Feedback Vibration Control of a Base-Isolated Building with Delayed Measurements Using H_∞ Techniques

H.R. Karimi, M. Zapateiro, N. Luo, J.M. Rossell

Abstract— In this paper we address the problem of vibration reduction of buildings with delayed measurements, where the delays are time-varying and bounded. We focus on a convex optimization approach to the problem of state-feedback H_∞ control design. An appropriate Lyapunov-Krasovskii functional and some free weighting matrices are used to establish some delay-range-dependent sufficient conditions for the design of desired controllers in terms of linear matrix inequalities (LMIs). The controller, which guarantees asymptotic stability and an H_∞ performance, simultaneously, for the closed-loop system of the structure, is then developed. The performance of the controller is evaluated through the simulation of an *n*-story base-isolated building.

I. INTRODUCTION

Vibration control has emerged as an important area of scientific and technological development in recent years. Developments in vibration control have allowed successful application of the concept in numerous areas. A variety of control techniques, such as LQR control, sliding mode control, backstepping control, H_2 control, H_∞ control, guaranteed-cost control and multi-objective control have been used in vibration systems (see [1]-[13]). In the field of dynamic systems and control, delays appear either in the state, in the control input, or in the measurements (see for instance the references [14]-[24] and the references therein). The presence of a delay in a system may be the result of some essential simplification of the corresponding process model. Generally, time delay exists inevitably in control systems, which mainly results from the following: (1) the time taken in the online data acquisition from sensors at different locations of the system; (2) the time taken in the filtering and processing of the sensory data for the required control force to the actuator; (3) the time taken by the

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actuator to produce the required control force. Therefore, how to analyze and synthesize dynamic systems with delayed arguments is a problem of recurring interest, as the delay may induce complex behaviors (oscillation, instability, bad performances) for the systems concerned (see [24]-[26] and the references therein). It is also worth citing that some appreciable pieces of work have been performed to design different control strategies such as LQR control, sliding mode control, backstepping control, QFT control, H_2 control, H_∞ control for vibration control of a building structure (see [1]-[5], [27]-[31]). However, the system performance and stability, simultaneously, are not investigated for a building structure with time-varying delayed measurements in these works. Up to now, to the best of the authors' knowledge, no results about a convex optimization method for the delay-range-dependent state-feedback H_∞ control problem of building structures with time-varying delayed measurements are available in the literature, which remains to be important and challenging. This motivates the present study.
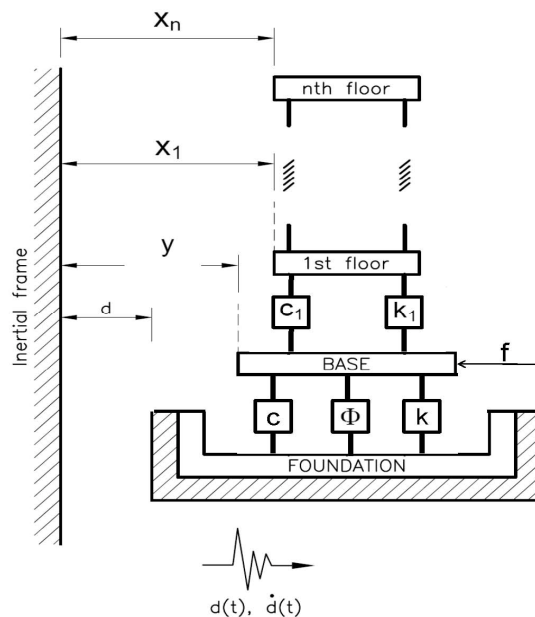

Fig. 1. Schematic of a base Isolated Structure.

In this paper, we further contribute to the development of delay-range-dependent state-feedback aspect of H_∞ control for vibration reduction in a building with delayed

measurements. The feedback loop is subject to a time-varying bounded delay within the sensors and the structure. Then we will present the application of the controller to the vibration control of a base-isolated building. The main merit of the proposed method lies in the fact that it provides a convex problem via introduction of additional decision variables such that the control gain can be found from the LMI formulations. By using an appropriate Lyapunov-Krasovskii method and some free weighting matrices, new sufficient conditions are established in terms of delay-range-dependent LMIs for the existence of desired controllers such that the resulting closed-loop system is asymptotically stable and satisfies a prescribed γ–level L_2–gain. Finally, simulation results are given to illustrate the usefulness of the proposed control methodology.

II. SYSTEM DESCRIPTION

Consider an uncertain n-story building whose base is isolated, as shown in Figure 1. The base is isolated by means of a frictional (passive) damper, Φ, and a control device with semi-active control input $f(t)$. Assume that the system is perturbed by an incoming earthquake. The structure dynamics can be divided into two subsystems, namely, the main structure (S_r) and the base (S_c) [27].

$$S_r : M\ddot{X}(t) + C\dot{X}(t) + KX(t) = [c_1, \underbrace{0, \cdots, 0}_{n-1}]^T \dot{y}(t) + [k_1, \underbrace{0, \cdots, 0}_{n-1}]^T y(t)$$

$$S_c : m\ddot{y}(t) + c\dot{y}(t) + ky(t) + f_{bf}(t) = f_{bg}(t) + f(t-h(t))$$

$$f_{bf}(t) = c_1(\dot{y}(t) - \dot{x}_1(t)) + k_1(y(t) - x_1(t))$$

$$f_{bg}(t) = -c\dot{d}(t) - kd(t) + \Phi(\dot{y}(t), \dot{d}(t))$$

$$\Phi(\dot{y}(t), \dot{d}(t)) = -\text{sgn}(\dot{y}(t) - \dot{d}(t))[\mu_{\max} - \Delta\mu e^{-\nu|\dot{y}(t) - \dot{d}(t)|}]Q$$

(1a-e)

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in \Re^n$ is the horizontal absolute floor displacement vector, $y \in \Re$ is the horizontal absolute base displacement, $d(t)$ and $\dot{d}(t)$ are the seismic excitation displacement and velocity, $f(t)$ is the active control force applied to the base level and $h(t)$ is an unknown time-varying delay in the measurement. Equation (1c) accounts for the dynamic coupling between the base and the main structure. Equation (1d) describes the forces introduced by the seismic excitation and the base isolation. Equation (1e) describes the dynamics of a frictional base isolator, where μ_{max} is the friction coefficient for high sliding velocity, $\Delta\mu$ is the difference between μ_{max} and the friction coefficient for low sliding velocity, ν is a constant and Q is the force normal to the friction surface. Parameters m, c and k are the mass, damping coefficient and stiffness of the base, while matrices M, C and K are those of the main structure as follows:

$$M = diag\{m_1, m_2, \cdots, m_n\}$$

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 & \cdots & 0 & 0 \\ -c_2 & c_2 + c_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -c_n & c_n \end{bmatrix},$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -k_n & k_n \end{bmatrix}.$$

(1f-h)

Assumption 1. The measurement time-varying delay $h(t)$ satisfies $h_1 \leq h(t) \leq h_2$ and $\dot{h}(t) \leq h_3$, where h_1 and h_2 are the minimum and maximum of $h(t)$, respectively, and h_3 is also the maximum of $\dot{h}(t)$.

Remark 1. The condition $\dot{h}(t) \leq h_3$ on time-varying delay $h(t)$ means that our method has no restriction on its derivative and can deal with any fast time-varying measurement delay.

Due to the base isolation, the movement of the main structure (S_r) is very close to the one of a rigid body. Then it is reasonable to assume that the inter-story motion of the main structure will be much smaller than the absolute motion of the base. Consequently, the following simplified equation of motion of the first floor is obtained:

$$m_1\ddot{x}_1(t) + c_1\dot{x}_1(t) + kx_1(t) = c_1\dot{y}(t) + k_1y(t) \qquad (1i)$$

In this work, it is assumed that only state variables of the base and the first floor system are measurable and the unknown seismic excitation $d(t)$ and $\dot{d}(t)$ are bounded and thus the unknown force $f_{bg}(t)$ in (1d) is bounded.

The following propositions about the intrinsic stability of the structure will be used in formulating the control law [27].

Proposition 1. The unforced main structure subsystem, i.e. (1a) with the null coupling term:

$$[c_1, 0, \ldots, 0]^T \dot{y} + [k_1, 0, \ldots, 0]^T y \equiv 0, \ t \geq 0 \qquad (1j)$$

is globally exponentially stable for any bounded initial conditions.

Proposition 2. If the coordinates $(y, \dot{y})$ of the base and the coupling term $[c_1, 0, \ldots, 0]^T \dot{y} + [k_1, 0, \ldots, 0]^T y$ are uniformly bounded, then the main structure subsystem is stable and the coordinates $(\mathbf{x}, \dot{\mathbf{x}})$ of the main structure are uniformly bounded for all $t \geq 0$ and any bounded initial conditions.

III. FORMULATION OF THE PROBLEM

The main objective of the controller design is to generate an active control force $f(t)$ that reduces the absolute base

displacement such that the base isolator can work safely in its elastic region. In order to design an H_∞ controller, we express the dynamics of the base (1b) and the first floor (1i) by the equations of the form

$$\begin{cases} \overline{M}\,\ddot{X}_{aug}(t) + \overline{C}\,\dot{X}_{aug}(t) + \overline{K}\,X_{aug}(t) = B_f\,f(t-h(t)) + B_g\,f_{bg}(t), \\ Z(t) = C_1\,X_{aug}(t) + C_2\,\dot{X}_{aug}(t) + D_1\,f(t-h(t)), \end{cases}$$

(2a,b)

with $\overline{M} = diag\{m_1, m\}$, $\overline{C} = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1+c \end{bmatrix}$, $\overline{K} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k \end{bmatrix}$,

$B_f = B_g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, where $X_{aug}(t) = [x_1, y]^T$ is the state vector,

$Z(t) \in \Re^s$ is the controlled output and $diag\{\cdots\}$ represents a block diagonal matrix. The matrices C_1, C_2 and D_1 have compatible dimensions and are defined in Section 5.

In the system (2), taking $\xi(t) := col(X_{aug}(t), \dot{X}_{aug}(t))$ yields an augmented system model, i.e., a first-order linear system:

$$\begin{cases} \dot{\xi}(t) = \overline{A}\,\xi(t) + \overline{B}_f\,f(t-h(t)) + \overline{B}_g\,f_{bg}(t), \\ Z(t) = \widetilde{C}\,\xi(t) + D_1\,f(t-h(t)), \end{cases}$$

(3a, b)

where

$$\overline{A} = \begin{bmatrix} 0 & I \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{C} \end{bmatrix}, \quad \overline{B}_f = \begin{bmatrix} 0 \\ \overline{M}^{-1}B_f \end{bmatrix},$$

$$\overline{B}_g = \begin{bmatrix} 0 \\ \overline{M}^{-1}B_g \end{bmatrix}, \quad \widetilde{C} := [C_1, C_2]$$

and $col\{\cdots\}$ represents a column vector.

Definition 1. The H_∞ performance measure of the system (3) is defined as

$$J_\infty = \int_0^\infty [Z^T(t)Z(t) - \gamma^2 f_{bg}^T(t)f_{bg}(t)] \; dt$$

where γ is a positive scalar.

The objective of this paper is to solve the following problem: For system (3) subject to the time-varying measurement delay, if all the states $X(t)$ and their derivatives are measurable, design a delayed state-feedback controller $f(t) = K_f\,\xi(t)$, where the matrix K_f is the controller gain to be determined such that

1) the resulting closed-loop system (3) is asymptotically stable;
2) under zero initial conditions and for all non-zero $f_{bg}(t) \in L_2[0,\infty)$, satisfies $J_\infty < 0$;

in this case, the system (3) is said to be asymptotically stable with an H_∞ performance measure.

IV. MAIN RESULTS

In this section, sufficient conditions for the solvability of the H_∞ control design problem are proposed using the Lyapunov method and an LMI approach ([12], [19], [21]).

Theorem 1. Consider the building vibration structure (3). For a given scalar γ under Assumption 1, there exists an H_∞ state-feedback control in the form of $f(t) = K_f\,\xi(t)$ such that the resulting closed-loop system is robustly asymptotically stable and satisfies the constraint $J_\infty < 0$, if there exist matrices $X_2, X_3, \hat{X}_1, \{\hat{N}_i\}_{i=1}^4$ and positive-definite matrices $X_1, \{\hat{R}_i\}_{i=1}^3$ satisfying the following LMI

$$\begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & \begin{bmatrix} \hat{N}_3 \\ 0 \end{bmatrix} & \begin{bmatrix} \overline{B}_g \\ 0 \end{bmatrix} & \begin{bmatrix} X_1\widetilde{C}^T \\ 0 \end{bmatrix} & \begin{bmatrix} h_2\hat{N}_1 & h_{12}\hat{N}_3 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} h_2 X_3^T & h_{12}X_3^T \\ h_2 X_2^T & h_{12}X_2^T \end{bmatrix} \\ * & \hat{\Pi}_{22} & \hat{N}_4 & 0 & (D_1\hat{X}_1)^T & [h_2\hat{N}_2 \quad h_{12}\hat{N}_4] & 0 \\ * & * & -\hat{R}_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66} & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{66} \end{bmatrix} < 0$$

(4)

with $h_{12} := h_2 - h_1$, $\hat{\Pi}_{66} = diag\{-h_2 X_1, -h_{12}X_1\}$ and

$$\hat{\Pi}_{11} = sym\left\{ \begin{bmatrix} X_3 & X_2 \\ \overline{A}X_1 - X_3 & -X_2 \end{bmatrix} X \right\} + diag\{\sum_{i=1}^3 \hat{R}_i, 0\},$$

$$\hat{\Pi}_{12} = \begin{bmatrix} \hat{N}_2^T - \hat{N}_1 - \hat{N}_3 \\ \overline{B}_f\hat{X}_1 \end{bmatrix}, \quad \hat{\Pi}_{22} = -(1-h_3)\hat{R}_3 - sym\{\hat{N}_4 + \hat{N}_2\}.$$

where the operator $sym(A)$ represents $A + A^T$. Then, the desired control gain is given by

$$K_f = \hat{X}_1 X_1^{-1} \text{ from LMI (4).} \qquad (5)$$

Proof. Firstly, we represent (3a) in an equivalent descriptor model form as

$$\begin{cases} \dot{\xi}(t) = \eta(t), \\ 0 = -\eta(t) + \overline{A}\,\xi(t) + \overline{B}_f K_f\,\xi(t-h(t)) + \overline{B}_g f_{bg}(t) \end{cases}$$

(6a,b)

Define the Lyapunov-Krasovskii functional

$$V(t) = \sum_{i=1}^5 V_i(t), \qquad (7)$$

where

$$V_1(t) = \xi(t)^T P_1 \xi(t) := [\xi(t)^T \quad \eta(t)^T] T P \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix},$$

$$V_2(t) = \sum_{i=1}^2 \int_{t-h_i}^t \xi(s)^T R_i\,\xi(s)\;ds, \quad V_3(t) = \int_{t-h(t)}^t \xi(s)^T R_3\,\xi(s)\;ds,$$

$$V_4(t) = \int_{-h_2}^0 \int_{t+\theta}^t \eta(s)^T R_4\,\eta(s)\;ds\,d\theta, \quad V_5(t) = \int_{-h_2}^{-h_1} \int_{t+\theta}^t \eta(s)^T R_5\,\eta(s)\;ds\,d\theta,$$

with $T = diag\{I, 0\}$ and $P = \begin{bmatrix} P_1 & 0 \\ P_3 & P_2 \end{bmatrix}$, where $P_1 = P_1^T > 0$.

Differentiating $V_1(t)$ in t we obtain

$$\dot{V}_1(t) = 2\xi(t)^T P_1 \dot{\xi}(t) = 2[\xi(t)^T \quad \eta(t)^T] P^T \begin{bmatrix} \dot{\xi}(t) \\ 0 \end{bmatrix}$$

$$= 2[\xi(t)^T \quad \eta(t)^T] P^T \{ \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{B}_f K_f \end{bmatrix} \xi(t-h(t)) + \begin{bmatrix} 0 \\ \overline{B}_g \end{bmatrix} f_{bg}(t) \}$$

(8)

Differentiating other Lyapunov terms in (7) give

$$\dot{V}_2(t) = \sum_{i=1}^{2} \xi(t)^T R_i \xi(t) - \xi(t-h_i)^T R_i \xi(t-h_i)$$ (9)

$$\dot{V}_3(t) = \xi^T(t) R_3 \xi(t) - (1-\dot{h}(t))\xi^T(t-h(t)) R_3 \xi(t-h(t))$$
$$\leq \xi^T(t) R_3 \xi(t) - (1-h_3)\xi^T(t-h(t)) R_3 \xi(t-h(t))$$ (10)

$$\dot{V}_4(t) = h_2 \eta(t)^T R_4 \eta(t) - \int_{t-h_2}^{t} \eta(s)^T R_4 \eta(s)\ ds$$

$$= h_2 \eta(t)^T R_4 \eta(t) - \int_{t-h(t)}^{t} \eta(s)^T R_4 \eta(s)\ ds - \int_{t-h_2}^{t-h(t)} \eta(s)^T R_4 \eta(s)\ ds$$ (11)

$$\dot{V}_5(t) = h_{12} \eta(t)^T R_5 \eta(t) - \int_{t-h_2}^{t-h_1} \eta(s)^T R_5 \eta(s)\ ds$$

$$= h_{12}\eta(t)^T R_5 \eta(t) - \int_{t-h_2}^{t-h(t)} \eta(s)^T R_5 \eta(s)\ ds - \int_{t-h(t)}^{t-h_1} \eta(s)^T R_5 \eta(s)\ ds$$ (12)

Moreover, from the Leibniz-Newton formula, i.e., $x(t) = x(t-h(t)) + \int_{t-h(t)}^{t} \dot{x}(s)\ ds$, the following equations hold for any matrices $\{N_i\}_{i=1}^{4}$ with appropriate dimensions:

$$2(\xi^T(t)N_1 + \xi^T(t-h(t))N_2)(\xi(t)-\xi(t-h(t)) - \int_{t-h(t)}^{t} \eta(s)\ ds) = 0 \quad (13)$$

$$2(\xi^T(t)N_3 + \xi^T(t-h(t))N_4)(\xi(t-h_1)-\xi(t-h(t)) - \int_{t-h(t)}^{t-h_1} \eta(s)\ ds) = 0 \quad (14)$$

From (8)-(12) and adding the left sides of equations (13) and (14) into $\dot{V}(t)$, we get

$$\dot{V}(t) = \sum_{i=1}^{5} \dot{V}_i(t) \leq 2[\xi(t)^T \quad \eta(t)^T] P^T \{ \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}\begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{B}_f K_f \end{bmatrix}\xi(t-h(t))$$

$$+ \begin{bmatrix} 0 \\ \overline{B}_g \end{bmatrix}f_{bg}(t)\} + \xi(t)^T(\sum_{i=1}^{3} R_i)\xi(t) - \sum_{i=1}^{2}\xi(t-h_i)^T R_i \xi(t-h_i) - (1-h_3)\xi^T(t-h(t))$$

$$\times R_3\ \xi(t-h(t)) + \eta(t)^T(h_2 R_4 + h_{12} R_5)\eta(t) - \int_{t-h(t)}^{t} \eta(s)^T R_4 \eta(s)\ ds$$

$$- \int_{t-h_2}^{t-h(t)} \eta(s)^T(R_4+R_5)\eta(s)\ ds - \int_{t-h(t)}^{t-h_1} \eta(s)^T R_5 \eta(s)\ ds$$

$$+ 2(\xi^T(t)N_1 + \xi^T(t-h(t))N_2)(\xi(t)-\xi(t-h(t)) - \int_{t-h(t)}^{t} \eta(s)\ ds)$$

$$+ 2(\xi^T(t)N_3 + \xi^T(t-h(t))N_4)(\xi(t-h_1)-\xi(t-h(t)) - \int_{t-h(t)}^{t-h_1} \eta(s)\ ds)$$

(15)

Now, to establish the H_∞ performance measure for the system (1), assume zero initial condition, then we have

$V(t)\big|_{t=0} = 0$. Consider the index J_∞ in Definition 1, then along the solution of (1) for any nonzero $d(t)$ there holds

$$J_\infty \leq \int_{0}^{\infty}[Z(t)^T Z(t) - \gamma^2 f_{bg}(t)^T f_{bg}(t)]\ dt - V(t)\big|_{t=0} + V(t)\big|_{t=\infty}$$

$$= \int_{0}^{\infty}[Z(t)^T Z(t) - \gamma^2 f_{bg}(t)^T f_{bg}(t) + \dot{V}(t)]\ dt$$

(16)

Substituting the term of $Z(t) = \widetilde{C}\xi(t) + D_1 f(t) = \widetilde{C}\xi(t) + D_1 K_f \xi(t-h(t))$ and the right hand side of inequality (15) into (16), we obtain

$$J_\infty \leq \int_{0}^{\infty}\vartheta^T(t)\Sigma\vartheta(t)\ dt - \int_{0}^{\infty}\int_{t-h_2}^{t-h(t)}\eta^T(s)(R_4+R_5)\eta(s)\ ds\ dt$$

$$- \int_{0}^{\infty}\int_{t-h(t)}^{t}(\vartheta^T(t)M_1 + \eta^T(s)R_4)R_4^{-1}(\vartheta^T(t)M_1 + \eta^T(s)R_4)^T\ ds\ dt$$

$$- \int_{0}^{\infty}\int_{t-h(t)}^{t-h_1}(\vartheta^T(t)M_2 + \eta^T(s)R_5)R_5^{-1}(\vartheta^T(t)M_2 + \eta^T(s)R_5)^T\ ds\ dt$$

(17)

where $\vartheta(t) := col\{\xi(t), \eta(t), \xi(t-h(t)), \xi(t-h_1), \xi(t-h_2), f_{bg}(t)\}$ is an augmented state vector and the matrix Σ is given by

$$\Sigma = \Pi + h_2 M_1 R_4^{-1} M_1^T + h_{12} M_2 R_5^{-1} M_2^T$$ (18)

where

$$\Pi = \begin{bmatrix} \Pi_{11} + \begin{bmatrix} \widetilde{C}^T\widetilde{C} & 0 \\ * & 0 \end{bmatrix} & \Pi_{12} + \begin{bmatrix} \widetilde{C}^T D_1 K_f \\ 0 \end{bmatrix} & \begin{bmatrix} N_3 \\ 0 \end{bmatrix} & 0 & P^T\begin{bmatrix} \overline{B}_g \\ 0 \end{bmatrix} \\ * & \Pi_{22} + (D_1 K_f)^T(D_1 K_f) & N_4 & 0 & 0 \\ * & * & -R_1 & 0 & 0 \\ * & * & * & -R_2 & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Pi_{11} = sym\{P^T\begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}\} + diag\{\sum_{i=1}^{3} R_i, h_2 R_4 + h_{12} R_5\},$$

$$\Pi_{12} = P^T\begin{bmatrix} 0 \\ \overline{B}_f K_f \end{bmatrix} + \begin{bmatrix} N_2^T - N_1 - N_3 \\ 0 \end{bmatrix},$$

$$\Pi_{22} = -(1-h_3)R_3 - sym\{N_4 + N_2\},$$

$$M_1 = col\{N_1, 0, N_2, 0, 0\},$$

$$M_2 = col\{N_3, 0, N_4, 0, 0\}.$$

Now, if $\Sigma < 0$, then $J_\infty < 0$ which means that the L_2−gain from the disturbance $f_{bg}(t)$ to the controlled output $Z(t)$ is less than γ. By applying Schur complement on the matrix Σ, one obtains $\Sigma < 0$ is equivalent to

$$
\begin{bmatrix}
\Pi_{11} & \Pi_{12} & \begin{bmatrix} N_3 \\ 0 \end{bmatrix} & P^T \begin{bmatrix} \bar{B}_g \\ 0 \end{bmatrix} & \begin{bmatrix} \tilde{C}^T \\ 0 \end{bmatrix} & \begin{bmatrix} h_2 N_1 \\ 0 \end{bmatrix} & h_{12} N_3 \\
* & \Pi_{22} & N_4 & 0 & (D_1 K_f)^T & h_2 N_2 & h_{12} N_4 \\
* & * & -R_1 & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma^2 I & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & -h_2 R_4 & 0 \\
* & * & * & * & * & * & -h_{12} R_5
\end{bmatrix} < 0
$$
(19)

It is also easy to see that the inequality above implies $sym(P_2^T) < 0$. Hence, the matrices P and P_2 are nonsingular. Then, according to the structure of the matrix P, the matrix $X := P^{-1}$ has the form

$$
X = \begin{bmatrix} X_1 & 0 \\ X_3 & X_2 \end{bmatrix},
$$
(20)

where $X_i = P_i^{-1}$ ($i = 1, 2$) and $X_3 = -X_2 P_3 X_1$. Let $\zeta = diag\{X^T, X_1, X_1, I, I, X_1, X_1\}$. Premultiplying ζ and postmultiplying ζ^T to the inequality (19), we obtain

$$
\begin{bmatrix}
\hat{\Pi}_{11} & \begin{bmatrix} \hat{N}_2^T - \hat{N}_1 - \hat{N}_3 \\ \bar{B}_f K_f X_1 \end{bmatrix} & \begin{bmatrix} \hat{N}_3 \\ 0 \end{bmatrix} & \begin{bmatrix} \bar{B}_g \\ 0 \end{bmatrix} & \begin{bmatrix} X_1 \tilde{C}^T \\ 0 \end{bmatrix} & \begin{bmatrix} h_2 \hat{N}_1 \\ 0 \end{bmatrix} & h_{12} \hat{N}_3 \\
* & \begin{pmatrix} -(1-h_3)\hat{R}_3 \\ -sym\{\hat{N}_4 + \hat{N}_2\} \end{pmatrix} & \hat{N}_4 & 0 & (D_1 K_f X_1)^T & h_2 \hat{N}_2 & h_{12} \hat{N}_4 \\
* & * & -\hat{R}_1 & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma^2 I & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & -h_2 \hat{R}_4 & 0 \\
* & * & * & * & * & * & -h_{12} \hat{R}_5
\end{bmatrix} < 0
$$
(21)

with $\hat{N}_i = X_1 N_i X_1$, $\hat{R}_i = X_1 R_i X_1$ and $\hat{\Pi}_{11} = sym\left\{ \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} X \right\}$
$+ diag \{\sum_{i=1}^{3} \hat{R}_i, 0\} + X^T . diag \{0, h_2 R_4 + h_{12} R_5\} . X$. Now, by considering $K_f X_1 = \hat{X}_1$, $R_4 = X_1^{-1}$, $R_5 = X_1^{-1}$ (to remove the present nonlinearities in the optimization technique) and applying Schur complement on the third term of the matrix $\hat{\Pi}_{11}$, the matrix inequality (21) is converted into a convex programming problem written in terms of LMI (4).

Remark 2. It was shown that the Lyapunov terms, i.e., $V_2(t)$, $V_4(t)$ and $V_5(t)$, result in delay-range-dependent criterions for the problem of H_∞ delayed control design.

V. NUMERICAL RESULTS

The controller is implemented with the following numerical values: the mass and stiffness of the base are $m = 6 \times 10^5$ kg, $k = 1.184 \times 10^7$ N/m, and the base damping ratio is 0.1, respectively; the main structure stiffness varies linearly from the first floor ($k_1 = 9 \times 10^8$ N/m) to the top floor ($k_{10} = 4.5 \times 10^8$ N/m); and the damping ratio is 0.05. The

frictional damper has the following values: $Q = \sum_{i=1}^{10} m_i$, $\mu_{max} = 0.185$, $\Delta\mu = 0.09$, and $v = 2.0$.
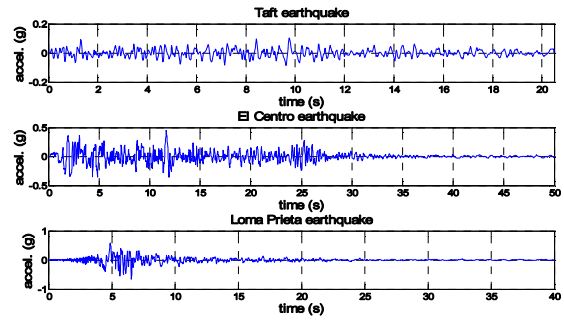

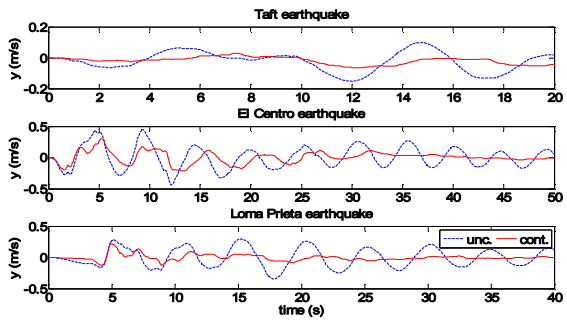Fig. 2. The Taft earthquake records.

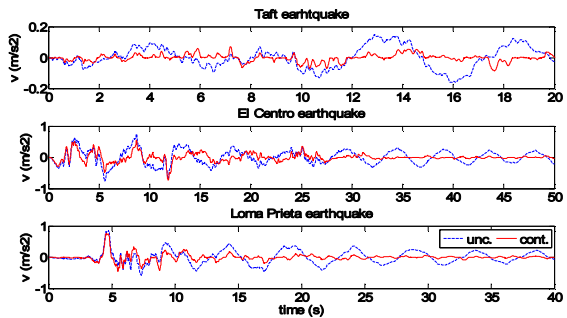Fig. 3. Relative base displacement under three different earthquakes.

Fig. 4. Relative base velocity under three different earthquakes.

The simulation is run by exciting the structure with the records of the Taft, El Centro and Loma Prieta earthquakes, as shown in Figure 2. A time-varying delay $h(t) = 0.03 + 0.01 \sin(50\pi t)$ is used.

To design an H_∞ state-feedback control law, LMI (4) is solved using Matlab LMI Control Toolbox [31] in the case of $C_1 = \varepsilon_1 . diag\{I_4, 0\}$, $D_1 = \varepsilon_2 .[0_{1\times4}, 0]^T$ with $\varepsilon_1 = \varepsilon_2 = 0.1$ and obtained the minimum value of the parameter γ in optimal H_∞ performance measure as $\gamma = 0.30$ with the control gain

$$
K_f = 10^8 [-9.1876 \quad 9.2539 \quad 0.0384 \quad -0.0560].
$$

The minimum and maximum allowable bounds of $h(t)$ and $\dot{h}(t)$ for guaranteeing the stability of the structure are obtained as $h_1 = 20 ms$, $h_2 = 40\ ms$ and $h_3 = 0.5\pi > 1$. Figures 3 and 4 show the results of the structure response (base relative displacement and velocity) in the cases there is no control ("unc.") and with the controller ("cont."). In both cases, a reduction is achieved when the active control device is integrated.

VI. CONCLUSIONS AND FUTURE WORK

In this paper we have addressed the problem of vibration reduction in a base-isolated building with delayed measurements, where the delays are time-varying and bounded. The controller was formulated following state-feedback H_∞ techniques. Some delay-range-dependent sufficient conditions for the design of a desired control were given in terms of linear matrix inequalities (LMIs). The controller, which guarantees asymptotic stability and an H_∞ performance, simultaneously, for the closed-loop system of the structure, was developed based on an appropriate Lyapunov-Krasovskii functional. The performance of the controller was evaluated by means of simulations in MATLAB/Simulink. Future work will investigate control designs for the structure under consideration by involving dynamics of semiactive actuators (MR dampers) which insert some nonlinear terms into the model.

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