

Essays on Portfolio Choice

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When working intensively on a project over many years one discovers the importance of "not putting all your eggs in one basket" (i.e. diversify). The asset in my "life-portfolio" with highest return and least risk is my family. I will especially thank my love, Hilde, and my parents, Kjersti and Reidar. Thank you for always being there for me!

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Trond

Introduction: Portfolio Choice

The four papers of my thesis elaborate on different aspects of portfolio choice. Several developments in society make portfolio choice an important research topic. During the last few decades the financial markets have experienced a (mainly policy-induced) move towards international integration, liberalization and product innovation. The rapidly aging population is creating large challenges to traditional institutions on which people have relied for retirement income. For example, many defined benefit occupational pensions plans face funding deficits. Government-run public pensions are also seriously under-financed; casting doubt on what people in many developed countries have anticipated would be their main pillar of retirement security. Pension reforms, growth in mutual fund participation, increasing importance of private pension funds, and large capital inflows to hedge funds are a few examples of trends that have impact on financial markets.

The main effect of the changes during the last few decades is that people are more responsible for their choice of portfolio of assets for retirement savings. A major source of retirement income may come from assets accumulated in a defined-contribution pension plan, or assets accumulated as a supplement to defined-benefit public or private pension plan. This individualized responsibility and the rapid change in financial markets result in an increased demand for advice on complex portfolio decisions.

The basic theoretical paradigm of portfolio choice is the mean-variance analysis developed by Markowitz (1952). This approach usefully emphasizes the ability of diversification to reduce risk. Mean-variance analysis has been a great success in practice, but mean-variance analysis relies on the assumption that investors only care about the distribution of wealth one period ahead. However, theoretical work from Mossin (1968), Samuelson (1969) and Merton (1969) pioneered multi-period (long-term) portfolio choice. Recently, advances in theory and numerical methods have made it possible to find solutions to complex (e.g. intertemporal hedging demands and non-tradable labor income) long-term portfolio choice problems (see e.g. Kim and Omberg (1996), Brennan, Schwartz, and Lagnado (1997), Viceira (2001), Cocco, Gomes, and Maenhout (2005) and Campbell and Viceira (2002)).

There are, however, systematic deviations of individual investor portfolios from the normative prescriptions of the standard portfolio choice (see e.g. Lease, Lewellen, and Schlarbaum (1974), Blume, Crockett, and Friend (1974), Blume and Friend (1975), Barber and Odean (2000), and Goetzmann and Kumar (2005)). A distinctive characteristic of the portfolio choice is the failure to diversify.¹ Both investors' personal characteristics and

¹Calvet, Campbell, and Sodini (2006) investigate the total financial asset portfolio of individuals (including mutual funds and cash) and find that the welfare cost of diversification is quite modest due to a

their behavioral biases influence their portfolio choices. For example, individual investors have a tendency to overweight familiar assets (Huberman, 2001) (e.g. home-bias (French and Porterba, 1991), local-bias (Coval and Moskowitz, 1999) or overweight own-company stock (Benartzi, 2001)).

The four papers that make up my thesis take different approaches to portfolio choice. The first paper, Expertise Bias, is an empirical paper investigating individuals' portfolio choice. The main finding is that individual investors have an excess weight (according to standard portfolio theory) in stocks related to their expertise. The investigation of this research question is possible due to a unique Norwegian data set that follows all Norwegian citizens. For each individual, the data set contains information that can connect his expertise (e.g. history of employment, experience, education, and wage) with his stock holding (all individual stocks). In addition, the data set includes many socioeconomic and portfolio variables.

The next two papers, Optimal Pension Insurance Design and Intergenerational Effects of Guaranteed Pension Contracts, focus on how financial institutions such as life insurance companies, manage individuals' pension savings. Optimal Pension Insurance Design documents that within a standard expected utility framework traditional pension contracts are not part of the optimal portfolio. However, the demand for the pension products may be explained through behavioral models (e.g. Cumulative Prospect Theory). The third paper documents an intergenerational cross-subsidization effect in guaranteed interest rate life and pension contracts. The subsidy may be large enough to explain why late generations hold guaranteed interest rate products as part of their optimal portfolio allocation.

In the last paper I investigate portfolio choice on an even more aggregate level, a country. In Strategic Asset Allocation for a Country, I offer advice on an investment strategy that captures the long-term relationship between the non-tradable assets and liabilities and the financial assets of a country. Instead of using contemporaneous correlation, I apply cointegration and duration matching to identify the long-term relationship between the non-tradable assets and liabilities and the financial assets.

The following subsections briefly describe the papers in the thesis:

”Expertise Bias”

Co-authored with Hans K. Hvide

We document a bias towards investing in stocks that are related to individuals' expertise. A unique register-based data set with microdata on individuals' characteristics such

substantial share of international stocks in mutual funds.

as history of employment, education and investment portfolio, shows that investors have an excess weight (according to standard portfolio models) of stocks that are in the same industry as employment, even after controlling for own-company stock and local bias. The excess weight is mainly driven by industry specific experience.

”Optimal Pension Insurance Design”

Co-authored with Helge A Nordahl

In this paper we analyze how the traditional life and pension contracts with a guaranteed rate of return can be optimized to increase customers’ welfare. Given that the contracts have to be priced correctly, we use individuals’ preferences to find the preferred design. Assuming CRRA utility, we cannot explain the existence of any form of guarantees. Through numerical solutions we quantify the difference (measured in certainty equivalents) to the preferred Merton solution of direct investments in a fixed proportion of risky and risk free assets. The largest welfare loss seems to come from the fact that guarantees are effective by the end of each year, not only by the expiry of the contract. However, the demand for products with guarantees may be explained through behavioral models. We use cumulative prospect theory as an example, showing that the optimal design is a simple contract with a life-time guarantee and no default option.

”Intergenerational Effects of Guaranteed Pension Contracts”

Co-authored with Helge A Nordahl

In this paper we show that there exist an intergenerational cross-subsidization effect in guaranteed interest rate life and pension contracts as the different generations partially share the same reserves. Early generations build up bonus reserves, which are left with the company at expiry of the contract. These bonus reserves function partly as a subsidy of later generations, such that the latter earn a risk-adjusted return above the risk-free rate. Furthermore, we show that this subsidy may be large enough to explain why late generations buy guaranteed interest rate products, which otherwise would not have been part of the optimal portfolio allocation.

”Strategic Asset Allocation for a Country”

Forthcoming in Financial Markets and Portfolio Management

This paper develops a simple strategic asset allocation model for a country with non-tradable assets and liabilities. Contemporaneous correlation does not capture the long-term

relationship between the non-tradable items and the financial assets. I apply cointegration and duration matching to better identify the long-term relationship. The model is applied to the case of Norway. Simulations suggest that Norway should implement a strategy which entails a higher proportion (than today's strategy) invested in stocks. Although the new strategy is superior in several criteria and as Norway reforms its social security system, there is still considerable risk that Norway will fail to meet its liabilities.

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Expertise Bias*

Trond M. Døskeland and Hans K. Hvide

Preliminary and Incomplete. Please Do Not Cite or Distribute.

Abstract

We document a bias towards investing in stocks that are related to individuals' expertise. A unique register-based data set with microdata on individuals' characteristics such as history of employment, education and investment portfolio, shows that investors have an excess weight in stocks that are in the same industry as employment, even after controlling for own-company stock and local bias.

JEL Classification: D83; G11; J24

Keywords: Home Bias, Familiarity, Informed Trading, Incomplete Information

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As time goes on, I get more and more convinced that the right method in investments is to put fairly large sums into enterprises which one thinks one knows something about and in management of which one thoroughly believes. It is a mistake to think that one limits one's risks by spreading too much between enterprises about which one knows little and has no special reason for special confidence. One's knowledge and experience is definitely limited and there are seldom more than two or three enterprises at any given time which I personally feel myself entitled to put full confidence
John Maynard Keynes

Invest within your circle of competence. It's not how big the circle is that counts, it's how well you define the parameters.
Warren Buffett

1 Introduction

The strong lack of diversification by individual investors, given the gains from diversification from theories such as Markowitz (1952), is an important yet unresolved empirical puzzle in financial economics. For example, in Barber and Odean's (2000) sample of investment accounts at a retail brokerage firm, the median account holds three stocks. Using data for the representative US household, Polkovnichenko (2006) finds that 80% of the households that are equity owners hold five or less stocks. This inclination to hold a small number of stocks, dubbed the "diversification puzzle" by Statman (2004), has been confirmed for other countries as well (Finland: Grinblatt and Keloharju (2001), Sweden: Bodnaruk, Kandel, Massa, and Simonov (2007)).¹

In order to better understand the investment decisions made by individual investors, this paper investigates the relation between stock market investments and professional experience. There are at least four theoretical arguments that link work experience and portfolio choice. First, an investor might wish to hedge against variations in labor income and therefore avoid investments in professionally related stocks to (e.g., Baxter and Jermann (1997); Cocco, Gomes, and Maenhout (2005)). Second, investors can more cheaply obtain asymmetric information about stocks that are related to their professional experience, and therefore have an expertise relative to the market in such stocks. This argument would be in line with Merton (1987), who states that "investors buy and hold only those securities about which they have enough information". Third, investors might be overconfident and have only a *perceived* expertise in professionally related stocks, due to e.g., an overestimation of the precision of

¹Even if individuals' stock holdings are not well-diversified, their "total portfolio", including e.g., fund investments and property in addition to stocks could be reasonably well-diversified. For Sweden, Calvet, Campbell, and Sodini (2006) suggest that for most individuals the welfare cost of lack of diversification of financial assets is quite limited.

ones's work-related knowledge. Psychological research suggests that people are more likely to show such overconfidence in difficult tasks and in cases of self-declared expertise (Heath and Tversky, 1991; Camerer and Lovo, 1999).² This argument therefore suggests that investors hold an excess weight in professionally related companies. Fourth, an investor might focus more on professionally related stocks because they are salient or more often mentioned in their work environment. Such a behavioral bias would be in line with Huberman (2001)'s cognitive bias for the familiar and would be reminiscent of Heath and Tversky (1991) who state that "people prefer to bet in a context where they consider themselves knowledgeable or competent than in a context where they feel ignorant or uninformed".³ Evidence for the existence of a familiarity bias is provided by Huberman (2001) and, indirectly, by Barber and Odean (2007), who show that individual investors tend to overinvest in attention-getting stocks.

In this paper, we document how stock investments relate to professional expertise. Also, we report some preliminary findings on which of the four theories outlined above, if any, has more support. To this end, we employ a unique data set from Norway that combines information on individuals' stock investments with rich sociodemographic information. For example, the data contains information on the work income and the work experience of the investors, including the industry code of their workplace. In addition to allowing us to create measures of expertise, the data also allows us to control for the fact that employees in listed companies might receive stocks as part of their compensation. This is important as we believe such investments could well have other reasons than creating a hedge or a high return on investment (such as creating incentives at the workplace, see e.g., Oyer and Schaefer (2005)). Since we have data on where the investors live, we can also control for the fact that they might have a preference for geographically close stocks.

As a base case, a stock is defined as an "expertise stock" if it has the same two-digit SIC code as the SIC code of the sector of the employment of the owner. We find that individuals overweight the amount of stock held in companies from the same industries as they work. For example, after excluding all workers that are employed in listed companies we find that the average share of investments in expertise stocks is around 25%. After controlling for the market capitalization of the industry we estimate the excess weight to be around 19%. This figure is constant across a variety of robustness checks, such as defining an expertise stock as being a stock in the same three digit industry, or controlling for a possible local bias through excluding investors that live close to companies they invest in.

Our analysis of which of the theories best can explain the patterns in the data is only preliminary but suggests some interesting features. First, the "expertise bias" revealed by the

²Some of the reason why, on average, investors sell stocks that outperform those they purchase (Odean, 1999; Barber and Odean, 2001), may be that investors that think they are experts, do bad trades.

³In fact, Heath and Tversky (1991) also argue that this bias "might also help explain why investors are sometimes willing to forego the advantage of diversification and concentrate on a small number of companies with which they are presumably familiar."

data suggests that hedging against labor income fluctuations is not the underlying motive behind portfolio selection. Second, the expertise bias increases in the "depth" of an individual's industry expertise, as measured by the fraction of recent years being employed in the industry. This finding further undermines the idea that investors hedge against labor market income but seems equally consistent with the other theories. Third, individuals with higher income have a stronger bias. If we think of a higher income as reflecting more human capital and better training (it is not insider trading because we leave out stock market investments in own firm) this finding seems more consistent with asymmetric information. Fourth, our analysis suggests that the expertise bias is more pronounced for larger and more liquid stocks. This goes against asymmetric information theory, as asymmetric information is plausibly more likely to occur with small and illiquid stock (e.g., Ivkovic and Weisbenner (2005)), but quite well with familiarity bias. Our future work on the paper, briefly outlined in Section 4, will amongst others investigate whether investments in expertise stocks are associated with an excess return, as this seems to be the clearest difference between the asymmetric information theory and the behavioral theories.

Even if many papers investigate biases in portfolio choice, there are to our knowledge no other papers that directly confront the relationship between work expertise and stock holdings. For example, Massa and Simonov (2006) find empirical evidence that households do not use their financial assets to hedge labor income risk. Furthermore they find that households with a large fraction of their portfolio in familiar (both geographically and professionally close) stocks hedge less labor income risk. The paper does not present results showing the size of familiarity, nor do they correct for own-company stock. Somewhat related, Benartzi (2001) shows that employees hold a high fraction of their pension plan savings in own stocks in spite of bad diversification properties of such investments and Moskowitz and Vissing-Jørgensen (2002) document that as a fraction of all public equity held, both directly and indirectly through mutual funds, IRAs, pension plan, and annuities and trusts, own-company stock accounts for about 30% (weighted by amount of total public equity invested).

Building on the large literature documenting that investors have a "home bias", i.e., a strong preference for domestic investments over international counterparts (see e.g. French and Porterba (1991), Kang and Stulz (1997) and Tesar and Werner (1995)), Ivkovic and Weisbenner (2005) find that individual investors tend to invest more - and earn higher returns - in stocks where they are geographically close. While we have not yet done a systematic attempt to measure a possible local bias in our data, we note that the expertise bias persists even when excluding individuals that live close to the companies they own. This, interestingly, suggests that the expertise bias we document and local bias are independent phenomena.⁴

⁴Also, as pointed out by DeMarzo, Kaniel, and Kremer (2004), an "excess" weighting on local stocks can be rationalized by the need for hedging against fluctuations in local consumption prices (real estate, health care etc). Since a hedging motive more unambiguously suggests to shy away from expertise stocks, linking expertise to investment behavior therefore might provide a cleaner testing ground for testing rational versus behavioral theories.

The paper is organized as follows. In Section 2, we present the data and provide some summary statistics. In Section 3, we define the basic measures of expertise and excess weightings, and discuss the individual characteristics that determines an expertise bias. In Section 4, we summarize the analysis and discuss future extensions.

2 Data and Summary Statistics

2.1 Data

Our sample covers all Norwegian individual investors with a positive stock holding at the end of 2002. The data is collected from three sources. First, information on stock holdings at the end of 2002 (and trading activity during 2002) have been collected from VPS, the Norwegian Central Securities Depository. Second, for each investor we have sociodemographic data (age, sex, income, wealth, geographical location, education, and employment) for the period 1986-2002, collected from Statistics Norway. Importantly for our purposes, the sociodemographic data includes yearly information on the 5-digit SIC sector the individual was employed in. Third, from Oslo Stock Exchange (OSE) we have firm specific information on the listed companies, such as stock returns and market capitalization.

2.2 Summary Statistics

The basic individual sample statistics are presented in Table 1. As shown in panel A there were 308,929 individual investors with a positive amount invested directly on Oslo Stock Exchange at the end of 2002. Almost 70% of the investors are male. The average age is 49 years, the average education level is 12.3 years, and the average work experience is 20.2 years. The average non-capital income is NOK 411,488. This is higher than for the average Norwegian worker (NOK 320,400).⁵

From panel B we find that the average investor holds a portfolio worth NOK 71,654 in direct stock investments. The median amount is considerably lower (NOK 12,939), reflecting the skewness in the distribution of the value of stock portfolios (similar to e.g. stock portfolios in the Survey of Consumer Finance (Heaton and Lucas, 2000)). The average investor holds only two stocks and the return on his portfolio in 2002 is somewhat worse than the return of the market portfolio (-32.9% versus -31.2% , respectively).

Panel C shows that the sample decreases when controlling for different effects. The starting sample is 308,929. The sample we will work most with contains 55,203 individuals. In this sample we only look at individuals working in the private sector. Furthermore there has to be a company listed at OSE with the same SIC code as the individual's SIC code. To control for own-company stock we are on the conservative side and erase all investors that work in a

⁵The rate of exchange was at the end of 2002 6.966 NOK/USD and 7.291 NOK/EUR.

Table 1: **Summary Statistics**

The table presents summary statistics for individuals holding stocks in Norway at the end of 2002. Panel A presents the summary statistics for the socioeconomic and portfolio variables. Gross Wealth is less stocks and real estate. Panel B shows some summary statistics about Oslo Stock Exchange (OSE). Finally, panel C presents the sample size for the different corrections. The total number of individual stock holders at the end of 2002 was 308,929. Since only individuals working in the private sector with SIC code and with a company listed in the same SIC code, can buy an expertise stock, the sample reduces to 153,278. The last two lines show the sample size after first correcting for workers in listed companies and finally correcting for local bias. The rate of exchange was at the end of 2002 6.966 NOK/USD and 7.291 NOK/EUR.

Panel A						
Variable	Mean	Std. Dev.	Median	5%	95%	N
Age	48.7	13.7	50	25	70	308,929
Length of Education	12.3	3.3	12	7	17	308,929
Length of Experience	20.2	10.9	22.3	2.3	32.1	308,209
Income	411,488	1,949,846	343,373	71,072	890,267	308,729
Gross Wealth	811,028	8,694,354	249,592	13,143	2,239,732	308,729
Real Estate	281,888	537,452	199,813	0	806,990	308,729
Debt	535,994	2,819,175	202,883	0	1,832,880	308,729
Value stock portfolio	71,654	1,726,850	12,939	352	212,164	308,929
Return (2002)	-0.329	-0.287	-0.309	-0.844	0.091	308,929
Diversification (Number of Stocks)	1.950	2.297	1	1	6	308,929
Turnover (Transactions 2002)	3.358	19.618	1	0	10	308,929
Panel B						OSE end 2002
Shareholders in company	3,686	8,059	1,359	256	15,557	202
Market cap, NOK thousand billion						513
Market cap, Privately held, NOK thousand billion						22.1
Return, Oslo Børs Benchmark Index adjusted for dividend						-31.1%
Panel C						
# individuals (ind.)						308,929
# ind. with SIC (private sector)						210,016
# ind. with SIC (private sector) and listed companies with same SIC						153,278
# ind. same as excluded if work in listed company						128,694
# ind. same as above but corrected for local bias						55,203

listed company. To control for local bias we erase all individuals investing in companies with headquarter closer than 100 km from investor's residence.

3 Portfolios by Expertise

3.1 Measure of Expertise

To operationalize the notion of expertise, we need to find a link between the individual's stock portfolio and his professional skills. As the main proxy we will utilize the SIC code of the company the investor is employed in and the SIC codes of the listed companies. For each individual working in the private sector our data set contains a five-digit SIC code. We also know the length of experience and in which sector the experience is earned. For each company

listed on Oslo Stock Exchange we also have the primary and up to two supplementary SIC codes. For example, for an individual that works in a company with the 2-digit SIC code 61 (Water transport) and invests in a company with the same 2-digit SIC code, we treat this investment as an expertise investment. We will show matching results for both the 2-digit and the 3-digit SIC code. Later, we also discuss income, length of (sector) experience and education as possible proxies for expertise.

3.2 Measure of Excess Weighting

We define a measurement that captures an investor’s preference for expertise stocks. For each investor, we calculate a measurement indicating the strength of his expertise bias. The value-weighted fraction of investor i ’s portfolio that is invested in expertise stocks is given by w_i^{act} . To control for the distribution of expertise stocks we subtract the value-weighted fraction of all stocks in the market that are considered to be within the investors expertise, w_i^{bench} . The excess weight in expertise stocks ($ewps_i$) is defined as:

$$ewps_i = w_i^{act} - w_i^{bench}. \quad (1)$$

An investor is said to have an excess weight in expertise stocks if the fraction of expertise stocks in his portfolio is greater than the benchmark w_i^{exp} (the fraction of available expertise stocks). An investor with experience within a sector that constitutes a large fraction of the market, whereas he invests in other sectors, will get a negative excess weight. A high number is typical for an investor having expertise within a sector with few listed companies, nevertheless the investor invests heavily in this sector.

3.3 Basic Evidence of Excess Weights

Table 2 presents the results from the mapping between the expertise of the investors and the related stocks. Our results show that there exist an excess weight in expertise stocks.

The table shows results for both a two-digit and a three-digit SIC code mapping. As we can see there is not much difference between those two alternatives. The sample is smaller for the three-digit case since there are less individuals that have the alternative to invest in stocks that are in the same three-digit SIC group as they are working in. Due to this small difference, we will in the rest of the paper show results for the two-digit mapping.

In the table we also show four alternative measurements for the excess weight in expertise stocks. The first metric, w^{act} , measures the excess weight without any correction for market weights. Thereafter we investigate three measurements with different definitions of the benchmark. The market portfolio may be all outstanding equity for a company, including the equity owned by e.g. institutional investors and foreign investors (Benchmark: $w^{bench,1}$). The market portfolio may also be the market value of all outstanding equity owned by the individual investors (Benchmark: $w^{bench,2}$) or the market value owned by individual investors

Table 2: **Expertise Bias (different measurements)**

The table reports different measurements for the average estimates of the excess weight in expertise stocks. A stock is defined as an expertise stock if it has the same SIC code as the employment of the owner. For all the four panels (A-D) we illustrate four different measurements of the expertise bias. The first w^{act} shows the average actual weight in expertise stocks. The second shows the actual weight less the share of the close sector of the total market ($w^{bench,1}$). In this measurement the total market is all outstanding equity. The third measurements shows the actual weight less the share of the close sector of the total market owned by individual investors ($w^{bench,2}$). The final measurement presents the actual weight less the share of the close sector of the market value of the shares owned by investors with SIC code (working in the private sector) ($w^{bench,3}$). In addition to present the results for the two-digit SIC code, we also match based on three-digit SIC code.

		Two-digit SIC					Three-digit SIC
Measure	Benchmark	Mean	Std. Dev.	Median	5%	95%	Mean
Panel A							
w^{act}		0.329	0.451	0	0	1	0.332
$ewes$	$w^{bench,1}$	0.258	0.411	-0.003	-0.109	0.985	0.276
$ewes$	$w^{bench,2}$	0.261	0.420	-0.014	-0.109	0.970	0.280
$ewes$	$w^{bench,3}$	0.260	0.422	-0.010	-0.117	0.960	0.281
						N = 153,278	N = 132,865
Panel B							
Not "listed" workers							
w_i^{act}		0.247	0.414	0	0	1	0.241
$ewes$	$w^{bench,1}$	0.200	0.390	-0.004	-0.109	0.985	0.209
$ewes$	$w^{bench,2}$	0.191	0.391	-0.018	-0.109	0.950	0.200
$ewes$	$w^{bench,3}$	0.190	0.392	-0.023	-0.117	0.943	0.201
						N = 128,694	N = 108,468
Panel C							
Not "listed" workers & Distance to closest stock > 100 km (62 miles)							
w^{act}		0.229	0.408	0	0	1	0.228
$ewes$	$w^{bench,1}$	0.177	0.381	-0.004	-0.120	0.983	0.194
$ewes$	$w^{bench,2}$	0.173	0.386	-0.020	-0.109	0.950	0.189
$ewes$	$w^{bench,3}$	0.172	0.387	-0.023	-0.117	0.943	0.189
						N = 55,203	N = 46,391
Panel D							
Not "listed" workers & Distance to closest stock > 402 km (250 miles)							
w^{act}		0.228	0.411	0	0	1	0.225
$ewes$	$w^{bench,1}$	0.177	0.376	-0.004	-0.109	0.939	0.197
$ewes$	$w^{bench,2}$	0.173	0.386	-0.020	-0.109	0.950	0.191
$ewes$	$w^{bench,3}$	0.171	0.387	-0.023	-0.117	0.943	0.192
						N = 11,264	N = 9,362

that have the opportunity to invest in expertise stocks (Benchmark: $w^{bench,3}$). As shown in table 2, there are small differences between these benchmarks. For the rest of the paper we will do as e.g. Ivkovic and Weisbenner (2005), we will use outstanding equity owned by the individual investors, $w^{bench,2}$, as the benchmark.

Panel A in table 2 summarizes the results for all investors working in an industry with a company listed on Oslo Stock Exchange end of 2002. The average excess weight in stocks in the same industry as their expertise is 26.1%. The fraction of individuals that has expertise bias, i.e., $ewes > 0$, is 25.7%. Earlier studies (e.g. Benartzi (2001)) have showed that employees invest a significant fraction in the own-company stock. The high fraction may be due to stock programs or active expertise bias. Since we are not able to separate those two effects, we

will be on the conservative side and erase all individuals that are working directly in a listed company. The identification of the individuals is possible since we have data on the hierarchy of the listed companies.⁶ We find that the sample size decreases from 153,278 to 128,694 and the excess weight decreases to 19.1%. The excess weight for the 24,584 individuals working in a listed company is 62.7%.

We also control for local bias (panel C and D). Since there are so many similarities between local bias and expertise bias, it is important to separate those two effects. If not, one could argue that expertise bias was just a part of local bias. The rational story behind both biases, is that individuals have more information about "close" stocks. Here, "close" can be both those companies with headquarter close to where the investor lives or those companies with business related (close) to the expertise of the individual. To correct for local bias we delete all investors investing in local stocks, thus if the investor has a stock with headquarter closer than a certain distance, we drop the investor from our sample. As the cutoff between local and nonlocal investments Ivkovic and Weisbenner (2005) use two different distances, 250 miles and 100 km. In panel C and D, we find that the excess weight in expertise stocks still remains even after controlling for local bias. The expertise fraction goes down by merely 2%. This suggest that expertise bias is something different than local bias.⁷

As a result of the above analysis our base case measurement indicates that a conservative estimate of excess weight in expertise stocks equal 17.3%. It ranges from -23.0% to $\approx 100\%$. The low extreme is typical of the individuals that chose to invest only in industries different from their expertise industry. The other extreme is typical of investors working in sectors with few listed companies, yet their entire portfolio is invested in their "close" industry. In appendix A we investigate the heterogeneity in expertise bias among different industries.

3.4 Expertise Bias and Individual Characteristics

Table 3 reports the results of fitting cross-sectional regressions of the expertise bias measurement on several explanatory variables. We identify the salient characteristics of investors who exhibit stronger preferences for expertise stocks. We let the measurement, $ewes$, where the benchmark is $w^{bench,2}$, be the dependent variable in the investor-level cross-sectional regressions.

An extensive set of variables are employed as independent variables. The variables that capture investors' expertise are used to estimate the effect of these variables on the expertise bias. Other socioeconomic characteristics and portfolio variables are used to control for the effects of investor preferences for certain types of stocks. For ease of interpretation, we group the independent variables into three distinct categories:

⁶For the local bias literature, we have not yet seen a proper way of dealing with own-company stock.

⁷Be aware, we can not tell much about the size of local bias from the change in the expertise measurement, for that we need another measurement.

Table 3: Determinants of Preferences for Expertise Stocks

The table reports the results of cross-sectional regressions of the expertise bias measurement on several explanatory variables. The expertise measurement is defined as in equation (1) where $w^{bench,2}$. The explanatory variable experience within sector is defined as % of the last seven years working in the same SIC code as working end of 2002. Length of experience is defined as the difference between end of 2002 and end of education. The part time dummy is one if working less than 30 hours per week. Portfolio Diversification is the logarithm of the number of individual stocks the investor held at the end of 2002. Portfolio Turnover is the number of trades that an investor executed during the last year. t - statistics are reported in parenthesis. Standard errors allow for heteroscedasticity. Mean values are converted to simple interpretable numbers (not log or divided on a constant).

Variable	Mean	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
Experience within sector (% in SIC working at end of 2002)	.751	.1671 (32.08)			.1687 (31.28)	.1744 (32.44)	.1711 (31.86)	.1699 (30.70)
Length of Experience	19.80		.0121 (15.14)	.0100 (11.48)	.0080 (9.23)	.0088 (10.24)	.0076 (8.77)	.0065 (7.37)
Length of Experience ²			-.0003 (-15.63)	-.0003 (-13.13)	-.0003 (-12.30)	-.0003 (-11.48)	-.0002 (-10.12)	-.0002 (-8.99)
ln (Length of Education)	12.34			-.1099 (-10.73)	-.0999 (-9.82)	-.0582 (-5.73)	-.0577 (-5.69)	-.0672 (-6.37)
Part-time dummy				-.0793 (-16.04)	-.0660 (-13.53)	-.0562 (-11.41)	-.0715 (-14.38)	-.0741 (-14.11)
ln Income	433,409			.0249 (7.96)	.0232 (7.46)	.0505 (15.28)	.0653 (18.83)	.0719 (19.36)
ln Gross Wealth (less stock and real estate)	714,694					-.0379 (-31.80)	-.0341 (-27.37)	-.0374 (-27.79)
Real Estate/100000	258,524						-.0026 (-4.18)	.0024 (-4.21)
Woman							.0710 (15.59)	.0501 (10.42)
Financial Income/100000	66,635						.0000 (.05)	-.0002 (-.89)
ln Value stock portfolio	35,721							.0325 (27.15)
ln Return	-.372							.0168 (6.12)
ln Portfolio Diversification	1.682							-.1219 (-37.08)
Portfolio Turnover	3.537							-.0012 (-10.33)
Portfolio Turnover ²								.000 (5.78)
Intercept		.1720 (101.31)	.1720 (100.70)	.1817 (97.24)	.1803 (98.40)	.1793 (98.38)	.1650 (83.33)	.1749 (85.89)
N	46,554	50,576	50,576	49,732	49,732	49,639	49,639	46,554
R ²		.016	.005	.012	.028	.047	.053	.093

- Expertise Variables: Industry-specific experience (2-digit sector working now), Length of experience, Length of education, Part-time dummy, Income.
- Socioeconomic Characteristics: Gender, Age, Financial Income, Gross Wealth, Real Estate.
- Portfolio Variables: Value of stock portfolio, Return, Portfolio diversification (number of different stocks), Portfolio turnover (number of transactions).

The first regression in Table 3 shows the relationship between experience within the 2-digit sector that the individual is employed in now (% of the last 7 years) and expertise bias. All the independent variables (except the dummy variables) are centered around the mean, thus an average investor with average experience within today's SIC sector, has an excess weight in stocks related to working sector at 17.2%. For an individual with 10% more industry specific experience, the excess weight increases with 1.7%(= $0.1 * 0.1671$). As one can see from the table, this relationship is consistent across all specifications. Thus, more experience within a sector results in holding a higher fraction of the portfolio in stocks from that industry.

General experience, defined as the difference between end of 2002 and end of education, has also a significant effect on the size of the bias. We achieve the best fit with a polynomial. For the univariate regression the effect increases up till about twenty years, and decreases thereafter. For twenty years experience the size of the two terms are about 12%. In the full specification to the right in the table, the effect is less (about 5%) and the largest effect comes with about 16 years experience.

The length of education variable shows that the more educated, the less biased. In the full model a ten percent increase in the average length of education (1.2 years) will result in almost one percent ($.0672/10 = 0.007$) reduction in the average bias. We drop the age variable since the sum of education and experience is highly correlated with age (78.4%). Not surprisingly, a part-time worker owns less stocks related to its industry.

It is difficult identifying exactly whether the individual has a job that makes him an expert or not. However, wage is a good proxy for the importance of the job and therefore how strong the expertise is. As expected, a higher wage results in a stronger expertise bias. Thus, a rational story where the most important workers have lowest asymmetric information towards the sector, is supported by the estimate.

All in all, the estimates for the expertise variables are quite intuitive. The most important variables, experience within the sector and income, indicate that industry specific knowledge and human capital result in an excess weight in stocks related to expertise. General experience is less important and only up to a certain level. Furthermore, the length of education seems to have a negative effect on expertise bias.

The second group of variables is socioeconomic characteristics. There we do not find any surprising results. The bias is smaller for richer individuals (holding the stock portfolio and real estate outside). We also find that women exhibit a larger bias than men (about 5%).

The last group of variables is portfolio variables. A bit surprisingly we do not find that investors with higher value of the stock portfolio have a better diversification, and therefore a smaller bias. The sign of the variable is similar even if we do not control for diversification, although the size of the sign is smaller. For portfolio diversification we achieve the intuitive finding that the more individual stocks in the portfolio, the weaker the bias is.⁸

Several robustness tests of the regressions in Table 3 have been performed. The regressions have been run with industry dummies. The signs and levels of the variables are equal and R^2 increases. Since the dependent variable is a truncated variable between the lowest value (minus the largest sector, sector 65, -23.0%) and the highest ($\approx 100\%$) values, we have also performed Tobit regressions. The results of these regressions exhibit the same pattern and levels of significance as the reported linear regressions. In Table 4 we present the correlations between the variables.

3.5 Expertise Bias and Firm Characteristics

A crude attempt to separate the asymmetric information and the familiarity hypothesis is to investigate what stocks expertise investors favor. Under an asymmetric information hypothesis, experts are likely to favor smaller and less liquid stocks. Under a familiarity hypothesis, experts overweight stocks that are more liquid and more covered by the media. To investigate for liquidity in a simple manner we count the number of transaction done by individual investors for each firm in 2002. We divide the sample into two groups, one with the most liquid stocks (the companies with most transaction and that constitutes half of the total amount of transactions) and one group with the least liquid stocks. The expertise bias is present for both groups. However, the excess weight is larger for the liquid group than for the least liquid group, 17.4% versus 9.3% , respectively.

A related firm characteristic is size. To provide a crude measurement of a possible size effect, we divide our listed companies into two groups sorted by size with about the same total market cap (market value of all outstanding equity owned by the individual investors). For both groups, one with large firms and one with small firms, we still achieve an excess weight. However, the excess weight is larger for the large firms (27.7% versus 6.5%). According to a rational information story this is a bit puzzling. Investors should have an excess weight in small companies since it is probably easier to obtain asymmetric information about small firms. However, the numbers are more in line with a behavioral story where large companies are more in media and individuals are more familiar with them. This saliency or availability might influence individuals such that they think they have an expertise related to the industry of the large familiar companies.

⁸Ivkovic and Weisbenner (2005) in table III use some of the same variables as we are using (log Household income, log Number of stocks and log Value of stocks) explaining local bias. The signs of the variables are the same.

Table 4: **Correlation Matrix**

The table shows the correlation matrix for the independent variables.

Variable	Correlation matrix												
N=46,554	Ex. se.	Expe.	Edu.	Part	Inc	Wea.	R.E.	Wo	Fin	sto	Ret	Div	Turn.
Experience within sector	1.00												
Experience	.247	1.00											
ln (Length of Education)	-.114	-.414	1.00										
Part-time dummy	-.079	.039	-.132	1.00									
ln Income	.071	.103	.285	-.382	1.00								
ln Gross Wealth	.092	.235	.094	-.056	.302	1.00							
Real Estate	.041	.163	.018	-.038	.194	.245	1.00						
Woman	.002	.041	-.113	.256	-.292	-.166	-.123	1.00					
Financial Income	.016	.049	.006	-.003	.035	.218	.139	-.026	1.00				
Value stock portfolio	.068	.090	.069	-.019	.141	.267	.081	-.027	.067	1.00			
Return	.078	.136	-.045	.033	-.065	-.007	-.007	.133	.011	.228	1.00		
Portfolio Diversification	.019	.024	.079	-.031	.132	.211	.077	-.145	.037	.435	-.217	1.00	
Portfolio Turnover	-.005	-.021	.022	-.020	.098	.046	.027	-.058	.007	.158	-.142	.187	1.00

3.6 Expertise Bias and Other Issues

Returns. Under asymmetric information, we would expect the experts to do well on their expertise trades, better than non-experts and better than themselves do at investing in other stocks. We have not looked at this question. The average return of the portfolios of the individual investors in this sample is -37.2% . This is both lower than for all individual investors (-32.4%) and the Oslo Børs Benchmark Index adjusted for dividend (-31.1%).⁹ However, we find that the larger the bias is, the better the return on the portfolio is. This is suggestive evidence in favor of the asymmetric information explanation of the expertise bias.

Trading behavior. The more the investors trade, the smaller is the bias. If we believe in the overconfidence experts hypothesis, we would assume that investors that think they are expert exploit this and trades more. Since they only are expert in one area we would expect a large bias. However, it seems that these investors believe they are experts in several areas and diversify more the more they trade. Another question is why do the experts do better (on their expertise stock) than non-experts, is it because they enter the stock earlier before a run or because they sell off earlier before a drop. Since the current data is cross-sectional, this question will have to await analysis.

4 Conclusion

In order to better understand the investment decisions made by individual investors, we have investigated the relation between stock market investments and professional experience. Our main finding is the existence of an "expertise bias": individual investors tend to bias their portfolio towards stocks that are related to individuals' work experience, even after controlling for own-company stock and local bias. Furthermore, the expertise bias increases in the "depth" of an individual's industry expertise, as measured by the fraction of recent years being employed in the industry. These findings do not seem consistent with individuals picking stocks primarily to hedge against labor market income fluctuations, but seems equally consistent with a number of theories such asymmetric information, overconfidence, and a preference for the familiar.

Amongst our other findings, we find that individuals with higher income have a stronger bias. If we think of a higher income as reflecting more human capital and better training (it is not insider trading because we leave out stock market investments in own firm) this seems more consistent with asymmetric information than with the behavioral theories. However, our analysis also suggests that the expertise bias is more pronounced for larger and more liquid stocks. This goes against asymmetric information theory, as asymmetric information is plausibly more likely to occur with small and illiquid stock (e.g., Ivkovic and Weisbenner

⁹Without any risk adjustment this may indicate that private investors are worse investors than the average investor at OSE. The difference between the small and the large sample may indicate that the investors working in listed companies or located close to the companies have a better return than the market.

(2005)), but squares quite well with the presence of a familiarity bias.

Our future work will attempt to refine our understanding of which theory that might better explain the patterns in the data. Perhaps most importantly, we plan to investigate whether investments in expertise stocks are associated with an excess return, as this seems to be the clearest difference between the asymmetric information theory and the behavioral theories. Related to that, we wish to investigate the trading behavior of individuals with an expertise further, to for example see whether such individuals tend to be better at timing the market.

Let us list some other questions we plan to pursue. First, our measure of expertise could be modified in various ways. One example would be to define expertise through type of education (e.g., oil engineer versus software engineer), or through some combination of length and type of education and work experience. Christiansen, Joensen, and Rangvid (2006) find that economists are more likely to hold stocks. However, they do not look at the individual stocks in the stock portfolio, neither do they examine the relationship between employment and stock holdings. Our preliminary analysis of the role of education type yields some interesting results. For example, one might expect that investors with a higher economics or finance education should diversify more, and consequently have a lower excess weight invested in their own sector. For economists in general we actually find a tiny larger excess weight than the average excess weight (20% versus 19%). Investors with an education within finance, banking and insurance (code 343, N= 2891), on the other hand, have an excess weight at 34%. Since these individuals should be aware of the gains from hedging, this is suggestive of these individuals having some degree of asymmetric information.¹⁰

Second, it is conceivable that trading patterns and returns are determined by some underlying (unobserved) characteristics correlated to expertise. Fixed-effect regression exploiting changes in job sector will be applied to tackle this question. With panel data we can moreover provide measures of the correlations between return on expertise stocks and labor income (and on expertise stocks and the other stocks in the portfolio) which will make us better able to evaluate the extent to which individuals hedge risk. Finally, although our findings suggest that expertise bias is independent of other documented biases, in particular a local bias, we wish to understand this question better. For example, since our data includes information both on professional experience and on locality, we plan to estimate the relative magnitudes of expertise bias and local bias.

¹⁰In contrast, if the investor has an accounting and taxation education (code 344, N=1257) the excess weight is only 6%. Within the natural science and technical educations we do not find large deviation from average bias, however investors with a computer science education or an electronic and automation education have large biases, 28% and 27% respectively (code 481, N=3520, and code 523, N=9391). A bit surprisingly, investors educated as building and civil engineer have a low bias, 11%, (code 582, N= 7412).

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Appendix A: Expertise Bias and Industries

In an asymmetric information story one might expect that in a "complicated" industry there is a larger expertise bias. In Table 5 we find that the excess weights vary across the different sectors. However, we find that quite knowledge demanding industries (e.g. radio, TV, communication equip., instruments, watches and clocks, post and telecommunications and financial intermed.) have a large bias. The significant sectors with negative excess weight (hotels and restaurants, motor vehicles, trailers and semi-trailers and real estate activities) are unambiguously "easy" industries. Sector 74 (other business activities) pick up all the non-identifiable individuals (about 13%) and companies. The excess weight in this sector is almost equal zero. That suggests that there is no systematic bias in our results.

The table also illustrate the effect from correcting for local bias. We show the results before and after correcting for local bias. In Norway there are three large furniture factories (code 36) situated in some distant small places. Before correcting for local bias the weight

Table 5: **Expertise Bias and Industries**

The table reports average estimates for the excess weight in expertise stocks sorted by industries. The industries are listed with SIC codes in parenthesis. The Comp. column shows the number of companies listed on OSE within this sector. The columns $w^{bench,1}$ and $w^{bench,2}$ list the fraction of the sector of the market. The next two columns show the excess weight (ewes) for the different sectors. The first shows the results when corrected for holding of own-company stock and the last shows the results when also correcting for local bias.

2-digit SIC code	Market			Corrected own-stock		Correct own-stock and local bias	
	Comp.	$w^{bench,1}$	$w^{bench,1}$	ewes	N	ewes	N
				$w^{bench,2}$		$w^{bench,2}$	
Fishing, fish farming, incl. services (5)	4	0.002	0.016	0.271	765	0.327	553
Oil and gas extraction, incl. serv. (11)	25	0.398	0.166	0.251	3572	0.235	2099
Mining of metal ores (13)	1	0.001	0.005	-0.000	39	0.004	20
Food products and beverages (15)	10	0.061	0.109	0.318	4419	0.262	2215
Wood and wood products (20)	2	0.016	0.019	0.243	1120	0.258	657
Pulp, paper and paper products (21)	2	0.016	0.019	0.064	453	0.124	114
Publishing, printing, reproduction (22)	6	0.064	0.079	0.436	4022	0.293	1060
Chemicals and chemical products (24)	5	0.266	0.164	0.521	2966	0.585	1412
Other non-metallic mineral prod. (26)	2	0.014	0.005	0.042	801	0.017	363
Basic metals (27)	4	0.153	0.080	0.686	3267	0.717	2478
Fabricated metal products (28)	1	0.000	0.000	0.005	1573	0.004	774
Machinery and equipment (29)	10	0.025	0.037	0.101	2427	0.065	992
Office machinery and computers (30)	1	0.000	0.001	-0.001	18	-0.001	5
Electrical machinery and apparatus (31)	5	0.002	0.008	0.039	815	0.020	335
Radio, TV, communication equip (32)	8	0.017	0.034	0.510	1175	0.592	300
Instruments, watches and clocks (33)	8	0.008	0.015	0.425	1211	0.287	371
Motor vehicles, trailers, semi-tr.(34)	1	0.000	0.000	0.105	1018	-0.000	205
Other transport equipment (35)	1	0.006	0.020	0.174	3840	0.228	2304
Furniture, manufacturing (36)	3	0.006	0.020	0.563	1564	0.213	420
Electricity, gas and water supply (40)	3	0.008	0.009	0.064	1870	0.002	986
Water supply (41)	1	0.000	0.000	-0.000	128	-0.000	70
Construction (45)	2	0.004	0.020	0.029	10245	0.027	5101
Wholesale trade, commision trade (51)	16	0.008	0.048	0.030	11012	0.017	4043
Retail trade, repair personal goods (52)	4	0.003	0.018	0.007	7687	0.009	3740
Hotels and restaurants (55)	2	0.003	0.029	-0.022	2498	-0.018	1325
Land transport, pipeline transport (60)	3	0.001	0.003	0.077	2988	0.042	1430
Water transport (61)	37	0.109	0.109	0.266	3069	0.128	1458
Air transport (62)	1	0.002	0.002	0.066	1397	0.121	580
Post and telecommunications (64)	5	0.096	0.050	0.616	6594	0.579	2517
Financial intermed., less insurance (65)	29	0.120	0.230	0.422	9661	0.359	3461
Insurance and pension funding (66)	1	0.014	0.010	0.225	4029	0.195	1907
Auxiliary financial intermediation (67)	3	0.001	0.014	0.026	1765	0.044	449
Real estate activities (70)	7	0.012	0.007	0.002	3012	-0.003	1012
Computers and related activities (72)	24	0.015	0.064	0.289	6706	0.246	2060
Research and development (73)	4	0.004	0.021	0.004	1692	0.001	775
Other business activities (74)	11	0.008	0.023	0.012	17111	0.015	6687
Cultural and sporting activities (92)	2	0.000	0.001	0.005	2165	0.008	925
Total					128,694		55,203

was as high as 56.3% and 1564 individuals invested in the furniture companies. However, when correcting for local bias the numbers went down to 21.3% and 420, respectively. The story behind is probably that there are a lot of people working in nearby companies related to the listed furniture companies (e.g. suppliers). These individuals invest in their local listed factory and disappear from the sample when we correct for local bias.

The "Market" columns in Table 5 reports the distribution of individuals' investments by the two-digit SIC code that would prevail if investors invest in the most obvious benchmark, the market portfolio. The largest industry is financial intermediation less insurance (code 65) with a fraction of 23.0% of the total market of individual investors.

Optimal Pension Insurance Design*

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Abstract

In this paper we analyze how the traditional life and pension contracts with a guaranteed rate of return can be optimized to increase customers' welfare. Given that the contracts have to be priced correctly, we use individuals' preferences to find the preferred design. Assuming CRRA utility, we cannot explain the existence of any form of guarantees. Through numerical solutions we quantify the difference (measured in certainty equivalents) to the preferred Merton solution of direct investments in a fixed proportion of risky and risk free assets. The largest welfare loss seems to come from the fact that guarantees are effective by the end of each year, not only by the expiry of the contract. However, the demand for products with guarantees may be explained through behavioral models. We use cumulative prospect theory as an example, showing that the optimal design is a simple contract with a life-time guarantee and no default option.

Keywords: Household Finance; Portfolio Choice; Life and Pension Insurance; Prospect Theory

JEL Classification: G11, G13, G22

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1 Introduction

In this paper we combine previous work on valuation of life and pension (L & P) insurance products with well-developed theories on individuals' preferences in order to optimize customers' utility. We analyze the welfare effects of different components of pension insurance contracts, including annual guarantees. We find that contracts that are closest to a linear payout function give highest welfare. An annual guarantee seems to reduce linearity and hence lower the customers' welfare. Finally, we show that a behavioral model accounting for loss aversion may explain the existence of some forms of guarantee. A simple model with life-time guarantee seems to work best in this case.

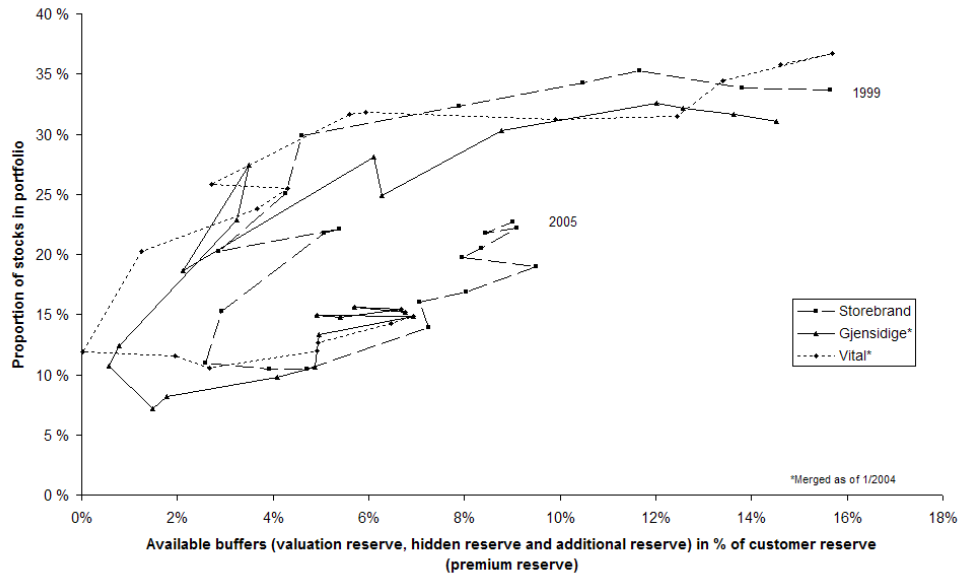
Our paper contributes to the field of household finance, defined by Campbell (2006) as how households use financial instruments to attain their objectives. Assuming that all prices are correct, we define a class of contracts, from which the customer can choose. Based on a set of preferences, the customer will then select his optimal contract. We assume that the customers' preferences can be described using the von Neumann and Morgenstern (1944) framework of expected utility. Furthermore, we use the conventional constant relative risk-aversion (CRRA) as our main representation of preferences.

From Borch (1962) we know that any utility function within the broader class of hyperbolic absolute risk aversion (HARA) utility function (including CRRA) induces linear sharing rules, meaning that each individual will get a fixed proportion of total wealth in any state of the economy. In our case, any kind of guarantee will inevitably lead to the customer receiving a higher proportion of total wealth in the states where the guarantee is effective. According to Borch, such a non-linear sharing rule will not be optimal.

However, it is likely that actual observed behavior will not coincide with the standard theories on optimal behavior as described above. As Campbell (2006) writes, "household finance poses a particular challenge to this agenda, because many households seek advice from financial planners and other experts, and some households make decisions that are hard to reconcile with this advice or with any standard model. One response to this is to maintain the hope that actual and ideal behavior coincide, but to consider non-standard behavioral models of preferences incorporating phenomena such as loss aversion and mental accounting". We alternatively explain the existence of guaranteed pension products by introducing behavioral models. We show that both a behavioral model within the expected utility framework (utility function with loss aversion) and outside, cumulative prospect theory (CPT), rationalize guaranteed features of the contract. We focus on CPT, initiated by Tversky and Kahneman (1992), since this model is the most developed and thoroughly investigated.

The main function of most modern life and pension insurance contracts is as a savings product, distributing financial market risk between customers and shareholders of the life insurance company. Despite the fact that there are no international standardized contracts, a number of common properties determine the risk sharing, e.g. asset allocation, guaranteed

interest rate, the profit sharing and the capital structure of the company.



Proportion of stocks in portfolio versus available buffers in % of customer reserve. Sources: Data from quarterly reports and analyst presentations.

Figure 1: Quarterly development of the asset allocation of Norwegian life insurers - 1999-2005.

Companies in the same market tend to follow each other closely when it comes to asset allocation (see Figure 1). Companies diverging from the "market standards" risk losing customers if their bet does not work as planned, while the upside is more limited. We use the conventional method of fixing asset allocation at the start of the contract as used e.g. by Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Miltersen and Persson (2003).

Guaranteed rates of return are normally defined in pension contracts as an annual property. Companies are obliged to grant a guaranteed amount in one year, and bonuses already distributed cannot be recalled and used as guaranteed return. However, as we will describe later, we also show a simplified contract, where guarantees are only effective at the expiry of the contract.

The return above the guarantee is shared between the company and the customer. In different countries this profit sharing is regulated by a number of different procedures, ranging from predetermined sharing rules to full company discretion from year to year (limited only by competitive pressure). As the market pressure is hard to assess in a theoretical model, we find it useful letting profit sharing be determined by a set of fixed rules, as in, e.g., Briys and de Varenne (2001). Again, more general versions allow for time-dependent but deterministic sharing rules or sharing rules as a function of some stochastic process.

We assume that the capital structure of the company is fixed only at time zero. In line

Company	Country	Year	Description	Consequences
Executive Life	USA	1991	At the end of the 1980s several US life insurers faced financial distress due to losses on real estate and junk bonds. Among the public, this led to a lack of confidence, causing a flood of surrenders. Executive Life had to file for chapter 11 protection after 1990 surrenders of more than 10 times the levels 4 years earlier.	Administration and run-off of the portfolio was administrated by the insurance supervisory authorities of the state of California and a new company, Aurura was set up by a French consortium. Still, policyholders who didn't surrender before the bankruptcy lost part of their promised amounts.
Nissan Mutual Life	Japan	1997	Nissan Mutual Life collapsed under the combination of high guarantees (70% of the liabilities yielded 5.5%) and low investment yield, due to both low interest rates and a bearish equity market. It was the first Japanese life insurer to go bankrupt after WW2. The equity of the company had probably been negative for several years.	Around $\frac{2}{3}$ s of the net losses to customers were covered by the policyholder protection program, a mandatory program for all Japanese insurers. A run-off-company, Aoba Life was established, and later acquired by the French company Artemis.
Equitable Life	UK	2002	The oldest mutual company in the world went down due to a combination of very high guarantees and wrong assessments of longevity risk in pension products. Failure to meet the guarantees and a lost court appeal to reduce guarantees almost caused Equitable to file for bankruptcy.	Customers faced large losses that despite complaints against supervisory authorities have not been compensated by the government. The active part (salesforce etc) of Equitable Life was sold to Halifax. In a compromise deal customers voted in favor of a rescue operation including a cut in payments to customers by appr. 20%.
Mannheimer	Germany	2003	In the first default scenario of a German insurer for more than 50 years, Mannheimer had to close acquisition of new business following large losses on the equity market after the millennium bubble. The group's non-life business also came under pressure.	Customers' claims were saved due to an issue of new capital by the Austrian insurer Uniqa who acquired a majority of the shares of Mannheimer.

Sources: Press clipping, annual reports, and Briys and de Varenne (2001), chapter 3.

Table 1: Overview of large life insurance financial distress situations.

with Miltersen and Persson (2003) we do not allow for dividend payments, nor any other form of capital changes. The company will default at the time where book equity is negative after guarantees are met. However, as we describe in Section 2, we also show simpler contracts, where bankruptcy (and guarantees) are only effective at expiry, or where shareholders will always pick up losses (unlimited responsibility). At that time the customers will take over all of the company's assets. Further compensation (rescue operations) from the government is not included. While in property & casualty insurance government supported guarantee funds frequently exist, such funds are rarely seen in life and pension insurance. Practice shows that such rescue operations can hardly be counted on, as in most of the larger recent defaults of life companies, governments have chosen not to intervene (see table 1 for details).

In line with most literature on this topic (e.g. Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Miltersen and Persson (2003)) we will not cover pure actuarial risk

elements, like mortality risk, disability risk, longevity risk, etc, or any type of administrative costs. Neither will we cover any part of the premium set aside to cover such elements, which means that we assume that the full initial payment from customers go into a form of savings account.

We define the contracts as correctly priced (as in Nielsen and Sandmann (2002), but with single premiums and payments) if the expected discounted payment of the contract is equal to the initial premium. Furthermore, for all contracts we ensure that pricing is correct by assigning a profit sharing that fits the other parameters. Individuals are then allowed to choose from the set of correctly priced contracts. As previously explained, we then use CRRA preferences to evaluate the contracts from the individuals' perspective.

There has been limited focus on whether L & P contracts are suited to satisfy customers' welfare. Previous research has focused on pricing life and pension insurance contracts. Only a few papers have used similar models to analyze welfare effects of guaranteed products. Brennan (1993) elaborates on the classical point made by Borch (1962) that guaranteed products will lead to a welfare loss, but without quantifying the effect further. Jensen and Sørensen (2001), and Consiglio, Saunders, and Zenios (2006) build on this point by quantifying the effects in various cases of life-time interest rate guarantees. We elaborate further the welfare effects of different contract design. To our knowledge, no one has previously investigated the value of contract design in a behavioral framework.

Our paper is organized as follows: In Section 2 we present the different features of our model. The numerical examples in Section 3 illustrate the efficiency loss of the different components of the contract. Section 4 contains the same analysis as Section 3 except that we use Cumulative Prospect Theory (CPT) instead of standard expected utility. Finally we conclude.

2 The Model

We assume a standard no-arbitrage economy with two assets, a risk free bank account, D_t and a risky equity index, S_t . The dynamics of the asset classes are given by:

$$dD_t = rD_t dt, \quad D_0 = d, \quad (1)$$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t, \quad S_0 = s, \quad (2)$$

where r is the constant risk-free interest rate, μ is the constant expected return on the equity index, σ is the constant volatility of the equity index, and Z_t is a standard Brownian motion. A proportion θ_t is invested in the equity index. We assume that the proportion of the equity index is fixed, i.e. that $\theta_t = \theta$. The dynamics of the total asset portfolio A_t under the real

probability measure P is then given by

$$dA_t = (rA_t + \theta(\mu - r)A_t)dt + \theta A_t \sigma dZ_t, \quad A_0 = a. \quad (3)$$

Design of fair contracts is done under the equivalent martingale measure Q (Harrison and Kreps, 1979), where the corresponding dynamics of the asset portfolio is given by

$$dA_t = rA_t dt + \theta A_t \sigma dZ_t^Q, \quad A_0 = a, \quad (4)$$

where Z_t^Q is a standard Brownian motion under Q .

Pension Contracts

We describe the following alternative pension contracts:

1. The customers directly choose the asset allocation, i.e., Merton's problem (Merton, 1971).
2. The customer return has a floor similar to a put option, to be called "implicit put".
3. The customer return has a floor, however the customer faces the risk of the company defaulting, e.g., the simple life insurance problem of Briys and de Varenne (1994).
4. The guarantees embedded in the product are realized on an annual basis, i.e., annual guarantees.

Capital Structure

Assets	Liabilities
A_0	$E_0 = (1 - \alpha)A_0$ $B_0 = 0$ $L_0 = \alpha A_0$

The table shows the balance sheet of the insurer at the start of the contract.

Table 2: Balance sheet at time $t = 0$

The liability side of the insurer's balance sheet at time t consists of the equity of the company, E_t , the reserves (customers' funds), L_t , and the bonus account, B_t (to be further described in Section 2.4). In the Merton problem (1 above) we define $E_t = 0$ and $B_t = 0$ for all t and in the cases of implicit put and simple life insurance (2 and 3 above) we define $B_t = 0$ for all t .

The initial balance sheet of the insurer (at time $t = 0$) is shown in table 2, where $(1 - \alpha)$ is defined as the proportion of equity to total assets at time $t = 0$.

Fair Contracts

The market value of the customers' funds, L_T , has a distribution depending on the contract design. We assume that the company only offers fair contracts to the customers (see, e.g., Brennan and Schwartz (1976) and Nielsen and Sandmann (2002)), i.e. solutions where

$$L_0 = \alpha A_0 = e^{-rT} E^Q(L_T). \quad (5)$$

Maximization Problem

For each of the fair pension contracts, we formulate a maximization problem

$$\max U = \max E(u(L_T)), \quad (6)$$

where u is the customer's utility function, with the usual assumptions that $u' > 0$ and $u'' < 0$. The decision variables over which the maximum is taken differ for the different pension contracts. Under CPT the U-function will be replaced by a V-function to be defined in Section 4.1.

2.1 The Merton Problem

The Merton problem consists of an individual investor who makes direct investments in the two assets described above. The customer's pay-out function is simply

$$L_T = A_T. \quad (7)$$

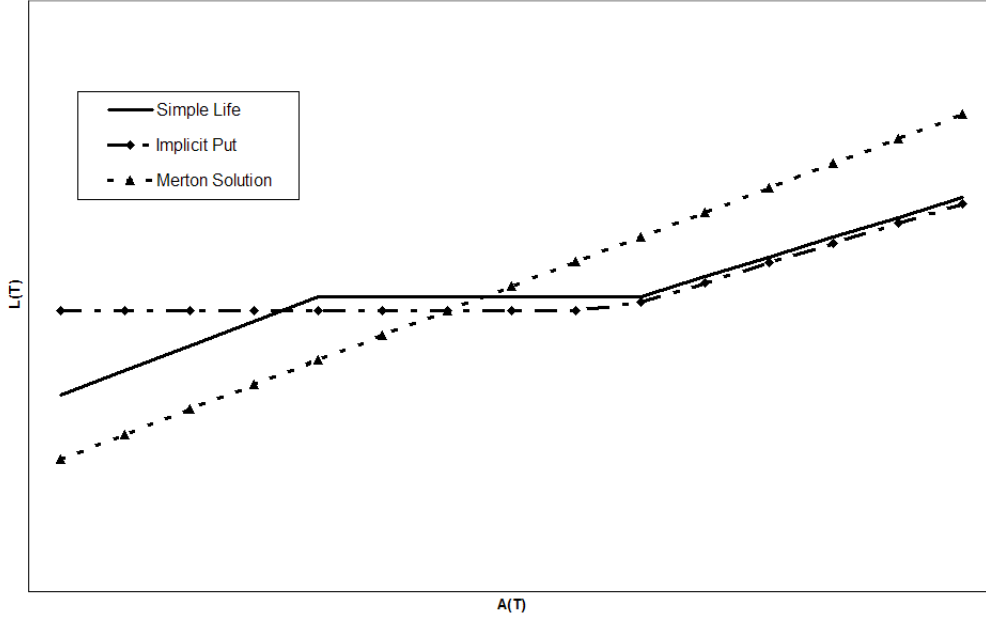
For the Merton case, all possible contracts satisfy the fair restriction in equation (5). We find the maximum in equation (6) over the asset allocation parameter θ .

2.2 Implicit Put

In the implicit put contract the customer has a guarantee on a promised amount at the expiry of the contract. If the assets of the company are insufficient to cover the guarantee, the customer has the right to extract the missing amount from the owners of the company. This is similar to a put option with no credit risk or the situation of any single company or product in a larger group setting, e.g., index-linked bonds as part of a wide menu of products in a financial conglomerate.

At date zero the company receives an initial amount of assets A_0 which it invests in a risk free asset (bank account - D) and an equity index (S). The investment comes from customers, providing an amount L_0 , and owners, providing an amount E_0 .

At the payout date T , the assets of the company are split according to the following rules:



The figure illustrates payoff patterns of different contracts at time T . Customer reserve (L_T) versus value of company (A_T).

Figure 2: Payoff patterns

1. The customer has a claim on his initial investment capitalized by a guaranteed rate g , in total amounting to $L_0 e^{gT}$.
2. The owner has a second priority claim on his proportion $(1 - \alpha)$ of the total assets at time T , amounting to $(1 - \alpha)A_T$.
3. The remaining profit is split with a proportion δ to the customer and $(1 - \delta)$ to the owners.

The payout structure is illustrated in Figure 2. Formalized, the payout function can be written as follows:¹

$$L_T = L_0 e^{gT} + \alpha \delta (A_T - \frac{1}{\alpha} L_0 e^{gT})^+. \quad (8)$$

We let δ be the residual parameter that makes the contract satisfy the fair restriction (equation 5). Thus, solving the market value of the contract of the customer at time $t = 0$ with respect to δ give us:

$$\delta = \frac{1 - e^{gT} e^{-rT}}{N(d_1) - e^{gT} e^{-rT} N(d_2)}, \quad (9)$$

¹In our model we assume all parameters are set at time 0, therefore no time index on the parameters g , θ , and α .

where

$$d_1 = \frac{(r - g + \sigma_A^2/2)T}{\sigma_A\sqrt{T}}, \quad d_2 = d_1 - \sigma_A\sqrt{T}, \quad \sigma_A = \theta\sigma. \quad (10)$$

We find that δ is independent of L_0 , α and A_0 .

We find the maximum in equation (6) over the asset allocation parameter θ and the guaranteed rate parameter g .

2.3 Simple Life Insurance

Contrary to the previous section, the simple life contract allows the company to default without any obligation for the owners to insert more capital. This is typical for a public company where life insurance is the main or only business. This type of contract was first described by Briys and de Varenne (1994). In Figure 2 we give a comparison of the form of L_T as a function of A_T for the three contracts given (Merton problem, implicit put, and simple life).

At the payout date T , the assets of the company are split according to the following formula:

$$L_T = A_T - (A_T - L_0e^{gT})^+ + \alpha\delta(A_T - \frac{1}{\alpha}L_0e^{gT})^+. \quad (11)$$

Again we let δ be the residual parameter that makes the contract satisfy the fair restriction (equation 5). Thus, solving the market value of the contract of the customer at time $t = 0$ with respect to δ give us:

$$\delta = \frac{\alpha - \alpha e^{gT} e^{-r(T)} N(d'_2) - 1 + N(d'_1)}{\alpha(N(d_1) - e^{gT} e^{-r(T)} N(d_2))}, \quad (12)$$

where

$$d'_1 = d_1 - \frac{\ln\alpha}{\sigma_A\sqrt{T}}, \quad d'_2 = d'_1 - \sigma_A\sqrt{T}, \quad (13)$$

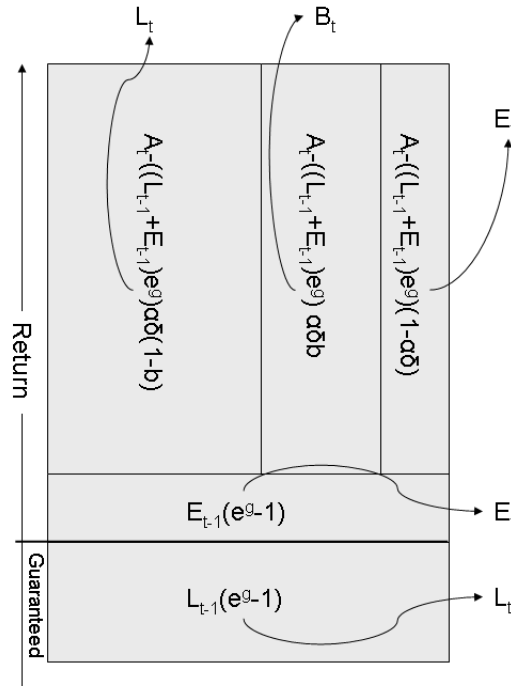
and δ is independent of L_0 and A_0 .

We find the maximum in equation (6) over the asset allocation parameter θ , the guaranteed rate parameter g , and the capital structure parameter α .

2.4 Annual Guarantees

As mentioned in the introduction, the existence of annual guarantees calls for a different treatment of contracts. We solve this by doing year-by-year-simulations and by declaring bankruptcy if book equity at the end of year turns out to be negative. In addition, bonuses are calculated at the end of each year and credited to the reserve. However, in order to keep the model as simple as possible, we do not allow for the company to pay dividends or to issue

new equity. Nor do we allow a company to run at negative equity for a period of time, even though this is commonly seen in practice.²



The figure illustrates how return is split between different types of capital. The first part of the return (guarantee) is allocated to the customer reserve, then a part is allocated to equity, while return above is split between customer reserve, bonus reserve, and equity.

Figure 3: Contract design

In order to provide buffers for companies to meet bad years in the security markets, regulators in most countries allow for (and to a certain extent require) the build up of buffers of capital that are yet to be allocated to customers' reserves. These buffers have different forms, importance and names from country to country, e.g. bonus reserves, value adjustment reserves, unrealized gains (reserves), fund for future appropriations, etc. We call them bonus reserves, B_t . Bonus reserves can be used if the achieved return is not sufficient for covering guaranteed returns.

Allocation to bonus reserves in practice is done in a number of ways, e.g., through allocating a proportion of bonuses each year, allocating unrealized gains on various types of securities, increasing the funds at the same rate as the other reserves, bringing the bonus reserve to a target level, etc. We shall use a simple allocation mechanism similar to the method described by Miltersen and Persson (2003). More sophisticated methods exist, see

²See Briys and de Varenne (2001), page 59 for anecdotal evidence.

e.g. Grosen and Jørgensen (2000) where allocations to the bonus reserves are also a function of a given target level (relative to reserves). However, for our purpose the gain of using such methods is limited.

In our model we credit the bonus reserves by a proportion of declared bonuses, b . Figure 3 illustrates the allocation rules. The bottom part of the return covers the guaranteed amount. If returns exceed the level of the guarantee, an amount will be used to cover a similar return on shareholders' capital. Then, if there is still something left, the remaining return will be split proportionally between equity, reserves, and bonus reserves.

Mathematically,

$$\begin{aligned}
L_t &= \begin{cases} A_t & \text{if } A_t \leq L_{t-1}e^g, \\ L_{t-1}e^g & \text{if } L_{t-1}e^g < A_t \leq L_{t-1}e^g + E_{t-1}e^g + B_{t-1}, \\ L_{t-1}e^g + \delta\alpha(1-b)(A_t - (L_{t-1}e^g + E_{t-1}e^g + B_{t-1})) & \text{if } A_t > L_{t-1}e^g + E_{t-1}e^g + B_{t-1}, \end{cases} \\
B_t &= \begin{cases} 0 & \text{if } A_t \leq L_{t-1}e^g + E_{t-1}, \\ A_t - L_{t-1}e^g - E_{t-1} & \text{if } L_{t-1}e^g + E_{t-1} < A_t \leq L_{t-1}e^g + E_{t-1} + B_{t-1}, \\ B_{t-1} & \text{if } L_{t-1}e^g + E_{t-1} + B_{t-1} < A_t \leq L_{t-1}e^g + E_{t-1}e^g + B_{t-1}, \\ B_{t-1} + \delta\alpha b(A_t - (L_{t-1}e^g + E_{t-1}e^g + B_{t-1})) & \text{if } A_t > L_{t-1}e^g + E_{t-1}e^g + B_{t-1}, \end{cases} \\
E_t &= A_t - L_t - B_t. \tag{14}
\end{aligned}$$

In the case of a bankruptcy ($A_t < L_{t-1}e^g$), customers will receive the full value of the company's assets (we assume no bankruptcy costs). We assume this is invested in the risk free asset, such that:

$$E_T = 0, \quad B_T = 0, \quad L_T = A_\tau e^{r(T-\tau)}, \tag{15}$$

where τ is the (stochastic) time of bankruptcy. The assumption that investments after bankruptcy are made solely in the risk free asset may give a penalty that is unrealistic.

For annual guarantees, with or without bonus reserves, closed form solutions are unavailable, and we have to rely on numerical solutions by simulation. We find the maximum in equation (6) with L_T replaced by $L_T + B_T$, over the asset allocation parameter θ , the guaranteed rate parameter g , the capital structure parameter α , and the bonus reserve parameter b .

3 Results with Expected Utility

3.1 Power Utility

Within the expected utility framework, we assume that the customer's utility belongs to the class of CRRA utility functions with a relative risk aversion coefficient γ . As shown by

Rubinstein (1976), this is consistent with our choice of the geometric Brownian motion as pricing process (given in Section 2). The utility can be described as a power utility function as the form

$$u(x) = \frac{1}{1-\gamma} x^{1-\gamma}. \quad (16)$$

3.2 Parameters

In the numerical example we use the following parameters, fixing α and b as constants:

$$\begin{aligned} A_0 = 5 & \quad r = 0.04 & \quad \mu = 0.065 & \quad \sigma = 0.15 \\ T = 5 & \quad \gamma = 3 & \quad \alpha = 0.9 & \quad b = 0.2 \end{aligned}$$

3.3 Results and Comparisons

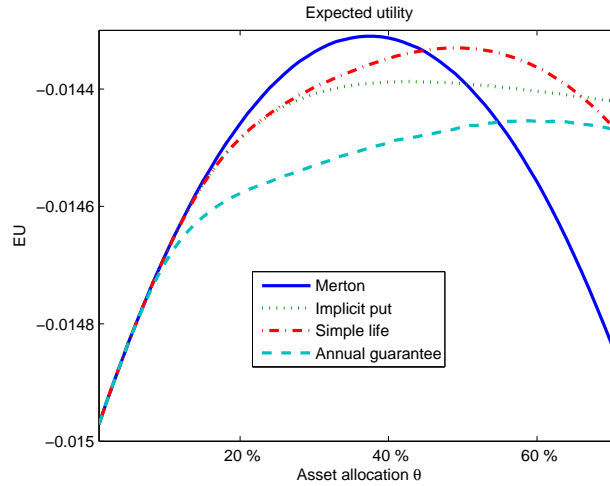


Figure 4: Optimal asset allocation under expected utility, given $g = 0.02$.

Figure 4 shows the performance of the four different contract types, given a guarantee of 2%.³ As a result of the fairness restriction given by equation (5), there exists one and only one fair contract for each θ . We find this contract by solving equation (5) for δ .⁴ We find that the highest utility is offered by the simple Merton type contract, investing directly in the risky and risk-free assets. The optimal allocation between the two assets has already been solved by Merton (1971). The optimal proportion of the risky asset is

$$\theta = \frac{\mu - r}{\gamma\sigma^2}. \quad (17)$$

³The results are found by Monte Carlo simulations with 100,000 paths.

⁴As there is one unknown parameter (δ), one equation (5) to be solved, and the value of the contract (RHS of (5)) is monotonously increasing with δ , multiple solutions can not exist. With our choice of parameters we have always been able to find a solution where $0 < \delta < 1$.

With our choice of parameters the solution is given as $\theta = 37\%$, which is confirmed by the numerical analysis.

As the simple life contract contains both upside potential and downside risk it is closer to the Merton solution and performs better than the implicit put contract, which has no downside risk. In the case of annual guarantees both early bankruptcies (because of the non-optimal risk-free investments) and a more restrictive guarantee structure (because of the asymmetric sharing rules) contribute to the utility being lower than for the other contract types.

In order to better compare the different cases we define the certainty equivalent (CEQ):

$$u(CEQ) = E[u(L_T)]. \quad (18)$$

We can interpret CEQ as the amount of wealth to be received at the horizon with certainty that would give the customer the same expected utility as he receives under the other strategies.

Pension contract	Merton		Implicit Put		Simple Life		Annual Guarantees	
	Optimal θ	CEQ	Optimal θ	CEQ	Optimal θ	CEQ	Optimal θ	CEQ
Guarantee g								
0.0 %	37 %	5.9111	37 %	5.9090	37 %	5.9096	43 %	5.8985
0.5 %	37 %	5.9111	37 %	5.9075	39 %	5.9090	46 %	5.8954
1.0 %	37 %	5.9111	38 %	5.9053	42 %	5.9082	48 %	5.8905
1.5 %	37 %	5.9111	40 %	5.9014	45 %	5.9078	55 %	5.8862
2.0 %	37 %	5.9111	42 %	5.8952	48 %	5.9069	58 %	5.8815
2.5 %	37 %	5.9111	47 %	5.8846	52 %	5.9061	63 %	5.8769
3.0 %	37 %	5.9111	70 %	5.8680	55 %	5.9055	68 %	5.8724

The table describes optimal asset allocation parameter θ and optimal certainty equivalents (CEQ) for different pension contracts and annual guarantees.

Table 3: Sensitivity analysis Expected Utility

Expanding the selection of possible guarantees, Table 3 shows the optimal asset allocation (i.e., giving the highest certainty equivalent, given the fairness restriction in equation (5)) for a selection of possible guarantees. The view that the Merton solution is optimal is confirmed by the fact that the other contracts perform better the lower the guarantees. In the limit, if the guarantees tend to $-\infty$, all contracts will be equal to the Merton solution, as there will be no binding guarantee. Hence, the optimal guarantee for the other contract types will be as low as possible. Figure 5 confirms this, showing that the utility in the implicit put case is highest for low guarantees and asset allocations with θ 's close to the Merton solution of 37%.⁵

⁵Even though results are only given for one chosen α and b , changing these parameters within reasonable limits ($0 < \alpha < 1, 0 < b < 1$) will not substantially impact our results. Typically a higher α in the case of simple life and annual guarantees will lead to contracts more similar to the Merton problem ($\alpha = 1$ gives the

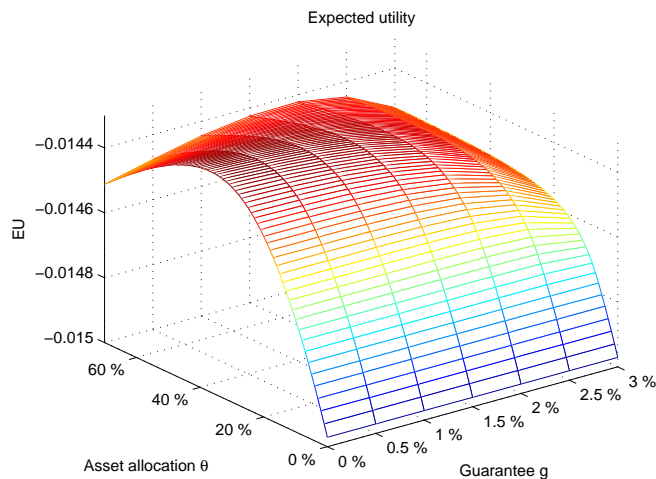


Figure 5: Utility of investing in the implicit put as a function of asset allocation, θ , and guarantee, g .

The certainty equivalents show that differences between the contract types are small, most certainty equivalents are not more than 1% lower than the one of the optimal Merton solution. The largest welfare loss comes from the inclusion of annual guarantees.

4 Results with Cumulative Prospect Theory (CPT)

4.1 Cumulative Prospect Theory (CPT)

In Section 3 we found that L & P insurance products are not optimal within the standard expected utility framework. This section is an attempt at using alternative behavioral models of human choice to explain the existence of guarantees. One of the most fully developed and thoroughly investigated models is the Tversky and Kahneman (1992) Cumulative Prospect Theory (CPT). It is a descriptive theory, based on experimental evidence, of how people evaluate risk.

CPT combines the concepts of loss aversion (LA) and a nonlinear rank-dependent weighting of probability assessments. The first concept, loss aversion, assumes the individuals are not taking absolute levels of wealth into account, but rather, gains and losses measured relative to a reference point. There is a value function defined over gains, similar to the utility function in expected utility. Over losses there is a loss aversion function that transforms the specific finding that individuals are much more sensitive to losses than to gains of the same magnitude. Here $\lambda > 1$ describes how much more sensitive an individual is to a loss relative to a gain. The LA function allows individuals to be risk averse over gains but risk seeking

Merton problem as the shareholders will always have zero capital, see e.g. Table 2), while a higher b in the annual guarantees problem leads to more smoothing, but has insignificant impact on utility.

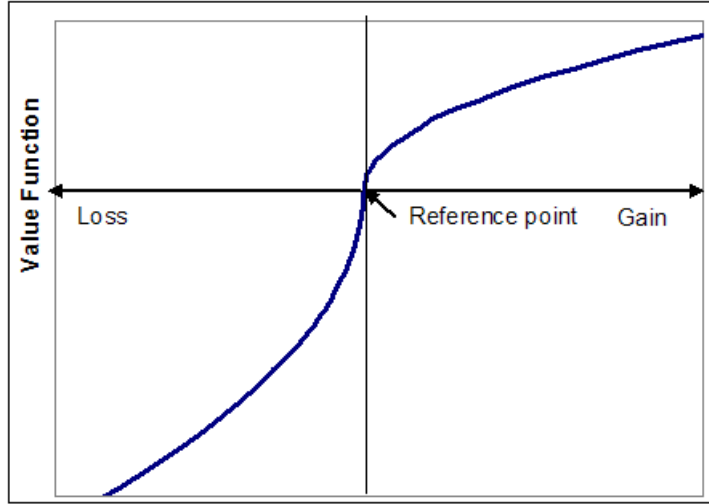


Figure 6: Value function for a loss averse investor

over losses, and for losses to matter more than gains. This is described by an S-shaped utility function, illustrated in Figure 6. The sensitivity to increasing gains or losses is measured by ϕ . Finally, there is a weighting function used to transform probability distributions into a function where individuals put more emphasis on extreme outcomes.

Cumulative prospect theory treats gains and losses separately. We define surplus wealth as current wealth relative to a reference point, Γ . The initial amount invested is frequently referred to as the reference point, hence we define $\Gamma = L_0$.

Assume a gamble is composed of $m + n + 1$ outcomes, $L_{T,-m} < \dots < \Gamma < \dots < L_{T,n}$, which occur with probabilities p_{-m}, \dots, p_n , respectively. The corresponding gamble can be denoted by the pair (L, p) , where $L = (L_{T,-m}, \dots, L_{T,n})$ and $p = (p_{-m}, \dots, p_n)$. We define

$$V^+(L; p) = w(p_n)u(L_{T,n}) + \sum_{k=1}^n \left[w\left(\sum_{j=0}^k p_{n-j}\right) - w\left(\sum_{j=0}^{k-1} p_{n-j}\right) \right] u(L_{T,n-k}), \quad (19)$$

and

$$V^-(L; p) = w(p_{-m})u(L_{T,-m}) + \sum_{k=1}^m \left[w\left(\sum_{j=0}^k p_{-(m-j)}\right) - w\left(\sum_{j=0}^{k-1} p_{-(m-j)}\right) \right] u(L_{T,-(m-k)}). \quad (20)$$

The preference value of the gamble (L, p) is given by

$$V(L; p) = V^+(L; p) + V^-(L; p) \quad (21)$$

where $V^+(L; p)$ measures contribution of gains, and $V^-(L; p)$ the contribution of losses. The

function $w(p)$ is a probability weighting function assumed to be increasing from $w(0) = 0$ until $w(1) = 1$. Prelec (1998) offers a single parameter version of the weighting function:

$$w(p) = e^{-(-\ln p)^\varphi} \quad (22)$$

where φ is a "free" parameter. The Prelec (1998) weighting function is almost identical to Tversky and Kahneman's weighting function. The key difference is that Prelec's specification is based on behavioral axioms rather than the convenience of the functional form. We note that with $\varphi = 1$, $w(p)$ degenerates to $w(p) = p$. Hence, we are back to the expected utility framework with a non-standard utility function. We will later use this as a special case, see Section 4.4.

Finally, the utility function is defined as follows:

$$u(L_T) = \begin{cases} u_G(L_T) = (L_T - \Gamma)^\phi & L_T \geq \Gamma, \\ \lambda u_L(L_T) = -\lambda(\Gamma - L_T)^\phi & L_T < \Gamma. \end{cases} \quad (23)$$

4.2 Parameters

$$\begin{array}{llllll} L_0 = 4.75 & r = 0.04 & \mu = 0.065 & \sigma = 0.15 & T = 5 \\ \alpha = 0.90 & b = 0.2 & \varphi = 0.75 & \lambda = 2.25 & \phi = 0.5 \end{array}$$

Estimates of the parameters of CPT can be found in several studies. A challenge for CPT is to move the empirical estimates from experimental data to real world choice scenarios. Tversky and Kahneman (1992) estimated $\phi = 0.88$, $\lambda = 2.25$, $\varphi_{\text{gain}} = 0.75$, and $\varphi_{\text{loss}} = 0.69$, but they used the parameter φ for a slightly different weighting function than we use. Camerer and Ho (1994) estimate $\phi = 0.32$ and $\varphi = 0.56$. Wu and Gonzalez (1996) also estimate the Prelec's weighting function yielding $\phi = 0.48$ and $\varphi = 0.72$. Based on all these different studies we assign the following figures to our free parameters: $\phi = 0.5$, $\varphi = 0.75$, and $\lambda = 2.25$. With ρ equal 0 the reference point is equal to the initial invested amount, $\Gamma = L_0$.

4.3 Results and Comparisons

In Figure 7 we compare the results of the four pension contracts in the case where the customers' preferences can be described by CPT. We find that guarantees are not effective for low θ s, hence all contract types give the same or almost the same results. For higher θ s the probability of large bankruptcies is the dominant feature of the contracts. This means that the implicit put (with no bankruptcies) performs the best, while the limited losses of annual guarantees also do fairly well. In the optimal θ , the implicit put is still the best, but simple life outperforms the annual guarantees, as losses are moderate and "unnecessary bankruptcies" are avoided.

To better compare the different cases we again define the certainty equivalent (CEQ) in

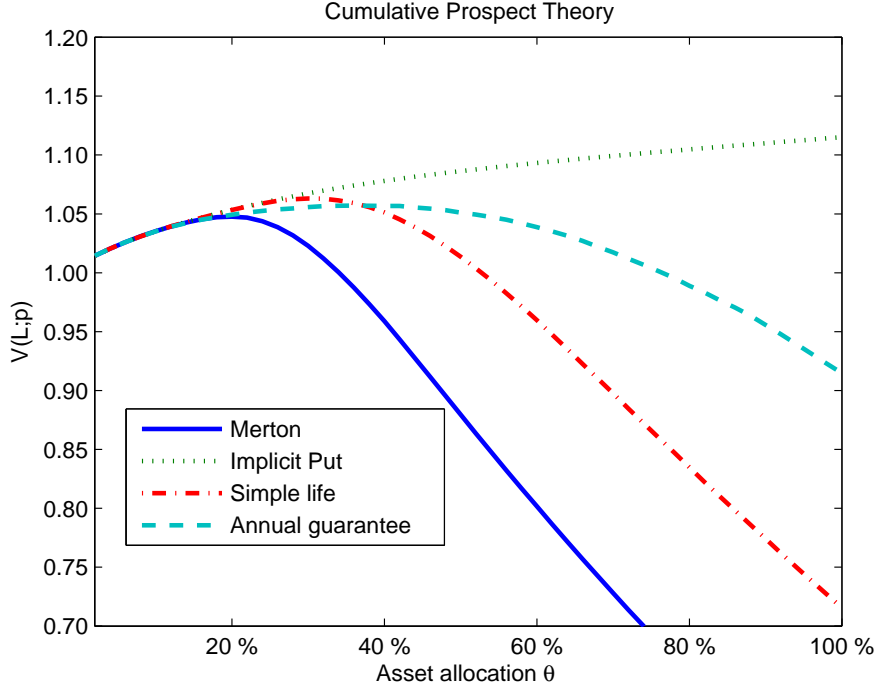


Figure 7: Optimal asset allocation under cumulative prospect theory, given $g = 0.02$.

a similar way as in Section 3.3:

$$V(CEQ) = V(L; p). \quad (24)$$

Table 4 shows the CEQ for the different contracts. Opposite to the situation with standard expected utility, the implicit put contract gives highest value. Hence, for the customer under CPT, the effect of combining no bankruptcies (losses) with the opportunity of taking high risk if $E(L_T)$ is high, is highly appreciated. It is worth noting that differences in terms of CEQs are larger than under expected utility.

Further details on each of the contracts are given in Table 4. As also shown in Figure 7, the solution to the Merton problem gives a more conservative asset allocation than under the classical expected utility. This is due to the high risk aversion around the reference point.

The optimal parameters of the favored implicit put contract are shown in Figure 8. Comparing this with Figure 5, we see that the optimal solution has shifted from minimizing the guarantee for an optimal θ to maximizing θ for an optimal guarantee which seems to be around 2%. Only in the case of a zero percent guarantee, the solution seems to be close to that of the Merton problem, with an optimal θ . Furthermore, there seems to be a large benefit of a guarantee larger than zero. This is due to the fact that our reference point assumes a return of zero percent. As the marginal utility at the reference point is infinite, it is beneficial to stay above this point by applying a positive guarantee.

Pension contract	Merton		Implicit Put		Simple Life		Annual Guarantees	
	Optimal θ	CEQ	Optimal θ	CEQ	Optimal θ	CEQ	Optimal θ	CEQ
Guarantee g								
0.0 %	18 %	5.8477	20 %	5.8510	20 %	5.8510	33 %	5.8694
0.5 %	18 %	5.8477	100 %	5.9145	25 %	5.8586	33 %	5.8738
1.0 %	18 %	5.8477	100 %	5.9651	28 %	5.8662	34 %	5.8738
1.5 %	18 %	5.8477	100 %	5.9897	29 %	5.8747	35 %	5.8740
2.0 %	18 %	5.8477	100 %	5.9957	29 %	5.8798	33 %	5.8675
2.5 %	18 %	5.8477	100 %	5.9830	28 %	5.8791	36 %	5.8565
3.0 %	18 %	5.8477	100 %	5.9488	25 %	5.8708	36 %	5.8399

The table describes optimal asset allocation parameter θ and optimal certainty equivalents (CEQ) for different pension contracts and annual guarantees.

Table 4: Sensitivity analysis Cumulative Prospect Theory

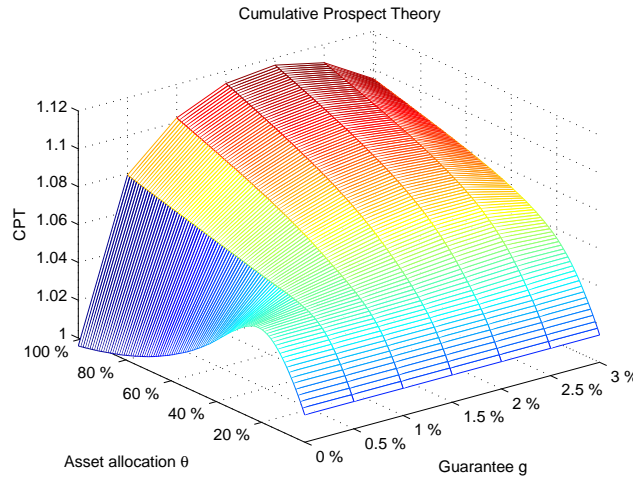


Figure 8: Utility of investing in the implicit put under CPT as a function of asset allocation θ and guarantee, g .

The solutions to the simple life and annual guarantees contracts both show inner optima for both g and θ . As the loss potential in the case of bankruptcy is lower under annual guarantees, the optimal asset allocation will be more aggressive in this contract.⁶ However, as shown in Figure 7, the default risk clearly penalizes high θ s.

4.4 Decomposition of Elements of CPT

In a number of ways the CPT differs from our standard CRRA utility. However, by carefully selecting parameters in the CPT function we show that each element of the CPT can be replaced by elements of the standard expected utility.

⁶Average loss given default in the case of $g = 2\%$ and optimal θ is 3.6% with the simple life contract and 2.3% with annual guarantees.

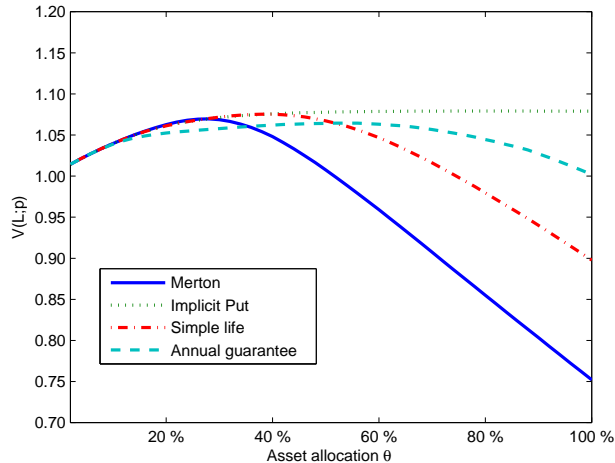


Figure 9: Optimal asset allocation with $\varphi = 1$, given $g = 0.02$.

By using the special case $\varphi = 1$ in equation (22), CPT is reduced to expected utility, but with a non-standard (loss aversion) utility function. The results of this special case are shown in Figure 9 to be similar to the results from Section 4.3. The implicit put contract is still the best, but now only by a tiny margin. Furthermore, we now find an internal optimum for the asset allocation, $\theta = 79\%$.

In the CPT utility function (23), letting $\Gamma = 0$, $\phi = 1 - \gamma$, and multiplying by $1/(1 - \gamma)$, the standard utility function (16) is obtained if L_T is strictly positive. As defined in equations (7), (8), (11), (14), and (15) this condition holds. In this case the only difference from the analysis under expected utility is the weighting function given as equation (22). The effect of this, shown in Figure 10, is similar to the case of $\varphi = 1$, hence the infinite marginal utility around the reference point is not the only reason for a guarantee being optimal. The difference from Section 3 is now that extreme high and low scenarios have a higher weight. In particular the higher values achieved by the implied put option (relative to the Merton problem) in the extreme low scenarios are now weighted higher than lower values achieved in more normal scenarios. Again, we find that the implicit put contract is the best, but this time with no internal optimum for θ .

The model also seems robust to changes in the weighting of losses relative to gains (λ). Tests with $\lambda = 1$ show that the conclusions are similar to those described in Section 4.3.

Finally, Gomes (2005) argues that the assumption implied by CPT that marginal utility decreases when final wealth approaches zero is unrealistic. He reformulates (23) to include another reference point $\underline{W} < \Gamma$, below which utility will again become concave, giving the

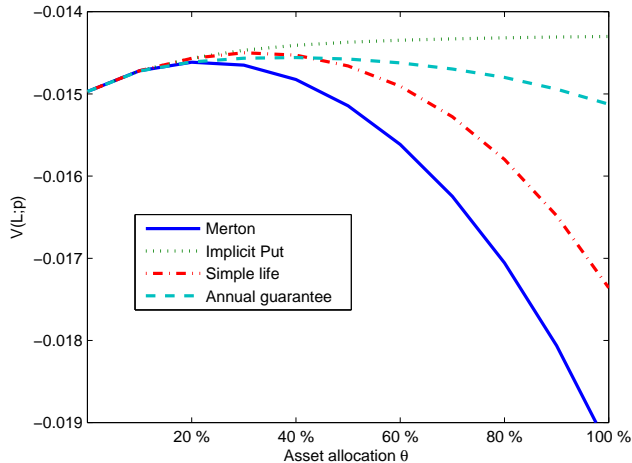


Figure 10: Optimal asset allocation with non-linear probability weighting, but no loss aversion, given $g = 0.02$.

new utility function

$$u(L_T) = \begin{cases} u_G(L_T) = (L_T - \Gamma)^\phi & L_T \geq \Gamma, \\ \lambda u_L(L_T) = -\lambda(\Gamma - L_T)^\phi & \underline{W} < L_T < \Gamma, \\ \frac{L_T^{1-\gamma}}{1-\gamma} - (\lambda(\Gamma - \underline{W})^\phi + \frac{W^{1-\gamma}}{1-\gamma}) & L_T \leq \underline{W}. \end{cases} \quad (25)$$

However, this will not change our findings as the increased impact of (large) losses will not punish the implicit put contract, which in this case will still be optimal.

5 Conclusion

We have presented a framework for optimizing pension insurance design by combining pricing principles with utility theory. Not surprisingly, the Merton solution is optimal with standard expected utility. Quantifying the impact in terms of CEQs, we find the largest loss when introducing annual guarantees.

With CPT implicit put outperforms the other alternatives. Contracts including both insurance against losses and stock market participation tend to give high expected utility. The contract design is now becoming more important in terms of CEQs. Annual guarantee contracts are still outperformed by simpler products.

Splitting the two most important features of CPT shows that both the new utility function (loss aversion) and the weighting function are able to explain that the implicit put contract is now the optimal. However, combining the two gives clearer and more powerful results than any one of them separately.

All in all we can not explain the demand for structured products in the framework of standard expected utility. A possible explanation may be that the customers' preferences include at least some elements included in CPT. However, potentially important features of the contracts, such as transaction costs, taxes, and actuarial elements are left for further research. Furthermore, more sophisticated models may include other sources of revenues, such as labor income or revenues from alternative pension system(s).

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Intergenerational Effects of Guaranteed Pension Contracts *

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Abstract

In this paper we show that there exist intergenerational cross-subsidization effects in guaranteed interest rate life and pension contracts as the different generations partially share the same reserves. Early generations build up bonus reserves, which are left with the company at expiry of the contract. These bonus reserves function partly as a subsidy of later generations, such that the latter earn a risk-adjusted return above the risk-free rate. Furthermore, we show that this subsidy may be large enough to explain why late generations buy guaranteed interest rate products, which otherwise would not have been part of the optimal portfolio allocation.

Keywords: Portfolio Choice; Life and Pension Insurance, Interest Rate Guarantees

JEL Classification: G11, G13, G22

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1 Introduction

Many households rely on life and pension (L&P) contracts to finance their retirement expenditures. We show that there exist an intergenerational cross-subsidization effect in guaranteed interest rate L&P contracts as the different generations partially share the same reserves. Because of the bonus reserves described below, early generations subsidize later generations, such that the latter earn a risk-adjusted return above the risk-free rate.

Previous research on L&P contracts has focused on the risk sharing between one customer and the company as well as pricing one customer's claim on the company. However, at any given point of time, the customer base of a company consists of many customers at different stages of the contract life cycle. In this paper we focus on the relationship between different generations of customers and the company.

We use a simple contract with annual guaranteed return in the fashion of Miltersen and Persson (2003). There is a large literature pricing L&P contracts. Some contributions illustrating different design of contracts are papers by Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Miltersen and Persson (2003). Our main complicating element will be the existence of a bonus reserve, which consists of funds allocated, but not yet guaranteed, to customers. This bonus reserve may or may not be individualized, so that customers may only receive a part, if any, of their proportional share at the expiry of the contract.

In a setting where different generations share the same reserves, at least two sources of cross-subsidization may occur. The bonus reserve is typically left in the company from the expiring generation to future generation. However, in the other direction is the effect that the new generation in a default scenario may end up paying a part of the obligation to the old generation. To test this hypothesis we calculate the expected risk-adjusted return (under the equivalent martingale measure Q) of the contract for each generation, given that the owners of the company is not able to extract any capital beyond the risk-adjusted return on capital. We find that the former effect is the larger, such that the net effect is that the later generations end up with the higher return.¹ The difference in return seems to be fairly small on an annual basis, but still significant over the lifetime of the contract. Changes in parameters have an impact on the size of the cross-subsidization, in particular the spread between risk-free interest rate and the guaranteed rate of return, as well as the crediting rate of the bonus reserve and the asset allocation contributes to these changes. The youngest generations benefit from high spreads, high allocations to the bonus reserves, and a conservative asset allocation, while opposite is the case for previous generations.

Risk-adjusted return different from cost of capital may in general raise the issue of arbitrage. However, in this case investors are normally households with a limited set of investment opportunities. Furthermore, shortselling pension contracts is not normally feasible, even though some repurchasing arrangements exists.

¹In our calibrated benchmark case bankruptcies only play a marginal role.

Most L&P companies have existed for ages.² At the time of investment of the first generations (which we will later show end with a return below cost of capital) only a limited set of investment opportunities existed, compared to today's market. Furthermore, there were no closed-fund investment opportunities, hence they had no way to contractually prevent new generations from entering the customers' fund of the L & P company.

Among the latest generations one could think of an arbitrage opportunity of buying the pension product and shortselling a replicating portfolio. However, L & P companies in general only allow for private investors or beneficiaries with limit investments, hence the shortselling capacity in a replicating portfolio is limited, and transaction costs will be high, particularly since the replicating portfolio also needs to be continuously rebalanced. Furthermore, L & P companies typically have a low degree of transparency in their investment, and therefore it will be difficult to find the optimal arbitrage strategy.

The cross-subsidization described above also provides an alternative explanation of the problem "why do households buy pension insurance?" Only a few papers look at the welfare effects of the contracts. Brennan (1993) elaborates on the classical point made by Borch (1962) that guaranteed products will lead to a welfare loss. According to Borch, we cannot explain the existence of these saving vehicles within an one-generation expected utility framework with HARA utility, as these contracts have a non-linear pay-out function. Jensen and Sørensen (2001), Consiglio, Saunders, and Zenios (2006) and Døskeland and Nordahl (2007) build on this point by quantifying the effects in various cases of interest rate guarantees. However, previous research assumes that all generations receive the same return. In the second part of this paper we expand the previous welfare studies by testing the impact of generation-based return in an individuals' portfolio choice model. Customers can choose between investing directly or indirectly via the guaranteed products.

We find that even when assuming standard preferences (constant relative risk aversion) utility maximization shows that pensions will be part of optimal portfolio. New generations benefit from the cross-subsidization. In addition, the return depends on market return during the same period as investment, but also during previous periods. Because of this, there will be an intergenerational diversification effect reducing the risk.

Explaining the choices of the previous generations is more difficult in our model. However, while today there exist a wide set of opportunities, previous generations clearly had a limited choice. It may be that in previous times guaranteed rate life and pension insurance products were purchased simply due to the lack of other alternatives.

The rest of our paper is organized as follows. In Section 2 we describe the multi-generation model of a pension insurance product. Section 3 provides support for our parametrization of the model. The numerical results of the cross-subsidization are given in Section 4, along with selected sensitivities. In Section 5 we provide a simple portfolio choice model and show optimal asset allocation for each generation, while Section 6 gives the conclusion and suggestions for

²Even though mergers and acquisitions frequently occur, the portfolios tend to prevail.

further research in this area.

2 The Model

Parameters of the Model	Notation
The Economy	
Value of one unit of the equity index at time t where all dividends are immediately reinvested into the index	S_t
Value of one unit of the risk-free bond account at time t	D_t
Constant risk-free rate	r
Constant expected return on the equity index	μ
Constant volatility of the equity index	σ
Standard Brownian motion	Z_t
The Company	
Value of the total asset portfolio at time t	A_t
Constant proportion of total asset portfolio, A_t , invested in the equity index	θ
The equity of the company at time t	E_t
The mathematical reserves (customers' funds) at time t	L_t
The bonus account (bonus reserves) at time t (to be further described)	B_t
Final wealth from insurance product to generation $t - T$ at time t	I_t^{t-T}
Cashflow to the investors of the company at time t	CF_t
The proportion of equity to total assets at time $t = 1$, $\frac{E_1}{A_1} = (1 - \alpha)$	$(1 - \alpha)$
The constant guaranteed rate	g
The proportion split of the remaining total profit (after guarantee and "α" split) to the households	δ
The effective proportion held in the equity index at time t	Θ_t
Proportion of declared bonuses credited the bonus reserves B_t	b
Payout ratio of bonus reserve	p
Discounted value of all cash flows	V
The Households	
Coefficient of relative risk aversion	γ
Overlapping generations of households, indexed by $h = 1, 2, \dots, H$	H
Term of the policy	T
Wealth at time t of generation h	W_t^h
The mathematical reserves of households of generation h at time t	L_t^h
Growth rate of the households' aggregate initial investments	v

Table 1: Definitions

In this section we formalize the modelling framework. We first describe the economy, then the L&P company including the insurance contract offered to households and finally the households.

2.1 The Economy

We assume a standard no-arbitrage economy with two assets, a risk free bank account, D_t and a risky equity index, S_t . The dynamics of the asset classes is then given by:

$$dD_t = rD_t dt, \quad D_0 = d \quad (1)$$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t, \quad S_0 = s \quad (2)$$

where r is the constant risk-free interest rate, μ is the constant expected return on the equity index, σ is the constant volatility of the equity index, and Z_t is a standard Brownian motion.

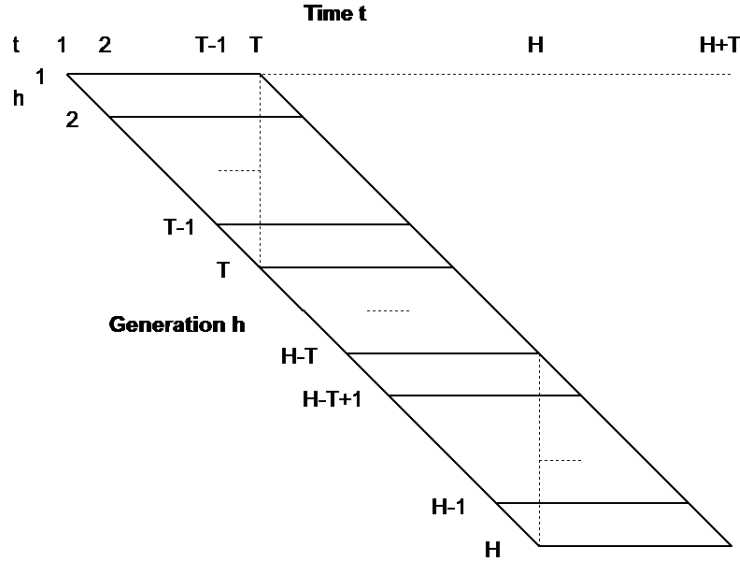


Figure 1: Generations

We create a model with H overlapping generations of households, indexed by $h = 1, 2, \dots, H$. On the x-axis in Figure 1 the different generations are listed. The y-axis illustrates the time line. Each generation uses the pension system for T periods. The wealth of the household at time T of generation h is given by W_T^h . We explore the implications of heterogeneity across generations.

2.2 The Company

We assume that there exists a financial intermediary, which we will refer to as "the company", offering pension contracts. The balance sheet development of the company is illustrated in Table 2 and 3. At time 1 only shareholders and generation 1 has invested in the company. The equity is then a proportion $(1 - \alpha)$ of the total assets, A_1 , of the company. The bonus reserve B_1 is zero at the initiation of the company. At the end of each subsequent year shareholders

Assets	Liabilities
A_1	$E_1 = (1 - \alpha)A_1$ $B_1 = 0$ $L_1 = \alpha A_1$

The table shows the balance sheet of the insurer at the set up date.

Table 2: Balance sheet at time $t = 1$

Assets	Liabilities
A_t	$E_t = (1 - \alpha)A_t$ B_t L_t^1 L_t^2 \dots L_t^H
	$\left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} L_t$

The table shows the balance sheet of the insurer at time t .

Table 3: Balance sheet at time t

will keep the proportion of equity constant at the level $(1 - \alpha)$, either by taking out dividends or by paying in capital. Furthermore, there will be a bonus reserve B_t , and all generations will have their specific allocated reserve L_t^h for generations $h = 1 \dots H$. The sum of reserves for all generations is labelled L_t . The balance sheet at time t is shown in Table 3.

A proportion θ_t of the company's assets is invested in the equity index. We will assume that the proportion of the equity index is fixed, i.e. that $\theta_t = \theta$. The dynamics of the total asset portfolio A_t under the real probability measure P is then given by

$$dA_t = (rA_t + \theta(\mu - r)A_t)dt + \theta A_t \sigma dZ_t, \quad A_1 = a. \quad (3)$$

In the discrete world we transform equation (3) to

$$A_t^{cum} = A_{t-1} e^{(r + \theta(\mu - r) - \frac{1}{2}\theta^2\sigma^2 + \theta\sigma(Z_t - Z_{t-1}))}, \quad (4)$$

or under the equivalent martingale measure, Q :

$$A_t^{cum} = A_{t-1} e^{(r - \frac{1}{2}\theta^2\sigma^2 + \theta\sigma(Z_t^Q - Z_{t-1}^Q))}. \quad (5)$$

Superscript *cum* indicates values before annual settlements.

2.3 The Pension Contract

In the setup of our model, we concentrate on the financial features of the contract, regarding the savings element as the most important.³ Even though there are no international standard contracts we try to make a simplified contract that will be close to products sold in most European (and some non-European) countries.

We assume that all parameters of the contracts are fixed at set up date. Furthermore, we assume correct initial pricing from the company's perspective, meaning that the company's average risk-adjusted return over the whole life time equals the risk-free rate.

The basis for calculating return to customers is the guaranteed rate of return. We use annual guarantees rather than life-time guarantees⁴, hence each year the customers' reserves will increase by a fixed rate g . In addition to that, customers may receive a bonus if the total return on customers' reserves and equity of the company exceeds g . The bonus will then be the surplus, less a proportional share ($\alpha\delta$) to shareholders to compensate for capital inserted and risk assumed by shareholders, as well as a proportional share b to the bonus reserves described below.

In order to provide buffers for companies to meet bad years in the security markets, regulators in most countries allow for (and to a certain extent require) the build up of buffers of capital that are yet to be allocated to customers' reserves. These buffers have different forms, importance and names from country to country, e.g. bonus reserves, value adjustment reserves, unrealized gains (reserves), fund for future appropriations, etc. We name them bonus reserves, B_t . Bonus reserves can be used if the achieved return is not sufficient of covering guaranteed returns.⁵ Furthermore, bonus reserves are not allocated to any specific generation. At the expiry of the contract, each generation will only be able to extract a part p of their proportional share of the bonus reserves.

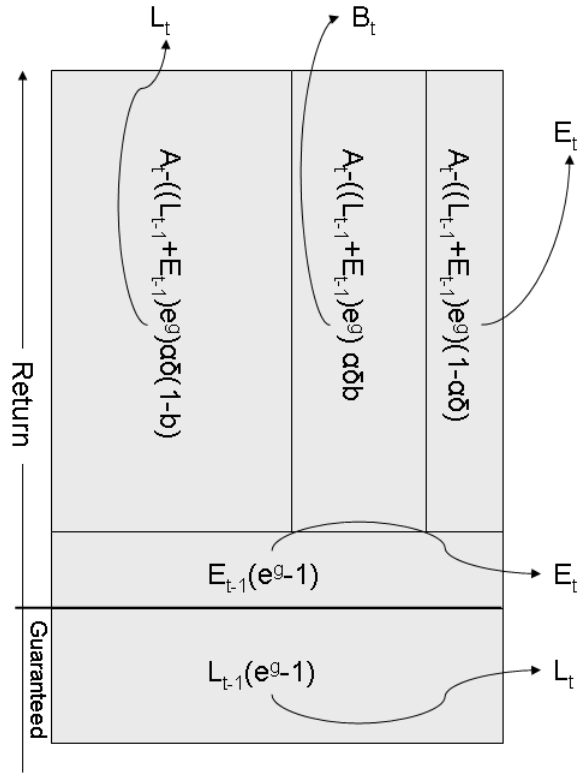
In our model we credit the bonus reserves by a proportion of declared bonuses, b . Figure 2 illustrates the allocation rules. The bottom part of the return covers the guaranteed amount. If returns exceeds the level of the guarantee, an amount will be used to cover a similar return on shareholders' capital. Then, if there still is something left, the remaining return will be split proportionally between equity, customers' reserves, and bonus reserves.

The model is initiated at time $t = 1$, where we define

³This means we will not cover pure actuarial risk elements, like mortality risk, disability risk, longevity risk, etc, or any type of administrative costs. Neither will we cover any part of the premium set aside to cover such elements, which means that we assume that the full initial payment from customers go into a form of savings account. Our contract will be based on a single premium and a single payment at expiry of the contract.

⁴As used e.g. by Grosen and Jørgensen (2000).

⁵In practice, allocation to bonus reserves is done in a number of ways, e.g. through allocating a proportion of bonuses each year, allocating unrealized gains on various types of securities, increasing the funds in the same rate as the other reserves, bringing the bonus reserve to a target level, etc. In order to find a common model, we use the simple allocation mechanism (proportion of declared bonuses) described by Miltersen and Persson (2003).



The figure illustrates how return is split between different types of capital. The first part of the return (guarantee) is allocated to the customer reserve, then a part is allocated to equity, while return above is split between customer reserve, bonus reserve, and equity.

Figure 2: Allocation of return

$$\begin{aligned}
 L_1^1 &= L_1 = \alpha A_1 \\
 E_1 &= (1 - \alpha) A_1 \\
 B_1 &= 0.
 \end{aligned}
 \tag{6}$$

We then initiate each generation at time $t = h$, when generation h does their investment:

$$L_t^h = L_1^1(1 + v)^{h-1} \quad \text{for all } t \text{ and } h \text{ such that } t = h,
 \tag{7}$$

where v is growth rate of the households' aggregate initial investments.

Each year after the initial year, generation h will lose the guaranteed amount if and only if total assets in the company is insufficient to cover the guaranteed amounts of generation

h and all previous generations.⁶ Furthermore, if assets are sufficient to cover the guaranteed amount to all generations, a corresponding return on the equity, and preservation of the bonus reserve, the customers earn a bonus. The bonus is the generation's proportional share of the reserves, multiplied by the customers' proportional share of the capital (α), the customers' share of profits (δ), the share being credited the reserves ($1 - b$), and the surplus in itself. Hence, for all t and h such that $h < t \leq h + T$

$$L_t^h = \begin{cases} 0 & \text{if } A_t \leq \sum_{j=1}^{h-1} L_{t-1}^j e^g \\ A_t - \sum_{j=1}^{h-1} L_{t-1}^j e^g & \text{if } A_t \leq \sum_{j=1}^h L_{t-1}^j e^g \\ L_{t-1}^h e^g & \text{if } A_t \leq (L_{t-1} + E_{t-1})e^g + B_{t-1} \\ L_{t-1}^h e^g + \frac{L_{t-1}^h}{L_{t-1}} \alpha \delta (1 - b) (A_t - (L_{t-1} + E_{t-1})e^g + B_{t-1}) & \text{if } A_t > (L_{t-1} + E_{t-1})e^g + B_{t-1}. \end{cases} \quad (8)$$

Finally, we let L_t^h be zero at all times where generation h has no investments:

$$L_t^h = 0, \quad \text{for all } t \text{ and } h \text{ such that } t > h + T \text{ or } t < h. \quad (9)$$

We can then sum all customers' reserves before cashflows made at the year end (marked by superscript cum):

$$L_t^{cum} = \sum_{j=1}^{t-1} L_t^j. \quad (10)$$

The bonus account is used when return on assets are not sufficient to cover the guarantee. Hence, if assets at the end of the year is low, the bonus account will be zero, or at least lower than the previous year. If assets are high, the bonus account will be credited a proportion b of the total bonus to customers. More formally,

$$B_t^{cum} = \begin{cases} 0 & \text{if } A_t \leq L_{t-1} e^g + E_{t-1} \\ A_t - L_{t-1} e^g - E_{t-1} & \text{if } A_t \leq L_{t-1} e^g + E_{t-1} + B_{t-1} \\ B_{t-1} & \text{if } A_t \leq L_{t-1} e^g + E_{t-1} e^g + B_{t-1} \\ B_{t-1} + \alpha \delta b (A_t - (L_{t-1} e^g + E_{t-1} e^g + B_{t-1})) & \text{if } A_t > L_{t-1} e^g + E_{t-1} e^g + B_{t-1}. \end{cases} \quad (11)$$

When contracts expire, customers may be allowed to extract a proportion p of their proportion of the bonus account. Hence, the new bonus account at the beginning of next year will be

$$B_t = B_t^{cum} \left(1 - \frac{L_t^{t-T}}{L_t^{cum}} p\right) \quad (12)$$

⁶Thus, the old generations will have the priority if the company risks default, but on the other hand they will leave some amount (part of the bonus reserve) in the company when their contract expire.

and the final wealth from the insurance product to the customer becomes

$$I_t^{t-T} = L_t^{t-T} + B_t^{cum} \frac{L_t^{t-T}}{L_t^{cum} p}. \quad (13)$$

After deposits from new customers and withdrawals for old ones with expiring contracts, the new reserves at the beginning of the next year will become

$$L_t = L_t^{cum} + L_t^t - L_t^{t-T}. \quad (14)$$

Equity at the end of year can be determined residually as

$$E_t^{cum} = A_t^{cum} - L_t^{cum} - B_t^{cum}. \quad (15)$$

However, at the beginning of the next year we assume the company to be recapitalized, such that the proportion of equity to customers' reserves stays constant over time. Hence,

$$E_t = L_t \frac{(1 - \alpha)}{\alpha} \quad (16)$$

and assets at the beginning of next year becomes

$$A_t = L_t + B_t + E_t. \quad (17)$$

Now, the cash flow to shareholders can be determined simply as the difference between equity at the end of a year and at the beginning of the next year:

$$CF_t = E_t^{cum} - E_t \quad (18)$$

with the corresponding function for value at time $t = 1$

$$V = \sum_{t=2}^{T+H} E^Q[CF_t] e^{-r(t-1)}. \quad (19)$$

For annual guarantees with bonus reserves closed form solutions of "fair" contracts are unavailable, and we have to rely on numerical solutions using 100,000 Monte-Carlo simulations. We define a fair contract as a contract where investors of the insurance company will be indifferent to whether the company makes the contract or not. To be able to find fair δ for a given set of the control variables, α, θ, g, b , we simulate m paths of the value of equity under the risk-neutral measure, Q ,

$$E_1 = V = \sum_{t=2}^{T+H} e^{-r(t-1)} E^Q[CF_t] \quad (20)$$

Since the value of equity is monotonically decreasing in δ , we can utilize Newton's method

(as described e.g. by Judd (1998), chapter 4.1) to find a fair δ for each contract.

2.4 The Households

We assume that households can be represented by a CRRA utility function with a relative risk aversion coefficient γ . Then the utility of generation, h , can be described as a power utility function on the form

$$u(W^h) = \frac{1}{1-\gamma} (W_{h+T}^h)^{1-\gamma}. \quad (21)$$

The households' maximization problem will be distributing the wealth between the risky and risk-free asset as well as the insurance product. The weights of the portfolio allocated to each of the assets are named ω_S , ω_D , and ω_I respectively. We assume a borrowing constraint and no short-selling, such that all weights are non-negative. Furthermore, we assume no continuous rebalancing. Even though the risky and risk-free assets are tradeable, the insurance asset can typically not be traded, at least not in portions, before expiry. The optimization problem can be formalized as

$$\max_{\omega_S, \omega_D, \omega_I} E[u(W^h)] \quad (22)$$

subject to

$$W^h = \bar{W}^h \left(\omega_S \frac{S_{h+T}}{S_h} + \omega_D \frac{D_{h+T}}{D_h} + \omega_I \frac{I_{h+T}^h}{L_h^h} \right) \quad (23)$$

$$\omega_S + \omega_D + \omega_I = 1 \quad (24)$$

$$\omega_S, \omega_D, \omega_I \geq 0. \quad (25)$$

We standardize the initial wealth \bar{W}^h to 1. Furthermore we know that $\frac{D_{h+T}}{D_h}$ is simply the risk free rate continuously compounded and we can replace it by e^{rT} . Hence we get from equation (23):

$$W^h = \omega_S \frac{S_{h+T}}{S_h} + \omega_D e^{rT} + \omega_I \frac{I_{h+T}^h}{L_h^h}. \quad (26)$$

3 Calibration

Table 4 reports our benchmark parameter values. We calibrate our model using data for a simplified, but typical contract in several European countries.

3.1 Parameters of the Economy

As our risk free rate, r , we use the long term rate of German government bonds. At Jan 1, 2006, the 30 year rate was 3.62%.⁷ We use 4.0% as our risk free rate. For the stock

⁷Source: Datastream.

Parameters of the Model	Notation	Benchmark Case
The Economy		
Risk-free rate	r	4.0 %
Equity premium of the equity index	μ	4.0 %
Volatility of the equity index	σ	16.0 %
The Company		
Proportion of total asset portfolio, A_t , invested in the equity index	θ	20 %
Customers' reserves to total assets at time $t = 1$ ($\alpha = \frac{L_1}{A_1}$)	α	90 %
The constant guaranteed rate	g	2.0 %
The proportion split of the remaining total profit to the households	δ	97.11%
Proportion of declared bonuses credited the bonus reserves B_t	b	30 %
Payout ratio of bonus reserve	p	36 %
The Households		
Overlapping generations of households, indexed by $h = 1, 2, \dots, H$	H	80
Term of the policy	T	20
Growth rate of the households' aggregate initial investments	v	2.0 %
Coefficient of relative risk aversion	γ	5

The table provides the benchmark case parameter values that are used to conduct the numerical analysis.

Table 4: Calibration

return process we consider a mean equity premium, $\mu = 4.0\%$, and a standard deviation, $\sigma = 16\%$. Considering an equity premium equal to 4% as opposed to the historical of 6% is a fairly common choice in this literature, see e.g. Cocco, Gomes, and Maenhout (2005), Yao and Zhang (2005) or Gomes and Michaelides (2005). Also notice that opposed to most other papers, we use a long-term interest rate.⁸

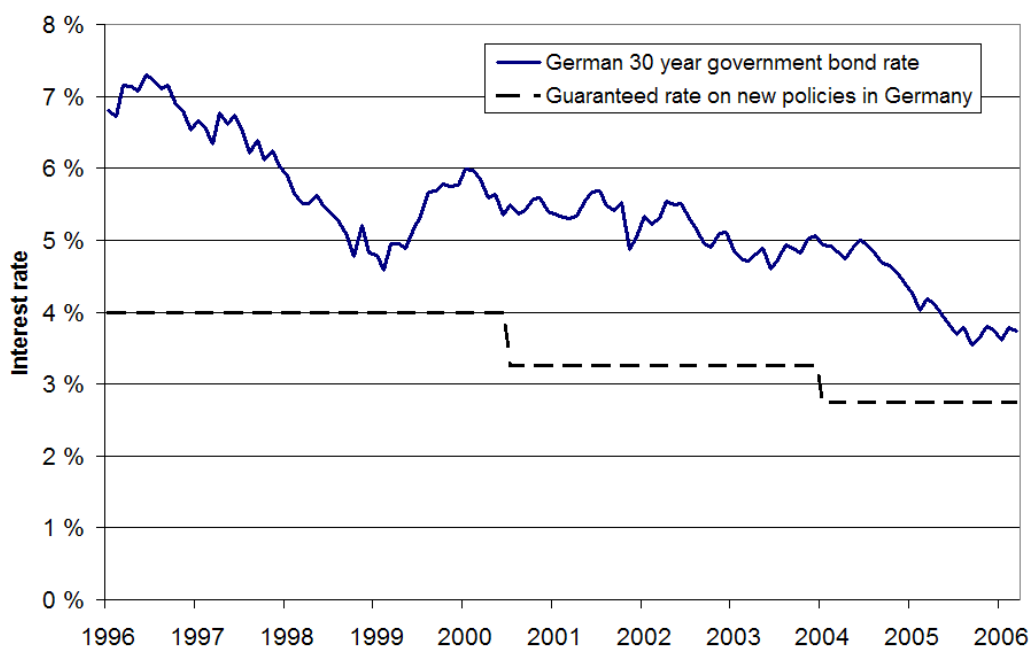
3.2 Guarantees

According to the 3rd European life assurance directive the guaranteed interest rate shall be maximum 60% of the interest rate of government bonds of the same currency, without defining this further.⁹ National authorities are left to make more detailed rules. However, currently most national regulations allow higher guaranteed rates than 60% of most euro government bond rates. A survey is provided in Table 5.

Historically, the spreads between guaranteed rates and government bond rates are much higher. An illustration from Germany is shown in Figure 3. In order to get a spread more in

⁸According to Dimson, Marsh, and Staunton (2004) the arithmetic nominal world equity return for 1900-2003 is 10.2% with a standard deviation of 16.9%. Mehra and Prescott (1985) found an arithmetic risk premium of about 6% above the short interest rate with a standard deviation at 16.6%.

⁹See article 17.



Sources: Datastream, CEIOPS, and Allianz Leben 2004 Annual Report

Figure 3: Interest rates

line with the historical average, and assuming that national regulators will eventually change their regulations in the direction of the 3rd European life assurance directive, we let the guaranteed rate be 2.0%.

3.3 Bonus Reserves

We illustrate our discussion on bonus reserves with the quite complicated German way of reserving (see Figure 4), as described by Allianz in their presentation to investors at the Allianz Capital Markets Day 14 Jul 2005.¹⁰ The 2004 surplus of Allianz was in total 4.9 bns euros, net of taxes, but including development of hidden reserves.¹¹ Of this, 3.2 bns were accounted on balance sheets, while 1.7 bns was related to the development of hidden reserves.

From the total surplus, 0.2 bns euros were transferred to the equity.¹² This gives a δ of 95.9%, which, however, has little relevance to us, as δ will be used as the balancing parameter for achieving correct pricing over the product life-cycle.

In total 2.0 bns euros were transferred to the mathematical reserves. This corresponds

¹⁰Presentation downloadable at www.allianz.com

¹¹We do not include development of hidden reserves in the loan portfolio, which may be quite substantial, but will prove impossible to quantify without information from the company itself.

¹²Furthermore, one could argue that parts of the hidden reserves in fact belongs to the shareholders. An alternative calculation will be taking into account only the on-balance-sheet-items when calculating δ , getting $\delta = 93.8\%$.

Country	Current Guaranteed Rate	
Austria	2.25 %	From 1 Jan 2006
Belgium	3.75 %	Other rates for durations shorter than 8 years
Denmark	2.0 %	
France	2.5 %	3 % for durations shorter than 8 years
Finland	2.5 %	
Germany	2.75 %	
Italy	2.0 %	From 1 Jan 2006
Netherlands	3.0 %	
Norway	3.0 %	Proposed reduction to 2.75 % for some contracts
Sweden	2.75 %	
Switzerland	2.5 %	Lower rates used in contracts with no mandatory guarantees

The table shows current guaranteed interest rates in continental Europe. Sources: CEIOPS, BPV (Switzerland).

Table 5: Current Guaranteed Interest Rates in Continental Europe

to $b = 57.4\%$. However, in a historical context this seems to be on the high side. 2004 was a good year when it comes to securities market development, causing a large increase in the hidden reserves. In a more normal year we think one can expect no significant increase in the hidden reserves. When using only the on-balance-sheet items we get $b = 33.3\%$. We round this and use $b = 30\%$ as our base case.

The bonus reserves in Allianz can be split in several parts. We have already noticed the split between on-balance-sheet items (reserves for bonuses, RfB) and off-balance sheet items (hidden reserves). In addition it will be useful to split the RfB into allocated RfB and terminal bonus fund (which can be individualized) and free RfB (which cannot be individualized). We assume that at expiry of the contracts, customers will only receive their proportional parts of the individualized funds. For Allianz, this corresponds to $p = 36\%$, which we will use as our base case. We note, however, that individualized bonus fund are only common in some European countries, hence we will also show scenarios with $p = 0$.

3.4 Asset Allocation and Capital Structure

In Table 6 we find an average allocation for European L&P companies at 3% in real estate, 22% in equity and 74% in bonds. We regard real estate as close to fixed income, thus based on European data we set our rounded asset allocation parameter, $\theta = 20\%$. As the asset allocation typically changes over time, we provide sensitivities to this in Section 4.3.

When it comes to capital structure, EU regulations specify a minimum solvency capital of 4% of mathematical reserves + 0.3% of sum insured. This would normally approximately

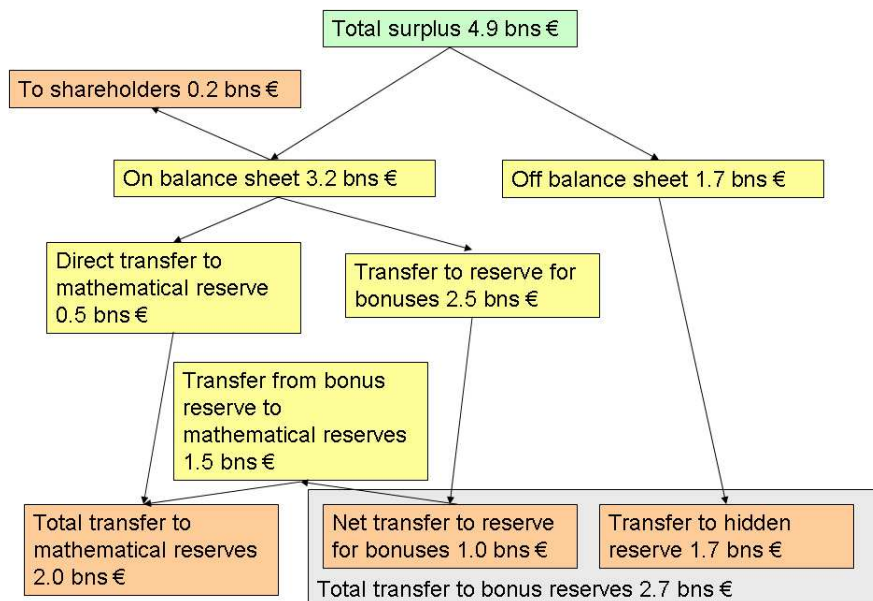


Figure 4: Distribution of surplus of Allianz Leben (2004 numbers)

correspond to $(1 - \alpha) = 0.05$. However, as in most countries companies tend to hold much more capital, we use instead $(1 - \alpha) = 0.1$.¹³

3.5 Parameters of the Households

In our model we use a single premium contract with a duration T of 20 years. In order to get a "going-concern-state", where old contracts expire, but at the same time new ones are written, it is necessary to use more than 40 generations. We use as number of generation $H = 80$.

Even in a "going-concern-state" there will be some growth in the assets of the company due to the increase in premiums paid by the newer generation. This is due to increase in population, inflation and real growth rates. The population growth in Europe is assumed to be zero or even negative in the future.¹⁴ Assuming no long-term real growth we limit our growth rate v to the inflation rate of 2% as is the target of ECB.

We start by presenting results for a quite common relative risk aversion $\gamma = 5$ (e.g. Gomes and Michaelides (2005)). Usually in the literature the range of γ is between 3 (e.g. Dammon, Spatt, and Zhang (2004)) and 10 (e.g. Cocco, Gomes, and Maenhout (2005)). Later on we will report results for different values of γ .

¹³Based on data from CEIOPS value weighted average of main countries is 9.2%.

¹⁴According to UN World Population 2004. See www.unpopulation.org.

	Real Estate	Equity	Bonds
Austria	4.0 %	33.9 %	62.2 %
Belgium	0.9 %	19.7 %	79.4 %
Germany	2.8 %	28.7 %	68.5 %
Denmark	2.6 %	32.8 %	64.6 %
Finland	8.1 %	30.3 %	61.6 %
France	2.9 %	11.3 %	85.8 %
Ireland	5.0 %	18.4 %	76.6 %
Italy	0.4 %	14.9 %	84.7 %
Netherlands	7.7 %	16.4 %	75.9 %
Norway	10.4 %	17.4 %	72.2 %
Average	3.4 %	22.5 %	74.1 %

The table shows the asset allocation for European life companies for 2004. The average is value weighted. Equity is the sum of shares, variable-yield securities, units in unit trusts and investments in affiliated enterprises. Bonds consists of debt securities, fixed income securities, investment pools, mortgage loans, other loans, deposits with credit institutions and deposits with ceding enterprises. Source: CEIOPS

Table 6: Asset allocation life companies 2004

4 Intergenerational Cross-subsidization

4.1 Result of the Benchmark Case

The main result of our benchmark case is given in Figure 5. We find that the expected risk-adjusted return is monotonically increasing with respect to generations and above the risk-free rate from generation 42. The return in the build-down period (only old customers leaving the company) of the last 20 generations seems unrealistically high, particularly for the very last generation. However, the build-down scenario may be unrealistic in itself, as companies will typically sustain.

Furthermore, we find low returns to generations in the build-up-phase of the first 20 generations. Customers in this phase should rather look for alternative investments in other product. In practise we see very few new life and pension companies selling guaranteed products in mature markets. Recently new companies in Western European markets, e.g. Mediolanum and MLP, have preferred unit-linked and other non-guaranteed products.

We will particularly focus on the difference between early and late generations in the going-concern phase from generation 20 to 60. We find an expected risk-adjusted return of 3.93% for generation 20, compared to 4.03% for generation 60. This return difference may seem small, but in a 20 year perspective it will still be significant, corresponding to an initial fee of 2% of invested capital.

In Figure 6 we illustrate the risk of each generation, measured by the standard deviation of the average annual return. We find that the risk increases in the same fashion as the

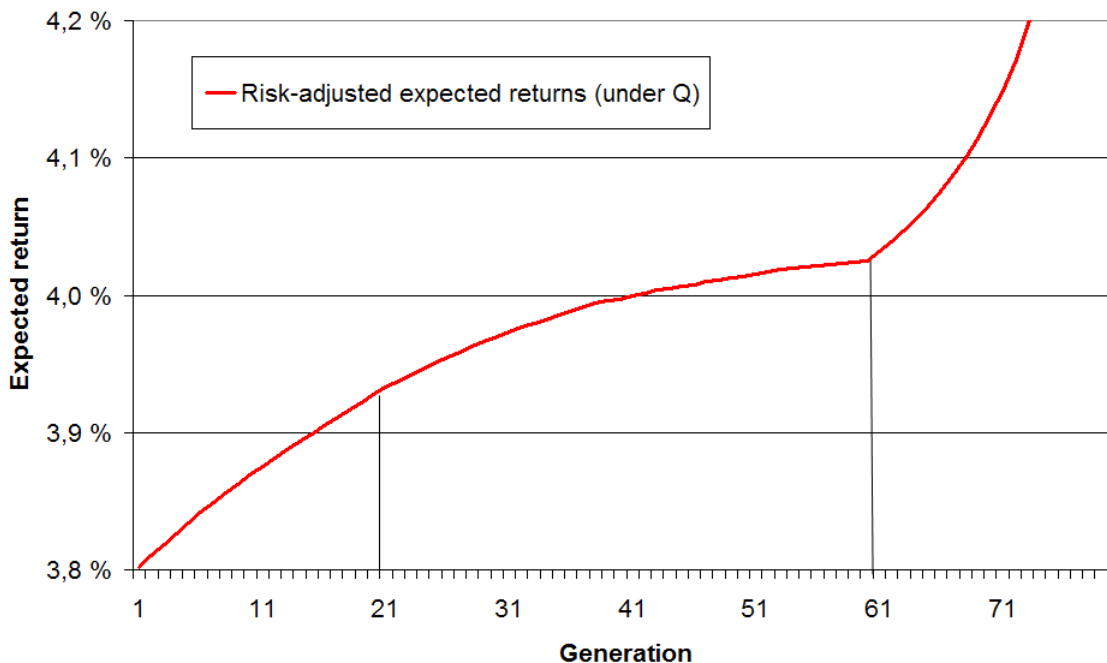


Figure 5: Expected return for different generations

The figure shows expected annual risk-adjusted return of the insurance contract for different generations 1 – 80. This return can be compared to the risk-free rate of 4%.

expected risk-adjusted return. Economically it seems that the return overcompensates for the risk, yielding a return above the risk-adjusted rate if there is enough risk taken.

The technical explanation is that each generation assumes investment risk of the investment period of the previous generation through the bonus reserve. If a generation faces high returns in the stock market it will leave behind a high bonus reserve causing high expected returns for the next generation(s).

In our model the first generations get no risk transferred from previous generations. However, they will be able to transfer some of the risk in their period to the next generations. These middle generations will assume risk from previous generations, but also be able to transfer risk to their followers. Finally, the last generations will assume risk from all previous generations as well as the full risk from their own investment period.

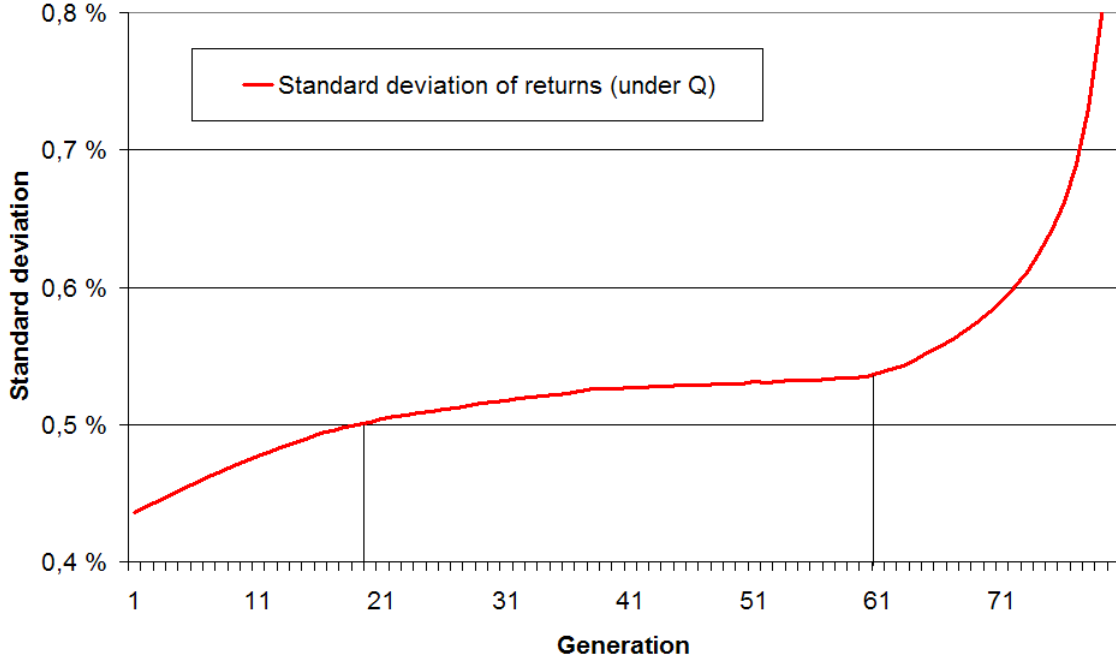


Figure 6: Standard deviation for different generations

The figure shows the standard deviation of the annual risk-adjusted return of the insurance contract.

4.2 Intergenerational Diversification

The returns of the different generations are dependent on both current and earlier periods stock market return. We illustrate this dependency by running OLS regressions where average yearly return, \tilde{r}^h , on the investment for the customer is the left-hand side variable. We split the return of generation $h = 20$ into two time periods of stock market return, $t = (1, 20)$, $r_{1,20}$ and $t = (21, 40)$, $r_{21,40}$:

$$\tilde{r}^{20} = \beta_0 + \beta_1 r_{1,20} + \beta_2 r_{21,40} + \varepsilon. \quad (27)$$

For generation 60 the average annual return, \tilde{r}^{60} , is influenced not only by the return for period $(1, 20)$ and $(21, 40)$, but also by the return for the periods $(41, 60)$ and $(61, 80)$, $r_{41,60}, r_{61,80}$, respectively:

$$\tilde{r}^{60} = \beta_0 + \beta_1 r_{1,20} + \beta_2 r_{21,40} + \beta_3 r_{41,60} + \beta_4 r_{61,80} + \varepsilon. \quad (28)$$

We can interpret β_i as how sensitive the customer's return is to the returns of the different time periods. The β coefficient measures the correlation between the customer's return and the respective period. We expect the β to decrease for more distant periods.

Panel a in Table 7 shows the regression for generation 20. If the stock market yields zero

Panel a	Regression generation 20		
	β_0	0.0335	[0.0335, 0.0335]
	β_1	0.0092	[0.0090, 0.0094]
	β_2	0.1357	[0.1355, 0.1359]

Panel b	Regression generation 60		
	β_0	0.0333	[0.0333, 0.0333]
	β_1	0.0013	[0.0009, 0.0016]
	β_2	0.0093	[0.0090, 0.0097]
	β_3	0.0253	[0.0250, 0.0257]
	β_4	0.1382	[0.1379, 0.1386]

This table shows two panels. Panel a illustrates regression for generation 20. In panel b we show the regression for generation 60. For each estimate of β , a 95% confidence interval is plotted.

Table 7: Intergenerational Diversification

return we would expect a return at 3.35%. If the average stock market return for the current period increases with 1% the customer's return increases with 0.14%. The customer's return is more than 14 times more sensitive to current periods return than the previous period.¹⁵

Generation 60 is dependent of 80 years stock market return, however, as shown in Panel b in Table 7, the most important period is not surprisingly the current period. We see that the previous period is more important for generation 60 than for generation 20. The reason is that the bonus reserve increases with time.

4.3 Sensitivities to the Benchmark Case

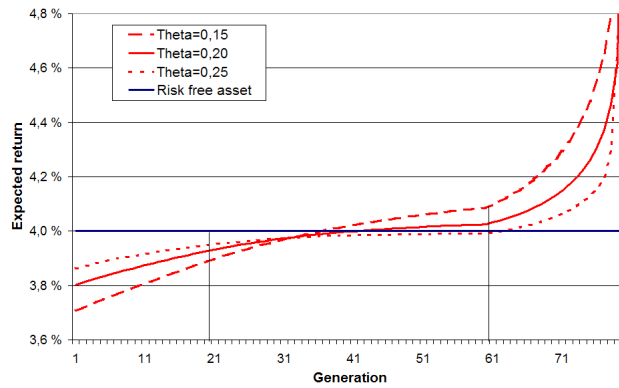


Figure 7: Sensitivity with respect to θ .

The effect of different alternative θ 's on the expected annual risk-adjusted return is illustrated in this figure. The benchmark case is $\theta = 0.20$.

¹⁵Since this regression is run on simulations the t-statistics are a function of numbers of simulations. With 100,000 simulations all the beta's are highly significant.

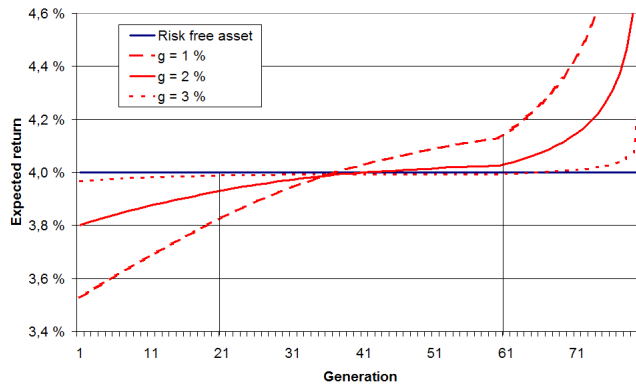


Figure 8: Sensitivity with respect to the guarantee g .

The figure illustrates how different levels of the guarantee g impacts the expected annual risk-adjusted return. The benchmark case is $g = 2\%$.

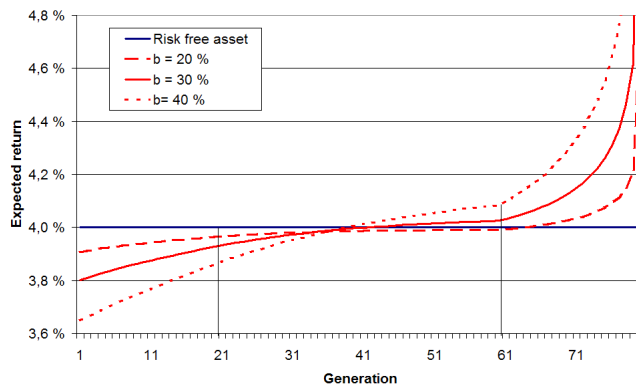


Figure 9: Sensitivity with respect to b .

In this figure we show how the proportion of declared bonuses credited the bonus reserves, b , impacts the expected annual risk-adjusted return of the insurance contract. The benchmark case is $b = 30\%$.

Given the different characteristics of different life and pension insurance markets, one common model will to a large degree have to build on averages. In Section 3 we show how some of the parameters vary across borders, in this section we will show how the level of different parameters will change the results of the model.

Changing the θ means changing the risk of the asset portfolio of the company. The higher the θ , the higher the risk. When assuming more risk, the bonus reserve will be more frequently used as low asset returns (lower than the guaranteed rate) becomes more frequent. This leads to a lower average level of the bonus reserve. As early generations wish to limit the build-up of bonus reserves, they will benefit from higher θ at the cost of later generations. In Figure 7 we give results for different levels of θ .

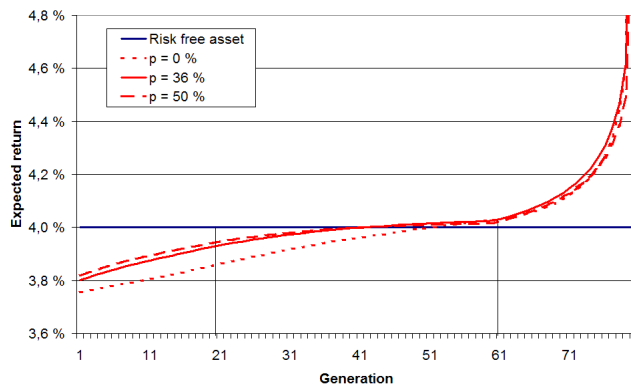


Figure 10: Sensitivity with respect to p .

This figure illustrates how the payout ratio of bonus reserve, p , impacts the expected annual risk-adjusted return of the insurance contract. The benchmark case is $p = 36\%$.

The finding that later generations benefit from a low θ may also give some explanation to low stock market exposure in most life and pension insurers. Intuitively one would think that companies should invest higher proportions in stocks to get closer to the optimal asset allocation for customers (see e.g. Døskeland and Nordahl (2007) for details). However, as companies prefer to satisfy new customers (the later generations) they may prefer a lower θ .

A low spread between the risk-free rate and the guaranteed rate will lead to slower build-up of bonus reserves. This is due both to lower expected profits of the company (roughly equal to the spread) and to the lower δ (less favorable profit sharing to customers) the company will allow to compensate the higher guarantee. In Figure 8 we show that the intergenerational cross-subsidization decreases when g is increased. The effect of decreasing r will be similar to the effect of increasing g . We note, however, that $g = 3\%$ imply a spread of only 1%, which is very low compared to the historical rates shown in Figure 3 and leaves the life insurance contract close to a bond contract.

The build-up of bonus reserves can also be influenced more directly by changing b . In Figure 9 we show that a higher b benefits the later generations. If b goes towards zero, there will be no bonus reserves causing differences between generations.

The impact of changing the pay-out-ratio p of the bonus reserve seems to be limited. In Figure 10 we show that the scenario with $p = 50\%$ is only marginally different from the benchmark case. Changing the payout ratio to $p = 0\%$ causes larger changes, this scenario yield lower returns to the first generations, while only the very last generations seem to benefit.

In Figure 11 we show the result that all generations benefit from a lower growth rate v . The first generations will still build up bonus reserves at the same pace as in the benchmark case. However, with a lower growth rate they will receive a larger proportion of the bonus reserve when the contract expires or in the case of low asset returns. The reason is that

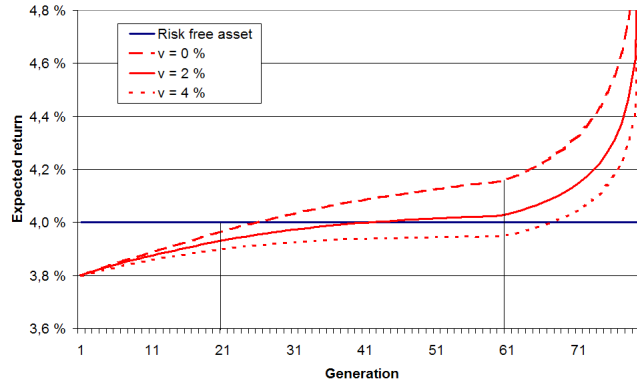


Figure 11: Sensitivity with respect to v .

This figure illustrates how the growth rate of households' aggregate initial investments, v , impacts the expected annual risk-adjusted return of the insurance contract. The benchmark case is $v = 2\%$.

subsequent generations' mathematical reserves are now smaller and the first generations' share of the total mathematical reserves is higher. This is only partially compensated for by a higher bonus reserve, as subsequent generations have only contributed to the build up of bonus reserves during a limited period.

Later generations will also receive this benefit, and in addition they will profit from the fact that the build-up of the ratio of bonus reserve to mathematical reserve now is faster, as the mathematical reserve grows more slowly. At the time of their initial investment their part of the bonus reserve will be larger per unit of investment, hence the "gift" from previous generation will have more impact. In Figure 11 we see that later generations get a higher benefit from a low growth rate than what the first generations do.

We note that the impact of the growth rate is the opposite of that of a pay-as-you-go pension system. While the pay-as-you-go system de facto produces a liability to be transferred from old generations to new ones, the guaranteed contracts produce an asset (the bonus reserve) to be transferred. Hence the guaranteed contracts may in some scenario work as a hedge of population growth risk of a pay-as-you-go system.

5 Optimal Portfolio Choice

In order to optimize individuals' portfolio choice as defined in Section 2.4, we run simulations under the real probability measure P . In Figure 12 we show how the expected return (given expectation for all generations at time 1, see below) develops over generations compared to the risk-adjusted return (under the equivalent martingale measure Q). As previously explained in Section 4.1 the standard deviation is higher for later generations. We note that the larger risk for later generations is compensated for by a larger risk premium measured by the difference

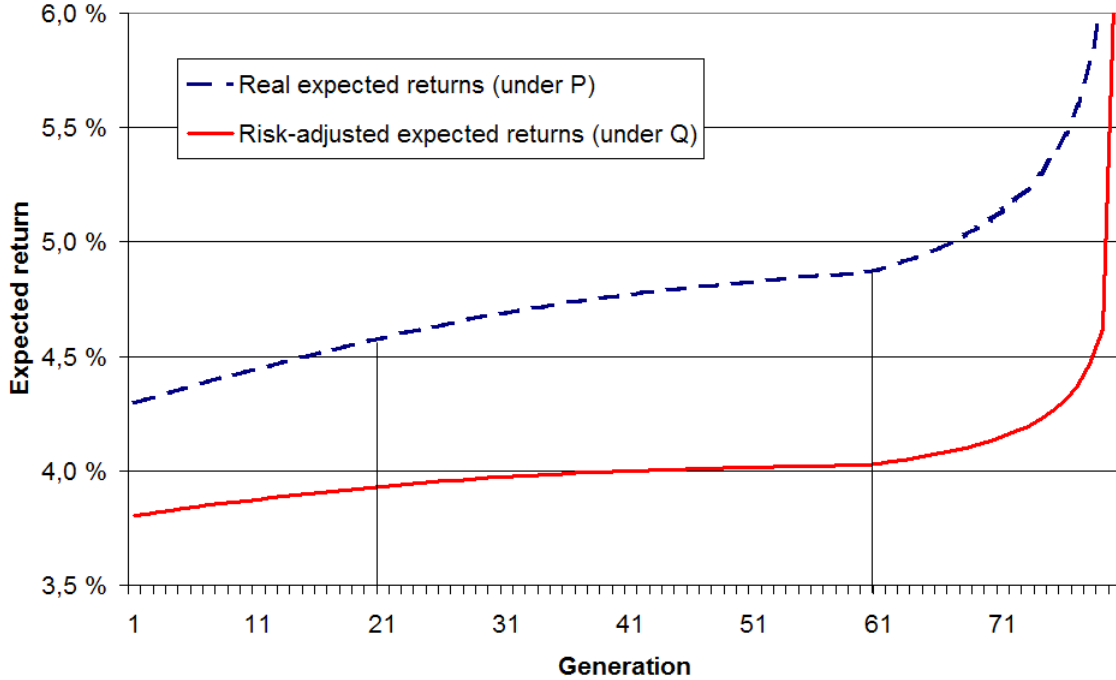


Figure 12: Real expected return for different generations.

In this figure we compare the real expected annual return (under P) for different generations with the similar expected risk-adjusted return (under Q).

between the return figures for each generation.

As we are interested in the life-cycle trend of the attractiveness of the contracts, we assume that the customers only know the expectation of the bonus reserve at time 1. The customers do not know the realization of the bonus contract, hence they can not start "timing" the contract by buying the contract only at high realizations of the bonus reserve. As we find that the expected return also depends on previous periods' market return, our expected return may be different from the expectation customers face at the time of investment. This makes sense in a setting where each generation is present behind a "veil of ignorance", they select a pension system (mix of e.g. public pensions, private pensions, and other savings products) to belong to some time ahead of the actual investment.

5.1 Optimality for Different Generations in the Benchmark Case

We maximize the household portfolio choice for each generation over three assets; the insurance asset, the risky asset and the risk-free asset as shown in equation (22). We would expect the early generations to prefer direct investments in the risky and risk-free asset, while later generations will prefer to invest in the life asset due to the higher expected returns.

In Figure 13 we show that the first 25 generations will prefer no investment in the insurance

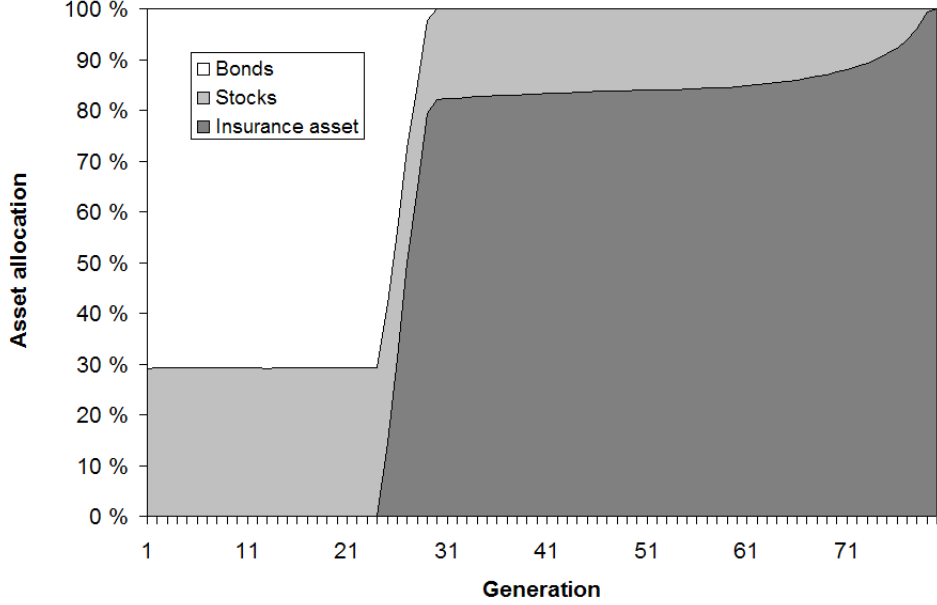


Figure 13: Optimal asset allocation for different generations, benchmark case.

This figure shows the optimal allocation to different assets classes for the different generations 1 – 80, given our benchmark case relative risk aversion coefficient $\gamma = 5$.

asset. In this period the expected return under Q is significantly below the risk-free rate (see Figure 5). The optimal allocation to the risky asset is approximately 29.2% which correspond to the Merton (1969) solution:¹⁶

$$\omega_S^* = \frac{\mu}{\gamma\sigma^2} = \frac{4\%}{5 \cdot 0.16^2} = 31.25\% \quad (29)$$

where ω_S^* is the optimal allocation to the risky assets. The other parameters are shown in Table 4.

More surprisingly the optimal solution shows that for generation 25 – 42 it is optimal to invest in the insurance asset even though the risk-adjusted return is lower than the risk-free rate. The reason is that the diversification effects considered in Section 4.2 benefits investments in the insurance asset combined with the risky asset. After generation 30 there is a slight increase in the optimal allocation to the insurance asset in later generations due to the increasing profitability of the life asset.

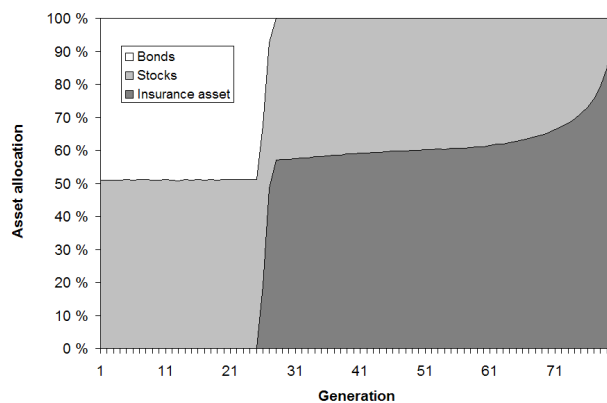


Figure 14: Optimal asset allocation for different generations with $\gamma = 3$.

This figure shows the optimal allocation to different assets classes, given the alternative $\gamma = 3$.

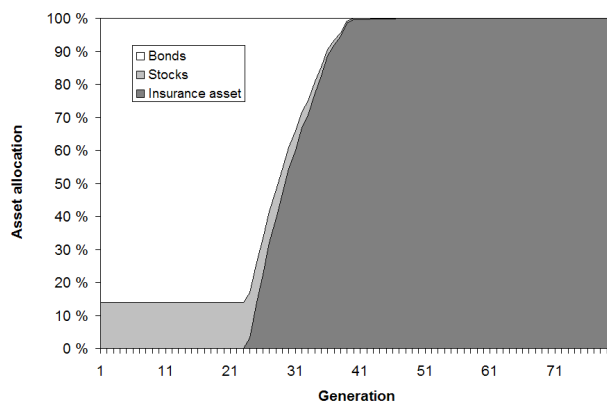


Figure 15: Optimal asset allocation for different generations with $\gamma = 10$.

This figure shows the optimal allocation to different assets classes, given the alternative $\gamma = 10$.

5.2 Optimality for Different Levels of Risk Aversion

The risk aversion parameter influences the optimal asset allocation. A lower risk aversion gives a higher allocation to the risky asset. For the earliest generations this drive down the allocation to the risk-free assets, while the generations after generation 25 mainly will reduce their exposure to the insurance asset. The optimal allocation in the case of $\gamma = 3$ is shown in Figure 14.

The results for an risk aversion parameter equals 10, is shown in Figure 15. Now, the generations 25 – 40 want to keep a positive proportion in all three available assets. The investment in the risky asset is optimal in order to keep some diversification with the insurance

¹⁶With the exception that due to the no rebalancing condition our solutions typically show a marginally lower investment in the risky asset.

asset. However, as the investor is now more risk-averse he wants to invest in the risk-free assets in order to keep the total risk down.

6 Conclusion

In this paper we investigate the return of different generations investing in a guaranteed interest rate life and pension contract. We use a numerical simulation model over 80 generations with realistically calibrated parameters of a typical European guaranteed rate contract, with the assumption of correct pricing over the life-time of the company. Our findings indicate that there exist a cross-subsidization from customers in early generations to customers in later generations. Furthermore, as returns for one generation depend also on return in previous periods, there is a time diversification effect built into the contract.

We also show that these effects are large enough to defend that a guaranteed rate contract is part of the optimal portfolio of the late generations. Hence our paper contributes to explaining why household invest in life and pension products even though they are not part of the optimal portfolio in a one-customer setting.

Future research in this area may expand our analysis to cover the question of whether private pensions should be included in a portfolio of pension systems. We have shown that there is a risk sharing effect between today's generation and earlier generation. This may add a dimension to today's system of pay-as-you-go and funded alternatives, where there is a risk sharing effect between today's generation and the younger generation.

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Strategic asset allocation for a country: the Norwegian case

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Abstract This paper develops a simple strategic asset allocation model for a country with non-tradable assets and liabilities. Contemporaneous correlation does not capture the long-term relationship between the non-tradable items and the financial assets. I apply cointegration and duration matching to better identify the long-term relationship. The model is applied to the case of Norway. Simulations suggest that Norway should implement a strategy which entails a higher proportion (than today's strategy) invested in stocks. Although the new strategy is superior in several criteria and as Norway reforms its social security system, there is still considerable risk that Norway will fail to meet its liabilities.

Keywords Strategic asset allocation · Social security system · Public pensions

JEL Classification G11 · H55

1 Introduction

One of the most important issues facing governments considering future pension liabilities is how to deal with an aging population. Many pension authorities utilize a traditional mean–variance solution with regard to their asset allocation choice. This paper argues that non-tradable items on the balance sheet alter the traditional allocation. To identify the long-term relationship between non-tradable items and financial assets I consider two alternatives, cointegration and duration matching. After calibrating the model to Norway, I show that both the

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cointegration and the duration matching suggest that Norway should increase the proportion invested in stocks today, and reduce it over time due to an aging population. The new strategies are superior to Norway's mean–variance strategy.

Most developed countries face a dramatic growth in their numbers of retired citizens and their governments usually carry significant responsibility for pensions. An important question for these countries is how to fund its pensions. To avoid undermining the sustainability of a pay-as-you-go insurance system the government must either substantially pre-fund future increases in its liabilities or significantly raise taxes.

In general most countries find it prohibitively expensive to pre-fund future liabilities, but countries with periods of unusually high revenues may do so. These relative increases in revenues can be due to exogenous changes in demand for their natural resources. In the late seventies, Norway discovered sizeable oil reserves. Having developed and invested in extraction technology, the Norwegian government currently receives significant non-renewable resource rents from these reserves. However, these revenues are expected to decrease over time and are predicted to be depleted by 2050. The capital gained from the oil is now invested in an oil fund (named the Pension Fund). The size of the fund today is about equal to Norway's GDP.

The central question that I address in this paper is: what does the optimal allocation strategy look like for a country? Even though the simulation exercise is specific to Norway, the qualitative results apply to other countries. I consider two cases of optimal asset allocation. The base endowment case disregards future obligations. The following optimal strategy corresponds to a constant mean-variance strategy. I then compare and contrast this with the second case in which non-tradable items on the balance sheet of the country are taken into account.

I apply the asset allocation theory directly to a nation's assets and liabilities.¹ In a "perfect" world we need not care about the asset allocation for a country. It is the households of the country that bear the risk of the investments. Building on the well-known result from [Modigliani and Miller \(1958\)](#) owners or households could, in principle, undo any allocation the nation decides upon through their personal portfolios. But imperfections such as taxes, transaction costs, and differences in information and expertise make the nation's allocation relevant to households.

To focus on the main effects of the different factors in the quite complex problem, I try to keep the model as simple as possible. Because the model satisfies the standard assumptions of portfolio theory, see e.g. [Campbell and Viceira \(2002b\)](#), I obtain closed-form solutions. I am not aware of any other paper investigating the same problem, although several papers, e.g. [Sharpe and Tint \(1990\)](#), [Rudolf and Ziemba \(2004\)](#), [van Binsbergen and Brandt \(2006\)](#), [Sundaresan and Zapatero \(1997\)](#), and [Boulier et al. \(1995\)](#), deal with a similar

¹ [Trovik \(2004b\)](#) also reviews the investment strategy for a country, but in contrast I suggest quantitative policy recommendations.

asset liability problem for corporate pension plans. For both corporate pension plans and countries, the under-funding of the pension liability is often a problem. In response to this under-funding, many governments have proposed a set of funding rule reforms to strengthen the pension system.

Traditional models for asset allocation use risk measures that are related to short-term variability in market value (contemporaneous correlation). For a long-term investor with long-term non-tradable assets and liabilities this is not appropriate since the long duration dimension is not captured. There are some papers investigating the long-term relationship between non-tradable assets and financial assets. We can divide the techniques for identifying the long-term relationship into two categories; cointegration and duration matching. [Baxter and Jermann \(1997\)](#) investigate the implications for portfolio choice of a cointegrated relationship between non-tradable human capital and GDP growth. Similarly, [Benzoni et al. \(2006\)](#) explore the implications of a cointegrated relationship between an investor's non-tradable human capital and financial assets. [Cardinale \(2003\)](#) looks for and finds a cointegrated relationship between financial assets and wages.

The second technique; duration matching between the investment portfolio and the liabilities, has existed for a while in the asset liability management literature (started probably by [Leibowitz 1986](#)). The matching utilizes the fact that we can find the average life (duration) of the cash-flows for both financial assets and non-tradable items. Duration is a well-known concept for bonds, but the literature has recently also started measuring duration for stocks (e.g. [Dechow et al. 2004](#); [Lettau and Wachter 2007](#); and [Santos and Veronesi 2004](#)).

Both techniques match the non-tradable assets and liabilities with the financial assets. The decomposition of non-tradable items into a stock-like and a bond-like part is constant over time for the cointegration solution. For the duration matching technique, the changing duration of the cash-flows over time allows for a time-varying relationship. However, for both techniques, a changing balance sheet results in a time-varying asset allocation strategy.

In the case of Norway I estimate the relationship between the financial assets and the largest non-tradable liability; pensions. I find that stocks are cointegrated with wages which again is linked to the pension liability. This implies that a larger fraction of the pension liability can be considered as an implicit short holding of stocks (stock-like). To neutralize this implicit holding, Norway should have a larger proportion than today (40%) of its financial assets invested in stocks. Due to the long duration of both stocks and the pension liability, duration matching also suggests that a large fraction of the pension liability is stock-like. However, since the large pension expenditures over time will steadily come closer, the duration of the pension liabilities will decrease, and the liability will become more bond-like. Thus, if we look at Norway as a pension plan, because of the demographic change, the plan will become more mature in the future and will result in a lower allocation to stocks over time.

From the calibration exercise of Norway I can simulate future balance sheets. Not surprisingly, I find that the inhabitants of Norway can rely on their social security system. However, the answer hinges on three conditions; lasting high

oil price, high growth in the world economy and finally that Norway succeeds in reforming its pension system. The likelihood that the two first conditions are jointly satisfied is small. The oil price and the growth in the economy are both volatile and often negatively correlated, and thus, as the sensitivity analysis in section 4 illustrates, the risk associated with Norway's future wealth is high. A stochastic oil price does not influence the shortfall probability much; the largest risk for Norway is a permanent low oil price. The wealth is less sensitive to the asset allocation decision than to oil price changes or investment returns. But opposite to the oil price and investment returns, the asset allocation is an influenceable variable.

The rest of the paper is organized as follows. Section 2 reviews the models for asset allocation. The optimal asset allocation strategy is identified for two optimization problems; the traditional mean-variance problem and the asset and liability problem. In Sect. 3, I calibrate and find the optimal allocation strategies for Norway. In Sect. 4, I provide a thorough sensitivity analysis with respect to the main factors that impact the nation's balance sheet. I answer the question as to whether the social security system in Norway is trustworthy. Section 4 concludes.

2 The model

I take the perspective of a government of a country. Traditionally in asset allocation, wealth is set equal to financial assets. However, for a country this is a too simple framework since it typically has non-financial assets and liabilities, e.g., tax income and/or pension expenditures. In this section I first describe the agent, which I assume is identical to a country. I then outline the traditional asset allocation model in which the country only takes financial assets into account. Finally, I compare and contrast this with a second case in which the country takes its future assets and liabilities into account.

2.1 The country

The balance sheet of the country is illustrated in Table 1. The country generates income and faces expenditures. I assume the balance sheet consists of an income asset, which is defined as the income stream $I = \{I_t\} > 0$ and the present

Table 1 Balance sheet of a country

Assets	Liabilities
$IA_t = PV_t(I)$	$L_t = PV_t(X)$
FA_t	W_t

The table shows a balance sheet for a country. Here IA_t and L_t are present value of an income and expenditure stream, respectively. Financial assets are given by FA_t . The surplus wealth of the country is W_t

value of all future income as $IA_t = PV_t(I)$.² For the country the stream could be taxes or revenue from a natural resource. The expenditures are defined as $X = \{X_t\} > 0$. The present value of all future expenditures sums to $L_t = PV_t(X)$. For a country, large parts of the expenditure stream could e.g. be the social security system, in particular a pension scheme.³ Financial assets, FA_t , is the value of the investment portfolio.

As in Sharpe and Tint (1990), one can interpret the country’s net wealth, W_t , as a surplus wealth. If the sum of the value of both assets, $IA_t + FA_t$, is less than the liability, L_t , net government wealth is negative, $W_t < 0$. In pension fund terminology; the country is under-funded. In such a situation the social security system is not trustworthy. The households may consider saving extra for pensions themselves and/or the government may cut government consumption or increase taxes.

2.2 The portfolio frontier

The model I specify is a partial equilibrium model. Often a country is a small player in the global financial markets; thus I assume financial returns are given. For simplicity, I restrict the investment universe to only two risky asset classes, stocks (S) and long-term bonds (B).⁴ Each asset has a simple gross return $R_i, i \in (s, b)$. The continuously compounded return or log return r_i I assume is

$$r_i \equiv \ln(R_i) \sim N(\mu_i, \sigma_i^2). \tag{1}$$

Here r_i is normally distributed with mean μ_i and variance σ_i^2 . Since r_i is IID normal the simple gross return is IID lognormal.

The government invests a fraction, α_t , of financial assets in stocks and a fraction $(1 - \alpha_t)$ in long-term bonds. The gross return on a portfolio of assets is a weighted average of the gross returns on the assets themselves, where the weight of each asset is the fraction of the portfolio’s value invested in that asset:

$$R_p = \alpha_t R_s + (1 - \alpha_t) R_b. \tag{2}$$

Since the log of a sum is different from the sum of logs, r_p does not equal $\alpha_t r_s + (1 - \alpha_t) r_b$. To calculate the continuous portfolio return, I use a Taylor series approximation of the nonlinear function relating log asset return to log

² The annual income for year t is given by I_t , and $I = \{I_t\}$ refers to the stream.

³ The income and expenditures are not netted because in Sect. 3 I study different effects of income and expenditures, respectively.

⁴ In fact, Norway is limited to invest in stocks and long-term bonds by a mandate from the Ministry of Finance. The argument is that these two asset classes exhibit long durations and therefore match Norway’s long-term liabilities. In Sect. 3.4, I illustrate how these assets actually match the liabilities. Since the Fund does not invest in risk free assets, one will not achieve the optimal solution in a typical mean–variance setting with risk free asset.

portfolio return, as suggested by [Campbell and Viceira \(2002b\)](#).⁵ In Appendix A, I find

$$r_{p,t+1} \approx \alpha_t r_s + (1 - \alpha_t) r_b + \frac{1}{2} \alpha_t (1 - \alpha_t) (r_s - r_b)^2. \quad (3)$$

The mean, $\mu_{p,t+1}$, and variance, $\sigma_{p,t+1}^2$, of the continuous portfolio return are derived in Appendix B. The continuous mean portfolio return is

$$\mu_{p,t+1} \approx \mu_b + \alpha_t (\mu_s - \mu_b) + \frac{1}{2} \alpha_t (1 - \alpha_t) [(\mu_s - \mu_b)^2 + \sigma_s^2 + \sigma_b^2 - 2\text{Cov}(r_s, r_b)], \quad (4)$$

which is an approximation of the expected gross portfolio return. The third term is an adjustment factor, which vanishes if α_t is either zero or one. The approximated portfolio variance derived in Appendix B is as follows:

$$\sigma_{p,t+1}^2 \approx \alpha_t^2 \sigma_s^2 + (1 - \alpha_t)^2 \sigma_b^2 + 2\alpha_t (1 - \alpha_t) \text{Cov}(r_s, r_b). \quad (5)$$

In addition to low portfolio return, $R_{p,t+1} < 1$, expenditures, X_t , can reduce next period's financial asset,

$$FA_{t+1} = [\alpha_t R_s + (1 - \alpha_t) R_b] (FA_t + I_t - X_t). \quad (6)$$

2.3 Wealth definitions

I find optimal asset allocation for two definitions of wealth. In the base case the country acts as an endowment fund, which disregards future obligations. In the second case the country takes its future assets and liabilities into account.

The base case: endowment

The government disregards the assets and liabilities ($IW_t = L_t = 0$ in Table 1), thus, the financial asset, FA_t , is equal to wealth, W_t . I assume the country prefers a high mean and a low variance of portfolio returns,

$$\max_{\alpha_t} E_t[U(W_{t+1})] = \max_{\alpha_t} \left[E_t[r_{p,t+1}] + \frac{1}{2} (1 - \gamma) \sigma_{p,t}^2 \right], \quad (7)$$

where γ is the risk aversion.⁶

⁵ In contrast to [Campbell and Viceira \(2002b\)](#) I approximate the portfolio return for two risky assets. I do a more accurate approximation than [Campbell and Viceira \(2002b\)](#), see Appendix A.

⁶ I assume the country has standard expected utility preferences. One might argue that the government of a country has behavioral preferences, e.g. dislikes losses more than liking gains. For an overview of the differences between standard expected utility and behavioral models, see [DeGiorgi and Hens \(2006\)](#).

Using the log portfolio return from Eq. (4) and the variance from Eq. (5), we face the following simple optimization problem:⁷

$$\begin{aligned} \max_{\alpha_t} & \left[\mu_{p,t+1} + \frac{1}{2}(1 - \gamma)\sigma_{p,t}^2 \right] \\ & = \max_{\alpha_t} [\mu_b + \alpha_t(\mu_s - \mu_b) + \frac{1}{2}\alpha_t(1 - \alpha_t)[(\mu_s - \mu_b)^2 - 2\text{Cov}(r_s, r_b) + \sigma_s^2 + \sigma_b^2] \\ & \quad + \frac{1}{2}(1 - \gamma)[\alpha_t^2\sigma_s^2 + (1 - \alpha_t)^2\sigma_b^2 + 2\alpha_t(1 - \alpha_t)\text{Cov}(r_s, r_b)]], \end{aligned} \tag{9}$$

that has the following optimal solution:

$$\alpha_t = \bar{\alpha} = \frac{\mu_s - \mu_b + \frac{1}{2}[(\mu_s - \mu_b)^2 + \sigma_s^2 + \sigma_b^2] + (\gamma - 1)\sigma_b^2 - \gamma\text{Cov}(r_s, r_b)}{(\mu_s - \mu_b)^2 - \gamma[2\text{Cov}(r_s, r_b) - \sigma_s^2 - \sigma_b^2]}. \tag{10}$$

Since $FA_t = W_t$, the fraction of financial asset, α_t , is equal to the fraction of wealth, $\bar{\alpha}$, invested in stocks. The solution is independent of wealth and time, i.e. a standard myopic mean-variance solution. Therefore the country should invest a constant fraction $\bar{\alpha}$ of wealth, independent of time, in stocks and $(1 - \bar{\alpha})$ in long-term bonds. The forthcoming Fig. 3 illustrates the solution for the Norwegian case.

Assets and liabilities

In this case I assume the government takes the whole balance sheet in Table 1 into account. In addition I assume, even if it is possible in some situations, the country does not trade its assets and liabilities. Empirically, most of the countries with natural resources choose not to sell their resources. One could argue that countries that borrow to pay pensions, have sold future tax income. However, in my analysis I assume that there are some externalities affecting the country and therefore do not trade its assets and liabilities.⁸

Decomposing non-tradable items

Theoretically, there may be merit in thinking of an investment strategy constructed to match non-tradable assets and liabilities. In practice, however, such a portfolio cannot be purchased in the market, e.g., for the pension liabilities,

⁷ As mentioned in Campbell and Viceira (2002b) I can rewrite (7) as

$$\max \left[\ln E_t[R_{p,t+1}] - \frac{\gamma}{2}\sigma_{p,t}^2 \right]. \tag{8}$$

The agent trades off simple gross return against the variance of the log return.

⁸ It might be that the government wants to control the resources, it may be hard to find a buyer, or the resources give effects that another owner will not utilize.

there are no bonds currently available where the income and capital are linked to increases in wage inflation. Therefore the investment strategy has to carry some residual risk.

In this setting we can use stocks and bonds to minimize residual risk. The question then arises as to what is the optimal allocation in bonds and stocks to minimized residual risk? In order to answer this question it is important to understand the long-term relationship between the non-tradable items and the financial assets.

The risk of any asset class with return R_i is measured with its covariance with the wealth:

$$\text{Cov}(R_i, W_{t+1}) = \text{Cov}(R_i, FA_{t+1}) + \text{Cov}(R_i, IA_{t+1}) - \text{Cov}(R_i, L_{t+1}). \quad (11)$$

If the asset class covaries negatively with the value of the income asset and positively with the value of the liability, it hedges the non-tradable items and should be rewarded.⁹

In the portfolio choice with labor income literature there is different evidence about the relationship between non-tradable labor income and financial assets. Most models attribute bond-like qualities to the future flow of labor income. That is, these models predict that, through their labor income, agents implicitly hold a large position in bonds, implying that they should take a more aggressive position in stocks with their financial assets (see [Jagannathan and Kocherlakota 1996](#); [Cocco et al. 2005](#); [Viceira 2001](#)). The authors find that there is a low correlation between stock market returns and changes in wages, so labor income is more like a bond.

However, as pointed out by [Benzoni et al. \(2006\)](#), the labor income specification in these models may be unnecessarily restrictive. Since the contemporaneous correlation between market returns and changes to aggregate labor income flow is low, as in data, the longer-term correlations are low as well. By allowing aggregate labor income to be cointegrated with dividends, [Benzoni et al. \(2006\)](#) find a significantly higher long-term correlation between returns on human capital and market returns. If labor income is more related to future stock returns, then the young should invest more in bonds, and their stock exposure should increase as they age. If you wait over longer horizons, the correlation between human capital (labor income) and physical capital (which should be related to the stock market) is very high. So over long horizons where you allow for “cointegration”, labor income is more like a stock.

As illustrated above there are several ways of specifying the relationship between a non-tradable asset and the financial assets. This is not only true for portfolio choice for individuals, but also for a country. There are different ways of specifying the relationship between the non-tradable items and

⁹ A standard example is a nominal long-term bond. This moves inversely with nominal interest rates. If the liability takes the form of long streams of fixed nominal payments, then they increase in value when nominal interest rates decline. The last term in Eq. (11) will reduce the covariance. Thus the bond is a good hedge for the liability.

Table 2 Balance sheet

	Assets	Liabilities
The table illustrates a balance sheet that in addition to showing the items, also divides the items into a stock and long-term bond part	$\beta_{ia,t}IA_t$ $(1 - \beta_{ia,t})IA_t$ α_tFA_t $(1 - \alpha_t)FA_t$	$\beta_{l,t}L_t$ $(1 - \beta_{l,t})L_t$ $\bar{\alpha}W_t$ $(1 - \bar{\alpha})W_t$

financial assets. In the case part of the paper, I will explain and show results for two different ways of estimating the relationship; cointegration and duration matching.

Both alternatives lead to a decomposition of the non-tradable item into a fraction of one of the financial assets. I find how much of the non-tradable item that is similar to stocks (stock-like) and how much is similar to long-term bonds (bond-like). Thus, I assume that the non-tradable items act as an implicit holding of one of the two asset classes. I let $\beta_{ia,t}$ denote the fraction of the income asset that is a substitute for stocks, and $(1 - \beta_{ia,t})$ denote the fraction that is a substitute for long-term bonds. The liability can be interpreted as a short position in either stocks or long-term bonds. I define $\beta_{l,t}$ as the fraction of the liability that is stock-like, and $(1 - \beta_{l,t})$ denotes the fraction that is bond-like. $\beta_{l,t}L_t$ is then the nation’s short position in stocks due to the liability.

This decomposition affects the optimal asset allocation strategy. The definition of wealth is the only difference in the optimization from the base case. From Eq. (10) we find that the fraction of *wealth* invested in stocks, $\bar{\alpha}$, is constant. This fraction is similar in both cases since neither the risk aversion nor the investment opportunity set changes. The total demand for stocks is still $\bar{\alpha}W_t$.¹⁰ Thus, in this case:

$$S_t = \bar{\alpha}W_t = \beta_{ia,t}IA_t + \alpha_tFA_t - \beta_{l,t}L_t. \tag{12}$$

Still, α_t denotes the optimal fraction of *the financial asset* invested in stocks. The amount invested in stocks is α_tFA_t . The optimal amount invested in stocks on the right-hand side of the balance in Table 2 has to be equal the amount on the left-hand side. Financial asset, FA_t , is used to match both sides. Solving Eq. (12) with respect to α_t give us:

$$\alpha_t = \bar{\alpha} + \frac{1}{FA_t}[(\bar{\alpha} - \beta_{ia,t})IA_t - (\bar{\alpha} - \beta_{l,t})L_t]. \tag{13}$$

Consequently, the fraction of financial asset held in stocks is dynamic even though the fraction of wealth held in stocks is constant. The three factors influencing the asset allocation strategy are, first, the endowment case solution,

¹⁰ The idea to review the optimal allocation this way is from Jagannathan and Kocherlakota (1996), Eq. (19).

Table 3 Investment strategies

Endowment	Assets and Liabilities	
$\frac{\partial \alpha_t}{\partial FA_t} = 0$	$(\bar{\alpha} - \beta_{ia,t})IA_t > (\bar{\alpha} - \beta_{l,t})L_t$	$\frac{\partial \alpha_t}{\partial FA_t} = -\frac{[(\bar{\alpha} - \beta_{ia,t})IA_t - (\bar{\alpha} - \beta_{l,t})L_t]}{FA_t^2} < 0$
	$(\bar{\alpha} - \beta_{ia,t})IA_t < (\bar{\alpha} - \beta_{l,t})L_t$	$\frac{\partial \alpha_t}{\partial FA_t} = -\frac{[(\bar{\alpha} - \beta_{ia,t})IA_t - (\bar{\alpha} - \beta_{l,t})L_t]}{FA_t^2} > 0$
	$(\bar{\alpha} - \beta_{ia,t})IA_t = (\bar{\alpha} - \beta_{l,t})L_t$	$\frac{\partial \alpha_t}{\partial FA_t} = 0$
Rebalance strategy	$(\bar{\alpha} - \beta_{ia,t})IA_t > (\bar{\alpha} - \beta_{l,t})L_t$	Contrarian
	$(\bar{\alpha} - \beta_{ia,t})IA_t < (\bar{\alpha} - \beta_{l,t})L_t$	Portfolio insurance
	$(\bar{\alpha} - \beta_{ia,t})IA_t = (\bar{\alpha} - \beta_{l,t})L_t$	Rebalance

The table shows the differentiate with respect to financial asset for both cases. At the bottom the different investment strategies are shown

$\bar{\alpha}$, second, the composition of the balance sheet, FA_t, IA_t, L_t , and, third, similarities between the financial assets and the items on the balance sheet, $\beta_{ia,t}$ and $\beta_{l,t}$.

The country allocates more of the financial asset to stocks than in the endowment case if $IA_t(\bar{\alpha} - \beta_{ia,t}) > L_t(\bar{\alpha} - \beta_{l,t})$. In this situation, large parts of the balance act as a substitute for long-term bonds, therefore one uses the financial asset to increase the amount of stocks held. To achieve the endowment solution in this extensive framework the country has to assume that $\beta_{ia,t} = \beta_{l,t} = \bar{\alpha}$. This implies that in the endowment case (ignoring the whole balance sheet) one implicitly assumes that a fraction $\bar{\alpha}$ of the non-tradable items is stock-like.

2.4 Investment strategy

In this context the investment strategy is defined as the rebalance strategy given changes in the financial asset, $\partial \alpha_t / \partial FA_t$.¹¹ The results for the two cases are shown in Table 3. For the endowment case, I find that the allocation is independent of the financial asset, which defines a rebalance strategy. The strategy for the assets and liabilities case is more complicated. Actually, one can achieve three investment strategies. In addition to the rebalance strategy, one has a contrarian and a portfolio insurance strategy. A contrarian strategy is similar to the rebalance strategy, but one should not only rebalance, but actually reduce the fraction invested in stocks as the financial asset increases. The last strategy is the portfolio insurance strategy. One should reduce the proportion invested in stocks as the market falls, and vice versa.¹² Only if $(\bar{\alpha} - \beta_{ia,t})IA_t = (\bar{\alpha} - \beta_{l,t})L_t$ is the strategy independent of FA_t .

¹¹ For a nice overview of the different strategies, see Perold and Sharpe (1988).

¹² In a market with mean-reversion, the rebalance strategy is especially favorable, and the portfolio insurance strategy expensive. In the latter, one has to sell after a downturn, and that is when one expects good returns.

The present values of the streams $\{I_t\}$ and $\{X_t\}$ may change. To find how the allocation changes with respect to the streams, I differentiate $\partial\alpha_t/\partial IA_t = \frac{\bar{\alpha}-\beta_{ia,t}}{FA_t}$ and $\partial\alpha_t/\partial L_t = -\frac{(\bar{\alpha}-\beta_{ia,t})}{FA_t}$. I will review examples of the strategies in Sect. 3.

2.5 Comparison

The asset allocation strategy is optimal given the setup and assumptions for each case, thus one cannot directly compare the level of expected utility. However, I will outline some measures of wealth for a country with income and expenditure streams. These criteria can be used to compare the different asset allocation strategies.¹³ I classify four main criteria, assuming comparison at time T :

- *Expectation*, $E(W_T) = E(FA_T) - IA_T - L_T$.
- *Volatility*, $STD(W_T) = STD(FA_T)$.
- *Value at Risk (VaR)*, the VaR number is the amount one will not lose more than with a defined probability in the next T years.
- *Shortfall risk* is the probability of ending up in year T with a lower wealth than a defined threshold.

In the simulation study in Sect. 3 I will return to the exact definition of the last two risk measures. There will always be a discussion of the best criteria, all the measures above assign a single real number to a probability distribution.¹⁴

3 The Norwegian case

As mentioned, Norway is special in some ways, and others not. The challenge of an aging population affects most developed countries. However, only a few countries have large revenues from natural resources. For Norway the revenues from oil and gas resources have to be invested carefully to meet increasing pension expenditures. I identify Norway's balance sheet and investigate which asset allocation strategy best suits Norway's social responsibilities.

3.1 The balance sheet

The Norwegian government has two sources of income: Gains from its petroleum, $\{O_t\}$, and taxes, $\{Ta_t\}$. $\{Ta_t\}$ tends to be stable over time, whereas $\{O_t\}$ diminishes due to depletion. Thus, the two income streams will develop differently over time. It is also useful to divide the expenditures, $\{X_t\}$, into

¹³ Not only cannot expected utility be used because of different definition of wealth, but also due to a positive probability of negative wealth. At first glance it may seem strange that a lognormally process can take negative values, but since in each period something is added and subtracted, it can become negative if the deficit on the national account is large enough, see Eq. (6).

¹⁴ For a thorough analysis of risk measures, see Artzner et al. (1999).

Table 4 Balance sheet Norway

Assets	Liabilities
$OW_t = PV_t(O)$	$PL_t = PV_t(Pe)$
$TW_t = PV_t(Ta)$	$RL_t = PV_t(Re)$
FA_t	W_t

The table illustrates the balance sheet of Norway. The assets are divided into three parts. Two non-tradable income streams and the financial asset. On the right-hand side the liabilities are divided into two non-tradable items. The surplus is denoted wealth, W_t

old age and disability pensions, $\{Pe_t\}$, and governmental services, $\{Re_t\}$. Pensions are expected to increase in volume, while $\{Re_t\}$ is assumed to be stable. Table 4 exhibits these assets and liabilities in addition to Norway's financial assets already accumulated, FA_t , and net government wealth, W_t .

The figures for the flows are collected from the National Budget. All the figures in this paper are in real terms. I assume a growth in GDP at 2% and discount the streams with 4%. The figures in the National Budget only consist of a single number, therefore I define the cash-flows as follows, with IA_t as example: $IA_t = PV(I) = \sum_{t=s}^T I_t(1+r_c)^{s-t}$ and $I_t = E_t[\tilde{I}_t]$. Here r_c is the discount rate for the cash-flow.¹⁵ I set the investment horizon, T, to 2050. As mentioned earlier, petroleum income is projected to decrease, pension expenditures to increase. Therefore the National Budget projection is named the "sharkjaw", which is replicated in Fig. 1. The solid lines reproduce the assumptions in the National Budget. The dotted and dashed lines represent alternative scenarios already argued for in other documents. In Sect. 4, the different scenarios are used in a sensitivity analysis. I assume all the streams reach steady state in 2050.

The reference path for the oil price in the National Budget of 2006 is based on the assumption that the price will be NOK 350 (USD 50 (assume NOK/USD 7)). In Fig. 1, I illustrate two alternative scenarios with an oil price at NOK 180 (USD 26) and NOK 230 (USD 33), respectively.¹⁶ The different scenarios may be interpreted as future curves. The future curve at NYMEX in October 2006 is above USD 58 as long as it is possible to trade contracts (until Dec 2012).

In the National Budget comprehensive estimates of the pension expenditures, $\{Pe_t\}$, are given.¹⁷ Figure 1 shows the net pension expenditures. In the

¹⁵ In order to simplify the analysis I use the same discount rate, r_c , for all the non-tradable cash-flows. The rate should dependent on the risk of each cash-flow. Since the non-tradable cash-flows are a substitute for a portfolio of stocks or long-term bonds, I assign a rate between the rate of return of long-term bonds and stocks.

¹⁶ The first figure is the same as used in the National Budget of 2005, whereas the latter is from the report: "Macroeconomic Perspectives for the Norwegian Economy – Challenges and Options", Ministry of Finance, November 2004. All figures are very conservative. The oil price in October 2006 is USD 58.

¹⁷ In the National Budget one does not take into account that the increased number of recipients of national insurance benefits will also increase taxes, thus the net effect will be lower than in the figure. In the white paper from the Government to Stortinget (the Parliament), December 2004, one set the net effect to 7/11 of gross increase in expenditures.

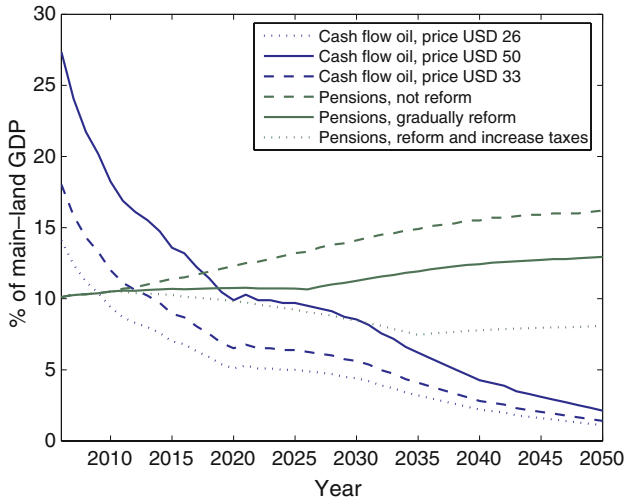


Fig. 1 Different scenarios for pensions and petroleum revenue A similar figure is presented in the National Budget. There it is named the “sharkjaw” (probably to emphasize for the reader the scary difference between the petroleum revenues and pensions). In the figure there are three scenarios for both streams. I review the scenarios in the sensitivity analysis in Sect. 4

same figure I plot three scenarios for the pension expenditures. The thick line is in line with the goal of the Pension Commission,¹⁸ where I reduce the overall expenditures in compliance with the pension reform, named ‘gradual reform’. At the end of the period the expenditures are reduced by 20%. The lowest line illustrates a scenario with increased taxes in addition to the reform. To show the effect of increased taxes I reduce the pension expenditures. The highest line illustrates the alternative that Norway will not be able to implement the reform. This could also be the situation where Norway implements the reform, but obtains unexpected expenses, e.g. high disability costs. The net present values in 2006 of the three alternatives are: NOK 6.1, NOK 4.8, and NOK 7.2 thousand billion, respectively.

Statistics Norway has projected the Norwegian population for the years 2006 to 2050.¹⁹ I find that for the different scenarios the population of working age will be about the same as today, while the group of elderly will almost double. The Ministry of Finance has taken the demographic development into consideration when projecting the pension expenditures. There will not be large changes in taxes or government expenditures due to demographic changes since the group of working people is almost constant. I let the future tax revenue $\{Ta_t\}$ and government expenditures $\{Re_t\}$ grow with GDP. As a proxy for initial tax income I use the figure in the National Budget for revenues exclusive petroleum activities. Similarly, I estimate government expenditures $\{Re_t\}$ by taking

¹⁸ The Pension Commission is a public committee that has proposed a reformed pension system. Their final report was presented January 2004, <http://www.pensjonsreform.no/english.asp>.

¹⁹ http://www.ssb.no/english/subjects/02/03/folkfram_en.

Table 5 Balance sheet Norway, 2006 and 2050

Balance sheet Norway start 2006	
Assets	Liabilities
Panel a	
$PV_{2006}(O) = 4.6$	$PV_{2006}(Pe) = 6.1$
$PV_{2006}(Ta) = 23.0$	$PV_{2006}(Re) = 20.7$
$FA_{2006} = 2.0$	$W_{2006} = 2.9$
Balance sheet Norway start 2050	
Panel b	
$PV_{2050}(O) = 1.6$	$PV_{2050}(Pe) = 9.9$
$PV_{2050}(Ta) = 34.0$	$PV_{2050}(Re) = 30.5$
$FA_{2050} = Z$	$W_{2050} = Z - 4.8$

This table shows two panels. Panel a illustrates the balance sheet of Norway for 2006. In panel b I show the balance sheet for 2050. In the latter panel the value of the financial asset is unknown, Z . All numbers are in NOK thousand billion

the expenditures exclusive of pension costs. The final figure I need is the value of financial asset. The main financial asset for Norway is the Pension Fund (before named Petroleum Fund). Thus, as a synonym of the financial asset I use the Pension Fund. I estimate this to be equal NOK 2.0 thousand billion.²⁰

With the assumptions above I am able to calculate the balance sheet shown in Table 5 for Norway in 2006 and 2050. The upper panel shows the balance for 2006. As a base case I assume that the government in Norway is able to reform its social security system. The estimated net government wealth is $W_t =$ NOK 2.9 thousand billion. Norway's pensions are currently over-funded. The lower panel of Table 5 shows the estimated balance for 2050. I investigate how the different asset allocation strategies between 2006 and 2050 influence the value of the financial assets for 2050, $FA_{2050} = Z$, and therewith wealth. To be over-funded in 2050, i.e. $W_{2050} > 0$, Norway's financial assets have to be larger than its liabilities, i.e. NOK 4.8 thousand billion.

The financial asset process

Each asset allocation strategy generates a financial asset process. Every period starts out with existing financial asset, FA_t . Thereafter the income streams; income from the petroleum sector, O_t , and tax income, T_t , are added. Since Norway has a pay-as-you-go pension system, the yearly pension expenditures, Pe_t are subtracted directly. The same is done with other expenditures, Re_t . The stochastic portfolio return generated by the asset allocation α_t is multiplied with

²⁰ Beyond the Norwegian Pension Fund, the government's most important financial asset is the state's direct ownership interests in enterprises. However, the dominating part of Norway's financial asset is the Pension Fund. The Fund has a global benchmark. Since the Fund is large relative to the Norwegian economy, the Fund is not allowed to invest in Norway. In the future the Fund will contain almost all Norway's financial asset, thus I use the benchmark for the Fund as the benchmark for the financial asset.

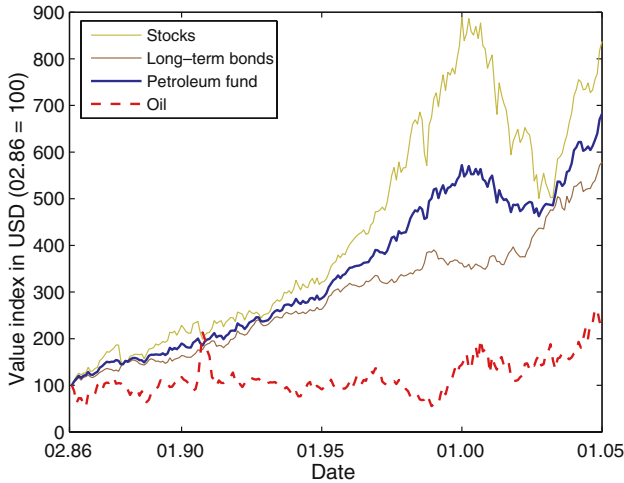


Fig. 2 Return on the Pension Fund versus oil The figure illustrates the nominal growth of four indices using data from 86 until 05. The composition of the stock and the long-term bond indices are equal to the reference portfolio of the Pension Fund published 09.04. I use data from FTSE for stocks, and Salomon Brothers for long-term bonds. The oil price is Brent Spot. All the data is collected from Datastream

the residual. Thus, the financial asset process will evolve as follows:²¹

$$FA_{t+1} = [\alpha_t R_s + (1 - \alpha_t) R_b](FA_t + O_t + T_t - Pe_t - Re_t). \quad (14)$$

Given the assumptions, Norway will run with a projected deficit on its national accounts after 2033, i.e. $O_t + T_t - Pe_t - Re_t < 0$. Thus, Norway’s goal has to be: Collect a large fortune within 2033 and try to cover the increasing deficit by high returns on financial asset.²² In addition, I investigate the effects from a stochastic oil income process, \tilde{O}_t , in Sect. 4.

3.2 The portfolio frontier

In Fig. 2 and Table 6 I show the yearly geometric nominal historical return of the Pension Fund for the time period 1986–2005.²³ The realized geometric real return of the Fund for the years 1997–2005 is 4.56%.²⁴ Finally, returns on World asset classes from Dimson et al. (2004) for the years 1900–2003 shown in

²¹ The government has made a decision rule, which gives the amount Norway can use of the Fund. I choose to disregard it, since I do not believe the government will follow it. Norway does not follow it today, and will probably not do so in the future with lower surplus on its national account.

²² If $FA_t + O_t + T_t - Pe_t - Re_t < 0$, i.e. the country has no cash, I assume Norway borrows abroad at the risk free rate, R_f . I set the real risk free rate equal $R_f - 1 = 1.5\%$. If this situation occurs after 2033 Norway will never balance its budget.

²³ These figures are fictitious since the Fund has only existed since 1997.

²⁴ Measured in terms of the Fund’s currency basket.

Table 6 Historical returns, standard deviations, and correlation coefficients

	Geometric return (%)	Standard deviation (%)	Correlation oil (%)
Stocks	11.2	14.9	0.1
Long-term bonds	9.3	7.9	4.1
Pension fund	10.4	8.3	
Oil	4.1	38.7	
Inflation US	3.0	1.1	
Inflation Norway	3.3	2.2	

The table describes the returns on six indices using data starting in 1986. The returns are nominal yearly return. The composition of the stock and the long-term bond indices are equal to the reference portfolio of the Pension Fund published 09.04. I use data from FTSE for stocks, and Salomon Brothers for long-term bonds. The oil price is Brent Spot. All the data is collected from Datastream

Table 7 Global investment returns (Dimson et al., 2004)

Asset	Geometric return (%)	Standard deviation (%)
Equities	5.7	17.4
Bonds	1.6	10.4
US bills	1.0	4.7

The table describes the returns in US dollars for sixteen-country world equity and bond indices. The time period is 1900–2003 and each country is weighted by its size. US inflation is used to convert the figures into real numbers. Source: (Dimson et al., 2004), p. 161

Table 8 Means, correlation coefficients, and standard deviations

	Means and correlation matrix		
	Geometric means	Correlation	
		Stocks	Long-term bonds
Stocks, S	$\mu_s = 5.0\%$	$\sigma_s = 17\%$	
Long-term bonds, B	$\mu_b = 2.0\%$	$\rho_{s,b} = 0.30$	$\sigma_b = 8.5\%$
Base case strategy 40% stocks and 60% It bonds	$\mu_{bc} = 3.55\%$	$\sigma_{bc} = 9.68\%$	
GMV 10.5% stocks and 89.5% It bonds	$\mu_{gmv} = 2.45\%$	$\sigma_{gmv} = 8.32\%$	

The table describes the returns I use as input to my model. I also show two portfolios, the base case and the global minimum variance (GMV) portfolio

Table 7 are used as a basis for specifying reasonable numbers for the expected returns and the covariance matrix for stocks and long-term bonds (see Table 8). My numbers are in line with investigated figures, but with lower returns. In particular, the stock return is more conservative.

In Fig. 3 the portfolio frontier is plotted. To achieve a 60:40 split between long-term bonds and stocks similar to the strategy of the Pension Fund defined by the Ministry of Finance, I need to set the risk aversion, γ , equal 5. The expected return of this base case strategy, α_{bc} , is $\mu_{bc} = 3.55\%$ and the volatility

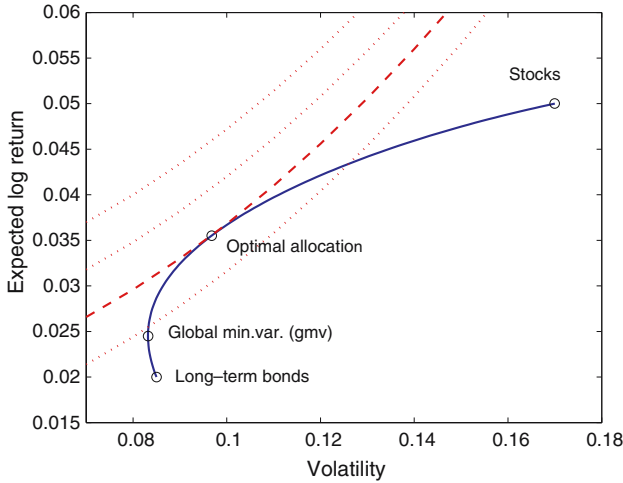


Fig. 3 Portfolio frontier The figure illustrates the portfolio frontier for stocks and long-term bonds. Shorting of stocks and bonds are not allowed. For our objective function I obtain a larger utility going northwest in the figure. The highest utility level given the frontier solves the optimal allocation problem

equal $\sigma_{bc} = 9.68\%$. The return is lower than the return achieved for the years 1997–2005 (4.56%).

3.3 The base case: endowment

Asset allocation

The first asset allocation strategy corresponds to the endowment case. Since not taking the non-tradable assets and liabilities into account, this is a constant proportion strategy. Recall from Eq. (10) that under the set of chosen parameters the allocation equals today’s strategy of the Fund, i.e. $\alpha_t = \alpha = \bar{\alpha} = 40\%$.

Based on simulations, I find that wealth increases for most outcomes. The solid line in Fig. 4a illustrates the expected value of wealth, which is expected to be NOK 17.1 thousand billion in 2050. Figure 4b shows the distribution of wealth for 2050. Table 9 summarizes the findings of this asset allocation strategy. For the last period there is a 2.9% chance of ending up not meeting the liabilities, i.e. $W_{2050} < 0$. The lowest line in Fig. 4a is the 97.5% Monte Carlo VaR.

3.4 Assets and liabilities

Recently, the Norwegian Pension Fund changed name from the Petroleum Fund. However, in its strategy the Fund does not take Norway’s pensions into account. In this section I implement the asset liability model developed in Sect. 2.3.

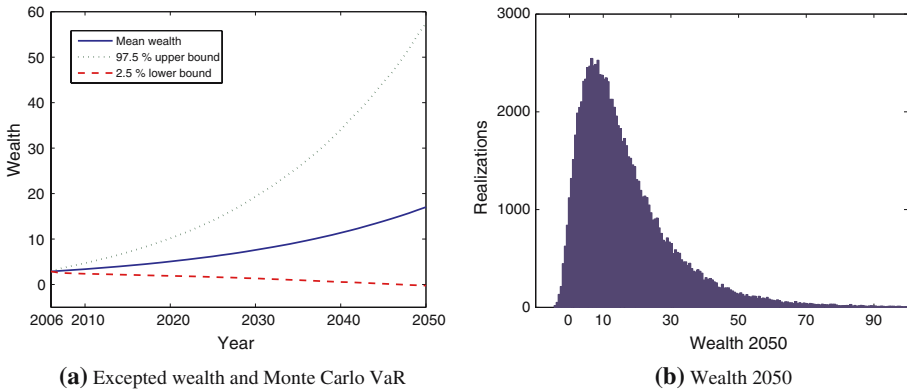


Fig. 4 Wealth **a** Illustrates the development of the wealth over time. One can be 97.5% sure to be above the *dashed line* and correspondingly sure to be below the *dotted line*. **b** Shows the distribution of wealth for 2050. If $FA_{2050} > 4.8$, then $W_{2050} > 0$. All numbers in NOK thousand billion

Table 9 Risk analysis

The table describes some characteristics of the distribution of wealth and financial asset for 2050. I simulate 100000 outcomes of the base case strategy. All numbers in NOK thousand billion

Endowment (base case)	
Mean(W_{2050})	17.1
Std(W_{2050})	15.6
Monte Carlo VaR	17.3
2.5th percentile	-0.2
97.5th percentile	57.5
$Pr(W_{2050} < 0)$	2.9%
$Pr(FA_{2050} < 0)$	0.0%

There are many different non-tradable items on the Norwegian balance sheet. It would be very complicated and little transparent to match all the items with the financial assets, therefore I choose the largest and most important non-tradable item; the pension liability. Even if Norway has large petroleum income, this asset is only one fifth of the pension liabilities in 2050. The surplus on the national account excluding petroleum and pensions, $Ta_t - Re_t$, is also several times less the pension liabilities.²⁵

²⁵ By assuming that the revenue streams from the surplus on the national account and from petroleum activities do not influence the asset allocation strategy, we implicitly assume that $\beta_{ow,t} = \beta_{net,t} = \bar{\alpha}$. In Sect. 3.4 I show formally that this assumption neutralize the effect from these two non-tradable cash-flows.

It is difficult to find a consistent link between both the surplus on the national account and the oil income and the financial assets. Since Norway does not hedge its oil income there is a strong relationship between the petroleum revenues and the oil price, however, the link between the oil price and the asset classes is ambiguous. The main reason for high oil price in the seventies was shortage of oil supply, today the high price is driven by large demand. The stock market performs different in these two situations. In the seventies there was a bear market, while now it is a more flat market. As shown in Table 6 contemporaneous correlation between the oil price and the asset classes shows no significant values. I have tested also for different time intervals and periods. Based on this framework, a third asset the Fund should consider investing in is financial assets that perform well when the oil is depleting, e.g. alternative energy technology.

I let $\beta_{pl,t}$ denote the stock-like fraction of the pension liability. I estimate the relationship by both cointegration and duration matching.

Cointegration

Cointegration is an econometric tool for testing long-run equilibrium. Time-series, all which achieve stationarity after differentiating, can have linear combinations which are stationary in levels. [Cardinale \(2003\)](#) investigates the relationship between pension liabilities and asset prices. With data from the UK, he finds that while short-run correlation evidence is less consistent, there is a consistent long-run link. In this section I will describe a model describing the relationship; pension liabilities and asset prices. The model builds on [Cardinale \(2003\)](#). Furthermore I will do a cointegration analysis similar to [Cardinale \(2003\)](#), but on data relevant for Norway.

[Cardinale \(2003\)](#) assumes that the level of accrued pension liability, PL , is proportional to the current level of the real wage, Y , times an appropriate bond, B :

$$PL \propto YB. \tag{15}$$

In my setting the relationship is between the pension liability of a country and an aggregate wage index, whilst in [Cardinale \(2003\)](#) the relationship is between a particular pension plan and the aggregate wage index. The latter connection may be more suspicious since there is much heterogeneity among the pension plans of different companies. In Norway the system of how pension rights are earned is formulated in terms of a calculation unit. The development of this unit is tied to the average of nominal prices and wages.

Taking logs of Eq. (15), we have

$$pl \propto y + b \tag{16}$$

where $l = \ln(L)$, $y = \ln(Y)$, and $b = \ln(B)$.

Another connection, is the cointegrating relationship:

$$y = \delta + \omega_s s + \omega_b b \tag{17}$$

where s is the level of the equity index, and b the level of the bond portfolio. The estimate of the long-term relationship between wages and stocks is ω_s , and between wages and bonds is ω_b , respectively.

Substituting Eq. (17) into Eq. (16) and rescaling the weights of stocks and bonds such that they sum to one, we achieve the following expression:

$$pl = C + \frac{\omega_s}{\omega_s + \omega_b + 1} s + \frac{\omega_b + 1}{\omega_s + \omega_b + 1} b \tag{18}$$

where C is a constant. If $\omega_s = \omega_b = 0$ then a bond matched portfolio is the best match for the liability in the long-run. Whether this is so in practice is an empirical question. Cointegration allows us to test the joint null hypothesis: $\omega_s = 0, \omega_b = 0$.

The financial assets are given by:

$$FA = \Theta_s S + \Theta_b B \tag{19}$$

where $\Theta_i, i \in (s, b)$ are quantities. We choose Θ_i to minimize the difference between assets and liabilities. Hence:

$$fa - pl = \ln \left[\Theta_s e^s + \Theta_b e^b \right] - \left[C + \frac{\omega_s}{\omega_s + \omega_b + 1} s + \frac{\omega_b + 1}{\omega_s + \omega_b + 1} b \right] \tag{20}$$

which can be used to calculate sensitivities to levels of stocks and long-term bonds and therefore to compute the minimum risk asset allocation. The sensitivities with respect to the levels of stock and bond prices are given by,

$$\frac{\partial(fa - pl)}{\partial s} = \frac{\Theta_s e^s}{FA} - \frac{\omega_s}{\omega_s + \omega_b + 1} = \beta_{pl} - \frac{\omega_s}{\omega_s + \omega_b + 1} \tag{21}$$

$$\frac{\partial(fa - pl)}{\partial b} = \frac{\Theta_b e^b}{FA} - \frac{\omega_b + 1}{\omega_s + \omega_b + 1} = (1 - \beta_{pl}) - \frac{\omega_b + 1}{\omega_s + \omega_b + 1}. \tag{22}$$

The fraction, $\Theta_s e^s / FA$, is equal to the stock-like fraction of the liability, β_{pl} . The minimum level of risk is found by setting these sensitivities to zero. Therefore, the minimum risk long-run asset allocation is:

$$\beta_{pl} = \frac{\omega_s}{\omega_s + \omega_b + 1}, \quad (1 - \beta_{pl}) = \frac{\omega_b + 1}{\omega_s + \omega_b + 1}. \tag{23}$$

In Table 10 I show both [Cardinale \(2003\)](#)'s and my own estimation of Eq. (17). [Cardinale \(2003\)](#) uses UK data, while I use Norwegian wages and international

Table 10 Cointegration

	Cardinale(2003)	My results
Wages		
Intercept	4.50 (0.54)	-0.40 (0.09)
Stocks, ω_s	0.48 (0.08)	0.41 (0.03)
Long-term bonds, ω_b	-0.57 (0.17)	-0.40 (0.06)

Both use real log terms. Cardinale: Quarterly 1964–2002 data. Wages, the Average Earnings Index (AEI). Stocks, FTSE All Share total return index. Long-term bonds, UK 10-year Government Bond total return Index. Døskeland: Yearly 1955–2005 data. Wages, labor cost index in total industry, collected from statistics Norway. The asset classes used are defined within the Ibbotson program as follows: stocks, standard and poor's 500 index (S&P 500). Long-term bonds, U.S. long-term Government Bond total return: Each year, a one-bond portfolio with a term of approximately 20 years and a reasonably current coupon is used

stock and bond indices since the Pension Fund invests in global portfolios. For many different tests and robustness checks, consult [Cardinale \(2003\)](#)'s paper. Here, I only give the results from the main [Engle and Granger \(1987\)](#) estimation. The similarity between the results is evidence for a long-term relationship. There is a positive correlation between real wages and the stock market. Long-term bonds are negative correlated with wages. All variables are significant. The error terms are stationary at 99% level.

As illustrated in Eq. (23), the fraction of stocks is given by β_{pl} . The average figures from both estimations result in $\beta_{pl} = 47\%$ and $1 - \beta_{pl} = 53\%$. The effects of the new allocation strategy will be illustrated together with the duration matching strategy in Sect. 3.4.

Duration matching

An alternative way to identify the long-term relationship between financial assets and the non-tradable pension liability is duration matching. First we find the average life (duration) of the cash-flows for both financial assets and non-tradable items, before we seek the allocation of the financial assets that best matches the liability. Instead of interpretation duration as average life, one can also think about duration as the sensitivity of an item's price to changes in some other variable.

Every item has several variables, I name them *key factors*, j , that influence the value. Similar to [Goodman and Marshall \(1988\)](#), I focus on two factors, inflation, $j = \pi$, and real interest rate, $j = r$. For each financial asset and the pension liability I find the inflation duration, D_π , and the real interest rate duration, $D_{r,t}$.²⁶ I assume all the durations are constant over time, except the real interest rate duration of the liability, $D_{pl,r,t}$. The latter duration changes because the large pension streams are getting closer and closer, which implies a lower duration over time.

Inflation affects the numerator of the valuation of pension liabilities as well as the denominator. Changing just the denominator is almost equivalent to just changing the real interest rate. Thus, the standard duration for liabilities is in fact a real interest rate duration. In Fig. 1 I plotted the cash-flow of Norway's pension liability. If I increase the discount rate by 1%, I find the interest rate duration, $D_{pl,r,t}$, plotted in Fig. 5. The duration decreases over time. Both [Goodman and Marshall \(1988\)](#) and [Siegel and Waring \(2004\)](#) have

²⁶ One can measure the duration of a bond, D_r with respect to the key factor, real interest rates, r . Specifically, modified duration is given by:

$$D_r = -\left(\frac{1}{B}\right)\left(\frac{\partial B}{\partial r}\right), \quad (24)$$

where B is the price of the bond. D_r is the percentage change in the price for a unit change in nominal yield.

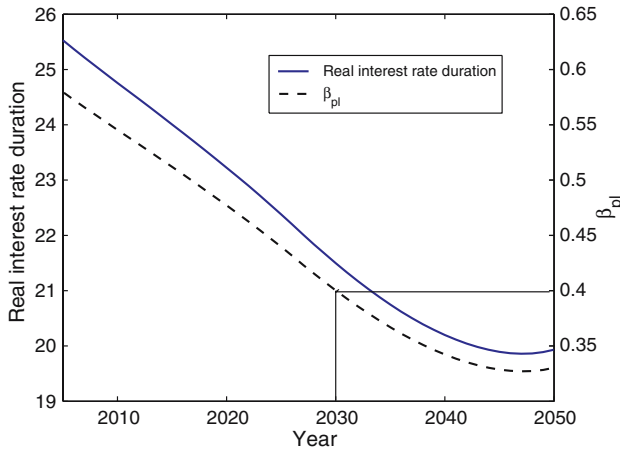


Fig. 5 Real interest rate duration and $\beta_{pl,t}$ for the pension liability The figure illustrates the development of the real interest rate duration and $\beta_{pl,t}$ over time

estimated the inflation interest rate duration for pension liabilities, $D_{pl,\pi}$.²⁷ I use the average of those two durations and set $D_{pl,\pi} = 6$. Figure 6 summarizes the estimated durations.

Before looking at the duration for nominal bonds, recall the connection between nominal interest rates n , expected inflation rate π , and the real interest rate r ; $n = \pi + r$. As pointed out by Siegel and Waring (2004), for nominal bonds the three duration definitions are approximately equal, $D_n \approx D_\pi \approx D_r$. For the price of a nominal bond it is the same whether the change in interest rate arises from changes in inflation or changes in real interest rate. In the mandate for the Pension Fund the modified duration should be between 3 and 7 years. For the last few years, the duration has been about 5. Thus, I set $D_{b,r} = D_{b,\pi} = 5$.

Stocks are often a hedge against inflation. With increasing inflation, the effect tends to be passed over to the customers in the prices of goods and services. Thus, the real value of the company is quite stable. This effect was first pointed out by Bodie (1976). The effect of a changing real discount rate has a larger influence on the present value of the firm, as it would for any long-term cash flows. The estimates for stock duration are not as precise as bond duration.²⁸

²⁷ I need to make the assumption that inflation and real interest rate developments in Norway and internationally are strongly correlated in the long-run. Consequently, I disregard currency risk. A monetary policy following an inflation target reduces the currency risk. In addition, wages in Norway cannot increase over time more than in other countries Norway competes with. As the oil reserves in Norway deplete, the country will not be as sensitive to the oil price as today. All these arguments indicate that the Norwegian economy will increasingly become that of a 'normal' country, i.e. follow international economic cycles.

²⁸ For a further discussion of estimating these durations, check Leibowitz et al. (1989) and Waring (2004). The concept of dual duration was first introduced by Leibowitz et al. (1989). A weakness with duration matching is that one does not adjust for risk. Two streams may have the same durations, but different market risk, see Santa-Clara (2004).

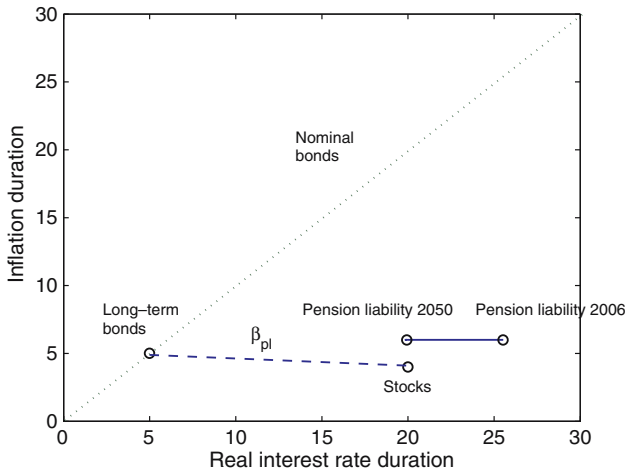


Fig. 6 Duration vector The figure illustrates the dual duration for stocks, nominal bonds, and the pension liability. The latter is illustrated as a *line*, since it is dependent on time. The bond portfolio of the Fund consists of nominal bonds with about 5 years duration, named long-term bonds. The combination of dual durations for stocks and long-term bonds is illustrated with a *dashed line*. The optimal mix of those two assets that best matches the pension liability is $\beta_{pl,t}$

Both Leibowitz et al. (1989) and Waring (2004) use estimates on a real interest rate duration of 20 years and an inflation duration of 4 years.²⁹ I use the same figures for the real interest rate duration for stocks, $D_{s,r} = 20$, and inflation duration, $D_{s,\pi} = 4$.

We find by looking at Fig. 6 that without considering risk, the pension liability is quite stock-like. The best match without risk would be an inflation protected bond with a long duration, e.g. Treasury inflation-indexed securities (commonly called TIPS). The problem with the benchmark of the Pension Fund is that the bonds are nominal and have a too short real interest rate duration.

Formally, the duration of the liability is matched to the duration of the financial assets (for key factor j) by the following equation,

$$D_{pl,j,t} = \beta_{j,t}D_{s,j,t} + (1 - \beta_{j,t})D_{b,j,t}. \tag{25}$$

Thus, the relationship $\beta_{j,t}$ is given by:

$$\beta_{pl,j,t} = \frac{D_{pl,j,t} - D_{b,j,t}}{D_{s,j,t} - D_{b,j,t}}. \tag{26}$$

Equation (26) holds for any key factor j of the liability. As mentioned, in this case there are two key factors, thus, I let the final $\beta_{pl,t}$ be a weighted sum of β_{π}

²⁹ A more advanced model could calculate different numbers for different stock indices, e.g. value and growth stocks as in Lettau and Wachter (2007).

and $\beta_{r,t}$:

$$\beta_{pl,t} = w\beta_{pl,r,t} + (1 - w)\beta_{pl,\pi} \tag{27}$$

where w denotes the subjective significance one put on the first factor, and $(1 - w)$ denotes the weight on the last key factor. I seek the closest match between the pension liability and a combination of stocks and long-term bonds. It is not the closest project since one weights the two dimensions with w . In today's monetary policy regime the inflation risk is not very large, thus w is quite high. I set it equal $2/3$ and then achieve a stock-like fraction of pension liability, $\beta_{pl,t}$, as plotted in Fig. 5, i.e. $\beta_{pl,2006} = 0.57$ and $\beta_{pl,2050} = 0.33$. Thus, over time the pension liability becomes more bond-like than stock-like. In the next section I show that a time-varying beta imposes a dynamic asset allocation.

Asset allocation

Two techniques have been used to find a relationship between the pension liability and the financial assets. For the cointegration technique the stock-like fraction of the liability, $\beta_{pl,t}$, is constant, equal to 47%. As shown in Fig. 5, duration matching gives us a time-varying relationship. In this section I will show the implications the different relationships have on the asset allocation strategy.

I find the proportion of financial assets invested in stocks α_t by calibrating Eq. (13) from Sect. 2.3 to the Norwegian case:

$$\alpha_t = \bar{\alpha} + \frac{1}{FA_t} [(\bar{\alpha} - \beta_{ow,t})OW_t + (\bar{\alpha} - \beta_{net,t})NET_t - (\bar{\alpha} - \beta_{pl,t})PL_t]. \tag{28}$$

Since I assume that the stock-like fraction of oil wealth and the surplus on the national account excluding petroleum revenue and pension expenditures, are similar to the constant proportion optimal asset allocation, $\beta_{ow,t} = \beta_{net,t} = \bar{\alpha}$, Eq. (28) simplifies to:

$$\alpha_t = \bar{\alpha} - \frac{(\bar{\alpha} - \beta_{pl,t})PL_t}{FA_t}. \tag{29}$$

The input parameters to Eq. (13) are constant, thus $\bar{\alpha}$ is still 40%. Since both PL_t and FA_t are always larger than zero, $\beta_{pl,t} > \bar{\alpha} = 40\%$, results in a larger allocation to stocks than the Endowment case, and vice versa. Hence, for the cointegration strategy the allocation to stocks will always be larger than the Endowment case. In contrast, for the duration matching case the allocation to stocks will be larger than the Endowment case before 2030 when $\beta_{pl,t} > \bar{\alpha} = 40\%$ and opposite afterwards.³⁰

Since the asset and liability strategies are functions of the financial assets (FA_t), the strategies have a distribution. By simulation, I can find the mean

³⁰ I still restrict α_t to be at the *efficient* frontier, thus the admissible investment area is between the global minimum variance (GMV) portfolio and 100% in stocks, $\alpha_t \in (\alpha_{gmw}, 100\%)$. If the allocation policy, Eq. (29), implies an allocation outside this area, I set the allocation equal either GMV or 100% in stocks.

allocations for all strategies (plotted in Fig. 7). Both asset and liability strategies impose that the Fund should start out with a high allocation to stocks and reduce it over time.

In Fig. 8, I compare the development of expected wealth for the three strategies. In addition the 2.5th and the 97.5th percentiles are plotted. Base case is

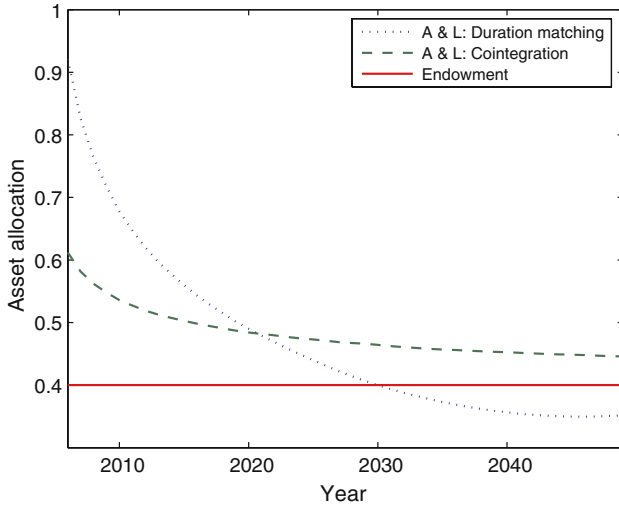


Fig. 7 Mean optimal allocation. This figure illustrates the mean allocation to stocks and long-term bonds for the Endowment strategy and the two Asset and Liability strategies; Duration matching and cointegration

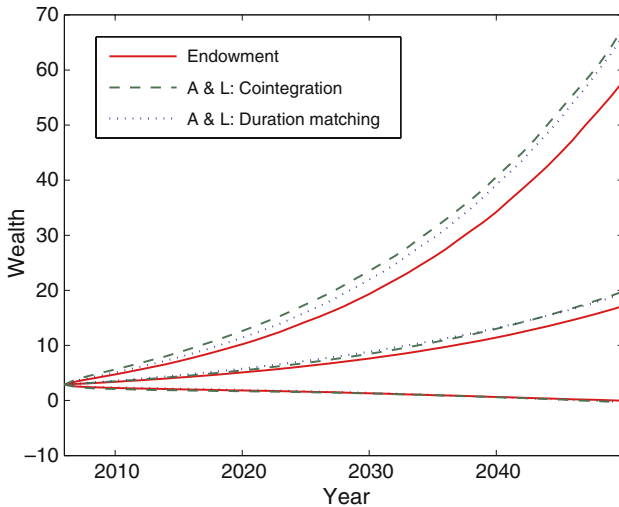


Fig. 8 Mean financial asset. This figure illustrates for all strategies, the simulated 97.5% percentile, the mean allocation, and the 2.5% percentile of wealth. The numbers are in NOK thousand billion

the solid lines, while the new ones are the dashed and dotted lines. The A & L strategies have a higher 97.5th percentile than base case. The expected wealth also gradually increases more for these strategies. The downside bounds are about the same for all cases. Table 11 shows that the asset and liability strategies give a higher mean wealth for 2050 than the Endowment strategy. The main reason is that these strategies give us the capacity to allocate more into stocks. However, these strategies also have the largest standard deviation. But since wealth is not normally distributed, standard deviation is not a good measure of risk.

In Fig. 9, I plot the empirical cumulative distribution function for wealth for 2050. Not surprisingly, the asset and liability strategies give the highest expected wealth. As shown in Fig. 9a, the large upside risk is part of the reason for the high mean and standard deviation of the A & L strategies. I do not find much

Table 11 Risk analysis

	Endowment	A & L:	
		Cointegration	Duration Matching
Mean(W_{2050})	17.1	19.8	19.4
Std(W_{2050})	15.6	17.9	18.5
Monte Carlo VaR	17.3	19.9	19.7
2.5th percentile	-0.2	-0.1	-0.3
97.5th percentile	57.5	66.3	67.8
$Pr(W_{2050} < 0)$	2.9%	2.6%	3.0%
$Pr(FA_{2050} < 0)$	0.0%	0.0%	0.0%
Mean($\text{mean}(\alpha)_t$)	40.0%	48.1%	47.0%

The table compares some characteristics of the three asset allocation strategies. All the measures are for the distribution of wealth and the financial asset in 2050. The numbers are in NOK thousand billion

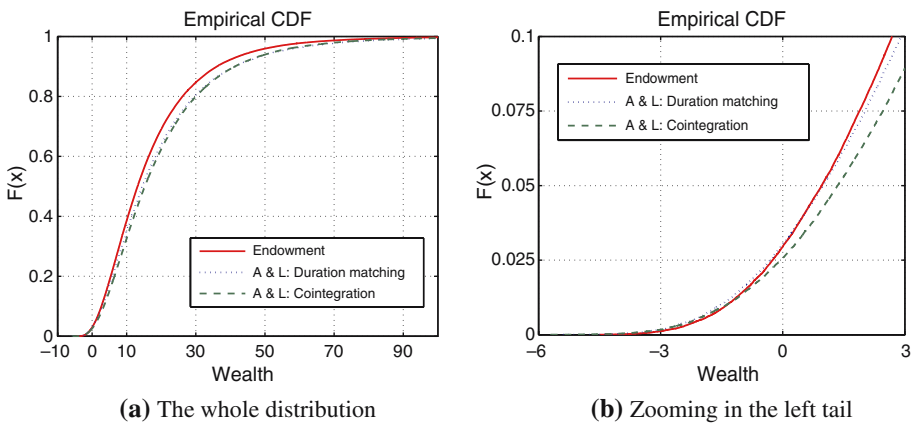


Fig. 9 Empirical cdf for financial asset 2050. This figure illustrates the cumulative distribution function of the financial asset year 2050. **a** Shows the whole distribution and **b** shows the left tail

difference between the strategies when I compare the downside risk, see Fig. 9b. If one uses Monte Carlo VaR to compare the strategies, one has to be careful. With different expected wealth, there are different starting points for the VaR values. All in all, the A & L strategies are better than Norway's strategy today.

4 Does Norway have problems meeting its liabilities?

There are a lot of factors, both influenceable and not influenceable, that affect the development of the nation's wealth. The government of Norway has mainly three alternative actions to manage its balance sheet. The first is to increase taxes, i.e. increase $\{Ta_t\}$. The second is to cut costs, e.g. the pension expenditures. As mentioned earlier, the Norwegian parliament has agreed to modernize the National Insurance Scheme. The system will lower the expenditures. The two first actions are in some sense the same. A lower pension can be viewed as an increased tax. The returns in the financial markets and the oil price are two variables Norway cannot influence, however, the third alternative; take on more risk, is a decision variable. In this section I do a sensitivity analysis with respect to all these important variables.

Panel a in Table 12 shows the sensitivity analysis with respect to the pension liability. So far I have assumed the government is able to carry through the pension reform. With or without increased taxes, the shortfall probability is low. However, it is often not that simple, since high taxes reduce the incentives to work, which again is followed by reduced tax revenues. High taxes may also cause high unemployment rates because many companies move out of the country. All this leads to a larger shortfall probability. If the government of Norway is not able to carry through the reform the shortfall probability is 21%.

The second factor I vary is the asset allocation. Unchanged asset allocation is similar to the results from the base case in panel a. In panel b I calculate three new alternatives. In the first Norway allocates everything to a risk free asset.³¹ The country will be sure of meeting its obligations. However, this is not a realistic alternative since one does not take the duration matching dimension into account. The GMV portfolio is the least risky asset allocation given that one invests on the portfolio frontier. In this alternative, the probability of a wealth lower than zero is 9.0%. With an "all-in" strategy, $\alpha = 100\%$, the shortfall probability is 5.0%. Thus, if Norway's goal is to minimize the probability of ending up with negative wealth and investing in risky assets, the optimal allocation to stocks is between the GMV and the stock portfolio. A possible solution is the base case since $Pr(W_{2050} < 0) = 2.9\%$.

Finally, panel c shows that the development of Norway's wealth is dependent on the returns from the financial markets. In addition to the base case

³¹ I assume the same return on the risk free asset as the rate Norway can borrow at, $R_f - 1 = 1.5\%$.

Table 12 Sensitivity analysis

Different pension expenditures, PL_t									
Status	Gradually reform and higher taxes			Base case, gradually reform			No reform		
	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D
Panel a:									
Mean(W_{2050})	25.1	24.1	27.4	17.1	19.8	19.4	10.6	15.9	12.9
Std(W_{2050})	16.6	18.9	19.2	15.6	17.9	18.5	14.6	16.8	17.3
Monte Carlo VaR	18.9	21.4	21.1	17.3	19.9	19.7	15.8	18.3	18.3
$Pr(W_{2050} < 0)$	0.0%	0.4%	0.0%	2.9%	2.6%	3.0%	20.6%	8.1%	18.9%
$Pr(FA_{2050} < 0)$	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.3%	0.4%
Different constant asset allocation strategies, α									
	100% in R_f			α_{gmv}			$\alpha = 100%$		
	End. = A&L			End. = A&L			End. = A&L		
Panel b:									
Mean(W_{2050})	2.1			7.9			57.4		
Std(W_{2050})	0			7.8			94.1		
Monte Carlo VaR	0			9.6			59.2		
$Pr(W_{2050} < 0)$	0.0%			9.0%			5.0%		
$Pr(FA_{2050} < 0)$	0.0%			0.0%			0.1%		
Change in returns, μ_s and μ_b									
R^P	Base case, $\mu_s = 5.0\%, \mu_b = 3.0\%$			1% lower return than base case			2% lower return than base case		
	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D
Panel c:									
Mean(W_{2050})	17.1	19.8	19.4	9.2	11.3	10.7	4.0	5.7	5.0
Std(W_{2050})	15.6	17.9	18.5	10.3	12.0	12.1	6.8	8.1	8.0
Monte Carlo VaR	17.3	19.9	19.7	11.4	13.4	13.0	7.5	9.1	8.6
$Pr(W_{2050} < 0)$	2.9%	2.6%	3.0%	11.3%	8.8%	10.9%	29.6%	22.6%	27.7%
$Pr(FA_{2050} < 0)$	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.2%

The table describes factors that influence the distribution of wealth in 2050. End. is short for the endowment case, A & L: C is short for the cointegration strategy, and A & L: D is short for duration matching alternative. All numbers in NOK thousand billion

scenario I investigate what happens if the return on stocks and bonds will be 1% or 2% lower than base case.³² For the endowment case the probability of not meeting the pension liabilities increases from 2.9 to 29.6%, if the return on financial asset decreases by 2%. The high sensitivity illustrates that Norway is very dependent on growth of the world economy. Even if I assume that the country takes prices as given in the model, the analysis shows that Norway has large incentives to arrange for growth in the world economy. It is also important that the Pension Fund continues to do a good job with active portfolio management;

³² The volatility is kept constant for all outcomes.

from 1998 to 2005 the Fund made an excess return of 0.50 yearly percentage points.

The oil price process

So far in the analysis I have treated the oil revenues, $\{O\}$, as an aggregate deterministic income stream. In the following analysis I split the stream into two parts, a quantity factor, Q_t , and a price factor, P_t , thus $O_t = Q_t P_t$. This decomposition is also a simplification. I assume the quantity path is deterministic.³³ For the oil price I will show results for both a deterministic and a stochastic price process. In panel a in Table 13 I illustrate the three different deterministic price levels shown in Fig. 1. In this setting $P(t)$ could be interpreted as the forward curve. Variability in the cash-flow from the petroleum sector has a large impact on Norway's future wealth. Norway's pension problems can be significantly reduced if the oil price continues to be as high as today. But a high oil price may cause low growth in the world economy, something that can lower the value of the financial asset. If this paper had been written with the projected oil price from last year's National Budget (USD 26), the conclusion would be that Norway with a 30% probability would not have met its liabilities, and not 3% as today.

When I allow for a stochastic oil price, \tilde{p}_t , I need to identify a reasonable price process. Schwartz (1997) investigates different models for the oil price process. I will, similarly to Trovik (2004a), implement the simplest model in Schwartz (1997). We assume the oil price follows an Ornstein-Uhlenbeck (mean-reverting) process. The discretized log of the price, \tilde{p}_t , for 1 year time interval is given by:

$$p_{t+1} = \theta(1 - e^{-\kappa}) + p_t e^{-\kappa} + \sqrt{\frac{1 - e^{-2\kappa}}{2\kappa}} \sigma_o \varepsilon \quad (30)$$

where ε is a standard normal variable, κ is the speed of reversion to the long-term level, θ , and σ_o is the instantaneous volatility.

As a base case in panel b in Table 13 I use the same parameters as Trovik (2004a), which again builds on Schwartz (1997) (the different parameters are shown in panel b–d in Table 13). The shortfall probability does not increase very much going from a deterministic model in panel a to a stochastic model in panel b. The main reason is that the process is quite stable (the half-life is $\ln(2)/\kappa = 1.73$). When I increase the volatility in panel c we see that the shortfall probability increases. As argued earlier in the paper it is hard finding a correlation between the oil price and the financial assets. In panel b and c, I assume a zero correlation; in the final panel d, I assume as a stress test, a correlation coefficient between the oil price and the financial assets, $\rho_{s,o}$, $\rho_{b,o}$ at

³³ In the real world the amount of oil and gas is uncertain. For example, it is possible that Norway will find more oil and gas than expected. For a discussion of Norway's Petroleum Revenue, see Trovik (2004a).

Table 13 Sensitivity analysis oil price

Different petroleum income, OW_t									
Oil price	Base case, $P_t = \text{USD}50$			$P_t = \text{USD}33$			$P_t = \text{USD}26$		
	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D
Panel a:									
Mean(W_{2050})	17.1	19.8	19.4	9.2	12.0	11.4	5.8	8.5	8.0
Std(W_{2050})	15.6	17.9	18.5	11.8	13.8	14.3	10.0	12.0	12.6
$Pr(W_{2050} < 0)$	2.9%	2.6%	3.0%	16.8%	11.8%	14.8%	30.4%	21.7%	26.2%
$Pr(FA_{2050} < 0)$	0.0%	0.0%	0.0%	0.4%	0.3%	0.4%	2.3%	1.7%	2.6%
Oil price	Base case, $E(\tilde{P}_t) = \text{USD}50$			$E(\tilde{P}_t) = \text{USD}33$			$E(\tilde{P}_t) = \text{USD}26$		
	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D
$\sigma_o = 30\%, \rho_{s,o} = \rho_{b,o} = 0, \kappa = 0.40, \theta = \ln P$									
Panel b:									
Mean(W_{2050})	17.2	20.0	19.9	9.2	11.9	11.4	5.8	8.4	8.0
Std(W_{2050})	16.2	18.5	18.9	11.7	13.9	14.5	9.9	12.1	12.7
$Pr(W_{2050} < 0)$	3.8%	3.1%	3.3%	18.8%	14.3%	16.6%	32.3%	24.9%	27.6%
$Pr(FA_{2050} < 0)$	0.0%	0.0%	0.0%	1.4%	0.7%	1.0%	3.3%	2.8%	3.3%
Oil price	Base case, $E(\tilde{P}_t) = \text{USD}50$			$E(\tilde{P}_t) = \text{USD}33$			$E(\tilde{P}_t) = \text{USD}26$		
	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D
$\sigma_o = 70\%, \rho_{s,o} = \rho_{b,o} = 0, \kappa = 0.40, \theta = \ln P$									
Panel c:									
Mean(W_{2050})	17.3	20.1	19.6	9.0	11.8	11.3	5.6	8.3	7.8
Std(W_{2050})	18.2	20.2	20.6	12.9	15.0	15.4	10.8	12.9	13.4
$Pr(W_{2050} < 0)$	6.2%	4.8%	6.0%	21.4%	16.0%	19.1%	33.9%	25.3%	29.4%
$Pr(FA_{2050} < 0)$	0.1%	0.1%	0.2%	1.6%	1.4%	2.1%	4.8%	3.7%	5.3%
Oil price	Base case, $E(\tilde{P}_t) = \text{USD}50$			$E(\tilde{P}_t) = \text{USD}33$			$E(\tilde{P}_t) = \text{USD}26$		
	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D	End.	A&L:C	A&L:D
$\sigma_o = 30\%, \rho_{s,o} = \rho_{b,o} = 0.5, \kappa = 0.40, \theta = \ln P$									
Panel d:									
Mean(W_{2050})	17.9	20.7	20.3	9.4	12.2	11.7	5.9	8.6	8.2
Std(W_{2050})	18.0	20.2	20.8	13.0	15.2	15.8	10.9	13.2	13.4
$Pr(W_{2050} < 0)$	5.1%	4.2%	5.2%	19.9%	15.1%	18.4%	32.8%	24.8%	28.8%
$Pr(FA_{2050} < 0)$	0.0%	0.0%	0.0%	1.2%	1.2%	1.7%	4.3%	3.5%	5.0%

The table describes how different alternatives of the oil price influence the distribution of wealth in 2050. End. is short for the endowment case, A & L: C is short for the cointegration strategy, and A & L: D is short for duration matching alternative. All numbers in NOK thousand billion

0.5. Even if a positive correlation is worse than zero correlation, this is not as bad as the high volatility scenario.

Not surprisingly, I find that Norway is very sensitive to the oil price. Therefore Norway should think through how to hedge against lower prices. The effect of the asset allocation strategy is not as important as the consequences of oil price changes. However, the asset allocation is a policy variable while the oil price is (almost) given.

5 Conclusion

In this paper I first show that when a country takes its non-tradable assets and liabilities into account, the resulting asset allocation strategy changes over time. Second, I purpose and estimate two alternative ways (cointegration and duration matching) of identifying the long-term relationship between the financial assets and the non-tradable pension liability. Applied to Norway, both techniques suggest that Norway should increase its holdings in stocks (relative to the strategy today) and reduce the allocation to stocks over time. Finally, I try to answer the important questions for Norway; is the social security system in Norway trustworthy? I find that the social security system is trustworthy if three conditions are satisfied; lasting high oil price, high growth in the world economy and finally that Norway succeeds in reforming its pension system. The likelihood that the two first conditions are jointly satisfied is small. If one or more of these conditions fail, the chances for Norway to meet its pension liability are dangerously close to fifty–fifty.

There are several ways of extending the model, e.g. households could have been included. A model based on Bodie et al. (1992), I think, is a good starting point for a more complex and realistic macroeconomic model. However, this expansion is left for future work.

Acknowledgements For helpful comments and suggestions, I thank two anonymous referees, Steinar Ekern, Michael Genser, Thore Johnsen, Tim Kehoe, Svein-Arne Persson, Kjetil Storesletten and Per stberg.

Appendix A: An approximation of the log portfolio return

I start out with Eq. (2). The gross portfolio return is given by

$$R_p = \alpha_t R_s + (1 - \alpha_t) R_b. \quad (31)$$

This can be rewritten as

$$\begin{aligned} \frac{R_p}{R_b} &= 1 + \alpha_t \left(\frac{R_s}{R_b} - 1 \right) \\ \ln \frac{R_p}{R_b} &= \ln \left(1 + \alpha_t \left(\frac{R_s}{R_b} - 1 \right) \right) \\ r_p - r_b &= \ln(1 + \alpha_t (\exp(r_s - r_b) - 1)). \end{aligned} \quad (32)$$

In Eq. (32), there is a nonlinear relation between the log excess return on stocks over long-term bonds, $(r_s - r_b)$, and the log excess returns on the portfolio over long-term bonds, $(r_p - r_b)$. I approximate this relation with a second-order Taylor expansion around $(r_s - r_b)$. I define a function $f(r_s - r_b)$:

$$f(r_s - r_b) = \ln(1 + \alpha_t (\exp(r_s - r_b) - 1)). \quad (33)$$

The derivatives of f is given by $f'_t(0) = \alpha_t$ and $f''_t(0) = \alpha_t(1 - \alpha_t)$. Inserting the result into the Taylor function, I evaluate the function f at $(r_s - r_b) = 0$:

$$\begin{aligned} f(r_s - r_b) &\approx f(0) + f'(0)(r_s - r_b) + \frac{1}{2}f''(0)(r_s - r_b)^2 \\ &= \alpha_t(r_s - r_b) + \frac{1}{2}\alpha_t(1 - \alpha_t)(r_s - r_b)^2. \end{aligned} \quad (34)$$

Appendix B: Mean and variance for the log portfolio return

The log return is given by:

$$r_{p,t+1} \approx \alpha_t r_s + (1 - \alpha_t)r_b + \frac{1}{2}\alpha_t(1 - \alpha_t)(r_s - r_b)^2. \quad (35)$$

Mean log portfolio

When taking the expected log portfolio return, I am cautious with the term $(r_s - r_b)^2$ in Eq. (35). [Campbell and Viceira \(2002a\)](#) replace the term $(r - r_f)^2$ (the return on the risky asset over the risk free asset) by σ^2 . Similarly to [Persson \(2004\)](#), I am more careful taking the expectation. Using the covariance identity the expectation of the last term in Eq. (35) can be written as:

$$\begin{aligned} E(r_s - r_b)^2 &= E(r_s^2) - 2E(r_s r_b) + E(r_b)^2 \\ &= (\mu_s - \mu_b)^2 + \sigma_s^2 + \sigma_b^2 - 2\text{cov}(r_s, r_b). \end{aligned} \quad (36)$$

Inserting Eq. (36) into Eq. (35) give us the following expression for expected log portfolio return

$$\mu_{p,t+1} \approx \mu_b + \alpha_t(\mu_s - \mu_b) + \frac{1}{2}\alpha_t(1 - \alpha_t)[(\mu_s - \mu_b)^2 + \sigma_s^2 + \sigma_b^2 - 2\text{cov}(r_s, r_b)]. \quad (37)$$

Variance log portfolio

$$\begin{aligned} \text{Var}(r_p) &\approx \alpha_t^2 \text{Var}(r_s) + (1 - \alpha_t)^2 \text{Var}(r_b) + 2\alpha_t(1 - \alpha_t) \text{Cov}(r_s, r_b) \\ &\quad + \frac{1}{4}\alpha_t^2(1 - \alpha_t)^2 \text{Var}(r_s - r_b)^2 + \alpha_t^2(1 - \alpha_t) \text{Cov}(r_s, (r_s - r_b)^2) \\ &\quad + \alpha_t(1 - \alpha_t)^2 \text{Cov}(r_b, (r_s - r_b)^2) \end{aligned} \quad (38)$$

Calculate: $\text{Var}(r_s - r_b)^2$. Define $r_d = r_s - r_b$, $\mu_d = \mu_s - \mu_b$, $\sigma_d^2 = \sigma_s^2 - \sigma_b^2 + 2\text{Cov}(s, b)$

$$r_d = \sigma_d X \sim N(\mu_d, \sigma_d^2), \quad r_d^2 = \sigma_d^2 X^2 \tag{39}$$

$$\text{Var}(r_d^2) = \sigma_d^4 \text{Var}(X^2) = \sigma_d^4 2(r + 2\mu_d) = \sigma_d^4 2(1 + 2\mu_d) \tag{40}$$

In the last equality, I use that X^2 is chi-squared $\chi^2(1, \mu_d)$, and that the variance is given by $\text{Var}(X^2) = 2(r + 2\mu_d)$ where r is the number of random variables (labelled the degrees of freedom). Thus, $\text{Var}(r_s - r_b)^2 = 2(1 + 2(\mu_s - \mu_b))(\sigma_s^2 + \sigma_b^2 + 2\text{Cov}(r_s, r_b))^2$.

Then I can calculate expression (38) except the last two terms.

I use the same method for both terms. Let r_i denote asset i where $i \in (s, b)$. r_d is still $r_s - r_b$. Define z as the zero mean variate, $r - \mu$.

$$\begin{aligned} \text{Cov}(r_i, r_d^2) &= \text{Cov}(r_i - \mu_i, (r_d - \mu_d + \mu_d)^2) = \text{Cov}(z_i, (z_d + \mu_d)^2) \\ &= \text{Cov}(z_i, z_d^2 + 2z_d\mu_d + \mu_d^2) = \text{Cov}(z_i, z_d^2) + 2\mu_d \text{Cov}(z_i, z_d) \\ &= 2\mu_d \text{Cov}(z_i, z_d) = 2(\mu_s - \mu_b) \text{Cov}(r_i, r_s - r_b) \end{aligned} \tag{41}$$

$\text{Cov}(z_i, z_d^2)$ is equal to zero because of the symmetry of the normal distribution, more explicit I can write $\text{Cov}(z_i, z_d^2) = E(z_i z_d^2) - E(z_i)E(z_d)^2 = 0$.

Setting $i = s$ I get

$$\text{Cov}(r_s, (r_s - r_b)^2) = 2(\mu_s - \mu_b)(\sigma_s^2 - \text{Cov}(r_s, r_b)), \tag{42}$$

and correspondingly if $i = b$:

$$\text{Cov}(r_b, (r_s - r_b)^2) = 2(\mu_s - \mu_b)(\text{Cov}(r_s, r_b) - \sigma_b^2). \tag{43}$$

The solution is then:

$$\begin{aligned} \text{Var}(r_p) &\approx \alpha_t^2 \sigma_s^2 + (1 - \alpha_t)^2 \sigma_b^2 + 2\alpha_t(1 - \alpha_t) \text{Cov}(r_s, r_b) \\ &\quad + \frac{1}{2} \alpha_t^2 (1 - \alpha_t)^2 (1 + 2(\mu_s - \mu_b))(\sigma_s^2 + \sigma_b^2 + 2\text{Cov}(r_s, r_b))^2 \\ &\quad + 2\alpha_t^2 (1 - \alpha_t)(\mu_s - \mu_b)(\sigma_s^2 - \text{Cov}(r_s, r_b)) \\ &\quad + 2\alpha_t(1 - \alpha_t)^2 (\mu_s - \mu_b)(\text{Cov}(r_s, r_b) - \sigma_b^2) \end{aligned} \tag{44}$$

I calculate how much each term constitutes to the total variance. α_t is set equal 0.5. Then the correction terms have largest impact on the total variance. I use the same figures as in the paper.

Term	% of total Var	% of total Var accumulated
$\text{Var}(r_p)$	100	
$\alpha_t^2 \text{Var}(r_s)$	63.48	63.5
$(1 - \alpha_t)^2 \text{Var}(r_b)$	15.81	79.3
$2\alpha_t(1 - \alpha_t)\text{Cov}(r_s, r_b)$	19.12	98.4
$\frac{1}{4}\alpha_t^2(1 - \alpha_t)^2 \text{Var}(r_s - r_b)^2$	0.22	98.6
$\alpha_t^2(1 - \alpha_t)\text{Cov}(r_s, (r_s - r_b)^2)$	1.62	100.2
$\alpha_t(1 - \alpha_t)^2 \text{Cov}(r_b, (r_s - r_b)^2)$	-0.19	100

If R_i had been normally distributed, then I would have an expression only consisting of the three first terms in (38). As we see in Table 5 these three terms consist of most of the variance. They constitute 98.4% of the variance.

Since the correct expression of the variance is very complicated, and the three first terms cover most of the variance, I use the three first terms in a simplified expression for the variance:

$$\text{Var}(r_p) \approx \alpha_t^2 \sigma_s^2 + (1 - \alpha_t)^2 \sigma_b^2 + 2\alpha_t(1 - \alpha_t)\text{Cov}(r_s, r_b). \quad (45)$$

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