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Adjustment Costs for Investments in the Abatement of Climate Change Under Uncertainty

Working Paper 2000:5

ISSN: 0804-452X

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Abstract

The paper study how optimal abatement of CO_2 emissions is affected by adjustment costs for investments in, and irreversibility of abatement capital. Optimal abatement is determined by a Ramsey model for economic growth, where uncertainy damage of climate change affects productivity. A numerical approximation of the solution of the dynamic programming model in continuous time indicate that uncertainty leads to a significant reduction in the level of abatement. Compared with a model that disregards adjustment costs and uncertainty, the level of emission cuts is primarily affected by the uncertainty, while the adjustment costs may be decisive for the question of when emission ought to be reduced. Irreversibility of abatement costs seems to have a limited impact on the optimal policy.

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7th April, 2000

1 Introduction

One of the most difficult questions arising in designing climate policy is how to encounter the large uncertainties. Although few question the possibility that emissions of greenhouse gases may lead to climatic changes, it is hard to tell what changes we may expect from given emission paths in the future. Future emissions are uncertain, and the resulting concentrations of greenhouse gases in the atmosphere are uncertain even if we knew the emissions with certainty. This makes it difficult to predict radiative forcing, and there is considerable uncertainty about the temperature change at a given level of forcing. However, these uncertainties are small compared with those related to assessments of impacts. In many cases, one have to limit the assessment of impacts to point out possible impacts which may turn out to be serious, but is likely not to happen. For example, there is a possibility that the Gulf current may be weakened significantly , and thereby leave Northern Europe colder as consequence of climate change. However the chance is small, and if not, Europe is likely to turn warmer. As a consequence, estimates of the economic damage of climate change are subject to uncertainties larger than usual in economic analysis.

Recommendations for climate policy need not be based on damage assessments, not at least because political targets, such as those in the Kyoto protocol, are usually expressed in terms of current emissions. Most of the economic analyses of climate policy have therefore focused on the question of achieving emission targets under given constraints. On the other hand, one of the most important questions remains how much of the emissions it is worthwhile to reduce. Conflicts among stakeholders may often be traced to this question, and the issue will turn out increasingly important in the future if countries not committed by the Kyoto protocol are to be included.

Whether to initiate early, aggressive actions to reduce greenhouse gas emissions, or to wait for more knowledge and perhaps obtain a lower degree of uncertainty is frequently discussed in climate policy, which also embeds many other issues raised in investment theory. Once installed, abatement capital, such as solar or wind power plants, cannot be utilised for alternative purposes, and investments in these kinds of technologies has thereby got a certain degree of irreversibity. In addition, substantial costs may be related to the implementation of the policy itself. Administration of indirect measures, such as targets or markets for emissions quotas, are costly. In addition, some measures will have to be carried out as public investments, thereby carrying with them a certain cost of public funds, costs that can be interpreted as adjustment costs

In this paper climate policy is considered as an investment decision that may be both irreversible and subject to adjustment costs. We integrate the unifying approach to investment decisions proposed by Abel and Eberly [4] in a Ramsey type of model for optimal growth, which includes feed-backs from the climate. Uncertainty in abatement costs (or effectiveness of abatement measures) and uncertainty in the impacts of climate change are studied separately.

2 Adjustment costs and irreversibility in climate policy

In neo-classical theory, investments is a mere derivation of the demand for capital in excess of the present stock. Investments is thereby determined by the price of capital. Jorgenson [9] pointed out that firms cannot instantaneously adjust its capital stock without additional costs. Eisner and Strotz [7] showed that frictions in the demand for capital may lead decision makers to give priority to old technologies over new ones, because new technologies implies additional costs related inter alia to installation, search and training. Therefore, a new technology will not be preferred to an old one unless the prospects for the new technology is strictly brighter than for the old. According to this theory, the demand for capital cannot be modelled as a simple derivation from the production function and the price of capital. There are certain resources foregone simply as a result of the investment.

Adjustment costs may clearly be significant for investments in climate measures. Maccini [11] classifies adjustment costs into internal and external costs. Internal adjustment costs include expenditures related to installation, training, and reseach and development. These may be substantial for countries that invest in climate measures. Over the past decade, emissions of CO_2 have become subject to great political concern, and is only beginning to represent a cost in economic activities. The economic incentives for searching alternative technologies with lower emissions of CO_2 are either recently implemented or not yet feasible. The need for research is partly reflected by the emphasis on climate research in industrialised countries.

Also private industries are increasing their engagement in the search for technologies that may reduce the emissions of CO_2. An implementation of new technologies requries training and will thereby add to the costs. Hence, CO_2 measures include the same internal adjustment costs as described in textbooks, and they represent expenditures that may become significant also in a national context. For example, bottom-up studies based on engineer's knowledge of the costs and the potentials for new technologies often point out that there may be substantial no-regret options to reduce CO_2 emissions. The startling question of why seemingly profitable investments are not undertaken may partly be explained by adjustment costs. In the case of climate policy, also governmental expenditures, such as the establishment of monitoring and control authorities, could be considered as resources foregone in the efforts to reduce emissions.

Most economic studies of climate change focus on reductions in CO_2 emissions. The Kyoto agreement adds, however, five gases, which are expected to contribute to approximately 30 percent of greenhouse gas emissions in the future, if the contribution from emissions are calculated over 100 years. Efforts to find alternatives to these emissions are picking up, and in some cases the propects for reducing the emissions of non-CO_2 gases at low costs with new technologies seem to be promising. Although investments in abatement of these gases are lower than for CO_2, the options may be important for single investors. Emissions of these gases have, however, been a free good till now, and a significant research in finding appropriate technological solutions remains to be done. The adjustment costs are therefore indisputable.

External adjustment costs accrue in cases where firms are monopsonists in the capital market. The external adjustment costs cannot therefore be measured directly, but represent certain market effects of the investments. It is difficult to say whether external adjustment costs will occur as a result of investments in climate policy. However, external costs may accrue if the investments are undertaken by public authorities, for instance as costs of public funds. In a number of cases, an implementation of climate policy may involve public investments, especially for reductions in other gases than CO_2. The emissions of these gases are seldom related directly to market transactions, such as for CO_2 where the bulk of emissions is attached to the use of fossil fuels. Hence, direct measures, or public investments, may apply, but imply certain additional costs if financed by public funds. In the present study, we analyse the effects of adjustment costs in a general context, and leave to later studies to provide realistic estimates of such costs.

Another property of investments, which distinguishes them from intermediate input in the production of goods and services, is that they represent sunk costs to some extent. Investment decisions cannot therefore be based on past and present evidence only, but must build on the expectations of the investors. Because of the uncertainty about future costs and incomes, one will have to consider the possibility that the investment turns out more or less beneficial than expected. While the possibility of higher profits than expected seldom (but sometimes) causes great problems, the possibility of a loss may be disasterous. The investor runs the risk of getting stuck with a huge capital cost and a low value of the capital stock.

A large share of the costs spent on climate measures involve investments with different degrees of irreversibility. Investments in new technologies with the only motive to cut emissions, cannot be replaced without costs if climate change turns out to be less devastating than expected. The vast uncertainties related to climate change need no further documentation. The possibility that the value of invested capital in abatement will be lower than expected is therefore significant.

In this study, all the reductions of CO_2 emissions are a result of investments. This is of course a simplification of the real life. For example, enhancement of sinks may be restricted to just let forests grow, a decision that may be redone immediately if one regrets. A tax on emissions of greenhouse gases may lead to adjustments in behaviour without major investments. Individuals may adjust back costlessly if the tax is withdrawn.

3 Optimal investments in climate measures

Adjustment costs and irreversibility have been analysed within two traditions of investment theory. Yoshikawa [19], Hayashi [8] and Abel [3] discuss the consequences of adjustment costs in a macroeconomic setting, and show that adjustment costs provide a rigorous basis for Tobin's so-called q-theory of capital [17]. McDonald and Siegel [12] and Pindyck [16] were the first to show how sensitive the investment decision is to uncertainty when subject to complete irreversibility. They base their results on the theory of optimal stopping, and Dixit and Pindyck [6] give a thorough presentation of different applications of the theories. Lucas and Prescott [10] include the irreversibility aspect in a model with adjustment costs. However, the first attempt to unify the two aspects were provided by Abel and Eberly [4], who analyse irreversible investments with adjustment costs in relation to the value of capital (Tobin's q). This allows the investment decision to be included in a set of macroeconomic relations, which is the track I follow in this paper.

The macroeconomic model is basically a version of the Ramsey-model which optimises the welfare of future consumption under given constraints on the state-variables. Define the value function

$$V(k_1, k_2, S, t) = \max_{x,y} E \int_t^{\infty} u(x_s) e^{-\delta s} ds. \qquad (1)$$

k_1 denotes the stock of real capital used for production of goods and services (productive capital), k_2 is the stock of abatement capital, and S is an indicator for the state of the environment. Subscripts for time are omitted for convenience. For instance, S might denote the level of concentrations of greenhouse gases in the atmosphere above the natural, or stable, level. This is usually defined as the concentrations in parts per million (ppmv) above the level from pre-industrial times. $u(x_s)$ is the atemporal utility of consumption level x at time s, and δ is the consumers' pure rate of impatience. To achieve an optimal solution, the decision makers control the consumption level and the cost of investing in abatement, y. At $t = 0$, k_1, k_2 and S are known to the decision maker.

The stock of productive capital develops according to the relation between production and allocation of the net national product in a closed economy,

$$dk_{1t} = [f(k_{1t}; S_t) - x_t - y_t]dt: \tag{2}$$

The net national product, $f(k_{1t}; S_t)$; is a function of the stock of productive capital and the concentrations of greenhouse gases. Hence, $f'_S < 0$, can be interpreted as the marginal damage cost of climate change.

(2) implies that investments in productive capital is modelled in the standard neo-classical way, with no adjustment costs. For abatement measures, we assume that adjustment costs are present. Investments in abatement capital add to the stock of abatement capital and depreciates by rate γ. Hence,

$$dk_{2t} = (i_t - \gamma k_{2t})dt$$

where i denotes the added stock of abatement capital. To add i to the capital stock, the investor will have to pay adjustment costs. The adjustment cost function proposed by Abel and Eberly [4] is applied here. It contains a constant term, C; which accrues if the investments in abatement is non-zero at t, but is zero if investments are zero. In other words, since one cannot decide an infinitely small investment without paying the constant amount C, there is a lower limit for the investment capital to be installed at each point in time.

In addition to the constant term, some adjustment costs also depend on the volume of investment, i, and on the capital stock at t. Hence, total abatement costs at t, if investments are non-zero, can be written as $y_t = c(i_t; k_{2t}; C)$. The dependency of the volume of new investments is related to the utilisation of the capacity of the economy at the time of investments. We therefore assume that $c'_i > 0$, and $c''_{ii} > 0$, while $c(i_t; k_{2t}; C)$ may either be increasing or decreasing in k_{2t}.

If there is no investments in abatement capital, all costs are zero. The fixed term C causes a discontinuity in the cost function at $i = 0$, which can be represented in the abatement cost function by a dummy, ν_t, where $\nu_t = 0$ when $i_t = 0$ and $v_t = 1$ else. Adjustment cost can thereby be written as

$$y_t = \nu_t c(i_t; k_{2t}; C); \tag{3}$$

which is called the augmented cost function.

Note that in order to have positive investments, marginal costs will have to exceed average costs, i.e. $c'_i > \hat{c}$. In principle, the investments may be both positive and negative. If the demand for new equipment is strictly positive, $i_t > 0$, the value of abatement capital needs to exceed the minimum price required to cover the average total investment costs, $\hat{c}$. Denote the price of purchasing abatement capital by p^+. Because of the uncertainty, it may be optimal to sell parts of the installed capital stock if climate change turns out to be less severe than expected, that is $i_t < 0$. The price obtained for previously installed capital in the second hand market, p^- cannot exceed the price of new capital. Hence, we must have $p^+ \geq p^-$.

We may interprete the difference between p^+ and p^- as the degree of irreversibility , since it represents the immediate loss imposed on the investor if the value of the abatement capital turns out lower than expected. The value of equipment in the second hand market will usually be positive, even for abatement capital. To keep the structure of the model as simple as possible, however, we assume that abatement capital is completely irreversible, that is, $p_t^i = 0, \forall t$.[1] Irreversibility thereby represents a second discrete element in the abatement cost function. Since we limit this study to the case of total irreversibility, we may include this aspect by redefining the dummy ν_t, such that $\nu_t = 0$ when $i_t \leq 0$ and $\nu_t = 1$ else.

The accumulation of abatement capital, $dk_{2t} = i_t - \gamma k_{2t}$ can be found by inverting the augmented adjustment cost function (3). We write $i = \Phi(\nu_t, y_t, k_{2t})$. The signs of the derivatives of $\Phi(\nu_t, y_t, k_{2t})$ follows from the assumptions about the shape of $c(i_t, k_{2t}, C)$, that is, $\Phi_y' > 0$ and $\Phi_{yy}'' < 0$ when $y > 0$, and zero when $y = 0$. Φ_2' depends on whether there are economies of scale in abatement capital. The restrictions on the augmented adjustment cost function implies that $\Phi(\nu_t, y_t, k_{2t}) = 0$ when $\nu = 0$. We assume that abatement capital evolves according to

$$dk_{2t} = \Phi(\nu_t, y_t, k_{2t})dt. \tag{4}$$

For simplicity, we assume that greenhouse gases accumulate in atmosphere with a constant rate of decay equal to α.[2] S_t evolves according to a stochastic process,

$$dS_t = [\alpha S_t + g(k_{1t}, k_{2t})]dt + \sigma_S S_t dz_t. \tag{5}$$

α is the rate of decay, which expresses the lifetime for the greenhouse gas in the atmosphere ($-1 < \alpha < 0$). For gases with long life-times, α is close to 0, which means that anthropogenic emissions accumulate in the atmosphere. In the case of CO_2, the natural decay of concentrations depends on the carbon cycles, and the assumption of a constant decay rate is a very rough one, indeed. It will, however, be sustained because we want to focus primarily on the investment decision. $g(k_{1t}, k_{2t})$ is the anthropogenic emissions, generated by economic activities. We assume that $g_1' > 0$, and $g_2' < 0$. The stochastic term dz_t is normally distributed, with $E(dz_t) = 0$, and σ_S is an expression for the standard deviation of the stochastic process. By its direct contribution to damages in the production function, the stochastic evolution of S_t can be interpreted as uncertainty in the damage costs.

Denote by X_i state variable i $(= k_1, k_2, S)$. Then, the Hamilton-Jacoby-Bellman equation is:

$$0 = \max_{x,y,\nu} E\{[u(x_t)e^{-\delta t} + V_t']dt + \sum_i V_i' dX_i + \frac{1}{2}\sum_i \sum_j V_{ij}'' dX_i dX_j\}.$$

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[1] The model applied here could be extended to include the case of partly reversible investments by setting an exogenous p^i which could be thought of as a price of scrapped equipment on the world market, and include the value of sold equipment in the national product.

[2] S is an indicator for the potential for damages of climate change, which is closely related to the concentrations of CO_2 in the atmosphere. The decay of CO_2 concentrations depends on the cabon cycles, which cannot properly be represented by constant decay rate. A constant decay rate for all emissions must therefore be taken as a very rough approximation.

In the last term, we have $Edk_1^2 = Edk_2^2 = Edk_1dk_2 = Edk_1dS = Edk_2dS = 0$. By Ito's formulae, $EdS^2 = (\sigma_S S)^2dt$. Denote by V_i' the derivative of the value function with respect to state variable i. To simplify notation, we write V_1' for the derivative with respect to k_1, and V_2' for the derivative with respect to k_2. Insert this, (2), (4) and (5) into the Hamilton-Jacoby-Bellman equation, and take expectations. Then, we obtain:

$$\begin{aligned} 0 &= \max_{x,y,\gamma} E\{[u(x)e^{-\delta t} + V_t'] + V_1'[f(k_1,S) - x - y] \\ &\quad + V_2'\psi(\gamma, y, k_2) + V_S'[\alpha S + g(k_1,k_2)] + \frac{(\sigma_S S)^2}{2}V_{SS}'', \end{aligned} \quad (6)$$

where we have dropped the time indicator, t. Maximisation of (6) with respect to x and y gives:

$$u_x'e^{-\delta t} = V_1', \quad (7)$$

and

$$V_2'\psi_y' = V_1'. \quad (8)$$

The partial derivatives of $V(k_1, k_2, S, t)$ express the marginal contribution, or the shadow prices, of the state variables to the value of the system at t. (7) states that the marginal contribution from productive capital is equal to the marginal value of consumption at all points in time. ψ_y' measures how much abatement capital will be installed on the margin by a dollar invested in abatement. In other words, (8) is the familiar first order condition, that if it is decided to make investments in abatement, a dollar spent on either capital type has to yield the same social benefit on the margin. The decision maker equates the marginal cost of adding abatement capital and the social price of abatement capital V_1'/V_2'. This corresponds to Tobin's q-theory of capital, and as Abel and Eberly [4] point out, the investment decision is thereby made without directly considering the uncertainty. The uncertainty is, however, reflected in the price of abatement capital, which is subject to market conditions.

Because of the shift parameter γ_t, also the capital installation function $\psi(\gamma_t, y_t, k_{2t})$ is discontinuous at $y = 0$. The decision maker will have to make a "discrete choice" as to whether to invest in abatement capital or not. Since new abatement capital must be expected to give a positive contribution to the system, (8) applies if and only if the expected return of new investments is positive. To assure optimality, the value of new abatement capital therefore has to be at least as high as the average cost of investments. To detemine the value of the shift parameter, define $\bar{y}$ as the value of y determined by (8) given k_{2t}, and $\gamma = 1$. Then we have

$$\begin{aligned} \gamma &= 0 \text{ when } q_2 < c/\bar{y} \\ \gamma &= 1 \text{ when } q_2 \geq c/\bar{y} \end{aligned} \quad (9)$$

(9) gives the critical value at which the switch from non-action to action is determined. For a thorough analysis of "the range of inaction", see Abel and Eberly [4] and Dixit and Pindyck [6].

(7), (8) and (9) can be regarded as atemporal conditions. For given prices, the level of consumption and abatement costs can be read directly. The atemporal conditions also give a direct answer to how adjustment costs affect the investment decision. The impact of uncertainty and irreversibility depends on how the system evolves over time, which is determined by the intertemporal conditions: The investment decision at t; given the optimal stock of abatement capital at t are made on the basis of the expectations at t. If a stochastic event occurs before $t + \Delta t$, which contributes to a reduction in the value of k_2, the stock of abatement capital is too large compared with the cases of certainty or perfect foresight, thereby pushing the value of abatement capital down. Irreversibility means that the effect of this event will last for a longer period than if the capital stock can be adjusted immediately.

The intertemporal conditions can be found by a two-step proceedure (see e.g. Pindyck [15]). First, we take the derivative of (6) with respect to each of the state variables, multiply through by dt and insert for the expected evolutions of k_1, k_2 and S:

$$\begin{aligned} 0 \;=\; & [(u'_x x'_1 e^{-\delta t} + V''_{t1})dt + V''_{11}dk_1 + V'_1(f'_1 - x'_1 - y'_1)dt + V'_{21}dk_2 \\ & + V'_2\Phi'_y y'_1 dt + V''_{S1}EdS + V'_S g'_1 dt + \frac{(\sigma_S S)^2}{2}V'''_{SS1}]; \end{aligned}$$

$$\begin{aligned} 0 \;=\; & [(u'_x x'_2 e^{-\delta t} + V''_{t2})dt + V''_{12}dk_1 + V'_1(-x'_2 - y'_2)dt + V'_{22}dk_2 \\ & + V'_2[\Phi'_2 + \Phi'_y y'_2]dt + V''_{S2}EdS + V'_S g'_2 dt + \frac{(\sigma_S S)^2}{2}V'''_{SS2}]; \end{aligned}$$

$$\begin{aligned} 0 \;=\; & [(u'_x x'_S e^{-\delta t} + V''_{tS})dt + V'_{1S}dk_1 + V'_1(f'_S - x'_S - y'_S)dt + V''_{2S}dk_2 \\ & + V'_2\Phi'_y y'_S dt + V''_{SS}EdS + V'_S\alpha dt + \frac{(\sigma_S S)^2}{2}V'''_{SSS} + \sigma^2 S V''_{SS}]: \end{aligned}$$

The second step aims at finding an expression for the transition probabilities for each of these diffusions. Since V'_i ($i = k_1; k_2$ and S) are solutions of a stochastic differential equation, they are themselves stochastic integrals for which the drift and diffusion coefficients are continuous in t (see e.g. Milliaris and Brock [13]). We may therefore assign an infinitesimal operator to each of them. In general terms, the infinitesimal operator is

$$AG(S) = G'_S + G'_t + \frac{1}{2}G''_{SS}dS^2$$

The transition probabilities can be expressed implicitly by the expected value at t of V'_i for some future point in time $t + \Delta t$. According to Dynkin's formulae, the expected value at $t = 0$ for a diffusion at τ can be expressed as

$$EG(S_\tau; \tau) = G(S_0; 0) + E\int_0^\tau AG(S; s)ds$$

If we choose $t = \tau$, the function $u(S;t) = EG(S_{t;}t)$ is differentiable wrt. t, and by Kolmogorov's backward eqation, we have

$$\frac{\partial u}{\partial t} = Au$$

where the right hand side is to be interpreted as A applied on S (Øksendal [14]).

The infinitesimal operator for V_i', Av_i, can be expressed as

$$Av_i = [V_{it}''dt + V_{i1}''dk_1 + V_{i2}''dk_2 + V_{iS}''EdS_t + \frac{(\sigma S)^2}{2}V_{iSS}'''dt] \tag{10}$$

By use of Kolmogorov's backward equation, which implies that $Av_i = \frac{\partial}{\partial t}[V_i']$; and replacing for $V_{it}''dt$ in the partial derivatives of the HJB-eqation above, we get,[3]

$$\frac{\partial}{\partial t}[V_1'] = -[u_x'x_1'e^{-\delta t} + V_1'(f_1' - x_1' - y_1') + V_2'\gamma_y'y_1' + V_S'g_1']; \tag{11}$$

$$\frac{\partial}{\partial t}[V_2'] = -[u_x'x_2'e^{-\delta t} + V_1'(-x_2' - y_2') + V_2'(\gamma_2' + \gamma_y'y_2') + V_S'g_2']; \tag{12}$$

$$\frac{\partial}{\partial t}[V_S'] = -[u_x'x_S'e^{-\delta t} + V_1'(f_S' - x_S' - y_S') + V_2'\gamma_y'y_S' + V_S'\alpha + \sigma^2 SV_{SS}'']. \tag{13}$$

where x_i' and y_i' denotes the partial derivative of the optimal consumption and optimal abatement costs with respect to state variable i.

Finally, replace V_i' by the first order conditions (7) and (8), and divide through by $V_1'; V_2'$; and V_S' respectively. Then the rate of change in the value of the state variables, or the discount rates, can be expressed as:

$$\rho_1 = -\frac{\frac{\partial}{\partial t}[V_1']}{V_1'} = f_1' + \frac{V_S'}{V_1'}g_1'; \tag{14}$$

$$\rho_2 = -\frac{\frac{\partial}{\partial t}[V_2']}{V_2'} = \frac{V_S'}{V_1'}\gamma_y'g_2' + \gamma_2'; \tag{15}$$

$$\rho_S = -\frac{\frac{\partial}{\partial t}[V_S']}{V_S'} = \frac{V_1'}{V_S'}f_S' + \alpha + \sigma^2 S\frac{V_{SS}''}{V_S'}; \tag{16}$$

which are the intertemporal optimum conditions for the values of productive capital, abatement capital and the concentrations of greenhouse gases when there is uncertainty about the damage. They depend on the unknown terms $\frac{V_S'}{V_1'}$

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[3] Note that the term G_t' in the expression for the infinitesimal operator includes the time derivatives of non-stochastic variables (k_1 and k_2), and that the function $u(S;t)$ is not the same as V_{it}'' when applied on V_i'.

and $\frac{V''_{SS}}{V'_S}$. $\frac{V'_S}{V'_1}$ can be interpreted as the relative (shadow) price of greenhouse gas concentrations. $\frac{V''_{SS}}{V'_S}$ is an expression for the curvature of the value function, that is, it expresses how sensitive the optimal solution is to variations in the concentrations of greenhouse gases.

(14) is the social return on productive capital. It contains the direct marginal productivity of capital in production subtracted by the relative value of emissions of greenhouse gases. Hence, the social return on capital is lower than the private return. This may justify e.g. taxation of emissions.

The main component of the return on abatement capital, given by (15), is the value of marginal emission cuts resulting from an additional dollar spent on abatement. The adjustment costs enter this expression by how productive abatement costs are in terms of investments in abatement capital. A small adjustment is also made for possible economies of scale in abatement capital. It should be noted that the return on productive capital and abatement capital are determined by widely different components. Hence, they are generally not equal. The relationship between them depends, inter alia, on the optimal path for investments in abatement determined by the intertemporal equilibrium conditions developed below.

The only term in which uncertainty comes in directly is condition (16), which characterises the path for the shadow value of greenhouse gas concentrations. Note that without uncertainty, ρ_S may be negative since α and f'_S both are negative. By its impact on the path of V'_S, uncertainty affects the value of productive capital and abatement capital indirectly. Since $\frac{V'_S}{V'_1}$ and $\frac{V''_{SS}}{V'_S}$ are unknown, we cannot say in what direction and how much so far. However, we can match the atemporal and the intertemporal optimum conditions in order to find the intertemporal equilibrium conditions under which the optimal solution evolves. According to (7), the derivative of V'_1 with respect to the time indicator is

$$\frac{\partial}{\partial t}[u'_x e^{-\delta t}] = [u''_{xx}\dot{x} - \delta u'_x]e^{-\delta t}.$$

Define $\mu_x = -xu''_{xx}/u'_x$, which is the inverse of the elasticity of substitution of consumption over time. Insert this and divide through by V'_1 to obtain the intertemporal equilibrium condition for the discount rate of productive capital

$$\rho_1 = \mu_x \frac{\dot{x}}{x} + \delta, \qquad (17)$$

which is the familiar Ramsey rule for optimal economic growth. The difference between (17) and the standard rule is only the deduction for the social loss of emissions due to higher concentrations of greenhouse gases included in ρ_1.

In a similar way, we can find the path of the value for abatement capital, or the discount rate for abatement capital, by the derivative of (8) w.r.t. time.

$$\frac{\partial}{\partial t}[V'_2] = \frac{[\frac{\partial}{\partial t}[V'_1]\Phi'_y + V'_1\Phi''_{yy}\dot{y}]}{(\Phi'_y)^2}.$$

Define $¹_y = y©^{00}_{yy}=©^0_y$; which characterises the curvature of the abatement cost function, and divide through by V^0_2. Then,

$$½_2 = ¹_y \frac{y}{y} + ½_1 \qquad (\text{when } º = 1); \tag{18}$$

which leads us closer to an answer of how abatement capital is to be discounted compared with productive capital: The discount rates are equal only if it is optimal to keep the level of abatement costs constant over time. Generally, this is not the case unless it is optimal not to investment in abatement ($º_t = 0 8t$). The result is therefore intuitive: The discount rate for state variables, such as concentrations of greenhouse gases, equals the social economic rate of return only if it is optimal not to invest in climate measures. In other words, to apply the economic rate of return to discount benefits from climate policy is indeed based on marginal considerations.

Another case in which $º_t = 0$ is along an optimal path where abatement is positive at some t, but may be zero because $q_2 < c=\bar{y}$. If so, we cannot take the derivative of (8), because $©^0_y = 0$. The value of abatement capital is, then, determined by the exisiting stock of abatement capital. The evolution of V^0_2 when $º_t = 0$ is found by optimising the system with respect to x, given that $y = 0$, and $k_2 > 0$. Then,

$$½_2 = ©^0_2 k_2 + \frac{V^0_S}{V^0_2} g^0_2 \qquad (\text{when } v = 0): \tag{19}$$

(18) and (19) will generally lead to a discount rate for abatement measures widely different from the economic rate of return. This relates to the optimal timing of action, which is different in the case of traditional economic investments and the case of investments in climate measures. For example, it is shown in Wigly, Richels and Edmonds [18] and Aaheim [2] that the social gain of postponing abatement efforts may be considerable. The reason is that significant damages of climate change will probably not occur for many years. Hence, the gains of aggressive actions are much higher in the future than they are now. Meanwhile, it is better to partly invest in productive capital and partly consume.

It has been held against this argument that future effects of climate change are uncertain. Hence, we do not know whether significant damages will happen in the short, medium or long term. The question is then how to allocate abatement efforts under uncertainty. To answer this question, we have to find a numerical solution of the model.

4 A numerical approximation

To achieve a numerical representation of the model, we have to find approximations of the unknown terms $\frac{V'_S}{V'_1}$ and $\frac{V''_{SS}}{V'_S}$. Recall that from (14) we have:

$$V'_S = \frac{V'_1(\rho_1 - f'_1)}{g'_1}. \tag{20}$$

To approximate $\frac{V''_{SS}}{V'_S}$ we assume that, over a short time interval, V'_S can be approximated by first order, i.e. $V'_S = a + bS$.[4] Then $V''_{SS} = b$. From (14) we can assign the following values to the parameters: $a = \frac{\rho_1 V'_1}{g'_1}$, and $b = -\frac{f'_1 V'_1}{g'_1 S}$. Given that these fractions can be regarded as constant over the interval $(t, t + \Delta t)$, we can write

$$\frac{V''_{SS}}{V'_S} \approx \frac{1}{(1 - \frac{\rho_1}{f'_1})S} \tag{21}$$

Inserting (21) into (16) we can write the discount rate for the value of greenhouse gas concentrations as

$$\rho_S = \frac{V'_1}{V'_S} f'_S + \alpha + \frac{\sigma^2}{1 - \frac{\rho_1}{f'_1}}$$

The model has two control variables and three state variables. We may eliminate ρ_S from the system by requiring that in intertemporal equilibrium, the time derviative V'_S in (20) has to be equal to ρ_S. The time-derivative of the right hand side of (20) divided through by V'_S yields

$$\rho_S = \rho_1 + \frac{\dot{\rho}_1 - [f''_{1t} + f''_{11}\dot{k}_1 + f''_{1S}\dot{S}]}{\rho_1 - f'_1} - \frac{g''_{11}}{g'_1}\dot{k}_1. \tag{22}$$

To simplify expressions for the numerical calculations, we assume that $\dot{\rho}_1 = \frac{\ddot{x}}{x} - (\frac{\dot{x}}{x})^2$ is negligible. Then, equating the right hand sides of (16) and (22) we find

$$\frac{V'_S}{V'_1} \approx \frac{f'_S - \frac{A}{g'_1}}{\rho_1 + \frac{g''_{11}}{g'_1}\dot{k}_1 - (\alpha + \frac{\sigma^2}{1 - \frac{\rho_1}{f'_1}})} \tag{23}$$

where

$$A = f''_{1t} + f''_{11}\dot{k}_1 + f''_{1S}\dot{S}.$$

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[4] From the expression for V'_S we see that there are alternatives to this. We have tried to make the assumption about constant a and b as correct as possible. The numerical calculations indicate an error in the interval -4 to 1 percent, clearly biased towards underestimating V'_S.

For simplicity we have assumed that the effect on emissions of abatement is independent on the level of production, e.g. additive, such that $g''_{12} = 0$ (end of pipe -solutions).

(14), (15), (23), (17) and (18) or (19) allow us to solve the variables $\rho_1, \rho_2, \frac{V'_1}{V'_S}, \frac{\dot{x}}{x}$ and $\frac{\dot{y}}{y}$. Thereby, we have sufficient conditions to solve the evolution of the optimal solution over time, or the intertemporal equilibrium. Adding the atemporal optimum conditions, we can assess the optimal control. To do a numerical analysis, we need initial and terminal conditions on the state variables. The initial values of productive capital, abatement capital and greenhouse gas concentrations are assumed to be known. For the terminal period, T, we apply a weak form of strong sustainability (see Asheim, [5] and Aaheim, [1]) requiring that

$$V'_{10}k_{10} + V'_{20}k_{20} = V'_{1T}k_{1T} + V'_{2T}k_{2T}$$

and

$$V'_{S0}S_0 = V'_{ST}S_T$$

The two conditions imply that we do not allow the present value of the capital stock to be lower in the terminal period than the initial value. The same applies for the value of concentrations of greenhouse gases. This limits the possibility to substitute between real capital and greenhouse gas concentrations.

The system evolves according to a stochastic process, which means that the level of the shadow prices, especially on concentrations, change as new information is gained. The two conditions above is therefore determined the first year, and the physical end points are sustained after the first year. This condition of sustainability is rather strong, and requires that irrespective of how the world develops, the sustainability criterion refers to one fixed year ($t = 0$). An alternative could be to relate the value of each component of wealth (real capital and greenhouse gas concentrations) to the year in which the decisions are made. Note also that the condition is somewhat artificial, since it does not involve any evaluation of the present. To evaluate the present in the light of sustainability is, however, beyond the scope of this paper.

The numerical model finds an optimal decision at each point in time. The optimality conditions developed above are based on continuous time, which means that rate of the change of each variable from t to $t + \Delta t$ has to be approximated. For this purpose, I have used a 2nd order Taylor expansion for each state, and for each control variable. The optimal solution follows from inital choices of consumption, x, and price of abatement capital, q_2, at each starting year, for example $t = 0$; which is the first year a decision is made. The optimal levels of x and q_2 at $t = 0$ are determined by means of the end points, where sustainability is required, and is based on the information available at $t = 0$. The same procedure is followed the next year when new information has arrived, i.e. when dz_0 is known. Under full certainty, the decisions for years after $t = 0$ coincide with the optimal paths evaluated at $t = 0$, since no new information occurs later. This can be called the perfect foresight solution.

Figure 1: Initial and terminal figures in four alternatives

Without perfect foresight, a stochastic event occurs between t and $t + \Delta t$. For example, when damages are uncertain, $S_{t+\Delta t}$ turns out different from what was expected at t. Adjusting for this new information, we then have to develop new paths for x and y in order to decide $x_{t+\Delta t}$ and $y_{t+\Delta t}$.

5 Results

The chocie of functionals of the model are shown in the appendix. The welfare function is log-linear, with factor equal to 0.125, and a time correction factor (rate of impatience) equal og 0.02. The model was calibrated with reference to data for Norway. The production function is a Cobb-Douglas function with real capital as only input. It was calibrated on the basis of a real return of 6 percent. Damage from climate change are assumed quadratic with a production loss of 2 percent of GNP at $2 \times CO_2$. The abatement cost function is also on the Cobb-Douglas form of installed equipment and stock of abatement capital, except that a constant term is added. The parameter for installation was set to 1.2, and -0.005 for the stock. The constant term is 0.25, i.e. 250 mill NOK in the base year.

The life-time of CO_2; α, was set to 1/120. The emissions coefficient were calibrated using emissions of CO_2 in Norway, while the parameteres for emission cuts were based on the assumption that the cost of 10 percent reduction of emissions amounts to 0.9 percent of GNP, while 30 percent reductions amounts to 3 percent of GNP. Note that adjustment costs add to these. The link between Norwegian emissions and concentrations of greenhouse gases are based on equal percentage cuts all over the world (see Aaheim [1] for a closer discussion).

The model were run over a period of 50 years, and table 1, column 1 shows the results of the main variables of interest in the reference case under full certainty. The optimal consumption level is 469 in year 0 and rises at a rate of approximately 2.5 percent per year to 1318 in year 51. Abatement starts at 6.37 bill NOK and rises to 415 bill NOK in year 50. This means that emissions

Figure 2: Abatement decisions with stochastic damage

are approximately zero in year 51, and concentrations rises from 356 to 464. The price of abatement capital rises relative to the price of real productive capital over the entire period, which is reflected by the discount rates. For real capital the discount rate starts at 5 percent an falls to 3.75 percent in year 51. For abatement capital it starts at 2 percent and rises to nearly 3 percent. Hence, abatement is gradually more emphasised over the entire period, making investments in abatement exponentially increasing.

Uncertainty was introduced by setting $\sigma_S = 0.02$. Roughly speaking, this implies that the standard deviation of annual correction of the concentrations level due to new information amounts to 2 ppmv. Column 2 to 4 in table 1 shows results for the expected optimal paths used as the basis for the decision at $t = 0$. The initial level of optimal abatement costs in the uncertain reference case falls substantially relative to the case of certainty (column 2 in table 1), starting at 1.88 bill NOK. It also implies a higher initial consumption level, but a somewhat slower growth. The element of postponing abatement is stronger under uncertainty, and the abatement costs in year 51 is higher than under certainty. Still, total abatement for the entire period is lower, and the expected level of concentration in year 51 is 506 ppmv; more than 40 ppmv higher than under certainty. The change of the abatement profile is reflected in the paths for the price of abatement capital, which starts at 1.30, and ends up at 3.21 relative to the price of real capital. Under certainty the figures are 1.62 and 3.16.

How sensitive the decision at $t = 0$ is for changes in abatement costs is also shown in table 1. Limitation of abatement options can be illustrated by a more rapidly increasing abatement cost curve. The result is that abatement shifts

Figure 3: Evolution of states

towards earier action, in order to level out differences in abatement costs over time. Hence, the initial price of abatement capital is higher at t = 0, than in the uncertain reference case. It is also somewhat higher at t = 51, but has fallen at a higher rate, and the expected concentrations are higher, 520, vs. 506 ppmv. The sensitivity to adjustment costs mainly results in a shift from abatement to consumption when adjustment costs increases. The exception is an increase in the constant term. Increasing the constant term from 250 mill NOK to 800 mill NOK resulted in a substantial postponement of abatement. With the exception of year 9, the price of abatement capital is below the lower limit the first 12 years, leaving the decision maker in the range of inaction. When abatement picks up, the activity is relatively high, starting at 5 and ends at 449 bill NOK in year 51. As a consequence, the concentrations are 520 in t = 51, which not much higher than in the uncertain reference case.

These results indicate that taking adjustment costs into account may change the profile of abatement rather much, but that the level of abatment is more dependent on uncertainty than on the size of the adjustment costs. The question remains as to what extent irreversibility may affect the solution. To see this, we have to develop a stochastic path for concentrations and compare it with the case of perfect foresight, on which the results in table 1, applicable for the decision at t = 0, are based.

Figures 2 to 4 show the evolution of states, optimal abatement and the relation between state and price of abatement capital in the uncertain reference case, and two alternative stochastical evolutions of states, alternatives A and B. The two alternatives differ only with respect to the stochastic events that

Figure 4: Value of abatement capital with stochastic damage

occur at each point in time. The characteristics of the solution can be traced after a few number of years, and we have therefore limited the time series to 10 years with decision making. The terminal year of all the decisions is, however, year 52 (2047). In both the stochastic cases, the damages turn out more serious than expected at $t = 0$; but in alt 9 the damages are gradually downgraded, and assumed lower than expected towards the end. To the extent that irrevesibility matters, the overinvestments in previous periods should then lead to a reduction in the price of abatement capital. However, as seen from figure 3, the investments follow the evolution of states remarkably well. A slight indication of irreversibitily can be found in years 7, when the concentrations are a little higher in the stochastic alt. 9 than in the perfect foresight case, but abatement costs are exactly the same. Another illustration of the possible effect of irreversibility can be found by comparing concentrations and q_2. see figure 4. A high effect of irreversibility would result in a substantially lower q_2 for the same year with approximately the same level of concentrations if there had been overinvestments in earlier years. However, this effect is hard to discover in figure 4. If concentrations are higher, the value of abatement capital is generally higher, and previous investments seem to have very little influence. This conforms with other studies of irreversibility, see e.g. Kolstad (1994) and Brekke and Lystad (1999).

6 Conclusions

Investments in the abatement of emissions of greenhouse gases are subject to extreme uncertainty, and may require substantial adjustment costs in order to be initiated. Optimal investments is characterised by the level of investments at each point in time, and by the investment profile over time. This paper studies the possible impact of uncertainty and adjustment costs on these characteristics of investments. The model applied in the paper leads to substantially lower investments as response to an increase of the uncertainty. This conclusions may be model specific and is explained by the way intertemporal welfare changes according to different outcomes of climate damage under optimal policy.

Adjustment costs do not affect the level of abatement substantially, but may be important for the investment profile. It was found that higher acceleration of costs as abatement increases advances abatement efforts. The total costs may turn out higher, but the amount of emission cuts goes down. High fixed costs makes it worthwhile to wait with an initiation of abatement. However, the level of abatement is somewhat higher than in the case of lower fixed costs, and hence, and the concentrations thereby turn out approximately the same.

Several studies have examined the role of irreversible costs with the use of alternative models. The present model uses a new model based on Abel and Eberly's (1994) approach, which unifies the analysis of abatement costs and irreversibility. The impact of irrevesibility on the abatement decisions was found to be very small. The same conclusion is, in general, found also in other studies.

Acknowledgement 1 Many thanks to Kjell Arne Brekke for critical remarks and constuctive suggestions. Also thanks to Asbjørn Torvanger for valuable comments.

A APPENDIX: Choice of functions

Social intertemporal welfare according to ordinary logaritmic stationary welfare:

$$V = \max E \int_t^{\infty} Ax^{\alpha} e^{-\delta s} ds$$

where $A > 0, 0 < \alpha < 1, \delta > 0.$ The choices in the reference case are ($A = 1, \alpha = 0.15, \delta = 0.02$)

The production function is Cobb-Douglas , with damage defined with reference to rate of damage, d, at $2 \times CO_2$ (560 ppm), with exponent ε. Rate of technological advancement is γ

$$f(k_{1t}, S_t, t) = B(1 - d(\frac{|S_t - 280|}{280})^{\varepsilon})e^{\gamma t} k_1^{\beta}$$

where $B > 0, d > 0, \varepsilon > 1, \gamma > 0, 0 < \beta < 1.$ The choices in the reference case are ($B = 34,3745, d = 0.02, \varepsilon = 2, \gamma = 0.015, \beta = .386254$).

The adjustment cost for investments in abatement capital is

$$c(i_t, k_{2t}) = i_t^{\mu_1} k_{2t}^{\mu_2} + C$$

where $\mu_1 \geq 1, \mu_2 \leq 0, C \geq 0$ (there may be arguments also for $\mu_2 > 0$). The choices in the reference cases are ($\mu_1 = 1.3, \mu_2 = -0.001, C = 0.5$).

Emissions converge emissions from production directly to concentrations of CO_2 :

$$g(k_{1t}, k_{2t}) = G_1 k_{1t}^{\beta} - G_2 k_{2t}^{\kappa}$$

where $G_1 > 0, G_2 > 0, 0 < \kappa \leq 1.$ The choices in the reference case were $G_1 = xx, G_2 = 0.02843, \kappa = 0.6826.$

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