Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 628295, 15 pages
doi:10.1155/2012/628295

Research Article

Fuzzy Investment Portfolio Selection Models Based on Interval Analysis Approach

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Received 3 November 2012; Accepted 7 November 2012

Academic Editor: Peng Shi

This paper employs fuzzy set theory to solve the unintuitive problem of the Markowitz mean-variance (MV) portfolio model and extend it to a fuzzy investment portfolio selection model. Our model establishes intervals for expected returns and risk preference, which can take into account investors' different investment appetite and thus can find the optimal resolution for each interval. In the empirical part, we test this model in Chinese stocks investment and find that this model can fulfill different kinds of investors' objectives. Finally, investment risk can be decreased when we add investment limit to each stock in the portfolio, which indicates our model is useful in practice.

1. Introduction

The Markowitz mean-variance (MV) portfolio optimization theory [1] proposed by Markowitz has been widely used, which leads the investment theory to a new era. In this model, the best portfolio is a maximized profit, subjected to reaching a specified level of risk, or a minimized variance, subjected to obtaining a predetermined level of expected return [2]. However, there have been persistent doubts about the estimates' performance. Many studies indicate the model has some shortage. For example, Michaud [3] have found that MV-optimized portfolios are unintuitive, and, therefore, their estimates should be promoted [4].

In portfolio selection problem, the variables such as expected return, risk, liquidity, and so forth cannot be predicted precisely and investors generally make portfolio decision according to experience and economic wisdom; therefore, deterministic portfolio selection is not a good choice for investors. Fuzzy set theory is thought to be a good method to solve this problem. For example, Bhattacharyya et al. [5] use fuzzy set theory to extend the investment portfolio model into a mean-variance-skewness (MVS) model, which is tested to be useful to explain the stock investment decision.

Our paper employs fuzzy set theory to extend Markowitz portfolio optimization theory by establishing return intervals and risk intervals. Because investors have difficulty to make a decision which is the best expected return or the lowest investment risk, they may set a portfolio which is in different hierarchical return and risk levels. Therefore, our paper can help investors to make investment decisions according to their return expectations level and risk preference level. Our method further minimizes the intuitive disadvantage of Markowitz mean-variance (MV) portfolio optimization method and thus can be more widely used in practice.

Our paper is structured as follows. Section 2 describes investment portfolio selection model and explain how to establish return intervals and risk intervals. Section 3 uses Chinese stock market and tests our models empirically. Section 4 makes some conclusions.

2. Fuzzy Investment Portfolio Selection Models

2.1. Markowitz MV Portfolio Optimization Model

Markowitz MV portfolio optimization theory assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile, that is, if for that level of risk an alternative portfolio exists which has better expected returns.

Markowitz MV portfolio optimization models are as follows:

$$\begin{aligned} &\max f(x) = \sum_{i=1}^{n} E(r_i) x_i, \\ &\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \le \omega, \\ &\sum_{i=1}^{n} x_i = 1, \\ &0 \le x_i \le \mu_i, \quad i = 1, 2, \ldots, n, \quad j = 1, 2 \ldots, n, \end{aligned} \tag{2.1}$$

$$\begin{aligned} &\min f(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j, \\ &\text{s.t.} \quad \sum_{i=1}^{n} E(r_i) x_i \ge r_0, \\ &\sum_{i=1}^{n} x_i = 1, \\ &0 \le x_i \le \mu_i, \quad i = 1, 2, \ldots, n, \quad j = 1, 2 \ldots, n. \end{aligned} \tag{2.2}$$

Model (2.1) explains a maximized expected return based on a certain risk level and model (2.2) delineates a minimized risk level conditional on a certain return level. In these two models, x_i is the investment portion of stock i in an investor's portfolio and r_i is the return of stock i. $R = (E(r_1), E(r_2), \dots, E(r_n))^T$ is the portfolio of expected returns, where $E(r_i)$ is the expected return of stock i. $\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$ is the portfolio variance, which represents investment risk, where σ_{ij} is the covariance of stocks i and j. ω is the maximum risk an investor can tolerate, μ_i is the maximum limit of stock i in investment portfolio, and r_0 is the minimum return an investor expects.

2.2. Risk Preference

Investment decisions are accompanied by much uncertainty, and risk preference varies according to an individual's personality and other reasons, such as age. Studies show that an individual's risk preference is not only affected by expected returns and risk, but also affected by gender, age, educations, wealth, health, marital factors, and so forth. In fact, investors' risk preferences are not stable; that is, different environment and social psychology can affect them as well. In general, an investor's risk appetite changes in a different environment. Therefore this section is to analyze the risk appetite.

Many investment decision models use the forms of utility function, but in some occasions, the utility function is difficult to be determined. In order to simplify the calculation and facilitate the interpretation of results, Mercurio proposes a utility function $U(W_M) = E(W_M) - A\,\mathrm{Var}(W_M)$. Based on this model, model (2.3) is constructed using risk preference coefficients, where $V = ((\sigma_{ij}^2)_{n\times n})$ is the covariance matrix of r_i, and $e = (1, 1, \dots, 1)^T$. This model solves the single-objective optimization problem of Markowitz's mean-variance portfolio model. One has

$$\begin{gathered} \min S(X) = (1-\beta)X^T V X - \beta X^T R, \\ \text{s.t} \quad X^T e = 1. \end{gathered} \tag{2.3}$$

In this model, β is defined as the coefficient of risk preference and it is nonnegative. In this model, if β is bigger, investment return is more important for investors. In particular, investors focus only on the income and do not care about risk when $\beta = 1$, while, it is exactly the opposite when β approaches zero. When investors become risk averse, that is, when $\beta = 0$, model (2.3) will be simplified to the minimum variance model of portfolio:

$$\begin{gathered} \min \sigma_p^2 = X^T V X, \\ \text{s.t} \quad X^T e = 1. \end{gathered} \tag{2.4}$$

The optimal solution for model (2.4) is $X_g = (V^{-1}{}_e)/(e^T V^{-1}{}_e)$. Further the optimal return and optimal variance of model (2.4) are as follows:

$$\begin{gathered} R_g = R^T X_g = \frac{R^T V^{-1} e}{e^T V^{-1} e}, \\ \sigma_g^2 = X_g^T V X_g = \frac{1}{e^T V^{-1} e}. \end{gathered} \tag{2.5}$$

Then we define $a = e^T V^{-1} e$, $b = R^T V^{-1} e$; the solution of model (2.4) can be further simplified as follows:

$$X_g = \frac{V^{-1}e}{a}, \qquad R_g = \frac{b}{a}, \qquad \sigma_g^2 = \frac{1}{a}. \tag{2.6}$$

Properties. If the covariance matrix is positive definite, model (2.3) has a unique optimal solution:

$$X^* = \frac{V^{-1}e}{e^T V^{-1} e} + \frac{\beta}{2(1-\beta)}\left[V^{-1}R + \frac{e^T V^{-1} R}{e^T V^{-1} e} V^{-1} e\right]. \tag{2.7}$$

In (2.7), X^* indicates the optimal portfolio of model (2.3).

Proof. If V is positive definite, then model (2.3) is convex quadratic. The necessary and sufficient conditions for the solution of convex quadratic are to meet the Kuhn-tucker conditions. Kuhn-tucker conditions for model (2.3) are

$$\begin{gathered} 2(1-\beta)VX^* - \beta R - \lambda e = 0, \\ X^{*T} e = 1. \end{gathered} \tag{2.8}$$

According to (2.8), we can deduce the portfolio

$$X^* = \frac{V^{-1}e}{e^T V^{-1} e} + \frac{\beta}{2(1-\beta)}\left[V^{-1}R + \frac{e^T V^{-1} R}{e^T V^{-1} e} V^{-1} e\right]. \tag{2.9}$$

According to (2.7), we can solve the optimal return of (2.3):

$$R_p = R^T X^* = \frac{(ac - b^2)\beta}{2a(1-\beta)} + \frac{b}{a}, \tag{2.10}$$

where $c = R^T V^{-1} R$.

Discussion about $ac - b^2$.

By definition we know that $a > 0$, $c > 0$, and the covariance matrix is positive semi-definite.

According to the Cauchy-Schwarz inequality, $ac - b^2 \geq 0$, then:

(1) when securities in the portfolio have the same expected returns, according to Cauchy-Schwarz inequality, $ac - b^2 = 0$;

(2) in the portfolio, if there are at least two securities whose expected returns are different, the expected returns of all securities in the portfolio are not identical.

According to the Cauchy-Schwarz inequality, $ac - b^2 > 0$.

Based on the above two properties, combined with the actual situation, we assume that the expected returns of all securities in the portfolio are not exactly the same, so $ac - b^2 > 0$. Thus, the risk preference decision parameter β can be computed as follows:

$$\beta = \frac{2(aR_p - b)}{ac - b^2 + 2(aR_p - b)}. \tag{2.11}$$

□

2.3. Fuzzy Investment Portfolio Model

Generally, an interval linear model is expressed as the following form:

$$\begin{aligned} &\max f(x) = f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} K_i x_i, \\ &\text{s.t.} \quad \sum_{i=1}^{n} Z_{ij} x_i \le M_j, \quad j = 1, 2, \ldots, m, \\ &x_i \ge 0, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{2.12}$$

In the above formula, K_i and Z_{ij} are interval numbers, and they are, respectively, expressed as $K_i = [\underline{k_i}, \overline{k_i}]$, $Z_{ij} = [\underline{z_{ij}}, \overline{z_{ij}}]$, where we will denote the feasible region of X by Ω, that is $X \in \Omega$.

After this, some scholars transformed the interval linear programming model into a two-goal programming problem and build a new planning model as follows:

$$\begin{aligned} &\max f_1(x) = f_1(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \underline{k_i} x_i, \\ &\text{s.t.} \quad X \in \Omega, \end{aligned} \tag{2.13}$$

$$\begin{aligned} &\max f_2(x) = f_2(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \overline{k_i} x_i, \\ &\text{s.t.} \quad X \in \Omega. \end{aligned} \tag{2.14}$$

Da and Liu [6] adopt a parameter α and use interval model to extend the above model into (2.15), which takes into account both investors' judgment and environment situation.

$$\begin{aligned} &\max f_3(x) = f_3(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \left[(1-\alpha) \underline{k_i} x_i + \alpha \overline{k_i} x_i \right], \\ &\text{s.t.} \quad X \in \Omega, \alpha \in [0, 1]. \end{aligned} \tag{2.15}$$

In (2.15), there are two kinds of specific situation, that is, $\alpha = 1$ and $\alpha = 0$. Further, based on investors' risk preference, there are two kinds of models: (2.16) represents a risk lover's investment and (2.17) represents a risk adverse investor's choice, where, $v_i = [\underline{v_i}, \overline{v_i}]$ means the risk range of stock i. One has

$$\begin{aligned} &\max U(x_i) = \sum_{i=1}^{n} r_i x_i, \\ &\text{s.t.} \quad X^T e = 1, \quad X \geq 0, \end{aligned} \tag{2.16}$$

$$\begin{aligned} &\min V(x_i) = \sum_{i=1}^{n} v_i x_i, \\ &\text{s.t.} \quad X^T e = 1, X \geq 0. \end{aligned} \tag{2.17}$$

As for investors, we can naturally think of that investors want to maximize the return on investment and minimize the investment risk in the portfolio, that is, meeting the above two models at the same time, but according to practical experience, we know that this is impossible, because the higher the expected return of the portfolio, their attendant risks will also be the greater; high yield is the compensation of high risk, which requires investors to choose a balance of investment returns and risks according to their own psychology.

Since returns are uncertain, the allocation of capital in different risky assets to minimize the risk and maximize the return is the main concern of portfolio selection [5]. Therefore, it is useful to take into account returns and risks together by using the risk preference coefficient β. Thus the above multitarget interval linear function can be converted into a parametric function. By applying the risk preference coefficient β, investment flexibility can be increased because investors can decide to set different proportion of earnings targets and risks targets. It also takes into account different investors' decision-making behaviors.

However, the expected returns and risks in portfolios should be intrinsically linked and discussed together. Chen et al. [7] simplify the variance constraints using fuzzy constraints:

$$\begin{aligned} &\max U(x_i) = \sum_{i=1}^{n} E(r_i) x_i, \\ &\text{s.t.} \quad \sum_{i=1}^{n} \sigma_i x_i \leq M + d(1-\alpha), \\ &\qquad X^T e = 1, \\ &\qquad \mu \geq X \geq 0, \\ &\qquad \mu = \{\mu_1, \mu_2, \ldots, \mu_n\}. \end{aligned} \tag{2.18}$$

In (2.18), α is fuzzy membership which an investor belongs to, $\alpha \in [0,1]$; the membership function is expressed as:

$$\mu(x_i) = \begin{cases} 1, & 0 \leq \sum_{i=1}^{n} \sigma_i x_i \leq M - d, \\ \dfrac{M - \sum_{i=1}^{n} \sigma_i x_i}{d}, & M - d \leq \sum_{i=1}^{n} \sigma_i x_i \leq M, \\ 0, & \text{others}, \end{cases} \tag{2.19}$$

where d is the tolerance of investors and σ_i is the standard deviation of the stock i's return. Chen et al. [7] adopt parameters in the fuzzy interval and thus the model is transformed into a linear function:

$$\begin{aligned} &\max U(x_i) = \sum_{i=1}^{n} \left[\underline{r_i}, \overline{r_i}\right] x_i , \\ \text{s.t.} \quad &\sum_{i=1}^{n} \left[\underline{\sigma_i}, \overline{\sigma_i} x_i\right] \leq \left[\underline{M} + d(1-\alpha), \overline{M} + d(1-\alpha)\right], \\ &X^T e = 1, \\ &\mu \geq X \geq 0, \\ &\mu = \{\mu_1, \mu_2, \ldots, \mu_n\}. \end{aligned} \tag{2.20}$$

This model can solve the calculation problems, but this model involves multiple parameters. Although parameters have the advantages in increasing flexibility, too many parameters result in decision-making errors and thereby increase uncertainty.

Risk preference is determined by the expected return and risk, which are expressed by interval numbers. We define risk preference coefficient as $\beta = [\underline{\beta}, \overline{\beta}]$ and use Markowitz method to calculate the stocks investment risk. Model (2.21) can estimate investors' risk preferences from investment decision-making behaviors, and then it compares the returns and risk together in the same model

$$\begin{aligned} \max f(x_i) &= \beta \sum_{i=1}^{n} r_i x_i - (1-\beta) \sum_{i=1}^{n} XVX^T \\ &= \left[\underline{\beta}, \overline{\beta}\right] \sum_{i=1}^{n} \left[\underline{r_i}, \overline{r_i}\right] x_i - \left[1-\overline{\beta}, 1-\underline{\beta}\right] \sum_{i=1}^{n} XVX^T \\ &= \sum_{i=1}^{n} \left[\underline{\beta r_i}, \overline{\beta}\overline{r_i}\right] x_i - \left[1-\overline{\beta}, 1-\underline{\beta}\right] \sum_{i=1}^{n} XVX^T s. \end{aligned} \tag{2.21}$$

Covariance matrix is $V = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \vdots & & & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix}$. All elements of the matrix are interval numbers, that is, $\sigma_{ij} = [\underline{\sigma_{ij}}, \overline{\sigma_{ij}}]$. So the formula $\sum_{i=1}^{n} XVX^T$ can be simplified as follows:

$$\begin{aligned} \sum_{i=1}^{n} XVX^T &= \{x_1 x_2 \cdots x_n\} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \vdots & & & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \{x_1 x_2 \cdots x_n\}^T \\ &= \left\{ \sum_{i=1}^{n} \left[\underline{\sigma_{i1}}, \overline{\sigma_{i1}}\right] x_i, \sum_{i=1}^{n} \left[\underline{\sigma_{i2}}, \overline{\sigma_{i2}}\right] x_i, \ldots, \sum_{i=1}^{n} \left[\underline{\sigma_{in}}, \overline{\sigma_{in}}\right] x_i \right\} \{x_1 x_2 \cdots x_n\}^T \end{aligned}$$

$$= x_1\sum_{i=1}^{n}\left[\underline{\sigma_{i1}},\overline{\sigma_{i1}}\right]x_i + x_2\sum_{i=1}^{n}\left[\underline{\sigma_{i2}},\overline{\sigma_{i2}}\right]x_i + \cdots + x_n\sum_{i=1}^{n}\left[\underline{\sigma_{in}},\overline{\sigma_{in}}\right]x_i$$
$$= \sum_{j=1}^{n}x_j\sum_{i=1}^{n}\left[\underline{\sigma_{ij}},\overline{\sigma_{ij}}\right]x_i. \tag{2.22}$$

Therefore, the portfolio model can be simplified as follows:

$$\begin{aligned}\max f(x_i) &= \beta\sum_{i=1}^{n}r_ix_i - (1-\beta)\sum_{i=1}^{n}XVX^T \\ &= \left[\underline{\beta},\overline{\beta}\right]\sum_{i=1}^{n}\left[\underline{r_i},\overline{r_i}\right]\ x_i - \left[1-\overline{\beta},1-\underline{\beta}\right]\sum_{j=1}^{n}x_j\sum_{i=1}^{n}\left[\underline{\sigma_{ij}},\overline{\sigma_{ij}}\right]x_i \\ &= \sum_{i=1}^{n}\left[\underline{\beta r_i},\overline{\beta r_i}\right]x_i - \sum_{j=1}^{n}x_j\sum_{i=1}^{n}\left[\left(1-\overline{\beta}\right)\underline{\sigma_{ij}},\left(1-\underline{\beta}\right)\overline{\sigma_{ij}}\right]x_i.\end{aligned} \tag{2.23}$$

The optimal estimators of (2.23) are defined as $b = [0,1]$, where b is the membership of the interval. A bigger b means higher expected return; that is, b reflects investor's confidence level on market expectations. Specifically when b is closer to 1, it indicates that investors are more optimistic about the market future. However, the expectation for the future market is extremely pessimistic when b is closer to 0. So the final form of the fuzzy investment portfolio model is expressed as follows:

$$\begin{aligned}\max f(x_i) &= \sum_{i=1}^{n}\left(b\overline{\beta r_i} + (1-b)\underline{\beta r_i}\right)x_i - \sum_{j=1}^{n}x_j\sum_{i=1}^{n}\left[b\left(1-\overline{\beta}\right)\underline{\sigma_{ij}} + (1-b)\left(1-\underline{\beta}\right)\overline{\sigma_{ij}}\right]x_i, \\ \text{s.t.}\quad & X^Te = 1, \\ & X \geq 0.\end{aligned} \tag{2.24}$$

By using (2.12), we can set an investment limit for each stock, and thus the model is expressed as follows:

$$\begin{aligned}\max f(x_i) &= \sum_{i=1}^{n}\left(b\overline{\beta r_i} + (1-b)\underline{\beta r_i}\right)x_i - \sum_{j=1}^{n}x_j\sum_{i=1}^{n}\left[b\left(1-\overline{\beta}\right)\underline{\sigma_{ij}} + (1-b)\left(1-\underline{\beta}\right)\overline{\sigma_{ij}}\right]x_i, \\ \text{s.t.}\quad & X^Te = 1, \\ & \mu_i \geq x_i \geq 0,\end{aligned} \tag{2.25}$$

where μ_i indicates the maximum amount of the stock i in sthe portfolio.

Based on the investment behavior analysis and clear understanding of their risk reference as well as future expectations of the market, investors can utilize this model to solve effective programs of the portfolio. Fuzzy theory is widely used to solve time varying problems [8–18]; therefore, the following section will adopt the model we generated to solve the practice problems.

3. Empirical Study on the Stock Investment Portfolio

3.1. Data

The principle of decentralization can reduce the nonsystematic risk of the portfolio. In accordance with the principle of decentralization, we use twenty stocks which are selected from different industries. In order to provide investors with multiple choices, these stocks have different enterprise growth rates.

We use Chinese stocks to test our fuzzy investment portfolio model. The primary data come from CSMAR database produced by GTA company. We select 20 stocks and the data include each stock's weekly opening prices and weekly closing prices for the 100 weeks from June 11, 2010, to May 18, 2012.

3.2. Variables

In order to compute the expected returns, we use the historical average return to estimate the expected return. On this basis, we can make interval estimation of the returns by utilizing the fuzzy statistical method and thus construct a number of reasonable income sets.

Since many stocks distribute a little dividend to investors, the weekly dividend can be ignored because the dividend will be very small after being divided into every week. So in this paper, we discard the dividend and only take into account the stock trading prices change. We use the price change during each week. The weekly return is calculated as

$$r_{it} = \frac{p_{it} - p_{it0}}{p_{it0}}, \tag{3.1}$$

where r_{it} is stock i's return during week t ($i = 1, 2, \ldots, 20$; $t = 1, 2, \ldots, 52$). Meanwhile, p_{it0} is the opening price of stock i on week t and p_{it} is the closing price of stock i on week t.

3.3. Interval Determined by Fuzzy Statistical Method

3.3.1. Determination of Return Intervals

As for the return interval, we compute it as follows.

(1) Based on the stocks' weekly returns for 100 weeks, we divide them into 10 subintervals. Then we count the number of actual return rate contained within each subinterval.

(2) The number of returns is the degree of membership of each interval.

(3) The final return interval is the interval which the sum of each median multiplied membership degree locates in.

The stock returns are listed in Table 1.

Table 1: The stock return intervals.

Stock name	Interval lower limit	Interval upper limit
Vanke	0.005224	0.031831
Konka	−0.00596	0.021606
Victor onward Textile	0.002509	0.031017
Universe	−0.00717	0.026362
Tellus	−0.03185	0.012672
Shenxin	−0.03593	0.02052
Nonfemet	−0.03978	0.010745
Xinmao S&T	−0.00754	0.022664
INT'L Group	−0.0213	0.01444
Jinlu	−0.0273	0.00901
ZY Environment	−0.01111	0.020413
DaTong Gas	−0.00474	0.021312
Advanced Technology	−0.00708	0.024332
Fujian Electric	−0.00968	0.018596
Sinosteel	−0.01617	0.044893
Huayi Brothers	−0.0378	0.022356
China Merchants Bank	−0.00997	0.013047
Xi'An Aero Engine	−0.0003	0.031385
Ping An	−0.0124	0.01055
Petro China	−0.0207	−0.0007

3.3.2. Determination of Risks Intervals

Based on Markowitz mean-variance model, we use the variance of the stock returns to measure the level of investment risk. Stock return variance is calculated as follows.

(1) Based on the stock' weekly returns for 100 weeks, we divide the returns into 10 subintervals and count the number of returns in each subinterval.

(2) The number of returns is the degree of membership of each interval.

(3) We define that a_i and b_i are the upper and lower bounds for each interval, respectively. Thus $\widehat{\mu}_u$ and $\widehat{\mu}_l$ are, respectively, the upper and lower bounds of the interval which the product of the mean of return range and the degree of membership are in.

Denote U as a fuzzy set, where $\{x_i = [a_i, b_i], i = 1, 2, \ldots, N\}$ comprises the set U. Thus the means can be expressed as $F\mu = [\mu_l, \mu_u]$. We define that $\{x_i = [a_i, b_i], i = 1, 2, \ldots, n\}$ is a random fuzzy sample from set U, where the mean of the sample is $F\overline{X} = [\widehat{\mu}_l, \widehat{\mu}_u]$, which is the return set. Further, we denote $c_i = (a_i + b_i)/2$, $l_i = |b_i - a_i|$, $c = (\mu_l + \mu_u)/2$, and $l = |\mu_u - \mu_l|$, $\widehat{c} = (\widehat{\mu}_l + \widehat{\mu}_u)/2$, $\widehat{l} = |\widehat{\mu}_u - \widehat{\mu}_l|$, then variance of the fuzzy set can be expressed as

$$F\sigma^2 = \left\langle \frac{\sum_{i=1}^{N} (c_i - c)^2}{N}, \frac{\sum_{i=1}^{N} (l_i - l)^2}{N} \right\rangle, \tag{3.2}$$

$$FS^2 = \left\langle \frac{\sum_{i=1}^{n} (c_i - \widehat{c})^2}{n-1}, \frac{\sum_{i=1}^{N} \left(l_i - \widehat{l}\right)^2}{n-1} \right\rangle. \tag{3.3}$$

Table 2: Variance based on fuzzy method.

Stock name	Variance
Vanke	0.001959
Konka	0.002526
Victor onward Textile	0.002652
Universe	0.002953
Tellus	0.003845
Shenxin	0.0056
Nonfemet	0.004228
Xinmao S&T	0.003022
INT'L Group	0.003809
Jinlu	0.00416
ZY Environment	0.002569
DaTong Gas	0.002516
Advanced Technology	0.00328
Fujian Electric	0.003027
Sinosteel	0.007421
Huayi Brothers	0.004313
China Merchants Bank	0.001113
Xi'An Aero Engine	0.003863
Ping An	0.001489
Petro China	0.000719

(4) Then we substitute the respective values into (3.3) to calculate the variance of stocks.

The results are in Table 2.

In this model, the risk of the stock is expressed by the covariance matrix than risk losing rate. In order to determine the interval for the variance the traditional approach is to directly select the range of variance fluctuation. Then it is expressed as $\alpha^2 \pm a$, where a is a given constant. But there are no studies about the determination of covariance interval. Therefore this paper defines the covariance by determining the variance of interval numbers. By using $a = 0.0001$, the covariance interval is determined.

3.4. Empirical Results

3.4.1. Estimation without Investment Limit

Next, in order to figure out the role of the investment limit on the portfolio, we also calculate the portfolio without setting maximum investment ratio and list the results in Table 3.

3.4.2. Estimation with Investment Limit

In order to show the role of risk controlling, we set a maximum investment ratio for each stock which depends on the investor's risk preference. The investor' risk preference can be calculated in accordance with investor's risk preference decision model. Given the different

Table 3: The investment portfolio of different market expectations without limits.

Stock name	$b = 0$	$b = 0.3$	$b = 0.5$	$b = 0.8$	$b = 1$
Vanke	1.0000	0.6763	0.6950	0.2345	0
Konka	0	0	0	0	0
Victor onward Textile	0	0.3237	0.3050	0	0
Universe	0	0	0	0	0
Tellus	0	0	0	0	0
Shenxin	0	0	0	0	0
Nonfemet	0	0	0	0	0
Xinmao S&T	0	0	0	0	0
INT’L Group	0	0	0	0	0
Jinlu	0	0	0	0	0
ZY Environment	0	0	0	0	0
DaTong Gas	0	0	0	0	0
Advanced Technology	0	0	0	0	0
Fujian Electric	0	0	0	0	0
Sinosteel	0	0	0	0.7655	1.0000
Huayi Brothers	0	0	0	0	0
China Merchants Bank	0	0	0	0	0
Xi’An Aero Engine	0	0	0	0	0
Ping An	0	0	0	0	0
Petro China	0	0	0	0	0
Fval	−0.0023	−0.0074	−0.0110	−0.0185	−0.0264

expectations of the investors on the market in the future, the optimal portfolio of investors can be calculated.

Firstly, we assume that the investor’s target return is $[10\%, 12\%]$. Then we calculated the interval of investors risk preference $[0.6031, 0.6454]$ using the risk preference decision parameter β. We can find that the investor is the slight risk preferences type. This paper sets the maximum investment ratio on each stock on the level of 0.2, that is, $\mu_i = 0.2$. Then the portfolio can be comprised of at least 5 stocks, which can relatively reduce investment risk. We assume the investor’s market expectations are, respectively, $b = 0$, $b = 0.3$, $b = 0.5$, $b = 0.8$, and $b = 1$. The results are listed in Table 4.

Table 4 shows that the investment portfolio only includes a few stocks. Some stocks such as Tellus and ZY Environment have not been selected regardless of the market expectations. The reason is that the expected returns of selected stocks are stable and they have higher expected return range. In Table 4, we find that the investment ratios of Vanke and Victor onward Textile are 0.2, respectively, because their expected return range is positive. Meanwhile, these two stocks are less risky. Meanwhile, the investment ratio of Advanced Technology is 0.2 as well, since the investment return of the Advanced Technology is more stable than that of other stocks regardless of economic situation. Investment proportion of DaTong Gas is high in bad economic environment. However, when the economy becomes better, the investment proportion gradually declines due to its instable return. Therefore this stock is a good choice for risk-averse investor.

In the contrary, the impact of the economic situation on Sinosteel is more significant, which is reflected by the big expected return interval. So the investment ratio gradually increases when the economic situation becomes better. The value shows a decreasing trend.

Table 4: The investment portfolio of different market expectations under the conditions of limited investment ratio.

Stock name	$b = 0$	$b = 0.3$	$b = 0.5$	$b = 0.8$	$b = 1$
Vanke	0.2000	0.2000	0.2000	0.2000	0.2000
Konka	0.1672	0.0632	0	0	0
Victor onward Textile	0.2000	0.2000	0.2000	0.2000	0.2000
Universe	0	0.0919	0.1047	0.1328	0.2000
Tellus	0	0	0	0	0
Shenxin	0	0	0	0	0
Nonfemet	0	0	0	0	0
Xinmao S&T	0	0	0	0	0
INT'L Group	0	0	0	0	0
Jinlu	0	0	0	0	0
ZY Environment	0	0	0	0	0
DaTong Gas	0.2000	0.1292	0.0477	0	0
Advanced Technology	0.0328	0.0658	0.0476	0.0672	0
	0	0.0499	0.2000	0.2000	0.2000
Fujian Electric	0	0	0	0	0
Sinosteel	0	0	0	0	0
Huayi Brothers	0.2000	0.2000	0.2000	0.2000	0.2000
China Merchants Bank	0	0	0	0	0
Xi'An Aero Engine	0	0	0	0	0
Ping An	0	0	0	0	0
Petro China	0	0	0	0	0
Fval	0.0011	−0.0044	−0.0089	−0.0160	−0.0208

Therefore the maximum value of the model will show an increasing trend as the economic situation improves.

By comparing Tables 3 and 4, we find that without the investment limit, the investment portfolio consists more of Vanke and Victor onward Textile, especially in the case of a poor economic situation. But when the economy becomes better, the investment portfolio becomes to include Sinosteel. It shows that Sinosteel is more significantly influenced by the economic situation. In conclusion, we find that the expected returns and risk are both increasing without investment restrictions. Therefore, the risk can be reduced if the investment limit is set in the portfolio.

In comparison to the current research on the investment portfolio, our model's results show the optimal selection portfolio for investors with different risk preference; that is, each investor can set an expected return and risk level and thus makes his/her decision according to this level. Meanwhile, our model can set the maximum risk limit. Through this restriction, investment risk can be under control in a certain level, because it's hard for investors to always find a minimum risk.

4. Conclusions

In this paper, we use fuzzy set theory to extend Markowitz mean-variance portfolio model and test this model in Chinese stock market. The results indicates that fuzzy set theory is useful to avoid the problems of Markowitz mean-variance portfolio model and takes into account different expected return levels and risk preference levels. The paper also uses 20 Chinese stocks to test the model's efficiency. We find that the risk can be minimized by our fuzzy investment portfolio model through adding the maximum investment portion. The model is finally proved to be useful in investment practice.

Fuzzy investment portfolio selection model can be used in many fields such as stock markets, futures market and stock index futures markets, and so forth. Further, the portfolio model can set more intervals according to investors' needs, which will be more detailed when intervals become smaller.

Acknowledgment

This study was sponsored by the National Science Foundation of China (nos. 71103049, 71271069, and 71031003), the Humanity and Social Science Research Foundation (10YJC790070), and the Research Fund for the Doctoral Program of Higher Education (20112302120038) of MoE.

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