s_t = s_s = s_b = 1$ exemplaric for region Ia. The dashed line represents the experimentally observed value.

Region IIa contains all other 12 choices and the choices $s_t = s_s = -1$ and $s_t = s_s = s_b = -1$ which give values to both regions. Figure (3.5) shows the characteristic graph of region IIa. The constraint in this region is given by $|rsin\alpha| < 0,3$.
Combining these two regions shows that independent of the mass sign of the quarks the following constraint is derived:

$$|rsin\alpha| \in [0, 0, 3) \qquad \text{or} \qquad |rsin\alpha| \in (0, 5, 1].$$

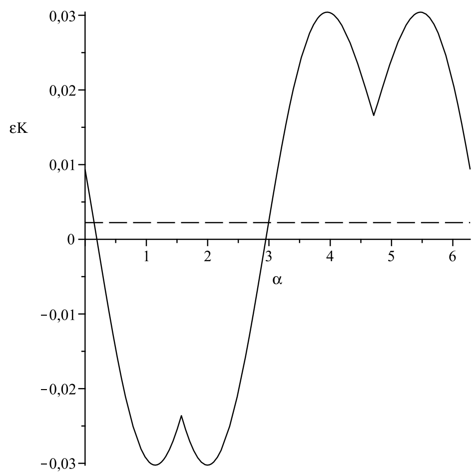


Fig. 3.5: The CP-violating parameter ϵ depending on α for a fixed mixing angle $\theta = 0, 15$, a fixed Top-mass $m_T = 1$ TeV, a fixed W-mass $m_{W_R} = 2, 5$ TeV and the sign choices $s_t = s_d = s_b = 1$ and $s_s = -1$ exemplaric for region IIa. The dashed line represents the experimentally observed value.

Most of the $|rsin\alpha|$-combinations in region IIa are actually far smaller than 0,3 and the region corresponds roughly to the region found in [43], which had the limitation $|rsin\alpha|$ <0,05. The loosening of the boundary could be explained by the new freedom of choice introduced via the two parameters θ and m_T describing the new vector-like singlet.

4. THE DECAY $B \to \mu^+\mu^-$ IN THE LRSM

In this chapter the expressions describing the B-meson decay to two muons in the LRSM are derived to one-loop level. The functions describing the loops are presented as extensions of the Inami-Lim-functions [66]. The enhanceability of the decay rate compared to that predicted by the Standard Model is discussed dependent on the new parameters. Finally the way in which the functions could be extended in order to take into account an additional vector-like heavy quark isosinglet is described.

4.1 Calculation of the contributing diagrams and the effective Lagrangian

Since the decay of Kaons and B-mesons to lepton pairs has not yet been observed, only an upper limit of the branching ratio is given experimentally. In this chapter the decay rate of a B-meson to a muon and an antimuon will be analysed. The experimentally-obtained upper boundary is given by [8]

$$B(B^0 \to \mu^+\mu^-)_{exp} < 1,5 \times 10^{-8}, \qquad B(B_s^0 \to \mu^+\mu^-)_{exp} < 4,7 \times 10^{-8}.$$

The value predicted by the Standard Model is roughly one to two orders of magnitude smaller than the upper boundary [67]

$$B(B^0 \to \mu^+\mu^-)_{SM} = (1,00 \pm 0,14) \times 10^{-10}, \qquad B(B_s^0 \to \mu^+\mu^-)_{SM} = (3,42 \pm 0,54) \times 10^{-9}.$$

There is therefore space for new physics to enhance the branching ratio.
The minuteness of the branching ratio is due to the fact that the mesons are only allowed to decay to leptons via the weak interaction, not via the electromagnetic interaction (chapter 4.2) and that the process can only be mediated at one-loop level, since the Z-boson does not couple to flavour-changing vertices.
Generally new particles such as the Z_2-boson from the LRSM could mediate the decay instead of the Standard Model Z-boson, but it will be shown that this contribution is negligible since it is suppressed by the Z_2-boson mass square. The functions presented in this chapter can easily be translated in order to describe the decay $B^+ \to \pi^+\nu\bar{\nu}$. The new particles therefore contribute via diagrams of second order in perturbation theory, as pictured in figure 4.1.

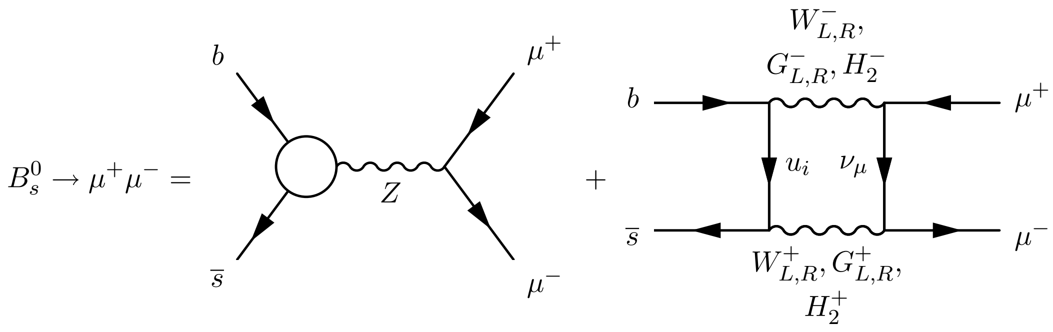
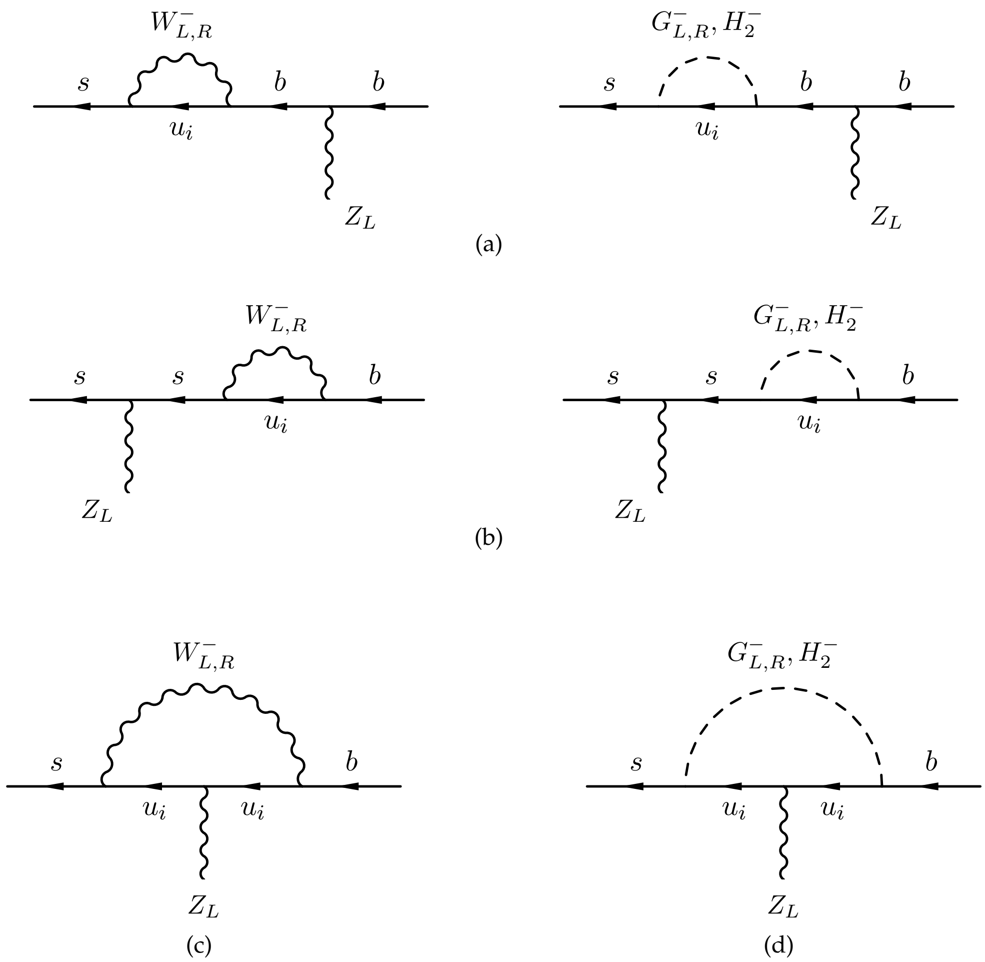


Fig. 4.1: Contributions to the effective Lagrangian for the decay $B_s^0 \to \mu^+\mu^-$.

The vertex corrections which contribute to the effective (Zsb)-vertex in the case of a left-right symmetric theory extended by an extra heavy Top-isosinglet are shown in figure 4.2. The same diagrams, including only Standard Model particles, have been discussed by T. and C. S. Lim [66].



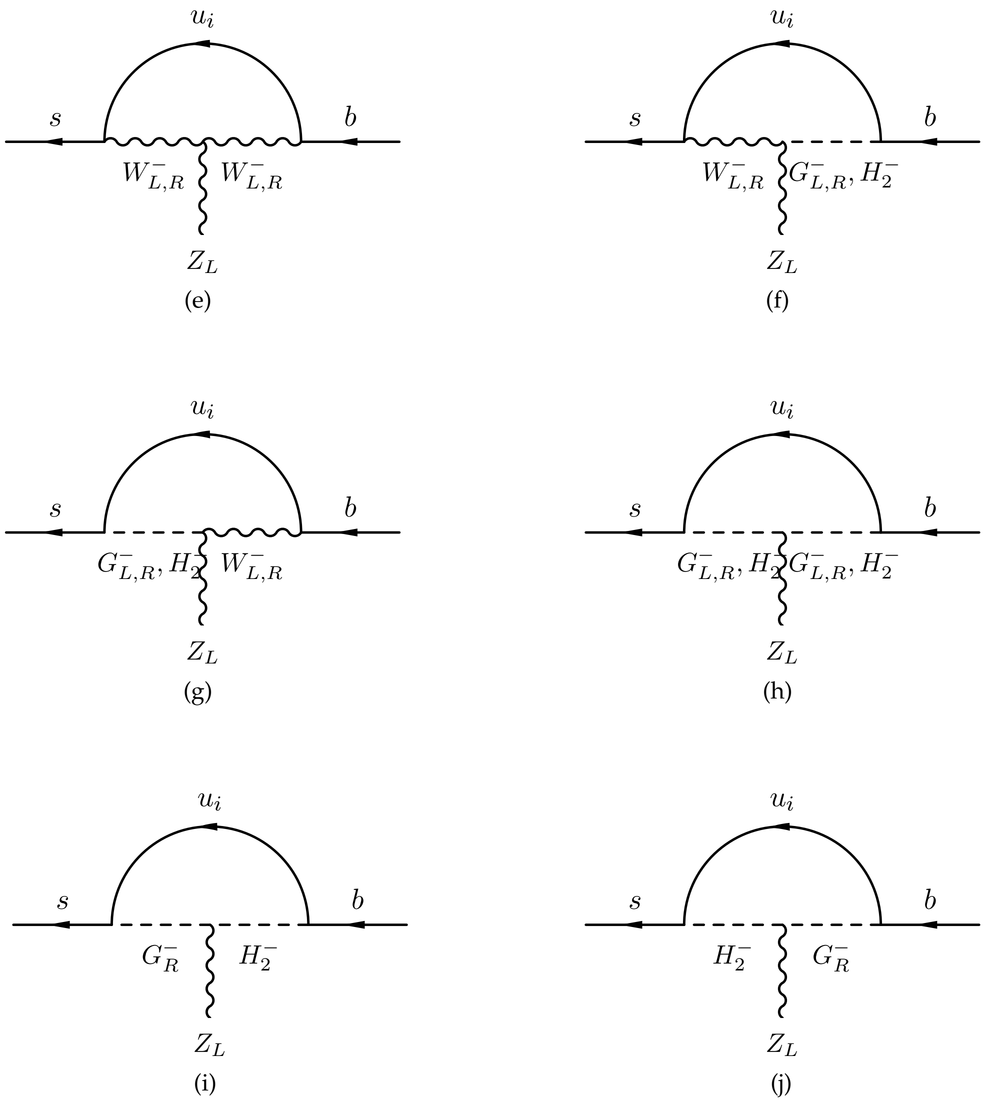


Fig. 4.2: Vertex corrections in the LRSM with $u_i \in \{u, c, t\}$ and potentially $u_i = T$.

Comparing these diagrams with those in [66] shows that there is one additional gauge boson (W_R) and two additional scalar particles (G_R and H_2) in these loops. Since the external quark masses are set to zero in the calculation, the gauge bosons W_L and G_L only allow left-handed external quarks, while the gauge boson W_R, as well as the scalar particles G_R and H_2 only allow right-handed external quarks. The diagrams (i) and (j) do not exist in the Standard Model.

First the contributions to the effective (Zsb)-vertex will be calculated. They are denoted by Γ^i_{L,R,H_2} where the exponent denotes the figure and the index the intermediate gauge boson (L,R stands for $W_{L,R}$ and H for H_2). The Feynman rules for the vertices have been taken from Appendix B. The integrals have been solved with the help of dimensional regularisation and are explicitly given in Appendix A.

The effective (Zsb)-vertex is thus given by

$$\Gamma_{Zsb} = \frac{g^3}{(4\pi)^2 cos\theta_W}\left(V^*_{Ljs}V_{Ljb}\sum_{i=a}^{h}\Gamma_{L(L)}(s\gamma_\mu\gamma_L b) + V^*_{Rjs}V_{Rjb}\sum_{i=a}^{l}\Gamma_{R(R),H(H),RH}(s\gamma_\mu\gamma_R b)\right)$$

with the following contributions:

$$\begin{aligned}
\Gamma_L^{a+b} &= \frac{1}{2}\left(\frac{1}{3}sin^2\theta_W - \frac{1}{2}\right)\left[\frac{x_i^2}{(x_i-1)^2}lnx_i - \frac{x_i}{x_i-1} - x_i f_1^L(x_i)\right] - (x_i \to x_u)\\
\Gamma_R^{a+b} &= \frac{1}{6}sin^2\theta_W\left[\frac{y_i^2}{(y_i-1)^2}lny_i - \frac{y_i}{y_i-1} - y_i f_1^R(y_i)\right] - (y_i \to y_u)\\
\Gamma_{H_2}^{a+b} &= -x_{H_2}\frac{1}{6}sin^2\theta_W z_i f_1^{H_2}(z_i) - (z_i \to z_u)\\
\Gamma_L^c &= \left(\frac{1}{3}sin^2\theta_W - \frac{1}{4}\right)\frac{x_i^2}{(x_i-1)^2}lnx_i + \frac{1}{2}\frac{x_i}{(x_i-1)^2}lnx_i - \left(\frac{1}{3}sin^2\theta_W + \frac{1}{4}\right)\frac{x_i}{x_i-1} - (x_i \to x_u)\\
\Gamma_R^c &= \frac{1}{3}sin^2\theta_W\frac{y_i^2}{(y_i-1)^2}lny_i - \frac{1}{2}\frac{y_i}{(y_i-1)^2}lny_i + \frac{1}{2}\left(1 - \frac{2}{3}sin^2\theta_W\right)\frac{y_i}{y_i-1} - (y_i \to y_u)\\
\Gamma_L^d &= -\frac{1}{6}sin^2\theta_W x_i f_2^L(x_i) + \frac{1}{4}\left(\frac{2}{3}sin^2\theta_W - 1\right)\left[\frac{x_i^2}{(x_i-1)^2}lnx_i - x_i - \frac{x_i}{x_i-1}\right] - (x_i \to x_u)\\
\Gamma_R^d &= \frac{1}{4}\left(\frac{1}{2} - \frac{2}{3}sin^2\theta_W\right)y_i f_2^R(y_i) + \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}sin^2\theta_W\right)\left[\frac{y_i^2}{(y_i-1)^2}lny_i - y_i - \frac{y_i}{y_i-1}\right] - (y_i \to y_u)\\
\Gamma_{H_2}^d &= \frac{1}{4}x_{H_2}\left(\frac{1}{2} - \frac{2}{3}sin^2\theta_W\right)z_i f_2^{H_2}(z_i)\\
&\quad + \frac{1}{2}x_{H_2}\left(\frac{1}{4} + \frac{1}{3}sin^2\theta_W\right)\left[\frac{z_i^2}{(z_i-1)^2}lnz_i - z_i - \frac{z_i}{z_i-1}\right] - (z_i \to z_u)\\
\Gamma_{LL}^e &= \frac{3}{2}(1 - sin^2\theta_W)\left[\frac{x_i^2}{(x_i-1)^2}lnx_i - \frac{1}{x_i-1}\right] - (x_i \to x_u)\\
\Gamma_{RR}^e &= -\frac{3}{2}sin^2\theta_W\left[\frac{y_i^2}{(y_i-1)^2}lny_i - \frac{1}{y_i-1}\right] - (y_i \to y_u)\\
\Gamma_{LL}^{f+g} &= sin^2\theta_W\left[\frac{x_i^2}{(x_i-1)^2}lnx_i - \frac{x_i}{x_i-1}\right] - (x_i \to x_u)\\
\Gamma_{RR}^{f+g} &= \sqrt{x_{W_R}}(\eta cos^2\theta_W - sin^2\theta_W)\left[\frac{y_i^2}{(y_i-1)^2}lny_i - \frac{y_i}{y_i-1}\right] - (y_i \to y_u)\\
\Gamma_{RH}^{f+g} &= \frac{x_i}{(x_i - x_{W_R})(x_i - x_{H_2})}lnx_i - \frac{x_{W_R}}{(x_i - x_{W_R})(x_{W_R} - x_{H_2})}lnx_{W_R}\\
&\qquad + \frac{x_{H_2}}{(x_i - x_{H_2})(x_{W_R} - x_{H_2})}lnx_{H_2} - (x_i \to x_u)\\
\Gamma_{LL}^h &= \left(\frac{1}{2} - sin^2\theta_W\right)\left[\frac{1}{4}\left(\frac{x_i^2}{(x_i-1)^2}lnx_i - \frac{x_i}{x_i-1}\right) - \frac{1}{4}x_i f_2^L(x_i)\right] - (x_i \to x_u)\\
\Gamma_{RR}^h &= -sin^2\theta_W\left[\frac{1}{4}\left(\frac{y_i^2}{(y_i-1)^2}lny_i - \frac{y_i}{y_i-1}\right) - \frac{1}{4}y_i f_2^R(y_i)\right] - (y_i \to y_u)\\
\Gamma_{HH}^h &= x_{H_2}\left(\frac{1}{2} - sin^2\theta_W\right)\left[\frac{1}{4}\left(\frac{z_i^2}{(z_i-1)^2}lnz_i - \frac{z_i}{z_i-1}\right) - \frac{1}{4}z_i f_2^{H_2}(z_i)\right] - (z_i \to z_u)
\end{aligned}$$

$$\Gamma^{i+j}_{RH} = \eta \left[\frac{1}{4} \left(\frac{x_i^2}{(x_i - x_{W_R})(x_i - x_{H_2})} ln x_i - \frac{x_{W_R}^2}{(x_i - x_{W_R})(x_{W_R} - x_{H_2})} ln x_{W_R} \right.\right.$$
$$\left.\left. + \frac{x_{H_2}^2}{(x_i - x_{H_2})(x_{W_R} - x_{H_2})} ln x_{H_2} \right) - \frac{1}{4} x_i \left(\frac{2}{\epsilon} + C^L + 1 \right) \right] - (x_i \to x_u)$$

The functions $f_1^{L,R,H_2}(x)$, $f_2^{L,R,H_2}(x)$ and C^{L,R,H_2} are given in (A.5), (A.6) and (A.7) and the following abbreviations have been used:

$$x_i = \frac{m_i^2}{M_{W_L}^2}, \quad y_i = \frac{m_i^2}{M_{W_R}^2}, \quad z_i = \frac{m_i^2}{M_{H_2}^2}, \quad \eta = \frac{M_{W_L}^2}{M_{W_R}^2}, \quad x_{W_R} = \frac{M_{W_R}^2}{M_{W_L}^2}, \quad x_{H_2} = \frac{M_{H_2}^2}{M_{W_L}^2}.$$

The Euler-Mascheroni constant is given by $\gamma_E \approx 0.5772$ [1].
The GIM mechanism has been used in the calculations of all the contributions, meaning that $\lambda_u = -\lambda_c - \lambda_t - \lambda_T$ has been inserted and that therefore the divergent contributions (which are not linear in any order of x_i) are cancelled. The divergences in contributions (c) and (e) have been removed using this method. The contributions solely including Goldstone bosons or the Higgs particle (diagrams a, b, d, h, i and j) include the divergent term ($1/\epsilon$) multiplied by the squared masses of the quarks and therefore cannot be removed by the GIM mechanism. Since the theory is renormalisable, the divergences cancel each other when summing over these contributions. It transpires that to the chosen order in ϵ and χ the divergences produced by the three particles G_L, G_R and H_2 cancel each other separately, meaning that the individual contributions of one particle could be studied without having to include the other particles in order to cancel divergences.

Summing these divergent contributions gives the following finite contributions:

$$\Gamma_L^{a+b} + \Gamma_L^d + \Gamma_{LL}^h = \frac{1}{2} \left(\frac{1}{3} sin^2\theta_W - 1 \right) \left[\frac{x_i^2}{(x_i-1)^2} ln x_i - \frac{x_i}{x_i - 1} \right] + \frac{1}{4} x_i - (x_i \to x_u)$$
$$\Gamma_R^{a+b} + \Gamma_R^d + \Gamma_{RR}^h = \frac{1}{2} \left(\frac{1}{2} + \frac{sin^2\theta_W}{3} - \frac{y_i}{4} \right) \frac{y_i^2}{(y_i-1)^2} ln y_i$$
$$+ \left(-\frac{1}{8} + \frac{sin^2\theta_W}{24} - \frac{5 y_i sin^2\theta_W}{24} \right) \frac{y_i}{y_i - 1} + C^R \frac{y_i}{8} - (y_i \to y_u)$$
$$\Gamma_H^{a+b} + \Gamma_H^d + \Gamma_{HH}^h = x_{H_2} \left[\frac{1}{4} \frac{z_i^2}{(z_i-1)^2} ln z_i + \frac{1}{2} \left(-\frac{1}{4} + \frac{3 sin^2\theta_W}{4} - z_i \left(\frac{1}{4} + \frac{5 sin^2\theta_W}{12} \right) \right) \frac{z_i}{z_i - 1} \right]$$
$$- (z_i \to z_u)$$

These contributions include negligible terms. In the following calculation, all non-Standard Model contributions will be analysed and terms of higher orders in ϵ (equivalent to higher orders in η or $x_{H_2}^{-1}$) will be neglected. The result is dependent on the inclusion of an extra Top-quark, since the quantities y_i and z_i are negligible for the Standard Model quarks, but can be of order one for an extra Top-quark.

Considering only the LRSM, the following contributions in addition to those of the Standard Model must be taken into account:

$$
\begin{aligned}
\Gamma_R^{a+b} + \Gamma_R^d + \Gamma_{RR}^h &= 0 \\
\Gamma_H^{a+b} + \Gamma_H^d + \Gamma_{HH}^h &= \frac{x_{H_2}}{2}\left[\frac{1}{2}\frac{z_i^2}{(z_i-1)^2} ln z_i + \left(-\frac{1}{4} + \frac{3 sin^2\theta_W}{4}\right)\frac{z_i}{z_i-1}\right] - (z_i \to z_u) \\
\Gamma_R^c &= 0 \\
\Gamma_R^e &= \frac{3}{2} sin^2\theta_W \frac{1}{y_i-1} - (y_i \to y_u) \\
\Gamma_{RR}^{f+g} &= \sqrt{x_{W_R}} sin^2\theta_W \frac{y_i}{y_i-1} - (y_i \to y_u) \\
\Gamma_{RH}^{f+g} &= -\frac{x_{W_R}}{(x_i - x_{W_R})(x_{W_R} - x_{H_2})} ln x_{W_R} \\
&\quad + \frac{x_{H_2}}{(x_i - x_{H_2})(x_{W_R} - x_{H_2})} ln x_{H_2} - (x_i \to x_u) \\
\Gamma_{RH}^{i+j} &= 0
\end{aligned}
$$

The contribution Γ_R^e becomes negligible when the u-quark mass is set to zero.
Considering an extra heavy quark in the LRSM negates only the contributions due to the diagrams (i) and (j). All other contributions must be taken into account in their entirety.

Summing up all contributions leads to the following:

$$
\Gamma_{L(L)} = \sum_i \Gamma_{L(L)}^i = \frac{1}{4}x_i - \frac{5}{4}\frac{1}{x_i-1} + \frac{1}{4}\frac{3x_i^2 + 2x_i}{(x_i-1)^2} ln x_i - (x_i \to x_u)
$$

The terms differ from those in [66] due to the fact that here a specific gauge (Feynman-t'Hooft) has been chosen.
The sum of the contributions including the new particles is displayed further down in the discussion (4.2, 4.3) summarised by the function $\tilde{C}_R$ which adds up all vertex corrections including the right-handed W-boson and the new scalar particle H_2.

Analysing the box diagram (figure 4.1) requires the coupling of the single-charged Higgs boson, the longitudinal modes of the W-bosons, and the W-bosons to the leptons. The derivation is analogous to that of the couplings to the quarks. The Feynman rules are obtained by replacing (u, c, t) with $(\nu_e, \nu_\mu, \nu_\tau)$ and (d, s, b) with (e, μ, τ) in (B.8, B.9, B.10, B.1). Applying these Feynman rules to the box diagram (figure 4.1), it transpires that all contributions with one or two intermediate particles being G_L, G_R or H_2 are linear or squared respectively in the neutrino mass. In this calculation there is assumed to be no mixing between lepton generations and the neutrino masses are assumed to be zero. Therefore all the aforementioned contributions are negligible and there are only four box diagrams left to be taken into account. These are the diagrams with two left-handed W-bosons A^{LL}, two right-handed W-bosons A^{RR} and the two diagrams with one left- and one right-handed W-boson A^{LR}, A^{RL}

$$\begin{aligned} A^{LL} &= -\frac{1}{2}\frac{x_i}{(x_i-1)^2} ln x_i + \frac{1}{2}\frac{1}{x_i-1} - (x_i \to x_u) \\ A^{RR} &= \eta\left[-\frac{1}{2}\frac{y_i}{(y_i-1)^2} ln y_i + \frac{1}{2}\frac{1}{y_i-1}\right] - (y_i \to y_u) \\ A^{LR} = A^{RL} &= \frac{1}{2}\left[-\frac{\eta ln\eta}{(\eta-1)(x_i\eta-1)} + \frac{\eta x_i ln x_i}{(x_i-1)(x_i\eta-1)}\right] \end{aligned}$$

which combine to give the contribution to the effective Lagrangian

$$\begin{aligned} A = \frac{g^4}{8(4\pi)^2 M_{W_L}^2} \sum_{i=u,c,t(,T)} & \left[V_{Ljs}^* V_{Ljb} A_{LL}(x_i)(s\gamma_\mu\gamma_L b) + V_{Lis}^* V_{Rib} A_{LR}(x_i)(\bar{s}\gamma_L b)\right. \\ & \left. + V_{Ris}^* V_{Lib} A_{RL}(x_i)(\bar{s}\gamma_R b) + V_{Rjs}^* V_{Rjb} A_{RR}(y_i)(s\gamma_\mu\gamma_R b)\right] . \end{aligned}$$

Analysing the sizes of the contributions above reveals that the contribution due to two right-handed W-bosons can be neglected even if the new massive Top-quark is considered. The contribution A^{LR} would be neglected if it were isolated, since its leading order is linear in η. It will be shown, however, that the contribution is enhanced due to its differing quark operator by a factor of around 32 and must therefore be taken into account when considering W-bosons which are not too large.

The effective Lagrangian writes

$$\begin{aligned} L_{eff} = & \frac{4G_F}{\sqrt{2}}\chi\left[\sum_{i=c,t(,T)}\left(\lambda_{Li}\tilde{C}_L(x_i)(\bar{s}_L\gamma_\mu b_L) + \lambda_{Ri}\tilde{C}_R(x_i)(\bar{s}_R\gamma_\mu b_R)\right)(\mu_L\gamma^\mu\mu_L)\right. \\ & \left. + \sum_{i=u,c,t(,T)}\left(A_{LR}(x_i)(\lambda_{LRi}(\bar{s}_R b_L)(\mu_L\mu_R) + \lambda_{RLi}(\bar{s}_L b_R)(\mu_R\mu_L))\right)\right] \end{aligned}$$

with

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_{W_L}^2}, \qquad \chi = \frac{\alpha}{4\pi s_W^2} = \frac{g^2}{(4\pi)^2},$$

and the functions $\tilde{C}_{L,R}$ summing up the vertex corrections (Γ) and the box corrections (A) while applying the GIM mechanism and setting the quantities x_u, y_u and z_u to zero. The GIM mechanism has not yet been used for the box contribution mediated by one left- and one right-handed boson, but will be applied in chapter 4.10.

The effective Lagrangian has been achieved by multiplying the effective (Zsb)-vertex by $1/M_{Z_L}^2$ and the vertex function of the $(Z_L\mu^+\mu^-)$-vertex [1].

$$L_{Z\mu\mu} = \frac{g}{c_W} Z_\alpha\left[\bar{\mu}_L\gamma^\alpha\left(-\frac{1}{2} + s_W^2\right)\mu_L + \bar{\mu}_R\gamma^\alpha s_W^2\mu_R\right]$$

Contributions caused by the intermediate particle Z_R have been neglected since the Z_R-mass is of the order of the mass of the right-handed W-boson (2.16).
The effective vertex function for initial left-handed quarks is given by

$$\tilde{C}_L(x_i) = \Gamma_L + A^{LL} = \frac{1}{4}x_i - \frac{3}{4}\frac{x_i}{x_i-1} + \frac{3}{4}\frac{x_i^2}{(x_i-1)^2}lnx_i. \tag{4.1}$$

The effective vertex function describing right-handed initial states splits into one contribution describing the intermediate c- and t-quark ($i = c, t$) where many contributions were of higher order in η and therefore neglected

$$\begin{aligned}\tilde{C}_R(x_i) &= \left[\Gamma^{a+b}_{H_2} + \Gamma^{d}_{H_2} + \Gamma^{h}_{H_2} + \Gamma^{f+g}_{RR} + \Gamma^{f+g}_{RH}\right] \\ &= \frac{1}{4}\frac{x_i^2 x_{H_2}}{(x_i - x_{H_2})^2}(lnx_i - lnx_{H_2}) - \frac{1}{8}x_{H_2}\frac{x_i}{x_i - x_{H_2}} \\ &\quad - \frac{x_{W_R}}{(x_i - x_{W_R})(x_{W_R} - x_{H_2})}lnx_{W_R} + \frac{x_{H_2}}{(x_i - x_{H_2})(x_{W_R} - x_{H_2})}lnx_{H_2} \\ &\quad + s_W^2\left[\frac{3}{8}x_{H_2}\frac{x_i}{x_i - x_{H_2}} + \sqrt{x_{W_R}}\frac{x_i}{x_i - x_{W_R}}\right]\end{aligned} \tag{4.2}$$

and one describing the new heavy Top-quark as an intermediate particle

$$\begin{aligned}\tilde{C}_R(x_T) = &\sum_{j=a}^{h}\left(\Gamma^j_R + \Gamma^j_{H_2}\right) \\ = &-\frac{1}{8x_{W_R}}\frac{x_T}{(x_T - x_{W_R})^2}(-x_T^2 + 2x_T x_{W_R} - 8\sqrt{x_{W_R}} + 4x_{W_R}^2)(lnx_T - lnx_{W_R}) \\ &- \frac{1}{8\sqrt{x_{W_R}}}\frac{x_T}{(x_T - x_{W_R})^2}(3\sqrt{x_{W_R}} - 8) - \frac{x_T C^R}{8x_{W_R}} \\ &- \frac{1}{4}\frac{x_T^2 x_{H_2}}{(x_T - x_{H_2})^2}(lnx_T - lnx_{H_2}) + \frac{1}{8}\frac{x_T}{x_T - x_{H_2}}(x + x_{H_2}) \\ &- x_T\left(\frac{lnx_T}{(x_T - x_{W_R})(x_T - x_{H_2})} + \frac{lnx_{H_2} - lnx_{W_R}}{x_{W_R} - x_{H_2}}\right) \\ &+ s_W^2\left[-\frac{1}{\sqrt{x_{W_R}}}\frac{x_T^2}{(x_T - x_{W_R})^2}(x_{W_R} + \sqrt{x_{W_R}} + 1)(lnx_T - lnx_{W_R})\right. \\ &\qquad + \frac{1}{24x_{W_R}}\frac{1}{x_T - x_{W_R}}\left(-5x_T^2 + x_i(29x_{W_R} + 24x_{W_R}^{3/2} + 24\sqrt{x_{W_R}})\right) \\ &\qquad \left. + \frac{1}{24}\frac{x_T}{x_T - x_{H_2}}(-10x_T + 9x_{H_2})\right].\end{aligned} \tag{4.3}$$

Considering the behaviour of $\tilde{C}$ under very large scalar boson masses reveals that not all terms decouple in the manner of the Appelquist-Carazzone theorem (chapter 1.8). There are two terms which arise from diagram (h) (describing two H_2-bosons) which converge when taking the limit of infinite scalar mass

$$\lim_{m_{H_2}\to\infty}\left[-x_{H_2}\frac{1}{4}\left(\frac{1}{2} - sin^2\theta_W\right)\frac{z_i}{z_i - 1} - (z_i \to z_u)\right] = \frac{1}{4}\left(\frac{1}{2} - sin^2\theta_W\right)(x_i - x_u).$$

The first term is half as large as the dominating leading top-quark contribution in the Standard Model (4.1). Since this result is in accordance neither with the decoupling theorem nor with the understanding of an effective theory, these terms will be abandoned in the further calculations in the expectation that the remaining terms nevertheless represent the behaviour of an extension of the Standard Model by trend.
The calculation will be split into two parts. First all but the constant contributions will be taken into account and the behaviour depending on the right-handed W-mass and the Higgs boson mass will be evaluated. The second analysis is based on the fact that the contribution due to the W_R- and H_2-boson can be considered in isolation, since the divergences left over from dimensional regularisation cancel for the two contributions separately. In line with the analysis in [43] the scalar boson mass is concluded to be large ($m_{H_2} > 15$ TeV) and thus the contributions due to the Higgs boson are assumed to decouple. Therefore in the second calculation only the W-boson contributions which behave correctly according to the decoupling theorem are considered.

4.2 The branching ratio Br($B_s \to \mu^+\mu^-$)

The branching ratio of the neutral B$_s$-meson decaying to a muon-antimuon pair is given by

$$Br(B_s \to \mu^+\mu^-) = \left|\langle\mu^+\mu^-|\, H_{eff}\, |B_s\rangle\right|^2 \tau_B$$

with the lifetime of the B meson τ_B.
Thus the calculation of the branching ratio involves the calculation of the following expectation values:

$$\langle\mu^+\mu^-|\, (\bar{s}\gamma_\mu\gamma_{L,R}b)(\mu\gamma^\mu\gamma_L\mu)\, |B_s\rangle = \langle\mu^+\mu^-|\, \bar{s}\gamma_\mu\gamma_{L,R}b\, |0\rangle\, \langle 0|\, \mu\gamma^\mu\gamma_L\mu\, |B_s\rangle \tag{4.4}$$

$$\langle\mu^+\mu^-|\, (\bar{s}\gamma_{L,R}b)(\mu\gamma_L\mu)\, |B_s\rangle = \langle\mu^+\mu^-|\, \bar{s}\gamma_{L,R}b\, |0\rangle\, \langle 0|\, \mu\gamma_L\mu\, |B_s\rangle \tag{4.5}$$

where the vacuum state $|0\rangle$ has been inserted. Rewriting the projection matrices ($\gamma_L = (1-\gamma_5)/2$ and $\gamma_R = (1+\gamma_5)/2$) and using

$$\langle 0|\, \bar{s}\gamma_\mu b\, |B_s\rangle = 0 \qquad \text{and} \qquad \langle 0|\, \bar{s}b\, |B_s\rangle = 0$$

because the strong interaction is parity-invariant [5] results in the first expectation value in equation (4.4)

$$\langle 0|\, \bar{s}\gamma_\mu\gamma_{L,R}b\, |B_s\rangle = \mp\frac{1}{2}\, \langle 0|\, \bar{s}\gamma_\mu\gamma_5 b\, |B_s\rangle = \pm\frac{i}{2} f_{B_s} p_\mu \tag{4.6}$$

with the form factor f_{B_s} discussed in Appendix C and the right-hand side of the equation taken from [5].
The first expectation value in equation 4.5 can be derived by contracting (4.6) with p_μ and using relation [1]

$$p_\mu \bar{s}\gamma^\mu\gamma_5 b = (m_s + m_b)\bar{s}\gamma_5 b \tag{4.7}$$

which can be derived from the Dirac equation.

$$\langle 0|\, \bar{s}\gamma_{L,R} b\, |B_s\rangle = \mp \frac{1}{2} \langle 0|\, \bar{s}\gamma_5 b\, |B_s\rangle = \pm i f_{B_s} \frac{m_{B_s}^2}{m_s + m_b}$$

Thus the expectation value of the effective Hamiltonian can be rewritten in the following way:

$$\begin{aligned}
&\langle \mu^+\mu^-|\, H_{eff}\, |B_s\rangle \\
&= V \Big[\lambda_L \tilde{C}_L \langle \mu^+\mu^-|\, (\bar{s}\gamma_\mu\gamma_L b)(\bar{\mu}\gamma^\mu\gamma_L\mu)\, |B_s\rangle + \lambda_R \tilde{C}_R \langle \mu^+\mu^-|\, (\bar{s}\gamma_\mu\gamma_R b)(\bar{\mu}\gamma^\mu\gamma_L\mu)\, |B_s\rangle \\
&\quad + A_{LR} \left(\lambda_{LR} \langle \mu^+\mu^-|\, (\bar{s}\gamma_L b)(\bar{\mu}\gamma_R\mu)\, |B_s\rangle + \lambda_{RL} \langle \mu^+\mu^-|\, (\bar{s}\gamma_R b)(\bar{\mu}\gamma_L\mu)\, |B_s\rangle\right)\Big] \\
&= \frac{V}{4} \Big[\Big(-\lambda_L\tilde{C}_L + \lambda_R\tilde{C}_R\Big) \langle \mu^+\mu^-|\, \bar{\mu}\gamma^\mu\gamma_L\mu\, |0\rangle \langle 0|\, \bar{s}\gamma_\mu\gamma_5 b\, |B_s\rangle \\
&\quad + A_{LR} \left(-\lambda_{LR} \langle \mu^+\mu^-|\, \bar{\mu}\gamma_R\mu\, |0\rangle + \lambda_{RL} \langle \mu^+\mu^-|\, \bar{\mu}\gamma_L\mu\, |0\rangle\right) \langle 0|\, \bar{s}\gamma_5 b\, |B_s\rangle\Big] \\
&= i\frac{V}{4} f_{B_s} \Big[\Big(\lambda_L\tilde{C}_L - \lambda_R\tilde{C}_R\Big) p_\mu \langle \mu^+\mu^-|\, \bar{\mu}\gamma^\mu\gamma_L\mu\, |0\rangle \\
&\quad + A_{LR}\frac{m_B^2}{m_s+m_b} \left(\lambda_{LR} \langle \mu^+\mu^-|\, \bar{\mu}\gamma_R\mu\, |0\rangle - \lambda_{RL} \langle \mu^+\mu^-|\, \bar{\mu}\gamma_L\mu\, |0\rangle\right)\Big] \\
&= i\frac{V}{8} f_{B_s} \Big[\Big(\lambda_L\tilde{C}_L - \lambda_R\tilde{C}_R\Big) p_\mu \left(\langle \mu^+\mu^-|\, \bar{\mu}\gamma^\mu\mu\, |0\rangle - \langle \mu^+\mu^-|\, \bar{\mu}\gamma^\mu\gamma_5\mu\, |0\rangle\right) \\
&\quad + A_{LR}\frac{m_B^2}{m_s+m_b} \left((\lambda_{LR}-\lambda_{RL}) \langle \mu^+\mu^-|\, \bar{\mu}\mu\, |0\rangle + (\lambda_{LR}+\lambda_{RL}) \langle \mu^+\mu^-|\, \bar{\mu}\gamma_5\mu\, |0\rangle\right)\Big]
\end{aligned} \tag{4.8}$$

with V being some prefactor including the factor $g^4/M_{W_L}^2$.
The muon matrix element is rewritten in the manner of [1] such that the first matrix element in the first row can be written as an electromagnetic current (j^α) and the second as an axial current (j_5^α) with

$$j^\alpha = \bar{\mu}\gamma^\alpha\mu \qquad \text{and} \qquad j_5^\alpha = \bar{\mu}\gamma^\alpha\gamma_5\mu.$$

Since the muon field follows the Dirac equation, the conservation of the vector current $p_\alpha j^\alpha = 0$ and a relation for the axial-vector current $p_\alpha j_5^\alpha = 2m_\alpha\bar{\mu}\gamma_5\mu$ can be derived [1] in a manner analogous to (4.7). Inserting these relations into (4.8) shows that the first term in the first row vanishes.
This also explains why the photon does not mediate the decay under consideration, since it produces only a non-axial current with the muons.
The Dirac spinors of an incoming particle and an outgoing anti-particle or vice versa are orthogonal, meaning that the first matrix element in the second row also vanishes. Thus the expectation value can be written proportional to one matrix element

$$\langle \mu^+\mu^-|\, H_{eff}\, |B_s\rangle = -i\frac{V}{4} m_\mu f_{B_s} \Big[\Big(\lambda_L\tilde{C}_L - \lambda_R\tilde{C}_R\Big) - uA_{LR}\left(\lambda_{LR}+\lambda_{RL}\right)\Big] \langle \mu^+\mu^-|\, \bar{\mu}\gamma_5\mu\, |0\rangle$$

with

$$u = \frac{m_{B_s}^2}{2m_\mu(m_s+m_b)} \approx 31,66.$$

Hence a quantity measuring the impact of the new physics due to the LRSM and the new quark is given by

$$\begin{aligned} A_s &= \frac{Br(B_s \to \mu^+\mu^-)_{LRSM(+Top)}}{Br(B_s \to \mu^+\mu^-)_{SM}} \\ &= \left| \frac{\sum_{i=c,t(,T)} \left(\lambda_{Li}\tilde{C}_L(x_i) - \lambda_{Ri}\tilde{C}_R(x_i)\right) - u\sum_{i=u,c,t(,T)} A_{LR}(x_i)\left(\lambda_{LRi} + \lambda_{RLi}\right)}{\sum_{i=c,t}\lambda_{Li}^{SM}\tilde{C}_L(x_i)} \right|^2 \end{aligned} \tag{4.9}$$

with

$$\lambda_{L,Ri} = V^*_{L,Ris}V_{L,Rib} \qquad \text{and} \qquad \lambda_{LRi} = V^*_{Lis}V_{Rib}$$

where the CKM matrix elements are given by

$$V_{Lcj} = V_{Lcj}^{SM}, \quad V_{Rcj} = V_{Rcj}^{LRSM}, \quad V_{L,Rtj} = V_{L,Rtj}^{SM,LRSM}cos\theta, \quad V_{L,Rtj} = V_{L,Rtj}^{SM,LRSM}sin\theta$$

for $j = s, b$ and the Standard Model and LRSM CKM matrices given in equation (1.33) and (2.24) respectively.
The contribution due to the LRSM plus the Top-quark can be written in terms of the masses of the new particles (m_{W_R}, m_{H_2} and m_T), the complex phase of the vev of the Higgs bidoublet in the LRSM α, the mixing angle between the Standard Model top-quark and the new heavy Top-quark and the mass signs ($s_d, s_s, s_b, s_u, s_c, s_t$ and s_T) which in this analysis are all set to 1.
Since all the left- and right-handed CKM matrix elements can be written as Standard Model CKM matrix elements multiplied by some phase dependent on α and for the t-and T-quark by either $sin\theta$ or $cos\theta$, the quantity A_s is independent of the CKM matrix elements if only one intermediate quark type or t and T are considered.
Using the main Standard Model contributions as a reference

$$\left|\lambda_{Lt}^{SM}\right| \tilde{C}_L(x_t) \approx 1,97\left|\lambda_{Lt}^{SM}\right|$$

all contributions with an absolute value smaller than $0,01\left|\lambda_{Lt}^{SM}\right|$ (for m_{W_R} =2 TeV and m_T =1 TeV) will be neglected. One must take into account that every contribution can be enhanced by the multiplication with another contribution due to the fact that the absolute value is taken. It transpires that the maximum contribution comes from the Top-quark vertex correction $\tilde{C}_L = 41,75$ for $m_T = 1$ TeV. This correction is suppressed by $sin^2\theta < x_T^{-1/2}$ (chapter 3.2.1) and thus the enhancement factor reduces to approximately 3,36. It transpires that the c-quark contributions to the vertex corrections (the functions $\tilde{C}_{L,R}$) can be neglected. In order to be able to use the GIM mechanism for the box contribution, the left- and right-handed CKM matrix elements must be rewritten

$$\begin{aligned} u \sum_{i=u,c,t,T} A_{LR}(x_i)\left(\lambda_{LRi} + \lambda_{RLi}\right) &= u \sum_{i=u,c,t,T} A_{LR}(x_i)\lambda_{Li}^{SM} f_i(\alpha,\theta) \\ &\stackrel{GIM}{=} \lambda_{Lc}^{SM}\left(A_{LR}(x_c)f_c(\alpha) - A_{LR}(x_u)f_u(\alpha)\right) \\ &\quad + \lambda_{Lt}^{SM}\left(A_{LR}(x_t)f_t(\alpha) - A_{LR}(x_u)f_u(\alpha)\right)cos^2\theta \\ &\quad + \lambda_{Lt}^{SM}\left(A_{LR}(x_T)f_t(\alpha) - A_{LR}(x_u)f_u(\alpha)\right)sin^2\theta \end{aligned} \tag{4.10}$$

with

$$f_u(\alpha) = e^{-i\gamma_3(\alpha)} + e^{i\gamma_2(\alpha)}, \qquad f_t(\alpha) = e^{i\gamma_3(\alpha)} + e^{-i(\gamma_2(\alpha)+2\gamma_3(\alpha))}$$
$$f_c(\alpha) = e^{-i\gamma_3(\alpha)} + e^{-i\gamma_2(\alpha)}$$

and the functions γ_1, γ_2 and γ_3 given in equation (2.25). Thus the functions $f_i(\alpha)$ can give a maximum enhancing multiplication factor of two. Evaluating the different contributions shows that the contributions due to the c-quark can be neglected (the first row of equation (4.10)).
The quantity A_s simplifies to

$$\begin{aligned} A_s &= \left| \frac{\sum_{i=t}\left(\lambda_{Li}\tilde{C}_L(x_i) - \lambda_{Ri}\tilde{C}_R(x_i)\right) - uA_{LR}(x_t)\left(\lambda_{LRt} + \lambda_{RLt}\right)}{\lambda_{Lt}^{SM}\tilde{C}_L(x_t)} \right|^2 \\ &= \left| 1 + \tilde{C}_L(x_t)^{-1}\left[-e^{-i(\gamma_2(\alpha)+\gamma_3(\alpha))}\tilde{C}_R(x_t) - u\left(A_{LR}(x_t)f_t(\alpha) - A_{LR}(0)f_u(\alpha)\right)\right]\right|^2 \end{aligned}$$

where the u-quark mass has been set to zero. Only the LRSM has been considered since the contributions including the Top-quark need to be extended (chapter 4.4). It is observable that the quantity A_s does not depend on the Standard Model value of the CKM matrix elements. The expression for A_s for only the LRSM can be achieved by setting $s_\theta = 0$.

4.3 Discussion of the branching ratio Br($B_s \to \mu^+\mu^-$) in the LRSM

Of the six terms which contribute to $\tilde{C}_R$, the second and the fifth are constant and are not taken into account, as discussed in chapter 4.1. The other four terms have been analysed for growing H_2- and W_R-masses and it transpires that only the t-quark contribution of the first and the sixth term need to be included. These contributions belong to the diagram with two intermediate Higgs bosons (h) and the diagram with one intermediate right-handed W-boson and one intermediate Higgs boson (f+g).
Thus the quantity A_s writes

$$\begin{aligned} A_s \approx \Bigg| 1 - \tilde{C}_L(x_t)^{-1}\Bigg[& e^{i(\gamma_2+\gamma_3)}\left(\frac{1}{4}\frac{x_t x_{H_2}}{(x_t - x_{H_2})^2}(lnx_t - lnx_{H_2}) + s_W^2\frac{\sqrt{x_{W_R}}}{x_t - x_{W_R}}\right) \\ & + \frac{u}{2}\left(\left(-\frac{\eta ln\eta}{(\eta-1)(y_t-1)} + \frac{\eta x_t lnx_t}{(x_t-1)(y_t-1)}\right)f_t(\alpha) - \frac{\eta ln\eta}{(\eta-1)}f_u(\alpha)\right)\Bigg]\Bigg| \end{aligned}$$

and the new contribution to the B-meson decay can be described by means of only three parameters: the new W-boson mass m_R, the single-charged Higgs mass m_{H_2} and the complex phase α.
Considering first the manifest LRSM ($\alpha = 0$) shows that A_s is bigger than one (meaning that the new branching ratio is bigger than the Standard Model one) and converges to one for large right-handed W-masses and large Higgs boson masses (figure 4.3).

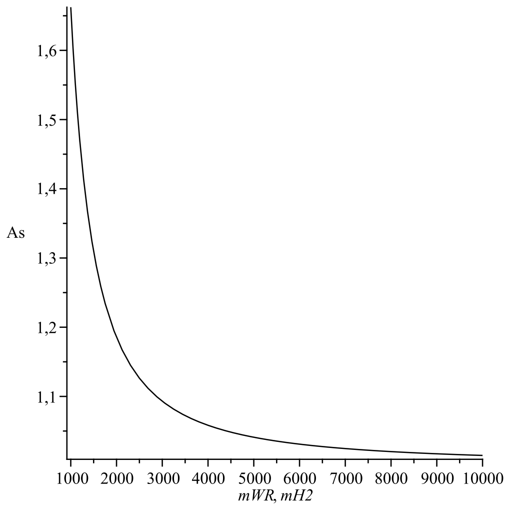


Fig. 4.3: A_s for the manifest LRSM. The masses are given in GeV.

It transpires that A_s is largest for small W-masses. Therefore the boson mass is fixed to the lowest boundary given in chapter 3.2.1 (m_{W_R} = 2,5 TeV). The dependence on α for three fixed H_2-boson masses is displayed in figure 4.4.

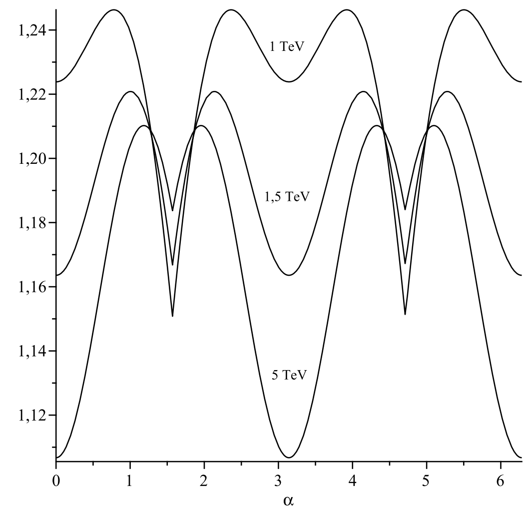


Fig. 4.4: A_s for a varying value of α and the fixed masses m_{W_R} = 2,5 TeV and m_{H2} = 1, 1,5 and 5 TeV.

Considering only $\alpha \in [0, \pi]$ reveals that A_s has two maxima with the same height and one minimum with $A_s > 1$. The position of the maxima depends on the scalar boson mass such that they grow closer together for growing boson masses. The minimum lies independently of m_{H_2} at $\alpha = \pi/2$. The value of the maxima is larger for a small Higgs boson mass, meaning that the maximum value for A_s under the given conditions is given by

$$A_s < 1,246 \qquad \text{for} \qquad \alpha = 0,78, \quad m_{W_R} = 2,5\,TeV \quad \text{and} \quad m_{H_2} = 1\,TeV.$$

The minimum increases for increasing m_{H_2} and finally converges to $A_s = 1,186$. However, one must bear in mind that the masses of the Higgs particle and the W-boson are related since both their main contributions are linear in v_R and that one therefore cannot consider $m_{H_2} \to \infty$ without $m_{W_R} \to \infty$. Letting both these masses go to infinity lets A_s converge to one (figure 4.5) and thus the growing nature of the minimum of A_s with growing m_{H_2} is only valid for a scalar mass being reasonably close to the W-mass. The same holds for the maximum, which converges if one allows only the scalar boson mass to tend towards infinity.

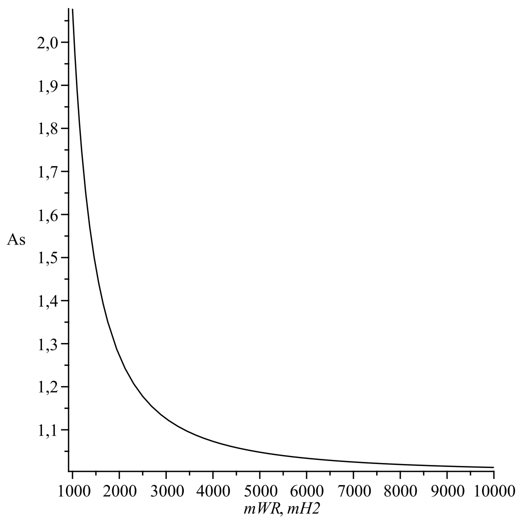


Fig. 4.5: The development of the minimum of A_s at $\alpha = \frac{\pi}{2}$ with growing W_R- and H_2-masses displayed in GeV.

Considering only the right-handed W-boson, the quantity A_s decreases with decreasing W-boson mass and therefore the upper boundary is given by

$$A_s < 1,21 \qquad \text{for} \quad \alpha = 1,21/1,93 \quad \text{and} \quad m_{W_R} = 2,5\,TeV \tag{4.11}$$

where the lower boundary on the W-mass [43] has been taken into account. Figure 4.6 shows the variation of A_s under the variation of α for a fixed W-mass. It transpires that for $\alpha \in [0, \pi]$ there are two maxima with the same height at $\alpha = 1,21$ and $\alpha = 1,93$ and two minima at $\alpha = \pi/2$ and $\alpha = \pi$ for every fixed W-mass.

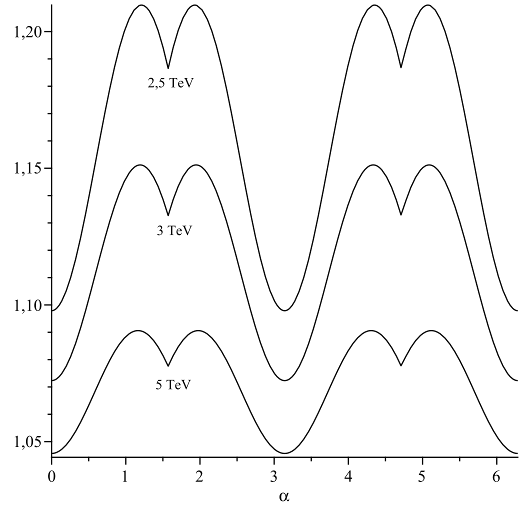


Fig. 4.6: A_s for a varying value of α and the fixed masses m_{W_R} = 2,5, 3 and 5 TeV.

Since the right-handed W-boson contributions follow the decoupling theorem, the quantity A_s converges to one if the W-mass tends towards infinity.
The mass sign choice initially selected in this calculation belongs to region II, defined in chapter 3.2.2. Taking into account the tighter boundary from [43] on the region $|rsin\alpha| < 0,05$ still lets the quantity A_s vary up to almost 10 %

$$A_s \approx 1.10 \qquad \text{for} \quad \alpha = 0.05 \quad \text{and} \quad m_{W_R} = 2,5\,TeV.$$

4.4 Discussion of the branching ratio Br($B_s \to \mu^+\mu^-$) in the LRSM extended by a Top-isosinglet

When calculating the contributions induced by an extra vector-like Top the contributions derived in chapter 4.1 are not sufficient and must be extended. The complication is due to the fact that the new CKM matrix (3.5) is not entirely unitary and therefore causes the following Z-boson interactions with the t- and T-quark, including flavour-changing neutral currents [21]:

$$L = -\frac{g}{2c_W} Z_\mu \left(cos^2\theta \bar{t}\gamma^\mu t + cos\theta sin\theta \bar{t}\gamma^\mu T + sin^2\theta \bar{T}\gamma^\mu T \right) + h.c.$$

Applying these Feynman rules to the diagrams listed in figure 4.2 alters the contributions (c) and (d).
Defining the function

$$B(x_i) = \Gamma^{(c)}(x_i) + \Gamma^{(d)}(x_i),$$

the contributions to the (Zsb)-vertex from the diagrams (c) and (d) write

$$\Gamma_{Zsb} = \frac{g^3}{(4\pi)^2 cos\theta_W} \left[\tilde{B}_L(x_t, x_T)(s\gamma_\mu\gamma_L b) + \left(\tilde{B}_R(y_t, y_T) + \tilde{B}_H(z_t, z_T) \right) (s\gamma_\mu\gamma_R b) \right]$$

with

$$\tilde{B}(x_t, x_T) = \lambda_t B(x_t) + \lambda_T B(x_T) - \lambda_t s_\theta^2 B(x_t) - \lambda_T c_\theta^2 B(x_T) + 2\sqrt{\lambda_t \lambda_T} c_\theta s_\theta B(x_t, x_T).$$

The first two contributions in the above equation have already been included in $\tilde{C}_R(x_t)$ and $\tilde{C}_R(x_T)$, while the other contributions must be added. The integrals solved in Appendix A can be used to calculate the contribution including the t- and T-quark. According to [21] all divergences due to dimensional regularisation cancel and the main contribution including the flavour-changing neutral coupling is positive. Therefore the value of A_s should be enhanceable when compared to the value of A_s calculated in the LRSM.
One must also be careful when calculating the contributions in the mass limit $m_T \to m_{W_R}$ so as to avoid unphysical divergences. These divergences arise from the fact that the masses must already be set equally when calculating the integrals in Appendix A, since the number of equal mass particles in the loop influences the Feynman parameterisation.

5. CONCLUSION

In this thesis I presented the Standard Model and discussed possible extensions, including the Left-Right Symmetric Model and the Little Higgs Model. The primary focus was the possibility of the existence of additional heavy quarks. Two ways in which these could be introduced were discussed.

The most straightforward method was found to be the introduction of an additional generation of quarks. In chapter 2.1 the parameterisation of an extended CKM matrix and boundaries on the new quark masses were described using data gleaned from recent literature as well as from earlier papers. It transpires that the allowed range of the new quark masses is narrow (2.1), since the quarks acquire their mass from the same Higgs doublet as the other six Standard Model ones. As the introduction of a fourth generation to the Standard Model is analysed extensively in the literature, no further calculations were required.

In chapter 2.1 the Lagrangian, the quark fields and the right-handed CKM matrix of the LRSM were extended so as to describe a Left-Right Symmetric Model with four generations of quarks. Given that the mass range of quarks would be roughly as narrow as the one for the fourth generation in the Standard Model, this possibility was again not explored any further.

The other promising method which allows the inclusion of a single additional quark is to introduce it as a vector-like isosinglet. This extra quark acquires the main contribution to its mass from another Higgs field and can therefore be far larger than the Standard Model quarks. Extensions of the Standard Model by an additional up-type or down-type quark have been explored in the literature, where the new particle field was directly coupled to a heavy mass scale. In this thesis the mass-giving Higgs field was explicitly introduced and an extra heavy up-type quark added to the Standard Model as well as to the LRSM. It transpires that the Top-quark can acquire its mass via a Higgs singlet when introduced to the Standard Model, whereas introducing a massive vector-like quark isosinglet to the LRSM requires an additional Higgs doublet. In effect, a model extended by an additional Top-quark receives two additional parameters: the Top-quark mass and the mixing angle θ.

In chapter 3.2.1 and 3.2.2 parameters of the LRSM with and without an additional heavy quark involved in the calculations of the B_d-meson mass-mixing, the CP-violating parameter ϵ_K and the B_s-meson decay to two muons were constrained when calculating those quantities including one-loop corrections.

It emerges that the B-meson mass-mixing and the CP-violating parameter only leave space for an LRS extension or an additional Top-quark if one allows the Standard Model parameters to alter. This alteration is a reasonable approach since the hadron matrix elements carry a large uncertainty and the determination of CKM matrix elements relies in part on the assumption of unitarity. Allowing the CKM matrix elements the possibility to alter by at most 10% gives the following two lower boundaries for the right-handed W-boson:

$$m_{W_R} > 2,2TeV \quad \text{(for region II and III)} \qquad \text{and} \qquad m_{W_R} > 2,5TeV \quad \text{(for region I and IV)}.$$

Using a slightly different approach, the same lowest boundary (2,5 TeV) has been derived in [43].

It was found that considering both the LRSM and an additional heavy quark leaves space for the new particles even without altering the Standard Model parameters, due to cancellations between the two new contributions. Therefore fixing the right-handed W-mass gives a an upper boundary to the Top-mass (table 3.4).

The mixing angle was constrained using the decoupling theorem such that one obtains the following relation for large Top-masses in the Standard Model extended by a Top-quark:

$$sin^2\theta < x_T^{-1/2}, \qquad \text{with} \qquad x_T = \frac{m_T^2}{m_{W_L}^2}.$$

and the tighter boundary in the LRSM extended by a Top-quark

$$sin^2\theta < (x_T ln x_T)^{-1/2}.$$

The last relation was used to derive a tendential lower boundary of the right-handed W-boson mass (table 3.5).

The analysis of the CP-violating parameter in the LRSM extended by a Top-quark shows that the quark mass sign choices can be split into two groups, one constraining the spontaneous CP-violating phase α to small and one to large values.

$$|rsin|\,\alpha \in [0, 0, 3) \qquad \text{or} \qquad |rsin|\,\alpha \in (0, 5, 1]$$

These two regions have also been found in [43] when only considering the LRSM but with tighter boundaries. Therefore this result again shows the enlarged space for new physics due to cancellations between the LRSM contributions and the Top-quark contributions.

In discussing the B-meson decay to two muons, the problem of imprecise hadron matrix elements and the model-dependent interpretation of measured CKM matrix element values was solved by introducing the new physics-measuring quantity A_s. This quantity divides the new contribution by that of the Standard Model and is (after some consideration of the magnitudes of the contributions due to different intermediate particles) independent of the hadron and the CKM matrix elements. It was shown that the branching ratio of the B-decay in the LRSM is enhanced for all choices of new parameters and all mass signs set

to plus one. Thus measuring the branching ratio could distinguish between the LRSM and the Standard Model. The enhancement becomes larger the smaller the W_R-mass and thus once the branching ratio is observed, a lower boundary on the right-handed W-mass can be derived. The maximum enhancement was shown to be around 20% for large α (region Ia, but a mass sign choice of region IIa) and 10% for small α (region IIa) when only considering one-loop corrections induced by the Standard Model and the right-handed W-boson.
A qualitative discussion on the ways in which to extend the calculation of the B-meson decay branching ratio in order to describe an additional Top-quark has been made. It was concluded that an enhancement of A_s is possible.

APPENDIX

A. INTEGRALS REQUIRED FOR LOOP CORRECTIONS

A.1 Integrals required for box diagrams

The function F_n (1.53) is solved explicitly for $n = 1, 2$.
The calculation is simplified using Feynman parameters. A general fraction can then be rewritten in the following way [1]:

$$\frac{1}{A_1^{m_1} A_2^{m_2} ... A_n^{m_n}} = \int_0^1 dx_1 ... dx_n \delta\left(\sum x_i - 1\right) \frac{\Pi x_i^{m_i - 1}}{[\sum x_i A_i]^{\sum m_i}} \frac{\Gamma(m_1 + ... + m_n)}{\Gamma(m_1) ... \Gamma(m_n)}$$

where x_i are the Feynman parameters. This equation can be shown explicitly for two Feynman parameters and the proof for n Feynman parameters can be obtained by induction. Applying this method to (1.53) gives

$$\begin{aligned} F_n(x_a, x_b, x_c, x_d) =& 2^{n/2}(-1)^{1-n/2} \int_0^1 dAdBdCdD \quad \delta(A + B + C + D - 1) \\ & \int \frac{d^4k}{(2\pi)^4} \frac{k^n}{[A(k^2 - x_a) + B(k^2 - x_b) + C(k^2 - x_c) + D(k^2 - x_d)]^4} \\ =& 2^{n/2}(-1)^{1-n/2} \int_0^1 dAdBdCdD \quad \delta(A + B + C + D - 1) \\ & \int \frac{d^4k}{(2\pi)^4} \frac{k^n}{[k^2 - (Ax_a + Bx_b + Cx_c + Dx_d)]^4}. \end{aligned}$$

The integration over the four-momentum k can be solved by Wick rotation which removes the minus sign of the time axis in the Minkowski metric by substituting

$$k^0 \equiv i k_E^0, \qquad \text{and} \qquad \vec{k} = \vec{k}_E.$$

This substitution is equal to a 90 degree rotation in the complex plane.
Changing the variable from k to k_E transforms the integral into Euclidean four-space and is therefore easily solvable. [1]

$$\begin{aligned} & F_n(x_a, x_b, x_c, x_d) \\ & = 2^{n/2}(-1)^{1-n/2} \int_0^1 dAdBdCdD \frac{\delta(A + B + C + D - 1)}{[A(k^2 - x_a) + B(k^2 - x_b) + C(k^2 - x_c) + D(k^2 - x_d)]^{2-n}} \end{aligned}$$

Performing the integral gives

$$\begin{aligned}
F_0(x_a,x_b,x_c,x_d) = {} & \frac{x_a lnx_a}{(x_a-x_b)(x_a-x_c)(x_a-x_d)} - \frac{x_b lnx_b}{(x_a-x_b)(x_b-x_c)(x_b-x_d)} \\
& + \frac{x_c lnx_c}{(x_c-x_d)(x_b-x_c)(x_a-x_c)} - \frac{x_d lnx_d}{(x_c-x_d)(x_b-x_d)(x_a-x_d)} \\
F_2(x_a,x_b,x_c,x_d) = {} & \frac{x_a^2 lnx_a}{(x_a-x_b)(x_a-x_c)(x_a-x_d)} - \frac{x_b^2 lnx_b}{(x_b-x_c)(x_a-x_b)(x_b-x_d)} \\
& + \frac{x_c^2 lnx_c}{(x_c-x_d)(x_b-x_c)(x_a-x_c)} - \frac{x_d^2 lnx_d}{(x_c-x_d)(x_b-x_d)(x_a-x_d)}.
\end{aligned}$$

These functions describe a loop with four different intermediate particles and are symmetrical under the exchange of any pair of particles.
The functions describing two different quarks and two similar gauge bosons or vice versa are obtained by the following limit:

$$F_n(x_a,x_b;x_c) = \lim_{x_d\to x_c} F_n(x_a,x_b,x_c,x_d)$$

These functions are only symmetrical under the exchange of the first two arguments which equates to the interchange of the two different quarks or the two different bosons.

$$\begin{aligned}
F_0(x_a,x_b;x_c) = {} & \frac{1}{x_a-x_b}\left[\frac{x_a lnx_a}{(x_a-x_c)^2} - \frac{x_b lnx_b}{(x_b-x_c)^2} + \frac{1}{x_b-x_c} - \frac{1}{x_a-x_c}\right] \\
& + lnx_c \frac{-x_c^2 + x_a x_b}{(x_b-x_c)^2(x_a-x_c)^2} \\
F_2(x_a,x_b;x_c) = {} & \frac{1}{x_a-x_b}\left[\frac{x_a^2 lnx_a}{(x_a-x_c)^2} - \frac{x_b^2 lnx_b}{(x_b-x_c)^2} + \frac{x_c}{x_b-x_c} - \frac{x_c}{x_a-x_c}\right] \\
& + x_c lnx_c \left[\frac{x_a}{(x_b-x_c)(x_a-x_c)^2} + \frac{x_b}{(x_b-x_c)^2(x_a-x_c)}\right]
\end{aligned}$$

Taking the limit of equal values in the first two arguments gives the contributions for loops including one type of quark and one type of gauge boson.

$$F_n(x_a,x_c) = \lim_{x_b\to x_a} F_n(x_a,x_b;x_c)$$

$$\begin{aligned}
F_0(x_a,x_b) = {} & \frac{x_a+x_b}{(x_a-x_b)^3}(lnx_b - lnx_a) + \frac{2}{(x_a-x_b)^2} \\
F_2(x_a,x_b) = {} & \frac{2x_a x_b}{(x_a-x_b)^3}(lnx_b - lnx_a) + \frac{x_a+x_c}{(x_a-x_c)^2}
\end{aligned}$$

If a box diagram in a theory with several W-bosons which only couple to left-handed fermions but have different masses is considered, the following function is needed:

$$c_0(x_a,x_b,x_c,x_d) = \left(1+\frac{x_a x_b}{4}\right) F_2(x_a,x_b,x_c,x_d) - 2x_a x_b F_0(x_a,x_b,x_c,x_d)$$

where x_a and x_b are functions of the masses of the quarks and x_c and x_d are functions of the masses of the gauge bosons.
In the case of two quarks and only the Standard Model boson the derived functions F_2 and F_0 can be inserted, which gives

$$c_0(x_i, x_j; 1) = \frac{x_i^2 lnx_i}{(x_i - x_j)(x_i - 1)^2}\left[1 - 2x_j + \frac{x_i x_j}{4}\right] \tag{A.1}$$

$$- \frac{x_j^2 lnx_j}{(x_i - x_j)(x_j - 1)^2}\left[1 - 2x_i + \frac{x_i x_j}{4}\right] \tag{A.2}$$

$$+ \frac{1}{(x_i - 1)(x_j - 1)}\left[1 - \frac{7x_i x_j}{4}\right] \tag{A.3}$$

with the limit

$$c_0(x; 1) = -xlnx\frac{8 + 24x + 45x^2}{4(x-1)^3} + \frac{4 + 4x - 15x^2 + x^3}{4(x-1)^2}. \tag{A.4}$$

Considering an N-dimensional CKM matrix the Wilson coefficients write

$$C_0 = \sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_i\lambda_j c_0(x_i, x_j; 1).$$

Writing down the first summand for each sum gives

$$C_0 = \lambda_1^2 c_0(x_1, x_1; 1) + \sum_{j=2}^{N}\lambda_1\lambda_j c_0(x_1, x_j; 1) + \sum_{i=2}^{N}\lambda_i\lambda_1 c_0(x_i, x_1; 1) + \sum_{i=2}^{N}\sum_{j=2}^{N}\lambda_i\lambda_j c_0(x_i, x_j; 1)$$

where x_1 is a function of the mass of the first generation quark, the u-quark.
Inserting the unitarity condition of the CKM matrix $\lambda_1 = -\sum_{i=2}^{N}\lambda_i$ gives

$$\begin{aligned} C_0 &= \sum_{i=2}^{N}\sum_{j=2}^{N}\lambda_i\lambda_j\left(c_0(x_1, x_1; 1) - c_0(x_i, x_1; 1) - c_0(x_1, x_j; 1) + c_0(x_i, x_j; 1)\right) \\ &= \sum_{i=2}^{N}\sum_{j=2}^{N}\lambda_i\lambda_j c_o'(x_i, x_j; 1). \end{aligned}$$

It is common procedure to approximate the mass of the u-quark as zero, since it is far smaller than all the other masses. Inserting $x_1 = 0$ into equation (A.3) and (A.4) gives

$$\begin{aligned} c_0'(x_i, x_j; 1) = x_i x_j \Bigg[&\frac{lnx_i}{(x_i - 1)^2(x_i - x_j)}\left(1 - 2x_i + \frac{x_i^2}{4}\right) - \frac{lnx_j}{(x_j - 1)^2(x_i - x_j)}\left(1 - 2x_j + \frac{x_j^2}{4}\right) \\ &- \frac{3}{4(x_i - 1)(x_j - 1)}\Bigg] \end{aligned}$$

with the limit

$$c_0'(x; 1) = \frac{x}{(x-1)^2}\left[1 - \frac{11x}{4} + \frac{x^2}{4} + \frac{3x^2 lnx}{2(x-1)}\right].$$

A.2 Integrals required for vertex corrections

Calculating the diagrams in figure 4.1 (c)-(j) one can initially set the external momentum to zero, since there are no intermediate d-type quarks and the limit is well-defined. The situation is more complicated when calculating the diagrams due to the quark self-energy (figure 4.1 (a) and (b)) and will therefore be performed explicitly (a similar derivation has been made in [5]).
In order not to arrive at a mathematically undefined s/d-quark propagator one must ascribe general momenta p_1 and p_2 to the external quarks. Afterwards one can set the quarks on mass shell ($p_1^2 = m_d^2$ and $p_2^2 = m_s^2$) and take the limit $m_s^2, m_d^2 \to 0$.
Considering the Higgs particle H_2 only the second diagrams of figure (a) and (b) contribute, therefore the Higgs scalar and the W-boson contribution are discussed separately.
First the following integral is defined:

$$\begin{aligned} I_{L,R,H}(p,m) &= \frac{p^\alpha}{p^2 - m^2} \int \frac{d^d k}{(2\pi)^d} \frac{k_\alpha}{(k^2 - m_i^2)((k-p)^2 - M_{W_{L,R,H}})} \\ &= \frac{p^\alpha}{p^2 - m^2} \int_0^1 dx \int \frac{d^d l}{(2\pi)^d} \frac{x p_\alpha}{(l^2 - \Delta_{L,R,H}(p^2))^2} \\ &= \frac{p^2}{p^2 - m^2} \tilde{I}_{L,R,H}(p^2) \end{aligned}$$

where the Feynman parameter description, dimensional regularisation and the substitutions

$$k_\alpha = l_\alpha + x p_\alpha \qquad \text{and} \qquad \Delta_{L,R,H}(p^2) = -x(1-x)p^2 + x M_{W_{L,R,H}} + (1-x) m_i^2$$

have been used. The following notation for the Z-boson current is also used:

$$J^\mu_{Z_L} = \frac{1}{c_W} \left[a_L \bar{u}_{Li} \gamma^\mu u_{Li} + b_L \bar{d}_{Li} \gamma^\mu d_{Li} + a_R \bar{u}_{Ri} \gamma^\mu u_{Ri} + b_R \gamma^\mu d_{Ri} \right]$$

where the coefficients $a_{L,R}$ and $b_{L,R}$ are written out in equation (B.2).
Thus one can write down the sum of diagram (4.1 (a) and (b)) where only the W-boson exchange is considered

$$(a)_{W_{L,R}} + (b)_{W_{L,R}} = \frac{g^3}{c_W} b_{L,R} (2 - \epsilon) \left[I_{L,R}(p_s, m_d) + I_{L,R}(p_d, m_s) \right] (\bar{s}_{L,R} \gamma^\mu d_{L,R}).$$

Setting the external quarks on shell ($p_{s,d}^2 = m_{s,d}^2$) gives

$$(a)_{W_{L,R}} + (b)_{W_{L,R}} = \frac{g^3}{c_W} b_{L,R} (2 - \epsilon) \frac{1}{m_s^2 - m_d^2} \left[m_s^2 \tilde{I}_{L,R}(m_s^2) - m_d^2 \tilde{I}_{L,R}(m_d^2) \right] (\bar{s}_{L,R} \gamma^\mu d_{L,R}).$$

Finally one can take the limit $m_s^2, m_d^2 \to 0$ where the following Taylor expansion of the integral function $\tilde{I}$ has to be inserted:

$$\tilde{I}_{L,R,H}(m^2 = 0) = \tilde{I}_{L,R,H}(0) + \sum_{n=1}^{\infty} \frac{\tilde{I}^{(n)}_{L,R,H}(0)}{n!} m^{2n}$$

It transpires that the only term left over is $\tilde{I}_{L,R}(0)$ since the term $(m_s^{2n} - m_d^{2n})/(m_s^2 - m_d^2)$ converges to zero. This can be seen by taking the limit of the quark masses one after another. Hence the contribution to the diagrams (a) and (b) due to the W-boson is given by

$$(a)_{W_{L,R}} + (b)_{W_{L,R}} = \frac{g^3}{c_W} b_{L,R}(2-\epsilon)\tilde{I}_{L,R}(0)(\bar{s}_{L,R}\gamma^\mu d_{L,R}).$$

The intermediate Higgs scalars G_L allow several contribution types, including contributions belonging to right-handed quark-states. The same is true for G_R producing contributions with initial left-handed quark states. However, it has been shown in [5] that all contributions proportional to integrals other than I go to zero for vanishing external quark momenta and masses. Thus the scalar contributions after taking this limit are given by

$$(a)_{G_{L,R}} + (b)_{G_{L,R}} = -\frac{2g^3}{c_W} b_{L,R} x_i \tilde{I}_{L,R}(0)(\bar{s}_{L,R}\gamma^\mu d_{L,R}).$$

The contribution resulting from the single-charged physical Higgs boson exchange is given by

$$(a)_H + (b)_H = -\frac{2g^3}{c_W} b_R x_i \tilde{I}_H(0)(\bar{s}_R\gamma^\mu d_R).$$

After rewriting the integrals for the discussed and all other diagrams using Feynman parameters and the Wick rotation, the integrals are solved in dimensional regularisation, meaning that the dimension is taken to be $d = 4 - \epsilon$. The resulting expression is subsequently expanded in ϵ such that all terms linear in the expansion variable are neglected. In performing these steps the following two expressions (taken from [1]) are used:

$$\begin{aligned}
\int \frac{d^d k_E}{(2\pi)^d} \frac{1}{(k_E^2 + \Delta)^n} &= \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2}} \\
\int \frac{d^d k_E}{(2\pi)^d} \frac{k_E^2}{(k_E^2 + \Delta)^n} &= \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 1}
\end{aligned}$$

The integrals required to calculate diagrams (a)-(j) are listed below, preceded by letters in parentheses denoting the diagrams in which they are used.

$$(a),(b): \quad \tilde{I}_{L,R,H}(0) = \frac{1}{(4\pi)^2} f_1^{L,R,H}(x_i)$$

$$\begin{aligned}
(c),(d): \quad &\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - m_i^2)^2 (k^2 - M_{W_L}^2)} = \frac{1}{(4\pi)^2}\left[f_2^L(x_i) - 1 + \frac{x_i}{(x_i-1)^2} ln x_i - \frac{1}{x_i - 1}\right] \\
(e),(h): \quad &\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - m_i^2)(k^2 - M_{W_L}^2)^2} = \frac{1}{(4\pi)^2}\left[f_2^L(x_i) - \frac{x_i}{(x_i-1)^2} ln x_i + \frac{1}{x_i - 1}\right] \\
(c),(d): \quad &\int \frac{d^d k}{(2\pi)^d} \frac{m_i^2}{(k^2 - m_i^2)^2 (k^2 - M_{W_L}^2)} = \frac{1}{(4\pi)^2}\left[\frac{x_i}{(x_i-1)^2} ln x_i - \frac{x_i}{x_i - 1}\right] \\
(f),(g): \quad &\int \frac{d^d k}{(2\pi)^d} \frac{m_i^2}{(k^2 - m_i^2)(k^2 - M_{W_L}^2)^2} = \frac{1}{(4\pi)^2}\left[-\frac{x_i^2}{(x_i-1)^2} ln x_i + \frac{x_i}{x_i - 1}\right]
\end{aligned}$$

$$
\begin{aligned}
(f),(g): &\int \frac{d^dk}{(2\pi)^d}\frac{m_i^2}{(k^2-m_i^2)(k^2-M_{W_R}^2)(k^2-M_{H_2}^2)} \\
&= \frac{x_i}{(4\pi)^2}\left[-\frac{x_i}{(x_i-x_{W_R})(x_i-x_{H_2})}lnx_i+\frac{x_{W_R}}{(x_i-x_{W_R})(x_{W_R}-x_{H_2})}\right. \\
&\qquad \left.-\frac{x_{H_2}}{(x_i-x_{H_2})(x_{W_R}-x_{H_2})}\right] \\
(i),(j): &\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-m_i^2)(k^2-M_{W_R}^2)(k^2-M_{H_2}^2)} \\
&= \frac{1}{(4\pi)^2}\left[\frac{2}{\epsilon}+C^L+1-\frac{x_i^2}{(x_i-x_{W_R})(x_i-x_{H_2})}lnx_i+\frac{x_{W_R}^2}{(x_i-x_{W_R})(x_{W_R}-x_{H_2})}\right. \\
&\qquad \left.-\frac{x_{H_2}^2}{(x_i-x_{H_2})(x_{W_R}-x_{H_2})}\right]
\end{aligned}
$$

with

$$f_1^{L,R,H_2}(x)=\frac{1}{\epsilon}+\frac{C^{L,R,H_2}}{2}+\frac{3}{4}-\frac{1}{2}\left[\frac{x^2}{(x-1)^2}lnx-\frac{1}{x-1}\right], \tag{A.5}$$

$$f_2^{L,R,H_2}(x)=\frac{2}{\epsilon}+C^{L,R,H_2}+1-\frac{x}{x-1}lnx, \tag{A.6}$$

$$C^{L,R,H_2}=-\gamma_E+ln(4\pi)-lnM_{W_{L,R},H_2}. \tag{A.7}$$

B. FEYNMAN RULES FOR VERTICES INCLUDING THE NEW LRSM GAUGE BOSONS

B.1 Feynman rules for the gauge bosons

The Feynman rules for the couplings of the gauge bosons to the quarks can be derived by inserting the covariant derivative (2.17) into the following Lagrangian:

$$L = \bar{Q}_{Li}(i\not{D})Q_{Li} + \bar{Q}_{Ri}(i\not{D})Q_{Ri}$$

Considering only the couplings which include a W-boson and inserting the rotation of the left- and right-handed quark fields (1.28) in order to achieve the quark mass eigenstates, the Lagrangian writes

$$\begin{aligned} L &= \frac{g}{\sqrt{2}} W^+_{L\mu} \bar{u}_{Li} V_{Lij} \gamma^\mu d_{Lj} + \frac{g}{\sqrt{2}} W^+_{R\mu} \bar{u}_{Ri} V_{Rij} \gamma^\mu d_{Rj} + h.c. \\ &= g \left(W^+_{L\mu} J^{\mu+}_{W_L} + W^+_{R\mu} J^{\mu+}_{W_R} \right) + h.c. \end{aligned} \tag{B.1}$$

with $J^{\mu+}_{W_L}$ and $J^{\mu+}_{W_R}$ given in (3.3) and (3.4), respectively.
Considering only the neutral boson part gives the following Lagrangian:

$$L = g(Z_{L\mu} J^\mu_{Z_L} + Z_{R\mu} J^\mu_{Z_R}) + eA_\mu J^\mu_{EM}$$

with

$$\begin{aligned} J^\mu_{Z_L} &= \frac{1}{c_W} \left[\left(\frac{1}{2} - \frac{2}{3} s^2_W \right) \bar{u}_{Li} \gamma^\mu u_{Li} + \left(-\frac{1}{2} + \frac{1}{3} s^2_W \right) \bar{d}_{Li} \gamma^\mu d_{Li} - \frac{2}{3} s^2_W \bar{u}_{Ri} \gamma^\mu u_{Ri} \right. \\ &\quad \left. + \frac{1}{3} s^2_W \bar{d}_{Ri} \gamma^\mu d_{Ri} \right] \end{aligned} \tag{B.2}$$

$$\begin{aligned} J^\mu_{Z_R} &= \frac{\sqrt{c^2_W - s^2_W}}{c_W} \left[-\frac{1}{6} \frac{s^2_W}{c^2_W - s^2_W} \bar{u}_{Li} \gamma^\mu u_{Li} - \frac{1}{6} \frac{s^2_W}{c^2_W - s^2_W} \bar{d}_{Li} \gamma^\mu d_{Li} \right. \\ &\quad \left. + \left(\frac{1}{2} - \frac{1}{6} \frac{s^2_W}{c^2_W - s^2_W} \right) \bar{u}_{Ri} \gamma^\mu u_{Ri} + \left(-\frac{1}{2} - \frac{1}{6} \frac{s^2_W}{c^2_W - s^2_W} \right) \bar{d}_{Ri} \gamma^\mu d_{Ri} \right] \\ J^\mu_{EM} &= \frac{2}{3} \bar{u}_{Li} \gamma^\mu u_{Li} - \frac{1}{3} \bar{d}_{Li} \gamma^\mu d_{Li}. \end{aligned} \tag{B.3}$$

The left-handed current (B.2) and the electromagnetic current (B.3) are the same as those derived in the Standard Model by means of the neutral boson part of the Standard Model covariant derivative (1.6).

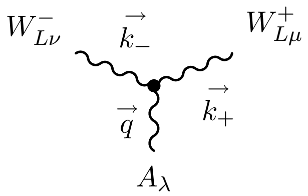
The triplet and quartic gauge boson coupling is introduced to the Lagrangian via the square of the field tensor

$$L = -\frac{1}{4} F_{Li\mu\nu} F^{\mu\nu}_{Li'} - \frac{1}{4} F_{Ri\mu\nu} F^{\mu\nu}_{Ri'}$$

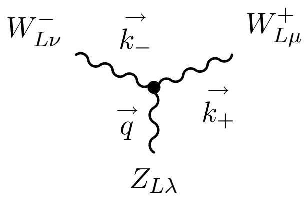
with

$$F_{L,Ri\mu\nu} = \partial_\mu W_{L,Ri\nu} - \partial_\nu W_{L,Ri\mu} + g\epsilon^{ijk} W_{L,Rj\mu} W_{L,Rk\nu}.$$

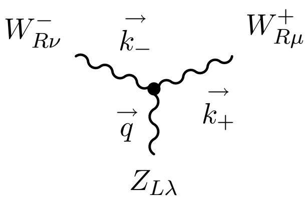
Considering only the triplet coupling results in the same outcome as in the Standard Model for the squared left-handed field tensor since W_{L1} and W_{L2} form the Standard Model W-bosons and $W_{L3} = c_W Z_L + s_W A$. This also means that there is no three gauge boson vertex at which the left-handed W-boson couples to Z_R. This, however, is caused by the fact that the mixing between Z_L and Z_R has been neglected otherwise there would be $(W_L W_L Z_1)$- and $(W_L W_L Z_2)$-vertices. The same holds for the W-bosons. In this approximation the left- and right-handed gauge bosons do not couple to each other.
Inserting the physical fields (2.14) into the Lagrangian mentioned above yields the following Feynman rules, where in all graphs the momenta are pointing inwards:



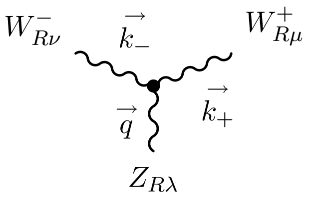
$$= ie[g_{\mu\nu}(k_+ + k_-)_\lambda + g_{\nu\lambda}(k_- - q)_\mu + g_{\lambda\mu}(q - k_+)_\nu]$$



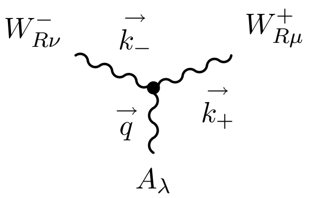
$$= igc_W[g_{\mu\nu}(k_+ + k_-)_\lambda + g_{\nu\lambda}(k_- - q)_\mu + g_{\lambda\mu}(q - k_+)_\nu]$$



$$= -ig\frac{s_W^2}{c_W}[g_{\mu\nu}(k_+ + k_-)_\lambda + g_{\nu\lambda}(k_- - q)_\mu + g_{\lambda\mu}(q - k_+)_\nu]$$



$$= -ig\frac{\sqrt{c_W^2 - s_W^2}}{c_W}[g_{\mu\nu}(k_+ + k_-)_\lambda + g_{\nu\lambda}(k_- - q)_\mu + g_{\lambda\mu}(q - k_+)_\nu]$$



$$= igs_W[g_{\mu\nu}(k_+ + k_-)_\lambda + g_{\nu\lambda}(k_- - q)_\mu + g_{\lambda\mu}(q - k_+)_\nu]$$

B.2 Feynman rules of the single-charged Higgs fields

There are four single-charged Higgs fields which enter the Lagrangian: $\phi_1^\pm$ and $\phi_2^\pm$ over the Higgs bidoublet (2.4), $\delta_R^\pm$ over the right-handed triplet Δ_R and $\delta_L^\pm$ over the left-handed triplet Δ_L (2.3). These four fields are not the physical mass eigenstates. In order to calculate those, the contributions due to traces of the squares of the covariant derivatives (2.6, 2.7, 2.8) and due to the Higgs potential (2.9) must be diagonalised. Since this calculation is fairly long but not particularly difficult, the result from [43] is used without further proof. It transpires that δ_L^+ does not mix with the other Higgs fields and is already the mass eigenstate of the massive Higgs boson H_1^+ with the following mass:

$$H_1^+ = \delta_L^+ \qquad \text{with} \qquad m_{H_1}^2 = (\rho_3 - 2\rho_1)v_R^2 + \frac{1}{2}\alpha_3\kappa^2$$

The other three Higgs fields combine to give another massive Higgs field H_2^+ and the longitudinal modes of the single-charged gauge bosons (W_L^+ and W_R^+) G_L^+ and G_R^+

$$H_2^+ = \phi_2^+ + \chi e^{i\alpha}\phi_1^+ + \epsilon\delta_R^+ \qquad \text{with} \qquad m_{H_2}^2 = \alpha_3(v_R^2 + \frac{1}{2}\kappa^2) \tag{B.4}$$

$$G_L^+ = -\chi e^{-i\alpha}\phi_2^+ + \phi_1^+ \qquad \text{with} \qquad m_{G_L}^2 = \xi m_{W_L}^2 \tag{B.5}$$

$$G_R^+ = -\epsilon\phi_2^+ + \delta_R^+ \qquad \text{with} \qquad m_{G_R}^2 = \xi m_{W_R}^2 \tag{B.6}$$

with the gauge parameter ξ which in these calculations is always taken as one (Feynman-t'Hooft gauge) and the abbreviations $\chi = \kappa'/\kappa$ and $\epsilon = \kappa/(\sqrt{2}v_R)$. The masses of the single-charged massive gauge bosons depend, as expected, on the vev of the Higgs fields and the

parameters of the Higgs potential.
The inverse transformation between the original and physical Higgs fields is given by

$$\begin{aligned}
\phi_2^+ &= \frac{1}{1+\chi^2+\epsilon^2}\left(H_2^+ - \chi e^{i\alpha}G_L^+ - \epsilon G_R^+\right) \approx H_2^+ - \chi e^{i\alpha}G_L^+ - \epsilon G_R^+ \\
\phi_1^+ &= \frac{1}{1+\chi^2+\epsilon^2}\left(\chi e^{-i\alpha}H_2^+ + (1+\epsilon^2)G_L^+ - \epsilon\chi e^{-i\alpha}G_R^+\right) \approx \chi e^{-i\alpha}H_2^+ + G_L^+ \\
\delta_R^+ &= \frac{1}{1+\chi^2+\epsilon^2}\left(\epsilon H_2^+ - \epsilon\chi e^{i\alpha}G_L^+ + (1+\chi^2)G_R^+\right) \approx \epsilon H_2^+ + G_R^+.
\end{aligned} \tag{B.7}$$

The approximations are necessary since the original transformations (B.6) taken from [43] are only up to single orders in ϵ and χ accurate.
The quark couplings to the Higgs fields emerge from the Yukawa couplings to the Higgs bidoublet. Thus δ_R^+ does not couple to the quarks. Considering only the charged components ϕ_1^- and ϕ_2^+ of the Higgs bidoublet and inserting the bidoublet into the Lagrangian describing the Yukawa interaction (2.18) for three generations of quarks gives

$$L = h_{ij}\left(\bar{u}_{Li}\phi_2^+ d_{Rj} + \bar{d}_{Li}\phi_1^- u_{Rj}\right) - \tilde{h}_{ij}\left(\bar{u}_{Li}\phi_1^+ d_{Rj} + \bar{d}_{Li}\phi_2^- u_{Rj}\right) + h.c..$$

The matrices h and $\tilde{h}$ can be rewritten as (chapter 2.2.5)

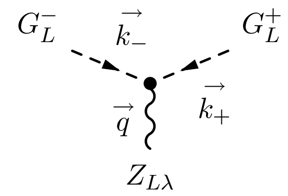
$$h = \frac{1}{\kappa}S_U\hat{M}_U \qquad \text{and} \qquad \tilde{h} = \frac{1}{\kappa}M_D - \frac{\kappa'}{\kappa^2}e^{i\alpha}S_U\hat{M}_U.$$

The parameter χ is determined by the largest quark mass-difference between the masses in one generation and is therefore of the order 10^{-2}. The order of ϵ is even smaller, considering the constraint on the W_R mass $m_{W_R} > 2,5$ TeV [43]. Therefore any contributions of order χ^2, ϵ^2, $\chi\epsilon$ and higher orders are neglected from now on.
Inserting the Higgs fields, the mass matrices and the rotated (to their mass eigenstates) quark fields (2.21) gives the Lagrangian for the quark coupling with the massive charged Goldstone bosons and the single-charged physical Higgs field

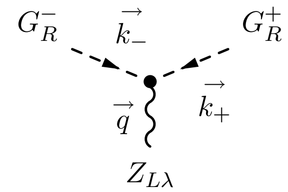
$$\begin{aligned}
L = \quad & \frac{g}{\sqrt{2}M_{W_L}}\bar{u}_i G_L^+(s_{u_i}m_{u_i}\gamma_L - m_{d_j}\gamma_R)V_{Lij}d_j && \text{(B.8)}\\
&+ \frac{g}{\sqrt{2}M_{W_R}}\bar{u}_i G_R^+(s_{u_j}m_{d_j}\gamma_L - m_{u_i}\gamma_R)V_{Rij}d_j && \text{(B.9)}\\
&- \frac{g}{\sqrt{2}M_{W_L}}\bar{u}_i H_2^+(s_{u_j}m_{d_j}\gamma_L - m_{u_i}\gamma_R)V_{Rij}d_j + h.c. && \text{(B.10)}
\end{aligned}$$

where the masses of the left- and right-handed W-boson are given in (2.11).

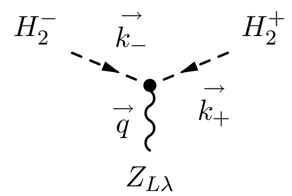
The coupling of the massive Goldstone bosons with other massive Goldstone bosons or gauge bosons enters the Lagrangian via the traces of the square of the covariant derivatives (2.6, 2.7, 2.8) where the approximate physical gauge and Higgs fields (2.14, B.7) must be inserted. In order to be consistent all higher orders of $\epsilon \approx M_{W_L}/M_{W_R}$ than order one have been neglected.



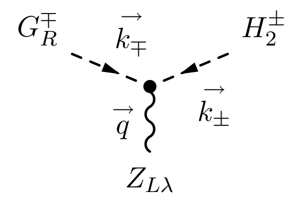
$$= \frac{ig}{2c_W}(c_W^2 - s_W^2)(k_- - k_+)_\lambda \tag{B.11}$$



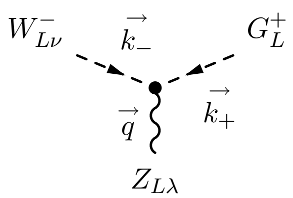
$$= -ig\frac{s_W^2}{c_W}(k_- - k_+)_\lambda \tag{B.12}$$



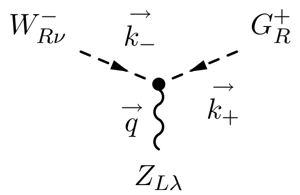
$$= \frac{ig}{2c_W}\left(1 - 2s_W^2\right)(k_- - k_+)_\lambda \tag{B.13}$$



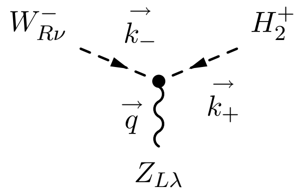
$$= \mp\frac{ig}{c_W}\frac{M_{W_L}}{M_{W_R}}(k_- - k_+)_\lambda \tag{B.14}$$



$$= ig\frac{s_W^2}{c_W}M_{W_L}g_{\nu\lambda} \tag{B.15}$$



$$= -ig\left(\frac{M_{W_L}^2}{M_{W_R}} c_W - M_{W_R}\frac{s_W^2}{c_W}\right) g_{\nu\lambda} \tag{B.16}$$



$$= \frac{ig}{c_W} M_{W_L} g_{\nu\lambda} \tag{B.17}$$

Comparing the Feynman rules (B.11, B.12, B.15, B.16) with those derived in [65] it transpires that (B.12, B.16) differ from the result given in [65]. This might be explained by different parametrisations of the Higgs or gauge fields. However, even if the Feynman rules in this thesis are not correct, their usage for loop diagram calculations should still be assured since the false terms are smaller than or of the order ϵ and (when used in loop diagrams) multiplied by at least another term of the order ϵ and therefore in these calculations neglected.
It is observable that (to the chosen order of perturbation theory) the longitudinal modes of the left- and right-handed W-bosons G_L and G_R couple neither to each other, nor to the right- and left-handed W-boson respectively. The single-charged Higgs boson H_2 behaves similarly to the right-handed Goldstone boson G_R, it does not couple to W_L and G_L but it does couple to W_R and G_R.

C. BOX DIAGRAM IN THE LRSM EXTENDED BY A HEAVY TOP-QUARK

The calculation of the box diagram-mixing in the LRSM extended by a heavy Top-quark is analogous to the calculation of the box diagram in the Standard Model performed in chapter 1.6.3. Since the number of intermediate particles increases, more diagrams must be taken into account. According to [43] the main contribution except for the Standard Model contribution with two left-handed W-bosons as intermediate particles is the contribution due to a left- and a right-handed W-boson as intermediate particles which in the Feynman-t'Hooft gauge can be presented as eight diagrams.

$$A = A_{W_L W_R} + A_{W_R W_L} + A_{W_L \phi_R} + A_{\phi_R W_L} + A_{W_R \phi_L} + A_{\phi_L W_R} + A_{\phi_L \phi_R} + A_{\phi_R \phi_L}$$

Inserting the Feynman rules (Appendix B) and using the notation (1.53) to refer to the functions derived in Appendix A and using the rule vertex= $-2iH_{eff}$ gives

$$\begin{aligned} H_{eff} =& \frac{G_F^2 M_{W_L}^2}{2\pi^2} \sum_{i,j=u,c,t,T} V_{Liq_2}^* V_{Riq_1} V_{Riq_2}^* V_{Liq_1} \eta_{LRij} \sqrt{x_i x_j} \\ & \left[\left(\frac{x_i x_j}{x_{W_R}} + 4 \right) F_0(x_i, x_j, x_{W_R}, 1) - \left(1 + \frac{1}{x_{W_R}} \right) F_2(x_i, x_j, x_{W_R}, 1) \right] O_{q_1 q_2 LR} \end{aligned}$$

with

$$O_{q_1 q_2 LR} = (\bar{q}_1 \gamma_L q_2)(\bar{q}_1 \gamma_R q_2).$$

The quarks q_1 and q_2 are determined by the considered meson and the correction factors η_{LRij} are analogous to the correction factors in the Standard Model. The different values for η depending on the meson-mixing process and the operator $O_{q_1 q_2}$ are listed in paper [68]. The effective Hamiltonian is equivalent to the Hamiltonian derived in [43]. The expectation value of the operator $O_{q_1 q_2 LR}$ depends on the considered meson and is for the cases of K^0-$\bar{K}^0$- and B_q-$\bar{B}_q$-mixing ($q = s, d$) given by [43]

$$\begin{aligned} \langle K^0 | \, O_{sdLR} \, | \bar{K}^0 \rangle &= -\frac{1}{2} m_K f_K^2 B_4(M) \left(\frac{m_K}{m_s(M) + m_d(M)} \right)^2 \\ \langle B_q | \, O_{qbLR} \, | \bar{B}_q \rangle &= -m_{B_q} f_{B_q}^2 B_4^q(M) \left[\frac{1}{12} + \frac{1}{2} \left(\frac{m_{B_q}}{m_b + m_q} \right)^2 \right] \end{aligned}$$

with the following constants taken from the Particle Data Group [8]:

$$m_K = 497,614 \pm 0,024 MeV, \qquad m_{B_s} = 5366.3 \pm 0.6 MeV, \qquad m_{B_d} = 5279.50 \pm 0.03 MeV$$

and [43]

$$m_s = 98MeV, \qquad m_d = 5MeV, \qquad m_b = 4.2GeV.$$

The values for $f\sqrt{B_4}$ are the ones with the largest uncertainty. The values predicted differ between 200±40 MeV [5] and 244±26 MeV [8]. The CKM elements are also provided with a certain error. Since the mass-difference of the mesons is measured very precisely it can be used to constrain either $f\sqrt{B_4}$ or the CKM matrix elements involved. Commonly Δ_{B_d} is used to constrain $|V_{td}|$. In order to achieve exactly the measured value of Δ_{B_d} for the Standard Model case while using the CKM matrix elements given by the Particle Data Group the value $f\sqrt{B_4}$ $=$ 205,2 MeV is chosen. The choice is justified by the argument that the calculations performed in this thesis are only of a systematic nature in order to determine the space left for an extra quark and are not performed in order to achieve exact values.
The value of M should be taken around the mass of the original particle, the meson. It is convenient to take $M = 5$ GeV for B_q-$\bar{B}_q$-mixing and $M < O(m_c)$ for K^0-$\bar{K}^0$-mixing [16].

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