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ENERGY PRICING MODELS

First published in 2014 by
PALGRAVE MACMILLAN®
in the United States—a division of St. Martins Press LLC,
175 Fifth Avenue, New York, NY 10010.

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Palgrave Macmillan is the global academic imprint of the above companies and has companies and representatives throughout the world.

ISBN: 978–1–137–37734–0

Library of Congress Cataloging-in-Publication Data

Energy pricing models : recent advances, methods, and tools / edited by Marcel Prokopczuk.
pages cm.
Includes bibliographical references and index.
ISBN 978–1–137–37734–0 (hardcover :alk. paper)
1. Energy industries–Prices. 2. Stochastic models. I. Prokopczuk, Marcel, editor.

HD9502.A2E54944 2014
333.79—dc23 2014022207

A catalogue record of the book is available from the British Library.

Design by Newgen Knowledge Works (P) Ltd., Chennai, India.

First edition: December 2014

10 9 8 7 6 5 4 3 2 1

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Efficient Pricing of Energy Derivatives

Anders B. Trolle

1 Introduction

In order to price, hedge, and risk-manage energy derivatives, it is critical to understand the dynamics of volatility in energy markets. Even a cursory look at the data shows that volatility is stochastic. For instance, over the past decade, the volatility implied from at-the-money (ATM) options on the front-month crude oil futures contract has varied between 13 percent (at the end of 2013) and 110 percent (at the end of 2008).[1] It is less obvious as to what extent volatility risk can be hedged by trading in the commodities themselves or, more generally, their associated futures, forward or swap contracts. In a comprehensive analysis of crude oil data, Trolle and Schwartz (2009) show that a significant component of volatility implied from options on futures contracts cannot be hedged by trading in the futures contracts themselves. In other words, a significant component of volatility is *"unspanned"* by the term structure of futures prices. It appears that unspanned stochastic volatility (USV) is also an important feature of other energy commodities.

In this chapter, I present a tractable framework, first developed in Trolle and Schwartz (2009), for pricing energy derivatives in the presence of USV. The model has several attractive features: First, it ensures a perfect fit to the initial futures term structure. Second, it has a fast and accurate Fourier-based pricing formula for European-style options on futures contracts, enabling efficient calibration to liquid plain-vanilla exchange-traded derivatives. Third, by specifying shocks to the futures term structure judiciously, the evolution of the futures curve can be described in terms of a low-dimensional affine state vector. This makes the model ideally suited for pricing complex energy derivatives and real options by simulation, where early exercise features can be handled using the least squares Monte Carlo (LSM) approach of Longstaff and Schwartz (2001); see, for example, Schwartz and Trolle (2010) for a real option application of the model.

Another source of market incompleteness is discontinuous moves in spot prices. For instance, Askari and Krichene (2008) and Larsson and Nossman (2011) find that jumps—in addition to stochastic volatility— is an important characteristic of crude oil prices. At the end of the chapter, I outline an extension of the framework that takes jumps in spot prices into account. The extended model retains the key attributes of the basic USV model.

Throughout the chapter, I focus on the risk-neutral dynamics of the model and efficient pricing of derivatives. Through a change of measure, one can obtain the actual/physical dynamics of the model, which would be relevant for risk-management applications. The change of measure also provides information on risk premia associated with volatility risk. I refer to Trolle and Schwartz (2009) for more discussion of these issues and to Trolle and Schwartz (2010) for an in-depth analysis of volatility risk premia in energy markets.

The model is based on the HJM framework of Heath et al. (1992). Other papers that rely on the HJM framework for modeling commodity derivatives include Cortazar and Schwartz (1994), Miltersen and Schwartz (1998), Crosby (2008), and Andersen (2010). Crosby (2008) considers jumps, while Andersen (2010) considers stochastic volatility.

An alternative approach to modeling commodity derivatives relies on specifying the (typically affine) dynamics of a limited set of state variables and deriving futures prices endogenously. Examples of this approach include Gibson and Schwartz (1990), Schwartz (1997), Schwartz and Smith (2000), and Casassus and Collin-Dufresne (2005). One drawback of this modeling approach is that it is very difficult to generate USV, because volatility is almost invariably completely spanned by the futures term structure.[2] In contrast, in the HJM modeling approach, USV arises naturally.

The chapter is structured as follows: Section 2 lays out notation and explains different modeling approaches. Section 3 considers a basic HJM model. Section 4 extends the model with USV. Section 5 describes the pricing of options on futures contracts. Section 6 shows the effect of model parameters on volatilities implied from options on futures contracts. Section 7 extends the model with jumps in spot prices. Section 8 concludes the chapter.

2 Preliminaries

Throughout the chapter, I work under the risk-neutral measure. Furthermore, I assume that interest rates are deterministic, which is fairly innocuous when pricing energy derivatives with short and intermediate maturities.[3] Let $S(t)$ denote the time-t spot price of the commodity and let $F(t, T)$ denote the time-t price of a futures contract maturing at time T. In commodity markets, the relation between spot and futures prices is determined by the cost of carry, which reflects interest rates, storage costs, as well as convenience yields associated with holding the physical commodity instead of a contract for future delivery of the same commodity. In the case of a constant continuously compounded cost of carry rate, δ, the relation between spot and futures prices is simply

$$F(t, T) = S(t)e^{\delta(T-t)}. \tag{1}$$

In the absence of arbitrage opportunities, futures prices are martingales under the risk-neutral measure (see, e.g., Duffie [2001]) from which it follows that $\frac{1}{dt}E_t\left[\frac{dS(t)}{S(t)}\right] = \delta$. More generally, I let the cost of carry vary stochastically, reflecting stochastic variation in convenience yields. Let $\delta(t)$ denote the time-t instantaneous spot cost of carry rate. Furthermore, let $y(t, T)$ denote the time-t instantaneous forward cost of carry rate at time T, defined such that futures prices are given by

$$F(t, T) = S(t) \exp\left\{\int_t^T y(t, u)du\right\}. \tag{2}$$

In the limit as $T \to t$, $y(t, t) = \delta(t)$.[4] It follows that the term structure of forward cost of carry rates can be inferred from the term structure of futures prices.

One strand of the commodity derivatives literature specifies the dynamics of $S(t)$ and $\delta(t)$ and derives futures prices endogenously. Another strand takes futures prices as given and specifies the dynamics of the entire futures curve, which is equivalent to specifying the dynamics of $S(t)$ and the entire forward cost of carry curve. This is the approach taken in this chapter.

3 A Basic HJM Model

I start with a basic HJM model where $S(t)$ and $y(t, T)$ have the following dynamics:

$$\frac{dS(t)}{S(t)} = \delta(t)dt + \sigma_S dW_1(t), \tag{3}$$

$$dy(t,T) = \mu_y(t,T)dt + \sigma_y(t,T)dW_2(t), \tag{4}$$

where $W_1(t)$ and $W_2(t)$ denote Wiener processes under the risk-neutral measure with correlation ρ.

For convenience, introduce the process

$$Y(t,T) = \int_t^T y(t,u)du, \tag{5}$$

the dynamics of which are given by

$$dY(t,T) = \left(-\delta(t) + \int_t^T \mu_y(t,u)du\right)dt + \int_t^T \sigma_y(t,u)du dW_2(t). \tag{6}$$

Then, from Equation (2), $F(t,T)$ is given by

$$F(t,T) = S(t)e^{Y(t,T)} \tag{7}$$

with the following dynamics:

$$\frac{dF(t,T)}{F(t,T)} = \left(\int_t^T \mu_y(t,u)du + \frac{1}{2}\left(\int_t^T \sigma_y(t,u)du\right)^2 + \rho\sigma_S \int_t^T \sigma_y(t,u)du\right)dt + \sigma_S dW_1(t) + \int_t^T \sigma_y(t,u)du dW_2(t). \tag{8}$$

Setting the drift in Equation (8) to zero (futures prices are martingales under the risk-neutral measure) and differentiating w.r.t. T yields

$$\mu_y(t,T) = -\sigma_y(t,T)\left(\rho\sigma_S + \int_t^T \sigma_y(t,u)du\right). \tag{9}$$

This condition on the drift of $y(t,T)$ is similar to the famous HJM drift conditions in term structure modeling. It has the important implication that the drift cannot be specified exogenously but is determined by the diffusion term, $\sigma_y(t, T)$. This is in contrast to traditional models of commodity derivatives, where both the drift and diffusion terms of $\delta(t)$ can be specified independently of each other.

The particular model depends on the choice of $\sigma_y(t, T)$. Throughout the chapter, I consider the following time-homogeneous specification

$$\sigma_y(t, T) = \alpha e^{-\gamma(T-t)}. \tag{10}$$

This choice has two advantages: First, it implies that long-term forward cost of carry rates are less volatile than short-term forward cost of carry rates, which seems intuitive. Second, the evolution of the futures curve can be described in terms of a low-dimensional affine state vector, as I now show. From Equation (9), it follows that the drift $\mu_y(t, T)$ is given by

$$\mu_y(t, T) = -e^{-\gamma(T-t)}\left(\rho\alpha\sigma_S + \frac{\alpha^2}{\gamma}\right) + \frac{\alpha^2}{\gamma}e^{-2\gamma(T-t)}. \tag{11}$$

Integrating Equation (4) and using the fact that $e^{-\gamma(T-u)} = e^{-\gamma(T-t)}e^{-\gamma(t-u)}$, one obtains

$$y(t, T) = y(0, T) + e^{-\gamma(T-t)}x(t) + \frac{\alpha^2}{2\gamma^2}e^{-2\gamma T}\left(e^{2\gamma t} - 1\right), \tag{12}$$

where

$$x(t) = -\int_0^t \left(\rho\alpha\sigma_S + \frac{\alpha^2}{\gamma}\right) e^{-\gamma(t-u)}du + \int_0^t \alpha e^{-\gamma(t-u)}dW_2(u). \tag{13}$$

It follows that $x(t)$ has the mean-reverting dynamics

$$dx(t) = \gamma(\theta - x(t))dt + \alpha dW_2(t), \quad x(0) = 0, \tag{14}$$

where $\theta = -\left(\rho\alpha\sigma_S/\gamma + \alpha^2/\gamma^2\right)$. Finally, from Equation (2) and using the fact that $\frac{F(0,T)}{F(0,t)} = \exp\left\{\int_t^T y(0, u)du\right\}$, futures prices are given by

$$F(t, T) = S(t)\frac{F(0, T)}{F(0, t)}\exp\left\{B(T - t)x(t) + A(t, T)\right\}, \tag{15}$$

where

$$B(T - t) = \frac{1}{\gamma}\left(1 - e^{-\gamma(T-t)}\right) \tag{16}$$

$$A(t,T) = \frac{\alpha^2}{4\gamma^3}\left(1 - e^{2\gamma t}\right)\left(e^{-2\gamma T} - e^{-2\gamma t}\right). \tag{17}$$

This model is the HJM equivalent of the two-factor Gibson and Schwartz (1990) model, in which the dynamics of $S(t)$ are given by Equation (3) and $\delta(t)$ (or, alternatively, the convenience yield) follows a mean-reverting Gaussian process. To see the equivalence, note that $\delta(t)$ is obtained by setting $T = t$ in Equation (12). It is straightforward to show that the dynamics of $\delta(t)$ are given by

$$d\delta(t) = \gamma(\theta_\delta(t) - \delta(t))dt + \alpha dW_2(t), \quad x(0) = 0, \tag{18}$$

with

$$\theta_\delta(t) = \frac{1}{\gamma}\frac{dy(0,t)}{dt} + y(0,t) - \frac{\rho\alpha\sigma_S}{\gamma} - \frac{\alpha^2}{2\gamma^2}\left(1 - e^{-2\gamma t}\right). \tag{19}$$

Therefore, the present model implies dynamics of $S(t)$ and $\delta(t)$ that are similar to Gibson and Schwartz (1990), with the exception that the mean-reversion level is time-dependent, due to the model matching the initial futures curve.

4 Stochastic Volatility

The basic HJM model has constant volatility and is not suited for the pricing of options and other nonlinear derivatives. I now extend the framework with stochastic volatility. The resulting model is equivalent to the SV1 model in Trolle and Schwartz (2009). $S(t)$ and $y(t,T)$ have the following dynamics:

$$\frac{dS(t)}{S(t)} = \delta(t)dt + \sigma_S\sqrt{v(t)}dW_1(t) \tag{20}$$

$$dy(t,T) = \mu_y(t,T)dt + \sigma_y(t,T)\sqrt{v(t)}dW_2(t) \tag{21}$$

$$dv(t) = \kappa(\theta - v(t))dt + \sigma_v\sqrt{v(t)}dW_3(t), \tag{22}$$

where $W_1(t)$, $W_2(t)$, and $W_3(t)$, denote correlated Wiener processes under the risk-neutral measure, with ρ_{12}, ρ_{13}, and ρ_{23} denoting pairwise correlations. This is the most general correlation structure that preserves the tractability of the model.

The model features unspanned stochastic volatility. The dynamics of futures prices are given by

$$\frac{dF(t,T)}{F(t,T)} = \sqrt{v(t)}\left(\sigma_S dW_1(t) + \int_t^T \sigma_y(t,u)du dW_2(t)\right). \tag{23}$$

Volatility of futures prices depends on $v(t)$, but since futures prices are only exposed to $W_1(t)$ and $W_2(t)$, while $v(t)$ is only exposed to $W_3(t)$, it is immediately clear that volatility risk cannot be completely hedged by trading in futures (or spot) contracts. To the extent that $W_1(t)$ and $W_2(t)$ are correlated with $W_3(t)$ (i.e., ρ_{13} and/or ρ_{23} are nonzero), volatility contains a spanned component, and volatility risk is partly hedgeable. If these correlations are both zero, volatility risk is completely unhedgeable.

By going through the same steps as in Section 3, one can derive the condition on $\mu_y(t,T)$ that must hold to ensure absence of arbitrage opportunities. This condition is given by (see also Trolle and Schwartz [2009]):

$$\mu_y(t,T) = -v(t)\sigma_y(t,T)\left(\rho_{12}\sigma_S + \int_t^T \sigma_y(t,u)du\right). \tag{24}$$

To model the dynamic of the futures curve in terms of a low-dimensional affine state vector, I again assume that $\sigma_y(t,T)$ is given by Equation (10). In this case, $y(t,T)$ is given by

$$y(t,T) = y(0,T) + \alpha e^{-\gamma(T-t)}x(t) + \alpha e^{-2\gamma(T-t)}\phi(t), \tag{25}$$

where

$$x(t) = -\int_0^t \left(\rho_{12}\sigma_S + \frac{\alpha}{\gamma}\right) e^{-\gamma(t-u)} v(u)du + \int_0^t e^{-\gamma(t-u)}\sqrt{v(u)}dW_2(u) \tag{26}$$

$$\phi(t) = \int_0^t \frac{\alpha}{\gamma} e^{-2\gamma(t-u)} v(u)du, \tag{27}$$

with the following dynamics:

$$dx(t) = \left(-\gamma x(t) - \left(\frac{\alpha}{\gamma} + \rho_{12}\sigma_S\right) v(t)\right) dt + \sqrt{v(t)}dW_2(t), \quad x(0) = 0 \tag{28}$$

$$d\phi(t) = \left(-2\gamma\phi(t) + \frac{\alpha}{\gamma}v(t)\right)dt, \quad \phi(0) = 0. \tag{29}$$

Consequently, futures prices are given by

$$F(t,T) = S(t)\frac{F(0,T)}{F(0,t)}\exp\{B(T-t)x(t) + C(T-t)\phi(t)\}, \tag{30}$$

where

$$B(T-t) = \frac{\alpha}{\gamma}\left(1 - e^{-\gamma(T-t)}\right), \tag{31}$$

$$C(T-t) = \frac{\alpha}{2\gamma}\left(1 - e^{-2\gamma(T-t)}\right). \tag{32}$$

Obtaining the expression for $\delta(t)$ from Equation (25), the dynamics of the log spot price, $s(t) \equiv \log(S(t))$, are given by

$$ds(t) = \left(y(0,t) + \alpha(x(t) + \phi(t)) - \frac{1}{2}\sigma_S^2 v(t)\right)dt + \sigma_S\sqrt{v(t)}dW_1(t). \tag{33}$$

It follows that futures prices are exponentially affine in $s(t)$, $x(t)$, and $\phi(t)$, which, along with $v(t)$, jointly constitute an affine state vector. Note that $\phi(t)$ is an "auxiliary," locally deterministic, state variable that captures the path information of $v(t)$. By augmenting the state vector with this variable, the model becomes Markovian.

Trolle and Schwartz (2009) consider extensions of the framework with multiple volatility factors. Those models are able to capture the empirical observation that some shocks to volatility are transitory, while others are more persistent.

5 Option Pricing

The pricing of European options on futures contracts is highly tractable. I continue with the case in which $\sigma_y(t,T)$ is given by Equation (10). For most exchange-traded products, options expire slightly before the expiry of the underlying futures contract.[5] Let $\mathcal{C}(t, T_0, T_1, K)$ denote the time-t price of a European call option expiring at time T_0 with strike K on a futures contract expiring at time T_1. Such an option can be priced quasi-analytically within the framework of this chapter. First, the dynamics of the log futures price, $f(t,T_1) \equiv \log(F(t,T_1))$, are given by

$$df(t,T_1) = -\frac{1}{2}\left(\sigma_S^2 + B(T_1-t)^2 + 2\rho_{12}\sigma_S B(T_1-t)\right)v(t)dt$$
$$+ \sqrt{v(t)}\,(\sigma_S dW_1(t) + B(T_1-t)dW_2(t))\,. \tag{34}$$

Next, using standard arguments, one can show that the characteristic function of $f(T_0,T_1)$ defined as

$$\psi(u,t,T_0,T_1) \equiv E_t\left[e^{iuf(T_0,T_1)}\right], \quad i=\sqrt{-1} \tag{35}$$

has the exponentially affine solution

$$\psi(u,t,T_0,T_1) = e^{M(T_0-t)+N(T_0-t)v(t)+iuf(t,T_1)}, \tag{36}$$

where $M(\tau)$ and $N(\tau)$ solve the following system of ordinary differential equations (ODEs)

$$\frac{dM(\tau)}{d\tau} = N(\tau)\kappa\theta \tag{37}$$

$$\frac{dN(\tau)}{d\tau} = N(\tau)\left(-\kappa + iu\sigma_v(\rho_{13}\sigma_S + \rho_{23}B(T_1-T_0+\tau))\right)$$
$$+\frac{1}{2}N(\tau)^2\sigma_v^2 - \frac{1}{2}(u^2+iu)\big(\sigma_S^2 + B(T_1-T_0+\tau)^2$$
$$+ 2\rho_{12}\sigma_S B(T_1-T_0+\tau)\big) \tag{38}$$

subject to the boundary conditions $M(0)=0$ and $N(0)=0$.

Finally, following Carr and Madan (1999), one can show that the Fourier transform of the modified call price

$$\widehat{\mathcal{C}}(t,T_0,T_1,K) = e^{\varphi\log(K)}\mathcal{C}(t,T_0,T_1,K)$$

can be expressed in terms of the characteristic function of $f(T_0,T_1)$.[6] Consequently, the modified call price (and from that the original call price) can be obtained by applying the Fourier inversion theorem. In particular, $\mathcal{C}(t,T_0,T_1,K)$ is given by

$$\mathcal{C}(t,T_0,T_1,K) = P(t,T_0)\frac{e^{-\varphi\log(K)}}{\pi}$$
$$\times\int_0^\infty \mathrm{Re}\left[\frac{e^{-iu\log(K)}\psi(u-(\varphi+1)i,t,T_0,T_1)}{\varphi^2+\varphi-u^2+i(2\varphi+1)u}\right]du,$$

where $P(t, T_0)$ denotes the time-t price of a zero-coupon bond maturing at time T_0.

The pricing approach here differs from the one in Trolle and Schwartz (2009) and has two advantages: First, it permits the use of the computationally efficient fast Fourier transform algorithm. Second, it only requires the evaluation of one integral (as opposed to two integrals).

Note that most exchange-traded options are American, whereas our pricing formula is for European options.[7] With a large number of options, calibration in real time is only feasible in the case of European options, necessitating a conversion of American prices to European prices prior to calibration. Trolle and Schwartz (2009) outline one approach for doing this, and show that it introduces minimal biases, at least for short-to-medium-term options that are at or out of the money.

6 Interpreting Model Parameters

Trolle and Schwartz (2009) conduct an extensive empirical analysis of the model using a panel data set of New York Mercantile Exchange (NYMEX) crude oil derivatives from January 1990 to May 2006 (4,082 business days). Each business day in the sample, they observe a volatility surface implied from options on crude oil futures contracts. Maturities are up to one year, and moneyness—defined as the option strike divided by the price of the underlying futures contract—is between 0.78 and 1.22. Given that the data set contains both cross-sections and time-series information, Trolle and Schwartz (2009) are able to estimate both the risk-neutral dynamics (from the cross-sections) and physical dynamics (from the time series) of the model.[8] Here, I focus on the risk-neutral dynamics, for which they obtain the following parameter values: $\kappa = 1.0125$, $\theta = 0.9877$,[9] $\sigma_v = 2.8051$, $\sigma_S = 0.2289$, $\alpha = 0.1373$, $\gamma = 0.7796$, $\rho_{12} = -0.8797$, $\rho_{31} = -0.0912$, and $\rho_{32} = -0.1128$. Note that ρ_{31} and ρ_{32} are relatively close to zero, implying that volatility is predominantly unspanned by the futures contracts.

Parameters σ_S, α, γ, and ρ_{12} impact the term structure of volatility. Panel A in Figure 1.1 shows the impact on the term structure of instantaneous futures volatility, while Panel B shows the impact on the term structure of implied volatility. The time-t instantaneous volatility of a futures contract with maturity $T - t$ is given by

$$\sigma_F(t, T) = \sqrt{v(t)}\sqrt{\sigma_S^2 + B(T-t)^2 + 2\rho_{12}\sigma_S B(T-t)}. \tag{39}$$

For the baseline set of parameters, where ρ_{12} is large and negative, the volatility term structure is downward-sloping. This is sometimes referred

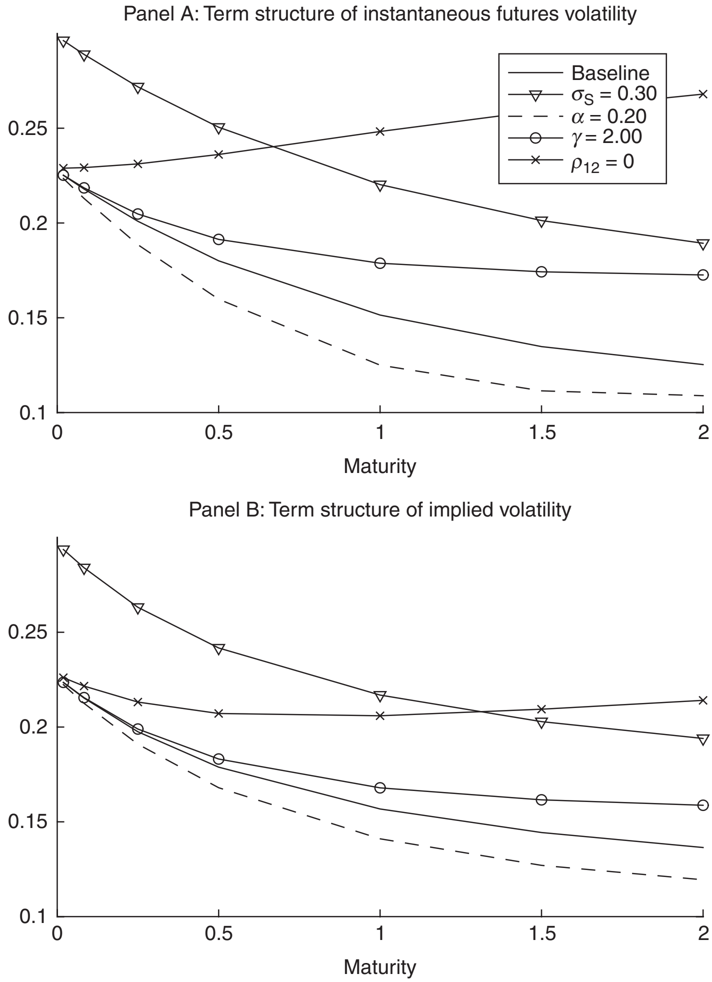

Figure 1.1 Impact of σ_S, α, γ, and ρ_{12} on volatility term structure.
This figure shows how σ_S, α, γ, and ρ_{12} impact the volatility term structure. Panel A shows the impact on the term structure of instantaneous futures volatility. Panel B shows the impact on the term structure of implied volatility. Baseline parameters of the model are those obtained by Trolle and Schwartz (2009) ($\sigma_S = 0.2289$, $\alpha = 0.1373$, $\gamma = 0.7796$, and $\rho_{12} = -0.8797$) and $v(t) = \theta$.

to as the "Samuelson effect" (see Samuelson [1965]). Increasing σ_S causes an almost parallel upward shift in the volatility term structure. Changing α affects the slope of the volatility term structure; in combination with ρ_{12} being large and negative, increasing α makes the volatility term structure more downward-sloping. A futures contract with an infinite maturity has an instantaneous volatility of $\sqrt{v(t)}\sqrt{\sigma_S^2 + \frac{\alpha^2}{\gamma^2} + \frac{2\rho_{12}\sigma_S\alpha}{\gamma}}$, and γ affects how fast the term structure approaches this level; increasing γ increases the speed of convergence. Finally, increasing ρ_{12} increases the slope of the volatility term structure, and for $\rho_{12} = 0$ the term structure is upward-sloping instead of downward-sloping.

The volatility smiles implied from options on futures contracts reflect the risk-neutral distributions of log futures returns. Figures 1.2–1.5 show how parameters σ_v, ρ_{13}, ρ_{23}, and κ impact the implied volatility smiles at

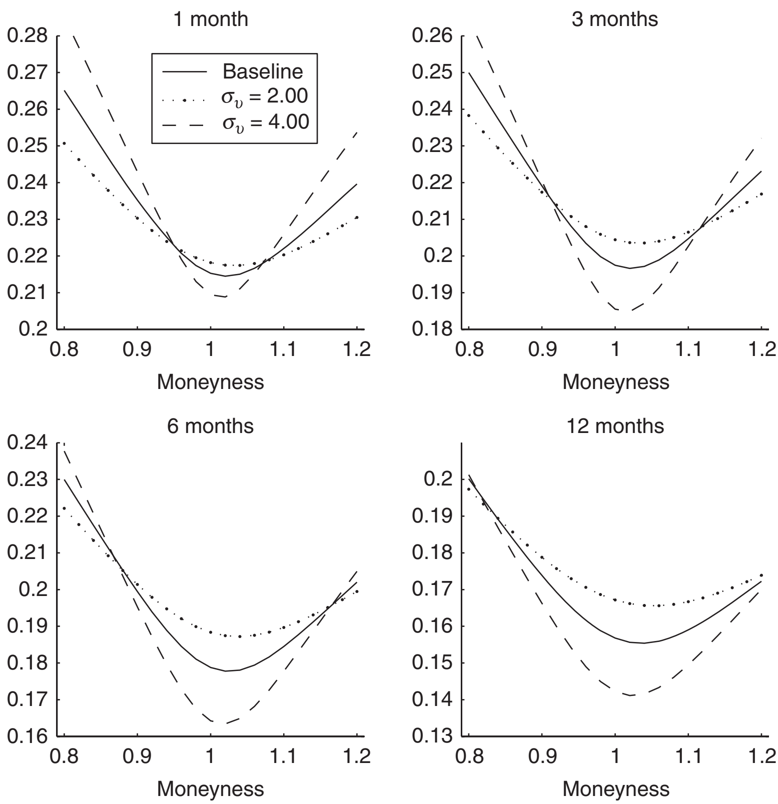

Figure 1.2 Impact of σ_v on implied volatility smiles.
This figure shows how σ_v impacts the implied volatility smiles at option maturities of 1, 3, 6, and 12 months. Baseline parameters of the model are those obtained by Trolle and Schwartz (2009) ($\sigma_v = 2.8051$) and $v(t) = \theta$. Moneyness is defined as an option strike divided by the price of the underlying futures contract.

maturities of 1, 3, 6, and 12 months. At the baseline parameters, there are pronounced volatility smiles at all horizons. This is due to the high value of σ_v, which induces significant excess kurtosis in the distribution of log returns. Also, the volatility smiles are skewed to the left at all horizons. This is due to the negative correlation between futures returns and innovations to volatility (caused by the negative values of ρ_{13} and ρ_{23}), which induces negative skewness in the distribution of log returns. Figure 1.2 shows how the curvature of the implied volatility smiles are impacted by σ_v. Figures 1.3 and 1.4 show how the skewness of the implied volatility smiles is impacted by ρ_{13} and ρ_{23}. ρ_{13} has an impact at all maturities,

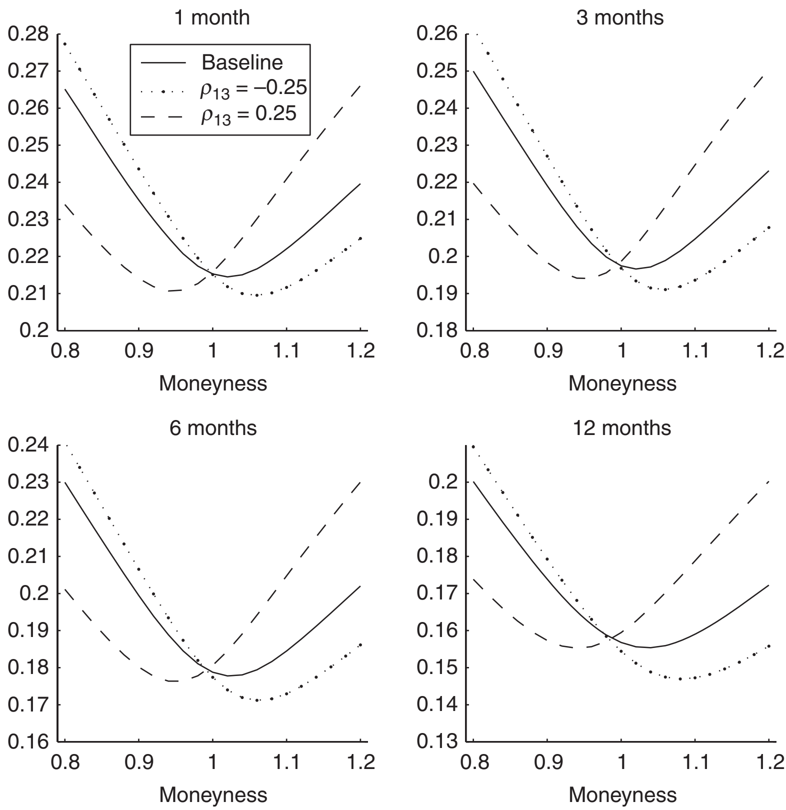

Figure 1.3 Impact of ρ_{13} on implied volatility smiles.
This figure shows how ρ_{13} impacts the implied volatility smiles at option maturities of 1, 3, 6, and 12 months. Baseline parameters of the model are those obtained by Trolle and Schwartz (2009) ($\rho_{13} = -0.0912$) and $v(t) = \theta$. Moneyness is defined as an option strike divided by the price of the underlying futures contract.

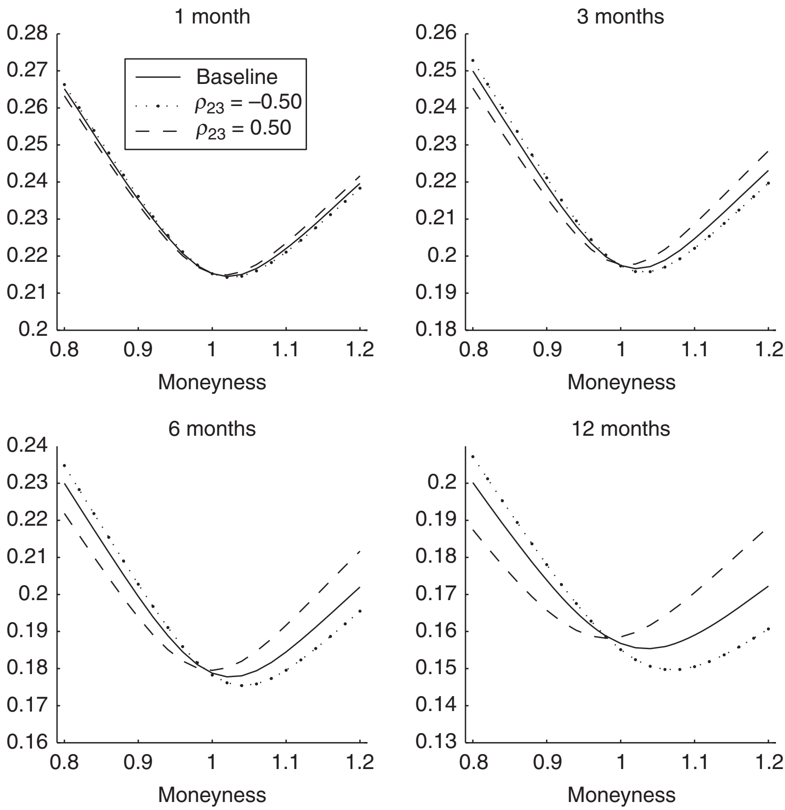

Figure 1.4 Impact of ρ_{23} on implied volatility smiles.
This figure shows how ρ_{23} impacts the implied volatility smiles at option maturities of 1, 3, 6, and 12 months. Baseline parameters of the model are those obtained by Trolle and Schwartz (2009) ($\rho_{23} = -0.1128$) and $v(t) = \theta$. Moneyness is defined as an option strike divided by the price of the underlying futures contract.

while the impact of ρ_{23} increases with maturity. Finally, Figure 1.5 shows how the implied volatility smiles are affected by κ. It is well known that in a stochastic volatility framework, the implied volatility smiles flatten out as the option maturity goes to zero or infinity. The maturity at which implied volatility smiles are most pronounced depends on the degree of mean-reversion in volatility; see Das and Sundaram (1999) for an analysis of the term structure of conditional moments in stochastic volatility models. Figure 1.5 shows that increasing κ makes longer-term smiles less pronounced.

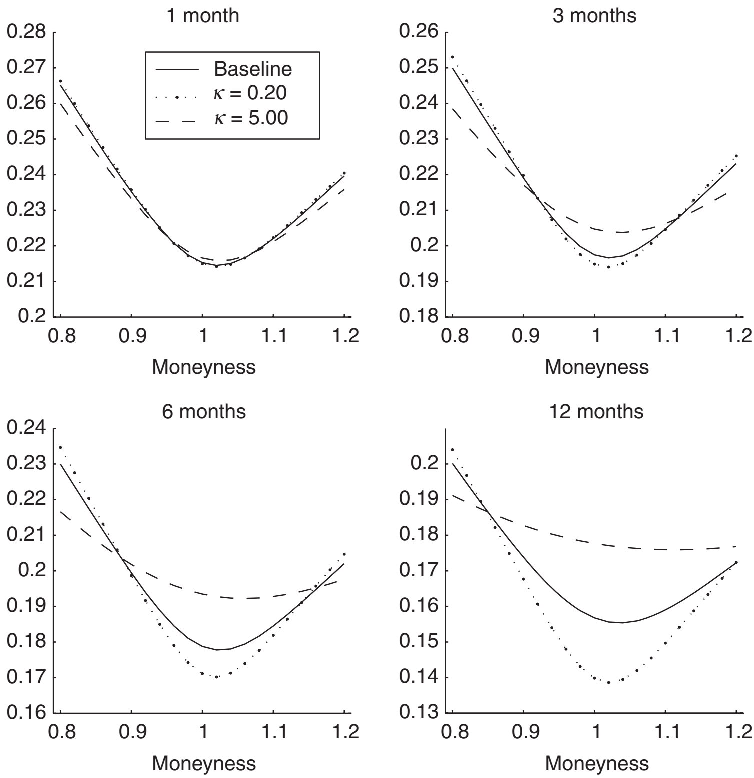

Figure 1.5 Impact of κ on implied volatility smiles.
This figure shows how κ impacts the implied volatility smiles at option maturities of 1, 3, 6, and 12 months. Baseline parameters of the model are those obtained by Trolle and Schwartz (2009) ($\kappa = 1.0125$) and $v(t) = \theta$. Moneyness is defined as an option strike divided by the price of the underlying futures contract.

7 Jumps

Several papers have documented jumps in energy prices; see, for example, Askari and Krichene (2008) and Larsson and Nossman (2011) for the crude oil market. In this section, I outline an extension of the framework that takes jumps in spot prices into account. I replace Equation (20) by

$$\frac{dS(t)}{S(t-)} = \delta(t)dt + \sigma_S\sqrt{v(t)}dW_1(t) + \left(e^z - 1\right) dq(t) - \overline{\mu}\lambda dt, \qquad (40)$$

where $q(t)$ is a Poisson jump counter (assumed independent from the Wiener processes) with instantaneous intensity λ.[10] Conditional on a jump occurring, z is the size of the jump in the log spot price and has the distribution

$$z \sim N(\mu_z, \sigma_z^2). \tag{41}$$

$\overline{\mu}$ is the expected percentage jump in the spot price and is given by

$$\overline{\mu} = E\left[e^z - 1\right] = e^{\mu_z + \frac{1}{2}\sigma_z^2} - 1. \tag{42}$$

In this extended model, the dynamics of futures prices are given by

$$\frac{dF(t,T)}{F(t-,T)} = \sqrt{v(t)}\left(\sigma_S dW_1(t) + \int_t^T \sigma_y(t,u)du dW_2(t)\right) + \left(e^z - 1\right) dq(t) - \overline{\mu}\lambda dt. \tag{43}$$

There are now two sources of market incompleteness: the USV component and the jump component.

With $\sigma_y(t,T)$ given by Equation (10), it is still the case that futures prices are exponentially affine in the three state variables $s(t)$, $x(t)$, and $\phi(t)$, which, along with $v(t)$, jointly constitute an affine state vector. Only now, the dynamics of $s(t)$ are given by

$$ds(t) = \left(y(0,t) - \overline{\mu}\lambda + \alpha(x(t) + \phi(t)) - \frac{1}{2}\sigma_S^2 v(t)\right) dt + \sigma_S\sqrt{v(t)}dW_1(t) + zdq(t). \tag{44}$$

Option pricing proceeds as in Section 5; only now, Equation (37) needs to be replaced by

$$\frac{dM(\tau)}{d\tau} = N(\tau)\kappa\theta + \left(e^{iu\mu_z - \frac{1}{2}u^2\sigma_z^2} - 1 - iu\overline{\mu}\right)\lambda. \tag{45}$$

When fitting the model to an implied volatility surface, the effect of the jump component is most pronounced at short option maturities. In particular, a negative mean jump size, μ_z, will add negative skewness to the implied volatility smiles. For typical parameter values, the effect of the jump component decays fairly rapidly as option maturity increases; see, for example, the discussion in Gatheral (2006) in the context of equity index options.

Many extensions that preserve the tractability of the model are possible. The jump intensity could be made an affine function of variance, $\lambda(t) = \lambda_0 + \lambda_v v(t)$. This captures the fact that jumps are more likely to occur when the market is more volatile. Also, following Duffie et al. (2000) for equity options, the variance process could be extended with (positive) jumps, possibly correlated with jumps in spot prices. Larsson and Nossman (2011) show that this is an important characteristic of the time series of crude oil prices.

8 Conclusion

In this chapter, I presented a tractable framework, first developed in Trolle and Schwartz (2009), for pricing energy derivatives in the presence of USV. Among the model features are (i) a perfect fit to the initial futures term structure, (ii) a fast and accurate Fourier-based pricing formula for European-style options on futures contracts, enabling efficient calibration to liquid plain-vanilla exchange-traded derivatives, and (iii) the evolution of the futures curve being described in terms of a low-dimensional affine state vector, making the model ideally suited for pricing complex energy derivatives and real options by simulation, where early exercise features can be handled using the LSM approach of Longstaff and Schwartz (2001). I also consider an extension of the framework that takes jumps in spot prices into account.

Notes

1. Throughout the chapter, implied volatilities are obtained using the Black (1976) model.
2. Following Collin-Dufresne and Goldstein (2002), one could derive parameter restrictions such that volatility is unspanned, but such restrictions are highly nonlinear and severely impact model flexibility.
3. In these cases, the pricing error that arises from not explicitly modeling stochastic interest rates is negligible, since the volatility of interest rates is typically orders of magnitudes smaller than the volatility of futures returns, and the correlation between interest rates and futures returns tends to be relatively low.
4. Because of the assumption that interest rates are deterministic, I do not distinguish between forward and future cost of carry; see Miltersen and Schwartz (1998).
5. For instance, in the case of crude oil, regular options expire three business days prior to the expiration of the underlying futures contract.
6. The control parameter φ must be chosen to ensure that the modified option price is square integrable, which is a sufficient condition for its Fourier transform to exist.
7. In some cases, such as crude oil, European options trade side by side with American options. In these cases, American options tend to be the most liquid.

8. The latter requires a specification of the change of measure from risk-neutral to physical.
9. Note that in the model, $\sigma_S, \alpha, \kappa\theta$, and σ_v are not simultaneously identified; see, e.g., the discussion of invariant affine transformations in Dai and Singleton (2000). Trolle and Schwartz (2009) normalize $\kappa\theta$ to one.
10. The $S(t)$-process is right-continuous. The value of S right before a jump at time t is the left limit $S(t-) = \lim_{u\uparrow t} S(u)$.

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