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BY

Endre Bjørndal, Mette Bjørndal AND **Victoria Gribkovskaia**

Norges Handelshøyskole

NORWEGIAN SCHOOL OF ECONOMICS

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Endre Bjørndal, Mette Bjørndal, Victoria Gribkovskaia[1]

In the Nordic day-ahead electricity market zonal pricing or market splitting is used for relieving congestion between a predetermined set of bidding areas. This congestion management method represents an aggregation of individual connection points into bidding areas, and flows from the actual electricity network are only partly represented in the market clearing. Because of several strained situations in the power system during 2009 and 2010, changes in the congestion management method have been considered by the Norwegian regulator. In this paper we discuss nodal pricing in the Nordic power market, and compare it to optimal and simplified zonal pricing, the latter being used in today's market. A model of the Nordic electricity market is presented together with a discussion of the calibration of actual market data for four hourly case studies with different load and import/exports to the Nordic area. The market clearing optimization model incorporates thermal and security flow constraints. We analyze the effects on prices and grid constraints and quantify the benefits and inefficiencies of the different methods. We find that the price changes with nodal pricing may not be dramatic, although in cases where intra-zonal constraints are badly represented by the aggregate transfer capacities in the simplified zonal model the nodal prices may be considerably higher on average and vary more than the simplified zonal prices. On the other hand nodal prices may vary less than the simplified zonal prices if aggregate transfer capacities are set too tightly. Allowing for more prices in the Nordic power market would make dealing with capacity limits easier and more transparent.

Keywords: Nodal pricing, Zonal pricing, Congestion management, Electricity market simulation

[1] NHH Norwegian School of Economics, Department of Business and Management Science, 5045 Bergen, Norway (e-mail: endre.bjorndal@nhh.no; mette.bjorndal@nhh.no; victoria.gribkovskaia@nhh.no (corresponding author)).

1. INTRODUCTION

The Nordic power market, represented by Nord Pool Spot, covers at the moment Norway, Sweden, Denmark, Finland and the Baltic countries. In the Nord Pool Spot day-ahead electricity auction (Elspot), power is traded for delivery each hour the next day. At present there are 15 large bidding areas (between one and five for each country), and prices are calculated for each area, taking into account the available transmission capacity between the areas. Since February 2014 the Nordic power market has been coupled with the North-Western European (NWE) power market, i.e. the day-ahead market from France to Finland is operated under a common day-ahead power price calculation using the Price Coupling of Regions (PCR[2]) solution. The congestion management of the European market coupling is similar to the one in the Nordic day-ahead market.

During 2009 and 2010 the Nordic market experienced several periods with very high prices. The high prices were predominantly the result of cold weather and nuclear production outages, but also low transfer capacities between the bidding areas. The low transfer capacities were not due to line outages or maintenance, but rather the result of system operators taking into account congestion within the areas. This practice has been questioned, and one of the remedies that have been suggested is to increase the number of prices (bidding areas) in the Nord Pool Spot market clearing, and to better reflect the real constraints of the power system in the network model used. Thus, the aim of our work, which has been commissioned by the Norwegian regulator (NVE), has been to investigate the effects of a market system which takes advantage of more information about the physical system in terms of capacities and flows, and the location of supply and demand bids.

Nord Pool Spot is a voluntary pool, although trades between Elspot bidding areas are mandatory. Nord Pool Spot covers about 80 % of the physical power in the Nordic region, and the pool is used not only for mandatory trades but also to increase the legitimacy of prices and to act as a counterpart. There are three types of bids available: hourly bids for individual hours, block bids that create dependency between hours, and flexible hourly bids, which are sell bids for hours with highest prices. Since the market is settled many hours before real time, imbalances occur, and they are settled by intraday trading in Elbas and in the regulation markets.

Our analysis of congestion management methods for the Nordic electricity market takes as a starting point the optimal power flow for a single hour. The efficiency of a specific market mechanism can then be evaluated based on the degree to which one can realize the optimal power flow. In the Nordic power market this is dependent on the formulation of practical rules for area price determination at Nord Pool Spot. These rules imply a number of simplifications and approximations compared to the optimal power flow benchmark. For a start, prices are not noted for each connection point (node) in the system, instead area prices are computed, that are uniform within larger areas of the network corresponding to the predetermined bidding areas. The number

[2] PCR is the initiative of seven European Power Exchanges, to develop a single price coupling solution to be used to calculate electricity prices across Europe, and allocate cross border capacity on a day-ahead basis.

of bidding areas and how the boundaries of these are exactly determined, therefore, affects economic efficiency. Another simplification is that market participants bid within each bidding area (zone) and not at each generation or load point. This results in uncertainty regarding the effects of a bid on the system, and consequently a possibility that the capacity control is imprecise. Likewise, the transmission capacities are often associated with transfer interfaces that include several transmission lines. This also results in a less accurate capacity control than if line capacities were used on individual level. In the whole Nordic system there is a practice of *moving* a transmission constraint within a price area to an area boundary, by reducing capacity between bidding areas. Previous work by [4] has shown that this is a practice that can be costly and greatly affects the level of the zonal prices in different regions.

By including a detailed network model in the price calculation, the system operators' setting of trade capacities becomes redundant. Prices and power flows can be calculated simultaneously for each hour based on an optimization procedure in which the social welfare based on the players' bids to the power exchange is maximized. However, this requires more and smaller zones at Nord Pool Spot in order to determine the exact location of production and consumption in the network.

In order to provide this type of analysis, we have constructed the OptFlow model (for a mathematical description we refer the reader to the Appendix), with a more detailed representation of the grid in the Nordic electricity market and a possibility to solve market clearing models with either nodal or zonal pricing. Different cases have been developed from real market data and the model is used to investigate questions like, for example, how the nodal prices are compared to different zonal prices for different market scenarios, and which constraints are binding or infeasible in the different cases.

The rest of the paper is organized as follows. A brief description of different locational price models is provided in Section 2. In section 3 we present and explain the market clearing models that we apply to the Nordic electricity market. Analysis and discussion of case results based on the four chosen data scenarios is presented in Section 4. Finally, summary and conclusions are provided in Section 5.

2. LOCATIONAL PRICE MODELS

The objective of a deregulated power market is efficiency in the short and long runs through a competitive short-term power market with efficient utilization of existing resources, as well as an optimal long-term development of the power system. The efficient short-run utilization of limited generation and transmission capacity can in principle be found by solving an optimal economic dispatch problem, where the difference between consumer benefits and production cost is maximized, subject to generation and transmission constraints. The latter include thermal and security constraints. Solving the optimal economic dispatch problem, we get a value of power for each location in the transmission system, and this is a benchmark that can be used for assessing different congestion management methods. In the following, we shortly describe the three locational price models that we analyze in this paper: nodal and, optimal and simplified, zonal prices. Nodal prices are based on the value of power obtained from an optimal economic dispatch problem. Simplified zonal prices and optimal zonal prices represent two different simplifications or approximations of nodal prices. A more formal description of the different locational price models can be found in the Appendix.

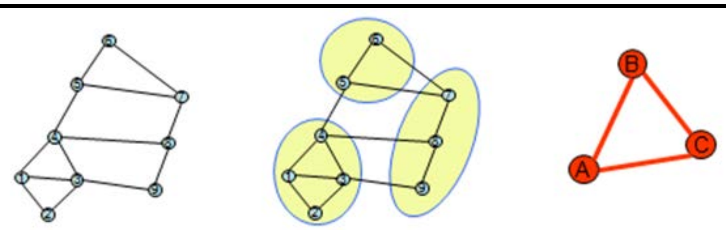
2.1. Nodal prices

According to [24] nodal prices are the locational prices consistent with the principle of prices equal to marginal cost in a power market. Nodal prices are the result of maximizing net social welfare given the physical constraints of the transmission network and transmission losses (if included in the model). The concept of contract networks, providing financial capacity rights, maintains the short-run efficiency of the market while respecting the specific physical conditions is introduced in [14].

Due to the fact that power flows in an electricity network obey certain physical laws and the nature of electricity flow is such that it cannot be routed and will take all available paths between origin and destination, nodal prices or locational marginal prices (LMPs) as they are frequently called possess some specific properties. A single limitation can induce price differences throughout the network. Contrary to common beliefs in the presence of congestion there may be flows from a high price node to a low price node, and addition of a new line may result in lower social surplus (see for instance [32]).

As an example, network representations underlying the different locational price models is provided in Figure 1. If the network on the left-hand side represents all injection and withdrawal points in a network, and all links between the nodes, the nodal pricing mechanism will result in one price for each of the nine connection points.

Figure 1: Locational prices – underlying network representations.



2.2. Zonal prices

Zonal pricing is a simplification of nodal pricing suggesting fewer prices than there are connection points in the network and some sort of aggregation. How this aggregation is specifically done is, however, not well defined. What is to be aggregated? It can be prices only or the physical network itself.

Optimal zonal prices

Aggregating prices only, i.e. economic aggregation, the topology of the network is represented in full while the prices within (pre)defined zones are required to be uniform. This is illustrated by the middle part of Figure 1, where the nine nodes have been divided into three groups, but the original network is still present. In this case, bids are given for nodes and the capacities are set for individual lines, but prices are determined for zones. We call these prices optimal zonal prices, since they are found by solving an optimal economic dispatch problem, with the extra requirements that prices of nodes belonging to the same zone are to be equal.

Optimal zonal prices have been studied by [2]. These prices are second best compared to optimal nodal prices. Different zonal divisions are preferred by different agents (producers and consumers in a node have opposing interests for instance) and grid revenues may be negative under optimal zonal prices. It is very difficult to find an optimal zonal division as it will depend on market characteristics, topology of the network and hourly costs among other factors. If there are too few zones, it may be impossible to find prices that are uniform within predefined areas and in addition clear the market subject to all relevant constraints. Network partitioning may also result in *adverse flows*, i.e. power flowing from high prices to low prices. The above factors decribe the complexity in establishing zonal boundaries especially when one needs to find a favourable network division that will be fixed for a longer period of time.

Simplified zonal prices

The area or zonal pricing used in the Nordic power market is better represented by the underlying network topology in the right-hand part of Figure 1. In this pricing approach, which we call simplified zonal pricing, the original detailed network and the three defined zones have been replaced by three aggregated nodes and some aggregated connections between these. We call it a physically aggregated network, since the network has been highly simplified thus neglecting the physical characteristics of the power flows. This type of aggregation is not straightforward for electricity networks. Injections and withdrawals in different nodes within a zone can in general have very different effects on the power system. It is also an open question how to determine the characteristics of the aggregated lines, i.e. admittance and capacity. Contrary to ordinary transportation networks where flows can be routed, an aggregated line consisting of two individual links may have flows in opposite directions. In such a case, both individual lines may be overloaded even if the sum of the flows is within the constraint.

In the simplified zonal pricing model, detailed information on nodal bids is lost, and constraints within a zone are not represented. Setting capacities on aggregated lines is difficult: if they are too restrictive, the power system may not be fully utilized, if they are too encouraging, the market outcomes may result in infeasible flows.

2.3. Electricity market modelling

Most electricity market models incorporating transmission constraints are based on the DC load flow (DCLF) model first developed by [24]. Here we do not provide a literature review of electricity market models in general but rather concentrate on presenting the efforts to model large-scale market models that have been tested on real data.

DCLF is applied in [27] to model the Austrian electricity system based on the detailed representation of the country's high-voltage grid. Since the Austrian market model did not employ any system-wide congestion management mechanisms nodal prices resulting from the model are used to assess concrete options for economically optimal market-based removal of congestion. The model has been tested for spesific winter days but the level of detail for the underlying data is however unclear. DCLF approach is applied the to a 13 node model of the electricity market in England and Wales by [12] with the aim of demonstrating benefits of nodal versus zonal (generator) and uniform pricing systems. The model is based on projected demand and capacity data; each node represents TSO-defined generator zones which generally coincide with constrained circuits, and the main power flows are represented by the inter-nodal links. Generation is given by simplified marginal costs, demand incorporates some medium-term elasticity and ten sets of curves are provided for each season representing different load levels.

A large-scale model of the European electricity market called ELMOD is presented in [18]. The full model incorporates under the welfare maximisation objective the DCLF with unit commitment, pumped storage units and wind energy input extending to a maximum time frame of 24 hours. It should be noted though that the model even though applied in many other works is rarely used in its full functionality. Another important consideration is that nodal demand functions are estimated from reference values, while supply functions are based on estimated marginal costs, and line characteristics are taken from literature. The basic version of ELMOD – static optimal dispatch – has been applied for studying the impacts of increasing wind generation in Germany under nodal pricing in [19], under comparison of nodal, zonal and uniform pricing approaches in [31], and while simulating congestion management in Germany in [17]. ELMOD is used as a cost minimisation model by fixing demand values for analysis of different scenarios for generation extension in [10].

With regards to zonal divisions, [11] analyse the effectiveness of zonal congestion management and different methods for aggregating nodes into zones on the basis of the model of the European electricity system developed in [32]. The NEULING model that calculates minimal generation dispatch and re-dispatch is used to test the effects of introducing zonal pricing in Europe under a number of proposed zonal configurations in [5]. Generation and load data used is given on a regional level. Model results are provided for 2015 and 2020 as reference years based on hourly LMPs.

3. A MODEL OF THE NORDIC ELECTRICITY MARKET

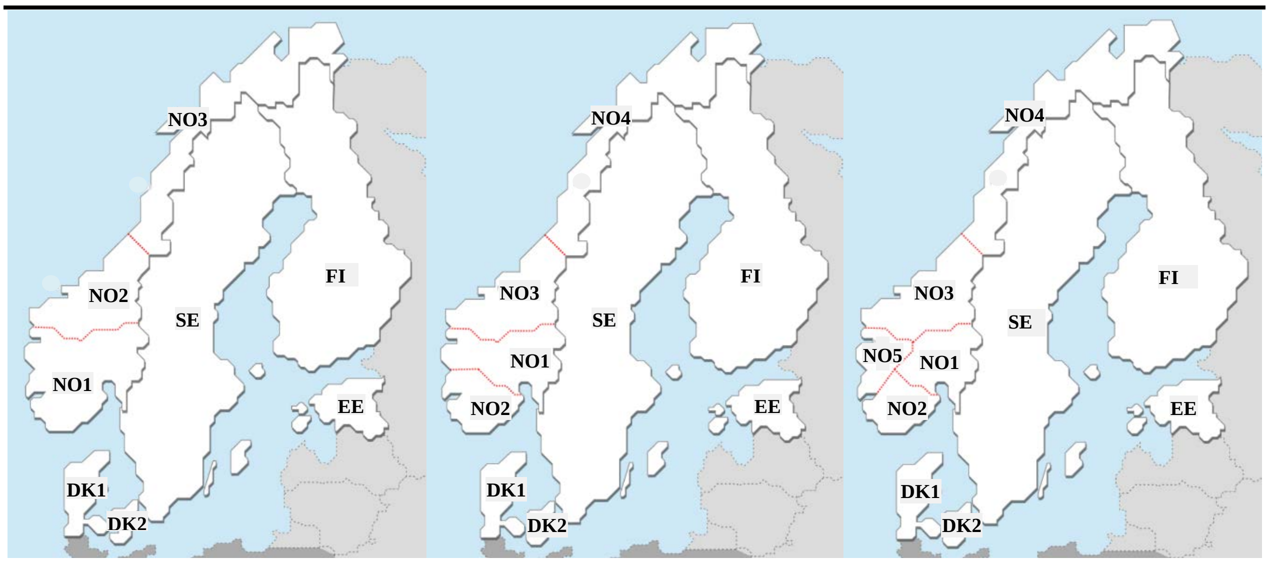
In order to analyze the effect of different congestion management methods, we have developed a model of the Nordic electricity market, OptFlow, and constructed a number of hourly supply/demand scenarios based on actual hourly market data from Nord Pool Spot. The scenarios differ with respect to supply and demand bid curves, as well as number of bidding areas and the transfer limits between them. The detailed topology and other network parameters are kept constant. Following Nord Pool Spot bidding rules, supply and demand curves are piece-wise linear. We consider supply and demand only for single hours, hence features involving multiple hours, such as block bids and ramping restrictions, are not included. Accepted block bids will sometimes be part of the bid curves, however, as price independent buy or sell bids. Since we apply different congestion management methods while keeping the bid curves constant, we implicitly assume that the choice of method does not affect the bid curves. In practice this might not be true, since the chosen method will affect prices, and hence the expected water values that are embedded in the bid curves. More detailed information about the model and discussion of the results from a number of hourly cases can be found in [3].

3.1. Transmission network models

Topology

The network topologies used in our pricing models are presented in Figure 2 and Figure 3. Zonal divisions that are applicable in our case analysis is illustrated by Figure 2. For the nodal pricing model the Norwegian part of the grid has 178 nodes and 242 lines, and it corresponds roughly to the Norwegian central transmission grid as seen in Figure 3. The Swedish part of the network is simpler, with 27 nodes and 42 lines, and is based on the Samlast model (see [25]). The other Nord Pool Spot price areas, DK1, DK2 and Finland, are represented by a single node each in both the nodal pricing and the zonal pricing models. The same is true for Estonia, when this price area is included in the data set.

Figure 2: Network topology at Nord Pool Spot in 2010.



13.04.2009 – 11.01.2010 11.01.2010 – 15.03.2010 15.03.2010 – 05.09.2011

Figure 3: Network topology – nodal pricing.

Power flows and thermal capacity constraints

We assume that the power flows of the nodal pricing model are found by using the DC approximation of the AC power flows (see [24]). The power flows depend on line voltages and admittances and must comply with the thermal capacities, which are based on voltages and maximal line currents. The specific formulas for admittance and thermal capacity are provided in the Appendix. Line parameters for the Norwegian part of the network have been supplied by the system operator Statnett, see [26], and the regulator NVE. Parameters for the Swedish network are taken from the Samlast model described in [25] and made available to us by NVE. In the nodal pricing model capacities for lines between countries have been set to actual Nord Pool Spot capacities, except for the lines between Sweden and Norway. The optimal zonal pricing model uses the same network model as the nodal pricing model, whereas the simplified zonal pricing model uses a simplified transport model of the flows between price areas combined with the Nord Pool Spot transfer capacities. A mathematical description of the pricing models can be found in the Appendix.

Security constraints

Security constraints have been included in the nodal pricing model for the Norwegian part of the network. Under the *N-1* criterion the transmission system should tolerate the outage of one component, whether it is a generation facility or a transmission line. The outage of a component will typically lead to a redirection of the power flows. These effects can be determined endogenously, as part of the optimal power flow problems, or it can be specified in advance by a heuristic set of constraints. An example of the former type of approach is found in [23], while Statnett's procedure, which we have applied, is an example of the latter type. The restrictions in Statnett's approach are termed *cut constraints*, and each constraint approximates the effect of a potential outage of one network component (see the Appendix for a formal description). We include in the nodal pricing model 38 cut constraints based on the description given in [26].

3.2. Bid curves

Cases

We have constructed a number of cases, which are based on actual hourly market solutions in the day-ahead market of Nord Pool Spot. These cases represent different market conditions with respect to Norwegian load and import, and the number of bidding areas. In the following, we focus on four hours in 2010 corresponding to Cases 1 to 4, for details study Table 1. Since bids to Nord Pool Spot are given only on area (zone) level, we do not have actual data for supply and demand curves on a more detailed level, i.e. matching the more disaggregated topology of our network model in Figure 3. Thus, for Norway and Sweden we have constructed bid curves on the node level, to be described in more detail below. An extended description of the approach can be found in [3]. For the remaining Elspot bidding areas we have used the actual bid curves from Nord Pool Spot.

Table 1: Overview of selected cases

No.	Date	Hour	Load	Import	Load (GWh)	Net import (GWh)	No. of bidding areas in Norway	SE, DK1, DK2, FI	Estonia
1	15.12.2010	19	High	Low	20.95	-3.2	5	x	x
2	07.10.2010	11	Medium	Low	14.22	-4.3	5	x	x
3	01.08.2010	6	Low	High	8.48	3.6	5	x	x
4	06.01.2010	10	Record		23.99	0.7	3	x	

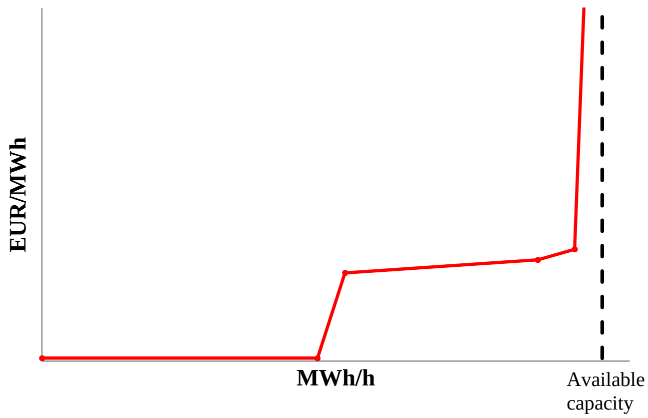
Generation bid curves

We have simplified the present market rules such that nodal generation bid curves have up to six linear segments, as illustrated in Figure 4 (in reality a bid curve may have more than 60 segments). The available generation capacity in each of the Norwegian nodes is set equal to available winter power, as detailed by the system operator in [26]. For the Swedish nodes we have made a rough estimation of nodal capacities based on the data for actual generation from the Swedish system operator (see www.svk.se), and data on the installed capacity from Nord Pool Spot (see [21]).

For most of the Norwegian nodes, as well as nodes in North Sweden with mostly hydro power, we use bid curves similar to the one shown in Figure 4. The first segment has a constant marginal cost of 2.5 Euros/MWh, and represents intermittent power generation like river hydro power plants and/or wind. The capacity in the first segment is set by taking into account the location of wind and river plants. The other parts of the curve represent the water values of the hydro power resources, and are found from the Nord Pool Spot bid curves.

We assume that most of the capacity in Mid-Sweden is thermal, with a constant marginal cost of 60 Euros/MWh. Bid curves for the thermal generation plants in Norway are set in the same way. For the three nuclear plants in Sweden we use a constant marginal cost of 4 Euros/MWh.

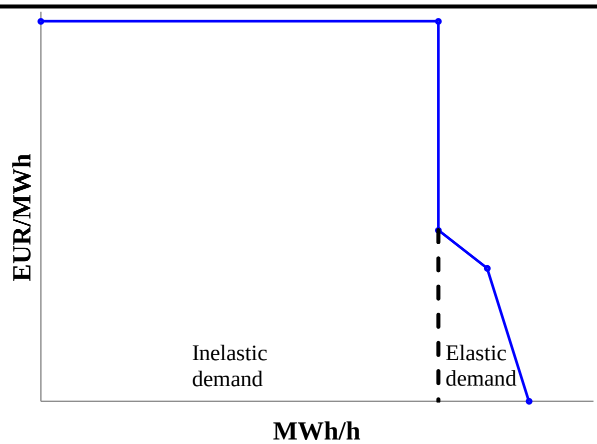
Figure 4: Generation bid curve example – hydro/wind power.



Load bid curves

For load we determine the bid curves for the hourly cases based on observed load, as reported by the system operators. The general shape of the demand curves is shown in Figure 5. The bid curve for each node has an inelastic part given by the vertical segment. This part is set equal to total load minus load for industrial consumers in the node. The elastic part of the bid curve may consist of up to two linear segments, as shown in the figure below, and the breakpoints are based on the Nord Pool Spot bid curves.

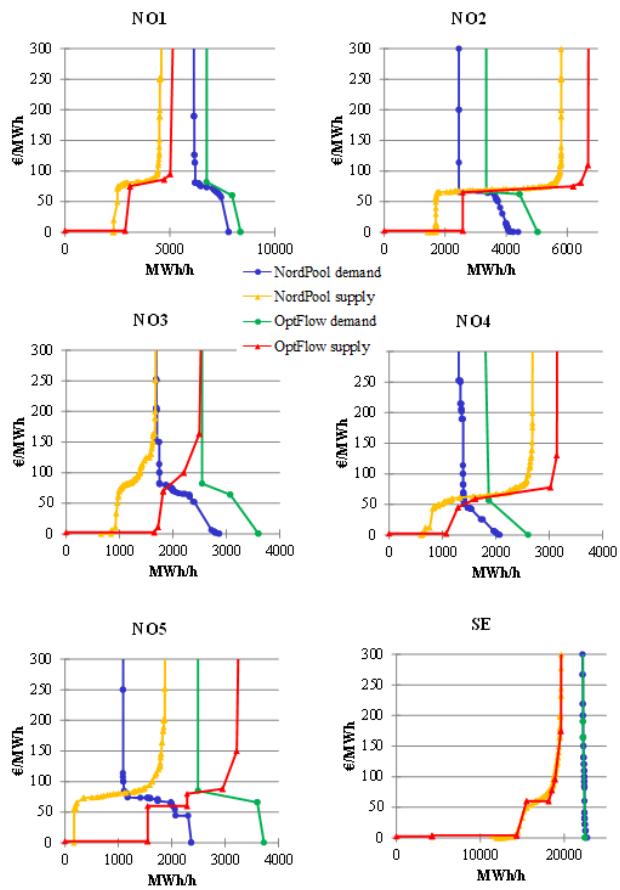
Figure 5: Load bid curve example.



Example – bid curves for Case 1

In Figure 6 we have visualised the nodal bid curves constructed in OptFlow and the actual Nord Pool Spot bid curves made up by aggregating the nodal bid curves within the bidding areas for Case 1. Bid curves are presented for the Swedish area, SE, and the five Norwegian areas, NO1 – NO5 applicable to this specific date in December 2010. We see that the shapes of the OptFlow curves reflect the replicas of the actual Nord Pool Spot bid curves quite closely; however, the volumes differ. This is so due to the fact that the Nord Pool Spot bid curves only include a fraction of the actual load/production, 70-80 % on average. In order to evaluate the effects of different pricing methods on the power system in more detail we need to model the total load, covering (as close to) 100 % of the supply and demand in the Norwegian and Swedish areas[3].

Figure 6: Nord Pool Spot and aggregate OptFlow bid curves for Norway and Sweden – Case 1.



[3] We have used data from Svenska Kraftnät to calibrate the Swedish bid curves. As can be seen from Figure 7, the calibrated curves are almost identical to the Nord Pool Spot bid curves. We expected higher volumes also for the Swedish curves, and the data allow for different interpretations. We have tested the effects of increasing the supply and demand volumes by approximately 10 %, the effects are small, and we have kept the curves as in Figure 7 for the analyses that follows.

In Table 2 we compare the actual Nord Pool Spot prices to the prices obtained from our model. Column (I) displays the actual prices of the Elspot market clearing. The corresponding OptFlow prices in columns (II) and (III) are computed using two different sets of bid curves: For the values in column (II), the actual bid curves submitted to Nord Pool Spot are used, whereas the numbers in column (III) result from simulating the Nord Pool Spot market clearing using our calibrated bid curves, i.e. aggregating the nodal bid curves, and clearing the market according to the simplified zonal pricing approach described in Section 2 and in the Appendix. For the computations we have used actual Nord Pool Spot capacities for (aggregate) inter-zonal connections. Intra-zonal capacity constraints, constraints related to Kirchhoff's second law, as well as security constraints, have all been relaxed. Hence, this OptFlow price calculation closely resembles the model actually used for the computation of the Elspot prices.

Table 2: Prices under the three market models – Case 1.

Bidding area	**(I) NPS actual area**	**(II) OptFlow prices with NPS bid curves**	**(III) OptFlow prices with calibrated bid**
NO1	104.56	104.56	105.63
NO2	104.56	104.56	105.63
NO3	130.50	130.51	130.70
NO4	130.50	130.51	130.70
NO5	104.56	104.56	105.63
DK1	130.50	130.51	130.70
DK2	130.50	130.51	130.70
SE	130.50	130.51	130.70
FI	130.50	130.51	130.70
EE	38.95	38.95	38.95

We see that Elspot prices (I) and area prices calculated by the OptFlow model with Nord Pool Spot bid curves (II) match exactly. This shows that the OptFlow model is capable of reproducing the Nord Pool Spot results when using the same bid curves. Moreover, the differences between the actual Elspot prices (I) and the OptFlow area prices calculated on the basis of the nodal bid curves (III) are quite small. Contrary to (I) and (II), the disaggregated bid curves cover 100 % of production and consumption, thus it is difficult to calibrate the bid curves so as to match the prices of the aggregated curves exactly. However, the relatively small differences between (I) and (III) show that the disaggregation we have developed works reasonably well in aggregate, although it still leaves a great deal of uncertainty with respect to how accurate the distribution of production and consumption is on the nodes within the bidding areas. This is as close to real market simulation as we can get with the data provided.[4]

Based on the limited data available on nodal bid curves, we conclude that the disaggregation in (III) is a reasonable starting point for analyzing different congestion management methods. In order to evaluate the effects on the power system, we need all production and consumption represented. Thus, column (III) is the starting point of our comparisons, i.e. column (III) will in our analysis represent the Nord Pool Spot area prices.

4. RESULTS FROM THE CASE STUDIES

4.1. Case 1: Security constraints are infeasible and relaxed

Hour 19 on December 15, 2010 had high load and export from Norway, and high prices (varying between 105-130 Euros/MWh) in all the Nord Pool Spot areas except for Estonia.

The optimal nodal prices for this hour show a tremendous price increase for some of the nodes in the Norwegian area NO5 compared to the simplified zonal prices in column (III) in Table 2. The nodes Arna, Fana, and Mongstad, all close to the city of Bergen, have prices equal to the price cap of Nord Pool Spot of 2000 Euros/MWh (the optimal zonal price in area NO5 is also close to the price cap). The extreme prices are caused by the cut constraints *Bergen 1* and *Bergen 2*, which impose the following requirements on combinations of lines in order to secure safe supply to the Bergen city area:

Bergen 1: flow (Fana-Samnanger) + flow (Evanger-Dale) $\leq$ 670 MW

Bergen 2: flow (Fana-Samnanger) + flow (Dale-Arna) $\leq$ 670 MW

Figure 7: Bid curves and market clearing prices and quantities for Arna node – Case 1.

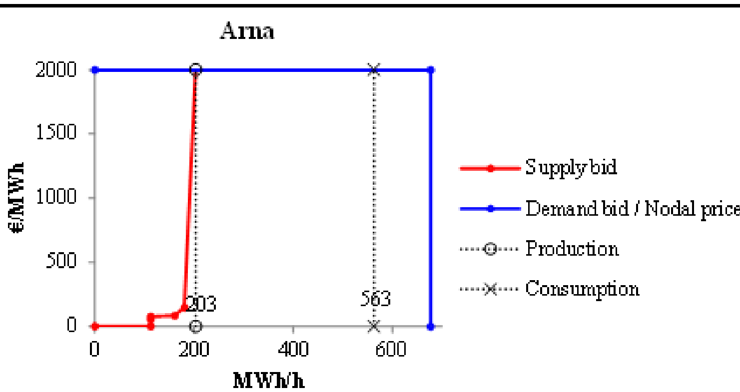


Figure 7 shows the market clearing prices and quantities for the Arna node. The red-coloured upward sloping curve is the supply curve, and the dotted vertical line close to its steepest part represents the quantity supplied at the market clearing price of 2000 Euros/MWh.

[4] Similar checks have been done on production, load and exchange data, for all the cases developed.

The demand curve in Arna is represented by the vertical blue line, showing an inelastic demand of 677 MWh/h. The Nord Pool Spot price cap of 2000 Euros/MWh is implemented in the OptFlow model, and can be illustrated in the Arna node by the horizontal blue colored segment of the demand curve. We see that the market clearing consumption quantity in the Arna node is on the horizontal extension of the demand curve, at 563 MWh/h. This is the optimal solution returned when allowing for nodal pricing and taking into account all constraints of the problem, i.e. both the thermal capacity constraints and the cut constraints imposed for security reasons. The solution is technically feasible in the OptFlow model, however, in economic terms, we are dealing with an infeasibility. The difference between the *inelastic* demand and the *market clearing* demand can be interpreted as the necessary curtailment of consumption in the Arna node in order to obtain a feasible flow.

This corresponds to the situation referred to in [6] and described in various other reports like [1] – for prolonged periods parts of the Norwegian power system has been operated at below agreed upon security standards due to high loads and/or lack of transmission capacity. In a nodal pricing system, this becomes very visible, as do the representations of the security constraints involved, sice the security constraints are explicitly modelled. In the following analyses of Case 1, we relax the infeasible cut constraints, i.e. the *Bergen 1* and *Bergen 2* cut constraints are removed in the OptFlow model. The relaxation will change the optimal nodal and zonal prices, while the simplified zonal prices will be unaffected, since the cut constraints are not directly included in this price calculation[5].

Table 3: Prices under relaxed Bergen security constraints – Case 1.

Bidding area	**Actual NPS**	**Zonal prices**		**Optimal nodal prices**		
		Simplified	**Optimal**	**Average**	**Min**	**Max**
NO1	104.56	105.63	140.51	139.25	139.21	139.40
NO2	104.56	105.63	110.00	139.23	139.23	139.24
NO3	130.50	130.70	141.04	139.59	139.45	139.91
NO4	130.50	130.70	88.19	80.74	74.77	126.58
NO5	104.56	105.63	140.35	135.65	125.22	139.24
DK1	130.50	130.70	124.77	139.23	139.23	139.23
DK2	130.50	130.70	120.61	139.23	139.23	139.23
SE	130.50	130.70	140.69	138.51	93.65	140.83
FI	130.50	130.70	138.17	137.09	137.09	137.09
EE	38.95	38.95	36.10	38.95	38.95	38.95

Table 3 shows four sets of prices. Actual Nord Pool Spot prices are given in the first price column (corresponding to (I) and (II) in Table 2), while the second and third columns show,

[5] In practice, the cut constraints may affect the import and export capacities that the system operators set between the bidding areas, and which are given to the Elspot market clearing.

respectively, the simplified and optimal zonal prices calculated by the OptFlow model. The simplified zonal prices correspond to (III) in Table 2, while optimal zonal prices take into account the specific location of all bids on the nodes and all constraints of the disaggregated power system. The three rightmost columns show descriptive statistics for the optimal nodal prices within each price zone.

All the prices are now below 141 Euros/MWh, and moving from simplified zonal prices to optimal zonal or nodal prices results in price increases in NO1, NO2, NO3, NO5, and FI. Prices decrease in NO4, while for the rest of the areas optimal prices vary around the simplified area prices or are fairly unaffected by the change (EE). Note that the price vectors are not directly comparable, since actual and simplified area prices do not take into account all constraints in the system, thus at these prices, the resulting flows will not necessarily comply with the system constraints.

Figure 8: Nodal prices and load quantities – Case 1.

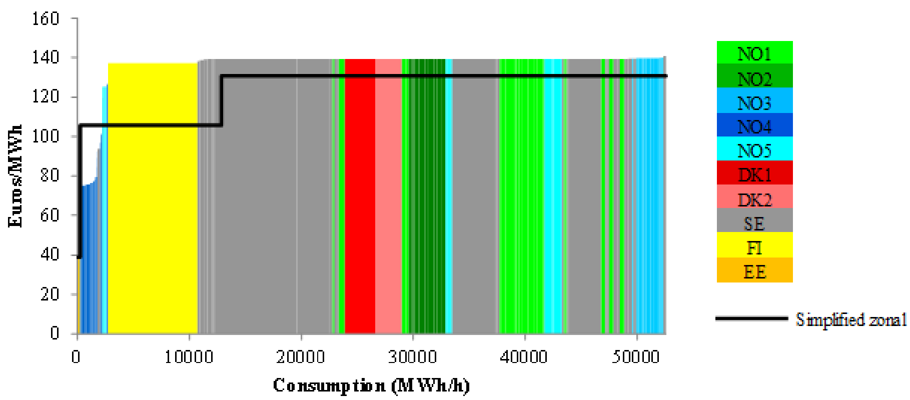


Figure 8 visualises the optimal nodal prices, sorted from the lowest to the highest weighted by consumption. The colours show which bidding area the nodes belong to, and the column widths represent load volumes. For a quick visual comparison of aggregate price differences, the simplified zonal prices are shown in a similar way[6]. Comparing the total volume weighted prices (i.e. the areas under the curves) we notice that for this hourly case, the nodal prices are on average higher than the simplified zonal prices. The reason for this is that the nodal prices include shadow prices for all transmission constraints (except for the relaxed cut constraints), whereas the simplified zonal prices do not, thus implying a solution that results in infeasible flows (see also Case 2). We also notice that the nodes in a specific bidding area like NO1 and NO5 are placed at several different locations along the first axis, i.e. some nodes are in the lower end whereas others are in the high price end of the price distribution, although for NO1-NO3, the nodal price differences within the zones are not very large. For optimal zonal prices the results are similar; prices in some locations increase while others decrease. On average, the optimal zonal prices are also higher than the simplified zonal prices.

[6] Since the simplified zonal prices are also sorted from the lowest to the highest, the curves cannot be compared directly for each MWh/h, in the sense that a specific point on the first axis may represent different locations in the two curves.

The histogram in Figure 9 describes the utilization of the lines' thermal capacity limits under the three pricing methods that we consider. The simplified zonal prices result from an optimization problem with a simplified network and power flow representation that does not take into account the physical laws. Consequently, the flows calculated in this optimization problem cannot be used to evaluate the effects of these prices on the real power system. Therefore, the power flows of the simplified zonal solution have been calculated using a two-stage approach, decribed in the Appendix. A similar procedure is used to find the flows when we clear the market assuming there are no network constraints (unconstrained market solution). Figure 9 shows the number of lines operating within different intervals of capacity utilization, and we distinguish between inter-zonal lines (red color) and intra-zonal lines (blue color). In the nodal price solution seven lines (four inter-zonal and three intra-zonal) are operated at between 90-100 % of their capacity. For the present high load case, we notice that, regardless of the congestion management method, most of the lines are operated well below their thermal capacity limits. We also note that the simplified zonal prices result in infeasible power flows over two out of 301 lines, one inter-zonal, and one intra-zonal.

Figure 9: Line capacity utilisation under the three market models – Case 1.

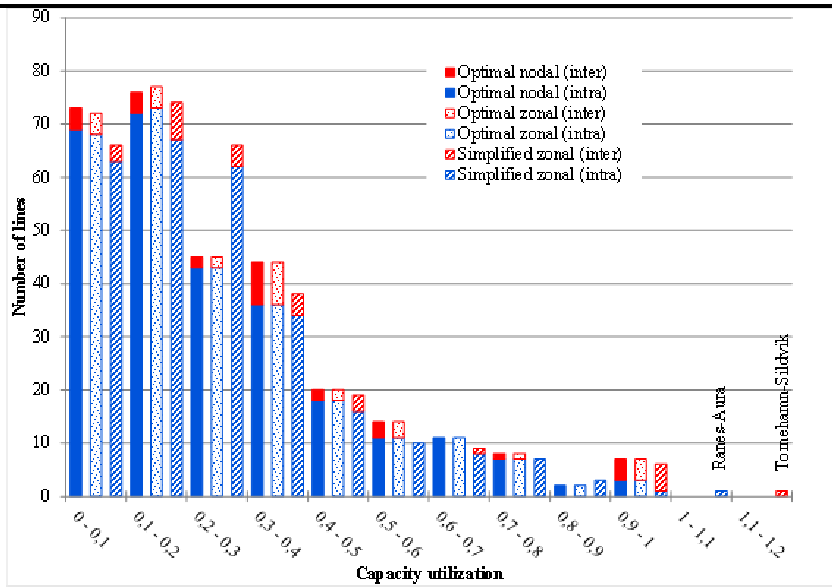
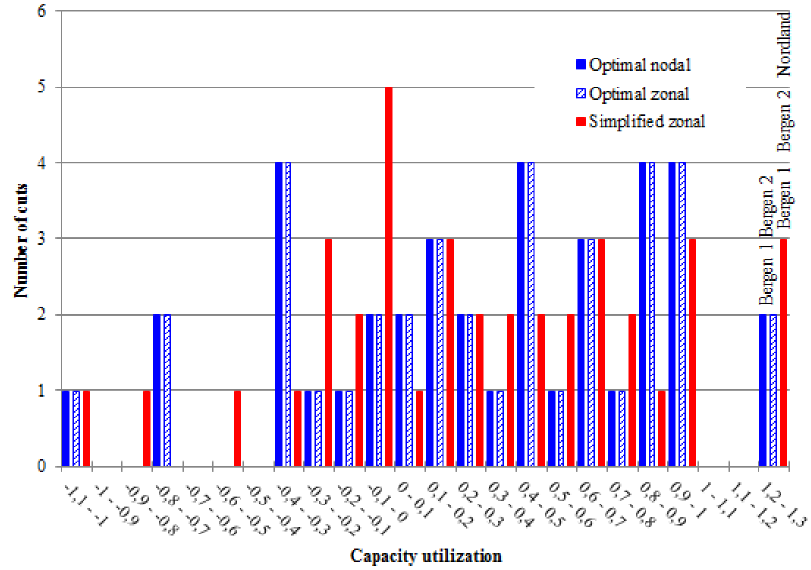


Figure 10 shows the utilization of the cut constraints for the different pricing methods, including the relaxed *Bergen* cuts. The relaxed cut constraints are overloaded in all three solutions, and in the nodal price solution they are overloaded by approximately 11 % and 15 %. All other cut constraints are fulfilled under the optimal nodal and optimal zonal prices.

Figure 10: Cut capacity utilisation under the three market models – Case 1.



The *Nordland* cut involving lines in NO3/NO4/Sweden is not satisfied in the simplified zonal solution in addition to two of the thermal constraints.[7]

Table 4 shows the changes in surplus compared to the unconstrained market solution. The absolute values of the consumer surpluses are not very meaningful, since demand is very inelastic, at least for high prices, and we have capped the consumer surplus at the price cap of 2000 Euros/MWh. Thus, the consumer surplus and the total social surplus are very much affected by the price cap, and we therefore focus on the changes in surplus compared to the unconstrained solution, the change in total surplus representing the congestion cost.

Table 4: Surplus differences (1000 Euros) – Case 1.

	Unconstrained	Simplified zonal	Optimal zonal	Nodal
Producers	6799.5	38.0	650.3	624.8
Consumers	99249.1	-111.7	-757.9	-761.4
Grid	0.0	68.6	90.3	120.9
Total	106048.7	-5.1	-17.3	-15,6
Infeasibilities	2 lines 1 cut	2 lines 1 cut	None	None

Bergen 1 and *Bergen 2* are overloaded in all solutions

Since the surpluses of the simplified zonal solution are not comparable to the corresponding surpluses of the optimal nodal and zonal solutions, that take into account all constraints, we have shown the number of overloaded thermal and security constraints in the last

[7] Note that the cut constraints are one-sided, so capacity utilization below – 100 % is not a problem.

row of the table. Alternatively, we could model counter trading in the regulation market, and take into account any efficiency effect from those. This is however, not straightforward. In order to obtain reasonable solutions in such a model, we need to make assumptions on the degree of flexibility in real time for the different producers and consumers. If we assume the same bid curves both for the day-ahead and the real time regulation market, and no start-up cost or other kinds of inflexibility, an optimal redispatch takes us to the optimal nodal price solution, which is clearly not realistic. In this paper, we have chosen to show the surpluses from the day-ahead market together with a summary of the infeasibilities that are left for the regulation market.

For the present case, we see that moving from simplified zonal prices to optimal zonal or nodal prices leads to a reduction in consumer surplus, and an increase in producer surplus and grid revenue. Since we disregard some constraints in the simplified zonal solution, the total surplus seems to go down, however this must be balanced by the infeasibilities that are left in the simplified zonal solution and which are dealt with in the optimal zonal and optimal nodal solutions. In the end, the infeasibilities must be taken care of in the simplified zonal solution too, this may be costly for society, and this cost is not reflected in Table 4.

We see that the difference between total surplus in the unconstrained solution and the optimal nodal price solution is 15600 Euros. This corresponds to the minimum possible congestion cost for this hour. The optimal zonal price solution would lead to a small increase in the congestion cost, to 17300 Euros.

4.2. Case 2: Simplified network constraints are too restrictive

Hour 11 on October 7, 2010 was characterized by medium sized Norwegian load and exports from Norway. The actual Nord Pool Spot prices were in the range of 50-56 Euros/MWh, which is about the same as the average system (unconstrained) price in 2010 of 53.06 Euros/MWh.

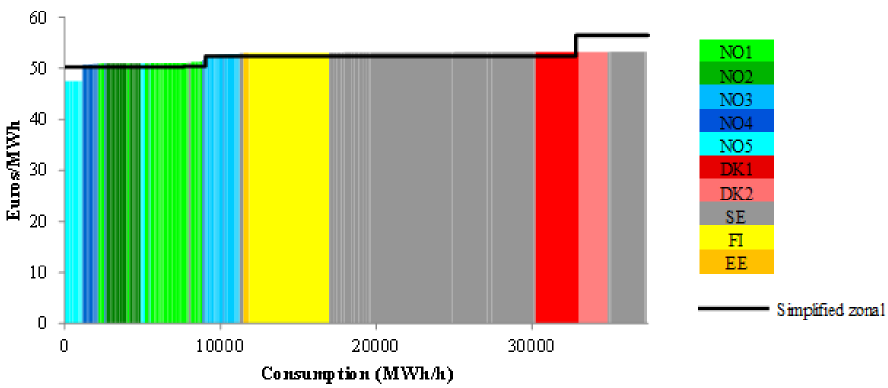
Table 5 compares the four sets of prices for Case 2. We see that when moving from simplified zonal prices to optimal nodal prices prices changes are rather small. The largest difference is for optimal zonal prices, and for Estonia.

Table 5: Prices – Case 2.

Bidding area	Actual NPS	Zonal prices Simplified	Zonal prices Optimal	Optimal nodal prices Average	Optimal nodal prices Min	Optimal nodal prices Max
NO1	50.04	50.25	51.81	51.12	50.95	51.81
NO2	50.04	50.25	51.06	51.02	51.01	51.04
NO3	52.28	52.40	52.56	52.60	52.09	52.84
NO4	50.32	50.35	55.22	50.97	50.61	52.65
NO5	50.04	50.25	51.04	48.23	47.45	51.04
DK1	56.48	56.48	51.65	53.24	53.24	53.24
DK2	56.48	56.48	56.35	53.24	53.24	53.24
SE	52.28	52.40	53.28	53.17	51.17	53.30
FI	52.28	52.40	53.02	53.02	53.02	53.02
EE	52.28	52.40	39.40	53.02	53.02	53.02

Figure 11 shows the optimal nodal prices, sorted from the lowest to the highest, and weighted by load volumes. We notice that the lowest and highest prices are reduced, and that in the middle part of the graph the nodal prices are very similar to the simplified zonal prices.

Figure 11: Nodal prices and load quantities – Case 2.



The histograms in Figure 12 and Figure 13 demonstrate that regardless of the congestion management method, most of the lines are operated well below their thermal capacity limits, and even the simplified zonal approach results in feasible power flows over all individual lines, i.e. no thermal constraints are violated. However, two of the cut constraints are violated in the simplified zonal solution. These are the *Fardal overskudd 1* and *Fardal overskudd 2* cuts in NO1/NO5. It is interesting to note that the first of these two constraints is not operated on its capacity limit neither in the optimal nodal solution nor in the optimal zonal solution.

Figure 12: Line capacity utilization under the three market models – Case 2.

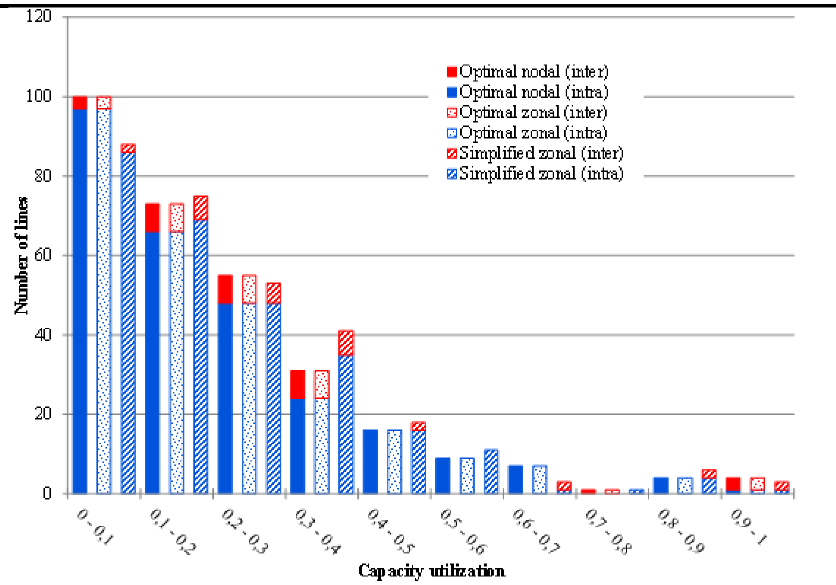


Figure 13: Cut capacity utilisation under the three market models – Case 2.

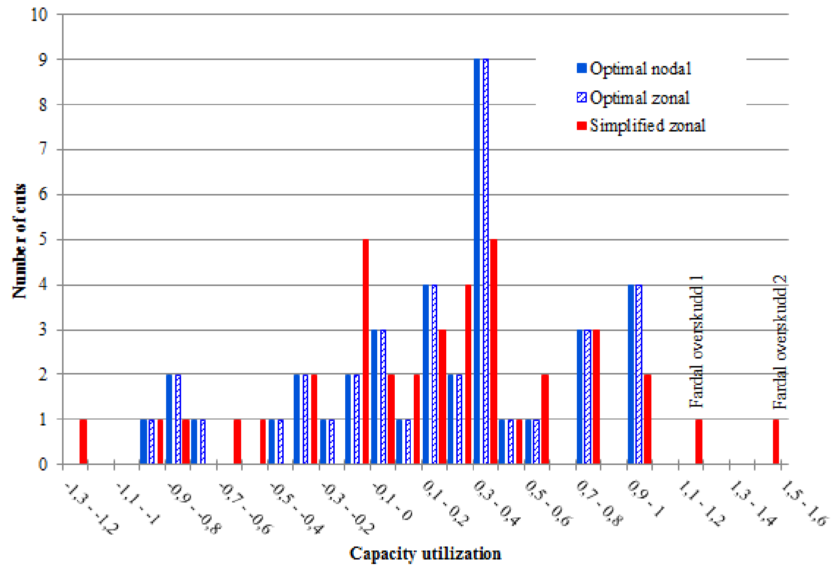


Table 6 shows that for this case that moving from simplified zonal prices to optimal zonal or nodal prices leads to a small increase in the total surplus, in addition to removing all infeasibilities. The simplified zonal solution will incur higher costs than under the other pricing methods since counter trading is needed in order to relieve the violated cut constraints.

Table 6: Surplus differences (1000 Euros) – Case 2.

	Unconstrained	Simplified zonal	Optimal zonal	Nodal
Producers	2364.2	21.1	42.9	28.8
Consumers	75841.2	-39.3	-49.9	-41.4
Grid	0.0	15.1	5.5	11.1
Total	78205.4	-3.1	-1.5	-1.5
Infeasibilities	4 lines 6 cuts	0 lines 2 cuts	None	None

4.3. Case 3: Intra-zonal constraints determine nodal prices

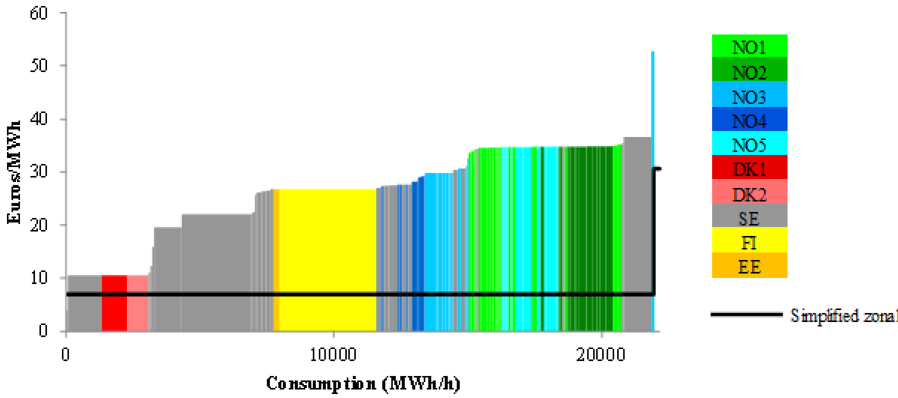
Hour 6 on the 1st of August 2010 was characterized by low loads and high imports to Norway. The actual Nord Pool Spot prices were low, roughly 7 Euros/MWh in all areas, except for Estonia.

Table 7 shows that in the simplified zonal price solution, all prices, except for Estonia, are equal. With nodal prices or optimal zonal prices there is more variation. As illustrated by Figure 14, nodal prices change a lot, and there is a considerable price increase in all nodes except for some in NO3, Sweden and Estonia. Optimal zonal prices are lower, as they are determined by some very low prices in DK1, DK2 and Estonia in particular.

Table 7: Prices – Case 3.

Bidding area	Actual NPS	Zonal prices		Optimal nodal prices		
		Simplified	Optimal	Average	Min	Max
NO1	7.29	7.00	6.98	34.54	32.14	35.21
NO2	7.29	7.00	43.50	34.81	34.75	34.87
NO3	7.29	7.00	6.99	27.52	2.50	52.74
NO4	7.29	7.00	6.49	28.14	26.97	29.94
NO5	7.29	7.00	10.10	34.75	34.70	34.79
DK1	7.29	7.00	0.00	10.53	10.53	10.53
DK2	7.29	7.00	0.40	10.53	10.53	10.53
SE	7.29	7.00	6.59	19.54	4.00	36.63
FI	7.29	7.00	6.35	26.82	26.82	26.82
EE	30.63	30.63	0.10	26.82	26.82	26.82

Figure 14: Nodal prices and load quantities – Case 3.



From Figure 15 we notice that the simplified zonal approach results in considerable overload on the two links Ranes – Trollheim (in NO3) and Ringhals – Göteborg (in Sweden). Since these are both intra-zonal constraints, and not well represented in the simplified zonal solution, this explains why the nodal prices change so much compared to the simplified zonal prices.

While the *Midt-Norge 2* cut in NO3/Sweden is operating on the capacity limit in the optimal nodal and zonal solutions, it is overloaded in the simplified zonal flow, as shown in Figure 16.

Figure 15: Line capacity utilisation under the three market models – Case 3.

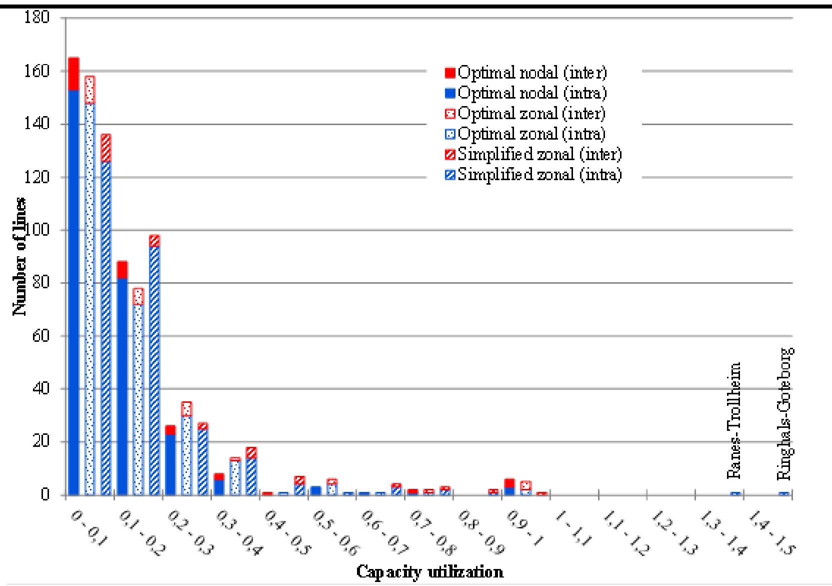
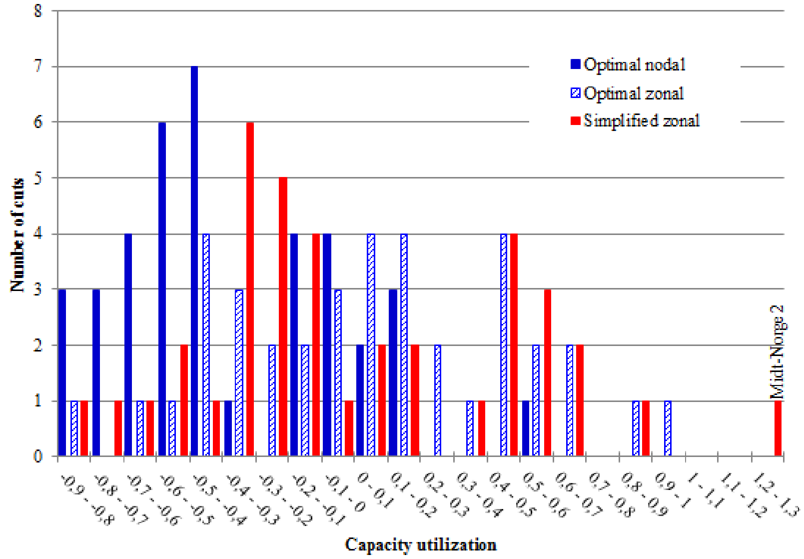


Figure 16: Cut capacity utilisation under the three market models – Case 3.



As can be seen from Table 8 nodal prices lead to a reduction in consumer surplus and an increase in grid revenue and producer surplus compared to simplified zonal prices. The changes are less for optimal zonal prices. Total surpluses are however not comparable since the simplified zonal solution is not feasible, with two thermal constraints and one security constraint being overloaded by 25 – 50 %. In this case the actual constraints of the system are badly represented by the defined zones, and the congestion cost in the optimal zonal solution more than doubles compared to the minimum congestion cost in the nodal solution.

Table 8: Surplus differences (1000 Euros) – Case 3.

	Unconstrained	Simplified zonal	Optimal zonal	Nodal
Producers	1382.6	-6.5	8.2	293.1
Consumers	39286.0	-0.1	-49.2	-424.8
Grid	0.0	6.0	9.5	116.8
Total	40668.6	-0.6	-31.4	-15.0
Infeasibilities	2 lines 1 cut	2 lines 1 cut	None	None

4.4. Case 4: Load is record high and nodal pricing removes infeasibilities

For 2010 this case represented a record high load of 23.99 GWh in the Norwegian area. In spite of this, prices were modest, ranging from 42 to 48 Euros/MWh, which was lower than the average system price for 2010. In this period, Norway was divided into three areas compared to five in the previous cases (ref. Figure 2).

This case is similar to Case 1, since the *Bergen 1* and *Bergen 2* cuts result in infeasibilities in the nodal price solution. As in the previous case, we choose to relax these security constraints in the following analyses.

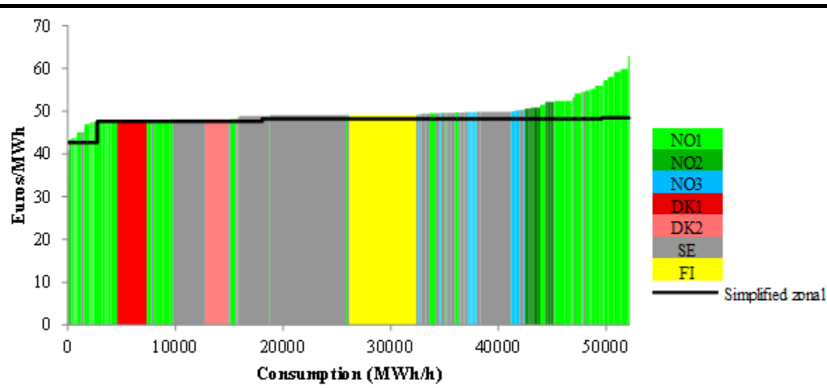
Table 9 shows that when moving from simplified zonal prices to optimal nodal prices, we see some significant price changes, especially for the Norwegian areas and Sweden, although the average nodal prices for the price zones are quite similar to the simplified zonal prices. Comparing the prices under simplified and optimal zonal solutions, we notice that the prices change considerably for DK2, but also for NO3 and SE.

Table 9: Prices – Case 4.

Bidding area	Actual NPS	Zonal prices		Optimal nodal prices		
		Simplified	Optimal	Average	Min	Max
NO1	47.54	47.95	47.22	49.95	41.02	62.92
NO2	48.45	48.45	48.02	49.38	33.88	53.25
NO3	48.20	48.14	62.74	49.93	49.21	50.50
DK1	42.65	42.65	42.74	47.56	47.56	47.56
DK2	48.20	48.14	115.00	48.09	48.09	48.09
SE	48.20	48.14	56.30	48.99	45.09	54.45
FI	48.20	48.14	49.39	49.09	49.09	49.09

Figure 17 illustrates this further. We notice that on average nodal prices are a little higher than simplified zonal prices. The variation in optimal nodal prices is not very large, but larger than for simplified zonal prices. The highest prices, but also some of the lowest, are attributed to NO1.

Figure 17: Nodal prices and load quantities – Case 4.



The histograms in Figure 18 and Figure 19 demonstrate the utilization of the capacity constraints. We notice that for this case the simplified zonal approach results in five overloaded lines, three of them intra-zonal, and two overloaded cut constraints, *Fardal overskudd 1* in NO3/NO5 and *BKK* in NO5, in addition to the two relaxed *Bergen* constraints (which flow are above allocated capacity in all three solutions).

Figure 18: Line capacity utilisation under the three market models – Case 4.

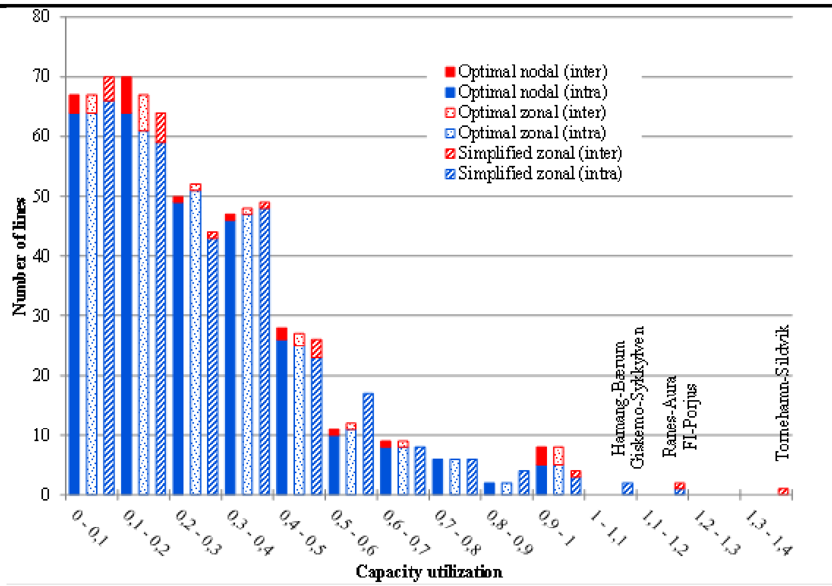
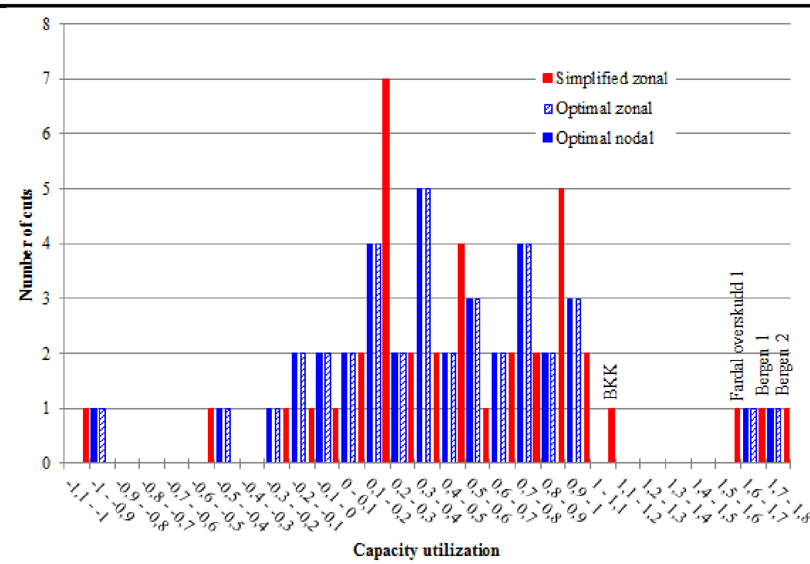


Figure 19: Cut capacity utilisation under the three market models – Case 4.



In Table 10 we see that moving from simplified zonal prices to optimal zonal or nodal prices leads to an increase in producer surplus and a decrease in consumer surplus. The grid revenue increases with optimal nodal prices while it becomes negative with optimal zonal prices. The total social surplus hardly changes, even if all the five individual overloaded constraints and the two infeasible cut constraints are taken care of with the optimal prices.

Table 10: Surplus differences (1000 Euros) – Case 4.

	Unconstrained	Simplified zonal	Optimal zonal	Nodal
Producers	3510.4	-11.8	363.5	43.1
Consumers	101332.0	6.2	-335.9	-92.4
Grid	0.0	5.1	-29.9	47.4
Total	104842.4	-0.6	-2.3	-1.8
Infeasibilities	7 lines 2 cuts	5 lines 2 cuts	None	None

Bergen 1 and *Bergen 2* are overloaded in all solutions

5. SUMMARY AND CONCLUSIONS

In this paper we have constructed a model of the Nordic electricity market in order to study three different pricing or congestion management methods: nodal pricing, simplified and optimal zonal pricing. We have calibrated hourly supply and demand curves based on actual Nord Pool Spot sales and purchase bids, as well as imports and exports with adjacent power markets. To obtain nodal bid curves, we have used data from the regulator and system operators on nodal production and exchange, information on generation technologies and capacities, and the location of energy-intensive industries.

We have simulated the effect that different congestion management methods have on the day-ahead market outcomes for specific hours with given bid curves, i.e. we assume that bids do not change even if the congestion management method does. The results that we report here are for the 4 calibrated bid scenarios from 2010, with varying prices, load and import and export levels. The calibrated nodal bid curves match relatively well with the aggregated Nord Pool Spot bid curves. However, the disaggregation depends on many assumptions, and may not reflect the actual nodal bid curves underlying the Nord Pool Spot bid curves, if these could be found. Thus, the simulations are evaluated with the calibrated nodal bid curves as the starting point.

Table 11 gives summary statistics for the four cases, and shows volume weighted average prices for load and generation, an overview of the remaining infeasible constraints that will have to be solved by counter trading in the regulation market, and an indication of the change in surplus for consumers, generators and grid if the day-ahead market was to be cleared at optimal nodal or zonal prices rather than the present area prices (simplified zonal prices).

The findings indicate that in many cases the price changes with nodal pricing are not dramatic (represented by Cases 1, 2, and 4) and sometimes are related to relatively small volumes of production and consumption (Cases 2 and 4). However, in situations where intra-zonal constraints are badly represented by the aggregate transfer capacities in the simplified zonal model, and it results in constraints being violated, the nodal prices may be considerably higher on average and vary more than the simplified zonal prices (Case 3). We also find examples where nodal prices vary less than simplified zonal prices. This may be the result of too tightly set aggregate transfer capacities. In this instance (i.e. Case 2) the surplus is higher and the infeasibilities disappear when we introduce nodal prices. Simplified zonal prices may also lead to the *wrong* constraints being violated compared to the nodal price solution, i.e. even if a constraint is not binding in the optimal nodal price solution, it may be overloaded in the simplified zonal price solution. Moreover, it becomes very visible when the security constraints cannot be fulfilled. This results in very high prices and curtailment of load. The alternative is to relax the security constraints (as we did for Cases 1 and 4).

Table 11: Overview case results.

Case no.	Congestion management method	Average price		Infeasibilities (incl. relaxed constr.)			Surplus relative to simplified zonal			
		Cons.	Prod.	Inter-zonal lines	Intra-zonal lines	Sec. cuts	Cons.	Prod.	Grid	Sum
1	Simpl. zonal	124.29	122.32	1	1	3				
	Opt. zonal	136.54	132.00			2	-	+	+	-
	Nodal	136.16	133.65			2	-	+	+	-
2	Simpl. zonal	52.39	52.06			2				
	Opt. zonal	55.55	53.91				-	+	+	+
	Nodal	52.51	52.20				-	+	-	+
3	Simpl. zonal	7.24	7.26		2	1				
	Opt. zonal	17.49	14.22				-	+	+	-
	Nodal	26.46	20.98				-	+	+	-
4	Simpl. zonal	47.81	47.80	3	2	4				
	Opt. zonal	54.73	55.07			2	-	+	-	-
	Nodal	49.69	48.80			2	-	+	+	-

Presented analyses proves that allowing for more prices in the Nordic power market would make dealing with capacity limits in the power system easier and more transparent. For an in-depth analyses of a nodal pricing model for the Nordic electricity market better data sets need to be established. We have had access to data for individual hours only, and information has been collected from many sources, not suited for this purpose originally. We would also recommend establishing data sets for longer periods (weeks or months), which would make it possible to take into account intertemporal considerations like block bids, ramping constraints and water values.

A topic for future research is to model counter trading realistically in order to take into account the cost of the constraints that are not resolved in the day-ahead scheduling and to better represent the current market solution. In general, it would be beneficial to find better ways to compare optimal solutions to those that include some infeasibilities.

APPENDIX: MATHEMATICAL MODELS

In this appendix we present the mathematical models that we have developed and applied for the simulation of market clearing under the three congestion management methods. The models take as input bids for generation and load, network topology and thermal and security constraints. Models' outputs are prices, generation and load quantities, and network flow.

Sets

N	Set of nodes.
L	Set of lines.
L^{DC}	Set of HVDC lines.
Z	Set of price areas (zones) in the simplified/optimal zonal pricing models.
N^z	Subset of nodes that are included in the price area $z \in Z$.
$CUTS$	Set of security cuts.

Parameters and functions

$p_i^s(\bullet)$	Function that represents the supply bid curve in node *i*.
$p_i^d(\bullet)$	Function that represents the demand bid curve in node *i*.
CAP_{ij}	Thermal capacity[8] limit of the line from *i* to *j*.
$CCAP_k$	Upper limit for the flow over a cut $k \in CUTS$.
CAP_{xz}	Upper limit on the flow from zone $x \in Z$ to zone $z \in Z$ in the simplified model.
H_{ij}	Admittance[9] of the line between the nodes *i* and *j*.
α_{ij}^k	A constant between 0 and 1 that represents the share of the flow over line (i, j) that is included in the cut constraint $k \in CUTS$.
β_{ij}^k	A constant that represents the share of the generation in node *i* that is deducted from the upper limit of the cut constraint $k \in CUTS$.
γ_{ij}^k	A constant that represents the share of the load in node *i* that is deducted from the upper limit of the cut constraint $k \in CUTS$.

Decision variables

q_i^s	Generation quantity (MWh/h) in node *i*.
q_i^d	Load quantity (MWh/h) in node *i*.
p_i	Price in node *i (*only used in the optimal zonal pricing model).
p_z	Price in zone *z (*only used in the optimal zonal pricing model).
f_{ij}	Flow of (real) power from node *i* to node *j*.
θ_i	Phase angle variable for node *i*.

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[8] Capacity here is calculated using the formula $\sqrt{3}V_{ij}I_{ij}^{MAX}$, i.e., using the voltage level and the maximal current of the line.

[9] Admittance here is calculated using the formula $\left(V_{ij}\right)^2 / \sqrt{X_{ij}^2 + R_{ij}^2}$, i.e., using the voltage level, reactance and resistance of the line, as suggested in [29].

Objective function

$$max_{q^d,q^s,f,\theta} \sum_{i \in N} \left[\int_0^{q_i^d} p_i^d(q)dq - \int_0^{q_i^s} p_i^s(q)dq \right] \tag{1}$$

The objective function maximizes total welfare, i.e., the total area under the load bid curves minus the total area under the generation bid curves.

Flow constraints

$$q_i^s - q_i^d = \sum_{j:(i,j) \in L} f_{ij} - \sum_{j:(j,i) \in L} f_{ji} \qquad i \in N \tag{2}$$

$$f_{ij} = H_{ij}(\theta_i - \theta_j) \qquad (i,j) \in L \backslash L^{DC} \tag{3}$$

$$\theta_1 = 0 \tag{4}$$

The power flow equations of the AC power networks are highly non-linear, but may be linearized under certain simplifying assumptions. In order to model the physical flow of power, we have used a linearized DC approximation. A large number of applications, like for example [7], [14], [15] and [30], exploit the DC approximation provided in [24] and [28]. The assumptions of the DC approximation are as following: a) only the real power balance is considered; b) the resistance of a line is negligible compared to the line's reactance and is thus set to 0 (lossless system); c) voltage angle differences across any line are small; d) voltage magnitudes are equal to 1 in a per-unit system. In our formulation equations (2) and (3) enforce Kirchhoff's first and second law, respectively, while (4) sets the phase angle for the slack bus equal to zero. Note that HVDC lines are not included in the loop flow constraint (3).

Thermal capacity constraints

$$-CAP_{ji} \leq f_{ij} \leq CAP_{ij} \qquad (i,j) \in L \tag{5}$$

Security cut constraints

$$\sum_{(i,j) \in L} \alpha_{ij}^k f_{ij} + \sum_{i \in N} \beta_i^k q_i^s + \sum_{i \in N} \gamma_i^k q_i^d \leq CCAP_k \qquad k \in CUTS \tag{6}$$

In the equations (6) we have implemented the heuristic security cut constraints that are described by Statnett in [26]. A cut *k* is defined by a set of transmission lines for which the total flow must not exceed $CCAP_k$$\mathrm{CCAP_k}$ in order to ensure feasibility of the production and consumption schedules even if some of the lines in the cut should fail. The relationship between the transmission line (i,j) and the cut *k* is given by the parameter α_{ij}^k. If $\alpha_{ij}^k = 1$, the power flow from *i* to *j* is included in full in cut *k*, whereas $\alpha_{ij}^k = 0\mathrm{\alpha_{ij}^k} = 0$ means that the flow is not included. Note that α_{ij}^k may also take on a value between 0 and 1, meaning that some portion of the flow from *i* to *j* is included in the cut constraint. An example of a cut constraint *k* could include the two lines (i,j) and (p,q), where line (i,j) is subject to failures. If the former line fails some of the power flowing through it would be redirected to the latter line. We could, e.g., have $\alpha_{ij}^k =$

$0.35\alpha_{ij}^{k} = 0\ \alpha_{ij}^{k} = 0{,}35$and $\alpha_{pq}^{k} = 1\alpha_{ij}^{k} = 0\alpha_{pq}^{k} = 1$, implying that 35 % of the flow from i to j will be added to the flow from p to q.

Cut capacities are sometimes adjusted based on observed production quantities in the nodes. For production in node *i* and cut *k* this adjustment is modelled using the constant β_i^k. Similarly, the capacity of a cut can be adjusted based on consumption in one or more nodes. For consumption in node *i* and cut *k* this adjustment is modelled using the constant γ_i^k.

OptFlow market clearing models and prices

The optimal nodal pricing model is defined by (1)-(6), and the resulting nodal prices are given by the shadow prices of the balance constraints in (2). Solving (1) only, gives the unconstrained solution, assuming there are no network constraints.

In the simplified zonal model we relax (3)-(6), i.e., the constraints relating to Kirchhoff's second law, thermal capacities, and security. We replace (3)-(6) with (7), i.e. constraints on inter-zonal flows. The resulting model is similar to the model currently used for market clearing by Nord Pool. Since the model has no intra-zonal capacity constraints, the shadow prices of (2) will be uniform within zones. Note that the inter-zonal capacities in (7) are set by the system operator, and we may have $CAP_{xz} < \sum_{(i,j)\in L\cap(N^x\times N^z)} CAP_{ij}$. Hence, the resulting model is not necessarily a proper relaxation of (1)-(6), and it is in fact possible that its objective function value is lower than in the optimal nodal pricing model.

$$-CAP_{zx} \leq \sum_{\substack{(i,j)\in L \\ i\in N^x \\ j\in N^z}} f_{ij} - \sum_{\substack{(i,j)\in L \\ j\in N^x \\ i\in N^z}} f_{ij} \leq CAP_{xz} \qquad x,z \in Z \tag{7}$$

The optimal zonal pricing model has the same objective function and constraints as the nodal pricing model, and in addition the prices are restricted by (8)-(10). Nodal prices are modelled explicitly by variable p_i, and (8) ensures uniform prices within zones. Constraints (9) and (10) relate the price in every node to the load and generation bid curves, respectively, for the particular node. Note that (9) and/or (10) might be satisfied with strict inequality for some nodes, which could give incentives for self-scheduling if optimal zonal prices were to be applied.

$$p_i = p_z \qquad i \in N^z, z \in Z \tag{8}$$

$$p_i \leq p_i^d(q_i^d) \qquad i \in N \tag{9}$$

$$p_i \geq p_i^s(q_i^s) \qquad i \in N \tag{10}$$

Computing the physical power flows

In the unconstrained and simplified zonal models, where Kirchhoff's second law is not imposed, it is necessary to compute the physical power flows in a second stage. Also, letting the flow over HVDC lines vary freely introduces too much freedom, and the solutions that we obtain with unrestricted HVDC flows are in many cases unreasonable.

Based on the nodal load and generation quantities from the first stage, we compute the final line flows, using a detailed network model given by (2)-(4), but without considering the capacity constraints in (5), except some restrictions with respect to flows over HVDC lines. Since the flow over an HVDC line can be controlled in practice we think it is reasonable to restrict its flow in the second stage. In our data set we only have HVDC lines between the zones. If an HVDC line is the only line connecting two zones, as is the case for the NO2-DK1 and DK2-SE lines, we fix the flow in the second stage based on the first stage results. In general, an HVDC line may be one of several lines connecting two zones, such as the FI-SE line (Fenno-Skan). Since there is also an AC connection between Finland and Sweden, the FI-SE flow from the first stage could represent either of the two connections between the two price areas. Instead of fixing the flow we impose the line's capacity constraint (5) in the second stage. A similar procedure is used to find the flows in the unconstrained solutions, imposing capacity constraints on the HVDC lines only.

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