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# ANALYZING RISK OF STOCK COLLAPSE IN A FISHERY UNDER STOCHASTIC PROFIT MAXIMIZATION

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## Abstract

*In commercial fisheries, stock collapse is an intrinsic problem caused by overexploitation or due to pure stochasticity. To analyze the risk of stock collapse, we apply a relatively simple Monte Carlo approach which can capture complex stock dynamics. We use an economic model with downward sloping demand and stock dependent costs. First, we derive an optimal exploitation policy as a feedback control rule and analyze the effects of stochasticity. We observe that the stochastic solution is more conservative compared to the deterministic solution at low level of stochasticity. For moderate level of stochasticity, a more myopic exploitation is optimal at small stock and conservative at large stock level. For relatively high stochasticity, one should be myopic in exploitation. Then, we simulate the system forward in time with the optimal solution. In simulated paths, some stock recovered while others collapsed. From the simulation approach, we estimate the probability of stock collapse and characterize the long term stable region.*

**Key words:** Stochasticity, Ensemble Kalman filter, Stock Collapse, Probability

**JEL Classification Codes** C61, Q22, Q57

## **Introduction**

Fear of rapid depletion of the world fish stocks is increasing and fisheries collapse is considered to be the result of purely economic or administrative mismanagement (Mullon et al 2005). The equilibrium theories hypothetically ensure that the fishing effort is maintained at, or below, a reasonable value, such that collapse should never occur. Nevertheless, it does. The rate of collapse has been stable since the 1950s indicating no improvement in the fisheries management (Mullon et al 2005). The bioeconomic models, which date back to the 1950s with the pioneering work of Gordon (1954), Schaefer (1954) and later works (see for example Clark 1973; Clark and Munro 1975; Hannesson 1975; Clark 1990), mainly focus on the deterministic framework and do not incorporate uncertainties in the resource growth. However, it is well perceived that the exploitation decisions based upon a deterministic model are suboptimal for the fish resources that are inherently stochastic in nature. Such decisions lead to a problem of overexploitation (Roughgarden and Smith 1996). The problem of overexploitation in fisheries is estimated to have increased over the last few decades despite substantial effort to improve management. FAO (2010) reported that the proportion of overexploited fisheries has increased from 10 percent in 1974 to 28 percent in 2008. The increasing trend of overexploitation makes the threat of collapses in fisheries worldwide more imminent. Overexploitation has already resulted in species collapse in several cases. Some species have been suppressed to such low levels that it no longer makes sense to continue commercial exploitation. The collapse of the Newfoundland North Atlantic cod during the 1990s (Hannesson 1996), the Norwegian herring during the 1960s (Lorentzen and Hannesson 2004), the Barents Sea capelin during the 1980s (Tereshchenko 2002) are some of the examples of the commercial collapse cases in point. The first two cases probably resulted from overexploitation; the last case resulted from stochastic events (Gjøsæter et al 2009).

Deterministic models provide poor guidance for the management of stochastic stocks even in the case of risk neutrality and in constant price conditions (Hannesson 1987). In other

words, it is crucial to understand the stochasticity in fish stock dynamics. The analysis of stochasticity in bioeconomic models began in the 1980s from the work of Reed (1979), Ludwig (1980), Lewis (1981), and Pindyck (1984) among many others. In recent years, further emphasis has been put on developing models for optimal management of these stochastic natural resources (Nøstbakken 2006; Agnarsson et al 2008; Sarkar 2009). Although the number of studies in bioeconomic modeling that include the stochastic dynamics are increasing, they are still not adequate. Many of these stochastic models are linear, resulting in bang-bang solutions or most rapid approach paths which are not always useful for practical purposes (Sandal and Steinshamn 1997b). Reality is, after all, not linear. We also think that the understanding of uncertainty in fish stock dynamics is limited. The challenge is at least two-fold: the quantification of the stochasticity in the stock dynamics and developing appropriate management plans accordingly.

There have been increasing efforts to analyze the risk of collapse in fisheries to improve the management and avoid the probability of the stock collapse (for example see Johnston and Sutinen 1996; Myers et al 1997; Hutchings 2000; Jonzén et al 2002; Escudero et al 2004; Hutchings and Reynolds 2004; Mullon et al 2005; Mitra and Roy 2006) . Although a complete closure or a substantial reduction of the fishing mortality is suggested for the recovery of a collapsed stock, not all stocks recover (Hutchings 2000; Hutchings and Reynolds 2004). In contrast to the perception that marine fish stocks are highly resilient to large population reductions, Hutchings (2000) claims that there is little evidence of rapid recovery from prolonged declines. *Jonzén et al (2002)* analyzed the risk of collapse in the Baltic cod fishery and concluded that a substantial reduction in the exploitation level is the only way of avoiding the overexploitation and consequently the risk of collapse. Similarly, Hutchings and Reynolds (2004) analyzed the consequences for recovery and risk of extinction of a collapsed fishery. They found that the reductions in fishing pressure, although clearly necessary for population recovery, are often insufficient for the recovery of a collapsed stock.

Most of the literature in fishery economics focused on the analysis of stock collapse due to overexploitation under uncertain environment and concluded that overexploitation is the single cause of the stock collapse. There is a lack of understanding of the ‘level of stochasticity’ and ‘stock collapse’. Mullon et al (2005) found that the stochastic shocks in the stock dynamics can drive the collapse of the stock even without the harvest. Acknowledging that there is stochasticity in the dynamics of a fish stock, one has a situation where the fish stock can collapse even without any fishing pressure (Field et al 2009). Collapse can occur for any initial stock level, but its probability decreases with increasing stock level. Similarly, the probability of collapse also depends on the strength of the stochastic noise even for an optimally exploited stock.

The quantification of the stochasticity in a non-linear model is a difficult task (Kugarajh et al 2006). In addition, selection of an appropriate estimation method is always pertinent to the accuracy of the estimated parameters. We employed a Kalman filter type approach to estimate drift and diffusion parameters simultaneously. First, we specified a biomass surplus production model for the fishery formulated as a stochastic differential equation. The stochastic term is geometric, which means that the size of the stochastic term increases with the stock level. Next, we estimated parameters with the ensemble Kalman filter. The ensemble Kalman filter is a data assimilation method which generalizes the Kalman filter to a large class of nonlinear models (Evensen 2003). A key part of the method is the use of a Markov chain Monte Carlo approach to solve the time evolution of the stochastic differential equation. It fits the stochastic differential equation to observations on the stock level and on catches in a sequential manner. The method lets us estimate time varying parameters such that a relatively simple model can capture complex stock dynamics. The economic parameters in the study were adopted from Agnarsson et al (2008).

With the specified bioeconomic model, we calculated the optimal harvest profile as a feedback control rule (Sandal and Steinshamn 1997b; Sandal and Steinshamn 1997a; Sandal and Steinshamn 2001). In a feedback approach, the control variable is a deterministic function of the state variable. In contrast to the commonly used time path

approach, the feedback approach (closed-loop) is superior when facing stochasticity and uncertainty (Agnarsson et al 2008). Feedback solutions take prevailing stock (states) as an input and therefore automatically respond to unexpected changes in the stock and adapt to the new situations (Sandal and Steinshamn 1997b; Sandal and Steinshamn 1997a). We apply a dynamic programming (DP) technique to obtain the optimal feedback solution. DP is a very useful method when considering management models under stochasticity (Sanchirico and Springborn 2011).

Although it is possible to derive the probability of collapse directly from the stock dynamics equation, it requires solving the Fokker-Planck equation. The Fokker-Planck equation governs the time evolution of the probability density of the stock, and for practical purposes it is hard to solve. That the stock dynamics depends on the harvest policy complicates matters. We employed a Monte Carlo approach. We simulated the system forward in time with the estimated biological parameters and the derived optimal feedback solution. We can directly estimate the probability of stock collapse by simply counting the paths that collapse over time.

## **The Model**

Following Sandal and Steinshamn (1997b) and Agnarsson et al (2008), we model the dynamics of the fish stock biomass ( $x$ ) as:

$$\dot{x} = \frac{dx}{dt} = f(x) - h \quad (1)$$

where  $\dot{x} = dx/dt$  is the instantaneous change in stock biomass, and  $f(x) = r x^2 (1 - x/k)$  is the modified logistic growth function. The  $x^2$  term makes the growth  $f(x)$  skewed to the right and indicates that the growth of the fish stock is slow when the stock is small.  $r$  denotes growth rate and  $k$  denotes the carrying capacity of the species.  $h$  is the rate of biomass harvest. By adding a stochastic term in equation (1), the general stochastic dynamic growth function can be written as:

$$dx = (f(x) - h)dt + \sigma_0(x)dB \quad (2)$$

In equation (2),  $f(x) - h$  is the drift term which explains the net growth in the deterministic case and the term  $\sigma_0$  is the stochastic parameter. The term  $dt$  is time increment and  $dB$  denotes the incremental Brownian motion which are i.i.d. with mean zero and variance  $dt$ . The term  $\sigma_0(x)dB$  represents the stochastic part of the stock growth relationship. We assume quasiconcavity in the growth function and  $f(0) = f(k) = 0$  and impose the natural restrictions  $x \geq 0$  and  $h \geq 0$ .

Let the instantaneous net revenue from the harvest of the stock biomass  $\pi(x, h)$  be given as:

$$\pi(x, h) = p(h)h - c(x, h) \quad (3)$$

where,  $p(h)$  is the inverse demand function and  $c(x, h)$  is the cost function. We assume the following:

$$\frac{\partial p(\cdot)}{\partial h} < 0; \quad \frac{\partial c(\cdot)}{\partial h} > 0; \quad \frac{\partial c(\cdot)}{\partial x} < 0.$$

The functional forms for the demand and cost functions are specified as:  $p(h) = p_0 - p_1h$

and  $c(x, h) = \frac{c h^\alpha}{x}$ . The functional forms for demand and cost functions are employed to fit

the model to a real world fishery, where the price of the harvest depends on the amount harvested and the cost of harvest depends on the stock biomass.

By substituting the values in equation (3), the profit function can be rearranged as:

$$\pi(x, h) = p_0h - p_1h^2 - \frac{ch^\alpha}{x} \quad (4)$$

where,  $p_0, p_1, c$  and  $\alpha$  are parameters.

Given the growth function and profit function, the management objective is to maximize expected net present value of the return from the harvest schedule over an infinite time horizon. Hence our objective is

$$\text{Max}_h E \left[ \int_0^\infty e^{-\delta t} \pi(x, h) dt \right] \quad (5)$$

subject to the dynamic constraint equation (2) and the appropriate natural restrictions on the state and policy variables. Here, the non-negative parameter  $\delta$  is discount rate and  $E$  is the expectation operator. We define the maximum expected value as a value function

$$W(x_0, t_0) = \text{Max}_h E \left[ \int_{t_0}^\infty e^{-\delta(t-t_0)} \pi(x, h) dt \right] \quad (6)$$

$$x(t = t_0) = x_0$$

and obtain the optimal solution by solving following the Hamilton-Jacobi-Bellman (HJB) equation for the current value  $V$ , defined by  $W(x, t) = e^{-\delta t} V(x)$ .

$$\delta V = \max_{h \geq 0} \left\{ \pi(x, h) + (f(x) - h)V_x + \frac{1}{2} (\sigma_0 x)^2 V_{xx} \right\} \quad (7)$$

The subscripts of  $V$  denote partial derivatives. It is difficult or impossible to solve the Hamilton-Jacobi-Bellman (HJB) equation analytically in our case. We approach the problem by using numerical approximation methods.

### **Numerical Approximation and Monte Carlo Simulation**

Our problem is strongly non-linear in the control. Analytic solutions to such problems are extremely rare and it is difficult to solve the Hamilton-Jacobi-Bellman (HJB) equation together with nonlinearity and boundary conditions. Numerical methods are the only viable alternatives. The Markov chain approximation approach is one of the most effective methods (Song 2008), which is based on probability theory. Numerical algorithms for optimal stochastic control problems of this kind can be found in Kushner and Dupuis (2001).

The numerical technique entails discretizing the state space for the HJB control problem (7), constructing transition probabilities for the controlled Markov chain by applying finite difference techniques and then iterating on the resulting discrete value transition equation with initial guess  $V_0$  for the value function. The combined approximation in policy space



and value space is more powerful. The iteration is carried out until the value function converges to the optimal value function (for details of the approximation refer to Kushner and Dupuis 2001).

With the optimal policy solution available in feedback form, we simulated the system forward in time. We carried out the Monte Carlo simulation for a range of initial stock levels. The simulation was performed for a thousand realizations for 500 years. At the end of the simulation horizon, a simulated path either reached a stable region, comparable to a deterministic steady state, or collapsed to a near zero stock level. The higher the initial stock level, the higher the share of simulated paths reached the stable region. For a given initial stock level, we treat the distribution of the simulated paths as an estimate of the probability of collapse.

### **Model Application: The Northeast Arctic Cod Fishery**

We used the data from the Northeast Arctic cod (*Gadus morhua*) fishery in the model. This is the main commercial fish species in Norway and considered to be the basis of the Norwegian commercial white fish industry (Kugarajh et al 2006). Northeast Arctic cod data from Barents Sea were obtained from the International Council for the Exploration of the Seas and employed to estimate the biological and economic parameters.

### ***Biological Parameters***

We use the ensemble Kalman filter (EnKF) approach to estimate biological parameters, namely the drift and diffusion parameters in equation (2). The EnKF is a data assimilation method which is widely used in physical applications like meteorology and oceanography, where phenomena typically have a chaotic nature. It has structural relationships to the classical Kalman filter, but extends to a large class of nonlinear models (see Burgers et al 1998 and references therein). The method was first suggested by Evensen (1994), while Burgers et al (1998) provided a theoretical clarification. Evensen (2003) reviewed both

theoretical developments and survey applications of the ensemble Kalman filter and related techniques.

The ensemble Kalman filter uses a Markov Chain Monte Carlo method to solve the fundamental equation for the time evolution of the probability density of the model state (Evensen 2003). The method applies to state space models with the dynamic equation (the state or model equation) written as a stochastic differential equation. The details of the algorithm for estimation can be found in the sources mentioned earlier. The estimated parameters are shown in figure 1. The parameter  $\sigma$  is estimated in the sense of Hansen and Penland (2007).

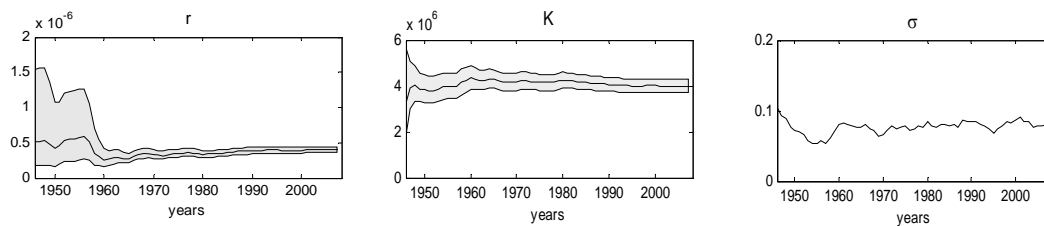


Figure 1: The biological parameters estimated using the ensemble Kalman filter approach.

Table 1. The functional forms and the estimated biological parameter values

Functions		Parameters	Descriptions	Values
Drift function	$F(x)=rx^2(1-x/k)$	$r$	Intrinsic growth rate	$4.0441 \times 10^{-7}$
		$k$	Stock carrying capacity	$3.9741 \times 10^6$
Diffusion function	$\sigma\theta(x)=\sigma\theta \times x$	$\sigma_0$	Volatility coefficient	$0.0763$

Although the stochastic process is estimated in the filtering procedure, we analyze effects of different sizes of the stochastic term. If appropriate management can reduce stochasticity, we are interested in the effect of small stochastic terms, while the precautionary principle may lead us to presume larger stochastic effects.

## ***Economic Parameters***

We obtained the economic parameters estimated by Agnarsson et al (2008), which were also estimated using data on the Northeast Arctic cod. The details of the functional forms and the estimated parameter values are presented in table 2.

Table 2. The functional forms and the economic parameter values

Functions		Parameters	Descriptions	Values
Price function	$P(h) = p_0 - p_1 \times h$	$p_0$	Price of the stock	12.65
		$p_1$	Strength of demand	0.00839
Cost function	$c(x, h) = c \times h^\alpha / x$	$c$	Cost of exploitation	5848.1
		$\alpha$	Harvest parameter	cost 1.1

Sources: adopted from Agnarsson et al (2008).

## **Results and Discussions**

### ***The Optimal Exploitation Policy***

The optimal exploitation policy in the deterministic case and in different levels of stochasticity is presented in figure 2. Different optimal behavior can be observed at different levels of stochasticity. When compared to the policy derived from the deterministic setup where the stochastic term has been set to zero, observe that the stochastic solution is more conservative at a low level of stochasticity ( $\sigma_0 = 0.3$ ). With increased level of stochasticity ( $\sigma_0 = 0.5$ ), the stochastic solution is conservative at high stock levels but it should be harvested earlier for small stock levels because the risk of extinction at small stock is high. The stochastic solution is more conservative than the deterministic solution at moderate to high stock levels, which is interesting along several dimensions. First, it is found that the geometric noise in the dynamics creates a downward

drag on the stock level, so expected surplus growth is smaller in the stochastic model than in the deterministic model. Second, the stochastic solution lines up with the precautionary principle which has been advocated by marine scientists and economists alike.

If the size of the stochastic term increases further ( $\sigma_0 \geq 0.7$ ), the stochastic solution approaches the myopic (infinite discounting) solution (Figure 2) because the probability of stock collapse is very high due to the strong noise in stock dynamics.

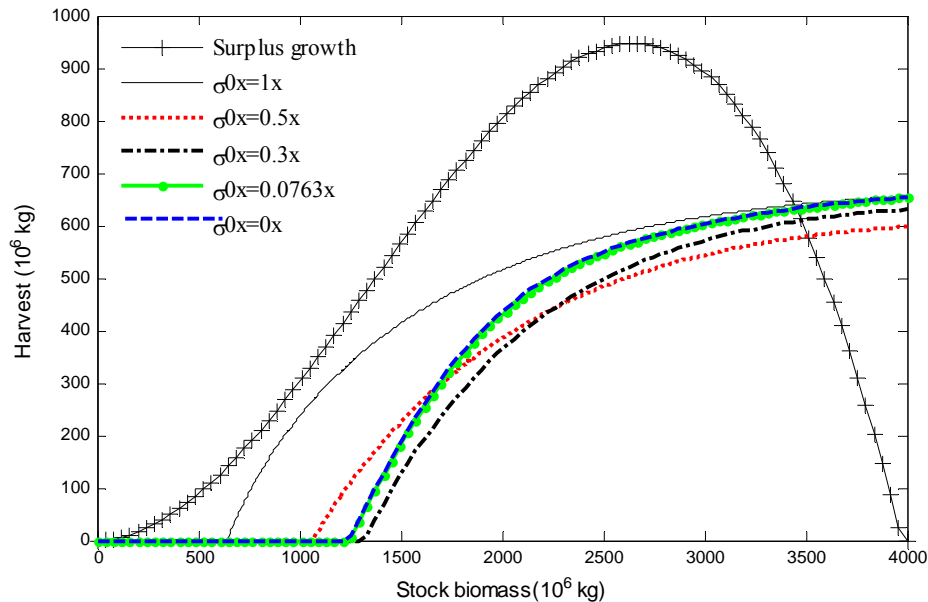


Figure 2: The feedback optimal exploitation paths for deterministic and stochasticity growth models. These optimal paths were obtained using 5 percent discount rate in all cases.

By applying a different level of stochasticity in the models, the optimal exploitation behavior at different levels of stochasticity can be generalized as:

*a. A low level of stochasticity ( $\sigma_0 \leq 0.4$ ) in the growth of biological stock leads to a conservative exploitation at all levels of stock biomass.*

b. A moderate level of stochasticity ( $0.4 < \sigma_0 < 0.7$ ) in growth of biological stock leads to a more myopic exploitation for small stock and conservative exploitation for large stock.

c. A high level of stochasticity ( $\sigma_0 \geq 0.7$ ) in the growth of biological stock leads to an exploitation that approaches the myopic policy at any levels of stock biomass.

We also compared the optimal solutions at different levels of stochasticity with the actual harvest data over the last 62 years (1946 - 2007). The optimal harvest and actual harvest are presented in figure 3. It can be noted that the actual harvest is fairly high compared to the optimal harvest models during most of the periods.

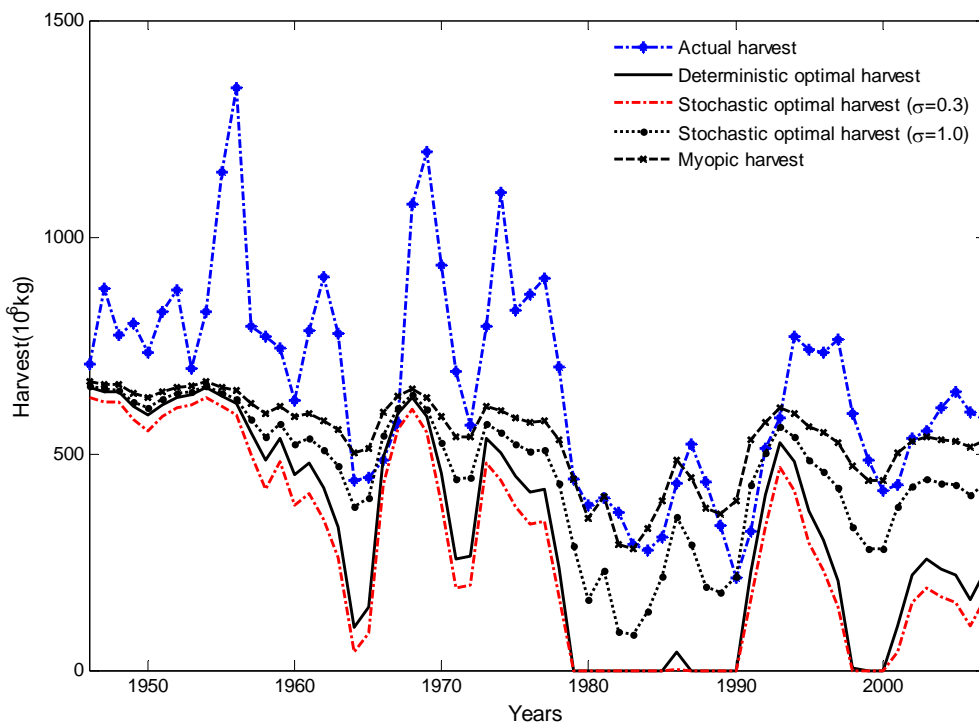


Figure 3: Actual versus optimal harvest of cod species in different growth models and stochastic levels.

Overexploitation can be observed over the whole period compared to the optimal catch identified in different models. It is very interesting to note that the historical harvest nearly

follows the very-high-level-stochastic model which is close to the myopic optimal model. The model explains that the historical harvest was ‘static optimum’ for the cod harvest contrary to optimal harvest except for some years such as during 1968-1970 and 1991-1994. However, the trends of historical catch and optimal catch are similar over most of the periods.

The general overexploitation of the fish stocks in Barents Sea could be the result of a policy that aims at a maximum sustainable yield (Agnarsson et al 2008) from a biological point of view and the economic aspect might have been ignored.

### ***Long-term Sustainable Optimal (LSO) Levels and Evolution of Stock Over Time***

After the Monte Carlo simulation (with the optimal solution), the paths that recovered and converged to a stable region, identified as the LSO region.

The LSO region in the deterministic setting is the steady state or the equilibrium level that can be achieved after a certain period of time if the stock is managed optimally. While in a stochastic setting, there is no equilibrium but most paths stabilize approximately at the same level after some period. This stable region, characterized as the mean of the stochastic realizations that relatively stabilizes after a certain period of time is defined as the stochastic LSO levels. In other words it can be defined as optimal stochastic stationary state (Smith 1986). The LSO stock level can be achieved either by allowing the stock to increase or by reducing the stock through exploitation.

We have attempted to obtain the LSO or the stationary state both in deterministic and stochastic settings. The steady state in the deterministic model in this study is characterized as:

$$x^* = 3453.4 \times 10^6 \text{ kg}$$

$$h^*(x^*) = 631.9271 \times 10^6 \text{ kg year}^{-1}$$

And the LSO level in stochastic setting ( $\sigma_x=0.3$ ) can be characterized by:

$$x^* = 3038.5 \times 10^6 \text{ kg}$$

$$h^*(x^*) = 577.59 \times 10^6 \text{ kg year}^{-1}$$

The LSO stock is lower in the stochastic model compared to the deterministic model due to the stochasticity induced downward drag. This can be easily shown for simplest case as follows:

Consider a simple single species surplus growth model  $dx = f(x)dt$  and add stochasticity to make the model more realistic as  $dx = f(x)dt + \sigma_0 x dB$ . Then by taking transformation  $y = \ln(x)$ , we get,  $dy = \left(\frac{f}{x} - \frac{1}{2}\sigma_0^2\right) dt + \sigma_0 dB$ . The term  $\frac{1}{2}\sigma_0^2$  creates an asymptotic downward drag on the stochastic growth. Therefore, the stochastic LSO for the stock is expected to be lower than the deterministic LSO. Subsequently, the optimal exploitation level becomes lower in the stochastic model, which has also been reported in the literature (for example, see Reed 1978).

In the simulation exercise, we have also looked at the time required for the stock to reach a stable level. We make a couple of observations. The system needs more time to stabilize if the initial stock level is small and needs more time to stabilize with stochasticity than without stochasticity.

Figures 4 and 5 show the evolution of the stock over time in the deterministic and stochastic models respectively. A large initial stock quickly approaches the LSO level because it is optimal to exploit due to low exploitation cost. Although the price decreases due to inverse demand function, the net revenue is still higher due to the decreased exploitation cost. On the other hand, a small initial stock takes longer to approach its LSO compared to a large stock level in both the deterministic and stochastic models but the duration to LSO is further prolonged in the stochastic case (Figures 4a and 5a).

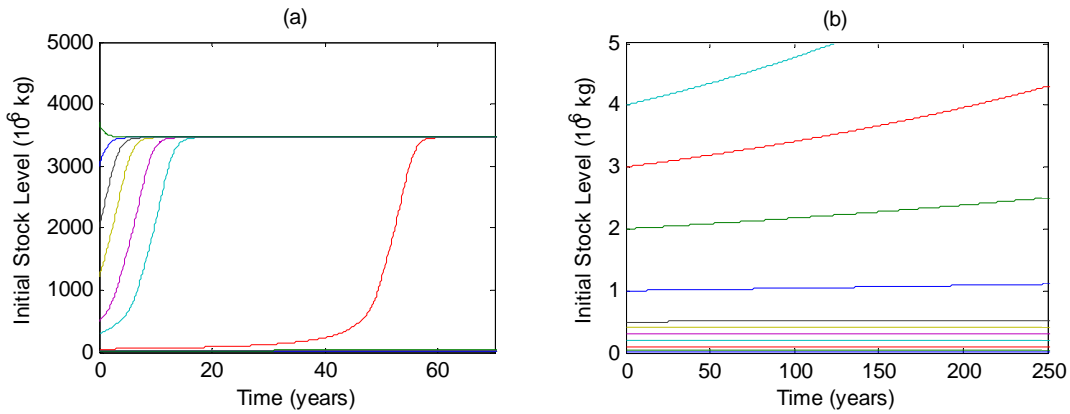


Figure 4: The deterministic evolution of the stock to LSO over time (a) large initial stock levels (b) small initial stock levels. If the initial stock level is very small, it may take many years to approach LSO level but the stock never collapses in deterministic growth stock.

Furthermore, Figure 5b demonstrates that there is a chance of extinction of a stochastic stock if it is small. Extinction may happen as a result of the stochasticity induced critical depensation. While Figure 4b shows that there is no problem of stock extinction in the case of the deterministic growth model, although, it may take several years to recover to LSO level (Figure 4b).

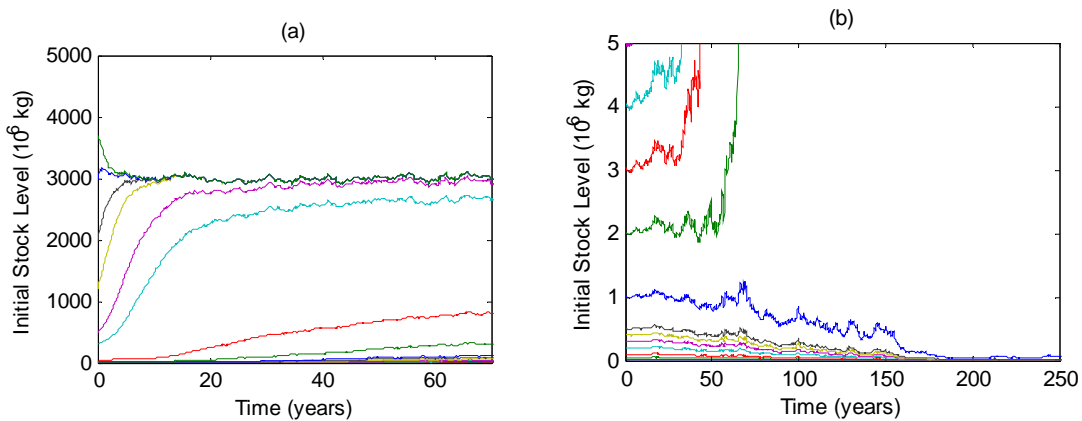


Figure 5: The stochastic evolution of the stock to LSO over time (a) large initial stock levels (b) small initial stock levels. The paths represent the mean of 1000 realizations, simulated over time. If the initial stock level is very small ( $<1 \times 10^6$  kg) the stock will collapse with high probability ( $P \cong 1$ ) i.e. there is almost no chance of stock recovery over the simulation horizon.



### *Analysis of the Risk of Stock Collapse*

To analyze the risk of extinction of the stock in the long run, we simulated the system forward in time from a range of initial stock levels with the stochastic model and optimal solution. As mentioned earlier, for high initial stock levels, all the paths recovered or persisted at healthy levels and approached the LSO levels in the long run. For small initial stock levels, all paths collapsed. But for some of the intermediate initial stock levels, some of the simulated paths collapsed, while other paths recovered or persisted at healthy levels. We identified the paths and grouped them into two: the collapsed group and the healthy group. We estimated the probability of collapse for a given stock level simply by observing how these paths are distributed into the groups. The precision of the estimate only depends on the number of paths simulated. A thousand realizations were simulated for 500 years from each initial stock level(s).

We observe that the risk of collapse or extinction (recovery) is higher (lower) for a smaller stock compared to a large stock level. Similarly, the risk of extinction is higher for a highly stochastic stock compared to the low stochastic stock which can be expected. The probability of stock collapse in the long run is presented in figure 6. The figure indicates that the probability of extinction approaches almost zero when the initial stock level is above  $100 \times 10^6$  kg at a low stochasticity ( $\sigma_0 = 0.1$ ) and above  $1000 \times 10^6$  kg if the stochasticity parameter is 0.3.

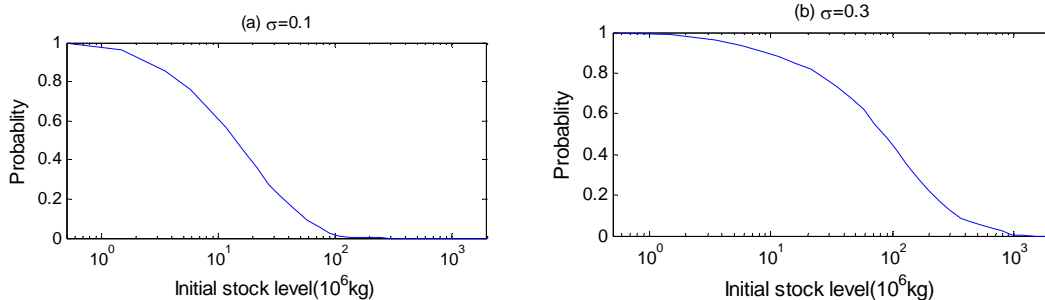


Figure 6: The probability stock collapse in the long run (500 years) under stochastic growth models (a) stochasticity 0.1 (b) stochasticity 0.3. The initial stock levels are given in a logarithmic scale on the x-axis and their collapse probabilities in the y-axis.

To add more clarity, we have also presented a table to illustrate the probability of extinction of different initial stocks at different degrees of stochasticity (table 3). It is clearly observed that the higher the stochasticity in the stock growth, the higher the probabilities of the stock collapse in the long run. A stock of biomass greater than  $100 \times 10^6$  kg could recover over time at a small degree of stochasticity ( $\sigma_0= 0.1$ ) but at a high degree of stochasticity ( $\sigma_0= 0.5$ ), there is a 75 percent probability that a stock could collapse even without exploitation in the long run. For a moderate level of stochasticity ( $\sigma_0= 0.3$ ), it could recover to a healthy stock level if the initial stock is above  $1000 \times 10^6$  kg.

Table 3. The probability of stock collapse in the long run at different degrees of stochasticity for different initial stock levels

Initial stock biomass level	Probability of stock collapse under different stochasticity levels		
	Pr( $\sigma_0= 0.1$ )	Pr( $\sigma_0= 0.3$ )	Pr( $\sigma_0= 0.5$ )
$0.5 \times 10^6$ kg	1.0	1.0	1.0
$5 \times 10^6$ kg	0.78	0.95	1.0
$10 \times 10^6$ kg	0.56	0.89	1.0
$50 \times 10^6$ kg	0.03	0.69	0.96
$100 \times 10^6$ kg	0.003	0.39	0.95
$120 \times 10^6$ kg	0	0.37	0.94
$250 \times 10^6$ kg	0	0.14	0.88
$1000 \times 10^6$ kg	0	0	0.79
$2000 \times 10^6$ kg	0	0	0.76

Viewed differently, the probability of stock collapse is more than 90 percent ( $P \geq 0.9$ ), if the stock is below  $3 \times 10^6$  kg at a low stochasticity level ( $\sigma_0= 0.1$ ), is below  $6 \times 10^6$  kg at moderate stochasticity level ( $\sigma_0= 0.3$ ) and is below  $200 \times 10^6$  kg for a high level of stochasticity ( $\sigma_0= 0.5$ ). The probability of stock collapse is less than 10 percent ( $P \leq 0.1$ ), if the stock is above  $30 \times 10^6$  kg at low degree of stochasticity and is above  $275 \times 10^6$  kg at

moderate degree of stochasticity. There is no chance of stock recovery if the stochasticity level is high ( $\sigma_0 > 0.5$ ) (Table 3).

### ***Stock Recovery Over Time***

In the previous section, we showed that there is some probability that a stochastic stock could recover in the long run. However, beside the probability of recovery, the time or the duration of stock recovery matters. The duration or total time required for the recovery of the stock in a stochastic model is important due to the underlying economic consequences of the resource stock. For example, a stock that recovers in infinite time does not have any economic importance. Therefore, in this section, we have calculated the probability of the stock recovery at different points in time. The probability of stock recovery at moderate level of stochasticity ( $\sigma_0=0.3$ ) is shown in the table 4.

Table 4. The probability of stock recovery in different initial stochastic stock levels at different points in time. Long run refers to the end of the simulation horizon, which was 500 years

Initial stock levels	Probability $\Pr(\sigma_0=0.3)$ of stock recovery in a given period			
	Within 5 years	Within 10 years	Within 25 years	Long run
$1 \times 10^6$ kg	0	0	0	0
$15 \times 10^6$ kg	0	0	0	0.15
$30 \times 10^6$ kg	0	0	0.04	0.26
$80 \times 10^6$ kg	0	0.03	0.24	0.53
$120 \times 10^6$ kg	0.03	0.14	0.37	0.63
$150 \times 10^6$ kg	0.11	0.33	0.55	0.72
$300 \times 10^6$ kg	0.41	0.69	0.83	0.90
$500 \times 10^6$ kg	0.72	0.85	0.97	0.99
$700 \times 10^6$ kg	0.94	0.98	0.99	1.0
$1000 \times 10^6$ kg	0.99	1.0	1.0	1.0

The probability of stock recovery within five years is zero for an initial biomass of about  $120 \times 10^6$  kg and there is 37 percent probability that it could recover within 25 years and 63 percent probability of it recovering in the long run. Similarly, an initial biomass level greater than  $500 \times 10^6$  kg, there is almost no collapse is observed in the long run, has only 72 percent probability of recovery in a short period (5 years). Therefore, a complete closure on the exploitation is essential for its recovery in the long run. At least an initial stock biomass of  $1000 \times 10^6$  kg could recover to its LSO level within five years. This means that any stock below  $1000 \times 10^6$  kg is required to close for fishing for at least five years.

We also analyze the relationship between the stochasticity and exploitation rate that cause stock collapse. We note two points from the previous sections: first, our model suggests that fishing moratorium is at a high stock level, approximately  $1200 \times 10^6$  kg of biomass and even lower in a stochastic model. Second, our simulation results indicate that there is risk of stock collapse when the stock is below  $1000 \times 10^6$  kg of biomass level at 0.3 stochasticity level. This indicates that there is no risk of stock collapse if the stock is managed optimally. However, if suboptimal exploitation occurs i.e. if we exploit the resource when it is below  $1000 \times 10^6$  kg, there is a probability that the stock could collapse or take longer to recover. Furthermore, we also observe that at high level of stochasticity ( $\sigma_0=0.5$ ), there is a higher probability that the stock can collapse even if managed optimally.

### **Concluding remarks**

A holistic approach should be applied to fisheries management in a stochastic environment. We demonstrate an approach to quantify stochasticity in fish stock dynamics. We derive the optimal, stochastic harvest profile, and demonstrate a method to assess the risk of collapse. We believe that policy makers would benefit from an increased appreciation of the effects of stochasticity, and the consequences of ignorance. The ignorance becomes apparent when we compare our solutions to historic catches. In periods, landings lie closer to the myopic solution than to both the stochastic and deterministic solutions.

Finally, we acknowledge that real world fisheries management is concerned with a limited number of fish stocks that are only a part of a larger ecosystem. To analyze the effect of stochasticity and to assess risk of stock collapse in an ecosystem framework is beyond our scope here. We are only on one of the first steps on a long ladder towards the ultimate goal of ecosystem management under uncertainty. But, it is an important step; a step in the right direction.

## References

**Agnarsson, S., R. Arnason, K. Johannsdottir, L. Ravn-Jensen, L.K. Sandal, S.I. Steinshamn and N. Vestergaard. 2008.** Comparative evaluation of the fisheries policies in Denmark, Iceland and Norway: multispecies and stochastic issues. 0803-4036, SNF, Bergen, Norway.

**Burgers, G., P. van Leeuwen and G. Evensen. 1998.** Analysis scheme in the ensemble Kalman filter *Monthly Weather Review*, 126(6): 1719 – 1724.

**Clark, C.W. 1973.** The economics of overexploitation *Science*, 181(4100): 630.

**Clark, C.W. 1990.** *Mathematical bioeconomics: the optimal management of renewable resources*, John Willey and Sons, Inc.

**Clark, C.W. and G.R. Munro. 1975.** The economics of fishing and modern capital theory: A simplified approach *Journal of Environmental Economics and Management*, 2(2): 92-106.

**Escudero, C., J. Buceta, F. de La Rubia and K. Lindenberg. 2004.** Extinction in population dynamics *Physical Review E*, 69(2): 021908.

**Evensen, G. 1994.** Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics *Journal of Geophysical Research*, 99(C5): 10143 - 10162.

**Evensen, G. 2003.** The ensemble Kalman filter: Theoretical formulation and practical implementation *Ocean dynamics*, 53(4): 343-367.

- FAO. 2010.** World review of fisheries and aquaculture. Fisheries Resources: Trends in Production, Utilization and Trade. The State of World Fisheries and Aquaculture.
- Field, I.C., M.G. Meekan, R.C. Buckworth and C.J.A. Bradshaw. 2009.** Susceptibility of sharks, rays and chimaeras to global extinction *Advances in marine biology*, 56(275-363).
- Gjørøseter, H., B. Bogstad and S. Tjelmeland. 2009.** Ecosystem effects of the three capelin stock collapses in the Barents Sea *Marine Biology Research*, 5(1): 40-53.
- Gordon, H.S. 1954.** The economic theory of a common-property resource: the fishery *The Journal of Political Economy*, 62(2): 124-142.
- Hannesson, R. 1975.** Fishery dynamics: a North Atlantic cod fishery *Canadian Journal of Economics*, 8(2): 151-173.
- Hannesson, R. 1987.** Optimal catch capacity and fishing effort in deterministic and stochastic fishery models *Fisheries Research*, 5(1): 1-21.
- Hannesson, R. 1996.** *Fisheries (mis)management: The case of the north Atlantic cod.*, Fishing News Books. Oxford, England.
- Hansen, J.A. and C. Penland. 2007.** On stochastic parameter estimation using data assimilation *Physica D: Nonlinear Phenomena*, 230(1–2): 88-98.
- Hutchings, J.A. 2000.** Collapse and recovery of marine fishes *Nature*, 406(6798): 882-885.
- Hutchings, J.A. and J.D. Reynolds. 2004.** Marine fish population collapses: consequences for recovery and extinction risk *BioScience*, 54(4): 297-309.
- Johnston, R.J. and J.G. Sutinen. 1996.** Uncertain biomass shift and collapse: implications for harvest policy in the fishery *Land Economics*, 500-518.
- Jonzén, N., M. Cardinale, A. Gardmark, F. Arrhenius and P. Lundberg. 2002.** Risk of collapse in the eastern Baltic cod fishery *Marine Ecology Progress Series*, 240(225-233).
- Kugarajh, K., L.K. Sandal and G. Berge. 2006.** Implementing a stochastic bioeconomic model for the North-East Arctic cod fishery *Journal of bioeconomics*, 8(1): 35-53.

**Kushner, H.J. and P. Dupuis. 2001.** *Numerical methods for stochastic control problems in continuous time*, Springer Verlag.

**Lewis, T.R. 1981.** Exploitation of a renewable resource under uncertainty *The Canadian Journal of Economics/Revue canadienne d'Economie*, 14(3): 422-439.

**Lorentzen, T. and R. Hannesson. 2004.** The collapse of the Norwegian Herring fisheries in the late 1950s and 60s crisis, adaptation and recovery. Discussion paper 2/2004, Center for Fisheries Economics, Institute for Research in Economics and Business Administration (SNF), Bergen, Norway.

**Ludwig, D. 1980.** Harvesting strategies for a randomly fluctuating population *ICES Journal of Marine Science: Journal du Conseil*, 39(2): 168-174.

**Mitra, T. and S. Roy. 2006.** Optimal exploitation of renewable resources under uncertainty and the extinction of species *Economic Theory*, 28(1): 1-23.

**Mullon, C., P. Fréon and P. Cury. 2005.** The dynamics of collapse in world fisheries *Fish and Fisheries*, 6(2): 111-120.

**Myers, R.A., J.A. Hutchings and N.J. Barrowman. 1997.** Why do fish stocks collapse? The example of cod in Atlantic Canada *Ecological applications*, 7(1): 91-106.

**Nøstbakken, L. 2006.** Regime switching in a fishery with stochastic stock and price *Journal of Environmental Economics and Management*, 51(2): 231-241.

**Pindyck, R.S. 1984.** Uncertainty in the theory of renewable resource markets *The Review of Economic Studies*, 51(2): 289.

**Reed, W.J. 1978.** The steady state of a stochastic harvesting model *Mathematical Biosciences*, 41(3): 273-307.

**Reed, W.J. 1979.** Optimal escapement levels in stochastic and deterministic harvesting models *Journal of Environmental Economics and Management*, 6(4): 350-363.

**Roughgarden, J. and F. Smith. 1996.** Why fisheries collapse and what to do about it *Proceedings of the National Academy of Sciences*, 93(10): 5078.

**Sanchirico, J.N. and M. Springborn. 2011.** How to get there from here: Ecological and economic dynamics of ecosystem service provision *Environmental and Resource Economics*, 1-25.

**Sandal, L.K. and S.I. Steinshamn. 1997a.** A feedback model for the optimal management of renewable natural capital stocks *Canadian Journal of Fisheries and Aquatic Sciences*, 54(11): 2475-2482.

**Sandal, L.K. and S.I. Steinshamn. 1997b.** A stochastic feedback model for optimal management of renewable resources *Natural Resource Modeling*, 10(1): 31-52.

**Sandal, L.K. and S.I. Steinshamn. 2001.** A simplified feedback approach to optimal resource management *Natural Resource Modeling*, 14(3): 419-432.

**Sarkar, S. 2009.** Optimal fishery harvesting rules under uncertainty *Resource and Energy Economics*, 31(4): 272-286.

**Schaefer, M.B. 1954.** Some aspects of the dynamics of populations important to the management of the commercial marine fisheries *Bulletin of the Inter-American tropical tuna commission*, 1(2): 27-56.

**Smith, J.B. 1986.** Stochastic steady-state replenishable resource management policies *Marine Resource Economics*, 3(2): 155-168.

**Song, Q.S. 2008.** Convergence of Markov chain approximation on generalized HJB equation and its applications *Automatica*, 44(3): 761-766.

**Tereshchenko, E.S. 2002.** The dynamics of population fecundity in Barents Sea capelin *ICES Journal of Marine Science: Journal du Conseil*, 59(5): 976.