Intergenerational Effects of Guaranteed Pension Contracts [*]

Trond M. Døskeland   Helge A Nordahl

September 28, 2006

Abstract

In this paper we show that there exist an intergenerational cross-subsidization effect in guaranteed interest rate life and pension contracts as the different generations partially share the same reserves. Early generations build up bonus reserves, which are left with the company at expiry of the contract. These bonus reserves function partly as a subsidy of later generations, such that the latter earn a risk-adjusted return above the risk-free rate. Furthermore, we show that this subsidy may be large enough to explain why late generations buy guaranteed interest rate products, which otherwise would not have been part of the optimal portfolio allocation.

Keywords: Portfolio Choice; Life and Pension Insurance, Interest Rate Guarantees
JEL Classification: G11, G13, G22

1 Introduction

Many households rely on life and pension (L&P) contracts to finance their retirement expenditures. We show that there exist an intergenerational cross-subsidization effect in guaranteed interest rate L&P contracts as the different generations partially share the same reserves. Because of the bonus reserves described below, early generations subsidize later generations, such that the latter earn a risk-adjusted return above the risk-free rate.

Previous research on L&P contracts has focused on the risk sharing between one customer and the company as well as pricing one customer's claim on the company. However, at any given point of time, the customer base of a company consists of many customers at different stages of the contract life cycle. We will focus this paper on the relationship between different generations of customers and the company.

We use a simple contract with annual guaranteed return in the fashion of Miltersen and Persson (2003). There is a large literature pricing L&P contracts. Some contributions illustrating different design of contracts are papers by e.g. Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Miltersen and Persson (2003). Our main complicating element will be the existence of a bonus reserve, which consists of funds allocated, but not yet guaranteed, to customers. This bonus reserve may or may not be individualized, so that customers may only receive a part, if any, of their proportional share at the expiry of the contract.

In a setting where different generations share the same reserves, at least two sources of cross-subsidization may occur. The bonus reserve is typically left in the company from the expiring generation to future generation. However, in the other direction is the effect that the new generation in a default scenario may end up paying a part of the obligation to the old generation.

To test this hypothesis we calculate the expected risk-adjusted return (under the equivalent martingale measure Q) of the contract for each generation, given that the company is not able to extract any capital beyond the risk-adjusted return on capital. We find that the former effect is the larger, such that the net effect is that the later generations end up with the higher return.[1] The difference in return seems to be fairly small on an annual basis, but still significant over the lifetime of the contract. Changes in parameters have an impact on the size of the cross-subsidization, in particular the spread between risk-free interest rate and the guaranteed rate of return, as well as the crediting rate of the bonus reserve and the asset allocation contributes to these changes. The youngest generations benefit from high spreads, high allocations to the bonus reserves, and a conservative asset allocation, while opposite is the case for previous generations.

Risk-adjusted return different from cost of capital may in general raise the issue of arbitrage. However, in this case investors are normally households with a limited set of investment opportunities. Furthermore, shortselling pension contracts is not normally feasible,

---

[1]In our calibrated benchmark case bankruptcies only play a marginal role.

even though some repurchasing arrangements exists.

Most L&P companies have existed for ages.[2] At the time of investment of the first generations (which we will later show end with a return below cost of capital) only a limited set of investment opportunities existed, compared to today's market. Furthermore, there were no closed-fund investment opportunities, hence they had no way to contractually prevent new generations from entering the customers' fund of the L & P company.

Among the latest generations one could think of an arbitrage opportunity of buying the pension product and shortselling a replicating portfolio. However, L & P companies in general only allow for private investors or beneficiaries with limit investments, hence the shortselling capacity in a replicating portfolio is limited, and transaction costs will be high, particularly since the replicating portfolio also needs to be continuously rebalanced. Furthermore, L & P companies typically have a low degree of transparency in their investment, and therefore it will be difficult to find the optimal arbitrage strategy.

The cross-subsidization described above also provides an alternative explanation of the problem "why do households buy pension insurance?" Only a few papers look at the welfare effects of the contracts. Brennan (1993) elaborates on the classical point made by Borch (1962) that guaranteed products will lead to a welfare loss. According to Borch, we cannot explain the existence of these saving vehicles within an one-generation expected utility framework with HARA utility, as these contracts have a non-linear pay-out function. Jensen and Sørensen (2001), Consiglio, Saunders, and Zenios (2006) and Døskeland and Nordahl (2006) build on this point by quantifying the effects in various cases of interest rate guarantees. However, previous research assume that all generations receive the same return. In the second part of this paper we expand the previous welfare studies by testing the impact of generation-based return in an individuals' portfolio choice model. Customers can choose between investing directly or indirectly via the guaranteed products.

We find that even when assuming standard preferences (constant relative risk aversion) utility maximization shows that pensions will be part of optimal portfolio. New generations benefit from the cross-subsidization. In addition, the return depends on market return during the same period as investment, but also during previous periods. Because of this, there will be an intergenerational diversification effect reducing the risk.

Explaining the choices of the previous generations is more difficult in our model. However, while today there exist a wide set of opportunities, previous generations clearly had a limited choice. It may be that in previous times guaranteed rate life and pension insurance products were purchased simply due to the lack of other alternatives.

The rest of our paper is organized as follows. In chapter 2 we describe the multi-generation model of a pension insurance product. Chapter 3 provides support for our parametrization of the model. The numerical results of the cross-subsidization are given in chapter 4, along with selected sensitivities. In chapter 5 we provide a simple portfolio choice model and

---

[2]Even though mergers and acquisitions frequently occur, the portfolios tend to prevail.

show optimal asset allocation for each generation, while chapter 6 gives the conclusion and suggestions for further research in this area.

2 The Model

Parameters of the Model	Notation
The Economy	
Value of one unit of the equity index at time t where all dividends are immediately reinvested into the index	S_t
Value of one unit of the risk-free bond account at time t	D_t
Constant risk-free rate	r
Constant expected return on the equity index	μ
Constant volatility of the equity index	σ
Standard Brownian motion	Z_t
The Company	
Value of the total asset portfolio at time t	A_t
Constant proportion of total asset portfolio, A_t, invested in the equity index	θ
The equity of the company at time t	E_t
The mathematical reserves (customers' funds) at time t	L_t
The bonus account (bonus reserves) at time t (to be further described)	B_t
Final wealth from insurance product to generation $t - T$ at time t	I_t^{t-T}
Cashflow to the investors of the company at time t	CF_t
The proportion of equity to total assets at time $t = 1$ ($\alpha = \frac{E_1}{A_1}$)	α
The constant guaranteed rate	g
The proportion split of the remaining total profit (after guarantee and "α" split) to the households	δ
The effective proportion held in the equity index at time t	Θ_t
Time of bankruptcy	τ
Proportion of declared bonuses credited the bonus reserves B_t	b
Payout ratio of bonus reserve	p
Discounted value of all cash flows	V
The Households	
Coefficient of relative risk aversion	γ
Overlapping generations of households, indexed by $h = 1, 2, \ldots, H$	H
Term of the policy	T
Wealth at time t of generation h	W_t^h
The mathematical reserves of households of age h at time t	L_t^h
Growth rate of the households' aggregate initial investments	v

Table 1: Definitions

In this section we formalize the modelling framework. We first describe the economy, then the L& P company including the insurance contract offered to households and the households.

2.1 The Economy

We assume a standard no-arbitrage economy with two assets, a risk free bank account, D_t and a risky equity index, S_t. The dynamics of the asset classes is then given by:

$$dD_t = rD_t dt, \qquad D_0 = d \tag{1}$$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t, \qquad S_0 = s \tag{2}$$

where r is the constant risk-free interest rate, μ is the constant expected return on the equity index, σ is the constant volatility of the equity index, and Z_t is a standard Brownian motion.
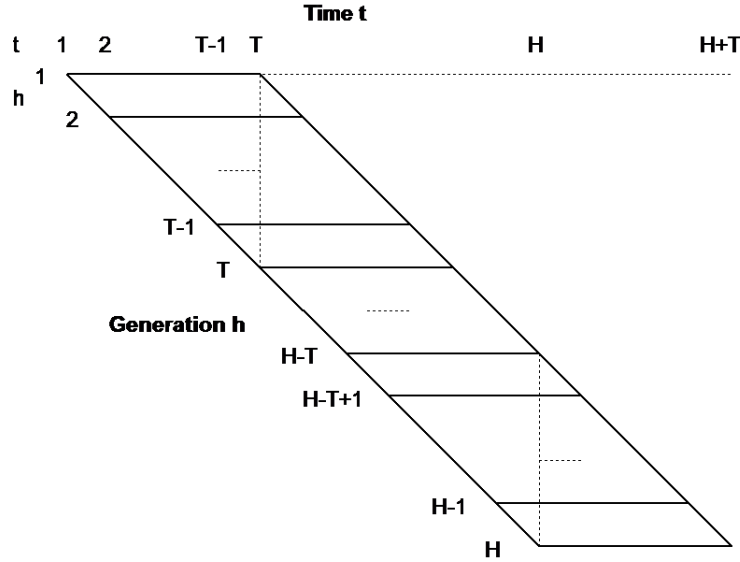

Figure 1: Generations

We create a model with H overlapping generations of households, indexed by $h = 1, 2, \ldots, H$. On the x-axis in figure 1 the different generations are listed. The y-axis illustrates the time line. Each generation uses the pension system for T periods. The wealth of the household at time T of generation h is given by W_T^h. We explore the implications of heterogeneity across generations.

2.2 The Company

We assume that there exists a financial intermediary, which we will refer to as "the company", offering pension contracts. The balance sheet development of the company is illustrated in tables 2 and 3. At time 1 only shareholders and generation 1 has invested in the company. The equity is then a proportion α of the total assets, A_1, of the company. The bonus reserve B_1 is zero at the initiation of the company. At the end of each subsequent year shareholders

Assets	Liabilities
	$E_1 = \alpha A_1$
A_1	$B_1 = 0$
	$L_1 = (1-\alpha)A_1$

The table shows the balance sheet of the insurer at the set up date.

Table 2: Balance sheet at time $t = 1$

Assets	Liabilities
	$E_t = \alpha A_t$
A_t	B_t
	L_t^1
	L_t^2 $\left.\begin{array}{c} \end{array}\right\} L_t$
	...
	L_t^H

The table shows the balance sheet of the insurer at time t.

Table 3: Balance sheet at time t

will keep the proportion of equity constant at the level α, either by taking out dividends or by paying in capital. Furthermore, there will be a bonus reserve B_t, and all generations will have their specific allocated reserve L_t^h for generations $h = 1 \ldots H$. The sum of reserves for all generations is labelled L_t. The balance sheet at time t is shown in table 3.

A proportion θ_t of the company's assets is invested in the equity index. We will assume that the proportion of the equity index is fixed, i.e. that $\theta_t = \theta$. The dynamics of the total asset portfolio A_t under the real probability measure P is then given by

$$dA_t = (rA_t + \theta(\mu - r)A_t)dt + \theta A_t \sigma dZ_t, \qquad A_1 = a. \tag{3}$$

In the discrete world we transform equation (3) to

$$A_t^{cum} = A_{t-1}e^{(r+\theta(\mu-r)-\frac{1}{2}\theta^2\sigma^2+\theta\sigma dZ_t)} \tag{4}$$

or under the equivalent martingale measure, Q:

$$A_t^{cum} = A_{t-1}e^{(r-\frac{1}{2}\theta^2\sigma^2+\theta\sigma dZ_t)}. \tag{5}$$

Superscript cum indicates values before annual settlements.

2.3 The Pension Contract

In the setup of our model, we concentrate on the financial features of the contract, regarding the savings element as the most important.[3] Even though there are no international standard contracts we try to make a simplified contract that will be close to products sold in most European (and some non-European) countries.

We assume that all parameters of the contracts are fixed at set up date. Furthermore, we assume correct initial pricing from the company's perspective, meaning that the company's average risk-adjusted return over the whole life time equals the risk-free rate.

The basis for calculating return to customers is the guaranteed rate of return. We use annual guarantees rather than life-time guarantees[4], hence each year the reserves will increase by a fixed rate g. In addition to that, customers may receive a bonus if the total return on customers' reserves and equity of the company exceeds g. The bonus will then be the surplus, less a proportional share $(\alpha\delta)$ to shareholders to compensate for capital inserted and risk assumed by shareholders, as well as a proportional share b to the bonus reserves described below.

In order to provide buffers for companies to meet bad years in the security markets, regulators in most countries allow for (and to a certain extent require) the build up of buffers of capital that are yet to be allocated to customers' reserves. These buffers have different forms, importance and names from country to country, e.g. bonus reserves, value adjustment reserves, unrealized gains (reserves), fund for future appropriations, etc. We name them bonus reserves, B_t. Bonus reserves can be used if the achieved return is not sufficient of covering guaranteed returns.[5] Furthermore, bonus reserves are not allocated to any specific generation. At the expiry of the contract, each generation will only be able to extract a part p of their proportional share of the bonus reserves.

In our model we credit the bonus reserves by a proportion of declared bonuses, b. Figure 2 illustrates the allocation rules. The bottom part of the return covers the guaranteed amount. If returns exceeds the level of the guarantee, an amount will be used to cover a similar return on shareholders' capital. Then, if there still is something left, the remaining return will be split proportionally between equity, reserves, and bonus reserves.

The model is initiated at time $t = 1$, where we define

---

[3]This means we will not cover pure actuarial risk elements, like mortality risk, disability risk, longevity risk, etc, or any type of administrative costs. Neither will we cover any part of the premium set aside to cover such elements, which means that we assume that the full initial payment from customers go into a form of savings account. Our contract will be based on a single premium and a single payment at expiry of the contract.

[4]As used e.g. by Grosen and Jørgensen (2000).

[5]In practice, allocation to bonus reserves is done in a number of ways, e.g. through allocating a proportion of bonuses each year, allocating unrealized gains on various types of securities, increasing the funds in the same rate as the other reserves, bringing the bonus reserve to a target level, etc. In order to find a common model, we use the simple allocation mechanism (proportion of declared bonuses) described by Miltersen and Persson (2003).
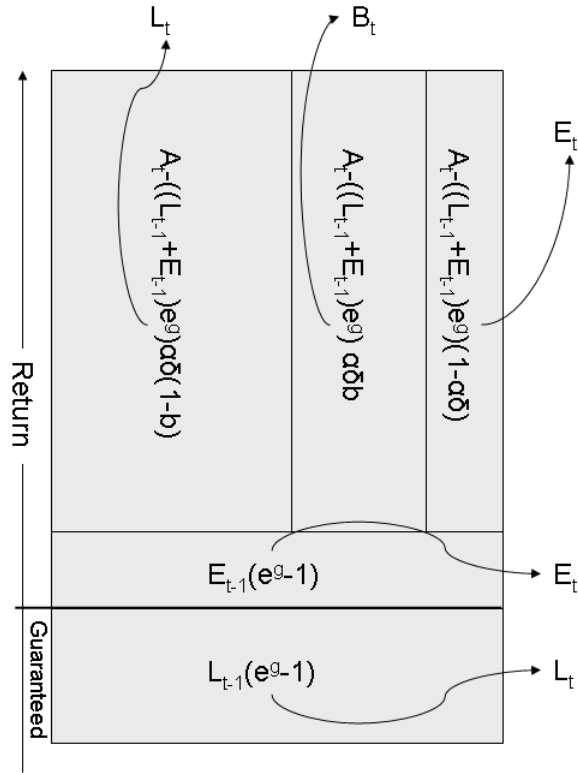
The figure illustrates how return is split between different types of capital. The first part of the return (guarantee) is allocated to the customer reserve, then a part is allocated to equity, while return above is split between customer reserve, bonus reserve, and equity.

Figure 2: Allocation of return

$$L_1^1 = L_1 = \alpha A_1$$
$$E_1 = (1 - \alpha)A_1 \qquad (6)$$
$$B_1 = 0.$$

We then initiate each generation at time $t = h$, when generation h does their investment:

For all t and h such that $t = h$:

$$L_t^h = L_1^1(1 + v)^{h-1} \qquad (7)$$

where v is growth rate of the households' aggregate initial investments.

Each year after the initial year, generation h will lose the guaranteed amount if and only if total assets in the company is insufficient to cover the guaranteed amounts of generation

8

h and all previous generations.[6] Furthermore, if assets are sufficient to cover the guaranteed amount to all generations, a corresponding return on the equity, and preservation of the bonus reserve, the customers earn a bonus. The bonus is the generation's proportional share of the reserves, multiplied by the customers' proportional share of the capital (α), the customers' share of profits (δ), the share being credited the reserves ($1 - b$), and the surplus in itself. Hence, for all t and h such that $h < t \leq h + T$

$$
L_t^h = \begin{cases} 0 & \text{if } A_t \leq \sum_{j=1}^{h-1} L_{t-1}^j e^g \\ A_t - \sum_{j=1}^{h-1} L_{t-1}^j e^g & \text{if } A_t \leq \sum_{j=1}^{h} L_{t-1}^j e^g \\ L_{t-1}^h e^g & \text{if } A_t \leq (L_{t-1} + E_{t-1})e^g + B_{t-1} \\ L_{t-1}^h e^g + \frac{L_{t-1}^h}{L_{t-1}} \alpha\delta(1-b)(A_t - (L_{t-1} + E_{t-1})e^g + B_{t-1}) & \text{if } A_t > (L_{t-1} + E_{t-1})e^g + B_{t-1}. \end{cases}
$$
(8)

Finally, we let L_t^h be zero at all times where generation h has no investments:

For all t and h such that $t > h + T$ or $t < h$

$$
L_t^h = 0. \tag{9}
$$

We can then sum all reserves before cashflows made at the year end (marked by superscript cum):

$$
L_t^{cum} = \sum_{j=1}^{t-1} L_t^j. \tag{10}
$$

The bonus account is used when return on assets are not sufficient to cover the guarantee. Hence, if assets at the end of the year is low, the bonus account will be zero, or at least lower than the previous year. If assets are high, the bonus account will be credited a proportion b of the total bonus to customers. More formally,

$$
B_t^{cum} = \begin{cases} 0 & \text{if } A_t \leq L_{t-1}e^g + E_{t-1} \\ A_t - L_{t-1}e^g - E_{t-1} & \text{if } A_t \leq L_{t-1}e^g + E_{t-1} + B_{t-1} \\ B_{t-1} & \text{if } A_t \leq L_{t-1}e^g + E_{t-1}e^g + B_{t-1} \\ B_{t-1} + \alpha\delta b(A_t - (L_{t-1}e^g + E_{t-1}e^g + B_{t-1})) & \text{if } A_t > L_{t-1}e^g + E_{t-1}e^g + B_{t-1}. \end{cases}
$$
(11)

When contracts expire, customers may be allowed to extract a proportion p of their proportion of the bonus account. Hence, the new bonus account at the beginning of next year will be

---

[6]Thus, the old generations will have the priority if the company risks default, but on the other hand they will leave some amount (part of the bonus reserve) in the company when their contract expire.

$$B_t = B_t^{cum}(1 - \frac{L_t^{t-T}}{L_t^{cum}}p) \tag{12}$$

and the final wealth from the insurance product to the customer becomes

$$I_t^{t-T} = L_t^{t-T} + B_t^{cum}\frac{L_t^{t-T}}{L_t^{cum}}p. \tag{13}$$

After deposits from new customers and withdrawals for old ones with expiring contracts, the new reserves at the beginning of the next year will become

$$L_t = L_t^{cum} + L_t^t - L_t^{t-T}. \tag{14}$$

Equity at the end of year can be determined residually as

$$E_t^{cum} = A_t^{cum} - L_t^{cum} - B_t^{cum}. \tag{15}$$

However, at the beginning of the next year we assume the company to be recapitalized, such that the proportion of equity to reserves stays constant over time. Hence,

$$E_t = L_t \frac{\alpha}{(1-\alpha)} \tag{16}$$

and assets at the beginning of next year becomes

$$A_t = L_t + B_t + E_t. \tag{17}$$

Now, the cash flow to shareholders can be determined simply as the difference between equity at the end of a year and at the beginning of the next year:

$$CF_t = E_t^{cum} - E_t \tag{18}$$

with the corresponding function for value at time $t = 1$

$$V = \sum_{t=2}^{T+H} E^Q[CF_t]e^{-r(t-1)}. \tag{19}$$

For annual guarantees with bonus reserves closed form solutions of "fair" contracts are unavailable, and we have to rely on numerical solutions using 100,000 Monte-Carlo simulations. We define a fair contract as a contract where investors of the insurance company will be indifferent to whether the company makes the contract or not. To be able to find fair δ for a given set of the control variables, α, θ, g, b, we simulate m paths of the value of equity under the risk-neutral measure, Q,

$$E_1 = \sum_{t=2}^{T+H} e^{-r(t-1)} E^Q[CF_t] = E^Q[V] \tag{20}$$

Since the value of equity is monotonically decreasing in δ, we can utilize Newton's method (as described e.g. by Judd (1998), chapter 4.1) to find a fair δ for each contract.

2.4 The Households

We assume that households can be represented by a CRRA utility function with a relative risk aversion coefficient γ. Then the utility of generation, h, can be described as a power utility function on the form

$$u(W^h) = \frac{1}{1-\gamma}(W_{h+T}^h)^{1-\gamma}. \tag{21}$$

The households' maximization problem will be distributing the wealth between the risky and risk-free asset as well as the insurance product. The weights of the portfolio allocated to each of the assets are named ω_S, ω_D, and ω_I respectively. We assume a borrowing constraint and no short-selling, such that all weights are non-negative. Furthermore, we assume no continuous rebalancing. Even though the risky and risk-free assets are tradeable, the insurance asset can typically not be traded, at least not in portions, before expiry. The optimization problem can be formalized as

$$\max_{\omega_S, \omega_D, \omega_I} E[u(W^h)] \tag{22}$$

subject to

$$W^h = \overline{W}^h (\omega_S \frac{S_{h+T}}{S_h} + \omega_D \frac{D_{h+T}}{D_h} + \omega_I \frac{I_{h+T}^h}{L_h^h}) \tag{23}$$

$$\omega_S + \omega_D + \omega_I = 1 \tag{24}$$

$$\omega_S, \omega_D, \omega_I \geq 0. \tag{25}$$

Without loss of generality we can standardize the initial wealth $\overline{W}^h$ to 1. Furthermore we know that $\frac{D_{h+T}}{D_h}$ is simply the risk free rate continuously compounded and we can replace it by e^{rT}. Hence we get from equation (23):

$$W^h = \omega_S \frac{S_{h+T}}{S_h} + \omega_D e^{rT} + \omega_I \frac{I_{h+T}^h}{L_h^h}. \tag{26}$$

3 Calibration

Table 4 reports our benchmark parameter values. We calibrate our model using data for a simplified, but typical contract in several European countries.

Parameters of the Model	Notation	Benchmark Case
The Economy		
Risk-free rate	r	4.0 %
Equity premium of the equity index	μ	4.0 %
Volatility of the equity index	σ	16.0 %
The Company		
Proportion of total asset portfolio, A_t, invested in the equity index	θ	20 %
The proportion of equity to total assets at time $t = 1$ ($\alpha = \frac{E_1}{A_1}$)	α	90 %
The constant guaranteed rate	g	2.0 %
The proportion split of the remaining total profit to the households	δ	97.11%
Proportion of declared bonuses credited the bonus reserves B_t	b	30 %
Payout ratio of bonus reserve	p	36 %
The Households		
Overlapping generations of households, indexed by $h = 1, 2, \ldots, H$	H	80
Term of the policy	T	20
Growth rate of the households' aggregate initial investments	v	2.0 %
Coefficient of relative risk aversion	γ	5

The table provides the benchmark case parameter values that are used to conduct the numerical analysis.

Table 4: Calibration

3.1 Parameters of the Economy

As our risk free rate, r, we use the long term rate of German government bonds. At Jan 1, 2006, the 30 year rate was 3.62%.[7] We use 4.0% as our risk free rate. For the stock return process we consider a mean equity premium, $\mu = 4.0\%$, and a standard deviation, $\sigma = 16\%$. Considering an equity premium equal to 4% as opposed to the historical of 6% is a fairly common choice in this literature, e.g. Cocco, Gomes, and Maenhout (2005), Yao and Zhang (2005) or Gomes and Michaelides (2005). Also notice that opposed to most other papers, we use a long-term interest rate.[8]
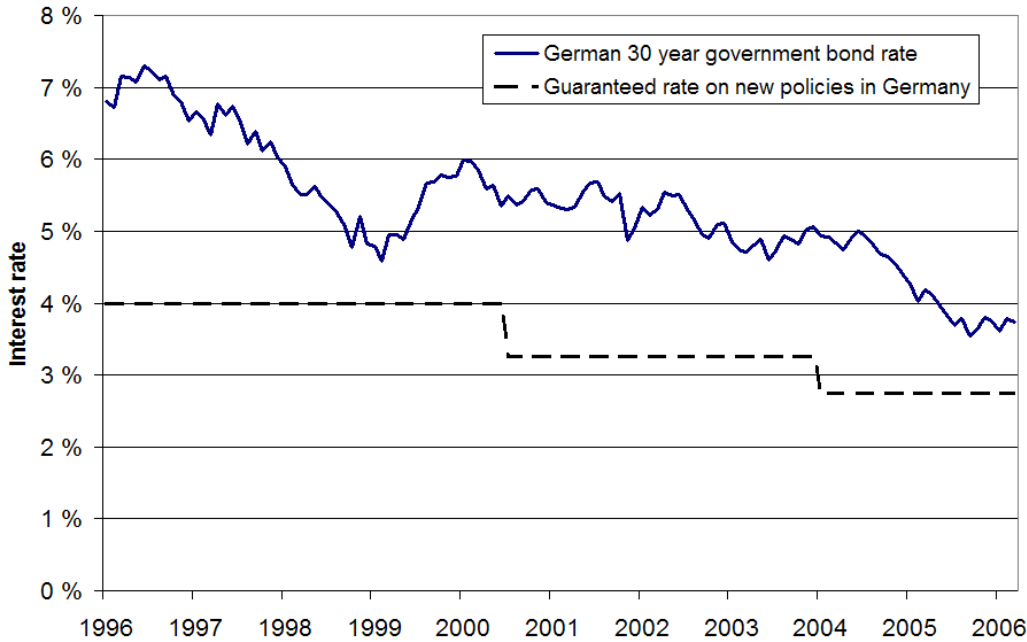
3.2 Guarantees

According to the 3rd European life assurance directive the guaranteed interest rate shall be maximum 60% of the interest rate of government bonds of the same currency, without defining this further.[9] National authorities are left to make more detailed rules. However, currently

---

[7]Source: Datastream.

[8]According to Dimson, Marsh, and Staunton (2004) the arithmetic nominal world equity return for 1900-2003 is 10.2% with a standard deviation of 16.9%. Mehra and Prescott (1985) found an arithmetic risk premium of about 6% above the short interest rate with a standard deviation at 16.6%.

[9]See article 17.

Sources: Datastream, CEIOPS, and Allianz Leben 2004 Annual Report

Figure 3: Interest rates

most national regulations allow higher guaranteed rates than 60% of most euro government bond rates. A survey is provided as table 5.

Historically, the spread between guaranteed rates and government bond rates are much higher. An illustration from Germany is shown in figure 3. In order to get a spread more in line with the historical average, and assuming that national regulators will eventually change their regulations in the direction of the 3rd European life assurance directive, we let the guaranteed rate be 2.0%.

3.3 Bonuses and Bonus Reserves

We illustrate our discussion on bonus reserves with the quite complicated German way of reserving (see figure 4), as described by Allianz in their presentation to investors at the Allianz Capital Markets Day 14 Jul 2005.[10] The 2004 surplus of Allianz was in total 4.9 bns euros, net of taxes, but including development of hidden reserves.[11] Of this, 3.2 bns were accounted on balance sheets, while 1.7 bns was related to the development of hidden reserves.

From the total surplus, 0.2 bns euros were transferred to the equity.[12] This gives an δ of

---

[10]Presentation downloadable at www.allianz.com

[11]We do not include development of hidden reserves in the loan portfolio, which may be quite substantial, but will prove impossible to quantify without information from the company itself.

[12]in addition one could argue that parts of the hidden reserves in fact belongs to the shareholders. An

Country	Current Guaranteed Rate	
Austria	2.25 %	From 1 Jan 2006
Belgium	3.75 %	Other rates for durations shorter than 8 years
Denmark	2.0 %	
France	2.5 %	3 % for durations shorter than 8 years
Finland	2.5 %	
Germany	2.75 %	
Italy	2.0 %	From 1 Jan 2006
Netherlands	3.0 %	
Norway	3.0 %	Proposed reduction to 2.75 % for some contracts
Sweden	2.75 %	
Switzerland	2.5 %	Lower rates used in contracts with no mandatory guarantees

The table shows current guaranteed interest rates in continental Europe. Sources: CEIOPS, BPV (Switzerland).

Table 5: Current Guaranteed Interest Rates in Continental Europe

95.9%, which, however, has little relevance to us, as δ will be used as the balancing parameter for achieving correct pricing over the product life-cycle.

In total 2.0 bns euros were transferred to the mathematical reserves. This corresponds to $b = 57.4\%$. However, in a historical context this seems to be on the high side. 2004 was a good year when it comes to securities market development, causing a large increase in the hidden reserves. In a more normal year we think one can expect no significant increase in the hidden reserves. When using only the on-balance-sheet items we get $b = 33.3\%$. We round this and use $b = 30\%$ as our base case.

The bonus reserves in Allianz can be split in several parts. We have already noticed the split between on-balance-sheet items (reserves for bonuses, RfB) and off-balance sheet items (hidden reserves). In addition it will be useful to split the RfB into allocated RfB and terminal bonus fund (which can be individualized) and free RfB (which cannot be individualized). We assume that at expiry of the contracts, customers will only receive their proportional parts of the individualized funds. For Allianz, this corresponds to $p = 36\%$, which we will use as our base case. We note, however, that individualized bonus fund are only common in some European countries, hence we will also show scenarios with $p = 0$.
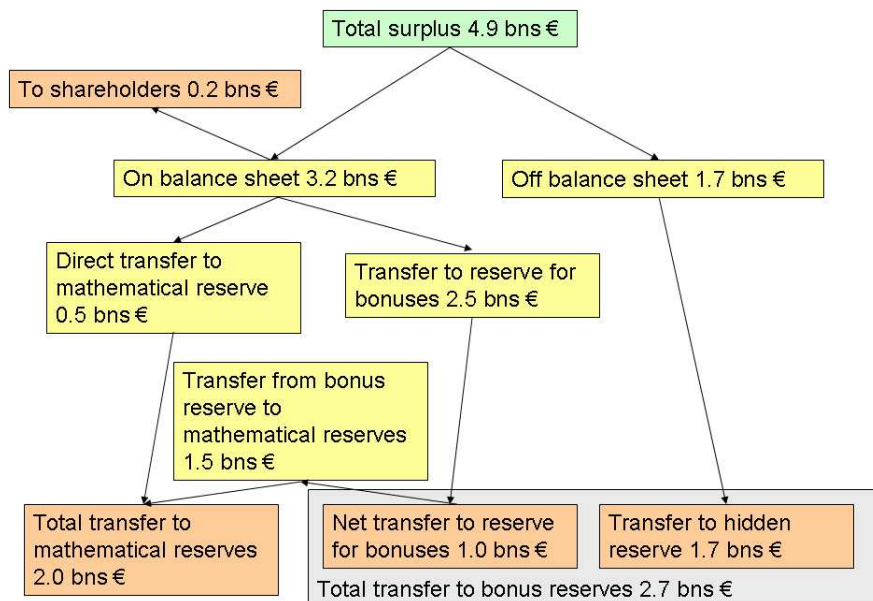
Total surplus 4.9 bns €

To shareholders 0.2 bns €

On balance sheet 3.2 bns €

Off balance sheet 1.7 bns €

Direct transfer to mathematical reserve 0.5 bns €

Transfer to reserve for bonuses 2.5 bns €

Transfer from bonus reserve to mathematical reserves 1.5 bns €

Total transfer to mathematical reserves 2.0 bns €

Net transfer to reserve for bonuses 1.0 bns €

Transfer to hidden reserve 1.7 bns €

Total transfer to bonus reserves 2.7 bns €

Figure 4: Distribution of surplus of Allianz Leben (2004 numbers)

3.4 Asset Allocation and Capital Structure

In table 6 we find an average allocation for European L&P companies at 3% in real estate, 22% in equity and 74% in bonds. We regard real estate as close to fixed income, thus based on European data we set our rounded asset allocation parameter, $\theta = 20\%$. As the asset allocation typically changes over time, we provide sensitivities to this in chapter 4.3.

When it comes to capital structure, EU regulations specify a minimum solvency capital of 4% of mathematical reserves + 0.3% of sum insured. This would normally approximately correspond to $\alpha = 0.05$. However, as in most countries companies tend to hold much more capital[13], we use instead $\alpha = 0.1$.

3.5 Parameters of the Households

In our model we use a single premium contract with a duration T of 20 years. In order to get a steady-state-period, where old contracts expire, but at the same time new ones are written, it is necessary to use more than 40 generations. We use as number of generation $H = 80$.

Even in a steady state there will be some growth in the assets of the company due to the increase in premiums paid by the newer generation. This is due to increase in population, inflation and real growth rates. The population growth in Europe is assumed to be zero or even negative in the future.[14] Assuming no long-term real growth we limit our growth rate v

---

alternative calculation will be taking into account only the on-balance-sheet-items when calculating δ, getting $\delta = 93.8\%$.

[13]Based on data from CEIOPS value weighted average of main countries is 9.2%

[14]According to UN World Population 2004. See www.unpopulation.org.

	Real Estate	Equity	Bonds
Austria	4.0 %	33.9 %	62.2 %
Belgium	0.9 %	19.7 %	79.4 %
Germany	2.8 %	28.7 %	68.5 %
Denmark	2.6 %	32.8 %	64.6 %
Finland	8.1 %	30.3 %	61.6 %
France	2.9 %	11.3 %	85.8 %
Ireland	5.0 %	18.4 %	76.6 %
Italy	0.4 %	14.9 %	84.7 %
Netherlands	7.7 %	16.4 %	75.9 %
Norway	10.4 %	17.4 %	72.2 %
Average	3.4 %	22.5 %	74.1 %

The table shows the asset allocation for European life companies for 2004. The average is value weighted. Equity is the sum of shares, variable-yield securities, units in unit trusts and investments in affiliated enterprizes. Bonds consists of debt securities, fixed income securities, investment pools, mortgage loans, other loans, deposits with credit institutions and deposits with ceding enterprizes. Source: CEIOPS

Table 6: Asset allocation life companies 2004

to the inflation rate of 2% as is the target of ECB.

We start by presenting results for a quite common relative risk aversion $\gamma = 5$ (e.g. Gomes and Michaelides (2005)). Usually in the literature the range of γ is between 3 (e.g. Dammon, Spatt, and Zhang (2004)) and 10 (e.g. Cocco, Gomes, and Maenhout (2005)). Later on we will report results for different values of γ.

4   Intergenerational Cross-subsidization

4.1   Result of the Benchmark Case

The main result of our benchmark case is given in figure 5. We find that the expected risk-adjusted return is monotonically increasing with respect to generations and above the risk-free rate from generation 42. The return in the build-down period of the last 20 generations seems unrealistically high, particularly for the very last generation. However, the build-down scenario may be unrealistic in itself, as companies will typically sustain.

Furthermore, we find low returns to generations in the build-up-phase of the first 20 generations. Customers in this phase should rather look for alternative investments in other product. In practise we see very few new life and pension companies selling guaranteed products in mature markets. Recently new companies in Western European markets, e.g. Mediolanum and MLP, have preferred unit-linked and other non-guaranteed products.

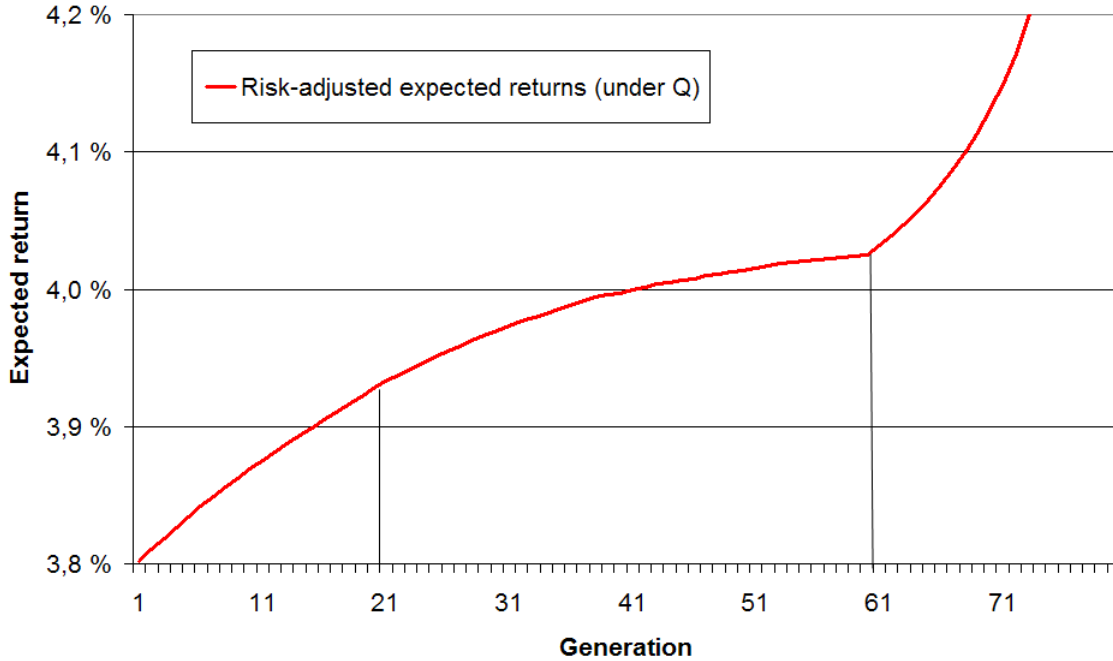We will particularly focus on the difference between early and late generations in the

Figure 5: Expected return for different generations

The figure shows expected annual risk-adjusted return of the insurance contract for different generations $1 - 80$. This return can be compared to the risk-free rate of 4%.

going-concern phase from generation 20 to 60. We find an expected risk-adjusted return of 3.93% for generation 20, compared to 4.03% for generation 60. This return difference may seem small, but in a 20 year perspective it will still be significant, corresponding to an initial fee of 2% of invested capital.

In figure 6 we illustrate the risk of each generation, measured by the standard deviation of the average annual return. We find that the risk develops in the same fashion as the average return. Economically it seems that the return overcompensates for the risk, yielding a return above the risk-adjusted rate if there is enough risk taken.

The technical explanation is that each generation assumes investment risk of the investment period of the previous generation through the bonus reserve. If a generation faces high returns in the stock market it will leave behind a high bonus reserve causing high expected returns for the next generation(s).

In our model the first generations get no risk transferred from previous generations. However, they will be able to transfer some of the risk in their period to the next generations. These middle generations will assume risk from previous generations, but also be able to transfer risk to their followers. Finally, the last generations will assume risk from all previous generations as well as the full risk from their own investment period.
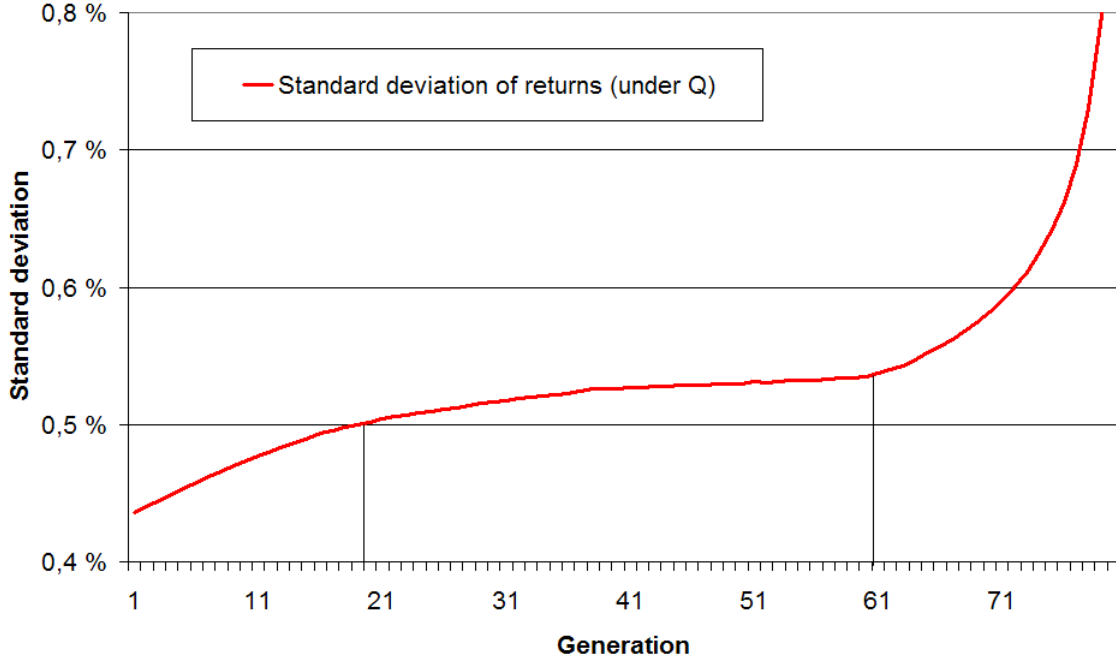
17

Figure 6: Standard deviation for different generations

We here show the standard deviation of the annual risk-adjusted return of the insurance contract.

4.2 Intergenerational Diversification

We can explain the return of different generations by the current and earlier periods stock market return. To quantify the size we run an OLS regression where average yearly return on the investment for the customer is the left-hand side variable.[15] We split the return of generation 20 into two time periods of stock market return, $t = (1, 20)$, $r_{1,20}$ and $t = (21, 40)$, $r_{21,40}$:

$$\tilde{r}_{20} = \beta_0 + \beta_1 r_{1,20} + \beta_2 r_{21,40} + \varepsilon. \tag{27}$$

For generation 60 the return, $\tilde{r}_{60}$, is influenced not only by the return for period $(0, 20)$ and $(21, 40)$, but also by the return for the periods $(41, 60)$ and $(61, 80)$, $r_{41,60}$, $r_{61,80}$, respectively:

$$\tilde{r}_{60} = \beta_0 + \beta_1 r_{0,20} + \beta_2 r_{20,40} + \beta_3 r_{40,60} + \beta_4 r_{60,80} + \varepsilon. \tag{28}$$

We can interpret β_i as how sensitive the customer's return is to the different timeperiods return. A $\beta > 0$ implies a diversification effect between the customer's return and the respective period. We expect the β to decrease for more distant periods.

---

[15] A more sophisticated model with non-linearity would probably be best, however the results we achieve are intuitive, implying a relative well-specified model.

Panel a	Regression generation 20	
β_0	0.0335	[0.0335, 0.0335]
β_1	0.0092	[0.0090, 0.0094]
β_2	0.1357	[0.1355, 0.1359]

Panel b	Regression generation 60	
β_0	0.0333	[0.0333,0.0333]
β_1	0.0013	[0.0009, 0.0016]
β_2	0.0093	[0.0090, 0.0097]
β_3	0.0253	[0.0250, 0.0257]
β_4	0.1382	[0.1379, 0.1386]

This table shows two panels. Panel a illustrates regression for generation 20. In panel b we show the regression for generation 60. For each estimate of β, a 95% confidence interval is plotted.

Table 7: Intergenerational Diversification

Panel a in table 7 shows the regression for generation 20. If the stock market yields zero return we would expect a return at 3.35%. If the average stock market return for the current period increases with 1% the customer's return increases with 0.14%. The customers return is over 14 times more sensitive to current periods return than the previous period.[16]

Generation 60 is dependent of 80 years stock market return, however, as shown in panel b in table 7, the most important period is not surprisingly the current period. We see that the previous period is more important for generation 60 than for generation 20. The reason is that the bonus reserve increases with time. Thus, both the stock market exposure $(\sum_{i=1}^4 \beta_i)$ and the diversification $(\rho_i = \beta_i \frac{\sigma_m}{\sigma_l})$ increases for every new generation.

4.3 Sensitivities to the Benchmark Case

Given the different characteristics of different life and pension insurance markets, one common model will to a large degree have to build on averages. In section 3 we show how some of the parameters vary across borders, in this section we will show how the level of different parameters will change the results of the model.

Changing the θ means changing the risk of the asset portfolio of the company. The higher the θ, the higher the risk. When assuming more risk, the bonus reserve will be more frequently used as low asset returns (lower than the guaranteed rate) becomes more frequent. This leads to a lower average level of the bonus reserve. As early generations wish to limit the build-up of bonus reserves, they will benefit from higher θ at the cost of later generations. In figure 7 we give results for different levels of θ.

The finding that later generations benefit from a low θ may also give some explanation to low stock market exposure in most life and pension insurers. Intuitively one would think

---

[16]Since this regression is run on simulations the t-statistics are a function of numbers of simulations. With 100000 simulations all the beta's are highly significant.
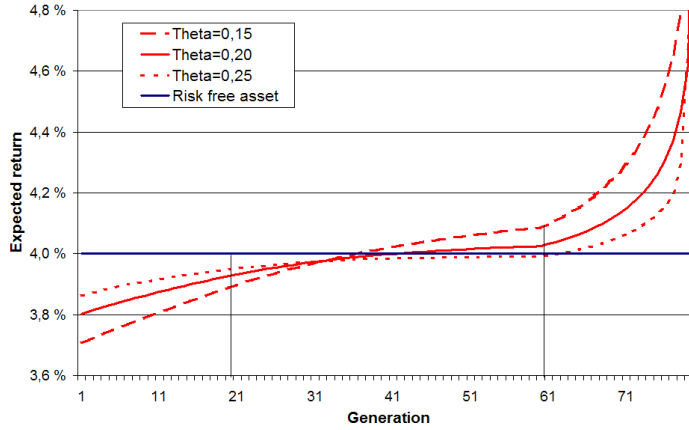
Figure 7: Sensitivity with respect to θ.
The effect of different alternative θ's on the expected annual risk-adjusted return is illustrated in this figure. The benchmark case is $\theta = 0.20$.

that companies should invest higher proportions in stocks to get closer to the optimal asset allocation for customers (see e.g. Døskeland and Nordahl (2006) for details). However, as companies prefer to satisfy new customers (the later generations) they may prefer a lower θ.
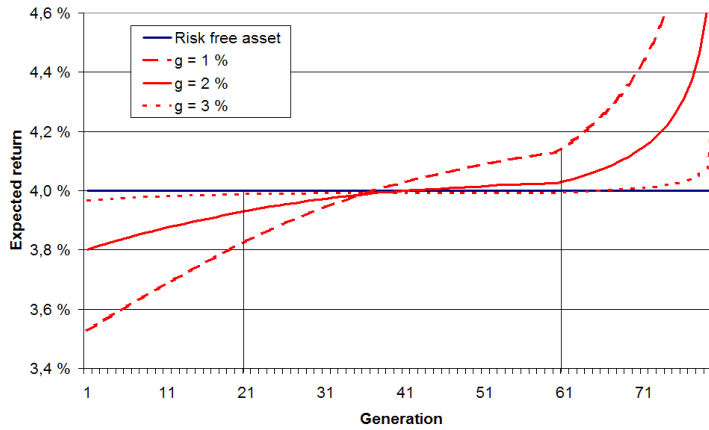

Figure 8: Sensitivity with respect to the guarantee g.
The figure illustrates how different levels of the guarantee g impacts the expected annual risk-adjusted return. The benchmark case is $g = 2\%$.

A low spread between the risk-free rate and the guaranteed rate will lead to slower build-up of bonus reserves. This is due both to lower expected profits of the company (roughly equal to the spread) and to the lower δ (less favorable profit sharing to customers) the company will allow to compensate the higher guarantee. In figure 8 we show that the intergenerational cross-subsidization decreases when g is increased. The effect of decreasing r will be similar to

20

the effect of increasing g. We note, however, that $g = 3\%$ imply a spread of only 1%, which is very low compared to the historical rates shown in figure 3 and leaves the life insurance contract close to a bond contract.
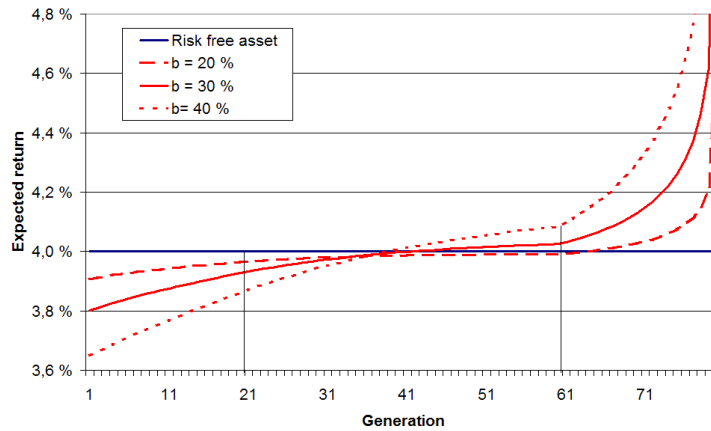

Figure 9: Sensitivity with respect to b.

In this figure we show how the proportion of declared bonuses credited the bonus reserves, b, impacts the expected annual risk-adjusted return of the insurance contract. The benchmark case is $b = 30\%$.

The build-up of bonus reserves can also be influenced more directly by changing b. In figure 9 we show the obvious result that a higher b benefits the later generations. If b goes towards zero, there will be no bonus reserves causing differences between generations.
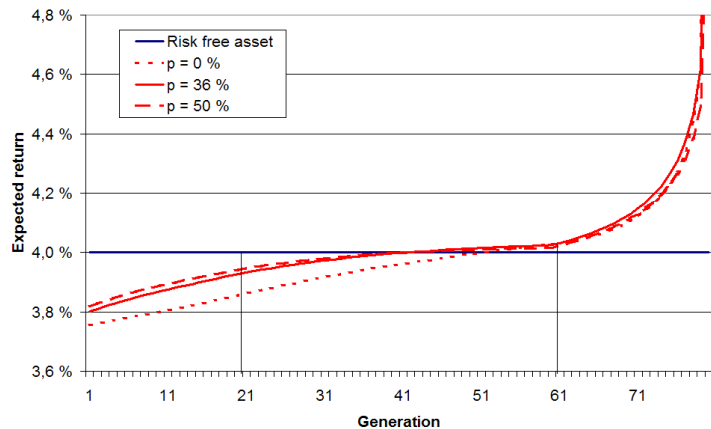
Figure 10: Sensitivity with respect to p.

This figure illustrates how the payout ratio of bonus reserve, p, impacts the expected annual risk-adjusted return of the insurance contract. The benchmark case is $p = 36\%$.

The impact of changing the pay-out-ratio p of the bonus reserve seems to be limited. In figure 10 we show that the scenario with $p = 50\%$ is only marginally different from the

benchmark case. Changing the payout ratio to $p = 0\%$ causes larger changes, this scenario obviously yield lower returns to the first generations, while only the very last generations seem to benefit.
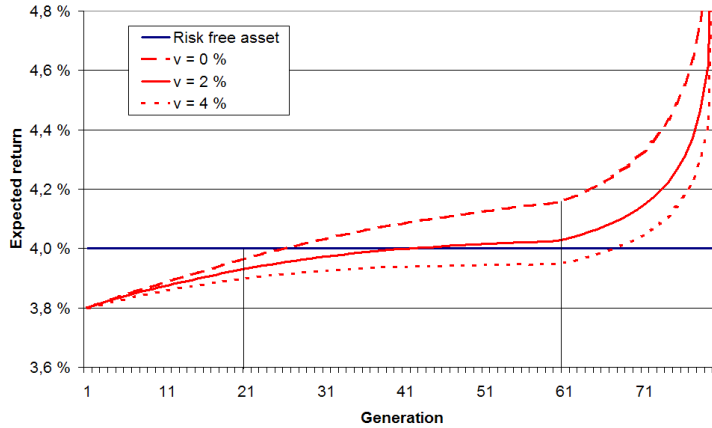

Figure 11: Sensitivity with respect to v.

This figure illustrates how the growth rate of households' aggregate initial investments, v, impacts the expected annual risk-adjusted return of the insurance contract. The benchmark case is $v = 2\%$.

In figure 11 we show the result that all generations benefit from a lower growth rate v. The first generations will still build up bonus reserves at the same pace as in the benchmark case. However, with a lower growth rate they will receive a larger proportion of the bonus reserve when the contract expires or in the case of low asset returns. The reason is that subsequent generations' mathematical reserves are now smaller and the first generations' share of the total mathematical reserves is higher. This is only partially compensated for by a higher bonus reserve, as subsequent generations have only contributed to the build up of bonus reserves during a limited period.

Later generations will also receive this benefit, and in addition they will profit from the the fact that the build-up of the ratio of bonus reserve to mathematical reserve now is faster, as the mathematical reserve grows more slowly. At the time of their initial investment their part of the bonus reserve will be larger per unit of investment, hence the "gift" from previous generation will have more impact. In figure 11 we see that later generations get a higher benefit from a low growth rate than what the first generations do.

We note that the impact of the growth rate is the opposite of that of a pay-as-you-go pension system. While the pay-as-you-go system de facto produces a liability to be transferred from old generations to new ones, the guaranteed contracts produce an asset (the bonus reserve) to be transferred. Hence the guaranteed contracts may in some scenario work as a hedge of population growth risk of a pay-as-you-go system.

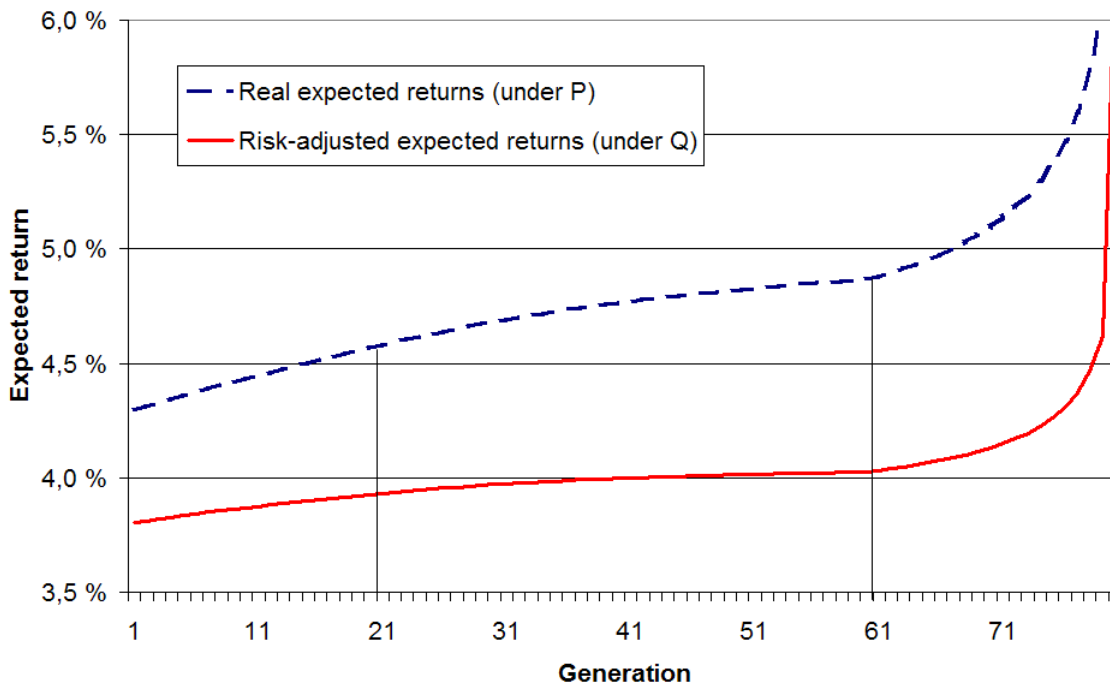# 5 Optimal Portfolio Choice

Figure 12: Real expected return for different generations.
In this figure we compare the real expected annual return (under P) for different generations with the similar expected risk-adjusted return (under Q).

In order to optimize individuals' portfolio choice as defined in section 2.4, we run simulations under the real probability measure P. In figure 12 we show how the expected return (given expectation for all generations at time 0, see below) develops over generations compared to the risk-adjusted return (under the equivalent martingale measure Q). As previously explained in section 4.1 the standard deviation is higher for later generations. We explained this mainly by the increased stock market exposure, shown by beta-values of the regression analysis in section 4.2. We note that the larger risk for later generations is compensated for by a larger risk premium measured by the difference between the return figures for each generation.

As we are interested in the life-cycle trend of the attractiveness of the contracts, we assume that the customers only know the expectation of the bonus reserve at time 1. The customers do not know the realization of the bonus contract, hence they can not start "timing" the contract by buying the contract only at high realizations of the bonus reserve. As we find that the expected return also depends on previous periods' market return, our expected return may be different from the expectation customers face at the time of investment. This makes

sense in a setting where each generation is present behind a "veil of ignorance", they select a pension system (mix of e.g. public pensions, private pensions, and other savings products) to belong to some time ahead of the actual investment.

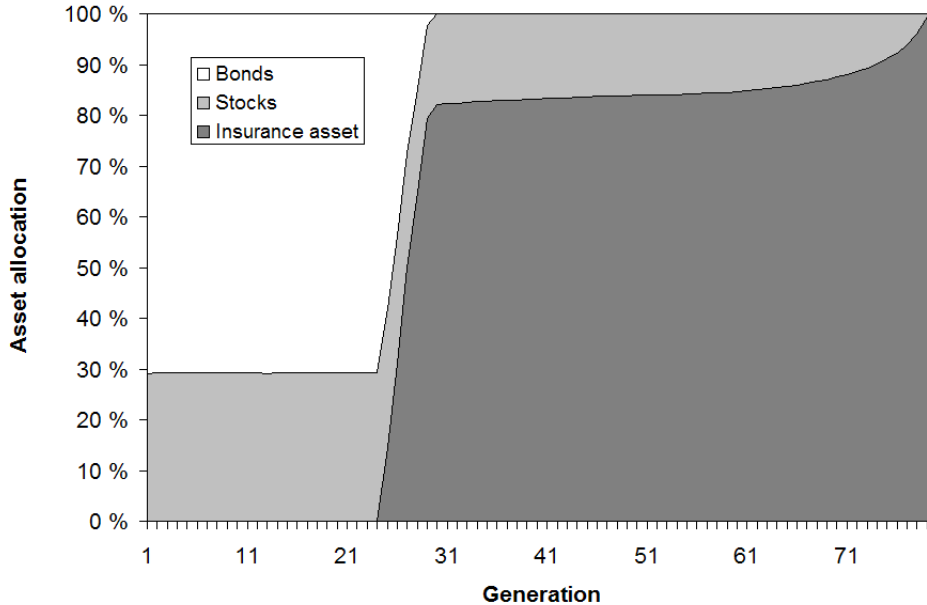## 5.1 Optimality for Different Generations in the Benchmark Case

Figure 13: Optimal asset allocation for different generations, benchmark case.
This figure shows the optimal allocation to different assets classes for the different generations $1 - 80$, given our benchmark case relative risk aversion coefficient $\gamma = 5$.

We maximize the household portfolio choice for each generation over three assets; the insurance asset, the risky asset and the risk-free asset as shown in equation 22. We would expect the early generations to prefer direct investments in the risky and risk-free asset, while later generations will prefer to invest in the life asset due to the higher expected returns.

In figure 13 we show that the first 25 generations will prefer no investment in the insurance asset. In this period the expected return under Q is significantly below the risk-free rate (see figure 5. The optimal allocation to the risky asset is approximately 29.2% which correspond to the Merton (1969) solution:[17]

$$\omega_S^* = \frac{\mu}{\gamma \sigma^2} = \frac{4\%}{5 \cdot 0.16^2} = 31.25\% \tag{29}$$

where ω_S^* is the optimal allocation to the risky assets and the other parameters are shown in

---

[17]With the exception that due to the no rebalancing condition our solutions typically show a marginally lower investment in the risky asset

table 4.

More surprisingly the optimal solution shows that for generation $25 - 40$ it is optimal to invest in the insurance asset even though the risk-adjusted return is lower than the risk-free rate. The reason is that the diversification effects considered in section 4.2 benefits investments in the insurance asset combined with the risky asset.

After generation 30 there is a slight increase in the optimal allocation to the insurance asset in later generations due to the increasing profitability of the life asset. We note that due to diversification effects and improved return potential, the effective stock market risk exposure is now slightly higher than the optimal allocation to the risky asset for the first generations. Using the β's of section 4.2 we get an exposure for generation 60 of

$$\Theta_{60} = \omega_S + \omega_L \sum_{i=1}^{4} \beta_i = 15.5\% + 84.5\% \cdot 17.4\% = 30.2\%. \tag{30}$$

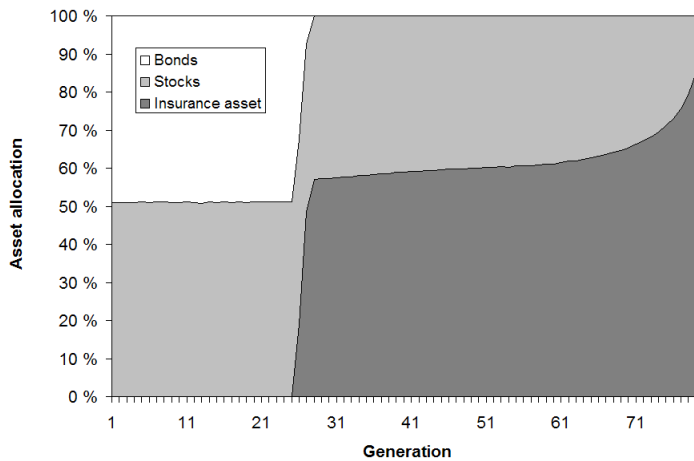## 5.2 Optimality for Different Levels of Risk Aversion

Figure 14: Optimal asset allocation for different generations with $\gamma = 3$.
This figure shows the optimal allocation to different assets classes, given the alternative $\gamma = 3$.

Changing the risk aversion parameter of course give some changes in the optimal asset allocation. A lower risk aversion gives a higher allocation to the risky asset. For the earliest generations this drive down the allocation to the risk-free assets, while the generations after generation 25 mainly will reduce their exposure to the insurance asset. The optimal allocation in the case of $\gamma = 3$ is shown in figure 14.

Increasing the risk aversion parameter to 10 gives more significant changes to the optimal allocation, as shown in figure 15. Now, the generations in the period $25-40$ want to keep some proportion in both the risk-free and risky asset. The investment in the risky asset is optimal
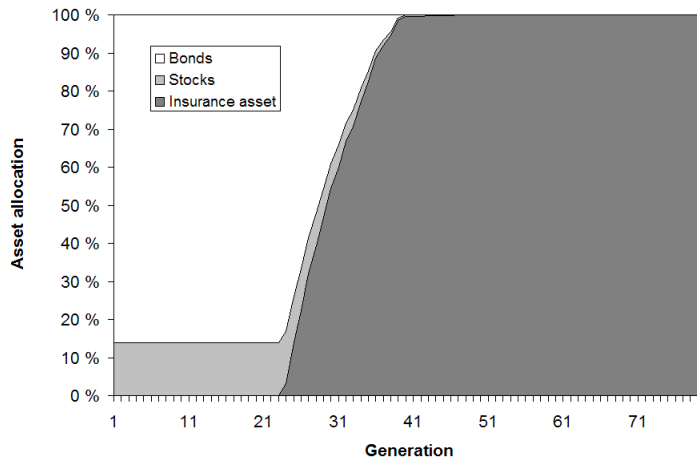
Figure 15: Optimal asset allocation for different generations with $\gamma = 10$.

This figure shows the optimal allocation to different assets classes, given the alternative $\gamma = 10$.

in order to keep some diversification with the insurance asset. However, as the investor is now more risk-averse he wants to invest in the risk-free assets in order to keep the total risk down.

6   Conclusion

In this paper we investigate the return of different generations investing in a guaranteed interest rate life and pension contract. We use a numerical simulation model over 80 generations with realistically calibrated parameters of a typical European guaranteed rate contract, with the assumption of correct pricing over the life-time of the company. Our findings indicate that there exist a cross-subsidization from customers in early generations to customers in later generations. Furthermore, as returns for one generation depend also on return in previous periods, there is a time diversification effect built into the contract.

We also show that these effects are large enough to defend that a guaranteed rate contract is part of the optimal portfolio of the late generations. Hence our paper contributes to explaining why household invest in life and pension products even though they are not part of the optimal portfolio in a one-customer setting.

Future research in this area may expand our analysis to cover the question of whether private pensions should be included in a portfolio of pension systems. We have shown that there is a risk sharing effect between today's generation and earlier generation. This may add a dimension to today's system of pay-as-you-go and funded alternatives, where there is a risk sharing effect between today's generation and the younger generation.

References

Borch, K., 1962, "Equilibrium in a Reinsurance Market," *Econometrica*, 30, 424–44.

Brennan, M. J., 1993, "Aspects of Insurance, Intermediation and Finance," *Geneva Papers on Risk and Insurance Theory*, 18, 7–30.

Cocco, J. F., F. J. Gomes, and P. J. Maenhout, 2005, "Consumption and Portfolio Choice over the Life-Cycle," *The Review of Financial Studies*, 18, 491–533.

Consiglio, A., D. Saunders, and S. A. Zenios, 2006, "Asset and Liability Management for Insurance Products with Minimum Guarantees: The UK Case," *Journal of Banking and Finance*, 30, 645–67.

Dammon, R. M., C. S. Spatt, and H. H. Zhang, 2004, "Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing," *Journal of Finance*, 65, 999–1038.

Dimson, E., P. Marsh, and M. Staunton, 2004, "Global Investment Returns Yearbook 2004," ABN-AMRO London Business School.

Døskeland, T. M., and H. A. Nordahl, 2006, "Optimal Pension Insurance Design," *Working paper*.

Gomes, F. J., and A. Michaelides, 2005, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, 60, 869–904.

Grosen, A., and P. Jørgensen, 2000, "Fair Valuation of Life Insurance Liabilities: The Impact of Interest Rate Guarantees, Surrender Options, and Bonus Policies," *Insurance: Mathematics and Economics*, 26, 37–57.

Hansen, M., and K. R. Miltersen, 2002, "Minimum Rate of Return Guarantees: The Danish Case," *Scandinavian Actuarial Journal*, 4, 280–318.

Jensen, B. A., and C. Sørensen, 2001, "Paying for minimum interest rate guarantees: Who should compensate who?," *European Financial Management*, 7, 183–211.

Judd, K. L., 1998, *Numerical Methods in Economics*, MIT Press.

Mehra, R., and E. C. Prescott, 1985, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145–61.

Merton, R. C., 1969, "Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case," *Review of Economics and Statistics*, 51, 247–57.

Miltersen, K. R., and S.-A. Persson, 2003, "Guaranteed Investment Contracts: Distributed and Undistributed Excess Return," *Scandinavian Actuarial Journal*, 4, 257–79.

Yao, R., and H. H. Zhang, 2005, "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints," *The Review of Financial Studies*, 18, 197–239.