# *Performance Evaluation of Channel Aggregation Strategies in Cognitive Radio Networks with Queues*

by

**Indika Anuradha Mendis Balapuwaduge**

*Supervised by*

*Professor Dr. Frank Yong Li &*

*Lei Jiao*

This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education.

University of Agder

Faculty of Engineering & Science

Department of Information & Communication Technology

Grimstad, Norway

May 2012

**Abstract**

With the growing usage of wireless communication devices, demand for the spectrum access is rapidly increasing. Therefore, an efficient spectrum management and spectrum access techniques are necessary and critical. However, studies on spectrum usage have revealed that most of the allotted spectrum is not used efficiently due to the static frequency allocation methods. With the evolution of cognitive radio, spectrum access techniques shift from static spectrum allocation to dynamic allocation with enhanced features such as spectrum sensing and spectrum adaptation.

In the first part of this thesis, we study several spectrum access techniques in cognitive radio networks, which have been developed with spectrum adaptation. The performance of cognitive radio systems are evaluated in terms of capacity, blocking probability and forced termination probability of the secondary network. Due to the strict priority over primary users, the performance of the secondary network is restricted. One of the successful solutions to further improve the system performance by increasing the capacity and decreasing the blocking and forced termination probabilities is the integration of a queuing model. Most of already designed queuing models for cognitive radio systems have been designed with certain limitations of performance. Therefore in this thesis, a bunch of techniques of performance improvement have been taken into account when designing the queuing model. The features: channel aggregation, spectrum handover, channel sharing, priority based queuing and heterogeneous traffic are considered together in order to model the queuing system as much as more realistic way which can further enhance the overall system performance.

In the second part of this thesis, we propose a queuing system referred to as Priority based Multiple Queue System (PMQS) which is designed with two queues separately for the real time and non-real time secondary user services. Channel access opportunities are distributed between two queues such a way that the real time services have the higher priority than elastic services. Two queuing approaches are introduced based on the queuing ability of the interrupted non-real time services. Continuous time Markov chain models are developed to evaluate the system performance in terms of capacity, blocking and forced termination probabilities of the secondary network. In addition, we explore the cost analysis of the proposed queuing model in terms of mean queuing delay. Other than that, spectrum utilization of the cognitive radio system is also evaluated. In order to minimize the associated queuing delay, a maximum value for the number of waiting lines inside a queue is set instead of an infinite queue size . Analytical results reveal that integration of the proposed queuing model could increase the capacity of the secondary network while decreasing the blocking probability. And also one of the proposed queuing methods can further decrease the forced termination rate of non-real time traffic. Associated queuing delay is controlled by proper selection of maximum queue sizes. For these reasons, it can be concluded that the proposed queuing model can be used to improve the system performance of multi-channel cognitive radio networks.

**Keywords:** *Cognitive radio, channel aggregation, spectrum adaptation, heterogeneous traffic, queuing system, Markov model, performance analysis*

# Preface

This thesis is the result of the Master Thesis, IKT-590 to fulfill the requirements of course content of MSc ICT, at the Faculty of Engineering and Science, University of Agder (UiA) in Grimstad, Norway. The research into the topic was started from the second week of January 2012 and ended on 31 May 2012. The course contains 30 credits towards the degree of MSc ICT. The main objective of this thesis is to analyze the performance of multi channel cognitive radio networks with an embedded queuing system.

The task of the completion of this thesis would have not been possible without the support of several individuals. I am greatly indebted to my supervisor, Professor Dr. Frank Y. Li, for his guidance, support and teaching me the process of good scientific research. Many thanks Professor Frank, for your always positive attitude and for helping me to improve my writing. My sincere gratitude goes to Lei Jiao who is a Doctoral Fellow of the ICT department in University of Agder, for his valuable comments on the thesis. He introduced me to many new ideas through several illuminating discussions pertaining to this thesis. Working with Lei has been very inspiring due to his dedication, innovative ideas and excellent organization. Last, but certainly not the least, I would like to express my gratefulness and humility to my parents, my brother and friends for their support during the Master thesis.

Indika Anuradha Mendis Balapuwaduge
University of Agder
Grimstad, Norway
31 May 2012

# Contents

# List of Figures

# List of Tables

# List of Abbreviations

| | |
|---|---|
| **BAQ** | Bandwidth Allocation Queue |
| **CBS** | Central Base Station |
| **CH** | Channel |
| **CRN** | Cognitive Radio Network |
| **CR** | Cognitive Radio |
| **CTMC** | Continuous Time Markov Chains |
| **DFA** | Dynamic Fully Adjustable |
| **DPA** | Dynamic Partially Adjustable |
| **DSA** | Dynamic Spectrum Access |
| **ESU** | Elastic Secondary User |
| **FCC** | Federal Communications Commission |
| **FCFS** | First-Come First-Serve |
| **HTTP** | HyperText Transfer Protocol |
| **ISM** | Industrial, Scientific and Medical |
| **ITU** | International Telecommunication Union |
| **LAN** | Local Area Network |
| **LCFS** | Last-Come First-Serve |
| **MAC** | Medium Access Control |
| **MAN** | Metropolitan Area Network |
| **MATLAB** | Matrix Laboratory |

| | |
|---|---|
| **NTIA** | National Telecommunications and Information Administration |
| **PMQS** | Priority-based Multiple Queuing System |
| **PU** | Primary User |
| **QoS** | Quality of Service |
| **QS** | Queue Selector |
| **RSS** | Random Selection for Services |
| **RSU** | Real-time Secondary User |
| **RV** | Random Variable |
| **SDR** | Software Defined Radio |
| **SMS** | Short Message Service |
| **STC** | Static |
| **SUQ** | Secondary User Queue |
| **SU** | Secondary User |
| **UWB** | Ultra Wide Band |
| **WAN** | Wide Area Network |
| **WRAN** | Wireless regional Area Network |

# Chapter 1: Introduction

*In this chapter, we attempt to provide background information regarding the concept of cognitive radio (CR) technology. Spectrum scarcity is briefly explained as the main reason behind the emergence of cognitive radio networks (CRNs). At the end of this chapter we describe motivate factors and the problem statement before outlining the thesis chapters.*

## 1.1 Background Information on Radio Spectrum

Radio Spectrum is a finite resource. It refers to the electromagnetic frequencies between 3 KHz and 300 GHz. Based on different requirements of different applications, the radio spectrum is divided into frequency bands. Table 1.1 indicates these bands and the corresponding frequency ranges [1]. Access to the spectrum is authorized by the telecommunication authorities in a country or region according to the regulations of International Telecommunication Union (ITU). Over the last decade, wireless communications have grown rapidly and more spectrum resources are needed to support various novel concepts of wireless services. High data rates become more and more necessary for normal professional needs or at home based users. Therefore, the demand for spectrum is increasing and frequency bands become congested specially in densely populated urban areas. Therefore, the issue of spectrum scarcity has become a major concern among wireless communications system designers.

The National Telecommunications and Information Administration's (NTIA) frequency allocation chart (Figure 1.1) indicates overlapping allocations over all of the frequency bands [2]. According to this figure, it is clear that all the frequency bands are exclusively allocated for particular services. However, the actual measurements show that the most of the allocated frequency bands are vastly underutilized [3] and a large number of spectrum holes exist under traditional fixed spectrum assignment. It is important to note that the static ways of frequency allocation is the main reason for this problem. Today, it is widely accepted that this type of static frequency allocations is as inefficient for radio spectrum management. Therefore, the difficulty of spectrum allocation is no longer a problem of searching an available frequency band. If the currently allocated spectrum is utilized in the most efficient manner, this problem can be solved. The most promising way to achieve this goal is the Dynamic Spectrum Allocation (DSA) instead of traditional static allocation methods [4].

Table 1.1: The Radio Spectrum.

| Designation | Frequency | Wavelength |
|---|---|---|
| ULF (Ultra Low Frequency) | 300 Hz to 3 kHz | 1000 km to 100 km |
| VLF (Very Low Frequency) | 3 kHz to 30 kHz | 100 km to 10 km |
| LF (Low Frequency) | 30 kHz to 300 kHz | 10 km to 1 km |
| MF (Medium Frequency) | 300 kHz to 3 MHz | 1 km to 100 m |
| HF (Medium Frequency) | 3 MHz to 30 MHz | 100 m to 10 m |
| VHF (Very High Frequency) | 30 MHz to 300 MHz | 10 m to 1 m |
| UHF (Ultra High Frequency) | 300 MHz to 3 GHz | 1 m to 10 cm |
| SHF (Super High Frequency) | 3 GHz to 30 GHz | 10 cm to 1 cm |
| EHF (Extremely High Frequency) | 30 GHz to 300 GHz | 1 cm to 1 mm |

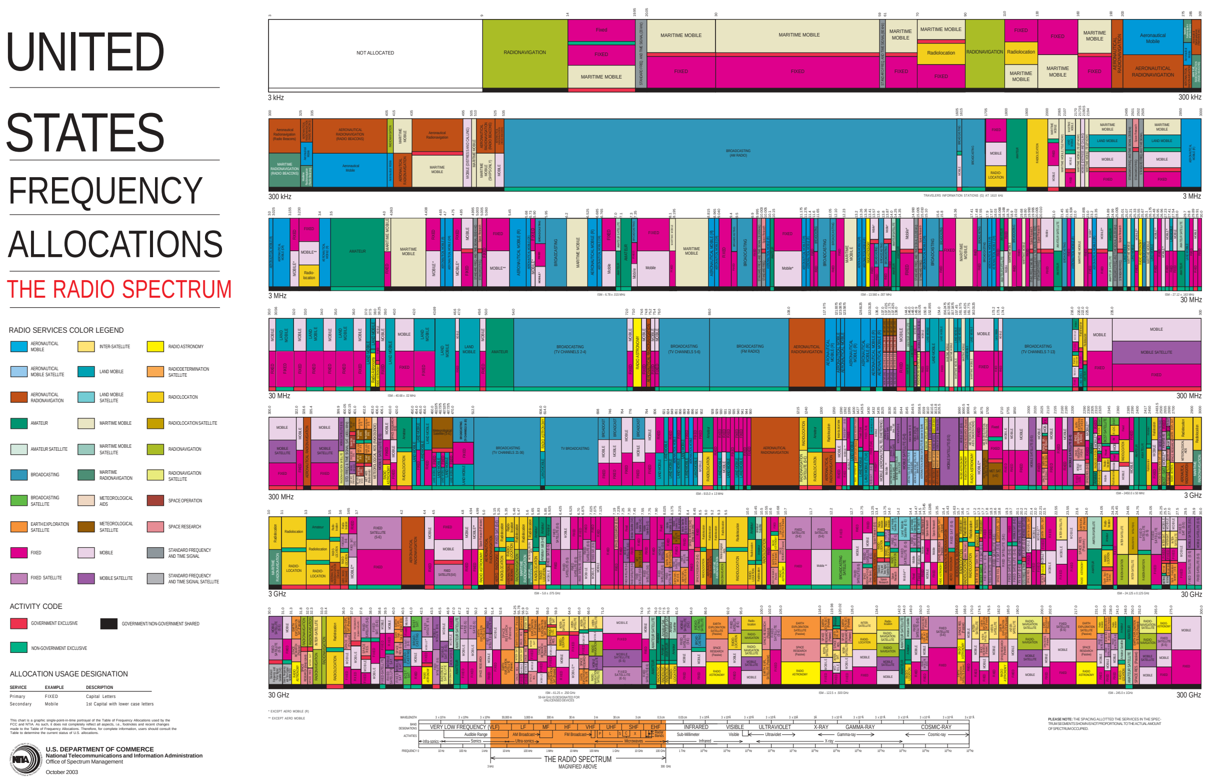


Figure 1.1: The NTIA's frequency allocation chart.

## 1.2 Methods to Increase Spectrum Availability

Most part of the radio spectrum is already allocated for existing wireless systems. Therefore, challenges and problems on how to allocate spectrum to more applications with unbalanced spectrum resources still exist which attract many attentions in research areas. The following approaches provide solutions to some extent for increasing the availability of spectrum for different wireless applications.

### 1.2.1 Unlicensed Spread Spectrum

In 1985, the ISM (industrial, scientific and medical) spectrum is allowed by the Federal Communications Commission (FCC) as an unlicensed radio band (2.4 - 2.5 GHz) [5]. Unlicensed spectrum is a very small portion of the entire radio spectrum. One of the main benefits of ISM band is that it allows the civil use of spectrum resources without paying for highly expensive spectrum resources. (But unlicensed users must follow the rules defined in the FCC regulations.) With the growth of private local area networks, more parties and technologies use unlicensed spectrum. Thus, it has become crowded and not all the users can access all the time. Since unlicensed spectrum does not provide performance guarantee of the systems, this is not a better solution for applications which need high QoS (Quality of Service). On the other hand, the interference caused by the radios of different applications in the same band which have close proximity locations is a downside of ISM band. Therefore, unlicensed spread spectrum cannot be considered as a perfect solution for the problem of spectrum scarcity.

Another approach to improve the spectrum usage is to allow secondary use of the frequency bands. The primary users who possess the license for the channels have the strict priority for channel access while secondary users will get access under several restrictions. One of the mechanisms to accomplish this is to share the spectrum without interfering with primary users. In UWB transmission, this is achieved by assigning restrictions on transmission power levels of secondary users.

### 1.2.2 Ultra Wide Band

The main difference between Ultra Wide Band (UWB) systems and other communication systems is that it operates across a very wide spectrum of several GHz frequencies [6]. But the combination of the features like lower transmit power level and pulsed date with wider bandwidth improves the transmission speed and decreases the interference level. However, UWB communications coexist with other wireless networking standards such as 802.11 LAN, 802.16 MAN and WAN. UWB radios use the frequency range from 3.1 GHz to 10.6 GHz. In addition to the above mentioned technologies, 57 - 64 GHz spectrum band (60 GHz Radios) is allocated for unlicensed usage as a measure of improving efficient spectrum utilization. This frequency range is used with applications which need high data rate wireless transmission. However, none the above mentioned techniques provide an effective solution towards the problem of spectrum scarcity. Cognitive Radio is the most promising solution we can have in this decade, to tackle with the issues of spectrum scarcity in a better way.

## 1.3 Cognitive Radio

Cognitive radio is a smart and novel approach for efficient channel utilization. Joseph Mitola introduced the concept of Cognitive radio in 1999 by his research work [7] as a method to reuse the underutilized spectrum in an opportunistic manner. Channel environmental sensing, followed by a knowledge based standard learning is the success behind this technology. Therefore, unused channels can be identified and those channels can be allocated for unlicensed users until the licensed users need the channel occupancy. Spectrum sensing, sharing and management are significant challenges of this technology. Among these challenges, spectrum sharing and management can be considered as the major concerns in this thesis. The FCC has defined a Cognitive radio as: "A radio that can change its transmitter parameters based on interaction with the environment in which it operates" [8]. The opportunistic users are called Secondary Users (SUs) or Cognitive Users and the licensed owners of the channels in the spectrum band are called Primary Users (PUs). In some papers PUs are referred as incumbent users or primary licensees. Secondary users can access the licensed spectrum during the absence time of a primary user. When the primary user arrives to a channel, the secondary user has to finish its transmission immediately and release the channel occupancy, since PUs have the strict priority over SUs.

### 1.3.1 Software Defined Radio

Software Defined Radio (SDR) can be considered as the key technology enabler for Cognitive radio. Most radios today have the limitation of lack of run time adaptivity to changes of the system [9]. But in SDR this limitation is overcome since its radio functionalities are performed by software modules. Since SDR technology has the ability of reconfiguration, radios can easily be switched among different functions and operations [10] depending on the status of the system. Actually, this is a basic requirement of a Cognitive radio network.

IEEE 802.22 Wireless Regional Area Network (WRAN) is the standard for using Cognitive radio techniques to allow sharing of licensed spectrum bands when the primary licensees are not present [11]. The spectrum where the IEEE 802.22 WRAN will use is already in use by TV services and wireless microphones and by using proper spectrum sensing, the issue of interference is mitigated. IEEE 802.22 WRAN uses the unused or white spaces within the television bands between 54 and 862 MHz, especially within rural areas where usage may be lower by employing Cognitive radio technology. To be sure, improvement of the spectrum utilization is not the only thing benefited from CR techniques. With the advancements of SDR, CR methods can be used to control interference of frequency bands in a way that more frequency bands could be utilized. And also, it provides technical background for operation on multiple frequencies with different power levels.

## 1.4 Motivation

Cognitive radio networks have gained increasing research importance due to its capability of improving the spectrum utilization. Efficient resource allocation in licensed frequency bands by using dynamic spectrum access is a one of the major research components in CRNs. We analyzed different dynamic channel access techniques which have been proposed by many research papers [12]-[17], [22]. Among those papers the dynamic channel aggregation strategy proposed in [22] has been designed for heterogeneous traffic environment and spectrum adaptation is also implemented by using channel sharing and channel adjustment. The results of that work prove that the proposed strategy can achieve better performance. Not only specific to the results in [22] but generally in CRNs, the blocking probability and the forced termination probability are increased due to the increase of primary user arrivals. The provision of very low probabilities for blocking and forced termination of secondary user services is a challenging goal for Cognitive radio systems. Motivated by need for an efficient queuing model which can achieve that goal, we are addressing in this thesis modelling and simulation approaches for different queuing strategies and investigate system performance of CRNs.

One important issue in Cognitive radios with a queuing scheme is to model the heterogeneous traffic of the secondary network with different priority levels depending on the QoS requirements. The other issue is to model the queuing system as it reduces the blocking probability and forced termination rate of secondary users. On the other hand, other performance parameters, i.e., capacity and the overall spectrum utilization have to be improved with the queuing model which is going to be developed. In addition, queuing delay has to be minimized. These issues became our targets of this thesis and motivated factors of our current research work.

## 1.5 Problem Statement

Spectrum management functions of the secondary network always depend on the PU activities. When the PU arrival rate increases, capacity of the secondary network decreases while blocking probability of new SU services increases. On the other hand, the ongoing SU services are forced to terminate when there are no enough channels upon a PU arrival. Therefore, the performance of SU services is restricted by the strict priority of PUs. In order to increase the system performance of the secondary network, dynamic spectrum access (DSA) methods incorporated with channel aggregation are introduced in many research attempts. In [13] channel bonding and aggregation methods are proposed for a multi-channel cognitive radio network in which the channels are not time slotted. Fig. 1.2 and 1.3 depict the results of the variation of blocking probability and forced termination probability in [13]. As shown in these figures, at higher arrival rates of primary and secondary users, both ongoing and new secondary users will experience service interruption and request blocking respectively. Even though channel aggregation shows better performance in terms of forced termination, it leads to increase blocking probability. Therefore, the necessity of an additional scheme to increase the overall performance arises.

With the evolution of research concepts, channel aggregation became an additive scheme which can increase the capacity of the Cognitive radio users. A majority of the dynamic channel access methods are associated

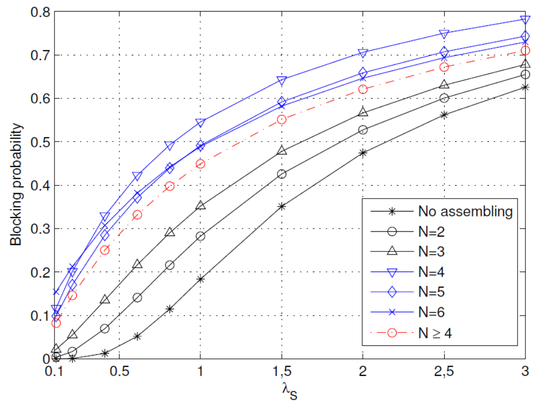


Figure 1.2: Blocking probability as a function of SU arrival rate, $\lambda_S$ [13].

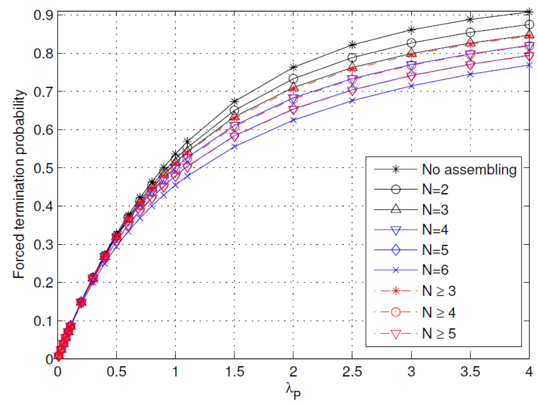


Figure 1.3: Forced termination probability as a function of PU arrival rate, $\lambda_P$ [13].

with channel aggregation strategies. The motivation behind this work is originated with the studies on existing DSA techniques. We studied dynamic channel aggregation techniques introduced by [22]. In that work, they have evaluated the system performance under different channel aggregation techniques in terms of capacity, blocking probability and forced termination probability. Due to the primary user arrivals, ongoing SUs have to terminate their transmission, if all other channels in the system have been occupied and there is no chance to share channels with other users. Forced termination probability increases dramatically, as the primary user arrival rate increases. And also new users have to be blocked if there are no ideal channels in the system. Blocking probability increases rapidly as the secondary user arrival rate increases. In order to experience better system performances these probabilities should be reduced as much as possible. Then the next natural question is "how we can reduce those probabilities to increase the system performance?"

This thesis proposes an integration of a queuing model with the existing Cognitive radio systems. The problem scenario of the proposed model is threefold.

- Decrease the call blocking probability of new CR service requests and the forced termination probability of ongoing non real-time services.
- Investigate the system performance in terms of secondary network's capacity and spectrum utilization.
- Address the the effect of queuing delay on system performance.

Consider a secondary user arrives to the system to start its service when there are no ways to obtain system resources. Then, the the SU service request is blocked by the system. However with the queuing model, blocked users are forwarded to the waiting positions of the queue until the required number of channels becomes idle. The services which have been interrupted due to PU arrivals also can be kept in the queue until free channels are available. However, certain types of interrupted services which need continuous transmission may not be queued since they need channel access without any interruptions. The queuing model proposed in this thesis is innovative in the sense that to the best of our knowledge, this is the first queuing model which analyses the system performance of a CRN which consists of the following features jointly.

- Dynamic spectrum access with channel assembling and spectrum adaptation techniques
- Multiple queues for heterogeneous traffic
- Priority based queuing disciplines
- Maximum queue size based on traffic type
- Queuing opportunity for forced terminated non-real time traffic

## 1.6 Thesis Definition

In this thesis, the main objective is to evaluate system performance of channel assembling strategies of multi-channel Cognitive radio networks which are integrated with queuing systems. As the initial task, we will study the existing channel assembling methods and their ability of spectrum adaptation. We target at introducing a new strategy for channel aggregation based on the existing methods. Then, we design a queuing system to improve the Cognitive radio network's spectrum utilization and the performance of the secondary network. M/M/C queuing model with First In First Out (FIFO) queuing discipline will be used in the design process of the targeted queue. The queuing model is designed in different approaches as it can be used to decrease the blocking probability of new users and minimize the forced termination probability of ongoing users. Queuing delay should be analyzed as the main cost introduced due to the queuing model. To evaluate all theses performance parameters mathematical expressions are derived by using Continuous Time Markov Chain (CTMC) modelling. Therefore the main tasks of the thesis can be summarized as follows.

- Study existing channel assembling methods and their ability of spectrum adaptation. At the end of this study, a new strategy for channel aggregation based on the existing methods is introduced. In addition, the existing queuing models which are designed for Cognitive radio networks are studied as the literature review. The disadvantages or limitations identified from those related work should be overcome at the proposed model of this thesis.
- Design a specific queuing model by considering heterogeneous traffic patterns to integrate with the existing systems. Since heterogeneous traffic is considered, multiple queues will be designed for different traffic types.
- Evaluate the system performance in terms of different parameters and validate those results by using MATLAB simulations.
- Produce convincing results based on the developed model which will be published later in a high quality conference or a journal.

## 1.7 Thesis Organization

The exposition in this thesis deals with the design of a queuing model with an improved dynamic spectrum access strategy for the existing cognitive radio networks which can enhance the system performance. A substantial section of this thesis has been concerned with Markov chain models and the numerical modelling of proposed queuing system. The thesis is conceived in 7 chapters and the main points of each chapter are summarized below.

**Chapter 1:**
In the first chapter, we briefly explain about the radio spectrum and the main issue associated with this resource, i.e., problem of spectrum scarcity. Available solutions for increasing the spectrum utilization are also described. Then, the Cognitive radio technology is explained briefly along with the concept of software defined radio. Finally the problem statement is presented with the thesis definition.

**Chapter 2:**
Chapter 2 introduces several dynamic spectrum access strategies already developed in Cognitive radio research field. In the first part of this chapter we discuss main functions of Cognitive radios and three types of DSA models. And several spectrum access strategies which have been developed with spectrum adaptation functions also discussed. The second part of Chapter 2 provides an in depth analysis of queuing theory basics and it introduces Kendall's notation with examples. The fundamental *Little's law* is explained which is an important principle in any kind of queuing analysis. The third part of this chapter explains the related work of this thesis. Different queuing models proposed by several research works are presented with appropriate comments.

**Chapter 3:**
Chapter 3 is devoted to explain the proposed queuing model with important assumptions. This chapter is an effort to highlight the proposed two queuing approaches, i.e., Case I and Case II. In addition to that, it also presents the important features of the queuing model in brief.

**Chapter 4:**
Chapter 4 contains a comprehensive analysis of the proposed queuing model and the dynamic channel access technique. In the first part of the chapter, functionality of the channel access technique over different PU and SU activities are described while the second part is allocated to explain the queue scheduling algorithm. The third part analyzes the state transitions of the Markov chain model and then closed form mathematical expressions to calculate the performance parameters of the proposed system are derived.

**Chapter 5:**
The numerical results are illustrated in Chapter 5. At the beginning of this chapter the accuracy of the derived theoretical models is assessed by simulations. Confidence intervals of the simulations are used in order to

emphasize the validity of the analytical models. The next sections of the chapter depict the system performance in terms of capacity, spectrum utilization, blocking probability, forced termination probability and average queuing delay. The optimum reasons for the behaviour of each performance parameter are explained. The system performance is compared between the proposed two queuing methods in order to determine the most suitable queuing mechanism for each parameter. In addition, analytical results are analyzed for different service distributions at the end of the chapter.

**Chapter 6:**
Chapter 6 recapitulate the major performance enhancements experienced by a Cognitive radio network as a result of the integration of the proposed queuing model. In addition, methods of controlling the associated queuing delay are discussed by highlighting the tradeoff between queuing delay and blocking probability. The effect of non-Poisson arrival pattern at the queue over the analytical models is also discussed.

**Chapter 7:**
Finally, Chapter 7 summarizes the main contributions and conclusions of the thesis and points out some directions for future research.

**Appendix A:**
The paper which is prepared based on this thesis work and which is planned to be submitted to *IEEE Transactions on Wireless Communications* most probably in Autumn 2012, is attached as **Appendix A**.

**Appendix B:**
The most important MATLAB codes of the developed programs are mentioned in **Appendix B**.

## 1.8 Chapter Summary

The Chapter began with highlighting the need for proper solution for the problem of spectrum scarcity. It was then explained that ISM frequency band and UWB as two radio systems which are developed to increase the spectrum availability for wireless applications. The chapter proceeds by bringing attention to the technologies of Cognitive radio and software defined radio. Motivate factors behind the thesis work is then explained by emphasizing the work done in [22]. In the problem statement, the problems of higher blocking probability and forced termination rate in an active primary and secondary user environment is conclusively illustrated. The main objective of the thesis is presented in the thesis definition and main activities are defined. The chapter ended by summarizing the thesis organization.

# Chapter 2: Basic Concepts & Related Work

*Cognitive radio can be considered as a new paradigm of wireless communication networks, to enhance the utilization of the radio frequency spectrum by opening up the under-utilized sections of the licensed spectrum for secondary usage. Cognitive capability and fluctuating spectrum are two main characteristics of CRNs. Due to the fluctuating nature of the available spectrum, dynamic spectrum sharing methods are used in CRNs. In the first part of this chapter we closely look at the available dynamic spectrum access methods for CRNs. During the second part of this chapter we analyze basic principles of general queuing systems which we considered as preliminary or necessary for understanding the analytical models in the next chapters. The last part of the chapter is dedicated for analyzing several channel access methods and queuing models which are already proposed for CRNs by different research attempts.*

## 2.1 Key Characteristics of CRNs

### 2.1.1 Cognitive Capability

In CR terminology, the temporary unused spectrum bands are known as *spectrum holes* or *white spaces*. The capability of identifying spectrum holes at a specific time or location is known as cognitive capability. As shown in the figure, according to the availability of the spectrum holes, secondary users have to dynamically access the channels [18].

### 2.1.2 Fluctuation of Spectrum

Communication resources available to CR users are highly dynamic. Because, the available spectrum is varying throughout the communication process due to the PU and SU activities. A licensed spectrum may be used by a PU at a time and it may become idle when the PU service is finished. Through the cognitive capability, SUs have to sense the spectrum and find idle bands which are not used by PUs at that time. When a PU appears on a channel which is occupied by an SU service, the interrupted SU has to vacate the channel immediately and find another idle channel. Therefore, the available spectrum for the SU services always changes depending on the user activities within the allowable spectrum band.

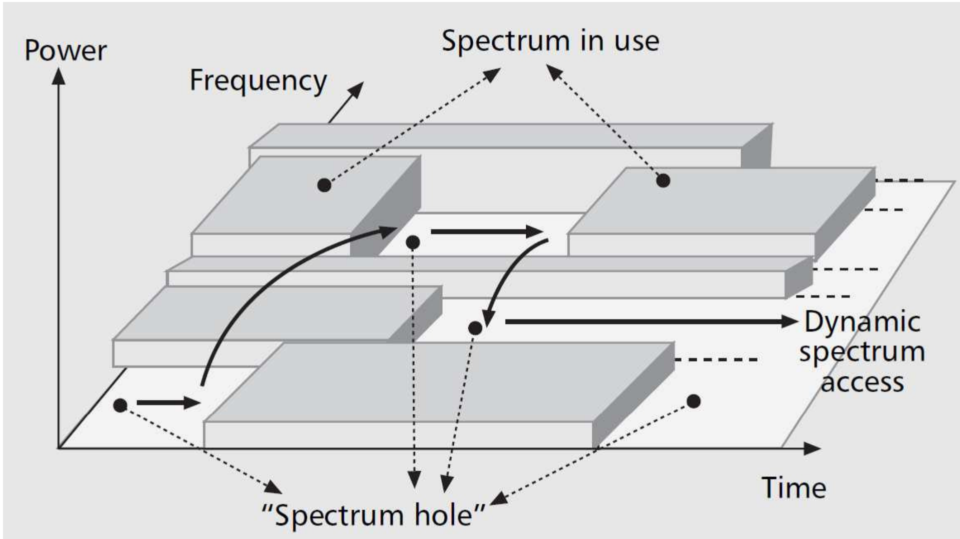


Figure 2.1: The Spectrum Hole Concept [18].

## 2.2 Functions of Cognitive Radio

The following main functions support intelligent and efficient dynamic spectrum access in CRNs. The main modules related with these functions are illustrated in Fig. 2.2.

- Spectrum sensing: By periodically sensing the intended frequency bands, CR users have to determine the status of the channels and events of the incumbent users. Spectrum sensing can be performed either by centralized base station or distributed architecture.
- Spectrum analysis: The data obtained through the proper spectrum sensing is used by the CR users to schedule and plan spectrum access. A decision to access a specific channel is made by optimizing the system performance in terms of capacity of the secondary network.
- Spectrum access: According to the decision made on spectrum analysis, white spaces are accessed by the unlicensed users. A cognitive medium access protocol is used to perform spectrum access by avoiding collisions with other users.
- Spectrum mobility: Based on the presence of incumbent users, unlicensed users need to change the operating frequency band which is known as *spectrum handover*. During the spectrum handover CR users should be able to continue their data transmission in the new band.

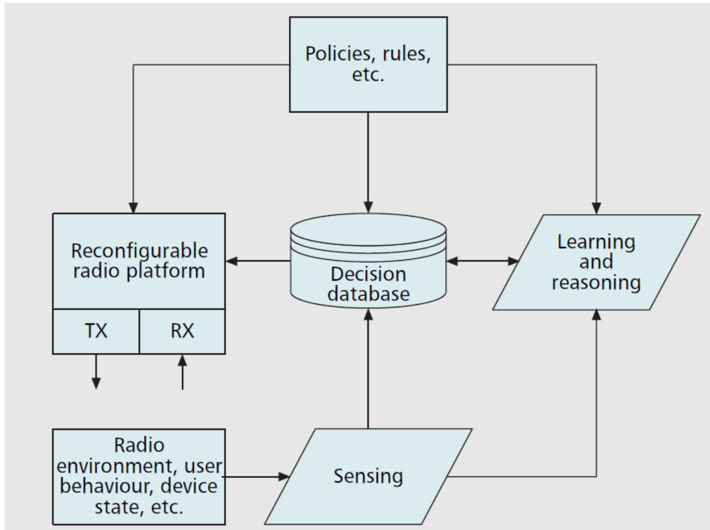


Figure 2.2: Cognitive Radio Functionality [19].

## 2.3 Dynamic Spectrum Access

The problem of underutilization of radio spectrum urges regulatory bodies to search for better spectrum management and allocation strategies. Thus, instead of the traditional static frequency allocation methods, dynamic spectrum allocation has become more stimulative in the research field. Dynamic Spectrum Access (DSA) strategies can be broadly categorized under three models [20].

- Open Sharing Model
- Hierarchical Access Model
- Dynamic Exclusive Use Model

In the open sharing model, the spectrum is open for access to all users. This model is already in use in the ISM band. In hierarchical access model, SUs can opportunistically access the channels when they are not occupied by PUs. However, in dynamic exclusive use model, a PU can grant access for his licensed frequency bands to an unlicensed user for a certain period of time. In this thesis our main attention is regarding the hierarchical access model since it is the most compatible approach with the legacy wireless systems. Moreover, this model can be categorized in to two approaches based on the transmission power regulations of the secondary networks.

- Spectrum underlay approach: If the PUs are occupying all the frequency bands almost all the time, SUs will not get an opportunity to transmit their data. To mitigate this disadvantage, in this case, the transmission power of the SU services keep below the interference temperature limit of primary users and signals are transmitted in a very wide frequency band like in UWB.

- Spectrum overlay approach: In overlay approach, there are no restrictions for the transmission power levels of SU services. But, SUs can access channels only during the absent period of the PU services [21].

## 2.4 The Poisson Process

The Poisson process is the main modelling tool used in this thesis for deriving mathematical expressions of the defined CR system with queuing models. The Poisson process is considered as a counting process that counts the number of incidents of a specific event through time and inter-arrival intervals have exponential distribution [23]. According to the definition of the Poisson process in [24], the counting process $\{N(t), t \geq 0\}$ is called a Poisson process with rate $\lambda$ if the inter-occurrence times $X_1, X_2, \cdots$ have a common exponential distribution function

$$P\{X_n \leq x\} = 1 - e^{-\lambda x}, \quad x \geq 0. \tag{2.1}$$

Therefore, in our analysis Poisson arrival process with exponential distributed service time is a main assumption regarding PU and SU arrivals and their service times. Later, in our queuing model, we have to examine an arrival process which consists of two types of arrivals. Thus, it is worth to analyze the following theorem on *merging of Poisson processes*.

**Theorem 2.1**
*Suppose that $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are independent Poisson processes with respective rates $\lambda_1$ and $\lambda_2$, where the process $\{N_i(t)\}$ corresponds to type $i$ arrivals. Let $N(t) = N_1(t) + N_2(t), t \geq 0$. Then the merged process $\{N(t), t \geq 0\}$ is a Poisson process with rate $\lambda = \lambda_1 + \lambda_2$.*

## 2.5 Fundamentals of Queuing System

There are several everyday examples that can be described as queuing systems, waiting line in a bank, computer networks, manufacturing systems and communications systems. A typical queuing system can be described considering three states, i.e., customers arriving for service, waiting for service if the servers are occupied and leaving the system after the service being finished. There is a standard notation for classifying queuing systems into different types based on the arrival pattern, distribution of the service time, queuing discipline and the number of servers. Since it was proposed by D. G. Kendall, it is named as *Kendall's Notation*.

### 2.5.1 Kendall's Notation

Kendall's notation is used to concisely define queue parameters. According to the Kendall's notation, a queuing system is described by the string A/B/X/Y/Z. However in most of the time, only first three symbols, i.e., A/B/X

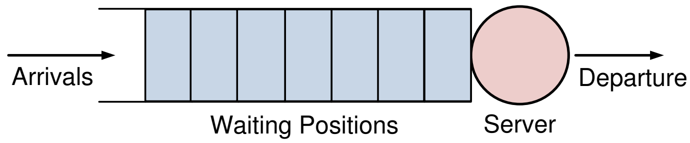


Figure 2.3: A Standard Graphical Notation for Queues.

is mentioned since they are the most important parameters to represent a queuing model. Table 2.1 indicates the meanings of each parameter with examples. In many practical situations, we assume that customers arrive according to a Poisson stream (exponential inter-arrival times) and their service times are independent and identically distributed. Service time can be deterministic or exponentially distributed. There may be a single server or a group of servers to process the service requests of the arrived customers. Practically, the number of waiting positions of a queuing system is limited. For example, a system with exponential inter-arrivals and service times, one server and having waiting rooms for $N$ customers is denoted as the four letter code M/M/1/N.

Table 2.1: Kendall's Notation.

| Parameter | Refers to | Examples |
|---|---|---|
| A | Inter-arrival distribution of the customers | M - Exponential, G - General<br>D - Deterministic |
| B | Service time distribution | M - Exponential, G - General<br>D - Deterministic |
| X | Number of parallel servers | $1, 2, \cdots, \infty$ |
| Y | Maximum number of customers allowed in the queue | $1, 2, \cdots, \infty$ |
| Z | Scheduling discipline or queuing strategy | FCFS, LCFS, Priority, RSS |

The rule that a server uses to select the next customer from the queue when the server is finished the service of the current customer is called queuing discipline. Commonly used queue disciplines are:

- FCFS - Service requests are served on a first-come first-serve basis. In FCFS, if a service request arrives when the queue is full then the system blocks the request.
- LCFS - Service requests are served in a last-come first-serve manner.
- Priority - According to the importance of the service, Service requests are served.
- RSS - Random Selection for Services
- FQ - Fair queuing does not consider the order or priority, but serve the service requests in a fair way.

## 2.6 Occupation Rate and Little's Law

The fraction of time that the server is working is called occupation rate or server utilization of the system. In some articles and books the terminology *traffic intensity* or *offered load* of the system also implies the same meaning. If the arrival rate is $\lambda$ and the mean service time is $1/\mu$, the occupation rate for a single server system $\rho$ is given by,

$$\rho = \frac{\lambda}{\mu} \tag{2.2}$$

Let the random variable $L(t)$ denotes the number of users in the system at time $t$ and let $S_n$ be the sojourn time of the $n^{th}$ user in the system. When the random variables $L$ and $S$ have the limiting distributions of $L(t)$ and $S_n$, then

$$\lim_{t\to\infty} \frac{1}{t} \int_{x=0}^{t} L(x)\, dx = E(L), \qquad \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} S_k = E(S), \tag{2.3}$$

where $E(L)$ and $E(S)$ denote the long-run average number of users in the system and the long-run average sojourn time respectively. Little's Law says that, the time average number of customers in a queuing system, $E(L)$, is equal to the rate at which customers arrive the system, $\lambda$ multiplied by the average sojourn time of a customer, $E(S)$ [25]. Therefore,

$$E(L) = \lambda E(S) \tag{2.4}$$

This equation is extremely useful in our analysis to calculate the average waiting time of a SU service inside a queue. And also it is more simple and general. This formula is general which means that, it can be applied for a queuing system regardless of the distribution of arrival pattern, distribution of inter-arrival times and number of servers in the system. The only requirement which needs to satisfy for applying Little's law is that the system should exist under steady state conditions.

## 2.7 Related Work - DSA Techniques

Hereafter, we describe three dynamic spectrum access techniques which have been implemented as spectrum overlay approaches. In those techniques, channel aggregation is a key feature which is used to increase the capacity of the secondary network. When there are more channels available in the target frequency band, SUs can utilize several channels instead of occupying a single channel. This can be performed in two ways depending on the availability of idle channels in the spectrum [26]. In the first case, channels which are adjacent to each other or contiguous channels (CH 3 and CH 4 in Fig. 2.4) could be bonded as one SU channel. On the other hand, channels which are not contiguous (CH 6 and CH 8 in Fig. 2.4) can be aggregated to form one SU channel. Therefore the difference between these two terms depends on the channel locations in the frequency spectrum. In this thesis, we use the term *channel aggregation (CA)* as a common representation for both ways.

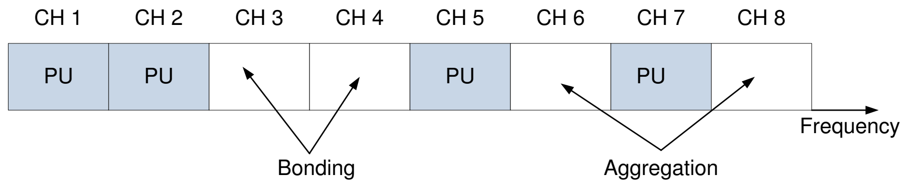


Figure 2.4: Channel Bonding and Aggregation.

In [12], two dynamic spectrum access strategies and one static strategy are proposed, and the performance of those strategies was evaluated in terms of forced termination probability and capacity of the secondary network. In the static strategy, once an SU service starts, the number of aggregated channels could not be changed. But in the dynamic strategies, the number of aggregated channels for an ongoing SU service is adjustable. Let $W$ and $V$ be the lower bound and the upper bound of the number of aggregated channels for SUs. $M$ and $T$ denote the total number of channels in the system and the total number of channels currently occupied (by SUs and PUs together) respectively. The main reason to select DSA techniques proposed in [12] for the discussion of this thesis can be explained as follows. The spectrum allocation methods proposed in [12] support dynamic spectrum access and they are featured with channel aggregation and real time spectrum adaptation. Spectrum adaptation is an important feature of spectrum access which means dynamically adapting to the spectrum status in a real-time manner in order to fully utilize the available radio resources. The DSA strategies proposed in [12] are explained under related work in subsection 2.7.1 - 2.7.3. Subsection 2.7.4 is devoted to give a short description of the main reference paper of this thesis.

### 2.7.1 Dynamic Fully Adjustable (DFA)

In DFA, if there are idle channels available upon a PU arrival, the interrupted SU will immediately release the channel to the licensed PU and continue its service on one of the idle channels. If there is no idle channel, the SU service which has the maximum aggregated channels will donate one channel to the interrupted SU service, given that it has more aggregated channels than the interrupted SU service and its remaining number is still not fewer than $W$. In the worst case, if all ongoing SU services have exactly $W$ channels upon a PU arrival, the interrupted SU service is forced to terminate.

On the other hand, when a new SU service arrives, the system should offer at least $W$ channels to accommodate the new arrival. Moreover, the new SU service may aggregate up to $V$ channels if there are enough channels available. If the number of idle channels is fewer than $W$, to accommodate the new SU service, the ongoing SU service with the maximum number of channels will donate channels as long as it can still have $W$ channels after donation. If the donated number is not enough, other ongoing SU services will collectively donate channels. If the number of idle channels plus the number that can be donated by ongoing SU services is still fewer than $W$, the new SU request is blocked.

As a result of service departures or forced termination of SU services, channels become idle. The ongoing SU service which has the minimum number of aggregated channels will utilize those idle channels up to $V$.

### 2.7.2 Dynamic Partially Adjustable (DPA)

DPA is different from DFA only regarding the PU arrival process. In DPA, when a PU arrives to a particular channel, if there is no idle channel in the system, the interrupted SU will adjust downwards its number of occupied channels. Furthermore, if PU arrives to a channel which is in use by an ongoing SU service with exactly $W$ channels and there are no other ideal channels, the interrupted SU service is forced to terminate. In other words, there is no channel sharing process with other ongoing SU services when a PU arrives in DPA. In fact, DPA has been designed for the purpose of reducing the complexity of channel assembling in DFA, since channel sharing involves more handshaking.

### 2.7.3 Static Strategy (STC)

In contrast with these two dynamic strategies which support spectrum adaptation, STC does not facilitate spectrum adaptation. In STC, upon the arrival of a new SU service, it can aggregate up to $V$ channels if $(M - T) \geq W$. When an arriving PU takes one of the channels currently used by a SU service, that SU service is simply dropped and all its occupied channels are released to the system. However, the newly released channels will not be re-allocated to other ongoing SU services, no matter whether they occupy $W$ channels or more.

In brief, DFA exhibits maximum flexibility of spectrum adaptation while DPA supports spectrum adaptation of all PU and SU activities except PU arrivals. Whereas STC does not allow spectrum adaptation for the sake of simplicity. According to the numerical results obtained in [12], dynamic strategies can achieve higher capacity in comparison with the static strategy.

### 2.7.4 Modeling and Performance Analysis of Channel Assembling in Multi-channel Cognitive Radio Networks with Spectrum Adaptation [22]

The channel assembling strategies proposed in [22] provides the basis for the dynamic channel aggregation strategy proposed for the queuing model in this thesis. In that work, two representative channel assembling strategies are proposed to evaluate the system performance considering heterogeneous SU traffic. According to the numerical results obtained in this paper, improved performances have been achieved by using dynamic channel aggregation in contrast with static channel aggregation. CTMC models for the Quasi-Stationary Regime (QSR), i.e., when PUs are relatively static compared with SU activities, are also introduced. In addition, the authors have evaluated system performance under various service distributions of PU and SU services.

## 2.8 Related Work - Queuing Models

Research interest on cognitive radio technology has grown greatly and most of the research works have dealt with performance improvement of secondary network while ensuring negligible interference for primary users. In most cases, performance of the cognitive radios has been evaluated with capacity and blocking probability of the secondary users. Also, a lot of scientific effort has been put into understanding and analyzing queuing systems for cognitive radio networks. However, the majority of those attempts mainly focus on the analysis of the sojourn time. Subsections 2.8.1 - 2.8.4 analyze the queuing models proposed by several scientific papers.

### 2.8.1 Opportunistic Scheduling for CRNs: A Queuing Analysis [27]

In [27], a queuing framework has been developed to analyze the performance of opportunistic access by CR users. The model is developed under infrastructure based dynamic spectrum access environment. The proposed scheme is assumed to be a time slotted with fixed slot size corresponding to the frame transmission. The arriving packets are modelled as a Batch Bernoulli process and the packets in the queue are served in FCFS order. By imposing resource constraints over new incoming connections the model allows to control the level of QoS. On the other hand, when the new CR user needs to start a new connection, the admission controller at the base station use the queuing model to decide the admission grant. Therefore, the proposed queuing model is utilized to achieving both QoS and admission control. The main focus of the numerical results is towards the mean packet delay and packet loss rate. The system performance has not been compared with a system without a queuing model.

### 2.8.2 Performance of a Cognitive Radio Network with Tolerable Service Degradation [28]

When a secondary user detects a primary user's arrival, it immediately vacates the channel and move in to another idle channel, if available. But if the CR users fail to detect the presence of the primary call, both users have to face with degraded service due to high interference level. In [28], several performance metrics are derived based on a queuing network model. Indeed this paper analyzes the performance of primary network by evaluating the service degradation experienced by primary users due to secondary user's incorrect spectrum sensing. A notable feature of this model is that each channel in the system is considered as a single server queuing system. A simple equal probability channel assignment strategy is assumed and the queued secondary calls access channels according to the FCFS order.

As shown in Fig. 2.5, total secondary user arrival rate to the queue has two parts. External (new) user arrival rate and the internal arrival rate from the other channels due to the SU switching among channels when PU arrivals take place. One of our comments on this model is that the arrival process to the queuing model cannot be considered as a Poisson process. Thus, an analytical proof or simulation confirmation is required to validate the solution.

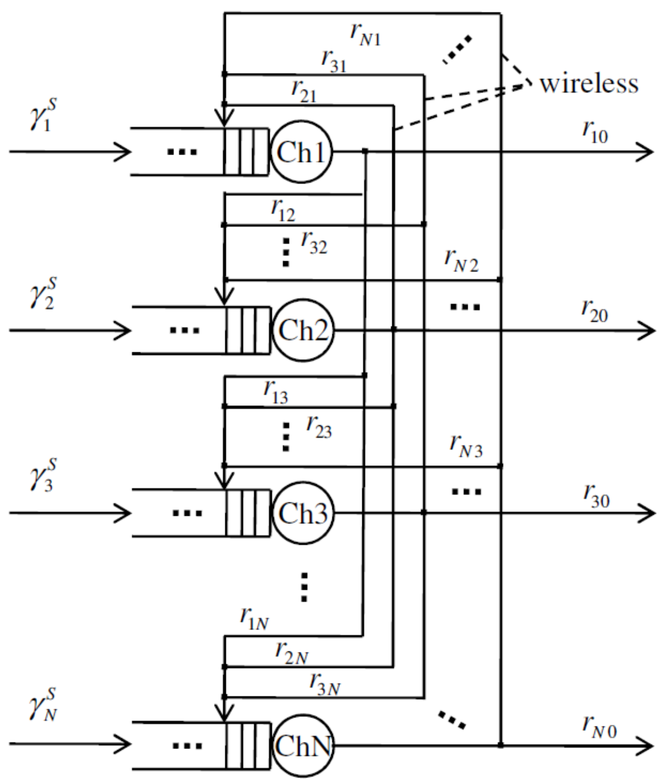


Figure 2.5: Queuing network model proposed in [28].

### 2.8.3 Markovian Queuing Model for Dynamic Spectrum Allocation in Centralized Architecture for Cognitive Radios [14]

In the proposed queuing model for a CR network in [14], access latency for the secondary users is evaluated as the main QoS parameter. In this solution, two queues are introduced named as secondary user queue (SUQ) and bandwidth allocation queue (BAQ). The queue which stacks the all SU arrivals referred as SUQ which is modelled as an M/M/1 queue and the SU requests entering the SUQ are served in FCFS order. In order to distribute the number of frequencies for the PUs and SUs, BAQ is used. A special feature of the BAQ is that both PU and SU requests are queued, so FCFS order could not be applied for BAQ since PUs have the strict priority over SUs. Due to the fact that the different types of services entering the queue, BAQ is modelled as M/G/S/N loss queuing model where $S$ is the number of available idle channels and $N$ denotes limited BAQ capacity. However, for the simplicity they have considered the special case where $S = N$. Access latency, i.e., the probability that the total time spent by a SU request to get the access grant and the blocking probability, i.e., the probability that an SU request will be denied are evaluated. But, in this work there is no comparative analysis with respect to a system without a queuing model.

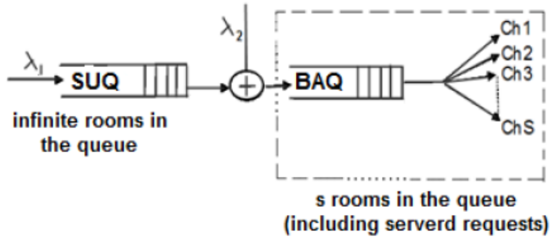


Figure 2.6: Queuing network model proposed in [14].

### 2.8.4 The Impact of Queue on Opportunistic-based Cognitive Radio Networks [29]

The effect of using a queue over an opportunistic based cognitive radio networks is investigated in [29]. The main target of the paper is to analyze the increase of the spectral efficiency by putting SU services in a queue. Both PU and SU networks are assumed to be infrastructure based and resource allocation is performed with access points. A simple channel access strategy is used without considering channel assembling. However, a system without queuing model is also considered in this work and numerical results obtained through the model are validated from the simulation results. According to the Fig. 2.7, the channel utilization of the proposed system has increased in contrast with the system without queues. However, the percentage of increment is not more than 5%. We believe that the usage of channel assembling techniques which support real time spectrum adaptation will further increase the performance of the queuing model.

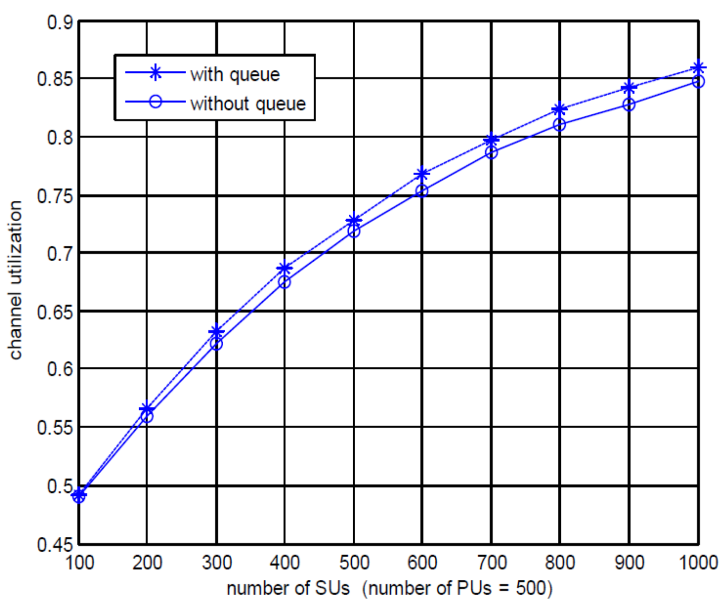


Figure 2.7: Channel utilization in two cases of with and without queue in [29].

## 2.9 Chapter Summary

At the beginning of this chapter we describe the basic characteristics and functions of cognitive radios. Moreover, the dynamic spectrum access methods are introduced by emphasizing its necessity in cognitive radio networks. The theoretical aspects of a queuing model are briefly explained in order to provide the necessary background of the queuing model proposed in the next chapter. One of the critical parameters, i.e., arrival process is highlighted by commenting the Poisson process since it is an important assumption when deriving the analytical models. The DSA techniques presented in the next part of this chapter provided us with a required background to the channel access strategy proposed in Chapter 4. We close the chapter by presenting several queuing models which are already proposed through several research papers which have been published in high quality conferences and journals. The limited number of features of associated channel access strategies and the limited area of performance evaluation are identified as restrictions of those models.

# Chapter 3: Proposed Priority-based Multiple Queue System (PMQS)

*Several queuing models developed for CRNs are analyzed in the previous chapter. By avoiding several limitations and restrictions of those models, we propose a priority based queuing approach for heterogeneous traffic in CRNs. Our proposed model consists of multiple queues for different service types. Channel access priorities for the queued customers are divided according to the service type. Chapter 3 will give the reader a complete understanding of the proposed queuing model and an entrance to the analytical models described in Chapter 4.*

## 3.1 System Model and Assumptions

In making an analytical model for a real-world application, it is always essential to make certain simplifying assumptions to make the mathematics tractable. On the other hand, too many assumptions are not recommended, since it would not be fitted with the real situation. Thus, in this thesis we made several assumptions which can represent a CR system reliably. We consider an infrastructure based cognitive radio system consisting a central base station (CBS) and multiple CR users. We consider two types of networks, i.e., primary and secondary networks. A primary network has the full priority to designated frequency bands and a secondary network can only access a frequency band when the corresponding primary network does not use that band. A primary network can be any licensed-band network such as TVs, radar or satellite. An unlicensed-band network such as an IEEE 802.11 wireless LAN can be a secondary network.

Resource allocation for the secondary network is performed by the CBS. The licensed spectrum band is divided in to $M \in \mathbb{Z}^+$ frequency channels and those channels are allocated for $M$ licensed users (primary users) and all channels are assumed to be statistically identical. Here $\mathbb{Z}^+$ denotes the set of positive integers. When these channels are not occupied by primary users, secondary users have the choice to access the channels with equal probability. Even though a primary user occupies only one channel for each service, secondary users can aggregate several channels to complete the service with high data rate. Both channel bonding and channel aggregation is allowed for SU services. Upon a PU service arrival, it will access one of the channels which are not occupied by other PUs with the same probability. The reliable detection of PU presence is an important feature of the secondary network [30]. Therefore, we assume perfect time synchronization and channel sensing capability in the CR network. A secondary user can detect the presence of a PU when a PU accesses a channel. Thus, the probability of channel sensing errors of the system is considered negligible. Database service is periodically updated with channel and queue status. The communication between the CBS and the database is maintained regularly. For example, if a new SU service arrives when there are no chances for accessing a channel, then CBS contact the database to check the availability of free positions in the queue. If free waiting positions are available in the queue, the new SU service is queued and the change is updated in the database.

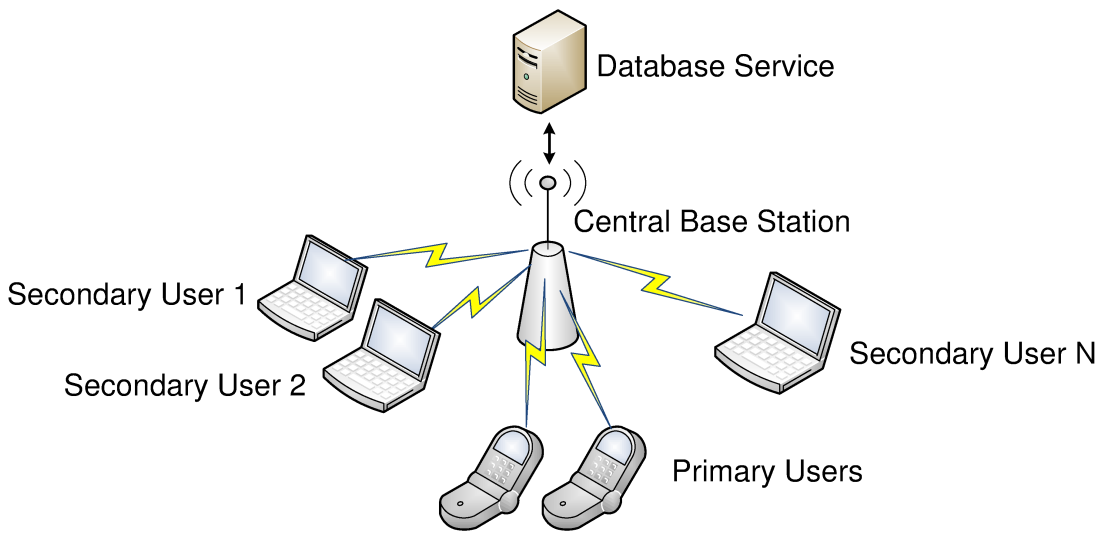


Figure 3.1: Infrastructure-based cognitive radio architecture.

## 3.2 Formulation of PMQS

In this thesis we propose multiple queues for different types of traffic distributions. In our analysis, we consider two types of SU traffic, i.e., elastic traffic and real time traffic. It is worth mentioning that the terminology *ESU* and *RSU* in this thesis are used to represent elastic SU services and real-time SU services respectively. However, the term *SU* commonly represents the both SU services. Elastic flows such as file transfer and email adjust their sending rate according to the existing network conditions. Therefore, in elastic traffic the service time can be reduced if more channels are aggregated. Real-time applications, such as video streaming and voice over IP [31] do not adjust its throughput in response to network conditions. Therefore, in our analysis the number of channels allocated for a real time service is fixed. That means, real time services are not fully adapted to the channel state changes. Other than this, the following assumptions are made in order to develop our analytical model.

- The arrivals of both PU and SU services are Poisson distributed with arrival rates $\lambda_P$, $\lambda_{S1}$ and $\lambda_{S2}$ for PU, ESU and RSU services respectively.
- The service time for both PU and SU services is exponentially distributed with service rates per channel $\mu_P$, $\mu_{S1}$ and $\mu_{S2}$ for PU, ESU and RSU respectively.

- All channels are homogeneous. Thus, the service rate of $k$ aggregated channels in secondary network equals $k\mu_S$ for elastic traffic.
- Services of the secondary network are independent of each other.
- The sensing and spectrum adaptation latency is negligible in comparison with the duration between consecutive service events.

The heterogeneous traffic considered in this thesis is mainly categorized into elastic and real time traffic. Therefore, we propose two separate queues for those SU services. The queue named as *EQ* is used for elastic traffic and the queue named as *RQ* is used for real time services. Since real time applications have the priority for channel access, *RQ* is considered as the *high priority queue* and *EQ* is considered as the *low priority queue*. Even though the users in RQ are assigned with higher priority, the priority level can be changed according to the service requirements. This feature will be discussed in Section 4.3.

### 3.2.1 Case I: Queuing Network Model without Feedback Link

Two queuing methods are proposed based on the user arrivals to the queues. In Case I, only the new users are allowed the access to the queuing system. Therefore, new ESU services can access waiting positions in the low priority queue and new RSU services can access waiting positions in the high priority queue. Already commenced SU services have no permission for accessing queues due to any reason. Fig. 3.2 illustrates the queuing model proposed with Case I.

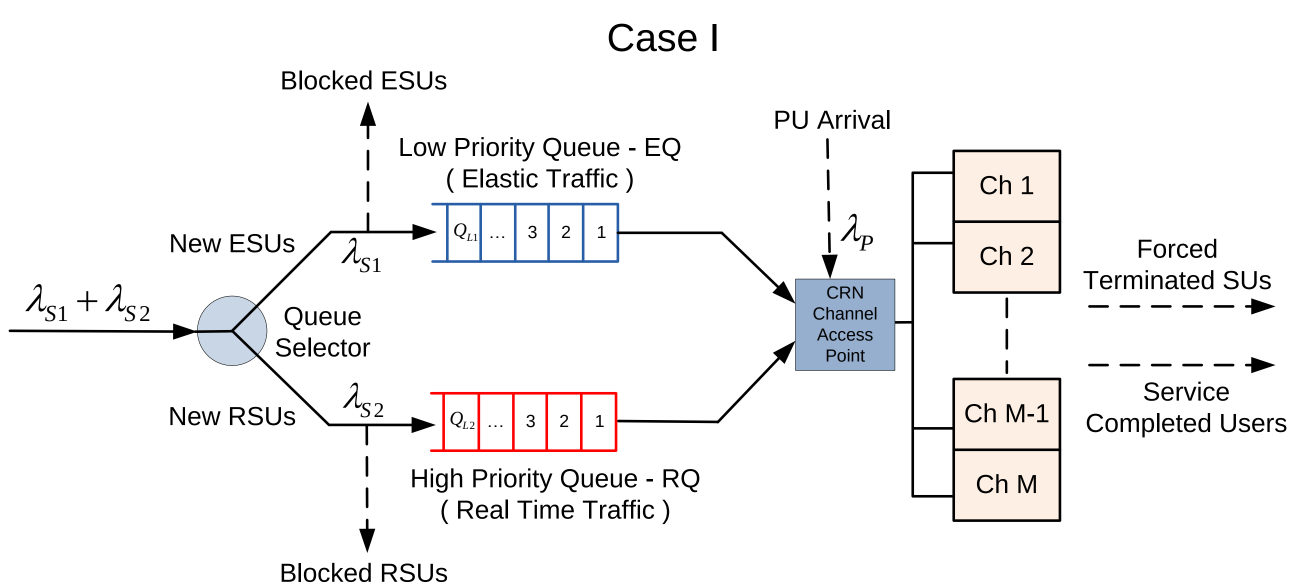


Figure 3.2: Proposed Queuing Model - Case I

### 3.2.2 Case II: Queuing Network Model with Feedback Link

But, in Case II, interrupted ESU services due to PU service arrivals, which cannot find any idle channel in the system by means of channel adaptation are also authorized to access EQ in addition to the new ESU services. In that case, *they are not actually forced terminated, but temporarily stopped the transmission*. However, if there is no enough waiting rooms in EQ, they are considered to be forced terminated. Other than this the rest of the functions of Case II are similar to Case I. Therefore, the interrupted RSU ongoing services will not be queued even though they cannot find an idle channel. Fig. 3.3 illustrates the queuing model proposed with Case II.

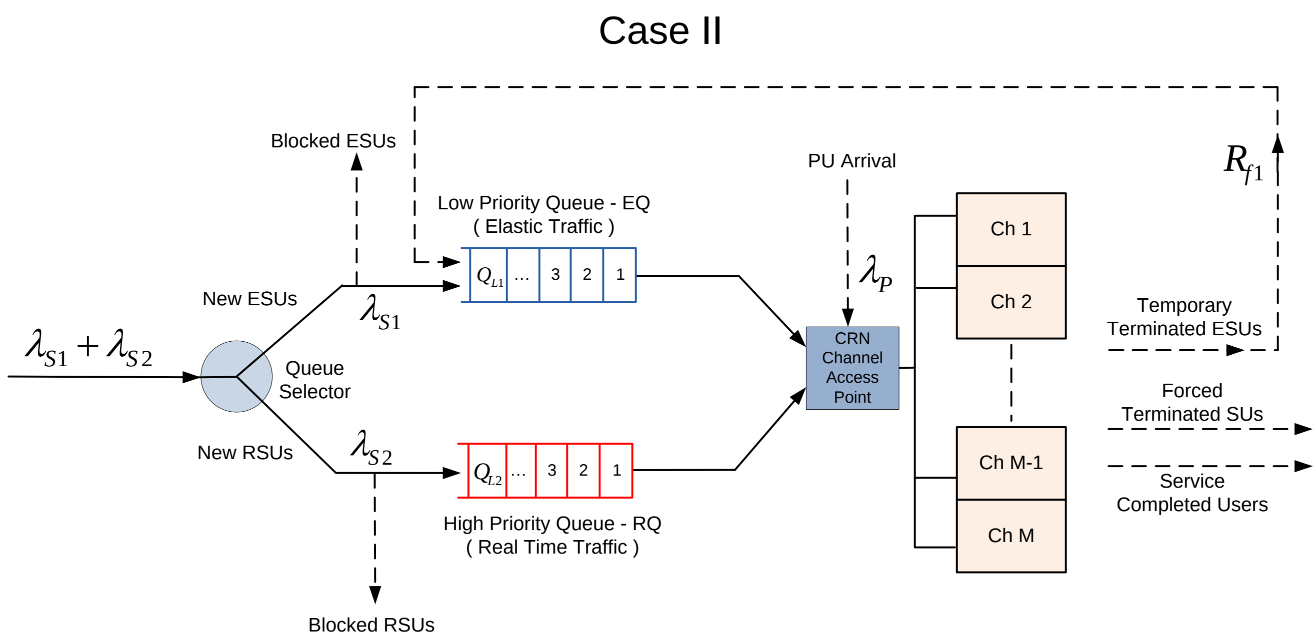


Figure 3.3: Proposed Queuing Model - Case II

Prior to presenting the features of these queuing models, the following notations are summarized.

$\lambda_{S1}$ = ESU arrival rate
$\lambda_{S2}$ = RSU arrival rate
$\lambda_P$ = PU arrival rate
$R_{f1}$ = ESU temporary termination rate
$M$ = Number of channels in the system
$Q_{L1}$ = Maximum queue size of low priority queue
$Q_{L2}$ = Maximum queue size of high priority queue

## 3.3 Basic Elements of Proposed Scenarios

### 3.3.1 Queue Selection

When a new SU service arrives, the queue selector (QS) categorizes the SU request as a low priority services or a high priority service. The categorization algorithm is defined by considering service type and their quality of service requirements. For example, if the newly arrived request is associated with VoIP service, QS identifies it as a real time application and directs it into high priority queue, i.e., RQ. Likewise all elastic traffic services are directed in to EQ and all real time applications are directed in to RQ. Even though the implementation of queue selection algorithms impose an additional cost on the operation, it is necessary to decide the application type because, multiple queues in the system operates with different priority levels. Otherwise, incoming SU services can decide the queue direction by using their own criteria, but it is not recommended for a infrastructure-based network architecture.

### 3.3.2 Maximum Queue Length

In this queuing model, we define maximum queue lengths, i.e., the number of waiting rooms for each queue. Real time applications such as VoIP and online video clips have strong QoS requirements such as minimum delay, exact synchronization and jitter of the information [32]. A large number of waiting rooms lead to large queuing delays. Therefore, the high priority queue, RQ should be equipped with less number of waiting rooms compared to the low priority queue. Non real time traffic such as electronic mail, file transfer, HTTP and backups benefits from large queues. The bigger the queue, it is less likely that packets will be dropped off the end of the queue. Therefore, the low priority queue, EQ is equipped with comparatively large number of waiting rooms. However, the average waiting time of an elastic traffic can be higher than that of real time traffic.

### 3.3.3 CRN Channel Access Point

One of the basic assumptions of this model is that the homogeneous spectrum environment. The spectrum is considered with licensed bands and each primary user has its own licensed frequency channel. And also, PUs have the strict priority for accessing channels at any time over SUs. Thus, PU services are not subjected for queuing and they are directly pointed to the respective channels. The decision of the CRN channel access point regarding the secondary network's channel access depends on the dynamic channel access strategy used in the CR system. Several channel access strategies are explained in the Section 2.7.

### 3.3.4 Queuing Discipline and Priority levels

If we consider the total SU arrivals, heterogeneous traffic is observable. But when we consider each queue separately, they process homogeneous traffic. Queue, EQ accepts only elastic traffic and queue, RQ accepts real time applications only. For homogeneous traffic, FCFS order is applied. Therefore, service requests in the both queues are processed with FCFS manner in which the first service in the queue is the first service that is processed. However, when channel access opportunities are available, that opportunities are divided between EQ and RQ with different priority levels. For example if 6 channels become idle, 4 channels are assigned for the real time services in RQ and the rest is assigned for the elastic traffic in EQ when the ratio of priority levels of RQ and EQ is given as 2:1. Due to this fact, the terminology *priority based multiple queue model (PMQS)* is used for representing the proposed solution. Otherwise none of the queues are processed with *priority queuing discipline*[1].

## 3.4 Chapter Summary

Even though the length of this chapter is comparatively short, it is worth remembering that the topics covered in the chapter is very important since it provides the description about the solution presented in this thesis. The basic and underlying assumptions made while designing the queuing model are listed prior to the presenting the solution. The proposed queuing model is explained in detail with perfect illustrations. It is classified in to two queuing approaches referred to as Case I and Case II. Case II is introduced as a suggestion to reduce the forced termination probability of the ongoing secondary user services. The chapter proceeds with the discussions on important features associated with the proposed queuing model. The need of a maximum queue size for each queue is also emphasized by considering the queuing delay. Finally, the term *PMQS* is introduced to the thesis reader as the designation used to represent the proposed queuing model.

[1]In priority queuing discipline, each service request is assigned a priority, and the service request with the highest priority goes first, regardless of the order of arrival.

# Chapter 4: Analytical Models

*In the first part of this chapter, we describe the channel access strategy designed for the CRN with the proposed queuing model. The latter part of the chapter is allocated to develop analytical models of the system after the analysis of state transitions in CTMCs. We derive mathematical equations to evaluate the system performances in terms of various parameters.*

## 4.1 Proposed Dynamic Channel Access Strategy

The channel access strategy that is utilized for the cognitive radio networks described in this thesis is based on the channel assembling strategy proposed by [22]. In [22], heterogeneous traffic is considered however, without any queuing approach. The dynamic channel assembling proposed in [22] uses both spectrum handover and adjustable channel aggregation. The DSA strategy proposed in this thesis is denoted as *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ and it is designed by combining the following three schemes.

- DSA methods proposed in [22] for heterogeneous traffic.
- The channel aggregation method mentioned in Section 2.7.1, i.e., DFA strategy.
- Newly designed channel access scheme for the queuing model.

## 4.2 *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$

Consider a CR network with $M$ PU licensed channels, where secondary users contend for the channel access when PUs are inactive. An ESU service can aggregate or bond several channels to increase its service rate. The lower and upper bounds of the number of aggregated channels for an ESU service are denoted as $W$ and $V$ where $W, V \in \mathbb{N}^+$. Therefore, an ESU can aggregate up to $V$ channels if it is available. However, ESU services are not processed unless it can aggregate at least $W$ channels. Depending on the channel status and the user activities, an ESU can dynamically adjust their number of aggregated channels. But for an RSU service the number of aggregated channels is always fixed. In our notations, $a \in \mathbb{N}^+$ represents the number of channels aggregated by an RSU service.

If a channel is currently used by a certain user, we called *the channel is in busy state*. If a channel is not occupied by any user then *the channel is in idle state*. The CRN described in this thesis involves three types of service events, i.e., PU, ESU and RSU arrival and departure. The channel accessibility and channel adaptation policies related to those service events are explained separately.

### 4.2.1 System Functions on PU Activities

In this subsection, we describe the system state adjustments performed due to the new arrivals of PU services and their departures. In order to illustrate the channel adaptation and queue allocations of the system, we use graphical representations for each description of specific events. Let $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ be the general state representation of the system where,

$j_{pu}$ = No. of PU services in the system at state $l$,
$j_k$ = No. of ESU services with $k$ aggregated channels at state $l$.
$g_a$ = No. of RSU services with $a$ aggregated channels at state $l$.

Consider a CR system with $M = 8, W = 1, V = 3$ and $a = 1$. The maximum queue sizes of the EQ and RQ are $Q_{L1} = 4$ and $Q_{L2} = 2$ respectively. The following depiction is used for represent the whole frequency band and the queues of the system.

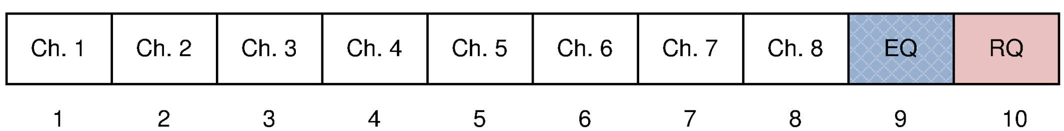


Figure 4.1: A graphical notation of the channels and queues of the system.

#### PU arrivals

When a PU arrives to a channel which is currently in idle state, that PU can start the transmission without any conflict and the secondary network will not face with any change. But if the PU arrival is occurred towards a channel in busy state due to an SU service with $k_1, W \leq k_1 \leq V$ aggregated channels, the interrupted SU has to vacate the channel immediately. The ESU service with the maximum number of aggregated channels, $k_2$ donate a channel for the interrupted SU service as long as its' remaining number of aggregated channels is still greater than or equals $W$, where $k_2 > k_1$. Otherwise, if there is no such ESU service which has $k_2$ aggregated channels where $k_2 > k_1$ then an interrupted ESU flexibly adjust downwards the number of assembled channels, as long as the remaining number of aggregated channels is still greater than or equals $W$. But if the interrupted SU service is an RSU, it is forced to terminate. When the interrupted ESU has exactly $W$ channel and there is no other ESU service which has more than $W$ aggregated channels, then it is forced to terminate if Case I is used for queuing model. If Case II is applied over the queuing process, that ESU service is temporary stopped and put in to EQ as long as EQ is not fully utilized. In the worst case, i.e., if EQ is also fully occupied, then corresponding ESU is forced to terminate. An important feature that should be mentioned about RSU services is that they do not donate channels for other SUs.

Now consider the state $x = \{j_1 = 1, j_2 = 2, j_3 = 0, j_a = 1, j_{pu} = 2, j_{eq} = 2, j_{rq} = 1\}$, i.e., there is an ESU services with single channel ($ESU_3$), two ESU services with 2 aggregated channels ($ESU_1$ and $ESU_2$), an RSU service and two PU services ($PU_1$ and $PU_2$). And also currently there are 2 ESU services and one RSU service are waiting inside EQ and RQ respectively. This scenario is illustrated in the following state diagram.

$$x = \{1,2,0,1,2,2,1\}$$

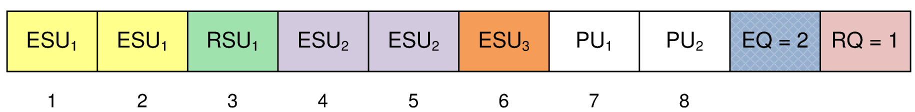


Figure 4.2: A graphical notation of the system state $x = \{1, 2, 0, 1, 2, 2, 1\}$.

If a PU arrives to Ch. 6, when the system is in the state shown in Fig. 4.2, $ESU_3$ has to vacate the channel and find an accessible channel according to the above described regulations. $ESU_1$ and $ESU_2$ has the maximum number of channel aggregation and they have more than $W$ channels. Therefore one of those services will donate a channel to the interrupted $ESU_3$. If $ESU_2$ donate a channel the output state of the system is shown in Fig. 4.3.

$$x = \{2,1,0,1,3,2,1\}$$

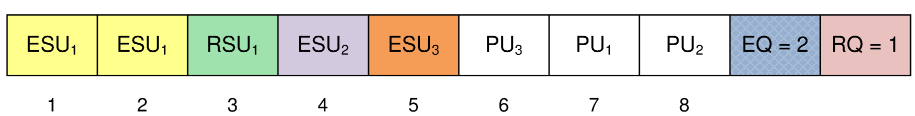


Figure 4.3: Channel adjustment after PU arrival to Ch. 6 in state $x = \{1, 2, 0, 1, 2, 2, 1\}$.

**PU departures**

Upon a departure of a PU service, the corresponding channel changes its state from busy to idle. When an idle channel appeared, high priority of accessing that channel is given to the waiting services in RQ. According to the FCFS discipline, the RSU service in the front end of RQ will get the chance to commence its service in the idle channel. If there are no real time services queued in RQ or $a > 1$, elastic traffic in EQ will receive the opportunity in the similar way if $W > 1$. If both queues will not access the channel then the ongoing ESU service which has the minimum aggregated channels, i.e., $k_{min}$, where $k_{min} < V$, can aggregate the new channel. For example, suppose the PU service, $PU_1$ departed when the system is in the state $x = \{j_1 = 1, j_2 = 2, j_3 = 0, j_a = 1, j_{pu} = 2, j_{eq} = 2, j_{rq} = 1\}$ as shown in Fig. 4.2. Since the first priority is given to the services in RQ according to the queue scheduling algorithm explained in Section 4.3, the front end RSU service ($RSU_2$) in RQ get the access for Ch. 7 as illustrated in Fig. 4.4. The queue, RQ becomes empty after this channel adjustment.

$x = \{1,2,0,2,1,2,0\}$

| ESU₁ | ESU₁ | RSU₁ | ESU₂ | ESU₂ | ESU₃ | RSU₂ | PU₂ | EQ = 2 | RQ = 0 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | |

Figure 4.4: Channel adjustment after PU departure in state $x = \{1, 2, 0, 1, 2, 2, 1\}$.

### 4.2.2 System Functions on SU Activities

The performance of the secondary network is not only relying on the PU activities, but also on the other SU events.

**SU arrivals**

When a new ESU service arrives, the service can commence if there are at least $W$ idle channels in the system whereas upon a RSU service arrival, system should allocate exactly $a$ channels. However, if there are not enough idle channels for newly arrived SU services, the ongoing ESU service which has the maximum number of channels will donate channels to the new users. In that case, after donation ongoing ESU should be able to keep at least $W$ channels, i.e., the lower bound of the channel aggregation. If the one with the maximum number cannot provide $W$ channels for new ESU service or $a$ channels for new RSU service by itself, the next ESU with the second maximum number will donate its channels and so on. If the ongoing ESUs collectively cannot provide the required number of channels, then the new service is put in the corresponding queue (ESUs are directed to EQ and RSUs are directed to RQ). However, if the queue is fully occupied the new service is blocked. One important fact to be mentioned here is that ESU services are not put in RQ even at a situation when EQ is fully occupied while RQ is empty. The same rule is applied for RSU services. As an example, consider an ESU arrival when the system in state $x = \{j_1 = 3, j_2 = 0, j_3 = 0, j_a = 2, j_{pu} = 3, j_{eq} = 4, j_{rq} = 2\}$ as shown in Fig. 4.5.

$x = \{3,0,0,2,3,4,2\}$

| ESU₁ | ESU₂ | ESU₃ | PU₁ | PU₂ | RSU₁ | RSU₂ | PU₃ | EQ = 4 | RQ = 2 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | |

Figure 4.5: A graphical notation of the system state $x = \{3, 0, 0, 2, 3, 4, 2\}$.

In this state, all the channels are occupied by licensed and unlicensed users and both queues are fully occupied. Since each ESU service has only one channel, there is no possibility for donating channels. Thus, the new SU request is blocked. On the other hand, if an RSU service arrived when the system is in state $x = \{j_1 = 2, j_2 = 2, j_3 = 0, j_a = 1, j_{pu} = 2, j_{eq} = 2, j_{rq} = 1\}$ as shown in Fig. 4.2. In this case, either $ESU_1$ or $ESU_2$ will donate a channel to the new $RSU_2$ and the resultant state is depicted in Fig. 4.6.

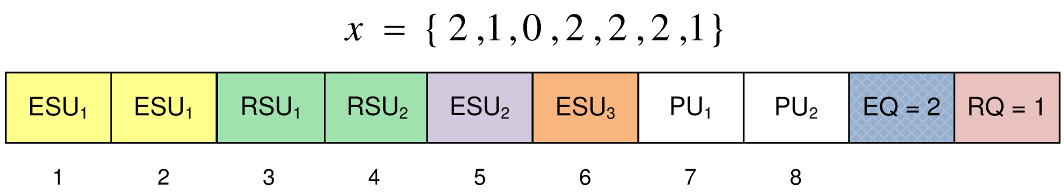


Figure 4.6: Channel adjustment after RSU arrival in state $x = \{1, 2, 0, 1, 2, 2, 1\}$.

### SU departures

Due to ESU and RSU departures, one or more channels become idle. The opportunity of accessing those channels divided between the waiting RSU and ESU services in the queues by using the queue scheduling algorithm explained in Section 4.3. By varying the priority factor $\beta$ the number of allocated channels for each queue can be altered. If one of the queues is empty, then all the idle channels are allocated for the services of other queue. If there are channels still in idle state even after all the waiting services in the queues obtain channel access, then the ongoing ESU service which has the minimum number of aggregated channels have the chance to aggregate them up to $V$. Since real time services should possess fixed number of channels, RSUs will not assemble newly idle channels. If the ESU with minimum number of channels aggregate $V$ channels and there are still vacant channels, the other ESUs will occupy the remaining ones by following the same principle.

For example, consider the departure of $ESU_2$ service when the system is in state $x = \{j_1 = 1, j_2 = 2, j_3 = 0, j_a = 1, j_{pu} = 2, j_{eq} = 2, j_{rq} = 1\}$ as shown in Fig. 4.2. In this case, 2 channels become idle. If $\beta = 2$ is applied in queue scheduling algorithm, $n_r = n_e = 1$. Therefore, these two idle channels are equally divided between the queues, so that the output state is depicted in Fig. 4.7. The waiting services at the front end of EQ and RQ queues are denoted as $ESU_4$ and $RSU_2$ respectively.

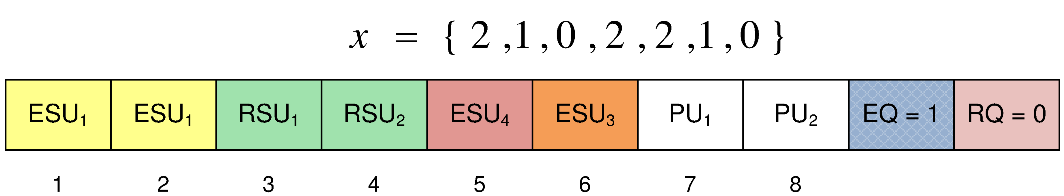


Figure 4.7: Channel adjustment after $ESU_2$ departure in state $x = \{1, 2, 0, 1, 2, 2, 1\}$.

When the queue scheduling algorithm allocates a certain number of channels for SU services in the queues, they are distributed among those SU services such a way that a maximum number of SU services will get the chance for service commencement. For example, if 10 channels are allocated for EQ when there are 5 ESU services in that queue while system parameter is $W = 2$, then all the ESU services in EQ will get the chance. Because the allocated number of channels is sufficient for all five ESU services in EQ in order to have the minimum requirement, i.e., 2 channels for service commencement.

## 4.3 Queue Scheduling Algorithm

Due to the departures of SU and PU services and forced termination of SU services, channels become idle in the system. Idle channels are allocated for the SU services waiting in the queues and ongoing ESU services. Ongoing ESU services get the opportunity for aggregating newly idle channels after the channel allocation procedure for the services in queues. If all the idle channels are allocated for the SU services in the queues, then channels are not allocated for ongoing ESU services. As mentioned in the previous chapter, the real time application requests in RQ have more priority than the elastic traffic requests in EQ. Let $\beta : 1$ be the ratio between priority levels assigned to RQ and EQ respectively where, $\beta \in \mathbb{Q}^+$ and $\beta \geq 1$. Here, $\mathbb{Q}^+$ represents the set of positive rational numbers. According to the QoS requirements of both elastic and real time traffic, the proper value for $\beta$ is determined. Although, a certain number of channels are initially available for the RSU services in the RQ, the maximum number of required channels to reach the upper bound of channel aggregation for all RSU services may be different. This fact is directly depend on the dynamic channel access strategy, *DSA* being used, minimum channel requirements and the current queue size of RQ. A similar argument is applied for ESU services in EQ. Consider the following notations which are used in the proposed queue scheduling algorithm.

| | | |
|---|---|---|
| $n$ | $=$ | The number of newly idle channels |
| $a$ | $=$ | The number of channels aggregated by an RSU service |
| $\beta : 1$ | $=$ | Ratio between priority levels assigned to RQ and EQ respectively, $\beta \geq 1$. |
| $n_r$ | $=$ | The number of channels available for waiting RSU Services in RQ<br>This parameter is calculated by the *line 2* or *line 5* in *Algorithm I* |
| $n_e$ | $=$ | The number of channels available for waiting ESU Services in EQ<br>This parameter is calculated by the *line 3* or *line 6* in *Algorithm I* |
| $n_{rq,\, max}$ | $=$ | The maximum number of channels required by all the RSU Services in RQ<br>This parameter is corresponding to the *line 8* in *Algorithm I* |
| $n_{eq,\, max}$ | $=$ | The maximum number of channels required by all the ESU Services in EQ<br>This parameter is corresponding to the *line 9* in *Algorithm I* |
| $n_{eq,\, min}$ | $=$ | The minimum number of channels required by at least one ESU Service in EQ<br>This parameter is corresponding to the *line 10* in *Algorithm I* |
| $n_E$ | $=$ | The number of channels actually used by waiting ESU Services in EQ |
| $n_R$ | $=$ | The number of channels actually used by waiting RSU Services in RQ |
| $n_O$ | $=$ | The number of channels allocated for ongoing ESU Services |
| $j_{eq}$ | $=$ | The number of ESU services currently in EQ |
| $j_{rq}$ | $=$ | The number of RSU services currently in RQ |

The allocation process of the $n$ newly idle channels among the services in EQ and RQ is illustrated in Algorithm I.

**Algorithm I**: Pseudo code for queue scheduling:

**Line** **Code**

**if** $n = 2k$ where, $k \in \mathbb{Z}^+$
  $n_r = \lfloor \frac{\beta}{\beta+1} n \rfloor$
  $n_e = n - n_r$
**elseif** $n = (2k+1)$ where, $k \in \mathbb{N}$
  $n_r = \lceil \frac{\beta}{\beta+1} n \rceil$
  $n_e = n - n_r$
**end**
$n_{rq,\ max} = f(DSA, a, j_{rq})$
$n_{eq,\ max} = f(DSA, W, V, j_{eq})$
$n_{eq,\ min} = f(DSA, W, V, j_{eq})$
**if** $n_{rq,\ max} \geq n_r$
  $n_R = \lfloor \frac{n_r}{a} \rfloor . a$
  $n_e = n - n_R$
    **if** $n_{eq,\ max} > n_e$
      **if** $n_e \geq n_{eq,\ min}$
        $n_E = n_e$
      **else**
        $n_E = 0$
      **end**
    **else**
      $n_E = n_{eq,\ max}$
    **end**
**else**
  $n_R = n_{rq,\ max}$
  $n_e = n - n_R$
    **if** $n_{eq,\ max} > n_e$
      **if** $n_e \geq n_{eq,\ min}$
        $n_E = n_e$
      **else**
        $n_E = 0$
      **end**
    **else**
      $n_E = n_{eq,\ max}$
    **end**
**end**
$n_Q = n_R + n_E$
**if** $n_Q < n$
  $n_O = n - n_Q$
**else**
  $n_O = 0$
**end**

**Example:** Consider a CRN which uses DSA based on the channel aggregation strategy described in Section 4.2, i.e., *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$. The CRN consists of $M$ channels where $M \in \mathbb{Z}^+$. These channels can be used by secondary services of heterogeneous traffic with the following parameters, i.e., $W = 4, V = 7$ and $a = 2$. Therefore an elastic service in the system can aggregate up to 7 channels while real time services always aggregate 2 channels. Consider a particular state at which there are 2 ESU services waiting in EQ and 3 services waiting in RQ until they will get the chance. Therefore $j_{eq} = 2, j_{rq} = 3$. Assume the value of $\beta$ is set as $\beta = 3/2$. From here onwards, we describe the proposed *Algorithm I* step by step.

- Suppose an ESU service of 7 aggregated channels is finished and those 7 channels become idle, i.e., $n = 7$. According to the *line 1* $n \neq 2k$ since there is no value of $k \in \mathbb{Z}^+$ such that $2k = 7$. But $n = 2k + 1$ is satisfied for $k = 3$ as in the *line 4*.
- Therefore, $n_r = \lceil \frac{\frac{3}{2}}{\frac{3}{2}+1} \times 7 \rceil = 5$ and $n_e = 2$ as in the *line 5* and *line 6*.
- The value of $n_{rq,\ max}$ is calculated by using *Algorithm II* and the values of $n_{eq,\ max}$ and $n_{eq,\ min}$ are calculated by using *Algorithm III*. According to the *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy, each real time service in the queue need only 2 channels. However one of the elastic service requests in the queue can aggregate up to 7 channels. Therefore, $n_{rq,\ max} = 2 \times 3 = 6$ and $n_{eq,\ max} = 7 \times 2 = 14$. Since the minimum number of channels required to provide channel access for at least one ESU service in the queue is $W$, $n_{eq,\ min} = 4$.
- Since $n_{rq,\ max} > n_r$, the number of channels actually allocated for the waiting RSU Services in RQ is $n_R = \lfloor \frac{5}{2} \rfloor \times 2 = 4$, according to the **If** condition in *line 11*. According to the *line 13*, $n_e = 7 - 4 = 3$.
- Since the *if* condition in *line 14* is satisfied due to $14 > 4$, we need to check the next condition in *line 15*. Since $n_e < n_{eq,\ min}$, the actually allocated channels for the ESU services in EQ is given by $n_E = 0$ as indicated in *line 18*.
- *Line 36* calculates the total number of channels allocated for queued SU services, i.e., $n_Q$. Since $n_Q = 4 + 0 = 4$ and $n_Q < n$, the system will allocate the rest of the idle channels, i.e., $n_O$ channels for ongoing ESU services. Therefore, 3 channels are allocated for them as indicated in *line 38*.

**Algorithm II**: Pseudo code for the calculation of $n_{rq,\ max}$ for *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy

| **Line** | **Code** |
|---|---|
| 1: | $n_{rq,\ max} = a \times j_{rq}$ |

**Algorithm III**: Pseudo code for the calculation of $n_{eq,\ max}$ and $n_{eq,\ min}$ for *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy

**Code**

**if** $j_{eq} \geq 1$
  $n_{eq,\ max} = V \times j_{eq}$
  $n_{eq,\ min} = W$
**else**
  $n_{eq,\ max} = 0$
  $n_{eq,\ min} = 0$
**end**

## 4.4 State Transitions

The following tables, Tables 4.1-4.5 summarize the state transitions associated with different events (SU/PU departures and SU arrivals) under different conditions related to both cases (i.e., Case I and Case II). However, the transitions from a generic state upon a PU arrival is not the same for Case I and Case II. Therefore, Table 4.6 and Table 4.7 summarize the state transitions associated with PU arrivals under different conditions related with Case I and Case II respectively. In these tables[1] the parameters $n,\ n_R$ and $n_E$ have the same meanings as described in *Algorithm I*.

Table 4.1: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a PU departure with $j_{pu}\mu_P$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| PU DP. An RSU in RQ uses the vacant channel. | $(j_W, \cdots, j_V, g_a+1, j_{pu}-1, j_{eq}, j_{rq}-1)$ | $j_{rq} > 0;\ j_{pu} > 0;\ a = 1.$ |
| PU DP. An ESU in EQ uses the vacant channel. | $(j_W+1, \cdots, j_V, g_a, j_{pu}-1, j_{eq}-1, j_{rq})$ | $j_{rq} = 0$ or $a > 1;\ j_{eq} > 0;\ j_{pu} > 0;$ $W = 1.$ |
| PU DP. An ongoing $ESU_k$ uses the vacant channel. | $(j_W, \cdots, j_k-1, j_{k+1}+1, \cdots, j_V, g_a, j_{pu}-1, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $a > 1;\ j_{eq} = 0$ or $W > 1;$ $j_{pu} > 0;\ j_k > 0,\ V > 1;$ $k = min\{r \mid j_r > 0, W \leq r < V\}.$ |
| PU DP. No SUs uses the vacant channel. | $(j_W, \cdots, j_V, g_a, j_{pu}-1, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $a > 1; j_{eq} = 0$ or $W > 1;$ $j_k = 0, W \leq k < V$ or $W = V;$ $j_{pu} > 0.$ |

[1]In these tables **DP** and **AR** indicate a departure event and an arrival event respectively. $\mathbf{ESU_k}$ denotes an ESU service with $k$ aggregated channels and $\boldsymbol{b(x)}$ denotes the total number of utilized channels at state $x$ by SUs and PUs.

Table 4.2:
Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a RSU departure with $ag_a\mu_{S2}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| RSU DP. An RSU in RQ uses all the vacant channels. | $(j_W, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}-1)$ | $j_{rq} > 0;\ g_a > 0;\ n_R = n = a.$ |
| RSU DP. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_a+1, \cdots, j_V, g_a-1, j_{pu}, j_{eq}-1, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ g_a > 0;\ n_E = n = a;$ $j_{eq} = 1,\ W \leq a \leq V$ or $j_{eq} > 1,\ W \leq a < 2W.$ |
| RSU DP. Two ESUs in EQ uses all the vacant channels. | $(j_W, \cdots, j_h+1, \cdots, j_l+1, \cdots, j_V, g_a-1, j_{pu}, j_{eq}-2, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ g_a > 0;\ n_E = n = a;$ $j_{eq} = 2,\ 2W \leq a \leq 2V,\ a = h+l,\ W \leq h, l \leq V$ or $j_{eq} > 2,\ 2W \leq a < 3W,\ a = h+l,\ W \leq h, l \leq V.$ |
| ... | ... | ... |
| RSU DP. An ongoing ESU with the minimum no. of channels, $h$ uses all the vacant channels. | $(j_W, \cdots, j_h-1, \cdots, j_l+1, \cdots, j_V, g_a-1, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $g_a > 0;\ h = min\{r \mid j_r > 0, W \leq r < V\};$ $l = a+h \leq V;\ V > W.$ |
| ... | ... | ... |
| RSU DP. All other ESUs use the vacant channels and reach the upper bound $V$. | $(0, \cdots, 0, \cdots, j_V+q, g_a-1, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $g_a > 0;\ q = \sum_{m=W}^{V-1} j_m;$ $a \geq \sum_{m=W}^{V-1}(V-m)j_m;\ V > W.$ |
| RSU DP. Other SUs cannot use the vacant channels. | $(j_W, \cdots, j_V, g_a-1, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;\ g_a > 0;$ $V > W,\ j_m = 0, \forall m < V$ or $W = V.$ |

Table 4.3:
Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a RSU arrival with $\lambda_{S2}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| RSU AR. Enough idle channels exist. | $(j_W, \cdots, j_k, \cdots, j_V, g_a+1, j_{pu}, j_{eq}, j_{rq})$ | $M - b(x) \geq a.$ |
| RSU AR. The ESU with the maximum no. of channels, $m$ donate $a$ channels. | $(j_W, \cdots, j_n+1, \cdots, j_m-1, \cdots, j_V, g_a+1, j_{pu}, j_{eq}, j_{rq})$ | $V > W; m = max\{r \mid j_r > 0, W+1 \leq r \leq V\};$ $n = m - [a - (M - b(x))],\ W \leq n < m.$ |
| RSU AR. Two ESUs with $m$ and $h$ channels collectively donate $a$ channels. | $(j_W+1, \cdots, j_n+1, \cdots, j_h-1, \cdots, j_m-1, \cdots, J_V, g_a+1, j_{pu}, j_{eq}, j_{rq})$ | $V > W;\ m = max\{r \mid j_r > 0, W+1 \leq r \leq V\};$ $h = max\{r \mid j_r > 0, W+1 \leq r \leq m-1\},\ j_m = 1$ or $h = m, j_m > 1;$ $n = h + m - [(W+a) - (M - b(x))],\ W \leq n < h.$ |
| ... | ... | ... |
| RSU AR. The new ESU request is put into the queue, RQ. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}+1)$ | $M - b(x) + \sum_{m=W+1}^{V}(m-W)j_m < a;$ $j_{rq} < Q_{L2}.$ |

Table 4.4:
Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a $\mathrm{ESU_k}$ departure with $kj_k\mu_{S1}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| $\mathrm{ESU_k}$ DP. An RSU in RQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_V, g_a + 1, j_{pu}, j_{eq}, j_{rq} - 1)$ | $j_{rq} \geq 1;\ j_k > 0;\ k = a;\ n_R = k.$ |
| $\mathrm{ESU_k}$ DP. Two RSUs in RQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_V, g_a + 2, j_{pu}, j_{eq}, j_{rq} - 2)$ | $j_{rq} \geq 2;\ j_k > 0;\ k = 2a;\ n_R = k.$ |
| ... | ... | ... |
| $\mathrm{ESU_k}$ DP. An RSU in RQ and an ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_h + 1, \cdots j_V, g_a + 1, j_{pu}, j_{eq} - 1, j_{rq} - 1)$ | $j_k > 0;\ k = a + h;\ j_{rq} \geq 1;\ n_R = a;$ $n_E = h;\ j_{eq} = 1,\ W \leq h \leq V$ or $j_{eq} > 1,\ W \leq h < 2W.$ |
| ... | ... | ... |
| $\mathrm{ESU_k}$ DP. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}$ $j_{eq} - 1, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_k > 0;\ n_E = n = k;$ $j_{eq} = 1,\ W \leq k \leq V$ or $j_{eq} > 1,\ W \leq k < 2W.$ |
| $\mathrm{ESU_k}$ DP. Two ESUs in EQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_h + 1, \cdots, j_l + 1, \cdots, j_V, g_a, j_{pu}, j_{eq} - 2, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_k > 0;\ n_E = n = k;$ $j_{eq} = 2,\ 2W \leq k \leq 2V,\ k = h + l,\ W \leq h, l \leq V$ or $j_{eq} > 2,\ 2W \leq k < 3W,\ k = h + l,\ W \leq h, l \leq V.$ |
| ... | ... | ... |
| $\mathrm{ESU_k}$ DP. An ongoing ESU with the minimum no. of channels, $h$ uses all the vacant channels. | $(j_W, \cdots, j_h - 1, \cdots, j_k - 1, \cdots, j_l + 1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $j_k > 0;\ h = min\{r \mid j_r > 0, W \leq r < V\};$ $l = k + h \leq V;\ V > W.$ |
| ... | ... | ... |
| $\mathrm{ESU_k}$ DP. All other ESUs use the vacant channels and reach the upper bound $V$. | $(0, \cdots, 0, \cdots, j_V + q, g_a, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0; j_{eq} = 0$ or $n_E = 0;$ $j_k > 0;\ q = \sum_{m=W}^{V-1} j_m - 1;$ $k \geq \sum_{m=W}^{V-1}(V - m)j_m - (V - k);\ V > W.$ |
| $\mathrm{ESU_k}$ DP. Other SUs cannot use the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $j_k > 0;\ V > W,\ j_m = 0, \forall m < V$ or $W = V.$ |

Table 4.5:
Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a ESU arrival with $\lambda_{S1}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| ESU AR. Enough idle channels exist. | $(j_W, \cdots, j_k + 1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $k = min\{M - b(x), V\} \geq W.$ |
| ESU AR. The ESU with the maximum no. of channels, $m$ donate $W$ channels. | $(j_W + 1, \cdots, j_n + 1, \cdots, j_m - 1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $V > W; m = max\{r \mid j_r > 0, W + 1 \leq r \leq V\};$ $n = m - [W - (M - b(x))];$ $W \leq n < m.$ |
| ESU AR. Two ESUs with $m$ and $h$ channels collectively donate $W$ channels. | $(j_W + 2, \cdots, j_n + 1, \cdots, j_h - 1, \cdots, j_m - 1, \cdots, J_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $V > W;\ m = max\{r \mid j_r > 0, W + 1 \leq r \leq V\};$ $h = max\{r \mid j_r > 0,\ W + 1 \leq r \leq m - 1\},\ j_m = 1$ or $h = m,\ j_m > 1;$ $n = h + m - [2W - (M - b(x))],\ W \leq n < h.$ |
| ... | ... | ... |
| ESU AR. The new ESU request is put into the queue, EQ. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq} + 1, j_{rq})$ | $M - b(x) + \sum_{m=W+1}^{V}(m - W)j_m < W;$ $j_{eq} < Q_{L1}.$ |

Table 4.6:
Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a PU arrival in Case I

| Activity | Dest. State | Trans. rate | Conditions |
|---|---|---|---|
| PU AR. A vacant channel exists. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\lambda_P$ | $M - b(x) > 0.$ |
| PU AR. An ESU$_k$ is interrupted and reduces its channels. | $(j_W, \cdots, j_{k-1}+1, j_k-1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{kj_k}{M-j_{pu}}\lambda_P$ | $V > W; b(x) = M;\ j_k > 0;\ k > W.$ |
| PU AR. ESU$_W$ is forced terminated. No spectrum adaptation. | $(j_W-1, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W \geq 1;\ b(x) = M;\ W = V$ or $W = 1$, $j_k = 0,\ W+1 \leq k \leq V$. Or $j_W = 1$; $b(x) = M; j_k = 0,\ W+1 \leq k \leq V.$ |
| PU AR. ESU$_W$ is forced terminated. RSU in RQ uses all the vacant channels. | $(j_W-1, \cdots, j_k, \cdots, j_V, g_a+1, j_{pu}+1, j_{eq}, j_{rq}-1)$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W \geq 1;\ b(x) = M;\ j_k = 0, \forall k > W;$ $j_{rq} > 0;\ W > 1;\ n_R = a = W-1.$ |
| PU AR. ESU$_W$ is forced terminated. Two RSUs in RQ uses all the vacant channels. | $(j_W-1, \cdots, j_k, \cdots, j_V, g_a+2, j_{pu}+1, j_{eq}, j_{rq}-2)$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W \geq 1;\ b(x) = M;\ j_k = 0, \forall k > W;$ $j_{rq} > 1;\ W > 1; n_R = 2a = W-1.$ |
| ... | ... | ... | ... |
| PU AR. ESU$_W$ is forced terminated and another ongoing ESU$_W$ uses all the vacant channels. | $(j_W-2, \cdots, j_l+1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1;\ b(x) = M;\ j_{rq} = 0$ or $n_R = 0$; $j_k = 0, \forall k > W;\ W > 1;$ $l = 2W-1 \leq V.$ |
| ... | ... | ... | ... |
| PU AR. ESU$_W$ is forced terminated and other all ESUs uses all the vacant channels and achieve $V$. | $(0, 0, \cdots, j_V+q, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1;\ b(x) = M;\ j_{rq} = 0$ or $n_R = 0$; $j_k = 0, \forall k > W;\ V > W > 1; q = j_W - 1;$ $W - 1 \geq (V-W)(j_W-1).$ |
| PU AR. RSU is interrupted and ESU$_k$ with max. channels, $k$ donates a channel. | $(j_W, \cdots, j_{k-1}+1, j_k-1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $V > W; b(x) = M;\ g_a > 0;\ k > W;$ $k = max\{r \mid j_r > 0,\ W+1 \leq r \leq V\}.$ |
| PU AR. RSU is forced terminated. No spectrum adaptation. | $(j_W, \cdots, j_k, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M; a = 1, j_k = 0,$ $W+1 \leq k \leq V$. Or $g_a > 0$; $b(x) = M;\ W = V.$ |
| PU AR. RSU is forced terminated. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k, \cdots, j_h+1, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}-1, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M;\ j_k = 0, \forall k > W;$ $a \geq W+1;\ n_E = h = a-1;$ $j_{eq} = 1,\ W \leq a-1 \geq V$ or $j_{eq} > 1,\ W \leq a-1 < 2W.$ |
| ... | ... | ... | ... |
| PU AR. RSU is forced terminated. An ESU$_W$ uses all the vacant channels. | $(j_W-1, \cdots, j_k+1, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M; a > 1,\ j_k = 0,$ $W+1 \leq k \leq V; j_W > 0,$ $k = a+W-1 \leq V;\ V > W.$ |
| ... | ... | ... | ... |
| PU AR. RSU is interrupted and ESU$_W$ is forced terminated. All ESUs uses the vacant channels and achieve $V$. | $(0, 0, \cdots, j_V+q, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M; a > 1,\ j_k = 0,$ $W+1 \leq k \leq V; q = j_W;$ $a-1 \geq (V-W)j_W;\ V > W.$ |

Table 4.7:
Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a PU arrival in Case II

| Activity | Dest. State | Trans. rate | Conditions |
|---|---|---|---|
| PU AR. A vacant channel exists. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\lambda_P$ | $M - b(x) > 0$. |
| PU AR. An ESU service with $k$ channels reduces its channels. | $(j_W, \cdots, j_{k-1}+1, j_k - 1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{kj_k}{M-j_{pu}}\lambda_P$ | $V > W$; $M = b(x)$; $j_k > 0$; $k > W$. |
| PU AR. The interrupted ESU$_W$ is put into the queue EQ and no spectrum adaptation | $(j_W - 1, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 0$; $b(x) = M$; $j_{eq} < Q_{L1}$; $W = V$ or $W = 1, j_k = 0, W+1 \le k \le V$. Or $j_W = 1$; $b(x) = M$; $j_k = 0, W+1 \le k \le V$. |
| PU AR. ESU$_W$ is put into the queue, EQ. Another ESU$_W$ uses the idle channels. | $(j_W - 2, \cdots, j_l + 1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $W > 1$; $l = 2W - 1 \le V$; $j_{eq} < Q_{L1}$. |
| ... | ... | ... | ... |
| PU AR. ESU$_W$ is put into the queue, EQ and other all ESUs uses the idle channels and achieve $V$ | $(0, 0, \cdots, j_V + q, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $V > W > 1$; $q = j_W - 1$; $j_{eq} < Q_{L1}$; $W - 1 \ge (V - W)(j_W - 1)$. |
| PU AR. The interrupted ESU$_W$ is forced terminated and no spectrum adaptation | $(j_W - 1, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 0$; $b(x) = M$; $j_{eq} = Q_{L1}$; $W = V$ or $W = 1, j_k = 0, W+1 \le k \le V$. Or $j_W = 1$; $b(x) = M$; $j_k = 0, W+1 \le k \le V$. |
| PU AR. ESU$_W$ is forced terminated and another $ESU_W$ uses the idle channels. | $(j_W - 2, \cdots, j_l + 1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $W > 1$; $l = 2W - 1 \le V$; $j_{eq} = Q_{L1}$. |
| ... | ... | ... | ... |
| PU AR. ESU$_W$ is forced terminated and other all ESUs uses the idle channel and achieve $V$. | $(0, 0, \cdots, j_V + q, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $V > W > 1$; $q = j_W - 1$; $j_{eq} = Q_{L1}$; $W - 1 \ge (V - W)(j_W - 1)$. |
| PU AR. RSU is interrupted and ESU$_k$ with maximum channels, $k$ donates a channel. | $(j_W, \cdots, j_{k-1}+1, j_k - 1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $V > W$; $M = b(x)$; $g_a > 0$; $k > W$; $k = max\{r \mid j_r > 0, W+1 \le r \le V\}$. |
| PU AR. RSU is forced terminated, No spectrum adaptation. | $(j_W, \cdots, j_k, \cdots, j_V, g_a - 1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $a = 1, j_k = 0$, $W+1 \le k \le V$. Or $g_a > 0$; $b(x) = M$; $W = V$. |
| PU AR. RSU is forced terminated. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k, \cdots, j_h + 1, \cdots, j_V, g_a - 1, j_{pu}+1, j_{eq} - 1, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $j_k = 0, \forall k > W$; $a \ge W+1$; $n_E = h = a - 1$; $j_{eq} = 1$, $W \le a - 1 \ge V$ or $j_{eq} > 1$, $W \le a - 1 < 2W$. |
| ... | ... | ... | ... |
| PU AR. RSU is forced terminated. An ESU$_W$ uses all the vacant channels. | $(j_W - 1, \cdots, j_k + 1, \cdots, j_V, g_a - 1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $a > 1$, $j_k = 0$, $W+1 \le k \le V$; $j_W > 0$, $k = a + W - 1 \le V$; $V > W$. |
| ... | ... | ... | ... |
| PU AR. RSU is interrupted and ESU$_W$ is forced terminated. All ESUs uses the vacant channels and achieve $V$. | $(0, 0, \cdots, j_V + q, g_a - 1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $a > 1$, $j_k = 0$, $W+1 \le k \le V$; $q = j_W > 0$, $a - 1 \ge (V - W)j_W$; $V > W$. |

## 4.5 CTMC Analysis

Due to various conditions and different events, it is difficult to illustrate a generalized state transition diagram of the system. Therefore, tables are used as the ideal representation for explaining all the possible state transitions. However for the completeness, the state transition diagrams for the simplest system, i.e., $W = V = a = Q_{L1} = Q_{L2} = 1$, are illustrated in Fig. 4.8 and 4.9. Since this is the simplest case, identical state transitions can be observed in both cases except the state transition occurred upon a PU arrival when the system is at state **B**. When the system is at state **B** upon a PU arrival, it switches to state **C** if Case I is used while it switches to state **E** if Case II is used.

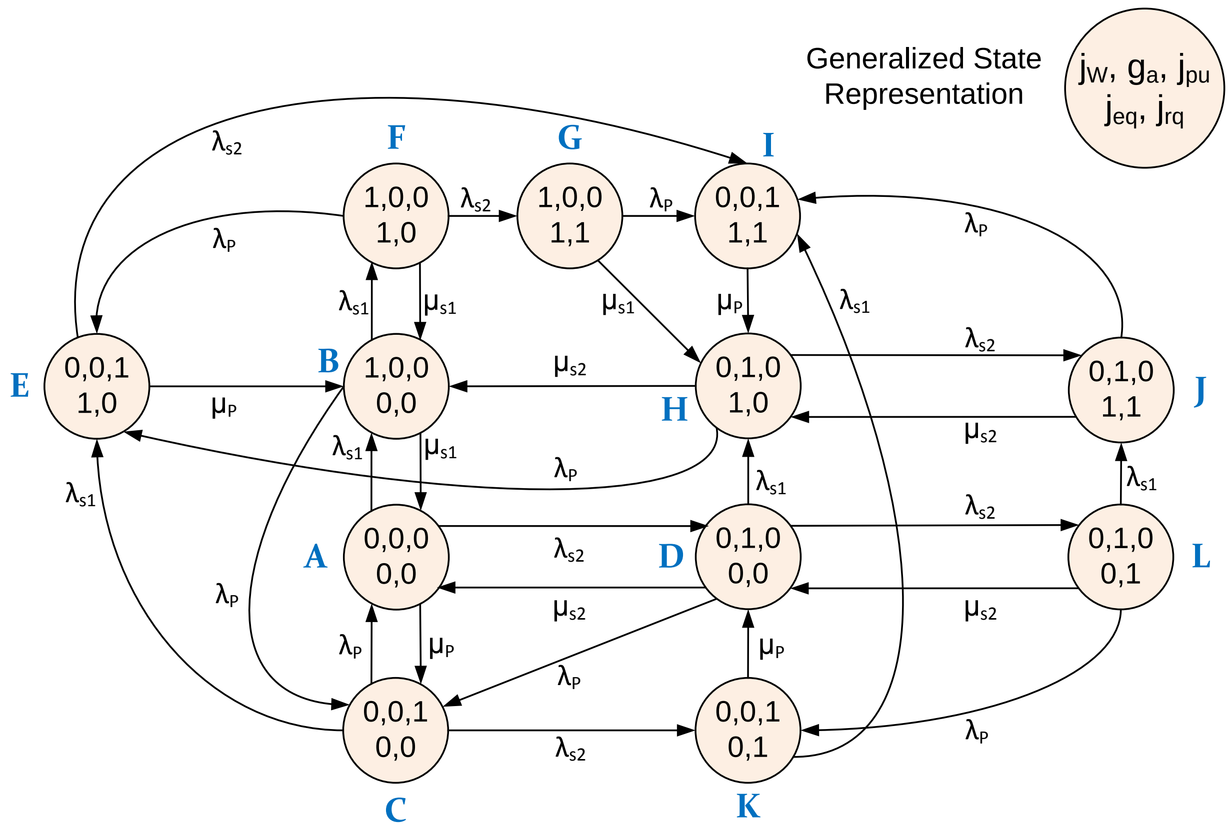


Figure 4.8: State transitions for Case I when $W = V = a = M = Q_{L1} = Q_{L2} = 1$

To model the proposed channel access strategy with the PMQS queuing model, continuous time Markov chains are developed under the assumptions explained in Section 3.1. Arrival and service patterns are the most important factors determining the behaviour of queuing networks. Customer arrival rate defines the number of customers enters into the system per unit time [33]. The arrival time of PU, ESU and RSU traffic follows an independent Poisson process. Also the service times are exponentially distributed. The random behaviour of the arrival and service patterns cause the queue length or the number of occupied waiting lines in the queue to vary. The notations used for the parameters of the CTMC models are listed in Table 4.8.

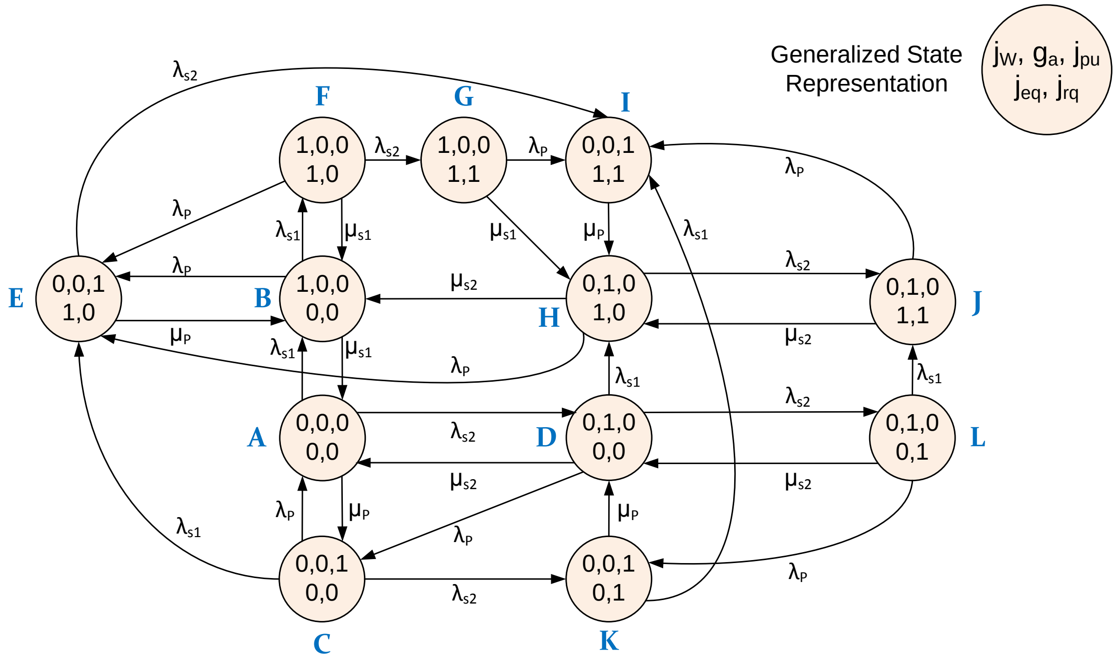


Figure 4.9: State transitions for Case II when $W = V = a = M = Q_{L1} = Q_{L2} = 1$

Table 4.8: Notations for System Statistics

| Parameter | Explanation |
|---|---|
| $\lambda_{S1}$ | ESU arrival rate |
| $\lambda_{S2}$ | RSU arrival rate |
| $\lambda_P$ | PU arrival rate |
| $\mu_{S1}$ | Average ESU service rate per channel |
| $\mu_{S2}$ | Average RSU service rate per channel |
| $\mu_P$ | Average PU service rate per channel |
| $j_{pu}$ | Number of PU services in the system at state $l$ |
| $j_k$ | Number of ESU services with $k$ aggregated channels at state $l$ |
| $g_a$ | Number of RSU services with $a$ aggregated channels at state $l$ |
| $j_{eq}$ | The number of ESU services currently in EQ |
| $j_{rq}$ | The number of RSU services currently in RQ |
| $W$ | Lower limit of the channel aggregation of ESU services |
| $V$ | Upper limit of the channel aggregation of ESU services |
| $a$ | Number of channels assembled by an RSU service |
| $L$ | Total number of states in the CTMC model |
| $\pi(x)$ | The steady state probability of state $x$ |

As mentioned in Section 4.2, the states of the CTMC models can be represented by $x = (j_W, j_{W+1}, \cdots j_V, g_a, j_{pu}, j_{eq}, j_{rq})$. Let $\mathcal{S}$ be the set of feasible states of the system, as

$$\mathcal{S} = \{x \mid j_W, j_{W+1}, \cdots j_V, g_a, j_{pu}, j_{eq}, j_{rq} \geq 0; b(x) \leq M\}, \tag{4.1}$$

where $b(x)$ denotes the total number of utilized channels at state $x$. Therefore,

$$b(x) = j_{pu} + ag_a + \sum_{k=W}^{V} kj_k. \tag{4.2}$$

Furthermore, $\pi(x)$ which stands for steady state probability of being in state $x$ can be calculated as follows. Given feasible states and their transition rates in a CTMC, we can construct the global balance equations and the normalization equation as expressed by Eq. (4.3) [34].

$$\boldsymbol{\pi Q} = 0, \ \sum_{x=1}^{L} \pi(x) = 1, \tag{4.3}$$

where $\boldsymbol{\pi} = [\pi(1), \pi(2), \cdots, \pi(x), \cdots, \pi(L)]$ is the steady state probability vector and $\boldsymbol{Q}$ denotes the transition rate matrix. The vector $\boldsymbol{\pi}$ is determined by solving Eq. (4.3). When the steady state probabilities are determined, the performance of the CRN can be evaluated in terms of different parameters. The derivations of mathematical expressions for those parameters are presented in the following subsections.

## 4.6 Analytical Models for Case I

In this section we derive closed form mathematical expressions to evaluate the performance parameters of the secondary network and the spectrum utilization of the cognitive radio network pertaining to the queuing method introduced as Case I in Section 3.2.1.

### 4.6.1 Secondary network capacity

In this thesis the terminology *capacity* implies the rate of service completions, i.e., the average number of service completions per time unit in secondary network [35, 36]. Let $\rho_1$ and $\rho_2$ be the capacity of ESU and RSU services respectively. Then,
capacity of ESU services,

$$\rho_1 = \sum_{x=1}^{L} \sum_{k=W}^{V} kj_k \mu_{S1} \pi(x), \tag{4.4}$$

capacity of RSU services,

$$\rho_2 = \sum_{x=1}^{L} ag_a \mu_{S2} \pi(x). \tag{4.5}$$

### 4.6.2 Spectrum Utilization

Due to the static frequency allocation methods, inefficient utilization of radio spectrum is a major issue [46]. Cognitive radio technology which is based on the dynamic spectrum access will solve this problem significantly. In our analysis, we develop a closed form expression to calculate the spectrum utilization of the system in terms of number of utilized channels in each state. In each state $x \in \mathcal{S}$ of the CTMC model, system possess $M$ channels and thereof $\left(j_{pu} + a g_a + \sum_{k=W}^{V} k j_k\right)$ number of channels are occupied by PUs and SUs. Therefore, the spectrum utilization of the CRN can be expressed as

$$\begin{aligned} U_{CRN} &= \sum_{x=1}^{L} \pi(x) \left( \frac{j_{pu} + a g_a + \sum_{k=W}^{V} k j_k}{M} \right), \\ &= \sum_{x=1}^{L} \pi(x) \frac{b(x)}{M}. \end{aligned} \tag{4.6}$$

### 4.6.3 Blocking Probability

An incoming ESU service will be blocked when the following conditions are satisfied, i.e., there are no adequate idle channels (at least $W$ idle channels) in the CRN to commence the service, there is no way of channel sharing with ongoing ESU services such a way that $W$ channels could be allocated for the new user and the queue, EQ is also fully occupied. The similar conditions are applied for calculating the blocking probability of incoming RSU services except the fact that they need exactly $a$ channels for commencing services. Let $P_{b1}$ and $P_{b2}$ be the blocking probability of ESU and RSU services respectively.
Blocking probability of ESU,

$$P_{b1} = \sum_{\substack{x=1, \\ M-b(x)+\sum_{k=W+1}^{V}(k-W)j_k<W, \\ j_{eq}=Q_{L1}}}^{L} \pi(x). \tag{4.7}$$

Blocking probability of RSU,

$$P_{b2} = \sum_{\substack{x=1, \\ M-b(x)+\sum_{k=W+1}^{V}(k-W)j_k<a, \\ j_{rq}=Q_{L2}}}^{L} \pi(x). \tag{4.8}$$

### 4.6.4 Forced Termination Probability of ESUs

The forced termination probability is the probability that an active process is forced to terminate the communication before the communication has regularly ended [37]. This parameter is an important dimension to evaluate the quality of an active communication session which is more sensitive against forced termination of an existing

connection. Therefore, forced termination represents a disruption of the service and it should be kept at a lower percentage as much as possible. In Case I, when a PU preempt an ESU service which has exactly $W$ aggregated channels, the ESU is forced to terminate if there is no at least one idle channel or chance of channel sharing with another ongoing ESU service. Therefore, the interrupted ESU ends communication before it finishes the communication. In order to obtain an expression for the forced termination probability, there is a need to firstly define the forced termination rate.
Mean forced termination rate of ESU,

$$R_{f1} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{M - j_{pu}} \pi(l). \tag{4.9}$$

The forced termination probability of ESUs, $P_{f1}$, can be expressed as the mean forced termination rate of ESUs, $R_{f1}$ divided by the mean admitted ESU rate, i.e., $\lambda^*_{S1}$.
Forced termination probability of ESU,

$$P_{f1} = \frac{R_{f1}}{\lambda^*_{S1}}, \tag{4.10}$$

where,

$$\lambda^*_{S1} = \lambda_{S1}(1 - P_{b1}),$$

$$P_{f1} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{(M - j_{pu})\lambda^*_{S1}} \pi(l). \tag{4.11}$$

### 4.6.5 Forced Termination Probability of RSUs

A forced termination of an RSU service occurs when a PU service preempts an RSU service and the preempted RSU service cannot detect an idle channel in the system or cannot share a channel with other ongoing ESUs. The forced termination probability of RSUs, $P_{f2}$, can be expressed as the mean forced termination rate of RSUs, $R_{f2}$ divided by the mean admitted RSU rate, i.e., $\lambda^*_{S2}$. Therefore,

$$P_{f2} = \frac{\text{Total RSU forced termination rate}}{\text{Total RSU connection rate}}$$

Mean forced termination rate of RSU is given by,

$$R_{f2} = \sum_{\substack{x=1, \\ M=b(x),\ j_{pu}<M, \\ j_r=0,\forall r>W, g_a>0}}^{L} \frac{\lambda_P a g_a}{M - j_{pu}} \pi(x). \tag{4.12}$$

Then, forced termination probability of RSU,

$$P_{f2} = \frac{R_{f2}}{\lambda^*_{S2}}, \tag{4.13}$$

where,

$$\lambda^*_{S2} = \lambda_{S2}(1 - P_{b2}), \tag{4.14}$$

$$P_{f2} = \sum_{\substack{x=1, \\ M=b(x), g_a>0 \\ j_{pu}<M,\ j_r=0, \forall r>W}}^{L} \frac{\lambda_P a g_a}{(M - j_{pu})\lambda^*_{S2}} \pi(x). \tag{4.15}$$

### 4.6.6 Mean Queue Length

The queue length depends on the arrival rates of both PUs and SUs and their service completion rates. Moreover, queue length is affected by the cognitive access protocol [38]. This fact is true in our analysis since Case II can be with a large queue length in contrast with Case I. However, at each state $x \in \mathcal{S}$, $j_{eq}$ and $j_{rq}$ are the queue lengths of the EQ and RQ respectively.
Therefore, average queue length of ESU services,

$$L_{EQ} = \sum_{\substack{x=1, \\ 0 \le j_{eq} \le Q_{L1}}}^{L} \pi(x) j_{eq}. \tag{4.16}$$

Average queue length of RSU services,

$$L_{RQ} = \sum_{\substack{x=1, \\ 0 \le j_{rq} \le Q_{L2}}}^{L} \pi(x) j_{rq}. \tag{4.17}$$

### 4.6.7 Average Queuing Delay

In a communication network, delay is an important parameter when QoS of the entire system is considered. In a general communication process, four types of delays are significant which includes the propagation delay, processing delay, queuing delay and service time [39]. Queuing delay refers to the waiting time in the queue and service time is the time the system takes to serve a particular user. In a cognitive radio system, these two delays are dominant since they account for a large percentage of the total delay. In this analysis, queuing delay which is equal to the average waiting time of an SU service in the queue is evaluated. As what is discussed in the previous subsection, the average queue lengths of EQ and RQ were given by Eq. 4.16 and 4.17 respectively. In Case I, the arrival rate of EQ, $\lambda_{EQ}$ is equal to the ESU arrival rate.

The arrival rate of the queue, EQ,

$$\lambda_{EQ} = \lambda_{S1}.$$

Using **Little's Law**, the average queuing delay of an ESU service,

$$D_{EQ} = \frac{L_{EQ}}{\lambda_{EQ}}. \tag{4.18}$$

Therefore,

$$D_{EQ} = \frac{\sum\limits_{\substack{x=1,\\ 0 \leq j_{eq} \leq Q_{L1}}}^{L} \pi(x) j_{eq}}{\lambda_{S1}}. \tag{4.19}$$

The arrival rate of the queue, RQ,

$$\lambda_{RQ} = \lambda_{S2}.$$

Using **Little's Law**, the average queuing delay of an RSU service,

$$D_{RQ} = \frac{L_{RQ}}{\lambda_{RQ}}.$$

Therefore,

$$D_{RQ} = \frac{\sum\limits_{\substack{x=1,\\ 0 \leq j_{rq} \leq Q_{L2}}}^{L} \pi(x) j_{rq}}{\lambda_{S2}}. \tag{4.20}$$

## 4.7 Analytical Models for Case II

Instead of direct termination, an interrupted ESU service in Case II is suspended to wait in EQ for accessing another idle channel. The derived mathematical equations for calculating the capacity and the blocking probability of the secondary network and the expressions for determining the mean queue length of both queues in Case II is the same as in Case I. And also, the equations for determining the forced termination probability and the mean waiting time of RSU services and spectrum utilization of the system are the same as in Case I. However, forced termination probability and the average queuing delay of an ESU service have to be analyzed separately for the two cases since the behaviour of the forced terminated ESU services in Case II is not the same as in Case I.

### 4.7.1 Preempted Probability of ESUs

We define the preempted probability as the probability that an ongoing ESU enters the EQ because of the arrival of a licensed PU service when there is no way of accessing an idle channel. In Case I preempted probability is not discussed since interrupted SU services are not put in the queue. Due to the same reason, Case II does not consider the preempted probability of RSU services. The preempted probability of ESUs, $P_{preempted}$, can be

expressed as the interrupted rate of ESUs when there is no way of accessing an idle channel, $R_{interrupt}$ divided by the mean admitted ESU rate, i.e., $\lambda^*_{S1}$.
Mean PU interrupted rate of ESU,

$$R_{interrupt} = \sum_{\substack{l=1,\\ M=b(x),\ i<M,\\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{M-i}\pi(l). \tag{4.21}$$

The preempted probability of ESU,

$$P_{preempted} = \frac{R_{interrupt}}{\lambda^*_{S1}}, \tag{4.22}$$

where,

$$\lambda^*_{S1} = \lambda_{S1}(1-P_{b1}).$$

Therefore

$$P_{preempted} = \sum_{\substack{l=1,\\ M=b(x),\ i<M,\\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{(M-i)\lambda^*_{S1}}\pi(l). \tag{4.23}$$

### 4.7.2 Forced Termination Probability of ESUs

In Case II, a preempted ESU service is forced terminated only if all the waiting lines of EQ are occupied by other ESU services. The forced termination probability of ESU in Case II, $P^{'}_{f1}$, can be expressed as the mean forced termination rate of ESUs, $R^{'}_{f1}$ divided by the mean admitted ESU rate, i.e., $\lambda^*_{S1}$.
Mean forced termination rate of ESU,

$$R^{'}_{f1} = \sum_{\substack{l=1,\\ M=b(x),\ i<M,\\ j_W>0,\ j_n=0,\ \forall n>W,\ j_{eq}=Q_{L1}}}^{L} \frac{\lambda_P W j_W}{M-i}\pi(l). \tag{4.24}$$

Forced termination probability of ESU,

$$P^{'}_{f1} = \frac{R^{'}_{f1}}{\lambda^*_{S1}}, \tag{4.25}$$

where,

$$\lambda^*_{S1} = \lambda_{S1}(1-P_{b1}),$$

$$P^{'}_{f1} = \sum_{\substack{l=1,\\ M=b(x),\ i<M,\\ j_W>0,\ j_n=0,\ \forall n>W,\ j_{eq}=Q_{L1}}}^{L} \frac{\lambda_P W j_W}{(M-i)\lambda^*_{S1}}\pi(l). \tag{4.26}$$

### 4.7.3 Average Queuing Delay of ESU Service

In Case II, the arrival rate to the low priority queue, EQ is higher than that of Case I due to admission of the interrupted ESU services. Thus, we have to modify Eq. 4.19 by following the similar procedure to calculate the average ESU waiting time in Case II. The arrival rate of EQ, $\lambda^{'}_{EQ}$ is equal to the ESU arrival rate plus ESU temporary termination rate.

$$\lambda^{'}_{EQ} = \lambda_{S1} + R_{interrupt}.$$

Using **Little's Law**, the average queuing delay of ESU service,

$$D^{'}_{EQ} = \frac{L_{EQ}}{\lambda^{'}_{EQ}}.$$

Therefore,

$$D^{'}_{EQ} = \frac{\sum\limits_{\substack{x=1, \\ 0 \leq j_{eq} \leq Q_{L1}}}^{L} \pi(x) j_{eq}}{\lambda_{S1} + \left( \sum\limits_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{M-i} \pi(l) \right)}. \tag{4.27}$$

## 4.8 Chapter Summary

The first part of the chapter is allocated for defining the dynamic channel aggregation strategy of the proposed queuing model which is designed based on the DSA techniques introduced in [12] and [22]. The system functions on PU and SU activities are defined with channel aggregation, channel sharing and spectrum handover possibilities. The opportunities of service commencement over the queued services are distributed between real time and elastic traffic services through the queue scheduling algorithm. The queue scheduling algorithm is developed with adjustable parameter called priority factor which is used to set the priority level for each queue. The chapter then listed the state transitions over each PU and SU event by considering both queuing methods. The output state and the transition rate at each state transition are listed in tables with corresponding conditions. Those tables are used to accomplish the duty of the system's state transition diagram which is not presented in this thesis due to its over-complexity. Afterwards, closed form expressions are derived for evaluating the system performance in terms of secondary network capacity, blocking probability, forced termination probability, mean queue length and average queuing delay. In addition, an expression is derived to calculate the overall spectrum utilization of the CRN.

# Chapter 5: Numerical Results & Evaluations

*This chapter focuses on analyzing the performance of multi-channel cognitive radio networks with proposed PMQS queuing model under both Case I and Case II. Numerical results are illustrated with discussions of the significant manifestations. Performance of the secondary network is analyzed in terms of its capacity, blocking probability, forced termination probability and the average queuing delay of SU services in both cases. In addition, the spectrum utilization of the allocated frequency band for the CRN also analyzed.*

*We consider a CRN with 6 channels (i.e., $M = 6$) in the allocated spectrum band. Unless otherwise stated, statistical parameters of licensed and unlicensed users are set as $\lambda_P = 1.0$, $\lambda_{S1} = 2.0$, $\lambda_{S2} = 1.0$, $\mu_P = 0.5$, $\mu_{S1} = 1.0$ and $\mu_{S2} = 1.0$. Configuration parameters of the dynamic spectrum access strategy used in the system, i.e., Dynamic $(a; W; V; Q_{L1}; Q_{L2})$ are fixed as $W = 1$, $V = 3$ and $a = 1$. Unless otherwise stated, the lengths of the queues are set as $Q_{L1} = 4$ and $Q_{L2} = 2$ and the ratio between priority levels assigned to RQ and EQ is set as 2:1, i.e., $\beta = 2$.*

## 5.1 Model Validation

To validate the analytical models, we first develop MATLAB codes to simulate PU and SU events over the proposed channel access strategy and compare the analytical results with the simulation results. However, for illustration clarity we depict the simulation results only in Section 5.1, although simulations have been performed for the all analysis of this chapter. Another important fact is that when we take simulation results it is important to have an idea of the preciseness of those observations. A confidence interval is a standard way of describing the precision of a measurement of some parameter [40]. In this section, the simulation results with confidence intervals are illustrated in shadow for the purpose of highlighting. However, we do not explain the reasons for the observed behaviour of the curves in Section 5.1, however, the next sections are allocated to accomplish that purpose. The sole target of this section is to present simulation results in order to validate the proposed analytical models.

### 5.1.1 Model Validation - Case I

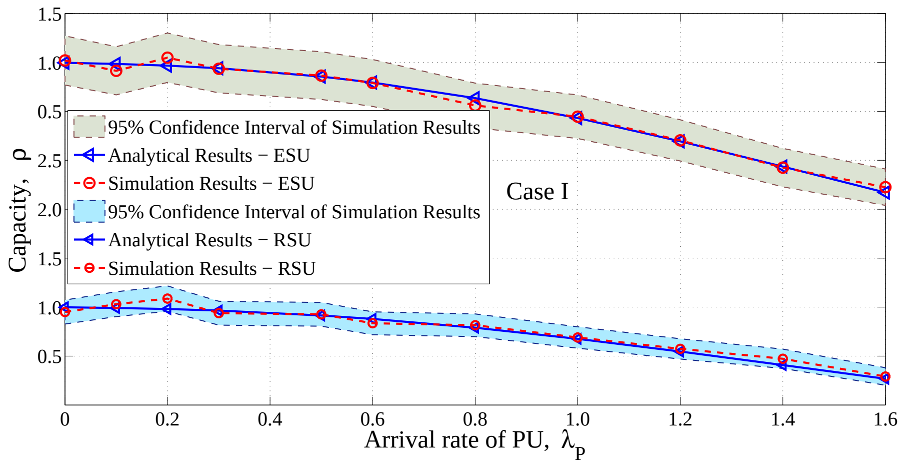


Figure 5.1: SU capacity as a function of PU arrival rate - Case I.

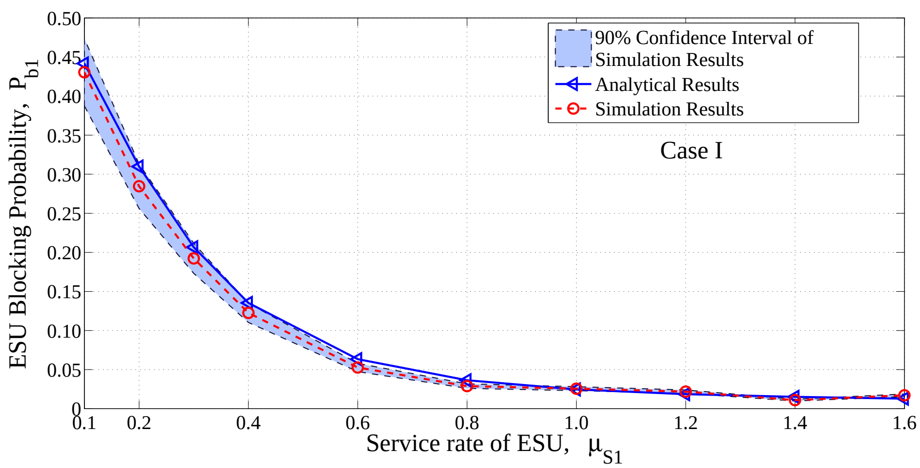


Figure 5.2: ESU blocking probability as a function of PU arrival rate - Case I.

Fig. 5.1 depicts the variation of secondary network capacity as a function of PU arrival rate. When more PU services arrive, the capacity of both ESU and RSU services decreases. As demonstrated in that figure analytical output exists within the 95% confidence intervals of the simulation results. In other words, our analytical model shows a precise matching with the obtained simulation results in terms of capacity calculation. And also, Fig. 5.2 and Fig. 5.3 illustrate that the analytical results representing the blocking probabilities of SU services stand within the 90% confidence interval. However, the analytical results do not show a full coincidence with the simulation results and the reasons can be due to main three factors. In our simulation, we model the proposed queuing structure and channel access protocol *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ for a CRN. PU and SU arrivals are generated with Poisson arrival process and set the duration of service time according to the exponential distribution. Then, to obtain a numerical value for a parameter, the simulation is performed for 500 time units,

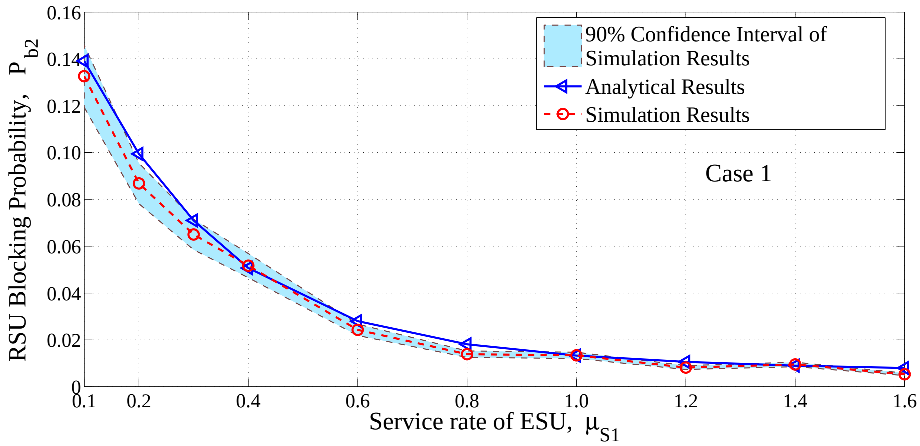


Figure 5.3: RSU blocking probability as a function of PU arrival rate - Case I.

i.e., $T_{total} = 500$. In other words, we examine the system behaviour for 500 time units and calculate the parameter values such as $\rho_1$, $P_{b1}$, etc. Each time unit denoted as $T$ is further divided into $n = 500$ sub-intervals. Therefore, the value of the smallest time unit denoted as $h$ in this simulation is $1/500 = 0.002$ time units. We analyze the system status at each sub-interval. The purpose of having a small time unit is to avoid simultaneous events. As an example, we assume that a SU departure and a PU arrival will not take place at the same time. However, in order to achieve this, the minimum time unit $h$ should be configured sufficiently small. Finally, we perform 15 rounds (instances) of simulations and obtained the mean value as the required result.

Thus, with the above mentioned facts, the correctness and the preciseness of the simulation results depends on the following three factors.

- When the number of simulation rounds is high, the reliability of the results is also high.
- When the total simulation time is large, the preciseness is high.
- When the number of sub-intervals per time unit is high, the correctness is high.

Therefore we believe that, with a large value of $T_{total}$ and a small value of $h$, the results will be more accurate if the average value is taken from a large number of simulation rounds.

### 5.1.2 Model Validation - Case II

Case II is different from Case I only regarding the action taken after the interruption of ongoing ESU services due to PU arrivals. The interrupted ESU services in Case II are allowed to be queued in EQ. If we consider these interruption events of ESU services as an arrival process, we need to ensure that it follows a Poisson process. The respective arrival rate equals to the interrupted rate of ESU services, i.e., $R_{interrupt}$ in Eq. 4.21. Therefore, arrivals to the low priority queue has two batches with arrival rates $\lambda_{S1}$ and $R_{interrupt}$. According to the *Theorem 2.1* stated in Section 2.4 two Poisson processes can be merged as one Poisson process [24]. Nevertheless, those

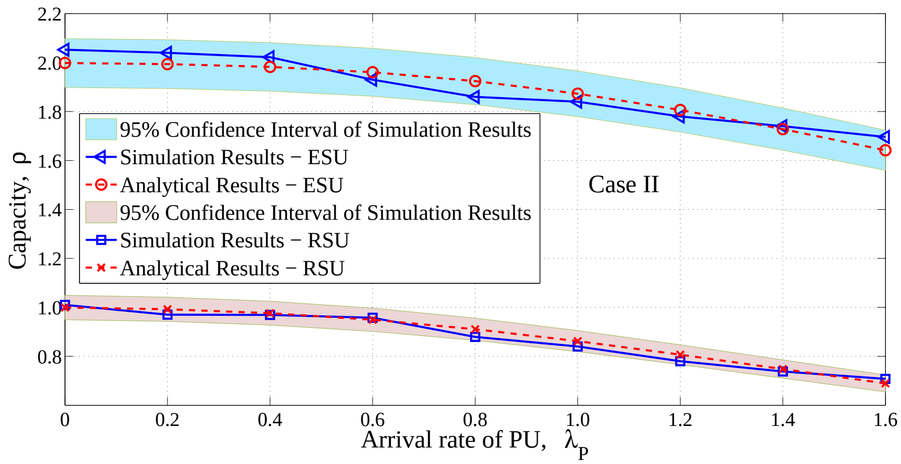


Figure 5.4: SU capacity as a function of PU arrival rate - Case II.

two arrival processes are not independent since PU interruption rate depends on the arrival rate of both ESU and PU services. However, in this thesis, the complete arrival process to the low priority queue is assumed to be a Poisson process. One of the purposes of analyzing simulation and analytical results obtained in Case II is to investigate the validity of this assumption.

According to Fig. 5.4 which shows the variation of the capacity over PU arrival rate, a close consistency between analytical and simulation results can be observed with a high confidence level. If the two types of traffic are compared, capacity variation of RSU services is closer to its simulation results than that of ESU services. The logical reason for this behaviour is unambiguous. The arrival process to the low priority queue consists of two processes and the arrival pattern of one of these processes is uncertain. But we can confirm that the arrival process to the high priority queue always follows a Poisson process. Therefore, analytical results obtained for RSU services in Case II show a good consistency with the simulation results.

## 5.2 Performance Evaluation for Exponential Distribution of Service Times

We have developed a queuing analytic framework to analyze the system performance of a CRN with dynamic channel access. In this section we analyze the numerical results obtained for various performance parameters under the assumption of Poisson arrival process with exponential distributed service times. The *Dynamic* $(a; W; V; Q_{L1}; Q_{L2})$ channel aggregation strategy proposed in this thesis should be able to enhance the performance of the existing cognitive radio networks in terms of several performance metrics, i.e., spectrum utilization of the CRN, capacity of the secondary network, blocking probability and forced termination probability of the system and average queuing delay. One of the main targets of this section is to investigate the significance of the queue structure over these performance metrics. First of all, spectrum utilization of the CR network is analyzed.

### 5.2.1 Spectrum Utilization

As we already know, cognitive radio has come out to as an efficient way of increasing spectrum utilization by making use of underutilized spectrum bands. To investigate the spectrum utilization of the system we analyze the impact of the proposed queuing model by varying the maximum queue length of both queues. According to the Fig. 5.5, it is clear that spectrum utilization increases due to the proposed queuing model. When the system is operated without the PMQS, spectrum utilization has the lowest value at any ESU arrival rate in contrast with the system's operation with PMQS. When the ESU arrival rate is increased the improvement of the spectrum utilization is more significant. For example, if the queue lengths of low and high priority queues equal to $Q_{L1=4}$ and $Q_{L2=2}$ respectively and when $\lambda_{S1} = 3.2$, spectrum utilization tends to increase about 8% in contrast with the system without queues. Even though the result is not illustrated, when the queue lengths are doubled, i.e., $Q_{L1=8}$ and $Q_{L2=4}$ spectrum utilization increases by $10\%$ compared with non-queuing system. Therefore it is clear that, even though the queue lengths are doubled, the percentage increment of spectrum utilization will not be doubled. In other words, the increase of spectrum utilization is not direct proportional to the queue lengths.

The increase of maximum queue size implies that the increase of the number of secondary users waiting for an opportunity of the service commencement. Therefore, when an ongoing service is finished, the newly idle channels are immediately acquired by the service requests which are queued in the queues. To increase the spectrum utilization, it is required to have new user requests when the states of the channels change to idle state from the busy state. Large queue lengths can fulfill this condition. Otherwise, channels will stay at idle state for a longer time intervals until the next user arrives, which cause to decrease the spectrum utilization. However this fact indeed depends on the strategy being used to access channels in the CRN.

On the other hand, numerical results reveal that under the identical conditions the CRN with Case II has the highest spectrum utilization in comparison to the CRN with Case I. The reason for this behaviour also can be explained by using the average queue size of the system. Since Case II permits the interrupted ESU services to be queued, the probability that the waiting positions of the low priority queue have been occupied by an ESU is higher than that of Case I. Due to this reason the probability of channels stay in idle state becomes low for a system at Case II. Then the results illustrated in Fig. 5.5 and 5.6 are the evidence of the improvement of spectrum utilization in Case II.

In Fig. 5.6 spectrum utilization has been evaluated by varying the elastic service rate. The reason for the continuous decrease of the spectrum utilization can be explained as follows. As ESU service rate, $\mu_{S1}$ grows, more elastic traffic could be finished within short time. Therefore, the probability that the channels stay in the busy state becomes low as $\mu_{S1}$ grows. In other words, channels will stay at the idle state for a comparatively large time intervals. Since the arrival rates of SUs and PUs are kept as constants in the analysis of Fig. 5.6, the probability of acquiring an idle channel by a new user also decreased when $\mu_{S1}$ increases. For these reasons, channels in the system are not utilized frequently. Thus, the final outcome is the continuous decrease of spectrum utilization.

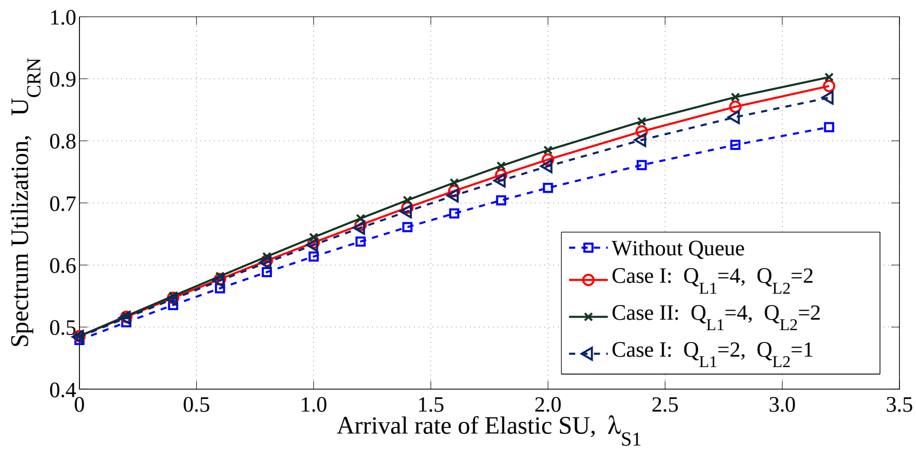


Figure 5.5: Spectrum utilization as a function of ESU arrival rate.

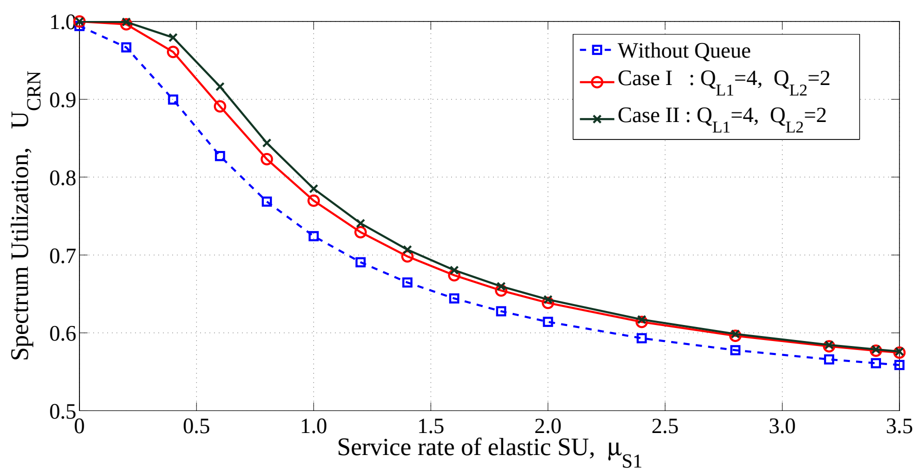


Figure 5.6: Spectrum utilization as a function of ESU service rate.

### 5.2.2 Secondary Network capacity

Fig. 5.7 depicts the system capacity of ESU services as ESU arrival rate $\lambda_{S1}$ varies. With the increasing arrival rate of ESU services, the capacity of ESU services initially increases dramatically. After that capacity grows smoothly and seems to be reached a maximum value. Initially capacity is increased with a high rate because, more ESU services are commenced at a high rate. Afterwards, all the channels in the system gradually close to be fully utilized, therefore capacity curves tend to reach a maximum value. The next significant observation is that the capacity increases due to the queuing model and it further can be increased when the maximum queue size is increased. Therefore, the capacity curve corresponding to the maximum queue size of 8, i.e., $Q_{L1} = 8$ always higher than that of $Q_{L1} = 4$. Large queue size means that more service requests can be stored. Since PU arrival rate is fixed in the analysis of Fig. 5.7, the probability of completing an ESU service is increased as more service requests are added to a queue. Due to this reason, capacity can be increased with large queue sizes.

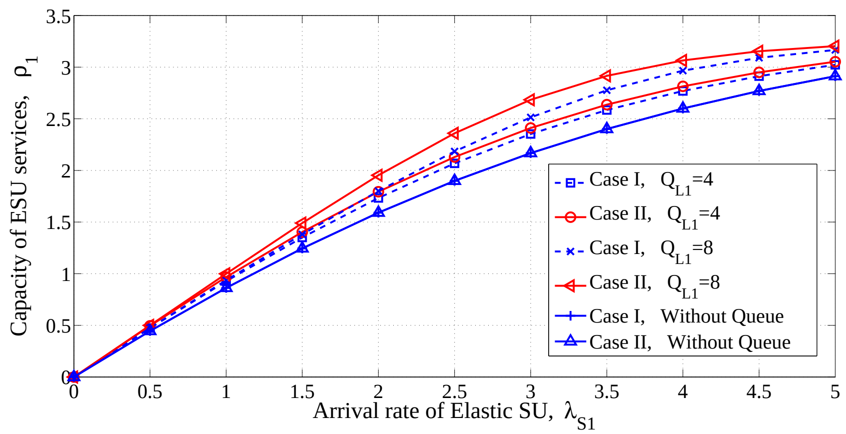


Figure 5.7: ESU capacity as a function of ESU arrival rate.

When the capacity is compared between the two cases, it is obvious that Case II gains more capacity than Case I. Since the interrupted ESU services are forced to terminate in Case I even when EQ is not fully utilized, they are not considered as service completions. Therefore, those services are not counted for capacity calculations. However, in Case II those interrupted services may have chances to finish their services later, since they are queued until they will have an opportunity to access system resources. Because of this reason, capacity of ESU services is improved in Case II than Case I. This fact can be again proved in another aspect, i.e., by analyzing the capacity of real-time services as illustrated in Fig. 5.8. In this figure we keep the ESU arrival rate at a constant value while RSU arrival rate is varied to test the capacity improvement. According to the obtained numerical results RSU capacity increases due to the queuing model. However there is no significant difference between RSU capacity curves corresponding to the two cases as we observed in the variation of capacity of ESU services. Functions of the RSU services in *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy is the same for both Case I and Case II since interrupted RSU services are not queued in both cases. Therefore, capacity of RSU services will not be improved in Case II as observed in ESU services.

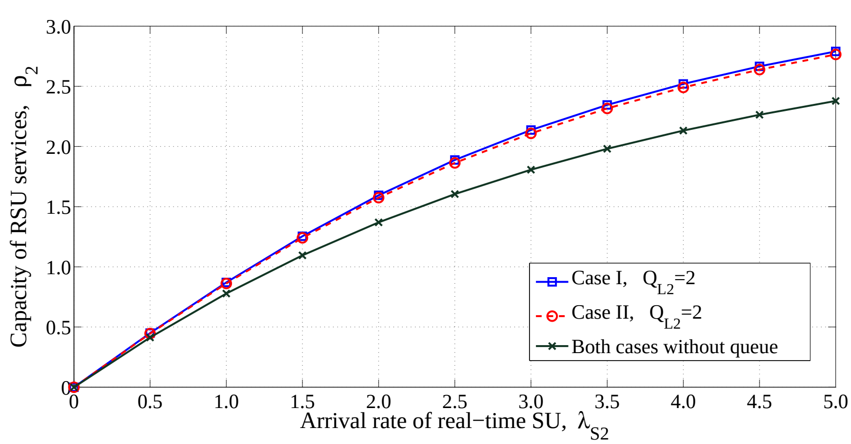


Figure 5.8: RSU capacity as a function of RSU arrival rate.

### 5.2.3 Blocking Probability

Fig. 5.9 depicts the ESU blocking probability as a function of PU service rate, $\mu_P$. It can be seen that the blocking probability dramatically decreases initially as $\mu_P$ grows. When PU service rate is smaller, only few PUs can finish their services per unit of time. Therefore a majority of the channels are occupied by respective licensed services. Therefore, the probability of blocking a new user request is high. On the other hand, the probability of a channel in the busy state decreases as $\mu_P$ increases. Then the probability of blocking a new ESU service also decreases with an increasing rate of $\mu_P$. As shown in Fig. 5.9, the blocking probability of ESU services can be further decreased with the proposed queuing strategies. When the queuing system is not integrated, the blocked calls are cleared if all the channels in the system are occupied and also there is no possibility of a channel sharing with other ongoing ESU services. Conversely, with the queuing model, the blocked calls due to insufficient channels are put in the queue until they will be offered with the required number of channels. For this reason, blocking probability is significantly reduced with the integration of queues. On the other hand, when the maximum queue size is increased more SUs can be queued without blocking their service requests. For this reason, ESU blocking probability shown in Fig. 5.9 decreases as queue size is increased.

To investigate the effect of Case II on blocking probability, Fig. 5.10 is included. As shown in that figure, the blocking probability of the system with Case II is higher than that of Case I. From the discussions on previous sections, we know that interrupted ESU services also allowed to be queued in EQ. In other words, the low priority queue in Case II is allocated for both new arrivals and interrupted services. Thus, the probability that the waiting positions of the queue being occupied by ESU services is high in Case II. For this reason, the chance of queuing becomes less for the new ESU service arrivals and therefore, the blocked service requests have to be cleared. The ultimate result is the increase of blocking probability in Case II than Case I.

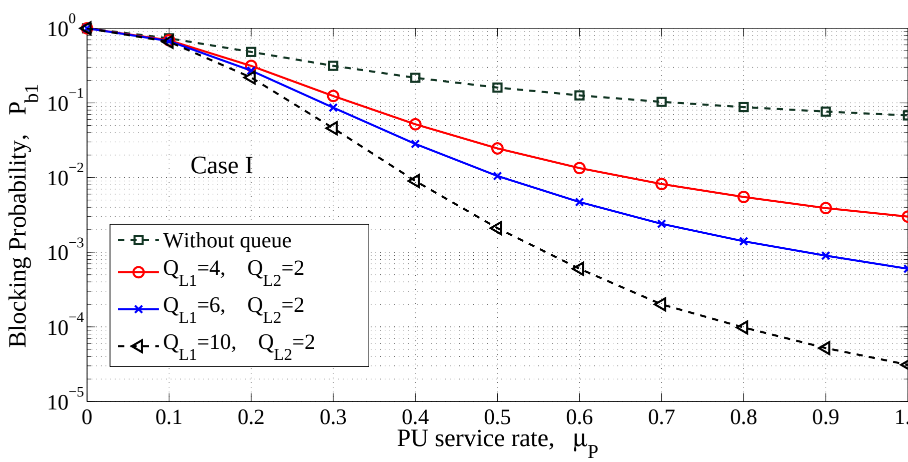


Figure 5.9: Blocking probability as a function of PU service rate - Case I.

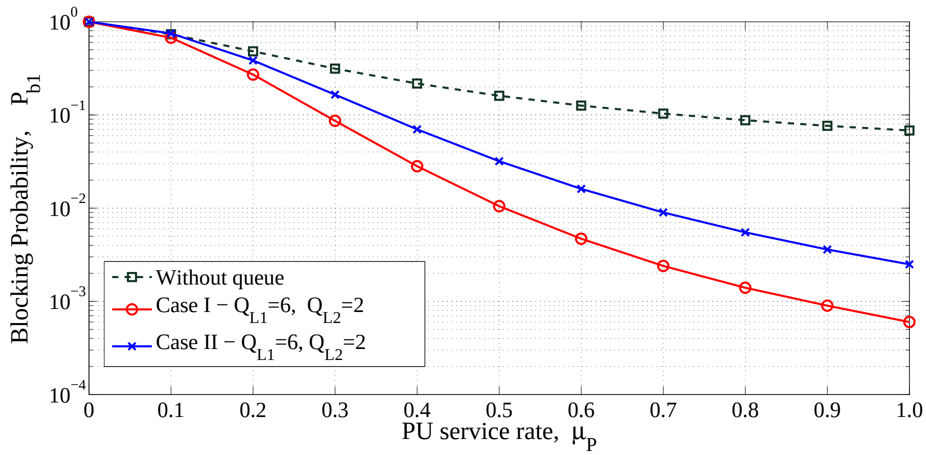


Figure 5.10: Blocking probability as a function of PU service rate - comparison between two cases.

### 5.2.4 Forced Termination Probability

In Fig. 5.11, the forced termination probability is shown as PU arrival rate varies. For ease in comparing the two cases, we illustrate forced termination probabilities of both cases in the same figure. We can observe from this figure that the forced termination probability increases as $\lambda_P$ increases in both cases. This behaviour is the same for the system without queuing. When $\lambda_P$ increases, PUs become more active in the network and due to this reason SU services will be forced to terminate. The next significant observation is that the rate of increasing of forced termination is decreasing with increasing $\lambda_P$. When $\lambda_P$ is comparatively low, certain channels in the system are occupied by SUs and with the growth of $\lambda_P$ they are forced to terminate. Nevertheless, at a higher $\lambda_P$ almost all the channels are occupied by PUs so that there are no ongoing SUs to be interrupted. Thus, the rate of increase of forced termination probability deteriorates with $\lambda_P$. According to the numerical results shown in Fig. 5.11, it is pointed out that the forced termination probability of the queuing system with Case I, is always higher than that of the system without queues. This result convince that Case I of the proposed queuing model cannot show better performance in terms of forced termination probability. The reason can be explained as follows.

To decrease the forced termination rate, either system should possess more bandwidth resources (channels) or less PU activities. In the numerical results shown in Fig. 5.11, the increase of PU arrival rate means more PU activities. On the other hand, when queues are integrated blocked SU services are not cleared but they are put into the queues. Then more interactions between SUs and PUs can occur in contrast with the system without queues. Due to this reason, more forced terminations can be observed in Case I. A fruitful method to obtain decreased forced termination probability of ESUs is proposed in Case II. In Case II the interrupted ESU services in which there is no way of accessing an idle channel, are put into the queue unless the queue is not fully utilized. Hence, the forced termination of ESU services should be reduced with Case II. As Fig. 5.11 highlights forced termination probability is greatly reduced in Case II. When the queue size is increased, better the performance. It is therefore imperative to stress that Case II can be highly recommended for a CRN which needs a low probability of forced terminations.

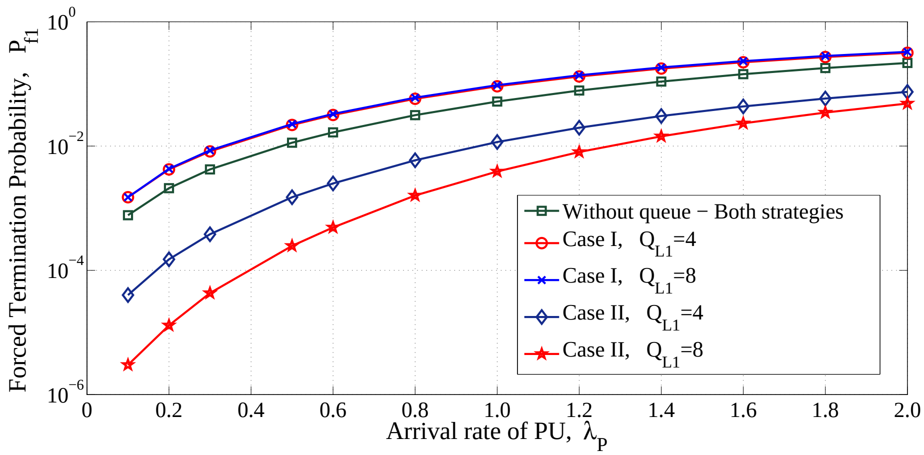


Figure 5.11: Forced termination probability as a function of PU arrival rate.

### 5.2.5 Average Queuing Delay

The main cost introduced in the proposed queuing schemes is the associated waiting time of SU services in the queue, i.e., queuing delay. Long queuing delays create a dilemma for the system to improve QoS. Thus, the analysis of tradeoffs has to be made between the average waiting time and other system performance metrics. If the capacity or blocking probability is only considered when designing the queuing model then queue length has to be increased. But secondary users will experience significant delays and the quality of service will degrade. If secondary users feel that they have to wait for a long time before the service commencement, they will not select the system at the next time. To study the effect of the queue size on the queuing delay we generate the numerical results shown in Fig. 5.12. As illustrated in this figure, average waiting time of ESU services increases with ESU arrival rate. When ESU arrival rate increases the number of ESU service requests in the queue also increases. Wherein most of the waiting rooms are allocated, the mean waiting time of a user will increase. Moreover, an increasing in the number of waiting positions in the queue has a similar effect on the average queuing delay. Apparently, in both cases when the maximum queue size is increased queuing delay also increases.

Fig. 5.12 also shows that the Case I performs better than Case II in terms of queuing delay. Because for a given maximum queue length, ESU services in Case II experience a larger queuing delay in contrast with Case I. In Case II, current queue size is larger than Case I since interrupted ESU services are also queued in addition to the new arrivals. Since average waiting time is direct proportional to the mean queue length as derived in Eq. (4.18), it is justified that Case II leads to a higher queuing delay. This is particularly important result, since it implies the trade off between the average waiting time and the forced termination probability explained in the previous subsection. That is, even though Case II contributes to significantly reduce the forced termination probability of ESU services, their waiting time will increase consequently. This allows us to explore certain optimization techniques to determine the optimal queue size for the proposed queuing models as it improves the system performance in terms of both performance metrics. This thesis does not develop such an optimization method however, it is in our research plan as a future work.

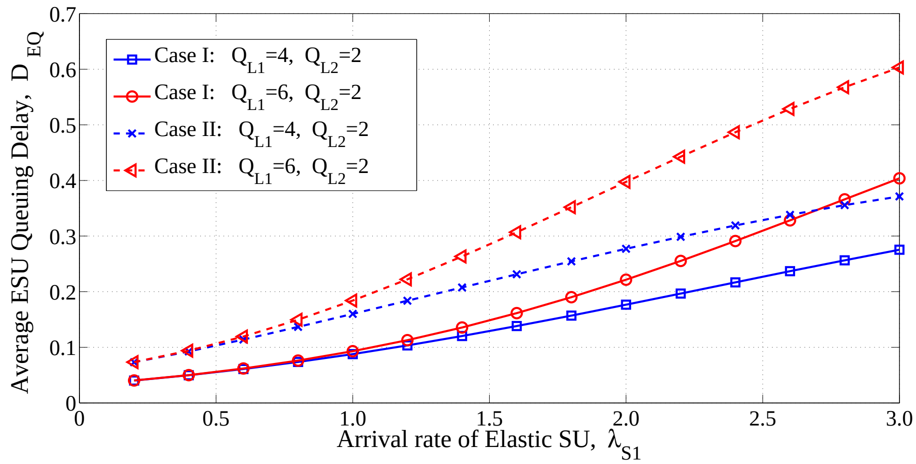


Figure 5.12: Average queuing delay as a function of ESU arrival rate.

## 5.3 Performance Evaluation with Log-normal Service Time Distributions

Thus far we have limited ourselves to Poisson arrival with exponential distributed service times. Even though the arrival process of SU services are modelled as a Poisson process and service times are assumed as exponentially distributed random variables (RV), the real world scenario can be deviate from those theoretical approximations [41]. This fact leads us to analyze the developed analytical models under different service time distributions. From this analysis we might be able to test the validity of the developed mathematical expressions once more. The results presented in Fig. 5.13 illustrate the secondary network capacity of the CR system with Case I under log normal distributed service time. For ease in comparison, we include the corresponding analytical results of the original exponential distribution in the same figure. The capacity variation corresponding to the log normal distribution is generated by simulations. However, it is worth to mention that the arrival process is still considered as a Poisson process.

In the scenario illustrated in Fig. 5.13, we assume that both mean and the variance of log normal distribution equal to those of the original exponential distribution. In MATLAB 2010b, log normal RVs can be generated via the function, *lognrnd*($\mu, \sigma$) where $\mu$ and $\sigma$ are the parameters of the required distribution. To determine those parameters the function *lognstat* which is defined in [42] is used. According to that function, if the mean value and the variance of the required log normal distribution are equal to $m$ and $\nu$ respectively then,

$$\mu = log\left[\frac{m^2}{\sqrt{\nu + m^2}}\right], \tag{5.1}$$

$$\sigma = \sqrt{log\left(\frac{\nu}{m^2 + 1}\right)}. \tag{5.2}$$

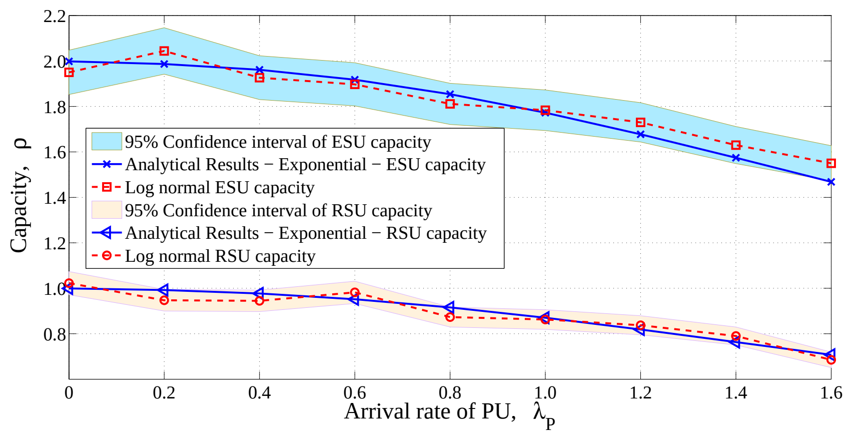


Figure 5.13: Capacity as a function of $\lambda_P$ with log normal distribution with the same variance as exponential distribution.

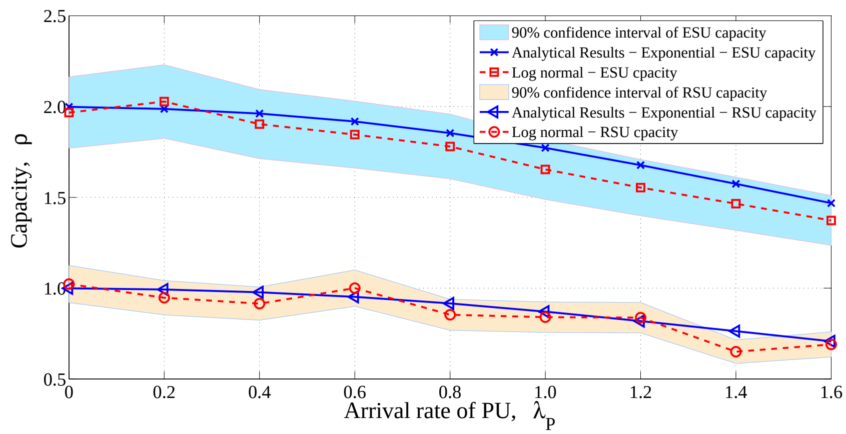


Figure 5.14: Capacity as a function of $\lambda_P$ with log normal distribution with a variance different from the exponential distribution.

The results presented in Fig. 5.13 show that the capacity of secondary network obtained using log normal distribution match quite closely to those of the analytical model. As demonstrated in that figure, analytical output of both ESU and RSU services exists within the 95% confidence intervals of the respective log normal simulation results. Hence, it is observed that these results are not sensitive to the type of service distributions as long as arrival pattern is still Poisson. Though the variance of the log normal distributions is kept as the same as in original exponential distributions, in real world scenario a high variability is observed in traffic flow sizes [22]. Therefore, we performed the same test by setting the variance of log normal distributions 20% higher than that of the exponential distributions. The obtained results are illustrated in Fig. 5.14. From this figure it can clearly be seen that the analytical results exist within the 90% confidence interval of the respective log normal simulation results. Although the confidence interval is broadened than the confidence levels in Fig. 5.13, the theoretical model still shows a good match with the simulation results.

## 5.4 Performance Comparison between Heterogeneous Traffic Types

For ease in understanding the performance of each queuing approach, the analysis presented in the previous sections mainly focuses on the performance of ESU services. It is intricate to determine the performances of RSU services since parameters of RSU services are fixed in those illustrations. In this section the comparison between two traffic types, i.e., ESU and RSU, is articulated under *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy. In the numerical results presented in this section the following parameters are kept as constants while only the PU arrival rate is varied as the independent parameter. $\lambda_{S1} = \lambda_{S2} = 2$, $\mu_{S1} = \mu_{S2} = 1$ and $\mu_P = 0.5$. The priority factor is assigned as $\beta = 2$ in the queue scheduling algorithm.

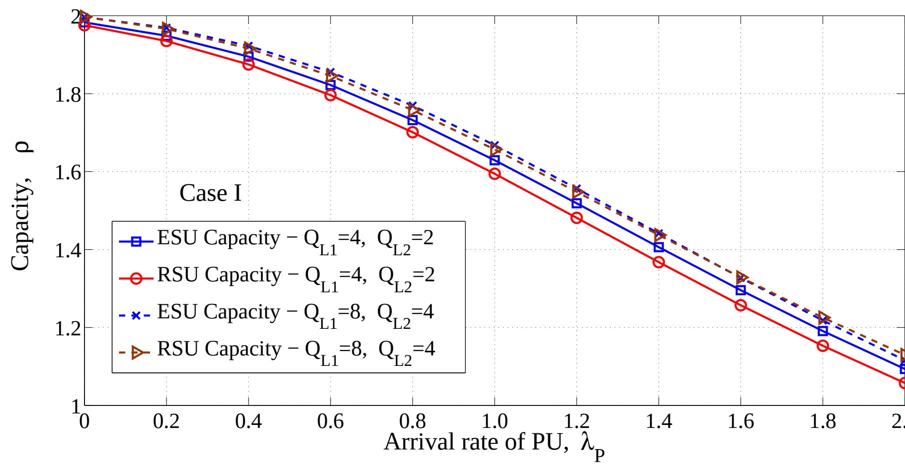


Figure 5.15: Capacity as a function of PU arrival rate.

As shown in Fig. 5.15, the secondary network capacity is gradually decreased with increasing PU arrival rate in both services. And also, when the maximum queue size is increased capacity also increases, since more SU services obtain the opportunity for service commencement. However, it can be observed that, although the arrival rate and the service rate of both services are the same, RSU capacity is slightly less than ESU capacity. This is mainly due to the queue size and the functioning ways of selected DSA method, i.e., *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy. The high priority queue, *RQ* has limited number of waiting positions compared to the low priority queue, *EQ* in order to reduce the queuing delay of real time applications. Thus, the number of service commencements of RSU services is reduced which affects for the reduction of capacity. On the other hand, RSU services do not show much spectrum adaptation in contrast with ESU services. However, it is clear that even though the maximum queue size of EQ is twice than that of RQ, the ESU capacity is not much higher than RSU capacity. For example, the ESU capacity at higher $\lambda_P$ values is not more than 8% of the capacity of RSUs. The dominant reason for this is the assigned higher priority for RSU services at the queue scheduling. Even though *RQ* is small, since the RSU services in *RQ* are given with high priority, service completion rate can be a higher value.

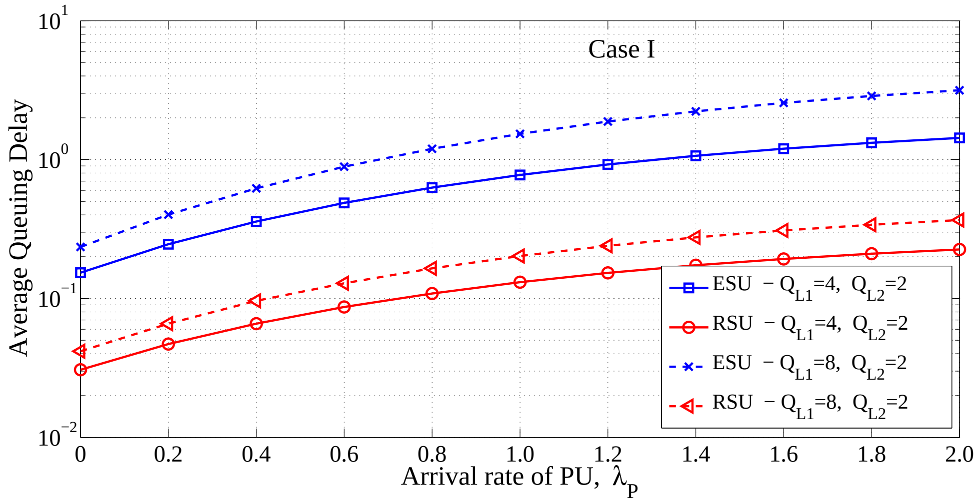


Figure 5.16: Average queuing delay as a function of PU arrival rate.

Fig. 5.16 depicts the average queuing delay of each service type with different queue sizes. The illustrated results explicitly show that the queuing delay experienced by ESU services is much higher than that of RSU services. As explained in *Little's law* due to large queue size of *EQ*, queuing delay dramatically increases. On the other hand, for the reason of low priority of elastic traffic, the ESU services in *EQ* have to wait for a longer time periods for commencing their services in contrast with RSU services in *RQ*. Due to those two reasons ESU services experience longer delays in the queue. As we already mentioned, maintaining less delay is an ideal condition for real time services. From Fig. 5.16, it is obvious that we could maintain a comparatively less delay for real time traffic in the CRN by designing the queue scheduling algorithm and setting the queue sizes in appropriate ways.

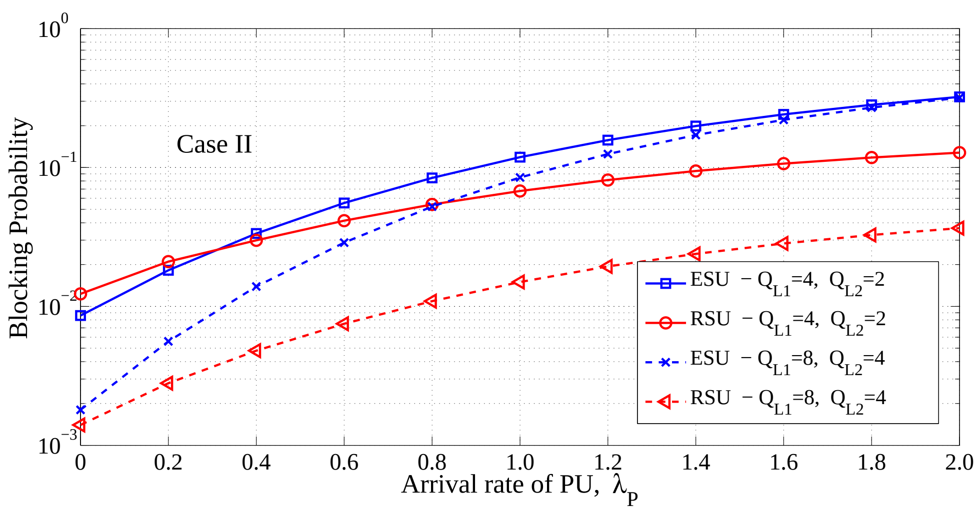


Figure 5.17: Blocking probability as a function of PU arrival rate - Case II.

Fig. 5.17 illustrates the blocking probability of SU services with respect to the arrival rate of primary users in Case II. Larger the queue sizes, the lower the blocking probability since blocked services can be queued without clearing the requests when the system resources are at busy state. But, when PU arrival rate increases blocking probability also increases since majority of the channels are in busy state. Those two observations are common for the both elastic and real-time traffic. However, the blocking probability of RSU services is higher than that of ESU services for lower values of $\lambda_P$. This is mainly because of the low queue size. When there is no enough space in the queue, the incoming RSU requests have to be blocked with comparatively higher rate than ESU requests. But at large $\lambda_P$ this behaviour is totally different since ESU blocking probability is higher than that of RSU even though the maximum queue size of *EQ* is large than the maximum queue size of *RQ*. The reason for this contradiction can be explained as follows. When the PU arrival rate in increased, the forced termination rate of ESU services also increases. In Case II, forced terminated ESU services are queued. Then, most of the waiting rooms of the queue are occupied by the force terminated services and therefore new ESU requests have to be blocked. Ultimately, the larger queue size does not show an advantage in terms of blocking probability for elastic traffic. However, there is no similar issue for RSU services.

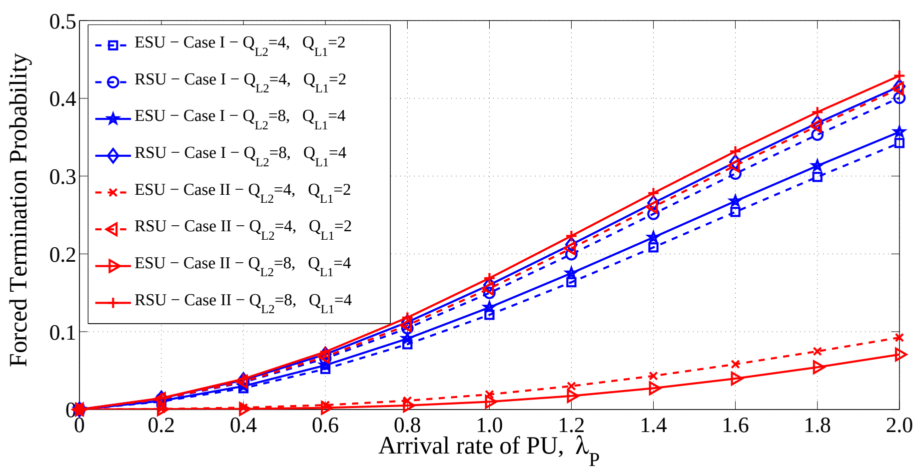


Figure 5.18: Forced termination probability as a function of PU arrival rate - comparison between two cases.

Forced termination probability of both services under two cases is illustrated in Fig. 5.18. When the queue possesses more waiting rooms in Case II, forced termination rate is slightly increases in both service types as already discussed in Section 5.2.4. But in Case II, ESU services show better performance since forced termination probability is significantly reduced. Since RSU functions of Case II are similar with Case I, there is no significant performance change of RSU forced termination probability.

## 5.5 Chapter Summary

To demonstrate the validity of derived analytical models, simulation results are depicted with confidence intervals. All most all the analytical results stood within the 90% confidence interval of simulation results pertaining to both queuing methods. Due to non-Poisson arrivals at the low priority queue, small deviations could also be observed in certain analytical outputs relevant to Case II. However, they are not significant deviations which can devaluate the model. The chapter proceeds by demonstrating the performance of proposed queuing model in terms of various parameters. The proposed system is always compared with the same CRN without a queuing model. Capacity of the secondary network and the overall spectrum utilization has increased due to the integrated queuing system. A significant decrease of the blocking probability is observed in Case I nevertheless, the forced termination rate is not decreased. Forced termination probability is greatly reduced with the queuing method introduced in Case II. Since associated queuing delay in Case II is higher than Case I, for delay critical applications Case II is not recommended.

The chapter ends by presenting the simulation results for secondary network capacity obtained under service time with log normal distributions. According to the results pertaining to those different distributions, the analytical models developed in this thesis can be considered as a robust reference model for analyzing the performance of CRNs.

# Chapter 6: Discussions

*This chapter is dedicated to discuss several important properties of the proposed queuing model and its performance. Prior to that, certain limitations of the existing methods briefly discussed. The queuing delay is also discussed in comparative way as it is the main consideration of the cost analysis of the proposed solution. Besides, the effect of non-Poisson arrivals on Markov modelling is discussed*

## 6.1 Fundamental Discussions on Proposed Model

This thesis began by highlighting the need for performance improvement of existing channel aggregation strategies designed for the multi-channel cognitive radio networks. Therefore it is of great importance to study the existing channel aggregation methods with enhanced features like spectrum adaptation. It is rare to find a related work that proposes novel DSA techniques which supports several enhanced features simultaneously. Most of the related publications have been considered with limited number of features. However, the dynamic channel aggregation strategy proposed by Lei Jiao et al. [22] is selected for analyzing since it covers a range of advanced features such as spectrum handover, channel adaptation and heterogeneous traffic modelling. A queuing system is identified as a main requirement to further increase the system performance of the DSA strategy proposed in [22]. An integration of a queuing model can be performed in many ways. The study of existing queuing models proposed in [14, 27, 28] for CRNs provided a proper background for this work. In addition, the models proposed in [43] and [44] which analyses different structures of queuing models have been studied. Those models have sophisticated designs as well as certain limitations. For example, some of the proposed methods have designed with focusing only on the capacity improvement of the secondary network. And some of the methods developed with analytical models solely targeting a decrease of packet loss and delay. Therefore, the proposed model in this thesis which analyzes the system performance in terms of various metrics such as capacity, blocking probability, forced termination, delay and spectrum utilization, is developed as it can be used to evaluate overall system performance of a CRN.

### Effect of maximum queue size

In our proposed model maximum queue sizes are defined for both queues according to the service type. The main objective of setting a finite queue size is to avoid longer queuing delays. Especially for the real time services queuing delay should be minimized as much as possible. Therefore, the queue size of the high priority queue is always set as a small value in contrast with the low priority queue. However, some research attempts have defined infinite buffer size for SU services to avoid packet dropping due to insufficient system resources [47]. For delay critical applications infinite buffer may cause significant effect on the QoS due to poor delay performance. For example voice services are rather sensitive to delay and short message services (SMS) are tolerant to delay [48]. Therefore, the most promising way of handling the queue size is two-fold. In the first method, priority queuing discipline should be applied instead of FCFS order. Then delay critical applications can be served first prior to the other applications regardless of the order of arrival. The next approach is the establishment of multiple queues as proposed in our queuing model in this thesis. In that solution, the queue which is allocated for delay critical applications should be designed with limited buffer size while the other queues can have infinite buffer sizes. Meanwhile, the high priority of resource allocation should be assigned to the queue with limited buffer size.

#### Effect of non-Poisson arrival process

It is well known that primary and secondary network behaviour of a CRN can be modelled using CTMCs, which is a powerful tool for modelling stochastic processes. The basic assumption required by the CTMC analysis in this thesis is that service requests arrive as a Poisson process. This requirement is naturally satisfied if the service requests are generated independently by a large number of users. The queuing approach defined as Case I satisfies these conditions while Case II does not since interrupted services are queued in the low priority queue. Since service interruption depends on the arrival rate of new services, it cannot be considered as an independent process. For example, when the SU arrival rate increases, service interruption rate also increases if PUs are relatively static. However, in similar cases of Erlang model with non-Poisson arrivals, it has proved that model is still valid under certain assumptions [49]. Due to this reason, the validity of the analytical models of Case II is essential for demonstrating convincing results. Since the analytical results show a good fit with the simulation results the model developed in Case II is validated. Due to time constraints, a mathematical investigation on non-Poisson process is not performed in this thesis.

## 6.2 Analysis on Numerical Results

The validity of the proposed analytical models is checked through the comparison between theoretical and simulation results. Analytical results show a good fit with the simulation results. But a slight deviation of simulation results is observed in Case II, nevertheless, analytical results are still inside the 90% confidence intervals of simulation output. This can be happened due to two main reasons. The main reason is that the non-Poisson arrival pattern at the low priority queue as the interrupted ESU services also eligible to be queued. The other reason is quite common for any simulation work, which is the limitation of simulation parameter selections and associated assumptions. For example, to avoid simultaneous events of different users, the repetitive time duration in which the system is tested by the simulation program should be set as small as possible. On the other hand, there is a limitation for setting the smallest time duration. Because, if the so-called time is too small as in the order of $10^{-8}$, the simulation program will spend a longer time period for generating the output.

The following Table 6.1[1] summarizes the performance parameters of elastic traffic[2] over the different queuing approaches. We find that the both cases perform better than the existing solution in terms of capacity, blocking probability and spectrum utilization. However, Case II demonstrates better performance than Case I in terms of capacity and spectrum utilization. Meanwhile it does not show much performance in terms of blocking probability in contrast with Case I. When forced termination probability is considered, only Case II shows improved performance. Even though the associated queuing delay is the common disadvantage in both cases in comparison to the existing system, Case I shows better delay performance than Case II.

Table 6.1: Performance Summary of the Proposed Queuing Methods

| Performance parameter of ESU service | Comparison between two cases: The best case | Comparison with the system without queuing system |
|---|---|---|
| Spectrum Utilization | Case II | Both cases perform better than $SYS_{NQ}$ |
| Capacity | Case II | Both cases perform better than $SYS_{NQ}$ |
| Blocking Probability | Case I | Both cases perform better than $SYS_{NQ}$ |
| Forced Termination Probability | Case II | Case II perform better than $SYS_{NQ}$ |
| Average Queuing delay | Case I | $SYS_{NQ}$ has no associated delay due to queuing. Therefore, the queuing delay is an disadvantage of both cases |

The performance of the two queuing methods can be analyzed in another aspect as demonstrated in Table 6.2. According to the SU and PU traffic behaviour and traffic type the most suitable queuing method can be recommended. Our recommendations in that table are totally based on the numerical results discussed in Chapter 5.

Table 6.2: Summary of Recommendations

| Traffic Behaviours | Recommended Case for better performance |
|---|---|
| SUs are relatively active, PUs are relatively static | Case I |
| SUs are relatively static, PUs are relatively active | Case II |
| Both SUs and PUs are active | Case I or Case II |
| Both SUs and PUs are static | Case I |
| Delay critical applications | Case I |
| Pure elastic traffic | Case II |

[1] $SYS_{NQ}$ denotes the cognitive radio system without proposed queuing scheme.

[2] Since there is no significant difference of system performance of real time services between the two cases, we do not explicitly summarized them.

# Chapter 7: Conclusions & Future Work

*In this chapter we conclude this thesis by summarizing main conclusions and our contributions. We also suggest some future research directions that could provide the next steps along the path to a practical and widely applicable cognitive radio systems.*

## 7.1 Conclusions

In the thesis definition, we expressed the main objective of the work of this thesis as the evaluation of system performance of multi-channel CRNs with queuing systems. In this final chapter, we will conclude by describing the progress made towards this objective fulfillment in terms of the development of the queuing model referred to as PMQS and the channel access strategy referred to as *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$. Queuing models proposed in various research activities are studied. However, certain limitations of those models over applicability on real scenarios are identified. To avoid such imperfections, PMQS was designed by including several features like channel aggregation, spectrum adaptation, heterogeneous traffic and priority based queue scheduling. The queuing scheme proposed in this thesis consists of separate queues allocated for different traffic types so that the FCFS discipline is easy to apply. The priority factor defined in the queue scheduling process is also influence on the performance of both services. By adjusting the value of the priority factor, system operator can distribute system resources between the services types.

The accuracy of the derived theoretical models is assessed by simulations. The obtained analytical results display a good fit with the exact simulation results confirming the correctness of the developed CTMC models. The numerical results show that the overall spectrum utilization and the capacity of the secondary network can be increased via the proposed model. Case II of the proposed model shows better performance in terms of spectrum utilization and the capacity than that of Case I. From the results pertaining to the blocking probability of SU services we can conclude that blocking probability can be further decreased in both cases with the increase of queue size. However, Case I shows better performance in terms of blocking probability than Case II. With the increase of queue sizes, even though the blocking probability is decreased the average queuing delay of a service has increased. Therefore, the maximum queue sizes have to be controlled according to the QoS requirements of the traffic type. Again Case I shows low queuing delays outperforming Case II. Thus, for the delay sensitive applications Case I is recommended due to the needs of minimized queuing delay.

The queuing method introduced as Case I does not show performance improvement in terms of forced termination probability of SU services. The main reason of presenting Case II is to decrease forced terminations in the secondary network. From the numerical results we can conclude that, a system with Case II queuing approach, significantly reduces the forced termination probability when the queue size is increased. Therefore, we can conclude that Case II as a better queuing method if PUs are more active in the CRN. And Case I is recommended for a system with less PU activities but more SU activities.

The findings of this thesis gives a deep insight into the performance improvement of the existing channel aggregation strategies in cognitive radio networks. By considering all the numerical results obtained for each performance parameter, we can conclude that the proposed priority based multiple queue system is an efficient queuing model which can be used to enhance the overall system performance. Therefore, the system design and demonstrated results are of importance to the cognitive radio research community.

## 7.2 Major Contributions

This thesis dealt with the exploration of performance improvement schemes for the existing dynamic channel access strategies in cognitive radio networks by means of designing a queuing system. The topics covered range from the analysis of existing models to the design and overall performance evaluation of the system. In this regard, the main contributions of this thesis are summarized below.

- A new priority based queuing scheme is proposed for multi-channel cognitive radio networks. The proposed model can increase the system performance in many aspects such as capacity and spectrum utilization.

- The functionality of one of the dynamic channel aggregation strategies proposed in recently published IEEE transaction paper [22] is extended towards a priority based queuing model for cognitive radio networks. The modified channel access strategy is designed with numerous enhancement features like spectrum handover, channel aggregation, channel sharing and queuing mechanisms for heterogeneous traffic. With the improved DSA technique, we have shown quantitative evidence that the proposed queuing model is able to increase the overall system performance.

- A novel queue scheduling mechanism is developed based on the priority factors of different traffic types. When a system consists of multiple queues, the next opportunities of service commencement are distributed among the waiting services in the queues according to the predefined priority levels. For this reason, the traffic with the lowest priority level also has the chance of acquiring the system resources depending on the parameter settings of the developed algorithm. Since the priority factor in our Algorithm I is a tunable parameter, the algorithm can be used as a generalized queue schedule for priority queuing with further improvements.

- Although the developed queue scheduling algorithm is specifically designed for cognitive radio systems, it is possible to extend for many applications which have multiple queues assigned for different customer requests. This would be particularly useful in different teletraffic models.

- The PMQS model is tested under different traffic distributions in addition to exponential distribution and those testing results show a good fitting between analytical and simulation results. For this reason we believe that the proposed queuing model, PMQS can be used to model real scenarios of cognitive radio networks with higher accuracy.

## 7.3 Recommendations for Future Work

The results of this thesis point to several interesting directions for future work.

The entire analysis presented in this thesis is based on the assumption that the service arrival process at the queue is a Poisson process with exponential distributed service times. Even though this assumption is totally valid regarding the proposed queuing approach of Case I, it is not quite agree with Case II as we mentioned in Section 5.1.2. Therefore, there is a need of further research to obtain exact mathematical definition for the arrival process of services at the queue in Case II.

In this thesis the Quasi-Stationary Regime (QSR) of the CRN is not analyzed due to time constraints. QSR is obtained with the assumption of infinitely slow PU activities compared to secondary network activities [22, 45]. In that case, interruption rate of SU services becomes negligible. Then the forced termination probability of proposed two queuing scenarios is interesting to be analyzed since their functionality is different from each other. Thus it is need to develop analytical models to study the performance of proposed queuing model in QSR.

Another important parameter that is not taken in to account in the proposed queuing model is that the maximum queuing delay for a SU service. Queuing delay has the most adverse effect on the delay performance of the system. If $D_{eq_{max}}$ is denoted as the maximum queuing delay that could be experienced by an ESU service in the low priority queue, then ESU services which are queued in the queue for more than $D_{eq_{max}}$ period will vacate the system as an unprocessed service. By introducing an upper bound for the queuing delay for a service, the optimum queue length for a queue can be properly determined depending on the probability that a service vacate the system as an unprocessed service.

In this thesis we concentrate our analysis for a single cell cognitive radio network. And the mobility of CR users is also not considered in the analysis. In our future work, the proposed queuing model can be analyzed with the mobility of CR users between different cells of the cognitive radio network. Other future research directions include the evaluation of system performance with different DSA techniques in addition to the channel access strategy proposed in the thesis. Therefore we are planning to extend the other channel aggregation strategies explained in Section 2.7 with PMQS queuing model.

# Bibliography


[1] ITU-R Recommendation V.431: *Nomenclature of the Frequency and Wavelength Bands used in Telecommunications*, International Telecommunication Union, Geneva, May 2005.

[2] Available: *http://www.ntia.doc.gov/files/ntia/publications/2003-allochrt.pdf*

[3] V. Alenta, R. Marsalek, G. Baudoin, M. Villegas, M. Suarez, and F. Robert, "Survey on Spectrum Utilization in Europe: Measurements, Analyses and Observations", in *Proc. 5th International Conference on Cognitive Radio Oriented Wireless Networks & Communications (CROWNCOM)*, Cannes, France, 2010.

[4] I. Akyildiz, W. Lee, M. Vuran, and S. Mohanty, "Next Generation-Dynamic Spectrum Access-Cognitive Radio Wireless Networks: A survey," *Computer Networks: The International Journal of Computer and Telecommunications Networking*, vol. 50, no. 13, pp. 2127-2159, 2006.

[5] N. Bao, and L. Shen, "Quick Sensing and Spectrum Sharing Strategy for Frequency Hopping Signal in ISM Band," in *Proc. International Conference on Wireless Communications & Signal Processing (WCSP)*, Nanjing, China, Nov. 2009.

[6] D. Cabric, M. S. W. Chen, D. A. Sobel, S. Wang, J. Yang, and R. W. Brodersen, "Novel Radio Architectures for UWB, 60 GHz, and Cognitive Wireless Systems", *EURASIP Journal on Wireless Communications and Networking archive*, vol. 2006, article ID 17957, pp. 1-18, Apr. 2006.

[7] J. Mitola and G. Q. Maguire, "Cognitive Radios: Making Software Radios more Personal", *IEEE Personal Communications*, vol. 6, no. 4, Aug. 1999.

[8] ET Docket No. 03-108, 20 FCC Rcd 5486, "Cognitive Radio Report and Order", Mar. 2005.
Summary available at http://edocket.access.gpo.gov/2005/05-8808.htm.

[9] P. Steenkiste, D. Sicker, G. Minden, and D. Raychaudhuri, "Future Directions in Cognitive Radio Network Research", pp. 6-8, NSF Workshop Report, Mar. 10, 2009.

[10] A. M. Wyglinski, M. Nekovee, and Y. T. Hou, *Cognitive Radio Communications and Networks - Principles and Practice*, pp. 3-8, Elsevier Inc., First Edition, 2009.

[11] C. R. Stevenson, G. Chouinard, Z. Lei, W. Hu, S. J. Shellhammer, and W. Caldwell, “IEEE 802.22: The First Cognitive Radio Wireless Regional Area Network Standard”, *IEEE Communications Magazine*, vol. 47, no. 1, pp. 130-138, Jan. 2009.

[12] L. Jiao, F. Y. Li, and V. Pla, “Dynamic Channel Aggregation Strategies in Cognitive Radio Networks with Spectrum Adaptation”, in *Proc. IEEE GLOBECOM*, Houston, TX, USA, Dec. 2011.

[13] L. Jiao, V. Pla, and F. Y. Li, “Analysis on Channel Bonding/Aggregation for Multi-channel Cognitive Radio Network”, in *Proc. European Wireless (EW)*, Lucca, Italy, Apr. 2010.

[14] P. Kaur, A. Khosla, and M. Uddin, “Markovian Queuing Model for Dynamic Spectrum Allocation in Centralized Architecture for Cognitive Radios”, *IACSIT International Journal of Engineering and Technology*, vol. 3, no. 1, Feb. 2011.

[15] E. W. M. Wong, and C. H. Foh, “Analysis of Cognitive Radio Spectrum Access with Finite User Population”, *IEEE Communication Letters*, vol. 13, no. 5, May 2009.

[16] H. Al-Mahdi, M. A. Kalil, F. Liers, and A. M. Thiel, “Increasing Spectrum Capacity for Ad hoc Networks using Cognitive Radios”, *IEEE Communication Letters*, vol. 13, no. 9, Sep. 2009.

[17] G. D. Nguyen, and S. Kompella, “Channel Sharing in Cognitive Radio Networks”, in *Proc. IEEE Military Communications Conference (MILCOM)*, pp. 2268-2273, San Jose, California, USA, Nov. 2010.

[18] I. F. Akyildiz, W. Y. Lee, M. C. Vuran, and S. Mohanty, “A Survey on Spectrum Management in Cognitive Radio Networks”. *IEEE Communication Magazine*, vol. 46, no. 5. , pp. 40-48, Apr. 2008.

[19] R.V. Prasad, P. Pawlczak, J. A. Hoffmeyer, and H. S. Berger,“Cognitive functionality in next generation wireless networks: standardization efforts,” *IEEE Communications Magazine*, vol. 46, no. 4, pp. 72-78, 2008.

[20] Q. Zhao and B. M. Sadler, “A survey of Dynamic Spectrum Access”, *IEEE Signal Processing Magazine*, vol. 79, no. 3. , pp. 79-89, May 2007.

[21] L. Bao Le, and E. Hossain, “Resource Allocation for Spectrum Underlay in Cognitive Radio Networks”, *IEEE Transactions On Wireless Communications*, vol. 7, no. 12, pp. 5306-5315 , Dec. 2008.

[22] L. Jiao, F. Y. Li, and V. Pla, “Modeling and Performance Analysis of Channel Assembling in Multi-channel Cognitive Radio Networks with Spectrum Adaptation”, *IEEE Transactions on Vehicular Technology, accepted for publication*, Mar. 2012.

[23] R. G. Gallager, *Discrete Stochastic Process*, pp. 32-35, Kluwer Academic Publishers, Boston, USA, 1996.

[24] H. C. Tijms, *A First Course in Stochastic Models*, pp. 02-06, John Wiley and Sons Inc., England, Second Edition, 2003.

[25] J. D. C. Little, and S. C. Graves, "Little's Law" Ch. 5, D. Chhajed and T. J. Lowe, eds. *Building Intuition: Insights From Basic Operations Management Models and Principles*, pp. 81-100, Springer, New York, 2008.

[26] C. Cordeiro, and M. Ghosh, "Channel Bonding versus Channel Aggregation", IEEE 802.22 WG document no. 22-06-0108-00-0000, July 2006.

[27] M. M. Rashid, M. J. Hossain, E. Hossain, and V. K. Bhargava,"Opportunistic Spectrum Scheduling for Multiuser Cognitive Radio: A Queuing Analysis", *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5259-5269, Oct. 2009.

[28] S. Tang, and B. L. Mark, "Performance of a cognitive radio network with tolerable service degradation", *Proc. 7th Int. Workshop on Design of Reliable Communication Networks (DRCN'09)*, Washington DC, USA, Oct. 2009.

[29] S. Homayouni, and S. A. Ghorashi, "The Impact of Queue on Opportunistic based Cognitive Radio Networks", *Journal of Computing*, vol. 3, no. 7, Jul. 2011.

[30] S. Tang, and B. L. Mark, "Performance Analysis of a Wireless Network with Opportunistic Spectrum Sharing", in *Proc. IEEE GLOBECOM*, pp. 4636-4640, Washington, DC, USA, Nov. 2007.

[31] M. H. Cheung, A. H. Mohsenian-Rad, V. W.S. Wong, and R. Schober, "Random Access for Elastic and Inelastic Traffic in WLANs", *IEEE Transactions On Wireless Communications*, vol. 9, no. 6, pp. 1861-1866, Jun. 2010.

[32] M. Gospodinov, "The Affects of Different Queuing Disciplines over FTP, Video and VoIP Performance", in *Proc. the 5th international Conference on Computer Systems and Technologies*, Rousse, Bulgaria, Jun. 2004.

[33] J. A. Sokolowski, and C. M. Banks, *Principles of Modeling and Simulation: A Multidisciplinary Approach*, pp. 75-78, John Wiley and Sons Inc., New Jersey, 2009.

[34] C. G. Cassandras, and S. Lafortune, *Introduction to Discrete Event Systems*, pp. 400-410, Second Edition, Springer, New York, 2008.

[35] W. Ahmed, J. Gao, H. A. Suraweera, and M. Faulkner, "Comments on Analysis of Cognitive Radio Spectrum Access with Optimal Channel Reservation", *IEEE Transactions on Wireless Communications*, vol. 8, no. 9, pp. 4488-4491, Sep. 2009.

[36] T. M. N. Ngatched, A. S. Alfa, and J. Cai, "Analysis of Cognitive Radio Networks with Channel Aggregation and Imperfect Sensing", in *Proc. the 20th International Conference on Computer Communications and Networks (ICCCN)*, Maui, Hawaii, Aug. 2011.

[37] H. Qing, X. Shaoyi, and J. Xiaojun, "Performance Evaluation of Secondary Users in Dynamic Spectrum Access System", in *Proc. IEEE 22nd International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Toronto, Canada, Sep. 2011.

[38] A. Laourine, S. Chen, and L. Tong, “Queuing Analysis in Multichannel Cognitive Spectrum Access: A Large Deviation Approach”, in *Proc. 29th IEEE International Conference on Computer Communications (INFOCOM)*, San Diego, California, USA, Mar. 2010.

[39] D. Wang, X. Li, and J. Y. McNair, “An Evaluation of the Performance Advantage of Diffserv with Respect To Voice over Ip Flows”, in *Proc. IEEE Military Communications Conference (MILCOM)*, Boston, MA, USA, Oct. 2009.

[40] D. T. Kaplan, “Making Statements with Precision: Confidence Intervals”, *Resampling Stats in MATLAB*, ISBN 0-9672088-0-7, Resampling Stats, Inc., Arlington, Virginia, 1999.

[41] D. Willkomm, S. Machiraju, J. Bolot, and A. Wolisz, Primary Users Behavior in Cellular Networks and Implications for Dynamic Spectrum Access, *IEEE Communications Magazine*, vol.47, no.3, pp.88-95, Mar. 2009.

[42] *lognstat Function* in Statistics toolbox, MATLAB R2012a documentation, available: http://www.mathworks.se/help/toolbox/stats/lognstat.html

[43] I. Suliman, and J. Lehtomaki, “Queuing Analysis of Opportunistic Access in Cognitive Radios”, *Second International workshop on Cognitive Radio and Advanced Spectrum Management (CogART)*, pp. 153-157, San Jose, Aalborg, Denmark, May 2009.

[44] M. Zhang, S. Jiang, G. Wei and H. Wang, “Performance Analysis of the Cognitive Radio Network with a Call Level Queue for Secondary Users”, in *Proc. 5th International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM)*, Beijing, China, Sep. 2009.

[45] S. K. Cheung, R. J. Boucherie, and R. Nuez-Queija, “Quasi-stationary Analysis for Queues with Temporary Overload,” in *Proc. 22nd International Teletraffic Congress (ITC)*, Amsterdam, Netherlands, Sep. 2010.

[46] W. Han, J. Li, Z. Tian, and Y. Zhang, “Dynamic Sensing Strategies for Efficient Spectrum Utilization in Cognitive Radio Networks,” *IEEE Transactions on Wireless Communications*, vol.10, no.11, pp.3644-3655, Nov. 2011.

[47] A. K. Farraj, S. L. Miller, and K. A. Qaraqe, “Queue Performance Measures for Cognitive Radios in Spectrum Sharing Systems,” *IEEE GLOBECOM Workshops (GC Wkshps)*, pp.997-1001, Houston, TX, USA, Dec. 2011.

[48] H. Tran, T. Q. Duong, H. J. Zepernick, “Average Waiting Time of Packets With Different Priorities in Cognitive Radio Networks, ”in *Proc. 5th IEEE International Symposium on Wireless Pervasive Computing (ISWPC)*, pp.122-127, Modena, Italy, May 2010.

[49] T. Bonald, “The Erlang Model with non-Poisson Call Arrivals”, in *Proc. the Joint International Conference on Measurement and Modeling of Computer Systems*, vol. 34, no. 1, pp. 276-286, Jun. 2006.

# Appendix A: Attached Publication

---

**Title:** Performance Analysis of Channel Aggregation Strategies in CR Networks with Queues

**Affiliation:** University of Agder, Faculty of Engineering and Science,
P.O. Box 509, 4898 Grimstad, Norway

**Submission status:** To be submitted to *IEEE Transactions on Wireless Communications, Autumn 2012*.

---

# Performance Analysis of Channel Aggregation Strategies in CR Networks with Queues

Indika A. M. Balapuwaduge
Dept. of Information and Communication Technology, University of Agder (UiA), N-4898 Grimstad, Norway
Email: iabala10@student.uia.no

*Abstract*—**With the evolution of cognitive radio, spectrum access techniques shift from static spectrum allocation to dynamic allocation with enhanced features such as spectrum sensing and adaptation. Channel aggregation is proposed in many MAC protocols to improve bandwidth utilization of cognitive users. Due to the strict priority over primary users, the performance of the secondary network is restricted. One of the successful solutions to further improve the system performance by increasing the capacity and decreasing the blocking and forced termination probabilities is the integration of a queuing model. In this paper, we propose a queuing system which is designed with two queues separately for the real time and non-real time secondary user services. Channel access opportunities are distributed between two queues such a way that the real time services have the higher priority than elastic services. Two methods are introduced based on the queuing ability of the interrupted non-real time services. Continuous time Markov chain models are developed to evaluate the system performance in terms of capacity, blocking and forced termination probabilities of the secondary network. In addition to the performance evaluation, we explore the cost analysis of the proposed queuing model in terms of mean queuing delay. The accuracy of the derived theoretical models is assessed by simulations. Analytical results reveal that integration of the proposed queuing model could increase the capacity of the secondary network while decreasing the blocking probability. And also one of the proposed queuing methods can further decrease the forced termination rate of non-real time traffic.**

**Keywords:** ***Cognitive radio, channel aggregation, spectrum adaptation, heterogeneous traffic, queuing system, Markov model, performance analysis***

## I. INTRODUCTION

Cognitive radio networks (CRNs) have gained increasing research importance due to its capability of improving the spectrum utilization. Efficient resource allocation in licensed frequency bands by using dynamic spectrum access (DSA) is a one of the major research components in CRNs. We analyzed different dynamic channel access techniques that have been proposed by many research papers [1 - 5]. In those methods, channel aggregation is a key feature which is used to increase the capacity of the secondary network. When there are more channels available in the target frequency band, secondary users (SUs) can utilize several channels instead of occupying a single channel. This can be performed in two ways depending on the idle channel availability in the spectrum [6]. In the first case, channels which are adjacent to each other or contiguous channels could be bonded as one SU channel. On the other hand, channels which are not contiguous can be aggregated to form one SU channel. Therefore the difference between these two terms depends on the locations of the channels in the frequency spectrum.

The dynamic channel aggregation strategy proposed in [7] has been designed for heterogeneous traffic environment and spectrum adaptation is also implemented by using channel sharing and channel adjustment. The results of that work prove that the proposed strategy can achieve better performance. Not only specific to the results in [7] but generally in CRNs, the blocking probability and the forced termination probability increase due to the increase of primary user arrivals. The provision of very low probabilities for blocking and forced termination of secondary user services is a challenging goal for cognitive radio (CR) systems. To further reduce the blocking probability and the forced termination probability of the channel aggregation strategies proposed in [7], and to increase the capacity of the secondary network, we present a queuing model with an improved channel access technique.

One important issue in cognitive radios with a queuing scheme is to model the heterogeneous traffic of the secondary network with different priority levels depending on the traffic QoS requirements. The other issue is to model the queuing system as it reduces the blocking probability and the forced termination rate of secondary users. On the other hand, the parameters like capacity[1] has to be improved with the queuing model.

The rest of this paper is organized as follows. Related work is explained in Sec. II and then the system model is introduced in Sec. III. Sec. IV and Sec. V describe the proposed dynamic channel aggregation strategy and the queue scheduling algorithm respectively. In order to analyze the system performance, CTMC models are developed in Sec. VI. Numerical results are illustrated in Sec. VII, before the paper is concluded in Sec. VIII.

## II. RELATED WORK

Research interest on cognitive radio technology has grown greatly and in most studies, performance of the cognitive radios has been evaluated with capacity and blocking probability of the secondary users. Also, a lot of scientific effort has been put into understanding and analyzing queuing systems

[1]In this paper, *capacity* implies the rate of service completions i.e., the average number of service completions per time unit in secondary network.

for cognitive radio networks. However, the majority of those attempts mainly focus on the analysis of the sojourn time. In [8], a queuing framework has been developed to analyze the performance of opportunistic access by CR users. The arriving packets are modelled as a Batch Bernoulli process and the packets in the queue are served in FCFS order. By imposing resource constraints over new incoming connections the model allows to control the level of QoS. On the other hand, when the new CR user needs to start a new connection, the admission controller at the base station use the queuing model to decide the admission grant. Therefore, the proposed queuing model has been designed to achieve both QoS and admission control.

In [9], several performance metrics are derived based on a queuing network model. Indeed this paper analyzes the performance of primary network by evaluating the service degradation experienced by primary users (PUs) due to secondary user's incorrect spectrum sensing. A notable feature of this model is that each channel in the system is considered as a single server queuing system. In the proposed queuing model for a CR network in [10], access latency for the secondary users is evaluated as the main QoS parameter. A special feature of the queuing models is that both PU and SU requests can queued, so FCFS order could not be applied for BAQ since PUs have the strict priority over SUs. The effect of using a queue opportunistic based cognitive radio networks is investigated in [11]. The main target of that paper is to analyze the increase of the spectral efficiency by putting SU services in a queue.

The channel assembling strategies proposed in [7] provides the basis for the queuing model and the dynamic channel aggregation strategy proposed in this paper. In that work, two representative channel assembling strategies are proposed to evaluate the system performance considering heterogeneous SU traffic. According to the numerical results obtained in that paper, better performance has been achieved by using dynamic channel aggregation in contrast with static channel aggregation.

## III. System Model and Assumptions

We consider an infrastructure based CRN consisting a central base station (CBS) and multiple CR users. The primary network has the full priority to designated frequency bands and the secondary network can only access a frequency band when the corresponding primary network does not use that band. Resource allocation for the secondary network is performed by the CBS. The licensed spectrum band is divided in to $M \in \mathbb{Z}^+$ frequency channels and those channels are allocated for $M$ licensed users (primary users) and all channels are assumed to be statistically identical. Here, $\mathbb{Z}^+$ denotes the set of positive integers. When these channels are not occupied by PUs, SUs have the choice to access channels with equal probability. Even though a PU occupies only one channel for each service, SUs can aggregate several channels to complete the service with high data rate. We also assume perfect channel sensing capability of the CR network so that SUs can detect when a primary user accesses a channel. Thus, the probability of channel sensing errors of the system is considered negligible.

In this paper we propose multiple queues for different traffic types. In our analysis, we consider two types of SU traffic, i.e., elastic traffic and real time traffic. Elastic flows such as file transfer and email adjust their sending rate according to network conditions. Real-time applications, such as video streaming and voice over IP [12] do not adjust its throughput in response to network conditions. Therefore, in our analysis the number of channels allocated for a real time service is fixed. It is worth mentioning that the terminology *ESU* and *RSU* in this paper are used to represent elastic SU services and real-time SU services respectively. However, the term *SU* commonly represents the both SU services. Other than this, the following assumptions are made in order to develop our analytical model.

- The arrivals of both PU and SU services are Poisson distributed with arrival rates $\lambda_P$, $\lambda_{S1}$ and $\lambda_{S2}$ for PU, ESU and RSU services respectively.
- The service time for both PU and SU services is exponentially distributed with service rates per channel $\mu_P$, $\mu_{S1}$ and $\mu_{S2}$ for PU, ESU and RSU respectively.
- All channels are homogeneous. Thus, the service rate of $k$ aggregated channels in secondary network equals $k\mu_S$ for elastic traffic.
- The sensing and spectrum adaptation latency is negligible in comparison with the duration between consecutive service events.

### A. Formulation of Queuing Model

We propose two separate queues for elastic and real time services. The queue named as *EQ* is allocated for elastic traffic and the queue named as *RQ* is allocated for real time services. Even though higher priority is assigned for RSU services in RQ, the priority level is adjustable according to the service requirements. Two queuing methods are proposed based on the user arrivals to the queues.

*1) Case I: Queuing Network Model without Feedback Link:* In Case I, only the new users are allowed the access to the queuing system. Therefore, new ESU services can access waiting positions in the low priority queue and new RSU services can access waiting positions in the high priority queue. Fig. 2 illustrates the queuing model proposed with Case I.

*2) Case II: Queuing Network Model with Feedback Link:* But, in Case II, interrupted ESU services due to PU service arrivals, which cannot find any idle channel in the system by means of channel adaptation are also authorized to access EQ in addition to the new ESU services. However, if there is no enough waiting rooms in EQ, they are considered to be forced terminated. Other than this the rest of the functions of Case II are similar to Case I. Fig. 3 illustrates the queuing model proposed with Case II. In these figures, $R_{f1}$ denotes the ESU temporary termination rate due to PU arrivals. $Q_{L1}$ and $Q_{L2}$ are the maximum queue sizes of EQ and RQ respectively.

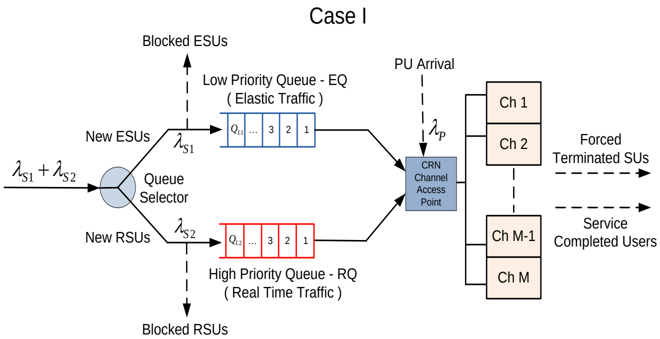


Fig. 1: Proposed Queuing Model - Case I

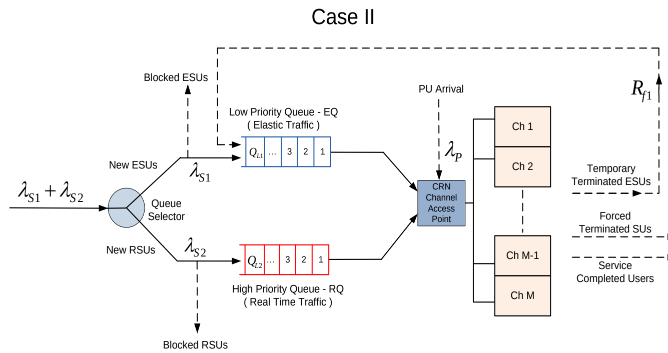


Fig. 2: Proposed Queuing Model - Case II

### B. *Queuing Discipline and Priority levels*

If we consider the total SU arrival process, heterogeneous traffic can be observed. Nevertheless, if each queue is considered separately, they process homogeneous traffic only. The queue, EQ process only elastic traffic and the queue, RQ process real time applications only. For homogeneous traffic, First Come First Serve (FCFS) order is applied. Therefore, the service requests in both queues are processed with FCFS manner in which the first service in the queue is the first service that is processed. However, when channel access opportunities are available, those opportunities are divided between EQ and RQ according to the assigned priority levels.

## IV. Proposed Dynamic Channel Access Strategy

The channel access strategy which is utilized for the cognitive radio network described in this paper, is based on the channel assembling strategy proposed by [7]. In [7], heterogeneous traffic is considered however, without a queuing structure. The dynamic channel assembling proposed in [7] uses both spectrum handover and adjustable channel aggregation. The DSA strategy proposed in this paper is denoted as *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$.

### A. Dynamic $(a, W, V, Q_{L1}, Q_{L2})$

Consider a CR network with $M$ PU licensed channels, where secondary users contend for the channel access when PUs are inactive. An ESU service can aggregate or bond several channels to increase its service rate [6]. The lower and upper bounds of the number of aggregated channels for an ESU service are denoted as $W$ and $V$ where $W, V \in \mathbb{N}^+$. Depending on the channel status and the user activities, an ESU can dynamically adjust their number of aggregated channels. But for an RSU service the number of aggregated channels is always fixed. In our notations, $a \in \mathbb{N}^+$ represents the number of channels aggregated by an RSU service. If a channel is currently used by a certain user, we called that *the channel is in busy state*. If a channel is not occupied by any user then *the channel is in idle state*. Let $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ be the general state representation of the system where,

$j_{pu}$ = No. of PU services in the system,
$j_k$ = No. of ESU services with $k$ aggregated channels,
$g_a$ = No. of RSU services with $a$ aggregated channels,
$j_{eq}\ j_{rq}$ = Current Queue Sizes of EQ and RQ.

### B. *PU arrivals*

When a PU arrives to a channel which is currently in idle state, that PU can start the transmission without any conflict and thus the secondary network will not face with any change. But if the PU arrival is occurred towards a channel in busy state due to an SU service with $k_1, W \leq k_1 \leq V$ aggregated channels, the interrupted SU has to vacate the channel immediately. The ESU service with the maximum number of aggregated channels, $k_2$ donate a channel for the interrupted SU service as long as its' remaining number of aggregated channels is still greater than or equals $W$, where $k_2 > k_1$. Otherwise, if there is no such ESU service which has $k_2$ aggregated channels where $k_2 > k_1$ then an interrupted ESU flexibly adjust downwards the number of assembled channels, as long as the remaining number of aggregated channels is still greater than or equals $W$ while if the interrupted SU service is an RSU, it is forced to terminate. When the interrupted ESU has exactly $W$ channel and there is no other ESU service which has more than $W$ aggregated channels, then it is forced to terminate if Case I is used for queuing model. If Case II is applied over the queuing process, that ESU service is temporary stopped and put in the EQ as long as EQ is not fully utilized. In the worst case, i.e., if the EQ is full, then corresponding ESU is forced to terminate.

### C. *PU departures*

Upon a departure of a PU service, the corresponding channel changes its state from busy to idle. When an idle channel appears, high priority of accessing the channel is given to the waiting services in RQ. According to the FCFS discipline, the RSU services in the front end of RQ will receive the chance to initiate its services. If real time services in RQ do not take the chance, then elastic traffic in EQ will receive the opportunity in the similar way. If both queues do not receive the chance, the ongoing ESU service which has the minimum aggregated channels, i.e., $k_{min}, \quad k_{min} < V$ can aggregate the new channel.

### D. SU arrivals

When a new ESU service arrives, the service can commence if there are at least $W$ idle channels in the system whereas upon a RSU service arrival, system should allocate exactly $a$ channels. However, if there are not enough idle channels for newly arrived SU services, the ongoing ESU service which has the maximum number of channels will donate channels to the new users. In that case, after donation ongoing ESU should be able to keep at least $W$ channels, i.e., the lower bound of the channel aggregation. If the one with the maximum number cannot provide $W$ channels for new ESU service or $a$ channels for new RSU service by itself, the next ESU with the second maximum number will donate its channels and so on. If the ongoing ESUs collectively cannot provide the required number of channels, then the new service is put in the corresponding queue (ESUs are directed to EQ and RSUs are directed to RQ). However, if the queue is fully occupied then the new service is blocked. One important fact to be mentioned here is that ESU services are not put in RQ even at a situation when EQ is fully occupied while RQ is empty. The same rule is applied for RSU services. And also RSU services do not donate channels for other SUs.

### E. SU departures

Due to SU departures, one or more channels become idle. The opportunity of accessing those channels divided between waiting RSU and ESU services in the queues by using queue scheduling algorithm explained in Section V. By varying the priority factor $\beta$ the number of allocated channels for each queue can be altered. If one of the queues is empty, then all the idle channels are allocated for the services of other queue. If there are channels still in idle state even after all the waiting services in the queues obtain channel access, then the ongoing ESU service which has the minimum number of aggregated channels has the chance to aggregate them up to $V$. Since real time services should possess fixed number of channels, RSUs will not assemble newly idle channels. If the ESU with minimum number of channels aggregate $V$ channels and there are still vacant channels, the other ESUs will occupy the remaining ones by following the same principle.

## V. Queue Scheduling Algorithm

Let $\beta : 1$ be the ratio between priority levels assigned to RQ and EQ respectively where, $\beta \in \mathbb{Q}^+$ and $\beta \geq 1$. Here, $\mathbb{Q}^+$ represents the set of positive rational numbers. Although, a certain number of channels are initially allocated for the RSU services in the RQ, the maximum number of required channels to reach the upper bound of channel aggregation for all RSU or ESU services may be different. The notations used in the proposed queue scheduling algorithm are listed in Table 1. The allocation process of the $n$ newly idle channels among the services in EQ and RQ is illustrated in Algorithm I. The parameters $n_{rq,\ max}$, $n_{eq,\ max}$ and $n_{eq,\ min}$ corresponding to lines 8,9 and 10 in Algorithm I are calculated by using Algorithm II and Algorithm III.

**Algorithm I**: Pseudo code for queue scheduling:

| **Line** | **Code** |
|---|---|
| 1: | **if** $n = 2k$ where, $k \in \mathbb{Z}^+$ |
| 2: | $n_r = \lfloor \frac{\beta}{\beta+1} n \rfloor$ |
| 3: | $n_e = n - n_r$ |
| 4: | **elseif** $n = (2k+1)$ where, $k \in \mathbb{N}$ |
| 5: | $n_r = \lceil \frac{\beta}{\beta+1} n \rceil$ |
| 6: | $n_e = n - n_r$ |
| 7: | **end** |
| 8: | $n_{rq,\ max} = f(DSA, a, j_{rq})$ |
| 9: | $n_{eq,\ max} = f(DSA, W, V, j_{eq})$ |
| 10: | $n_{eq,\ min} = f(DSA, W, V, j_{eq})$ |
| 11: | **if** $n_{rq,\ max} \geq n_r$ |
| 12: | $n_R = \lfloor \frac{n_r}{a} \rfloor .a$ |
| 13: | $n_e = n - n_R$ |
| 14: | **if** $n_{eq,\ max} > n_e$ |
| 15: | **if** $n_e \geq n_{eq,\ min}$ |
| 16: | $n_E = n_e$ |
| 17: | **else** |
| 18: | $n_E = 0$ |
| 19: | **end** |
| 20: | **else** |
| 21: | $n_E = n_{eq,\ max}$ |
| 22: | **end** |
| 23: | **else** |
| 24: | $n_R = n_{rq,\ max}$ |
| 25: | $n_e = n - n_R$ |
| 26: | **if** $n_{eq,\ max} > n_e$ |
| 27: | **if** $n_e \geq n_{eq,\ min}$ |
| 28: | $n_E = n_e$ |
| 29: | **else** |
| 30: | $n_E = 0$ |
| 31: | **end** |
| 32: | **else** |
| 33: | $n_E = n_{eq,\ max}$ |
| 34: | **end** |
| 35: | **end** |
| 36: | $n_Q = n_R + n_E$ |
| 37: | **if** $n_Q < n$ |
| 38: | $n_O = n - n_Q$ |
| 39: | **else** |
| 40: | $n_O = 0$ |
| 41: | **end** |

TABLE I: Notations used in Algorithm I

| | |
|---|---|
| $n$ | No. of newly idle channels |
| $a$ | No. of channels aggregated by an RSU service |
| $\beta : 1$ | Ratio between priority levels assigned to RQ and EQ. |
| $n_r$ | No. of channels available for RSU Services in RQ |
| $n_e$ | No. of channels available for ESU Services in EQ |
| $n_{rq,\ max}$ | The maximum number of channels required by all the RSU Services in RQ |
| $n_{eq,\ max}$ | The maximum number of channels required by all the ESU Services in EQ |
| $n_{eq,\ min}$ | The minimum number of channels required by all the ESU Services in EQ |
| $n_E$ | No. of channels actually used by ESU Services in EQ |
| $n_R$ | No. of channels actually used by RSU Services in RQ |
| $n_O$ | No. of channels allocated for ongoing ESU Services |
| $j_{eq}$ | Current Queue Size for ESU Services |
| $j_{rq}$ | Current Queue Size for RSU Services |

**Algorithm II**: Pseudo code for the calculation of $n_{rq,\ max}$ for *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy

**Code**

$n_{rq,\ max} = a \times j_{rq}$

**Algorithm III**: Pseudo code for the calculation of $n_{eq,\ max}$ and $n_{eq,\ min}$ for *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ strategy

**Code**

**if** $j_{eq} \geq 1$
  $n_{eq,\ max} = V \times j_{eq}$
  $n_{eq,\ min} = W$
**else**
  $n_{eq,\ max} = 0$
  $n_{eq,\ min} = 0$
**end**

## VI. CTMC Analysis

To model the proposed channel access strategy with the queuing model, continuous time Markov chains (CTMC) are developed. Let $L$ be the number of states of the CTMC and $\pi(x)$ be the steady sate probability of the state $x$. As earlier mentioned, the states of the CTMC models can be represented by $x = (j_W, j_{W+1}, \cdots j_V, g_a, j_{pu}, j_{eq}, j_{rq})$. Let $\mathcal{S}$ be the set of feasible states of the system, as

$$\mathcal{S} = \{x \mid j_W, j_{W+1}, \cdots j_V, g_a, j_{pu}, j_{eq}, j_{rq} \geq 0; b(x) \leq M\}, \tag{1}$$

where $b(x)$ denotes the total number of utilized channels at state $x$. Therefore,

$$b(x) = j_{pu} + a g_a + \sum_{k=W}^{V} k j_k. \tag{2}$$

Furthermore, $\pi(x)$ which stands for steady state probability of being in state $x$ can be calculated as follows. Given feasible states and their transition rates in a CTMC, we can construct the global balance equations and the normalization equation as expressed by the following equation.

$$\boldsymbol{\pi} \boldsymbol{Q} = 0, \quad \sum_{x=1}^{L} \pi(x) = 1, \tag{3}$$

where $\boldsymbol{\pi} = [\pi(1), \pi(2), \cdots, \pi(x), \cdots, \pi(L)]$ is the steady state probability vector and $\boldsymbol{Q}$ denotes the transition rate matrix. The vector $\boldsymbol{\pi}$ is determined by solving Eq. (3). When the steady state probabilities are determined, the performance of the CRN can be evaluated in terms of different parameters.

### A. Analytical Models for Case I

In this subsection we derive a analytical models to evaluate the system performance of Case I.

*1) Secondary network capacity:* In this report the terminology *capacity* implies the rate of service completions, i.e., the average number of service completions per time unit in secondary network. Let $\rho_1$ and $\rho_2$ be the capacity of ESU and RSU services respectively. Then, capacity of ESU services,

$$\rho_1 = \sum_{x=1}^{L} \sum_{k=W}^{V} k j_k \mu_{S1} \pi(x), \tag{4}$$

capacity of RSU services,

$$\rho_2 = \sum_{x=1}^{L} a g_a \mu_{S2} \pi(x). \tag{5}$$

*2) Blocking probability:* An incoming ESU service will be blocked when the following conditions are satisfied, i.e., there are no adequate idle channels to commence the service, there is no way of channel sharing with ongoing ESU services and the queue, EQ is also fully occupied. The similar conditions are applied for calculating the blocking probability of incoming RSU services except the fact that they need exactly $a$ channels for commencing services. Let $P_{b1}$ and $P_{b2}$ be the blocking probability of ESU and RSU services respectively.
Blocking probability of ESU,

$$P_{b1} = \sum_{\substack{x=1, \\ M-b(x)+\sum_{k=W+1}^{V}(k-W)j_k < W, \\ j_{eq}=Q_{L1}}}^{L} \pi(x). \tag{6}$$

Blocking probability of RSU,

$$P_{b2} = \sum_{\substack{x=1, \\ M-b(x)+\sum_{k=W+1}^{V}(k-W)j_k < a, \\ j_{rq}=Q_{L2}}}^{L} \pi(x). \tag{7}$$

*3) Forced termination probability of ESU:* In Case I, when a PU preempt an ESU service which has exactly $W$ aggregated channels, the ESU is forced to terminate if there is no at least one idle channel or chance of channel sharing with another ongoing ESU service. Therefore, the interrupted ESU ends communication before it finishes the communication. The forced termination probability of ESUs, $P_{f1}$, can be expressed as the mean forced termination rate of ESUs, $R_{f1}$ divided by the mean admitted ESU rate, i.e., $\lambda_{S1}^{*}$.

Mean forced termination rate of ESU,

$$R_{f1} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{M - j_{pu}} \pi(l). \tag{8}$$

Forced termination probability of ESU,

$$P_{f1} = \frac{R_{f1}}{\lambda_{S1}^{*}}, \tag{9}$$

where,

$$\lambda_{S1}^{*} = \lambda_{S1}(1 - P_{b1}),$$

$$P_{f1} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{(M - j_{pu}) \lambda_{S1}^{*}} \pi(l). \tag{10}$$

*4) Forced termination probability of RSU:* A forced termination of an RSU service occurs when a PU service preempts an RSU service and the preempted RSU service cannot detect an idle channel in the system or cannot share a channel with other ongoing ESUs. The forced termination probability of RSUs, $P_{f2}$, can be expressed as the mean forced termination rate of RSUs, $R_{f2}$ divided by the mean admitted RSU rate, i.e., $\lambda^*_{S2}$. The mean forced termination rate of RSU is given by,

$$R_{f2} = \sum_{\substack{x=1, \\ M=b(x),\ j_{pu}<M, \\ j_r=0, \forall r>W, g_a>0}}^{L} \frac{\lambda_P a g_a}{M - j_{pu}} \pi(x). \tag{11}$$

Then, forced termination probability of RSU,

$$P_{f2} = \frac{R_{f2}}{\lambda^*_{S2}}, \tag{12}$$

where,

$$\lambda^*_{S2} = \lambda_{S2}(1 - P_{b2}), \tag{13}$$

$$P_{f2} = \sum_{\substack{x=1, \\ M=b(x), g_a>0 \\ j_{pu}<M,\ j_r=0, \forall r>W}}^{L} \frac{\lambda_P a g_a}{(M - j_{pu})\lambda^*_{S2}} \pi(x). \tag{14}$$

*5) Mean queue length:* At each state $x \in \mathcal{S}$, $j_{eq}$ and $j_{rq}$ are the queue lengths of EQ and RQ respectively.
Therefore, mean queue length of ESU services,

$$L_{EQ} = \sum_{\substack{x=1, \\ 0 \le j_{eq} \le Q_{L1}}}^{L} \pi(x) j_{eq}. \tag{15}$$

Mean queue length of RSU services,

$$L_{RQ} = \sum_{\substack{x=1, \\ 0 \le j_{rq} \le Q_{L2}}}^{L} \pi(x) j_{rq}. \tag{16}$$

*6) Average queuing delay of SU service:* In this analysis, queuing delay which is equal to the average waiting time of an SU service is evaluated. As what is discussed in the last subsection, the average queue lengths of EQ and RQ were given by Eq. (15) and Eq. (16) respectively. In Case I, the arrival rate at EQ, $\lambda_{EQ}$ is equal to the ESU arrival rate.
The arrival rate at the queue, EQ,

$$\lambda_{EQ} = \lambda_{S1}.$$

Using **Little's Law** [13], the average queuing delay of ESUs,

$$D_{EQ} = \frac{L_{EQ}}{\lambda_{EQ}}.$$

Therefore,

$$D_{EQ} = \frac{\sum_{\substack{x=1, \\ 0 \le j_{eq} \le Q_{L1}}}^{L} \pi(x) j_{eq}}{\lambda_{S1}}. \tag{17}$$

The arrival rate of the queue, RQ,

$$\lambda_{RQ} = \lambda_{S2}.$$

Using **Little's Law**, the average queuing delay of RSUs,

$$D_{RQ} = \frac{L_{RQ}}{\lambda_{RQ}}.$$

Therefore,

$$D_{RQ} = \frac{\sum_{\substack{x=1, \\ 0 \le j_{rq} \le Q_{L2}}}^{L} \pi(x) j_{rq}}{\lambda_{S2}}. \tag{18}$$

### *B. Analytical Models for Case II*

Instead of direct termination, an interrupted ESU service in Case II is suspended to wait in EQ for accessing another idle channel. The derived mathematical equations for calculating the capacity and the blocking probability of the secondary network and the expressions for determining the mean queue length of both queues in Case II is the same as in Case I. And also, the equations for determining the forced termination probability and the mean waiting time of RSU services are the same as in Case I. However, forced termination probability and the average queuing delay of an ESU service have to be analyzed separately for the two cases since the behaviour of the forced terminated ESU services in Case II is not the same as in Case I.

*1) Preempted probability of ESUs:* We define the preempted probability as the probability that an ongoing ESU enters the EQ because of the arrival of a licensed PU service. In Case I preempted probability is not discussed since interrupted SU services are not put in the queue. Due to the same reason, Case II does not consider the preempted probability of RSU services. The preempted probability of ESUs, $P_{preempted}$, can be expressed as the interrupted rate of ESUs when there is no way of accessing an idle channel, $R_{interrupt}$ divided by the mean admitted ESU rate, i.e., $\lambda^*_{S1}$.
Mean PU interrupted rate of ESU,

$$R_{interrupt} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{M - i} \pi(l). \tag{19}$$

The preempted probability of ESU,

$$P_{preempted} = \frac{R_{interrupt}}{\lambda^*_{S1}}, \tag{20}$$

where,

$$\lambda^*_{S1} = \lambda_{S1}(1 - P_{b1}).$$

Therefore

$$P_{preempted} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{(M - i)\lambda^*_{S1}} \pi(l). \tag{21}$$

*2) Forced termination probability of ESU:* In Case II, a preempted ESU service is forced terminated only if all the waiting lines of EQ are occupied by other ESU services. The forced termination probability of ESU in Case II, $P_{f1}^{'}$, can be expressed as the mean forced termination rate of ESUs, $R_{f1}^{'}$ divided by the mean admitted ESU rate, i.e., $\lambda_{S1}^{*}$. Mean forced termination rate of ESU,

$$R_{f1}^{'} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W,\ j_{eq}=Q_{L1}}}^{L} \frac{\lambda_P W j_W}{M-i}\pi(l). \tag{22}$$

Forced termination probability of ESU,

$$P_{f1}^{'} = \frac{R_{f1}^{'}}{\lambda_{S1}^{*}}, \tag{23}$$

where,

$$\lambda_{S1}^{*} = \lambda_{S1}(1 - P_{b1}),$$

$$P_{f1}^{'} = \sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W,\ j_{eq}=Q_{L1}}}^{L} \frac{\lambda_P W j_W}{(M-i)\lambda_{S1}^{*}}\pi(l). \tag{24}$$

*3) Average waiting time of ESU service:* In Case II, the arrival rate to the low priority queue, EQ is higher than that of Case I due to admission of the interrupted ESU services. Thus, we have to modify Eq. (18) by following similar arguments to calculate the average ESU waiting time in Case II. The arrival rate of EQ, $\lambda_{EQ}^{'}$ is equal to the ESU arrival rate plus ESU temporary termination rate.

$$\lambda_{EQ}^{'} = \lambda_{S1} + R_{interrupt}.$$

Using **Little's Law**, the average waiting time of ESUs,

$$D_{EQ}^{'} = \frac{L_{EQ}}{\lambda_{EQ}^{'}}.$$

Therefore,

$$D_{EQ}^{'} = \frac{\displaystyle\sum_{\substack{x=1, \\ 0\ \le\ j_{eq}\ \le\ Q_{L1}}}^{L} \pi(x) j_{eq}}{\lambda_{S1} + \left( \displaystyle\sum_{\substack{l=1, \\ M=b(x),\ i<M, \\ j_W>0,\ j_n=0,\ \forall n>W}}^{L} \frac{\lambda_P W j_W}{M-i}\pi(l) \right)}. \tag{25}$$

## VII. Numerical Results & Discussions

In this section numerical results are presented to illustrate the performance of the proposed queuing model with two cases. We consider a CRN with 6 channels (i.e., $M = 6$) in the allocated spectrum band. Unless otherwise stated, statistical parameters of licensed and unlicensed users are set as $\lambda_P = 1.0$, $\lambda_{S1} = 2.0$, $\lambda_{S2} = 1.0$, $\mu_P = 0.5$, $\mu_{S1} = 1.0$ and $\mu_{S2} = 1.0$. Configuration parameters of the dynamic

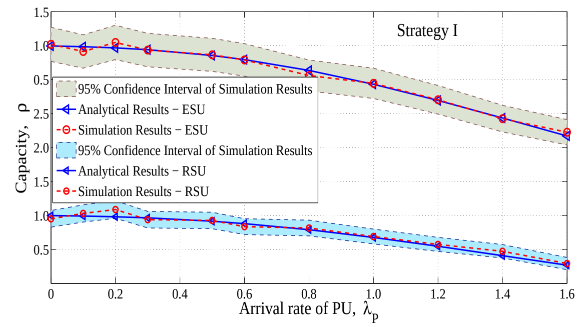


Fig. 3: SU capacity as a function of PU arrival rate - Case I.

spectrum access strategy used in the system, i.e., Dynamic $(a; W; V; Q_{L1}; Q_{L2})$ are fixed as $W = 1$, $V = 3$ and $a = 1$. Unless otherwise stated, the lengths of the queues are set as $Q_{L1} = 4$ and $Q_{L2} = 2$ and priority factor of the queue scheduling algorithm is set as $\beta = 2$.

### A. Model Validation

To validate the analytical models, we first develop MATLAB codes to simulate PU and SU events over the proposed channel access strategy and compare the analytical results with the simulation results. However, for illustration clarity we depict the simulation results only in this section, although simulations have been performed in all analysis of this section. Fig. 3 depicts the variation of secondary network capacity as a function of PU arrival rate. When more PU services arrive, the capacity of both ESU and RSU services decreases. As demonstrated in that figure, analytical output exists within the 95% confidence intervals of the simulation results. In other words, our analytical model shows a precise matching with the obtained simulation results.

### B. Performance Evaluation

The *Dynamic* $(a; W; V; Q_{L1}; Q_{L2})$ channel aggregation strategy proposed in this paper should be able to enhance the performance of the existing cognitive radio networks in terms of several performance metrics.

*1) Secondary Network capacity:* Fig. 4 depicts the system capacity of the ESU service network as ESU arrival rate $\lambda_{S1}$ varies. With the increasing arrival rate of ESU services, the capacity of ESU services initially increases dramatically. After that capacity grows smoothly and seems to be reached a maximum value. Initially capacity is increased with a high rate because, more ESU services are commenced at a high rate. Afterwards, all the channels in the system gradually close to be fully utilized, therefore capacity curves tend to reach a maximum value. The next significant observation is that the capacity increases due to the queuing model and it further can be increased when the maximum queue size is increased. Therefore, the capacity curve corresponding to the maximum queue size of 8, i.e., $Q_{L1} = 8$ always higher than that of $Q_{L1} = 4$. Large queue size means that more service requests can be stored. Since PU arrival rate is fixed in the analysis of Fig. 4, the probability of completing an ESU service is increased as more service requests are added

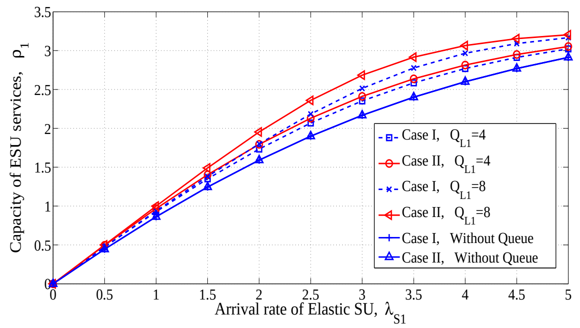


Fig. 4: ESU Capacity as a function of ESU arrival rate.

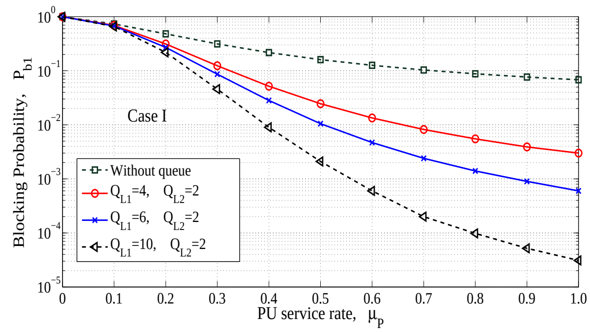


Fig. 5: Blocking probability as a function of PU service rate - Case I.

to a queue. Due to this reason, capacity can be increased with large queue sizes.

When the capacity is compared between the two cases, it is obvious that Case II gains more capacity than Case I. Since the interrupted ESU services are forced to terminate in Case I even when EQ is not fully utilized, they are not considered as service completions. Therefore, those services are not counted for capacity calculations. However, in Case II those interrupted services may have chances to finish their services later, since they are queued until they will have an opportunity to access system resources. Because of this reason, capacity of ESU services is improved in Case II than Case I.

*2) Blocking Probability:* Fig. 5 depicts the ESU blocking probability as a function of PU service rate, $\mu_P$. The blocking probability dramatically decreases initially as $\mu_P$ grows. When PU service rate is smaller, only few PUs can finish their services per unit of time. Therefore a majority of the channels are occupied by respective licensed services. Therefore, the probability of blocking a new user request is high. On the other hand, the probability of a channel in the busy state decreases as $\mu_P$ increases. Then the probability of blocking a new ESU service also decreases with an increasing rate of $\mu_P$. As shown in Fig. 5, the blocking probability of ESU services can be decreased with the proposed queuing strategies. With the queuing model, the blocked calls due to insufficient channels are put in the queue until they will be offered with the required number of channels. For this reason, blocking probability is significantly reduced with the integration of queues. On the other hand, when the maximum queue size is increased more SUs can be queued without blocking their service requests. For this reason, ESU blocking probability shown in Fig. 5 decreases as queue size is increased.

*3) Forced Termination Probability:* In Fig. 6, the forced termination probability is shown as PU arrival rate varies. We can observe from this figure that the forced termination probability increases as $\lambda_P$ increases in both cases. This behaviour is the same for the system without queuing. When $\lambda_P$ increases, PUs become more active in the network and due to this reason SU services will be forced to terminate. The next significant observation is that the rate of increasing of forced termination is decreasing with increasing $\lambda_P$. When $\lambda_P$ is comparatively low, certain channels in the system are occupied by SUs and with the growth of $\lambda_P$ they are forced to terminate. Nevertheless, at a higher $\lambda_P$ almost all the channels are occupied by PUs so that there are no ongoing SUs to be interrupted. Thus, the rate of increase of forced termination probability deteriorates with $\lambda_P$. According to the numerical results shown in Fig. 6, it is pointed out that the forced termination probability of the queuing system with Case I, is always higher than that of the system without queues. This result convince that Case I of the proposed queuing model cannot show better performance in terms of forced termination probability. The reason can be explained as follows.

To decrease the forced termination rate, either system should possess more channels or less PU activities. In the numerical results shown in Fig. 6, the increase of PU arrival rate means more PU activities. On the other hand, when queues are integrated blocked SU services are not cleared but they are put into the queues. Then more interactions between SUs and PUs can occur in contrast with the system without queues. Due to this reason, more forced terminations can be observed in Case I. A fruitful method to obtain decreased forced termination probability of ESUs is proposed in Case II. In Case II the interrupted ESU services in which there is no way of accessing an idle channel, are put into the queue unless the queue is not fully utilized. Hence, the forced termination of ESU services should be reduced with Case II. As Fig. 6 highlights forced termination probability is greatly reduced in Case II.

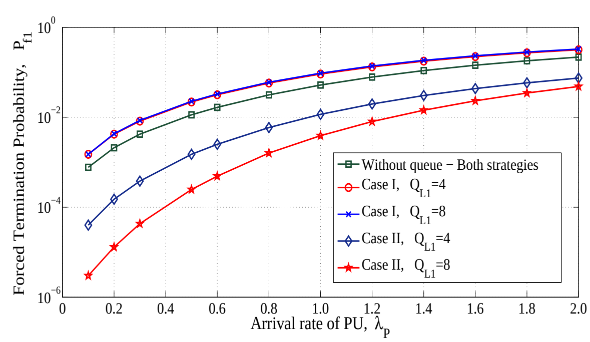


Fig. 6: Forced termination probability as a function of PU arrival rate.

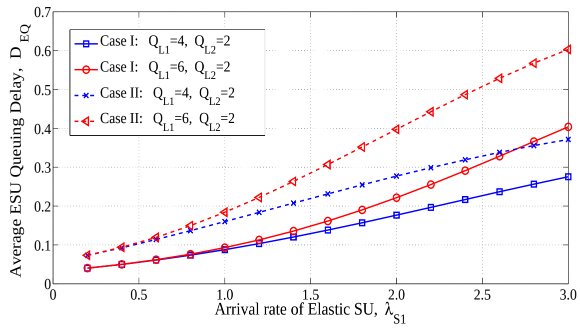


Fig. 7: Average queuing delay as a function of ESU arrival rate.

*4) Average queuing delay:* The main cost introduced in the proposed queuing schemes is the associated waiting time of SU services in the queue, i.e., queuing delay. To study the effect of the queue size on the queuing delay we generate the numerical results shown in Fig. 7. As illustrated in this figure, average waiting time of ESU services increases with ESU arrival rate. When ESU arrival rate increases the number of ESU service requests in the queue also increases. Wherein most of the waiting rooms are allocated, the mean waiting time of a user will increase. Moreover, an increasing in the number of waiting positions in the queue has a similar effect on the average queuing delay. Apparently, in both cases when the maximum queue size is increased queuing delay also increases.

Fig. 7 also shows that the Case I performs better than Case II in terms of queuing delay. Because for a given maximum queue length, ESU services in Case II experience a larger queuing delay in contrast with Case I. In Case II, current queue size is larger than Case I since interrupted ESU services are also queued in addition to the new arrivals. Since average waiting time is direct proportional to the mean queue length, it is justified that Case II leads to a higher queuing delay. This is particularly important result, since it implies the trade off between the average waiting time and the forced termination probability explained in the previous subsection. That is, even though Case II contributes to significantly reduce the forced termination probability of ESU services, their waiting time will increase consequently.

## VIII. Concluding Remarks

We have proposed a queuing analytic framework to evaluate the system performance of a multi-channel cognitive radio network with channel access based on dynamic channel aggregation and spectrum adaptation. The queuing structure proposed in this paper consists of separate queues allocated for different traffic types and thus, FCFS discipline is easy to apply. The numerical results which are validated through the exact simulation results show that the overall spectrum utilization and secondary network capacity can be increased via the proposed model. With the increase of queue sizes, blocking probability can further be decreased, however the average queuing delay of a service is increased. Therefore, the maximum queue sizes have to be controlled according to the QoS requirements of the traffic type. Since the defined two cases of queuing methods are different in terms of the functionality of the interrupted elastic SU services, the forced termination probability of the model shows different results in two cases. Even though Case II reduces forced terminations of ESUs, Case I leads to increase it. Conversely, when Case II is used blocking probability will be high. Therefore, we recommend to use Case II if PUs are more active in the CRN and Case I is recommended for a system with less PU activities but more SU activities.

## References


[1] E. W. M. Wong, and C. H. Foh, "Analysis of Cognitive Radio Spectrum Access with Finite User Population", *IEEE Communication Letters*, vol. 13, no. 5, May 2009.

[2] H. Al-Mahdi, M. A. Kalil, F. Liers, and A. M. Thiel, "Increasing Spectrum Capacity for Ad hoc Networks using Cognitive Radios", *IEEE Communication Letters*, vol. 13, no. 9, Sep. 2009.

[3] G. D. Nguyen, and S. Kompella, "Channel Sharing in Cognitive Radio Networks", *in Proc. of IEEE Military Communications Conference (MILCOM)*, pp. 2268-2273, San Jose, California, USA, Nov. 2010.

[4] P. Kaur, A. Khosla, and M. Uddin, "Markovian Queuing Model for Dynamic Spectrum Allocation in Centralized Architecture for Cognitive Radios", *IACSIT International Journal of Engineering and Technology*, vol. 3, no. 1, Feb. 2011.

[5] L. Jiao, F. Y. Li, and V. Pla, "Dynamic Channel Aggregation Strategies in Cognitive Radio Networks with Spectrum Adaptation", *in Proc. of IEEE GLOBECOM*, Houston, TX, USA, Dec. 2011.

[6] L. Jiao, V. Pla, and F. Y. Li, "Analysis on Channel Bonding/Aggregation for Multi-channel Cognitive Radio Network", *in Proc. of European Wireless (EW)*, Lucca, Italy, Apr. 2010.

[7] L. Jiao, F. Y. Li, and V. Pla, "Modeling and Performance Analysis of Channel Assembling in Multi-channel Cognitive Radio Networks with Spectrum Adaptation", *IEEE Transactions on Vehicular Technology, accepted for publication*, Mar. 2012.

[8] M. M. Rashid, M. J. Hossain, E. Hossain, and V. K. Bhargava,"Opportunistic Spectrum Scheduling for Multiuser Cognitive Radio: A Queuing Analysis", *IEEE Transactions On Wireless Communications*, vol. 8, no. 10, pp. 5259-5269, Oct. 2009.

[9] S. Tang, and B. L. Mark, "Performance of a cognitive radio network with tolerable service degradation", *Proc. 7th Int. Workshop on Design of Reliable Communication Networks (DRCN'09)*, Washington DC, USA, Oct. 2009.

[10] P. Kaur, A. Khosla, and M. Uddin, "Markovian Queuing Model for Dynamic Spectrum Allocation in Centralized Architecture for Cognitive Radios", *IACSIT International Journal of Engineering and Technology*, vol. 3, no. 1, Feb. 2011.

[11] S. Homayouni, and S. A. Ghorashi, "The Impact of Queue on Opportunistic based Cognitive Radio Networks", *Journal of Computing*, vol. 3, no. 7, Jul. 2011.

[12] M. H. Cheung, A. H. Mohsenian-Rad, V. W.S. Wong, and R. Schober, "Random Access for Elastic and Inelastic Traffic in WLANs", *IEEE Transactions On Wireless Communications*, vol. 9, no. 6, pp. 1861-1866, Jun. 2010.

[13] J. D. C. Little, and S. C. Graves, "Little's Law" Ch. 5, D. Chhajed and T. J. Lowe, eds. *Building Intuition: Insights From Basic Operations Management Models and Principles*, pp. 81-100, Springer, New York, 2008.


## Appendix

The Tables II-VIII summarize the state transitions associated with different events with different conditions. In these tables[1] the parameters $n$, $n_R$ and $n_E$ have the same meanings as described in *Algorithm I*.

[1]In these tables **DP** and **AR** indicate a departure event and an arrival event respectively. $ESU_k$ denotes an ESU service with $k$ aggregated channels.

TABLE II: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a PU departure with $j_{pu}\mu_P$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| PU DP. An RSU in RQ uses the vacant channel. | $(j_W, \cdots, j_V, g_a + 1, j_{pu} - 1, j_{eq}, j_{rq} - 1)$ | $j_{rq} > 0;\ j_{pu} > 0;\ a = 1.$ |
| PU DP. An ESU in EQ uses the vacant channel. | $(j_W + 1, \cdots, j_V, g_a, j_{pu} - 1, j_{eq} - 1, j_{rq})$ | $j_{rq} = 0$ or $a > 1;\ j_{eq} > 0;\ j_{pu} > 0;\ W = 1.$ |
| PU DP. An ongoing $ESU_k$ uses the vacant channel. | $(j_W, \cdots, j_k - 1, j_{k+1} + 1, \cdots, j_V, g_a, j_{pu} - 1, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $a > 1;\ j_{eq} = 0$ or $W > 1;$ $j_{pu} > 0;\ j_k > 0,\ V > 1;$ $k = min\{r \mid j_r > 0, W \leq r < V\}.$ |
| PU DP. No SUs uses the vacant channel. | $(j_W, \cdots, j_V, g_a, j_{pu} - 1, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $a > 1; j_{eq} = 0$ or $W > 1;$ $j_k = 0, W \leq k < V$ or $W = V;$ $j_{pu} > 0.$ |

TABLE III: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a RSU departure with $ag_a\mu_{S2}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| RSU DP. An RSU in RQ uses all the vacant channels. | $(j_W, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq} - 1)$ | $j_{rq} > 0;\ g_a > 0;\ n_R = n = a.$ |
| RSU DP. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_a + 1, \cdots, j_V, g_a - 1, j_{pu}, j_{eq} - 1, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ g_a > 0;\ n_E = n = a;$ $j_{eq} = 1,\ W \leq a \leq V$ or $j_{eq} > 1,\ W \leq a < 2W.$ |
| RSU DP. Two ESUs in EQ uses all the vacant channels. | $(j_W, \cdots, j_h + 1, \cdots, j_l + 1, \cdots, j_V, g_a - 1, j_{pu}, j_{eq} - 2, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ g_a > 0;\ n_E = n = a;$ $j_{eq} = 2,\ 2W \leq a \leq 2V,\ a = h + l,\ W \leq h, l \leq V$ or $j_{eq} > 2,\ 2W \leq a < 3W,\ a = h + l,\ W \leq h, l \leq V.$ |
| ... | ... | ... |
| RSU DP. An ongoing ESU with the minimum no. of channels, $h$ uses all the vacant channels. | $(j_W, \cdots, j_h - 1, \cdots, j_l + 1, \cdots, j_V, g_a - 1, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $g_a > 0;\ h = min\{r \mid j_r > 0, W \leq r < V\};$ $l = a + h \leq V;\ V > W.$ |
| ... | ... | ... |
| RSU DP. All other ESUs use the vacant channels and reach the upper bound $V$. | $(0, \cdots, 0, \cdots, j_V + q, g_a - 1, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $g_a > 0;\ q = \sum_{m=W}^{V-1} j_m;$ $a \geq \sum_{m=W}^{V-1} (V - m) j_m;\ V > W.$ |
| RSU DP. Other SUs cannot use the vacant channels. | $(j_W, \cdots, j_V, g_a - 1, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;\ g_a > 0;$ $V > W,\ j_m = 0, \forall m < V$ or $W = V.$ |

TABLE IV: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a $ESU_k$ departure with $kj_k\mu_{S1}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| $ESU_k$ DP. An RSU in RQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_V, g_a + 1, j_{pu}, j_{eq}, j_{rq} - 1)$ | $j_{rq} \geq 1;\ j_k > 0;\ k = a;\ n_R = k.$ |
| $ESU_k$ DP. Two RSUs in RQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_V, g_a + 2, j_{pu}, j_{eq}, j_{rq} - 2)$ | $j_{rq} \geq 2;\ j_k > 0;\ k = 2a;\ n_R = k.$ |
| ... | ... | ... |
| $ESU_k$ DP. An RSU in RQ and an ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_h + 1, \cdots j_V, g_a + 1, j_{pu}, j_{eq} - 1, j_{rq} - 1)$ | $j_k > 0;\ k = a + h;\ j_{rq} \geq 1;\ n_R = a;$ $n_E = h;\ j_{eq} = 1,\ W \leq h \leq V$ or $j_{eq} > 1,\ W \leq h < 2W.$ |
| ... | ... | ... |
| $ESU_k$ DP. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}$ $j_{eq} - 1, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_k > 0;\ n_E = n = k;$ $j_{eq} = 1,\ W \leq k \leq V$ or $j_{eq} > 1,\ W \leq k < 2W.$ |
| $ESU_k$ DP. Two ESUs in EQ uses all the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_h + 1, \cdots, j_l + 1, \cdots, j_V, g_a, j_{pu}, j_{eq} - 2, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_k > 0;\ n_E = n = k;$ $j_{eq} = 2,\ 2W \leq k \leq 2V,\ k = h + l,\ W \leq h, l \leq V$ or $j_{eq} > 2,\ 2W \leq k < 3W,\ k = h + l,\ W \leq h, l \leq V.$ |
| ... | ... | ... |
| $ESU_k$ DP. An ongoing ESU with the minimum no. of channels, $h$ uses all the vacant channels. | $(j_W, \cdots, j_h - 1, \cdots, j_k - 1, \cdots, j_l + 1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $j_k > 0;\ h = min\{r \mid j_r > 0, W \leq r < V\};$ $l = k + h \leq V;\ V > W.$ |
| ... | ... | ... |
| $ESU_k$ DP. All other ESUs use the vacant channels and reach the upper bound $V$. | $(0, \cdots, 0, \cdots, j_V + q, g_a, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0; j_{eq} = 0$ or $n_E = 0;$ $j_k > 0;\ q = \sum_{m=W}^{V-1} j_m - 1;$ $k \geq \sum_{m=W}^{V-1} (V - m) j_m - (V - k);\ V > W.$ |
| $ESU_k$ DP. Other SUs cannot use the vacant channels. | $(j_W, \cdots, j_k - 1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $j_{rq} = 0$ or $n_R = 0;\ j_{eq} = 0$ or $n_E = 0;$ $j_k > 0;\ V > W,\ j_m = 0, \forall m < V$ or $W = V.$ |

TABLE V: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a ESU arrival with $\lambda_{S1}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| ESU AR. Enough idle channels exist. | $(j_W, \cdots, j_k+1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $k = min\{M - b(x), V\} \geq W$. |
| ESU AR. The ESU with the maximum no. of channels, $m$ donate $W$ channels. | $(j_W+1, \cdots, j_n+1, \cdots, j_m-1, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $V > W$; $m = max\{r \mid j_r > 0, W+1 \leq r \leq V\}$; $n = m - [W - (M - b(x))]$; $W \leq n < m$. |
| ESU AR. Two ESUs with $m$ and $h$ channels collectively donate $W$ channels. | $(j_W+2, \cdots, j_n+1, \cdots, j_h-1, \cdots, j_m-1, \cdots, J_V, g_a, j_{pu}, j_{eq}, j_{rq})$ | $V > W;\ m = max\{r \mid j_r > 0, W+1 \leq r \leq V\}$; $h = max\{r \mid j_r > 0,\ W+1 \leq r \leq m-1\},\ j_m = 1$ or $h = m,\ j_m > 1$; $n = h + m - [2W - (M - b(x))],\ W \leq n < h$. |
| . . . | . . . | . . . |
| ESU AR. The new ESU request is put into the queue, EQ. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}+1, j_{rq})$ | $M - b(x) + \sum_{m=W+1}^{V}(m-W)j_m < W$; $j_{eq} < Q_{L1}$. |

TABLE VI: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a RSU arrival with $\lambda_{S2}$ transition rate

| Activity | Dest. State | Conditions |
|---|---|---|
| RSU AR. Enough idle channels exist. | $(j_W, \cdots, j_k, \cdots, j_V, g_a+1, j_{pu}, j_{eq}, j_{rq})$ | $M - b(x) \geq a$. |
| RSU AR. The ESU with the maximum no. of channels, $m$ donate $a$ channels. | $(j_W, \cdots, j_n+1, \cdots, j_m-1, \cdots, j_V, g_a+1, j_{pu}, j_{eq}, j_{rq})$ | $V > W$; $m = max\{r \mid j_r > 0, W+1 \leq r \leq V\}$; $n = m - [a - (M - b(x))],\ W \leq n < m$. |
| RSU AR. Two ESUs with $m$ and $h$ channels collectively donate $a$ channels. | $(j_W+1, \cdots, j_n+1, \cdots, j_h-1, \cdots, j_m-1, \cdots, J_V, g_a+1, j_{pu}, j_{eq}, j_{rq})$ | $V > W;\ m = max\{r \mid j_r > 0, W+1 \leq r \leq V\}$; $h = max\{r \mid j_r > 0, W+1 \leq r \leq m-1\},\ j_m = 1$ or $h = m, j_m > 1$; $n = h + m - [(W+a) - (M - b(x))],\ W \leq n < h$. |
| . . . | . . . | . . . |
| RSU AR. The new ESU request is put into the queue, RQ. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}+1)$ | $M - b(x) + \sum_{m=W+1}^{V}(m-W)j_m < a$; $j_{rq} < Q_{L2}$. |

TABLE VII: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a PU arrival in Case I

| Activity | Dest. State | Trans. rate | Conditions |
|---|---|---|---|
| PU AR. A vacant channel exists. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\lambda_P$ | $M - b(x) > 0$. |
| PU AR. An $ESU_k$ is interrupted and reduces its channels. | $(j_W, \cdots, j_{k-1}+1, j_k-1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{kj_k}{M-j_{pu}}\lambda_P$ | $V > W$; $b(x) = M;\ j_k > 0;\ k > W$. |
| PU AR. $ESU_W$ is forced terminated. No spectrum adaptation. | $(j_W-1, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W \geq 1;\ b(x) = M;\ W = V$ or $W = 1$, $j_k = 0,\ W+1 \leq k \leq V$. Or $j_W = 1$; $b(x) = M; j_k = 0,\ W+1 \leq k \leq V$. |
| PU AR. $ESU_W$ is forced terminated. RSU in RQ uses all the vacant channels. | $(j_W-1, \cdots, j_k, \cdots, j_V, g_a+1, j_{pu}+1, j_{eq}, j_{rq}-1)$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W \geq 1;\ b(x) = M;\ j_k = 0, \forall k > W$; $j_{rq} > 0;\ W > 1;\ n_R = a = W-1$. |
| . . . | . . . | . . . | . . . |
| PU AR. $ESU_W$ is forced terminated and another ongoing $ESU_W$ uses all the vacant channels. | $(j_W-2, \cdots, j_l+1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1;\ b(x) = M;\ j_{rq} = 0$ or $n_R = 0$; $j_k = 0, \forall k > W;\ W > 1$; $l = 2W-1 \leq V$. |
| . . . | . . . | . . . | . . . |
| PU AR. $ESU_W$ is forced terminated and other all ESUs uses all the vacant channels and achieve $V$. | $(0, 0, \cdots, j_V+q, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1;\ b(x) = M;\ j_{rq} = 0$ or $n_R = 0$; $j_k = 0, \forall k > W;\ V > W > 1; q = j_W - 1$; $W - 1 \geq (V-W)(j_W-1)$. |
| PU AR. RSU is interrupted and $ESU_k$ with max. channels, $k$ donates a channel. | $(j_W, \cdots, j_{k-1}+1, j_k-1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $V > W; b(x) = M;\ g_a > 0;\ k > W$; $k = max\{r \mid j_r > 0,\ W+1 \leq r \leq V\}$. |
| PU AR. RSU is forced terminated. No spectrum adaptation. | $(j_W, \cdots, j_k, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M;\ a = 1, j_k = 0$, $W+1 \leq k \leq V$. Or $g_a > 0$; $b(x) = M;\ W = V$. |
| PU AR. RSU is forced terminated. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k, \cdots, j_h+1, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}-1, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M;\ j_k = 0, \forall k > W$; $a \geq W+1;\ n_E = h = a-1$; $j_{eq} = 1,\ W \leq a-1 \geq V$ or $j_{eq} > 1,\ W \leq a-1 < 2W$. |
| . . . | . . . | . . . | . . . |
| PU AR. RSU is forced terminated. An $ESU_W$ uses all the vacant channels. | $(j_W-1, \cdots, j_k+1, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M; a > 1,\ j_k = 0$, $W+1 \leq k \leq V; j_W > 0$, $k = a + W - 1 \leq V;\ V > W$. |
| . . . | . . . | . . . | . . . |
| PU AR. RSU is interrupted and $ESU_W$ is forced terminated. All ESUs uses the vacant channels and achieve $V$. | $(0, 0, \cdots, j_V+q, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0;\ b(x) = M;\ a > 1,\ j_k = 0$, $W+1 \leq k \leq V; q = j_W$; $a - 1 \geq (V-W)j_W;\ V > W$. |

TABLE VIII: Transitions from a generic state $x = \{j_W, j_{W+1}, \cdots, j_k, \cdots, j_V, g_a, j_{pu}, j_{eq}, j_{rq}\}$ of *Dynamic* $(a, W, V, Q_{L1}, Q_{L2})$ upon a PU arrival in Case II

| Activity | Dest. State | Trans. rate | Conditions |
|---|---|---|---|
| PU AR. A vacant channel exists. | $(j_W, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\lambda_P$ | $M - b(x) > 0$. |
| PU AR. An ESU service with $k$ channels reduces its channels. | $(j_W, \cdots, j_{k-1}+1, j_k-1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{kj_k}{M-j_{pu}}\lambda_P$ | $V > W$; $M = b(x)$; $j_k > 0$; $k > W$. |
| PU AR. The interrupted $ESU_W$ is put into the queue EQ and no spectrum adaptation | $(j_W-1, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 0$; $b(x) = M$; $j_{eq} < Q_{L1}$; $W = V$ or $W = 1$, $j_k = 0$, $W+1 \le k \le V$. Or $j_W = 1$; $b(x) = M$; $j_k = 0$, $W+1 \le k \le V$. |
| PU AR. $ESU_W$ is put into the queue, EQ. Another $ESU_W$ uses the idle channels. | $(j_W-2, \cdots, j_l+1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $W > 1$; $l = 2W-1 \le V$; $j_{eq} < Q_{L1}$. |
| ... | ... | ... | ... |
| PU AR. $ESU_W$ is put into the queue, EQ and other all ESUs uses the idle channels and achieve $V$ | $(0, 0, \cdots, j_V+q, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $V > W > 1$; $q = j_W - 1$; $j_{eq} < Q_{L1}$; $W - 1 \ge (V-W)(j_W-1)$. |
| PU AR. The interrupted $ESU_W$ is forced terminated and no spectrum adaptation | $(j_W-1, \cdots, j_k, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 0$; $b(x) = M$; $j_{eq} = Q_{L1}$; $W = V$ or $W = 1$, $j_k = 0$, $W+1 \le k \le V$. Or $j_W = 1$; $b(x) = M$; $j_k = 0$, $W+1 \le k \le V$. |
| PU AR. $ESU_W$ is forced terminated and another $ESU_W$ uses the idle channels. | $(j_W-2, \cdots, j_l+1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $W > 1$; $l = 2W-1 \le V$; $j_{eq} = Q_{L1}$. |
| ... | ... | ... | ... |
| PU AR. $ESU_W$ is forced terminated and other all ESUs uses the idle channel and achieve $V$. | $(0, 0, \cdots, j_V+q, g_a, j_{pu}+1, j_{eq}+1, j_{rq})$ | $\frac{Wj_W}{M-j_{pu}}\lambda_P$ | $j_W > 1$; $b(x) = M$; $j_k = 0, \forall k > W$; $V > W > 1$; $q = j_W - 1$; $j_{eq} = Q_{L1}$; $W - 1 \ge (V-W)(j_W-1)$. |
| PU AR. RSU is interrupted and $ESU_k$ with maximum channels, $k$ donates a channel. | $(j_W, \cdots, j_{k-1}+1, j_k-1, \cdots, j_V, g_a, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $V > W$; $M = b(x)$; $g_a > 0$; $k > W$; $k = max\{r \mid j_r > 0,\ W+1 \le r \le V\}$. |
| PU AR. RSU is forced terminated, No spectrum adaptation. | $(j_W, \cdots, j_k, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $a = 1, j_k = 0$, $W+1 \le k \le V$. Or $g_a > 0$; $b(x) = M$; $W = V$. |
| PU AR. RSU is forced terminated. An ESU in EQ uses all the vacant channels. | $(j_W, \cdots, j_k, \cdots, j_h+1, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}-1, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $j_k = 0, \forall k > W$; $a \ge W+1$; $n_E = h = a-1$; $j_{eq} = 1$, $W \le a-1 \ge V$ or $j_{eq} > 1$, $W \le a-1 < 2W$. |
| ... | ... | ... | ... |
| PU AR. RSU is forced terminated. An $ESU_W$ uses all the vacant channels. | $(j_W-1, \cdots, j_k+1, \cdots, j_V, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $a > 1$, $j_k = 0$, $W+1 \le k \le V$; $j_W > 0$, $k = a+W-1 \le V$; $V > W$. |
| ... | ... | ... | ... |
| PU AR. RSU is interrupted and $ESU_W$ is forced terminated. All ESUs uses the vacant channels and achieve $V$. | $(0, 0, \cdots, j_V+q, g_a-1, j_{pu}+1, j_{eq}, j_{rq})$ | $\frac{ag_a}{M-j_{pu}}\lambda_P$ | $g_a > 0$; $b(x) = M$; $a > 1$, $j_k = 0$, $W+1 \le k \le V$; $q = j_W > 0$, $a-1 \ge (V-W)j_W$; $V > W$. |

# Appendix B: MATLAB Programs

## B.1 Generating Total State Space of CTMC Model

```
File Name: All_State_Space
clc;
clear all;

w                       % Minimum Number of channels for a SU
n = 4;                  % Let v = w + (n-2)
v_m = w + (n-2);        % Maximum Number of channels for a SU
M = 6;                  % Maximum Number of channels available
N = 0;                  % Total Number of channels used at the start
RQ=2;
EQ=4;

for i=0:(n-2)
ww(i+1)=fix(M/(w+i));
end
ww(n)=M;
ww

p1=[];                  % PU Analysis
for j=1:(ww(n)+1)
    p1(j)=j-1;
end
p1
pp1=size(p1);

p2=[];                  % SU index 1 analysis
for j=1:(ww(1)+1)
    p2(j)=j-1;
end
p2
pp2=size(p2);

p3=[];                  % SU index 2 analysis
for j=1:(ww(2)+1)
    p3(j)=j-1;
end
p3
pp3=size(p3);
```

```
p4=[];                % SU index 3 analysis
for j=1:(ww(3)+1)
    p4(j)=j-1;
end
p4
pp4=size(p4);

p5=[];                % RSU Analysis
for j=1:(ww(n)+1)
    p5(j)=j-1;
end
p5
pp5=size(p5);

p6=[];                % RSU Queue Analysis
for j=1:(RQ+1)
    p6(j)=j-1;
end
p6
pp6=size(p6);

p7=[];                % ESU Queue Analysis
for j=1:(EQ+1)
    p7(j)=j-1;
end
p7
pp7=size(p7);

k=0;
for j4=1:pp1(2)
for j3=1:pp2(2)
for j2=1:pp3(2)
for j1=1:pp4(2)
for j5=1:pp5(2)
for j6=1:pp6(2)
for j7=1:pp7(2)

    k=k+1;
    a=[p1(j4) p2(j3) p3(j2) p4(j1) p5(j5) p6(j6) p7(j7)];

sum = 0;
for i=1:(n-1)
        sum=((w+(i-1))*a(i+1))+sum;
end

N = sum+a(1)+a(5);
if N<=M
    r(k,:)=a;
    N;
else
    k=k-1;
end

end
end
```

```
end
end
end
end
end

r(:,[1 2]) = r(:,[2 1]);    %----------------------------
r(:,[2 3]) = r(:,[3 2]);    %   W W+1 W+2 ---- PU ORDER
r(:,[3 4]) = r(:,[4 3]);    %----------------------------
r(:,[4 5]) = r(:,[5 4]);
r(:,[6 7]) = r(:,[7 6]);

size_r=size(r);

% ++++++++++++++++++ Removing Unfeasible States  +++++++++++++++++++++++++++++++++++++++++++++++++++++

for i=1:size_r(1)
    T=TOTAL_USED_CH2( r(i,:),n,w );
    if (T<6 && r(i,6)>0) || (T<6 && r(i,7)>0) || (T<6 && r(i,1)>0) || (T<6 && r(i,2)>0) ||
    (r(i,3)>0 && r(i,6)>0) || (r(i,3)>0 && r(i,7)>0) || (r(i,2)>0 && r(i,6)>0) || (r(i,2)>0 && r(i,7)>0)
        rr(i,:)=0;
    else
        rr(i,:)=1;
    end
end
rr;
size_rr=size(rr)


i=1;
for k=1:size_rr(1)
   if rr(k,:)==0
       c1(i,:)=k;
       i=i+1;
   else
       i=i;
   end
end
c1;

 size_c1=size(c1);

 for k=1:size_c1(1)
     cc2(k,:)=c1(k,:)-(k-1);
 end
 cc2;

 size_cc2=size(cc2);

 for k=1:size_cc2(1)
     r(cc2(k,:),:)=[];
 end
r;
size_r=size(r)
```

```
% +++++++++++++++++ End of Removing Unfeasible States  +++++++++++++++++++++++++++++++++++++++++++++

for u=1:size_r(1)
    a=r(u,:);
    r11(u,:)=PU_ARRIVAL_N41(w, n, M, a, EQ, RQ);
    r12(u,:)=PU_ARRIVAL_N42(w, n, M, a, EQ, RQ);
    r13(u,:)=PU_ARRIVAL_N43(w, n, M, a, EQ, RQ);
    r14(u,:)=PU_ARRIVAL_N44(w, n, M, a, EQ, RQ);
    r2(u,:)=SU_ARRIVAL_N4(w, n, M, a, EQ, RQ);
    r3(u,:)=PU_DEPARTURE_N4(w, n, M, a, EQ, RQ);
    r4(u,:)=SU_DEPARTURE_N43(w, n, M, a, EQ, RQ);
    r5(u,:)=SU_DEPARTURE_N42(w, n, M, a, EQ, RQ);
    r6(u,:)=SU_DEPARTURE_N41(w, n, M, a, EQ, RQ);
    r7(u,:)=SU_DEPARTURE_N44(w, n, M, a, EQ, RQ);
    r8(u,:)=PSU_ARRIVAL_N4(w, n, M, a, EQ, RQ);
end

save variables.mat
```

## B.2 Calculation of Steady State Probabilities

```
File Name:Steady_State_Prob
clc;
clear all;

load variables.mat              % Load data from file "All_State_Space"

E=eye(size_r(1),size_r(1));     % Identity Matrix E

k=0; u1=0; u2=0; u3=0; u4=0;
u5=0; u6=0; u7=0; u8=0;

vw=zeros(size_r(1),size_r(1));   % Matrix to be Generated
v11=zeros(size_r(1),size_r(1));  % Matrix corresponding to PU Arrivals
v12=zeros(size_r(1),size_r(1));  % Matrix corresponding to PU Arrivals
v13=zeros(size_r(1),size_r(1));  % Matrix corresponding to PU Arrivals
v14=zeros(size_r(1),size_r(1));  % Matrix corresponding to PU Arrivals
v2=zeros(size_r(1),size_r(1));   % Matrix corresponding to SU Arrivals
v3=zeros(size_r(1),size_r(1));   % Matrix corresponding to PU Departures
v4=zeros(size_r(1),size_r(1));   % Matrix corresponding to SU Departures from 3rd index
v5=zeros(size_r(1),size_r(1));   % Matrix corresponding to SU Departures from 2nd index
v6=zeros(size_r(1),size_r(1));   % Matrix corresponding to SU Departures from 1st index
v7=zeros(size_r(1),size_r(1));   % Matrix corresponding to SU Departures from 4th index
v8=zeros(size_r(1),size_r(1));   % Matrix corresponding to PSU Arrivals

%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%
Us=1;                           % SU Service Rate for elastic traffic
Us2=1;                          % SU Service Rate for real time traffic
Up=0.5;                         % PU Service Rate
LambdaS=2;                      % SU Arrival Rate for elastic traffic
```

```
LambdaS2=1;                      % PSU Arrival Rate for real time traffic
LambdaP=1.4;                     % PU Arrival Rate
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)                % PU Arrivals into index 1
    k=0;                         % size_r(1)= Number of rows
    u1=0;
while u1==0
    k=k+1;
    if (r11(i,:)-r(k,:)==0)      % Check each row of vector "r" until
                                 % similar to r11 element r11(i,:)
        u1=k;
    else
        u1=0;
    end
    N=TOT_USED_CH( r11(i,:),r(i,:),n,w );
end


T=TOTAL_USED_CH( r(i,:),n,w );    % This function check weather the above r11 and r elements
  % are adentical or not

if r(i,1)==0 && r(i,2)==0 && r(i,3)==0 && r(i,4)==0
    v11(i,k)=LambdaP*(N)*15/100;
else
v11(i,k)=LambdaP*(N)*r(i,1)*w/T;  % Mark corresponding entry in the matrix v[]

end
end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)                 % PU Arrivals  into index 2
    k=0;                         % size_r(1)= Number of rows
    u1=0;
while u1==0
    k=k+1;
    if (r12(i,:)-r(k,:)==0)       % Check each row of vector "r" until
                                  % similar to r12 element r12(i,:)
        u1=k;
    else
        u1=0;
    end
    N=TOT_USED_CH( r12(i,:),r(i,:),n,w );
end

T=TOTAL_USED_CH( r(i,:),n,w );
if r(i,1)==0 && r(i,2)==0 && r(i,3)==0 && r(i,4)==0
    v12(i,k)=LambdaP*(N)*30/100;
else
v12(i,k)=LambdaP*(N)*r(i,2)*(w+1)/T;           % Mark corresponding entry in the matrix v[]
end
end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)                 % PU Arrivals into index 3
```

```
    k=0;                            % size_r(1)= Number of rows
    u1=0;
while u1==0
    k=k+1;
    if (r13(i,:)-r(k,:)==0)         % Check each row of vector "r" until
                                        % similar to r13 element r13(i,:)
        u1=k;
    else
        u1=0;
    end
    N=TOT_USED_CH( r13(i,:),r(i,:),n,w );
end

T=TOTAL_USED_CH( r(i,:),n,w );
if r(i,1)==0 && r(i,2)==0 && r(i,3)==0 && r(i,4)==0
    v13(i,k)=LambdaP*(N)*40/100;
else
v13(i,k)=LambdaP*(N)*r(i,3)*(w+2)/T;     % Mark corresponding entry in the matrix v[]
end
end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)                  % PU Arrivals into index 4
    k=0;                                % size_r(1)= Number of rows
    u1=0;
while u1==0
    k=k+1;
    if (r14(i,:)-r(k,:)==0)             % Check each row of vector "r" until
                                        % similar to r14 element r14(i,:)
        u1=k;
    else
        u1=0;
    end
    N=TOT_USED_CH( r14(i,:),r(i,:),n,w );
end

T=TOTAL_USED_CH( r(i,:),n,w );
if r(i,1)==0 && r(i,2)==0 && r(i,3)==0 && r(i,4)==0
    v14(i,k)=LambdaP*(N)*15/100;
else
v14(i,k)=LambdaP*(N)*r(i,4)*w/T;    % Mark corresponding entry in the matrix v[]
end
end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)                 % SU (Elastic traffic)Arrivals
    k=0;
    u2=0;
while u2==0
    k=k+1;
    if (r2(i,:)-r(k,:)==0)
        u2=k;
    else
        u2=0;
    end
```

```
    N=TOT_USED_CH( r2(i,:),r(i,:),n,w );
end

v2(i,k)=LambdaS*(N);
end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)                % PU Departures
    k=0;
    u3=0;
while u3==0
    k=k+1;
    if (r3(i,:)-r(k,:)==0)
        u3=k;
    else
        u3=0;
    end
    N=TOT_USED_CH( r3(i,:),r(i,:),n,w );
end

v3(i,k)=r(i,5)*Up*(N);
end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%


for i=1:size_r(1)               % SU Departures from 3rd index
    k=0;
    u4=0;
while u4==0
    k=k+1;
    if (r4(i,:)-r(k,:)==0)
        u4=k;
    else
        u4=0;
    end
   N=TOT_USED_CH( r4(i,:),r(i,:),n,w );
end

v4(i,k)=(w+2)*Us*r(i,3)*(N);
end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)               % SU Departures from 2nd index
    k=0;
    u5=0;
while u5==0
    k=k+1;
    if (r5(i,:)-r(k,:)==0)
        u5=k;
    else
        u5=0;
    end
    N=TOT_USED_CH( r5(i,:),r(i,:),n,w );
end
```

```
v5(i,k)=(w+1)*Us*r(i,2)*(N);
end
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)               % SU Departures from 1st index
    k=0;
    u6=0;
while u6==0
    k=k+1;
    if (r6(i,:)-r(k,:)==0)
        u6=k;
    else
        u6=0;
    end
    N=TOT_USED_CH( r6(i,:),r(i,:),n,w );
end

v6(i,k)=(w)*Us*r(i,1)*(N);
end
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)               % PSU Departures from 4th index
    k=0;
    u7=0;
while u7==0
    k=k+1;
    if (r7(i,:)-r(k,:)==0)
        u7=k;
    else
        u7=0;
    end
    N=TOT_USED_CH( r7(i,:),r(i,:),n,w );
end

v7(i,k)=(w)*Us2*r(i,4)*(N);
end
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=1:size_r(1)                % PSU (Real Time Traffic)Arrivals
    k=0;
    u8=0;
while u8==0
    k=k+1;
    if (r8(i,:)-r(k,:)==0)
        u8=k;
    else
        u8=0;
    end
    N=TOT_USED_CH( r8(i,:),r(i,:),n,w );
end

v8(i,k)=LambdaS2*(N);
end
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%
```

```
vw=v11+v12+v13+v14+v2+v3+v4+v5+v6+v7+v8; % Transition Matrix Generation - 1st Step

% Diagonal Elements set as the minus summation of rest of
                                            % the elements in that row
vv=0;
for i=1:size_r(1)
    vv=0;
    for ii=1:size_r(1)
    vv=vv+vw(i,ii);
    end
    vv;
    vw(i,i)=-vv;
end
vw;                     % Transition Matrix
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

P=vw/(max(max(abs(vw)))+1);
PP=E+P;
PPP=PP^1000;

vvv=0;
for i=1:size_r(1)
    vvv=0;
    for ii=1:size_r(1)
    vvv=vvv+PPP(i,ii);
    end
    vvv;

end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

ccc=transpose(PPP(1,:));
size_ccc=size(ccc);
```

## B.3 Calculation of Performance Metrics

```
% Capacity Calculation - Elastic Traffic

rho1=0;
for kk=1:size_r(1)
        Z=TOTAL_USED_SUCH( r(kk,:),n,w );   % Total No. of channels used by Elastic Traffic
        rho1=(Z*ccc(kk)*Us)+rho1;
end

% Blocking_Probability_Calculation_for_Elastic_Traffic

BlockProb1=0;
for kk=1:size_r(1)
    Z=TOTAL_USED_SUCH( r(kk,:),n,w );
    ZZ=Z+r(kk,5)+r(kk,4);                   % Total No. of channels used by all users
```

```
    if ZZ==M && r(kk,2)==0 && r(kk,3)==0 && r(kk,6)==EQ
        BlockProb1=BlockProb1+ccc(kk);
    else
        BlockProb1=BlockProb1;
    end
end

% Forced_Termination_Probability_Calculation_for_Elastic_Traffic

FT1=0;
for kk=1:size_r(1)
     Z=TOTAL_USED_SUCH( r(kk,:),n,w );
     ZZ=Z+r(kk,5)+r(kk,4);                  % Total No. of channels used by all users
     ZZZ=Z+r(kk,4);                         % Total No. of channels used by all SU users

     if ZZ==M && r(kk,2)==0 && r(kk,3)==0 && ZZZ>0
             FT1=(w*r(kk,1)*ccc(kk)*LambdaP/(ZZZ))+FT1;
     else
         FT1=FT1;
     end
end
FTB1=FT1/((1-BlockProb1)*LambdaS);

Y1=LambdaS*(1-BlockProb1)*(1-FTB1);         % To verify the correctness of calculation, Y1=rho1

% Average Queuing Delay for Elastic Traffic

QueueLength1=0;
for kk=1:size_r(1)
    QueueLength1=ccc(kk)*r(kk,6)+QueueLength1;
end
QL1=QueueLength1;
QD1=QL1/(LambdaS);

%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%
% Capacity Calculation - Real_Time_Traffic
rho2=0;
for kk=1:size_r(1)
    rho2=(r(kk,4)*ccc(kk)*Us2)+rho2;    % r(kk,4) = Total No. of channels used by ESUs
end


% Blocking_Probability_Calculation_for_Real_Time_Traffic

BlockProb2=0;
for kk=1:size_r(1)
    Z=TOTAL_USED_SUCH( r(kk,:),n,w );
    ZZ=Z+r(kk,5)+r(kk,4);
    if ZZ==M && r(kk,2)==0 && r(kk,3)==0 && r(kk,7)==RQ
        BlockProb2=BlockProb2+ccc(kk);
    else
        BlockProb2=BlockProb2;
    end
end
```

```
% Forced_Termination_Probability_Calculation_for_Real_Time_Traffic

FT2=0;
for kk=1:size_r(1)
     Z=TOTAL_USED_SUCH( r(kk,:),n,w );
     ZZ=Z+r(kk,5)+r(kk,4);
     ZZZ=Z+r(kk,4);

     if ZZ==M && r(kk,2)==0 && r(kk,3)==0 && ZZZ>0
             FT2=(1*r(kk,4)*ccc(kk)*LambdaP/(ZZZ))+FT2;
     else
         FT2=FT2;
     end
end
FTB2=FT2/((1-BlockProb2)*LambdaS2);


Y2=LambdaS2*(1-BlockProb2)*(1-FTB2);    % To verify the correctness of calculation, Y2=rho2

%Average Queuing Delay for Real time Traffic
QueueLength2=0;
for kk=1:size_r(1)
    QueueLength2=ccc(kk)*r(kk,7)+QueueLength2;
end
QL2=QueueLength2;
QD2=QL2/(LambdaS2);

%Spectrum Utilization
U_c=0;
for kk=1:size_r(1)
     Z=TOTAL_USED_SUCH( r(kk,:),n,w );
     ZZ=Z+r(kk,5)+r(kk,4);
     U_c=U_c+(ccc(kk)*ZZ);

end
U_CRN=U_c/M;
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%
```

## B.4 Simulation Program for Case I

```
File Name: Simulation_Case_I
clc;
clear all;

A2=0;                          % PU Distribution parameters
B2=0;    % PU Distribution parameters

muS1=0.4;                      % Service rate of ESU.
muS2=1;                        % Service rate of RSU.
muP=0.5;                       % 1/\muP = average call duration.
```

```
lambdaS1=2;                     % ESU Arrival rate.
lambdaS2=1;                     % RSU Arrival rate.
lambdaP=1;                      % PU Arrival Rate.

K=6;                            % Number of available channels.
T_max=500;                      % The total duration of the simulation.
QE=4;
QR=2;
n1=250000;                      % No. of total clicks within T_max.
n2=250000;
X_0=0;

[X,T]=call_placing_pu(X_0,lambdaP,muP,K,T_max);
A2=transpose(T);
B2=transpose(X);
size_PU=size(A2);

CC2(1)=0;
for j=2:size_PU(1)
    if B2(j)>B2(j-1)
        CC2(j)=1;
    else
        CC2(j)=5;               % PU Departure - Event ID is 5.
    end
end
%CC2(1)=0;
C2=transpose(CC2);
PU=[A2, B2, C2];                % A2 is Time, B2 is no. of PUs at time A2, C2 is the event ID

h1=T_max/n1;
p1 =lambdaS1*h1;
ESUA_Nr=binornd(1,p1,n1,1);     % Binormial distribution with parameters p1.
ESUA_Time1(1,:)=0;
for i=2:n1
    ESUA_Time1(i,:)=h1+ESUA_Time1(i-1,:);          % ESU Clicking Times.
end
E1=6*ones(n1,1);                                   % ESU Arrival - Event ID is 6.

for i=2:n1
    y1=rand;
    if y1>0.5
        ESUA_Time2(i,:)=ESUA_Time1(i,:)+0.0001;    % 0.001 is add or minus in order to avoid
    else                                           % simulataneous events.
        ESUA_Time2(i,:)=ESUA_Time1(i,:)-0.0001;
    end
end

h2=T_max/n2;
p2 =lambdaS2*h2;
RSUA_Nr=binornd(1,p2,n2,1);
RSUA_Time1(1,:)=0;
for i=2:n2
    RSUA_Time1(i,:)=h2+RSUA_Time1(i-1,:);
end
E2=7*ones(n2,1);                                   % RSU Arrival - Event ID is 7.
```

```
for i=2:n2
    y=rand;
    if y>0.5
        RSUA_Time2(i,:)=RSUA_Time1(i,:)+0.00005;    % 0.005 is add or minus in order to
    else                                             % avoid simulataneous events.
        RSUA_Time2(i,:)=RSUA_Time1(i,:)-0.00005;
    end
end

ESUA=[ESUA_Time2,ESUA_Nr,E1];
RSUA=[RSUA_Time2,RSUA_Nr,E2];
X1=[PU;ESUA;RSUA];
X2=sortrows(X1);

X2(1,:)=[];                                          % To remove 1st row of X2.
X2(2,:)=[];                                          % To remove 2nd row of X2.
%X2(3,:)=[];

size_X2=size(X2);                                    % Dimensions of X2.
X2;
%*******************************************************************

m=zeros(6,6);                   % 6x6 matrix. 6 is No. of channels.1st column for channel ID.
                                % 2nd column for current time.
                                % 3rd column to identify SU service presence and its No. of ch.
                                % 4th column to identify identify PU service presence .
                                % 6th column for identify the remaining service length of SU.

Q=zeros(1,2);                   % Q=[EQ RQ]

Q(1,1)=0;                       % No. of ESU services in the queue.
Q(1,2)=0;                       % No. of RSU services in the queue.
QE_Length=0;
QR_Length=0;
rho1=0;                         % Capacity of the ESU .
rho2=0;                         % Capacity of the RSU.
FTB1=0;
FTB2=0;
BP1=0;
BP2=0;
Total_ESU=0;
Total_RSU=0;
m;
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

for i=2:size_X2(1)
    T=TotalUsers(m);            % T is the Total used channels by SUs and PUs.

     TSU=Total_SU_Users(m);
     T1= Total_ESU1_Users(m);
     T2= Total_ESU2_Users(m);
     T3= Total_ESU3_Users(m);
     T4= Total_RSU_Users(m);
     T5=6-T;
```

```
    yt1=T1/TSU;
    yt2=T2/TSU;
    yt3=T3/TSU;
    yt4=T4/TSU;

    i3=su_with_index1(m);
    i4=su_with_index2(m);
    i7=su_with_index3(m);
    i8=rsu_with_index4(m);
    i9=pu_with_index0(m);

% PU ARRIVALS TO INDEX 1
if X2(i,3)==1

    y=rand;
    if y<=yt1


    if T<6
         m(i9,4)=1;
         m(i9,5)=exprnd(1/muP);
    elseif T==6 && i3<=6
        if i7<=6
         m(i9,4)=1;
         m(i9,5)=exprnd(1/muP);
         m(i7,3)=2;
        elseif i4<=6
         m(i9,4)=1;
         m(i9,5)=exprnd(1/muP);
         m(i4,3)=1;
        else
         m(i9,4)=1;
         m(i9,5)=exprnd(1/muP);
         m(i3,3)=0;
         m(i3,6)=0;
         FTB1=1+FTB1;
        end

    elseif T==6 && i3>6
        if i8<=6
         m(i9,4)=1;
         m(i9,5)=exprnd(1/muP);
         m(i8,3)=0;
         m(i8,6)=0;
         FTB2=1+FTB2;
        elseif i7<=6
         m(i9,4)=1;
         m(i9,5)=exprnd(1/muP);
         m(i7,3)=2;
        elseif i4<=6
         m(i9,4)=1;
         m(i9,5)=exprnd(1/muP);
         m(i4,3)=1;
        end
```

```
    end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

  %elseif X2(i,3)==2                        % PU ARRIVALS TO INDEX 2
     elseif y>yt1 && y<=(yt1+yt2)

     if T<6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
     elseif T==6 && i4<=6
         if i7<=6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
          m(i7,3)=2;
         else
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
          m(i4,3)=1;
         end

     elseif T==6 && i4>6
         if i3<=6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
          m(i3,3)=0;
          m(i3,6)=0;
          FTB1=1+FTB1;
         elseif i8<=6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
          m(i8,3)=0;
          m(i8,6)=0;
          FTB2=1+FTB2;
         elseif i7<=6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
          m(i7,3)=2;

         end
     end

%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%
 % elseif X2(i,3)==3                        % PU ARRIVALS TO INDEX 3
   elseif y>(yt1+yt2) && y<=(yt1+yt2+yt3)
     if T<6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
     elseif T==6 && i7<=6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
          m(i7,3)=2;
     elseif T==6 && i7>6
         if i8<=6
          m(i9,4)=1;
          m(i9,5)=exprnd(1/muP);
```

```
        m(i8,3)=0;
        m(i8,6)=0;
        FTB2=1+FTB2;
         elseif i3<=6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i3,3)=0;
        m(i3,6)=0;
        FTB1=1+FTB1;
       elseif i4<=6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i4,3)=1;

       end
   end
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

%elseif X2(i,3)==4                        % PU ARRIVALS TO INDEX 4
  elseif y>(yt1+yt2+yt3) && y<=(yt1+yt2+yt3+yt4)
   if T<6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
   elseif T==6 && i8<=6
       if i7<=6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i7,3)=2;
       elseif i4<=6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i4,3)=1;
       else
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i8,3)=0;
        m(i8,6)=0;
        FTB2=1+FTB2;
       end
   elseif T==6 && i8>6
    if i3<=6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i3,3)=0;
        m(i3,6)=0;
        FTB1=1+FTB1;
    elseif i7<=6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i7,3)=2;

       elseif i4<=6
        m(i9,4)=1;
        m(i9,5)=exprnd(1/muP);
        m(i4,3)=1;
```

```
        end
     end
     end


%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%
 elseif X2(i,3)==6 && X2(i,2)>0              % ESU Arrival
      Total_ESU=1+Total_ESU;
      T=TotalUsers(m);
      i3=su_with_index1(m);
      i4=su_with_index2(m);
      i7=su_with_index3(m);
      i6=su_with_index0(m);
      if T<=3
          m(i6,3)=3;
          m(i6,6)=exprnd(1/muS1);
      elseif T==4
          m(i6,3)=2;
          m(i6,6)=exprnd(1/muS1);
      elseif T==5
          m(i6,3)=1;
          m(i6,6)=exprnd(1/muS1);
      elseif T==6
          if i7<=6
             m(i6,3)=1;
             m(i6,6)=exprnd(1/muS1);
             m(i7,3)=2;
          elseif i4<=6
             m(i6,3)=1;
             m(i6,6)=exprnd(1/muS1);
             m(i4,3)=1;
          elseif Q(1,1)<QE
             Q(1,1)=Q(1,1)+1;
             QE_Length=QE_Length+1;
          else
              BP1=1+BP1;
          end
      end

%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

 elseif X2(i,3)==7 && X2(i,2)>0            % RSU Arrival
      Total_RSU=1+Total_RSU;
      T=TotalUsers(m);
      i3=su_with_index1(m);
      i4=su_with_index2(m);
      i7=su_with_index3(m);
      i6=su_with_index0(m);
      if T<=5
          m(i6,3)=4;
          m(i6,6)=exprnd(1/muS2);
      elseif T==6
          if i7<=6
             m(i6,3)=4;
```

```
            m(i6,6)=exprnd(1/muS2);
            m(i7,3)=2;
         elseif i4<=6
            m(i6,3)=4;
            m(i6,6)=exprnd(1/muS2);
            m(i4,3)=1;
         elseif Q(1,2)<QR
            Q(1,2)=Q(1,2)+1;
            QR_Length=QR_Length+1;
         else
             BP2=1+BP2;
         end
     end
 else
     m=m;
 end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++%

% PU Service Length Consideration at each time
 for ii=1:6
     if m(ii,4)>0
         T_diff=(X2(i,1)-X2(i-1,1));                % T_diff is the time difference between current
             m(ii,5)=m(ii,5)-(1*T_diff);            % time and the previous time.
             %++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
             if m(ii,5)<=0 && m(ii,4)==1            % PU departure with 1 channel
                  m(ii,4)=0;                        % Clear pu service index
                  m(ii,5)=0;                        % Clear pu service length
                  %++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
                      i3=su_with_index1(m);         % Index of the matrix row in which the
                        % ESU with 1 aggregated ch.
                      i4=su_with_index2(m);         % Index of the matrix row in which the
                               % ESU with 2 aggregated ch.
                      if Q(1,2)>0                   % If there is RSU service in the queue.
                        Q(1,2)=Q(1,2)-1;
                        m(ii,6)=exprnd(1/muS2);     % RSU service in the queue gets the chance.
                        m(ii,3)=4;
                      elseif Q(1,1)>0               % If there is ESU service in the queue.
                        Q(1,1)=Q(1,1)-1;
                        m(ii,6)=exprnd(1/muS1);     % ESU service in the queue gets the chance.
                        m(ii,3)=1;
                      elseif i3<=6                  % ESU service with 1 aggregated ch. adds new ch.
                        m(i3,3)=2;
                      elseif i4<=6                  % ESU service with 2 aggregated ch. adds new ch.
                        m(i4,3)=3;
                      else
                        m=m;
                      end
             end
     end
 end

%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
for ii=1:6
     if (m(ii,3)>0 && m(ii,3)<4)                    % ESU service length reduction with time.
         T_diff=(X2(i,1)-X2(i-1,1));                % T_diff is the time difference between current
```

```
        m(ii,6)=m(ii,6)-(m(ii,3)*T_diff);       % time and the previous time.

        if m(ii,6)<=0 && m(ii,3)==1             % SU departure with 1 aggregated channel
             rho1=1+rho1;
             m(ii,3)=0;                         % Clear su service index.
             m(ii,6)=0;                         % Clear SU service length.
                i3=su_with_index1(m);           % Index of the matrix row in which the
                  % ESU with 1 aggregated ch.
                i4=su_with_index2(m);           % Index of the matrix row in which the
                         % ESU with 2 aggregated ch.
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

             if Q(1,2)>0                        % If there is RSU service in the queue.
                Q(1,2)=Q(1,2)-1;
                m(ii,6)=exprnd(1/muS2);         % RSU service in the queue gets the chance.
                m(ii,3)=4;
             elseif Q(1,1)>0                    % If there is ESU service in the queue.
                Q(1,1)=Q(1,1)-1;
                m(ii,6)=exprnd(1/muS1);         % ESU service in the queue gets the chance.
                m(ii,3)=1;
             elseif i3<=6                       % ESU service with 1  ch. adds new 1 channel.
                m(i3,3)=2;
             elseif i4<=6                       % ESU service with 2  ch. adds new 1 channel.
                m(i4,3)=3;
             else
                m=m;
             end
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
        elseif m(ii,6)<=0 && m(ii,3)==2         % SU departure with 2 aggregated channel.
             rho1=1+rho1;
             m(ii,3)=0;                         % Clear su service index.
             m(ii,6)=0;                         % Clear SU service length.

                i3=su_with_index1(m);
                i4=su_with_index2(m);

             if Q(1,2)>0 && Q(1,1)>0            % If there are both RSU and ESU services
               % in the queue
                Q(1,2)=Q(1,2)-1;                % one of the each service has the chance.
                Q(1,1)=Q(1,1)-1;
                m(ii,3)=1;                      % ESU service in the queue gets the chance.
                m(ii,6)=exprnd(1/muS1);
                i5=rsu_with_indexO(m);
                   if i5<=6
                     m(i5,6)=exprnd(1/muS2);    % RSU service in the queue gets the chance.
                     m(i5,3)=4;
                   end
             elseif Q(1,1)==0 && Q(1,2)>1
                Q(1,2)=Q(1,2)-2;
                m(ii,6)=exprnd(1/muS2);         % RSU service in the queue gets the chance.
                m(ii,3)=4;
                i5=rsu_with_indexO(m);
                   if i5<=6
                     m(i5,6)=exprnd(1/muS2);
                     m(i5,3)=4;
```

```
                    end
              elseif Q(1,2)==0 && Q(1,1)>1
                  Q(1,1)=Q(1,1)-2;
                  m(ii,6)=exprnd(1/muS1);
                  m(ii,3)=1;
                  i6=su_with_index0(m);
                     if i6<=6
                       m(i6,6)=exprnd(1/muS1);
                       m(i6,3)=1;
                     end
              elseif Q(1,2)==0 && Q(1,1)==1
                  Q(1,1)=Q(1,1)-1;
                  m(ii,6)=exprnd(1/muS1);
                  m(ii,3)=2;
              elseif Q(1,1)==0 && Q(1,2)==1
                  Q(1,2)=Q(1,2)-1;
                  m(ii,6)=exprnd(1/muS2);
                  m(ii,3)=4;
                  i3=su_with_index1(m);
                  i4=su_with_index2(m);
                     if i3<=6
                       m(i3,3)=2;
                     elseif i4<=6
                       m(i4,3)=3;
                     end
              elseif Q(1,1)==0 && Q(1,2)==0 && i3<=6
                  m(i3,3)=3;
              elseif Q(1,1)==0 && Q(1,2)==0 && i4<=6
                  m(i4,3)=3;
                  i4=su_with_index2(m);
                     if i4<=6
                       m(i4,3)=3;
                     end
              else
                  m=m;
              end
%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

     elseif m(ii,6)<0 && m(ii,3)==3                      % SU departure with 3 aggregated channel
              rho1=1+rho1;
              m(ii,3)=0;                                 % Clear su service index
              m(ii,6)=0;                                 % Clear SU service length

                  i3=su_with_index1(m);
                  i4=su_with_index2(m);

              if Q(1,1)>0 && Q(1,2)>1
                  Q(1,2)=Q(1,2)-2;
                  Q(1,1)=Q(1,1)-1;
                  m(ii,6)=exprnd(1/muS1);
                  m(ii,3)=1;
                  i5=rsu_with_index0(m);
                     if i5<=6
                       m(i5,6)=exprnd(1/muS2);
                       m(i5,3)=4;
```

```
         end
      i5=rsu_with_index0(m);
         if i5<=6
           m(i5,6)=exprnd(1/muS2);
           m(i5,3)=4;
           end

  elseif Q(1,1)>=1 && Q(1,2)==1
      Q(1,2)=Q(1,2)-1;
      Q(1,1)=Q(1,1)-2;
      m(ii,6)=exprnd(1/muS1);
      m(ii,3)=1;
      i6=su_with_index0(m);
         if i6<=6
           m(i6,6)=exprnd(1/muS1);
           m(i6,3)=1;
         end
      i5=rsu_with_index0(m);
         if i5<=6
           m(i5,6)=exprnd(1/muS2);
           m(i5,3)=4;
         end

  elseif Q(1,1)==1 && Q(1,2)==0
      Q(1,1)=Q(1,1)-1;
      m(ii,6)=exprnd(1/muS1);
      m(ii,3)=3;

  elseif Q(1,1)==2 && Q(1,2)==0
      Q(1,1)=Q(1,1)-2;
      m(ii,6)=exprnd(1/muS1);
      m(ii,3)=2;
      i6=su_with_index0(m);
         if i6<=6
           m(i6,6)=exprnd(1/muS1);
           m(i6,3)=1;
         end

  elseif Q(1,1)==3 && Q(1,2)==0
      Q(1,1)=Q(1,1)-3;
      m(ii,6)=exprnd(1/muS1);
      m(ii,3)=1;
      i6=su_with_index0(m);
         if i6<=6
           m(i6,6)=exprnd(1/muS1);
           m(i6,3)=1;
         end
      i6=su_with_index0(m);
         if i6<=6
           m(i6,6)=exprnd(1/muS1);
           m(i6,3)=1;
         end

  elseif Q(1,1)==0 && Q(1,2)==3
      Q(1,2)=Q(1,2)-3;
```

```
    m(ii,6)=exprnd(1/muS2);
    m(ii,3)=4;
    i5=rsu_with_indexO(m);
       if i5<=6
         m(i5,6)=exprnd(1/muS2);
         m(i5,3)=4;
              end
    i5=rsu_with_indexO(m);
       if i5<=6
         m(i5,6)=exprnd(1/muS2);
         m(i5,3)=4;
       end

elseif Q(1,1)==0 && Q(1,2)==2
        Q(1,2)=Q(1,2)-2;
        m(ii,6)=exprnd(1/muS2);
        m(ii,3)=4;
        i5=rsu_with_indexO(m);
              if i5<=6
                  m(i5,6)=exprnd(1/muS2);
                  m(i5,3)=4;
              end
        i3=su_with_index1(m);
        i4=su_with_index2(m);
              if i3<=6
                  m(i3,3)=2;
              elseif i4<=6
                  m(i4,3)=3;
              end

elseif Q(1,1)==0 && Q(1,2)==1
        Q(1,2)=Q(1,2)-1;
        m(ii,6)=exprnd(1/muS2);
        m(ii,3)=4;
        i3=su_with_index1(m);
        i4=su_with_index2(m);
              if i3<=6
                  m(i3,3)=3;
              elseif i4<=6
                  m(i4,3)=3;
                   i4=su_with_index2(m);
                      if i4<=6
                           m(i4,3)=3;
                      end
              end


elseif Q(1,1)==0 && Q(1,2)==0
        i3=su_with_index1(m);
        i4=su_with_index2(m);
              if i3<=6
                  m(i3,3)=3;
                  i3=su_with_index1(m);
                  i4=su_with_index2(m);
```

```
                                        if i3<=6
                                             m(i3,3)=2;
                                        elseif i4<=6
                                             m(i4,3)=3;
                                        end

                                elseif i4<=6
                                    m(i4,3)=3;
                                    i4=su_with_index2(m);
                                         if i4<=6
                                            m(i4,3)=3;
                                         end
                                    i4=su_with_index2(m);
                                         if i4<=6
                                            m(i4,3)=3;
                                         end
                                end
                  end


           end
%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
% If index is RSU service
        elseif (m(ii,3)>0 && m(ii,3)==4)
                T_diff=(X2(i,1)-X2(i-1,1));              % T_diff is the time difference between
                m(ii,6)=m(ii,6)-(1*T_diff);              % current time and the previous time.

                    if m(ii,6)<=0 && m(ii,3)==4          % SU departure with 1 aggregated channel
                        rho2=1+rho2;                     % Capacity of the ESU
                        %v2=X2(i,1);                     % SU departed Time
                        m(ii,3)=0;                       % Clear su service index
                        m(ii,6)=0;                       % Clear SU service length

                        i3=su_with_index1(m);
                        i4=su_with_index2(m);

                        if Q(1,2)>0
                                %m(ii,3)=0;              % Clear su service index
                                Q(1,2)=Q(1,2)-1;
                                m(ii,6)=exprnd(1/muS2);
                                m(ii,3)=4;
                        elseif Q(1,1)>0
                                %m(ii,3)=0;              % Clear su service index
                                Q(1,1)=Q(1,1)-1;
                                m(ii,6)=exprnd(1/muS1);
                                m(ii,3)=1;
                        elseif i3<=6
                                m(i3,3)=2;
                        elseif i4<=6
                                m(i4,3)=3;
                        else

                                m=m;
                        end
                    end
```

```
        end


    end                                 % .................% End of SU departure occurances
    m;


%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

end                                     %..................% End of whole Loop

cap1=rho1/T_max
cap2=rho2/T_max
FTP11=FTB1/(Total_ESU-BP1)
FTP22=FTB2/(Total_RSU-BP2)
BLOCKProb11=BP1/Total_ESU
BLOCKProb22=BP2/Total_RSU
```