Heavy Meson Decays with Soft Gluon Effects

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Chapter 1

Introduction

The current theory of particle physics, known as the Standard Model (SM), was developed over a period of 30 years. While this theory has been very successful at predicting experimental results, there are both experimental and theoretical reasons to believe that it is incomplete. The fact that it is incomplete, implies that the Standard Model is a low energy approximation to a more complete higher energy theory.

Possible extensions to the SM have been proposed in the form of new particles or new forces. However, direct searches for new particles at accelerators have not detected any new particles or forces beyond those predicted by the Standard Model. If there is new physics at an energy that is out of the range of current accelerators, it is still possible to detect the effects from quantum fluctuations in the loop calculations, on processes, such as particle decay rates that occur at the lower, accessible energies.

This thesis is focused on calculating the nonleptonic weak decay rates of mesons. It is important to calculate the decay rates to test the SM prediction against measurements, but also to look for new physics. The effects of new physics will increase with energy so that the effects on the decay rates at accessible energies will be small. In particle decays that have a high rate in the SM, the effect of the new physics will be too small to be detected against the background. In rare decays, where the SM decay rate is small, the effect of new physics, even when small, might stand out against the background.

While the weak and electromagnetic (EM) interactions can be calculated using perturbation techniques, this becomes difficult at low energies, $<\sim 1$ GeV, with the strong interaction. Unlike the EM force, the coupling constant for the strong force decreases with higher energy. Perturbation expansion can be used at high energy. At the lower energy scale of confinement of the quarks into mesons, the coupling constant is too large for perturbation techniques to apply. These calculations can be done numerically by solving the full equations on a lattice, called lattice QCD. But this is computationally expensive and each decay mode must be calculated separately. Another complication is that the QCD Lagrangian is written in terms of quarks and gluons. However, at the energy scale ~ 1 GeV the quarks are confined into bound states of quarks (mesons and baryons), which are the particles that are measured. Another approach is to use effective theories, which are simpler, low energy approximations to the full QCD theory.

The current work focuses on calculations of the decays of heavy mesons (B, D) to light pseudoscalar mesons (π, K, η) and to light vector mesons (ρ, K^*). While these particles decay via the weak interaction, the decay process also includes QCD effects from the exchange of gluons. These calculations can be done using chiral quark models that include the mesons and effects of the soft gluons on the interactions.

The framework for the decay calculations is the operator product expansion, which allows seperation of the energy scales of the decay. . Heavy Light chiral perturbation theory extends this to include the heavy mesons (B,D). The chiral quark model is based on chiral perturbation theory, in which the degrees of freedom are the light mesons (π, K, η) and the coupling between them. The quark models include coupling between the mesons and the light quarks and soft gluons. Including the coupling of soft gluons to the light quarks allows calculation of the effects of the gluons on the decays. Heavy light chiral quark model extends the chiral quark model to include heavy mesons and the heavy quarks (b, c).

The current work uses a chiral quark model which is extended to include vector mesons, to calculate decay modes of B and D mesons to vector and pseudoscalar mesons. The calculations include soft gluon effects,using the gluon condensate, and gluon fusion production of η'. In the first paper, we calculate the decay amplitude of B mesons to two pions with a comparison of the factorizable and nonfactorizable decay modes. The second paper presents a calculation of the form factors for D meson decays to both vector and pseudoscalar mesons. The modified chiral quark model is used to calculate the form factors. The third paper is a calculation of the decay of D mesons to vector mesons and the η and η' meson, and the contribution to the η' mode from gluon fusion effects.

The following sections describe the models that are used in the calculations. Chapter 4 introduces effective field theory and the nonleptonic Fermi theory for weak decays. Chapter 5 describes the chiral perturbation theories and their extensions to include high energy and high mass particles. Chapter 6 describes the different quark models that are used. Chapter 7 describes the extensions of the quark models to include light vector mesons and high energy vector mesons and the method used to determine the couplings in the model. Chapter 8 gives a brief review of the 3 papers in the thesis and a summary of the results.

Chapter 2

The Standard Model

The Standard Model, which is the current theory that is used to treat the fundamental particles of matter and the interactions of the electromagnetic, weak and strong forces is a quantum field theory which combines quantum physics with relativity [29]. The Standard Model (SM) is a relativistic quantum field theory. The gauge fields enter the Lagrangian in the covariant derivative which is defined from the requirement of gauge invariance.

2.1 Particle content of the Standard Model

There are two main classes of particles in the standard model; fermions which are the matter particles and gauge bosons which are the force mediators. To experimental limits, the fermions, which include leptons and quarks, are point particles with no discernible structure, but carry a spin of 1/2. The gauge bosons carry a spin of 1 and include the massless photons and gluons, and the massive weak gauge bosons, W and Z.

The matter particles are spin 1/2 fermions. The six quarks (u, d, s, c, b, t), and six leptons (e,ν_e,μ,ν_μ,τ,μ_τ) are grouped into 3 families of two quarks and two leptons each,

$$\begin{pmatrix} u & e^- \\ d' & \nu'_e \end{pmatrix}, \begin{pmatrix} c & \mu^- \\ s' & \nu'_\mu \end{pmatrix}, \begin{pmatrix} t & \tau^- \\ b' & \nu'_\tau \end{pmatrix}. \tag{2.1}$$

Each family is grouped into left handed SU(2) doublets of quarks and leptons, and right handed singlet states;

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, u_R, d'_R, e_R, (\nu_e)_R\,. \tag{2.2}$$

The right and left handed particles are projections of the mass particle states, with, for example

$$e_R = R\,e = \frac{(1+\gamma^5)}{2}\,e \tag{2.3}$$

where $R = (1+\gamma^5)/2$ and $L = (1-\gamma^5)/2$ are the right and left handed projection operators. These are given as mass eigenstates. Because the mass eigenstates differ from the weak eigenstates, the weak eigenstates (d', s', b') can be written as linear combinations of the mass eigenstates related by the CKM matrix [23],

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \mathrm{V}_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{2.4}$$

While the quarks are the degrees of freedom in the SM Lagrangian, at low energies, they do not exist as free particles, but have only been observed in bound states of quark-antiquark pairs, $q\bar{q}$, (the mesons) and in bound state of 3 quarks or 3 antiquarks, qqq and $\bar{q}\bar{q}\bar{q}$ (the hadrons). Because, at low energies, the quarks only exist in bound states and free quarks do not exist, it is not simple to define a quark mass. The constituent quark mass is taken to be the average energy of a quark bound in a hadron in the ground state, while the current mass is the mass that appears in the Lagrangian. This is given in Eq. (3.1).

2.2 Gauge interactions of the Standard Model

The interactions between particles arise from the gauge symmetries in the Lagrangian. The gauge symmetry group for the Standard Model is

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y. \tag{2.5}$$

Here, $\mathrm{SU}(3)_C$ color is the gauge group for the strong interaction and $\mathrm{SU}(2)_L \times \mathrm{U}(1)_L$ is the gauge group for the electroweak interaction.

As an example, the QED interaction is generated by a local $\mathrm{U}(1)$ gauge transformation, with the quarks and leptons represented by a free fermion field $\psi(x)$. The Lagrangian density for $\psi(x)$ is

$$\mathcal{L} = \bar{\psi}(i\gamma \cdot \partial - m)\psi. \tag{2.6}$$

Under a local $\mathrm{U}(1)$ gauge transformation the field $\psi(x)$ transforms as

$$\psi(x) \rightarrow \psi'(x) = U\psi(x) = e^{i\alpha(x)}\psi(x), \tag{2.7}$$

where ψ is a fermion matter field (quark, lepton) and

$$U = e^{i\alpha(x)} \in U(1). \tag{2.8}$$

The derivative acting on the field $\psi(x)$ will transform as

$$\partial_\mu \psi(x) \rightarrow e^{i\alpha(x)}[\partial_\mu \psi(x) + i\psi(x)(\partial_\mu \alpha(x))]. \tag{2.9}$$

The Lagrangian density transforms as

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L} + i\bar{\psi}(\gamma \cdot \partial\alpha(x))\psi. \tag{2.10}$$

We can formulate a Lagrangian that is invariant under the gauge transformation if we introduce a vector field A_μ, which transforms as

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha, \tag{2.11}$$

which defines the covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu \tag{2.12}$$

where A_μ is called a gauge field and e is the coupling constant. The covariant derivative acting on the field $\psi(x)$ will transform under local U(1) gauge transformations the same way as the field ψ,

$$D_\mu \psi \rightarrow (D'_\mu \psi') = U(D_\mu \psi). \tag{2.13}$$

The Lagrangian density with the covariant derivative

$$\mathcal{L} = \bar{\psi}(i\gamma \cdot D - m)\psi\,, \tag{2.14}$$

is invariant under the U(1) gauge transformation. This brings in the gauge field A_μ with interactions with the matter fields $\psi(x)$ (quarks and leptons).

Similarly, invariance under the local $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ transformations introduces a covariant derivative that includes the gauge fields A^a_μ, W^a_μ and B_μ of the $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ groups respectively,

$$D_\mu = \partial_\mu \,+\, ig_s t^a A^a_\mu \,+\, igW^a_\mu \frac{\sigma^a}{2} \,+\, \frac{i}{2} g' Y B_\mu. \tag{2.15}$$

The matrices t_a are the 8 generators of the SU(3) color group. The matrices $\sigma^a/2$ are the 3 generators of the weak SU(2) group.

The $\mathrm{SU}(3)$ gauge field has 3 color charges and 8 gauge bosons. The gluon field tensor is given by

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ig_s f^{abc} A^b_\mu A^c_\nu, \tag{2.16}$$

where f^{abc} are the structure constants for the $SU(3)$ group. The field tensor for the weak force mediators is

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + ig\varepsilon^{abc} W^b_\mu W^c_\nu, \tag{2.17}$$

ε^{abc} are the structure functions for the $SU(2)$ group and for the $\mathrm{U}(1)_Y$ group bosons the field tensor is

$$F^Y_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{2.18}$$

Because the $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)$ groups are non-Abelian the terms in (2.16) and (2.17) with the structure constants will generate self coupling between the gauge bosons for the strong and weak interactions. The $\mathrm{U}(1)_Y$ group is Abelian and the structure constants are zero so there is no self interaction term with the B_μ gauge bosons.

2.3 Spontaneous symmetry breaking

While the gluons and photons are massless, the weak vector bosons W^+, W^- and Z have nonzero mass. Adding a mass term for the vector boson to the Lagrangian would violate the gauge symmetry. However, masses can be generated by spontaneous breaking of the weak gauge symmetry. A spontaneous symmetry breaking occurs when a symmetry that is inherent in the Lagrangian is broken by the ground state of a system. The spontaneous breaking of the weak gauge symmetry, which gives mass to the weak vector bosons W^+, W^- and Z, is achieved by adding a scalar field that couples to the weak and electromagnetic fields [18, 15, 22, 17]. This field, which is commonly called the Higgs field, is a complex scalar field with a general form,

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \langle\phi\rangle_0 + \Delta\phi, \tag{2.19}$$

with a nonzero ground state, $\langle\phi\rangle_0$. The Lagrangian density terms pertaining to the Higgs field are given by

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi), \tag{2.20}$$

where the potential is

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2. \tag{2.21}$$

The $\mathrm{SU}(2)_L$ x $\mathrm{U}(1)_Y$ symmetry is broken by the Higgs field acquiring a nonzero vacuum expectation value when $\mu^2 < 0$. The ground state, which gives a minumum for the potential $V(\phi)$, is

$$\langle\phi\rangle_0 \equiv \langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \tag{2.22}$$

with $v = \sqrt{-\mu^2/\lambda}$, where $\mu^2 < 0$ and $\lambda > 0$.

Because the Higgs field couples to SU(2) and U(1) fields, the covariant derivative

$$D_\mu\,\phi(x) = \left[\partial_\mu + igW^a_\mu\frac{\sigma^a}{2} + \frac{i}{2}g'B_\mu Y\right]\phi(x) \tag{2.23}$$

includes the SU(2) and U(1) gauge fields, $W^a_\mu(x)$ and $B_\mu(x)$. The Lagrangian then generates mass terms for the gauge bosons from the coupling of the vacuum expectation value of the Higgs field to the gauge boson fields. Taking this term on the ground state yields

$$(D_\mu\phi_0)^\dagger(D^\mu\phi_0) = \frac{v^2}{2}\left[\left(\frac{g}{2}\right)^2\left(W_1^2 + W_2^2\right) + \left(-\frac{g}{2}W_3 + \frac{g'}{2}B\right)^2\right]. \tag{2.24}$$

The fields W_1 and W_2 are written in the basis with the charged W-boson fields,

$$W^\pm = \frac{W_1 \mp iW_2}{\sqrt{2}}. \tag{2.25}$$

The fields W_3^μ and B^μ are written in terms of the Z boson and the photon field A^μ

$$Z^\mu = \frac{gW_3^\mu - g'B^\mu}{\sqrt{g^2 + g'^2}} \tag{2.26}$$

and

$$A^\mu = \frac{gW_3^\mu + g'B^\mu}{\sqrt{g^2 + g'^2}} \,. \tag{2.27}$$

It is common to express the Z and A fields in terms of a rotation,

$$\begin{aligned} Z &= \cos\theta_W W^3 - \sin\theta_W B \\ A &= \sin\theta_W W^3 + \cos\theta_W B, \end{aligned} \tag{2.28}$$

where θ_W is the weak mixing angle defined by

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \,, \quad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{2.29}$$

The W bosons have a mass

$$M_W = g\frac{v}{2}, \tag{2.30}$$

and the Z boson has a mass

$$M_Z = \frac{M_W}{\cos\theta_W}. \tag{2.31}$$

The photon field, A remains massless.

Chapter 3

Framework for calculation of weak decays of heavy mesons

The standard model Lagrangian is written in terms of quarks and leptons. At lower energies $\lesssim 1-2$ GeV, the quarks are in bound states of mesons, which are quark-antiquark pairs, or baryons, which are composed of 3 quarks (or 3 antiquarks). The particles that are measured in experiments are leptons and mesons and baryons. The mesons and baryons decay in strong and weak interactions. This work is focused on the weak decay of heavy mesons to lighter mesons. The weak interaction in these decays have QCD effects that arise from the emission of gluons from the quarks in the mesons. The QCD coupling increases at lower energies and at the energy scale of the meson decays, perturbative QCD is used down to a scale ~ 1 GeV. Our calculations are done using effective theories with mesons and their interactions together with quark models that give the coupling of the quarks to the mesons.

The quarks are generally classified in two groups, the light quarks u, d, s, which have masses $m \ll 1$ GeV and the heavy quarks, c, b and t which have mass $m > 1$ GeV. The current masses of the quarks are [4]

$$\begin{aligned} m_u &= 0.002 \text{ GeV} \qquad & m_c &= 1.275 \text{ GeV} \\ m_d &= 0.005 \text{ GeV} \qquad & m_b &= 4.6 \text{ GeV} \\ m_s &= 0.095 \text{ GeV} \qquad & m_t &= 173 \text{ GeV}. \end{aligned} \tag{3.1}$$

The mesons, bound states of a quark and an antiquark, are also classified as light or heavy. The light mesons (π,K, η) are bound states of the light quarks (u, d, s). The heavy mesons (B,D) are bound states containing at least one heavy quark (c, b). The top quark t, is heavy and decays too quickly to form mesonic bound states.

The decay rates are calculated in terms of the probability amplitude, $\mathcal{M}$, of decay process. The amplitude, $\mathcal{M}(P \to P_1 P_2)$, for a decay of a particle P to two decay products is calculated from the Lagrangian by

$$\mathcal{M}(P \to P_1 P_2) = \langle P_1 P_2 | \mathcal{L} | P \rangle. \tag{3.2}$$

The decay rate for a decay to two pseudoscalar mesons $P \to P_1 P_2$ is given by

$$\Gamma(P \to P_1 P_2) = \frac{1}{8\pi} \frac{|\vec{P}|}{M^2} |\mathcal{M}|^2. \tag{3.3}$$

For decays to one pseudoscalar and one vector meson, $P \to P_1 V$, the decay rate is given by

$$\Gamma(P \to P_1 V) = \frac{1}{8\pi} \frac{|\vec{P}|^3}{M^2} |\mathcal{M}|^2. \tag{3.4}$$

For a two body decay, where M is the mass of the initial particle and m_1 and m_2 are the masses of the decay products, the energy of the decay product is

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}. \tag{3.5}$$

The 3-momentum of the decay products, $|\vec{P}| = |\vec{P}_1| = |\vec{P}_2|$ is

$$|\vec{P}| = \frac{[(M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2)]^{1/2}}{2M}. \tag{3.6}$$

For the calculations of the amplitudes, we use effective theories and quark models. An effective field theory is a theory with the same symmetries as the full theory which is applicable in a limited energy range. In the meson decays that we consider, there are several energy scales.

At low energies $\lesssim 1$ GeV, where the quarks are confined to bound states, the degrees of freedom are the mesons. The effective theory for the light mesons (π,K,η), is Chiral Perturbation Theory (χPT). The Heavy Light Chiral Perturbation Theory HLχPT has as degrees of freedom the light mesons of the χPT and the heavy B and D mesons. These effective theories are based on QCD with the quark degrees of freedom integrated out.

At higher energies > 1 GeV the degrees of freedom are the quarks which can be treated with perturbative QCD. In this energy range effective theories are used. The Heavy Quark Effective Theory (HQET) includes the heavy c and b quarks. The Large Energy Effective Theory (LEET) is and effective theory for high energy light quarks.

Quark models are a bridge between the higher energy theories, where the degrees of freedom are quarks, and the low energy chiral perturbation theories, which are in terms of light mesons. They include interactions between the mesons and the quarks. The chiral quark model (χQM) includes interactions between the light mesons (π,K,η) and the light quarks, (u, d, s). The heavy light chiral quark model (HLχQM) extends the (χQM) to include the heavy mesons (B, D) and the heavy quarks (b, c). While the effective theories have parameters that are determined by QCD, the chiral quark models have parameters, such as the coupling constants and constituent masses, that must be determined by matching calculations of currents with known or measured quantities.

In the current work, an extension to the (χQM) is used, which includes couplings with the vector mesons (ρ,K^*,ω) which are bound states of 2 quarks in a state with spin 1.

3.1 The η and η' mesons

The light meson octet (π, K, η) contains the light mesons which includes the meson η_8. There also exists a singlet state η_1. The physical states which are measured, are the η and η' which are mixtures of the SU(3) singlet (η_1) and octet (η_8) states. In terms of the quark content, these states are

$$\eta_1 = \frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s}) \tag{3.7}$$

and

$$\eta_8 = \frac{1}{\sqrt{6}}(u\overline{u} + d\overline{d} - 2s\overline{s}). \tag{3.8}$$

The physical states, η and η', are defined as

$$\left(\begin{array}{c} \eta \\ \eta' \end{array} \right) = \left(\begin{array}{cc} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{array} \right) \left(\begin{array}{c} \eta_8 \\ \eta_1 \end{array} \right). \tag{3.9}$$

Here, θ is the η mixing angle. The η mixing angle has been measured to be $\theta = 13.7$ degrees [33], but the ideal mixing angle of $\theta = 19.5$ degrees is often used. Because the η' is a neutral, singlet state, in addition to the u, d and s quark content, η' also includes an admixture of $c\overline{c}$ quark states and gluon states. The η' has up to 26% gluon fraction [24]. Due to this gluon component the η' can be produced by the fusion of two gluons [20]. The production of η' by gluon fusion has been measured at the RHIC experiment [21].

The η and η' can also be written in the quark flavor basis, η_q and η_s,

$$\eta_q = \frac{1}{\sqrt{2}}(u\overline{u} + d\overline{d}) \tag{3.10}$$

$$\eta_s = s\overline{s} \tag{3.11}$$

$$\left(\begin{array}{c} \eta \\ \eta' \end{array} \right) = \left(\begin{array}{cc} cos\phi & -sin\phi \\ sin\phi & cos\phi \end{array} \right) \left(\begin{array}{c} \eta_q \\ \eta_s \end{array} \right). \tag{3.12}$$

ϕ has been measured to be $\phi \sim 39.3$ degrees [1].

Chapter 4

Effective Field Theory at the Quark Level

An effective field theory is usually, a low energy approximation to a higher energy theory. Effective theories are generally used when they have variables or degrees of freedom that are more appropriate or easier to use in a limited energy region. For example, chiral perturbation theory is used for energies below ~ 1 GeV, where the quarks are not free, but are bound into mesons and baryons. The degrees of freedom for chiral perturbation theory are the light mesons (π,K,η) rather than the quarks and gluons of the full QCD theory.

An effective theory is only applicable in a limited energy range and in general will not have the correct high energy behavior. The Lagrangian will include all the terms that are compatible with the the symmetry of the theory. The coefficients of these terms can generally be calculated from the full, high energy theory. However, for QCD theory, at low energies perturbation techniques cannot be used, due to the strong coupling constant. For low energy effective theories involving QCD, the coefficients are treated as free parameters of the theory and related to experimental data or data from lattice QCD calculations.

One can define an effective weak Lagrangian to describe weak interactions at low energies. In this effective theory, the W bosons and the top quark are removed as explicit degrees of freedom, i.e. they are integrated out. This is an effective field theory with n_f active quarks, so at scales above m_b, there are 5 quark flavors, while at the scale of m_c there are 4 quark flavors. The couplings in the effective action are suppressed by the masses of the heavy degrees of freedom that are integrated out.

4.1 Fermi theory of weak interactions

An example of an effective theory is the Fermi theory of weak interactions. To lowest order, the weak interaction is represented by an exchange of a single W boson between two weak currents. The currents have the form

$$J^\mu = \overline{u}_i \gamma^\mu (1-\gamma_5) V_{ij} d_j \, , \tag{4.1}$$

where u_i and d_j are quark fields, $u_1 = u, u_2 = c, u_3 = t$ and $d_1 = d, d_2 = s, d_3 = b$. V_{ij} is the CKM matrix defined in Eq. 2.4. These are the weak quark currents. There are also weak leptonic currents which are not involved in the hadronic meson decays. The weak, 4 quark interactions have the form

$$\mathcal{L}_{weak} = J^\mu g_W^2 \, D_{\mu\nu}(W) J^{\dagger\nu}, \tag{4.2}$$

where $D_{\mu\nu}(W)$ is the W propagator which, in the Feynman gauge, is given by

$$D_{\mu\nu}(W) = \frac{-g_{\mu\nu}}{p^2 - M_W^2 + i\epsilon}\,. \tag{4.3}$$

In the limit where $(p^2) \ll M_W^2$, the W boson propagator can be written

$$D_{\mu\nu}(W) = \frac{g_{\mu\nu}}{M_W^2} + \mathcal{O}(q^2/M_W^2). \tag{4.4}$$

This leads to an effective Lagrangian that includes a 4-fermion interaction

$$\mathcal{L}_{Fermi} = -\frac{G_F}{\sqrt{2}} J_\mu J^\mu, \tag{4.5}$$

where G_F is the effective coupling

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{4M_W^2}. \tag{4.6}$$

Equation (4.6) is a matching condition that gives a relation between the parameters of the full electroweak theory (g_W, M_W) and the coupling in the effective theory, G_F. It would be possible to expand the W propagator in powers of p^2/M_W^2 to get operators of higher dimension, but we leave out these operators in the meson decay calculations.

In a typical weak decay, the scale is set by the mass of the decaying hadron, for example the decaying meson have mass $M_B \sim 5$ GeV or $M_D \sim 2$ GeV and the decay products have mass $m_\pi \sim 140$ MeV and $m_K \sim 500$ MeV. These are all small compared to the mass $M_W \approx 80$ GeV of the W boson.

4.2 Operator product expansion OPE

In the operator product expansion (OPE) the Lagrangian is expressed as an expansion in the product of local operators [34]. For the weak interactions, the products of the quark current operators that interact (via the W exchange) are expanded into a series of local operators Q_i multiplied by Wilson coefficients, C_i,

$$\mathcal{L}_{weak}^{eff} = \sum_i C_i Q_i\,. \tag{4.7}$$

The Wilson coefficients $C_i = C_i(\mu)$ depend on the renormalisation scale μ of the interactions, and represent the strength that a given operator contributes to the amplitude. The Wilson coefficients are calculated using perturbative QCD and the renormalization group equations.

The OPE gives a factorization of the short and long distance physics. The Wilson coefficients $C_i(\mu)$ contain all the information about the short distance dynamics of the theory, at energy scales greater or equal to μ. They depend on the properties of the particles that have been integrated out of the effective theory. The factorization implies that the coefficients are independent of the external states, i.e., the C_i's are the same for all external particles.

In the operator product expansion, the weak effective Lagrangian for nonleptonic, weak decays, including QCD and electroweak corrections, can be written as a sum of operators [16]:

$$\mathcal{L}^{eff}_{Weak} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) Q_i(\mu). \tag{4.8}$$

While the weak effective Lagrangian will in general contain operators for all possible quark currents we generally only include the Q_i which are the relevant to the particular decay in question. For the quark model where μ is below the charm quark mass, the Lagrangian operators (Q_i's) contain only the light quarks, u,d,s.

We will focus on the first two operators which are dominating at the energy scale of the meson decays, on the order of 1 GeV. I.e., the Wilson coefficients of the first 2 operators, C_1 and C_2, are much larger than the coefficients of the other operators. For example, For the $\Delta C = 1$ processes, which is relevant for D meson decays, the operators are:

$Q_1 = (\overline{s}_i c_j)(\overline{u}_j d_i)$
$Q_2 = (\overline{s}_i c_i)(\overline{u}_j d_j)$
$Q_3 = (\overline{s}_i c_i)\sum_q(\overline{u}_j d_j)_{V-A}$
$Q_4 = (\overline{s}_i c_j)\sum_q(\overline{u}_j d_i)_{V-A}$
$Q_5 = (\overline{s}_i c_i)\sum_q(\overline{u}_j d_j)_{V+A}$
$Q_6 = (\overline{s}_i c_j)\sum_q(\overline{u}_j d_i)_{V+A}$.

The current $(\overline{s}_i c_j)$ is defined as $(\overline{s}_i\gamma^\mu(1-\gamma^5)c_j)$, where the i and j are color indices. For the $\Delta B = 1$ processes, which are relevant for B meson decays, the operators are:

$Q_1 = (\bar{b}_i u_j)(\overline{u}_j d_i)$
$Q_2 = (\bar{b}_i u_i)(\overline{u}_j d_j)$
$Q_3 = (\bar{b}_i d_i)\sum_q(\overline{q}_j q_j)_{V-A}$
$Q_4 = (\bar{b}_i d_j)\sum_q(\overline{q}_j q_i)_{V-A}$
$Q_5 = (\bar{b}_i d_i)\sum_q(\overline{q}_j q_j)_{V+A}$
$Q_6 = (\bar{b}_i d_j)\sum_q(\overline{q}_j q_i)_{V+A}$.

The operator Q_2 corresponds to the tree level diagram, the operator Q_1 is generated by the tree level diagram with a gluon correction, shown in figure 4.1.

The operator $Q_1 = (\overline{s}_i c_j)(\overline{u}_j d_i)$, after a Fierz transformation, can be written

$$Q_1^F = Q_1 = (\overline{s}_i d_i)(\overline{u}_j c_j) \tag{4.9}$$

We can also write Q_1 in terms of Q_2 by using the properties of the color $SU(3)_C$ generators,

$$\delta_{ij}\delta_{ln} = \frac{1}{N_C}\delta_{in}\delta_{lj} + 2t^a_{in}t^a_{lj}\,. \tag{4.10}$$

Q_1 is then

$$Q_1 = \frac{1}{N_C}(\overline{s}_i c_i)(\overline{u}_j d_j) + 2(\overline{s}_i c_i)^a(\overline{u}_j d_j)^a = \frac{1}{N_C}Q_2 + 2Q_2^c \tag{4.11}$$

here $(\overline{s}_i c_i)^a = (\overline{s}_i\gamma^\mu(1-\gamma^5)t^a c_i)$ is a colored quark current, t^a is a color matrix. The operator Q_2^c is a product of two colored currents, $Q_2^c = (\overline{s}_i c_i)^a(\overline{u}_j d_j)^a$.

The operator Q_2 can also be written in terms of Q_1. After a Fierz transformation, Q_2 is

$$Q_2 = (\overline{s}_i d_j)(\overline{u}_j c_i)\,. \tag{4.12}$$

Then using the transformation properties in Eq. (4.10), Q_2 becomes

$$Q_2 = \frac{1}{N_C}(\overline{s}_i d_i)(\overline{u}_j c_j) + 2(\overline{s}_i d_i)^a(\overline{u}_j c_j)^a = \frac{1}{N_C}Q_1 + 2Q_1^c\,. \qquad (4.13)$$

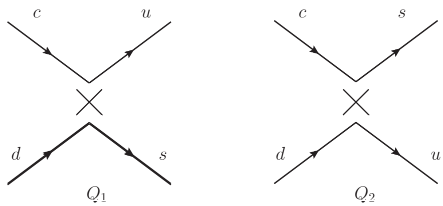

Figure 4.1: The Q_1 and Q_2 are local 4 quark operators.

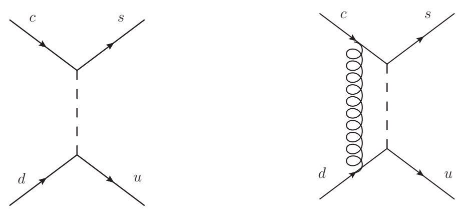

Figure 4.2: Contributions to the Q_1 and Q_2 operators. In the limit where $M_W \gg \mu$, the diagram on the left generates the Q_2 operator. The diagram on the right generates the Q_1 operator and a contribution to Q_2.

4.3 Factorizable and nonfactorizable contributions

To calculate amplitudes, we start with the factorization framework, where the amplitude is written in terms of a product of two factorizable currents [8]. As an example the decay $D^0 \to K^0\pi^0$ has the underlying quark transition $\bar{c} \to \bar{s}d\bar{u}$. The amplitude written in terms of the operators Q_1 and Q_2 is

$$\mathcal{M}(D^0 \to K^0\pi^0) = \langle K^0\pi^0|\mathcal{L}|D^0\rangle = \frac{G_F}{\sqrt{2}}V_{CKM}\,\langle K^0\pi^0|(C_1Q_1 + C_2Q_2)|D^0\rangle \qquad (4.14)$$

As an example of factorization, the term in the amplitude with Q_1 can be written in terms of quark currents

$$\langle K^0\pi^0|Q_1|D^0\rangle = \langle K^0\pi^0|(\overline{s}_i\gamma^\mu(1-\gamma^5)d_i)(\overline{u}_j\gamma^\mu(1-\gamma^5)c_j|D^0\rangle\,. \qquad (4.15)$$

Using the vacuum saturation approximation, this term can be written as a product of currents,

$$\begin{aligned}\langle K^0\pi^0|Q_1|D^0\rangle = &\langle K^0\pi^0|(\overline{s}_i\gamma^\mu(1-\gamma^5)d_i)|0\rangle\langle 0|(\overline{u}_j\gamma^\mu(1-\gamma^5)c_j|D^0\rangle \\ &+\langle K^0|(\overline{s}_i\gamma^\mu(1-\gamma^5)d_i)|0\rangle\langle \pi^0|(\overline{u}_j\gamma^\mu(1-\gamma^5)c_j|D^0\rangle\,.\end{aligned} \tag{4.16}$$

These are shown in figure (4.3). The term in the amplitude with operator Q_2 is

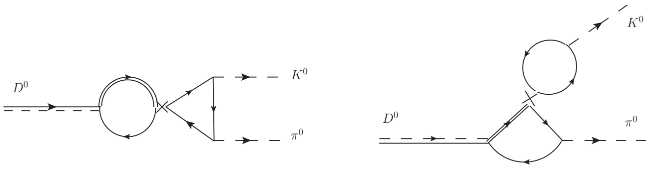

Figure 4.3: The factorizable currents in $\langle K^0\pi^0|Q_1|D^0\rangle$. The diagram on the left corresponds to the current $\langle K^0\pi^0|(\overline{s}_id_i)|0\rangle\langle 0|(\overline{u}_jc_j)|D^0\rangle$ and the diagram on the right corresponds to the current $\langle K^0|(\overline{s}_id_i)|0\rangle\langle\pi^0|(\overline{u}_jc_j)|D^0\rangle$.

$$\langle K^0\pi^0|Q_2|D^0\rangle = \langle K^0\pi^0|(\overline{s}_i\gamma^\mu(1-\gamma^5)c_i)(\overline{u}_j\gamma^\mu(1-\gamma^5)d_j|D^0\rangle\,. \tag{4.17}$$

This term cannot be written as a product of two currents due to the quark transitions in the operator. We then use equation (4.13) to write Q_2 in terms of Q_1 and a colored current,

$$\begin{aligned}\langle K^0\pi^0|Q_2|D^0\rangle = &\frac{1}{N_C}\langle K^0\pi^0|Q_1|D^0\rangle \\ &+2\,\langle K^0\pi^0|(\overline{s}_i\gamma^\mu(1-\gamma^5)t^a d_i)(\overline{u}_j\gamma^\mu(1-\gamma^5)t^a c_j|D^0\rangle\,.\end{aligned} \tag{4.18}$$

The first term is proportional to the Q_1 contribution. The second term is not factorizable, we write it as a product of quasi-factorizable colored currents,

$$\begin{aligned}&\langle K^0\pi^0|\overline{s}_i\gamma^\mu(1-\gamma^5)t^a d_i)(\overline{u}_j\gamma^\mu(1-\gamma^5)t^a c_j)|D^0\rangle = \\ &\quad\langle K^0\pi^0|(\overline{s}_i\gamma^\mu(1-\gamma^5)t^a d_i)|0\rangle\langle 0|(\overline{u}_j\gamma^\mu(1-\gamma^5)t^a c_j|D^0\rangle \\ &\quad+\langle K^0|(\overline{s}_i\gamma^\mu(1-\gamma^5)t^a d_i)|0\rangle\langle\pi^0|(\overline{u}_j\gamma^\mu(1-\gamma^5)t^a c_j|D^0\rangle\,.\end{aligned} \tag{4.19}$$

This is the nonfactorizable contribution, which is shown in figure (4.4). The matrix elements of

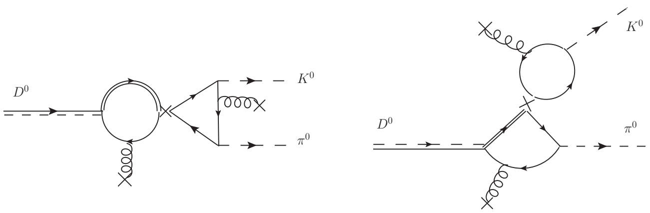

Figure 4.4: The nonfactorizable currents in $\langle K^0\pi^0|Q_1|D^0\rangle$. The diagram on the left corresponds to the current $\langle K^0\pi^0|(\overline{s}_id_i)^a|0\rangle\langle 0|(\overline{u}_jc_j)^a|D^0\rangle$ and the diagram on the right corresponds to the current $\langle K^0|(\overline{s}_id_i)^a|0\rangle\langle\pi^0|(\overline{u}_jc_j)^a|D^0\rangle$.

the noncolored currents in (4.16) are well known. The matrix elements of the currents with the colored operators, Q_1^c and Q_2^c, are calculated using effective theories and chiral quark models.

Chapter 5

Effective theories at the meson level

The effective theories that we use for nonleptonic meson decays, depend on the energy scale of the specific decays. Various models are used depending on the energy scale, shown in figure 5.1. At high energies, $\mu \sim 80$ GeV, the full Standard Model, with QCD and EW interaction is used. The degrees of freedom are the free quarks, u, d, s, c, t, b, gluons, W and Z bosons, see fig 5.1. At energies below $\mu \sim 5$ GeV, the Heavy Quark Effective Theory (HQET) can be used. This is generated by integrating out the top quark and the heavy W mesons, leaving the degrees of freedom, the c, b quarks. The light quarks quarks in this energy range are treated using the Large Energy Effective Theory. Here the degrees of freedom are hard, light quarks, u, d, s.

At lower energies, $\mu < 1$ GeV, the quarks exist in bound states of mesons. These are calculated with chiral perturbation theory (χPT), where the degrees of freedom are the light meson states π, K, η and with heavy-light chiral perturbation theory (HLχPT), where the degrees of freedom include the heavy mesons B and D.

The chiral quark models are a bridge between the theories with quark degrees of freedom at high energies, and the chiral perturbation theories with mesons, at lower energies. The degrees of freedom in the chiral quark models are quarks, and the heavy and light mesons.

The decay of a heavy, B or D, meson to a light π, K, η, ρ meson will involve several energy scales, $\mu \sim 5$ GeV for the B meson, $\mu \sim 2$ GeV for the D meson and $\mu < 1$ GeV for the light mesons and confinement effects. We use the operator product expansion to seperate the energy scales of the interactions.

5.1 Chiral Perturbation Theory, χPT

Chiral perturbation theory, χPT is an effective field theory for the pseudoscalar mesons (π,K, η) containing light quark flavors (u, d, s) [30]. It has a chiral symmetry in the limit where the light quarks are approximately massless. Treating the light quarks as massless is, in some cases, a valid approximation because the masses of the light quarks are much smaller than the masses of the mesons. The Chiral Lagrangian is a representation of QCD with the heavy quark(c, b, t) and gluon degrees of freedom integrated out. The form of the Lagrangian is determined by the $SU_L(3) \times SU_R(3)$ chiral symmetry plus a quark mass matrix ($\mathcal{M}_q$) term that breaks the chiral invariance. The perturbation expansion is in terms of the momentum which is small, instead of the strong coupling constant which is large.

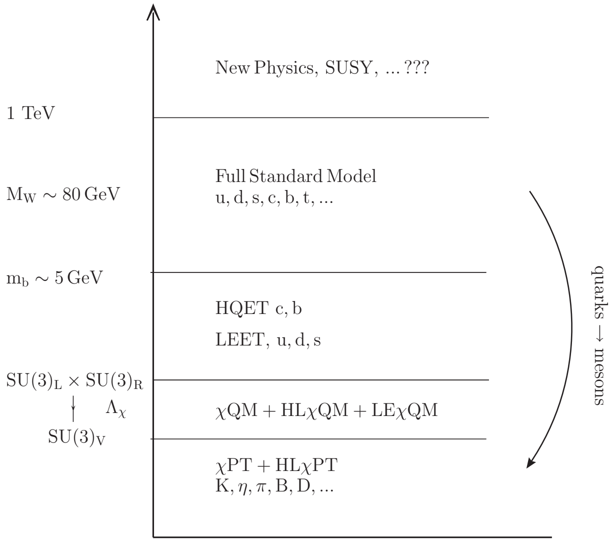

Figure 5.1: The effective theories and quark models used at various energy scales.

The χPT Lagrangian includes the light meson octet (π, K, η) in a 3 by 3 matrix,

$$\Sigma \equiv \xi \cdot \xi = \exp\left(\frac{2i}{f}\Pi\right) \; ; \quad \Pi = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{bmatrix}. \tag{5.1}$$

The effective Lagrangian to lowest order is

$$\mathcal{L}_{\chi PT} = \frac{f^2}{4} Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \frac{f^2 B_0}{2} Tr(\mathcal{M}\Sigma^\dagger + \Sigma\mathcal{M}^\dagger). \tag{5.2}$$

where $\mathcal{M}$ is the quark mass matrix,

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \tag{5.3}$$

f is the pion decay constant and B_0 is related to the quark condensate, $\langle 0|\bar{q}_i q_j|0\rangle = -f^2 B_0/2\delta_{ij}$. Expanding Σ in powers of Π produces the free pion Lagrangian plus higher order terms.

5.2 Heavy Quark Effective Theory, HQET

Another approximate symmetry of QCD is in the limit of heavy quark masses. In the decay of a heavy quark ($Q = b, c$), take the limit where $m_Q \to \infty$ where m_Q is the mass of the heavy quark. While this is a good approximation for the b quark, it can be problematic for the c quark, which has a lighter mass. For decays of heavy to light mesons we use Heavy Quark Effective

Theory (HQET) [26, 5].

In the limit $m_Q \to \infty$, the interactions of the light quark with the heavy quark in the heavy mesons are independent of the heavy quark mass m_Q and the quark spin. The heavy quark becomes a static source of color in its rest frame which is flavor and spin independent. A bound state including a heavy quark ($Q = b, c$) interacts with the light quarks (u, d, s) through low energy gluons. There are corrections on the order of $1/m_Q$.

This theory uses the mass of the heavy quark as an expansion parameter, giving predictions in the limit $m_Q \to \infty$. Q_v is a heavy quark (b or c) with velocity v and mass m_Q,

$$Q_v(x) = e^{-im_Q x\cdot v} P_+(v) Q(x). \tag{5.4}$$

Here, $P_+(v)$ is the projection operator of the momentum state:

$$P_+(v) = \frac{\gamma \cdot v + 1}{2}. \tag{5.5}$$

To lowest (zero) order in $1/m_Q$, the Lagrangian is

$$\mathcal{L}^{(0)} = \overline{Q}_v v \cdot D Q_v + \mathcal{O}(1/m_Q). \tag{5.6}$$

The velocity of v_μ in the heavy quark rest frame has the form, $v_\mu = (1, \vec{0})$, so that $v \cdot v = 1$ and the momentum is written $p = m_Q v + k$. Here $k \ll m_Q$ is the residual momentum. The heavy quark carries most of the energy of the hadron, and is nearly on-shell. The residual momentum k is a measure of how far off shell the heavy quark is.

In the limit $m_Q \to \infty$, the heavy quark propagator is modified:

$$\lim_{m_Q\to\infty} \frac{\gamma \cdot p_Q + m_Q}{P_Q^2 - m^2} = \lim_{m_Q\to\infty} \frac{m_Q v\cdot\gamma + \gamma\cdot k + m_Q}{m_Q^2 v^2 + 2m_Q v\cdot k + k^2 - m_Q^2} = \frac{P_+(v)}{v\cdot k} + \mathcal{O}(1/m_Q). \tag{5.7}$$

The heavy quark propagator can be written

$$S_Q = \frac{P_+(v)}{v \cdot k}. \tag{5.8}$$

To first order in $1/m_Q$, the Lagrangian is

$$\mathcal{L}^{(1)} = \frac{1}{2m_Q}\overline{Q_v}(-C_m g/2\sigma \cdot G + (iD_\perp)^2_{eff})Q_v + \mathcal{O}(m_Q^{-2}). \tag{5.9}$$

$D_\perp$ is the derivative orthogonal to the heavy quark velocity, and $\sigma \cdot G = \sigma^{\mu\nu} G^a_{\mu\nu} t^a$ is the chromo-magnetic term. $C_m = 1$ at tree level.

5.3 Heavy-Light Chiral Perturbation Theory (HLχPT)

HLχPT is based on heavy Quark Effective Field Theory (HQEFT) [8, 3], where to lowest (zeroth) order in m_Q the 0^- and 1^- heavy mesons are degenerate and H_v is the corresponding heavy $(0^-, 1^-)$ meson field doublet

$$H_v = P_+(v)\,(\gamma \cdot P^* - i\gamma_5\, P_5)\ , \tag{5.10}$$

where $P_+(v) = (1 + \gamma \cdot v)/2$ is a projection operator, and v is the velocity of the heavy quark. P^*_μ is the 1^- field and P_5 the 0^- field. The HLχPT Lagrangian,

$$\mathcal{L}_{\mathrm{HL}\chi\mathrm{PT}} \;=\; -Tr(\bar{H}_v\, iv_\mu \partial^\mu H_v) \;+\; Tr(\bar{H}_v{}^a\, H_v^b\, v_\mu\, \mathcal{V}^\mu_{ba}) \;-\; g_{\mathcal{A}}\, Tr(\bar{H}_v{}^a\, H_v^b\, \gamma_\mu\, \gamma_5 \mathcal{A}^\mu_{ba})\ , \tag{5.11}$$

includes these heavy meson fields, coupled to the vector and axial fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$. These are given by

$$\mathcal{V}_\mu \equiv \frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \quad , \quad \mathcal{A}_\mu \equiv -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)\ , \tag{5.12}$$

where

$$\xi \;=\; \exp\{i\Pi/f\} \tag{5.13}$$

and Π is defined in Eq. (5.3). Based on the symmetry of HQEFT, take the current of one heavy and one light quark, integrate out the quarks. This gives the bosonised current for the system with one heavy quark and one light quark ($Q_v\bar{q}$) is [7, 31, 19]:

$$\overline{q_L}\, \gamma^\mu\, Q_v \;\longrightarrow\; \frac{\alpha_H}{2} Tr\left[\xi^\dagger \gamma^\alpha L\, H_v\right]\ , \tag{5.14}$$

where Q_v is a heavy quark field, v is its velocity, and H_v is the corresponding heavy meson field. This bosonized current is compared with the matrix elements defining the meson decay constants f_H (where $H = B, D$), which are the same when QCD corrections below m_Q are neglected [26, 13].

5.4 Large Energy Effective Theory, LEETδ

The LEET (Large energy Effective Theory) was developed for calculations of heavy to light meson decays, such as B or $D \to \pi$. In these decays the light meson has a high energy compared to the mass. LEET includes high energy light quarks and pseudoscalar mesons. LEET was developed by Dugan and Grinstein [11], to study the $B \to D\pi$ decays, where the outgoing pion is highly energetic. Charles et al. [6] applied this to semileptonic heavy to light meson transitions. In LEET, the light quark in the decay has an energy that is large compared to the QCD sale and the mass of the light meson. In this limit, the interactions of the hard quark with the soft light quarks are independent of mass and flavor. The momentum of the decaying heavy meson is defined to be $P{\equiv}Mv$, where $v = (1, 0, 0, 0)$. The momentum of the hard light meson in the rest frame of the decaying meson is $P' = En$ where $n = (1, 0, 0, 1)$ and $n^2 = 0$. Unfortunately, the combination of the standard version of LEET with χQM will lead to infrared divergent loop integrals for $n^2 = 0$. Therefore, we use a modified version where instead of $n^2 = 0$, we use $n^2 = \delta^2 \neq 0$, with $\delta = \nu/E$ where $\nu \sim \Lambda_{QCD}$, such that $\delta \ll 1$. This construction is called LEETδ. This version of LEET uses almost light-like vectors

$$n = (1, 0, 0, +\eta) \qquad ; \quad \tilde{n} = (1, 0, 0, -\eta)\ , \tag{5.15}$$

where $\eta = \sqrt{1-\delta^2}$. This gives

$$n^\mu + \tilde{n}^\mu = 2v^\mu \ \ , n^2 = \tilde{n}^2 = \delta^2 \, , \, v\cdot n = v \cdot \tilde{n} = 1 \ \ , n\cdot\tilde{n} \ = 2-\delta^2 \, . \tag{5.16}$$

The LEETδ Lagrangian is [25],

$$\mathcal{L}_{LEET\delta} \ = \ \bar{q}_n \left(\frac{\gamma\cdot\tilde{n}+\delta}{N} \right) (in\cdot D) q_n \ + \ \mathcal{O}(E^{-1}) \ , \tag{5.17}$$

where $N^2 = 2n \cdot \tilde{n}$. Here, q_n is a hard, light quark

$$q_n(x) = e^{-iEn\cdot x} \mathcal{P}_+ q(x) \, . \tag{5.18}$$

This is analogous to the heavy quark field in (5.4). The projection operators are

$$\mathcal{P}_+ = \frac{1}{N^2} \gamma\cdot n (\gamma\cdot\tilde{n}+\delta) \quad , \, \mathcal{P}_- = \frac{1}{N^2} (\gamma\cdot\tilde{n}-\delta)\gamma\cdot n \ \ , \tag{5.19}$$

The quark propagator is then

$$S_n(k) \ = \ \frac{\gamma\cdot n}{N(n\cdot k)} \ \ , \tag{5.20}$$

which reduces to the LEET propagator in the limit $\delta \to 0$.

This can be used with light mesons with high energy (on the scale of the heavy mesons). In the limit where the light quark energy is large ($E \to \infty$) the flavor and spin of the high energy light quark are not seen by the low energy quarks in the hadron.

In the LEET limit ($M_H \to \infty$ and $E \to \infty$), the form factors for the $H \to P$ can be parametrised as [6]:

$$\langle P | \bar{q} \gamma^\mu Q_v | H \rangle \ = \ 2E \left(\zeta \, n^\mu + \zeta_1 \, v^\mu \right) \ . \tag{5.21}$$

The form factors for the vector current for the $H \to V$ transition can be written,

$$\langle V | \bar{q} \gamma^\mu Q_v | H \rangle = 2iE\zeta_\perp \, \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \epsilon^*_\sigma \, . \tag{5.22}$$

For the axial current, the corresponding matrix element has the form

$$\langle V | \bar{q} \gamma^\mu \gamma_5 Q_v | H \rangle \ = \ 2E\zeta_\perp^{(a)} \left[\epsilon^{*\mu} \ - \ (\epsilon^* \cdot v) \, n^\mu \right] \ + \ 2m_V \zeta_{||} (\epsilon^* \cdot v) \, n^\mu \ \ . \tag{5.23}$$

Here the form factor $\zeta_\perp^{(a)}$ is equal to $\zeta_\perp$ to leading order, and $\zeta_\perp^{(a)}$ and $\zeta_{||}$ scale in the same manner as $\zeta_\perp$ and ζ.

Chapter 6

Quark Models: Bridge between quark and meson models

While chiral perturbation theories give the Lagrangian in terms of mesonic states, the quark models include the quark and gluon degrees of freedom. The χQM is based on χPT and includes the light mesons (π, K, η), the light quarks (u, d, s) and an interaction term. The Heavy Light Chiral Quark Model,(HLχQM), extends the χQM to include the heavy mesons (B, D) and the heavy quarks (b, c) and interactions. The interaction terms in these quark models introduce model parameters, in the form of couplings that cannot be determined from symmetry considerations alone. These model parameters are determined by relating calculations to known quantities. These quark models can be used together with the gluon couplings to calculate decays with soft gluon emissions. For example the nonfactorizable amplitudes in $B \to \pi\pi$ decays in Paper 1 where the quark loops emit soft gluons which form a gluon condensate. Another application is calculating decay modes where the soft gluons emitted from the quark loops produce an η', as in the $D \to V\eta'$ decay modes in Paper 3.

6.1 Chiral Quark Model χQM

The Chiral Quark Model (χQM) contains the light mesons from χPT (π, K, η) and adds interactions with the light quarks (u, d, s) [31]. The Lagrangian contains quarks, Goldstone bosons of the broken $SU(3)_R \times SU(3)_L$ symmetry (the pseudoscalar meson octet π, K, η) and gluons. The χQM can be used to bridge the gap between high energy perturbative QCD and low energy χPT. The chiral quark model can be used to calculate the coefficients of Chiral Perturbation Theory. The model parameters are the quark condensate, gluon condensate and the quark masses.

The χQM Lagrangian includes the chiral perturbation theory Lagrangian with a term added for the interaction between the light quarks (u, d, s) and the light meson octet

$$\mathcal{L}_{\chi QM} = \overline{q}(i\gamma^\mu D_\mu - M_q)q - m(\overline{q}_R\Sigma^\dagger q_L + \overline{q}_L\Sigma q_R), \tag{6.1}$$

where q contains the light quarks (u, d, s), M_q is the light quark mass matrix, and includes the masses (m_u, m_d, m_s). The light meson octet is

$$\xi \cdot \xi = exp(i2\Pi/f) = \Sigma\,. \tag{6.2}$$

Integrating out the quarks will result in the chiral perturbation theory Lagrangian, Eq. (5.2).

The chiral quark model can also be formulated with flavor rotated quark fields,

$$\chi_L = \xi^\dagger q_L\,,\ \ \chi_R = \xi q_R\,, \tag{6.3}$$

where ξ is the light meson octet given in equation (5.3). The Lagrangian with the rotated quark fields is

$$\mathcal{L}^{rot}_{\chi QM} = \overline{\chi}[\gamma\cdot(iD+\mathcal{V}) + \gamma\cdot\mathcal{A}]\chi - m\overline{\chi}\chi\,, \tag{6.4}$$

where m is the constituent mass due to chiral symmetry breaking, and the vector and axial fields are given by

$$\mathcal{V}_\mu = \tfrac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger) \tag{6.5}$$

$$\mathcal{A}_\mu = -\tfrac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)\,. \tag{6.6}$$

The interaction term proportional to m in the Lagrangian (6.1) becomes a pure constituent mass term.

6.2 Including soft gluons in the quark models

The chiral quark model Lagrangian includes colored currents where the light quarks in the loops can emit soft gluons as in equation (4.19). We need to include these gluon emissions in the model. We will include the gluon emissions by using an effective propagator for a light quark interacting with a external gluon field, shown in figure 6.1. We use the method of Novikov et al. [27] for treating soft gluons. In the Fock-Schwinger gauge the gluon field can be written as an expansion in the momentum,

$$A^a_\mu(k) = -i\frac{(2\pi)^4}{2}G^a_{\rho\mu}\frac{\partial}{\partial_{k_\rho}}\delta^{(4)}(k)\ +\ \ldots\,. \tag{6.7}$$

and we keep only the lowest order term. The coupling of the soft gluon to the light quark is then approximated with the gluon at zero momentum,

$$-\frac{1}{2}g_s t^a\gamma^\mu G_{\mu\rho}(0)\frac{\partial}{\partial_{k_\rho}}\Big|_{k=0}\ +\ \ldots \tag{6.8}$$

For the case of a light quark emitting a single soft gluon, we use an effective propagator for a light quark moving in a gluonic background keeping only the first order in the gluon field [32]. This is calculated by applying the gluon vertex in equation (6.8) to the quark propagator. The quark propagator is

$$S(p) = \frac{1}{\gamma\cdot p - m} \tag{6.9}$$

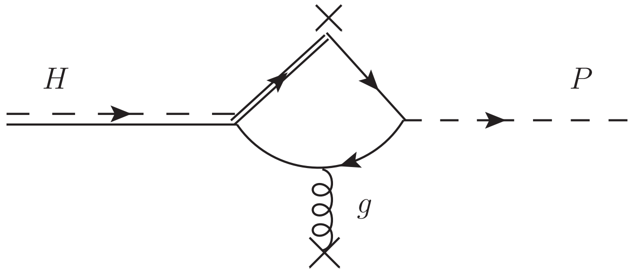

Figure 6.1: $H \to P$ current in the χQM with emission of a soft gluon from the light quark in the loop.

The propagator for a quark with momentum p emitting a soft gluon with momentum k is then

$$S_1(p,G) = -\frac{1}{2} g_s t^a G_{\mu\rho}(0) \frac{\partial}{\partial_{k_\rho}}\Big|_{k=0} S(p)\gamma^\mu S(p-k) \tag{6.10}$$

The gluon momentum is then set to zero, for the soft gluon emission and the propagator to lowest order is

$$S_1(p,G) = -\frac{g_s}{4} t^b G^b_{\mu\rho} \frac{\{\sigma^{\mu\rho}, (\gamma\cdot p + m)\}}{(p^2-m^2)^2}\,, \tag{6.11}$$

where $\{a,b\} = ab + ba$ is the anticommutator. The higher order terms in the expansion of the gluon vertex in equation (6.8) correspond to higher numbers of gluons emitted from the quark. The coupling of a gluon to a heavy quark is suppressed by $1/m_Q$. To lowest order the derivative of the heavy quark propagator gives a factor of the heavy quark velocity v^μ, therefore the vertex is proportional to $v^\mu v^\nu G_{\mu\nu}$ and $G_{\mu\nu}$ is antisymmetric so that $v^\mu v^\nu G_{\mu\nu} = 0$.

Two gluons which are emitted from the quark loops can combine to form a gluon condensate. This is approximated by replacing the two gluon fields with the gluon condensate,

$$g_s^2 G^a_{\mu\nu} G^b_{\alpha\beta} \to \frac{4\pi^2}{(N_c^2-1)} \delta^{ab} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}), \tag{6.12}$$

where $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$ is the vacuum value of the gluon condensate. There are higher order terms in the gluon condensate, but we will use only the lowest order term. This method is used to calculate the nonfactorizable amplitudes, where one gluon is emitted from each colored current. This method is used in Paper 1 to calculate the nonfactorizable contribution to $B \to \pi\pi$ decays and in Paper 3 to calculate the nonfactorizable contribution to $D \to PV$ decays.

6.3 Heavy Light Chiral Quark Model, HLχQM

The heavy light chiral quark model, HLχQM, combines the χQM with HQET to include heavy-light mesons [19, 8, 31]. These are mesons with a heavy quark (c or b) and a light quark (u,d,s). The Lagrangian contains both quark and meson fields with a term that couples the heavy-light mesons with heavy and light quark fields,

$$\mathcal{L}_{HL\chi QM} = \mathcal{L}_{HQEFT} + \mathcal{L}_{\chi QM} + \mathcal{L}_{Int} \tag{6.13}$$

$$\mathcal{L}_{Int} = -G_H[\overline{\chi}_a \overline{H}^a_v Q_v + \overline{Q}_v H^a_v \chi_a] + \frac{1}{2G_3} Tr[\overline{H}^a_v H^a_v]. \tag{6.14}$$

H^a_v is the heavy meson field, that includes both pseudo scalar and vector heavy mesons, $H^a_v \equiv P_+(P^a_\mu \gamma^\mu - iP^a_5 \gamma_5)$, where a is a SU(3) flavor index, Q_v is the reduced heavy quark field in (5.4). G_H and G_3 are coupling constants. The quark-meson coupling G_H is determined within the HLχQM to be [19]

$$G_H^2 \; = \; \frac{2m}{f_\pi^2} \rho \; , \tag{6.15}$$

where ρ is a hadronic quantity of order one. If we start with the HLχQM Lagrangian in Eq. (6.14) and integrate out the quarks, the result is the HLχPT Lagrangian in Eq. (5.11).

Non-factorizable (color-suppressed) effects in non-leptonic decays can be accounted for with gluon condensates. This model is used for example in $B - \overline{B}$ mixing and in $B \to D\overline{D}$ decays, and in the $B \to D\eta'$ decay [14, 12].

6.4 Large energy chiral quark model LEχQM

The Large energy chiral quark model (LEχQM) combines the high energy light mesons and quarks from LEET to the light quarks in the χQM [25]. This model is used to calculate heavy to light currents, for example $B \to \pi$ in Paper 1 and the $D \to \pi, K, \eta$ currents in Paper 2 and Paper 3.

For hard light and chiral quarks coupling to a hard light meson multiplet field M, we extend the ideas of χQM and HLχQM, and assume that the energetic light mesons couple to light quarks with a derivative coupling to an axial current:

$$\mathcal{L}_{\text{int}q} \; \sim \; \bar{q}\, \gamma_\mu \gamma_5 (i\, \partial^\mu M)\, q \; . \tag{6.16}$$

The light energetic mesons are described by an octet $3{\times}3$ matrix field $M = \exp{(+iEn \cdot x)}\, M_n$, where M_n has the same form as Π in (5.3):

$$M_n \;\; = \;\; \begin{pmatrix} \frac{\pi^0_n}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & \pi^+_n & K^+_n \\ \pi^-_n & -\frac{\pi^0_n}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & K^0_n \\ K^-_n & \bar{K}^0_n & -\frac{2\eta_n}{\sqrt{6}} \end{pmatrix} \; . \tag{6.17}$$

Here π^0_n, π^+_n, K^+_n etc. are the hard meson fields with momentum $\sim En^\mu$.

Combining (6.16) with the replacement $\partial^\mu \; \to \; iE\, n^\mu$ yields the LEχQM interaction Lagrangian:

$$\mathcal{L}_{LE\chi QM} \;\; = \;\; G_A\, E \bar{\chi}\, (\gamma \cdot n)\, Z\, q_n \; + \; h.c. \; , \tag{6.18}$$

where q_n is the field corresponding to an energetic light quark having momentum fraction close to one as in Eq. (5.17), and χ represents a soft quark. G_A is an unknown coupling which is

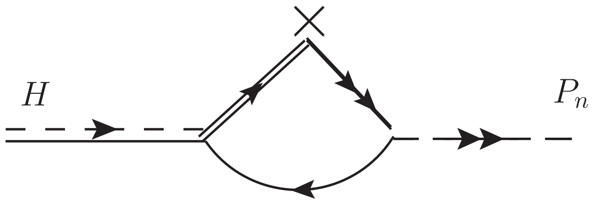

Figure 6.2: $H \to P_n$ current in the LEχQM. The hard pion P_n couples to the hard light quark q_n and a light chiral quark χ.

determined by relating G_A to a known meson decay, for example $B \to \pi$. The field Z contains the hard meson fields M_R and M_L and the soft meson fields ξ,

$$Z = \xi M_n \, R - \xi^\dagger M_n \, L \ . \tag{6.19}$$

The value of G_A can be determined by calculating the current for the heavy meson to light meson transition, $H \to P$ within LEχQM. Within the model, the current for heavy meson to hard light meson is,

$$J^\mu_{tot}(H_v \to M_n) = -2i\zeta \sqrt{\frac{E}{M_H}} \ \mathrm{Tr} \left\{ \gamma^\mu L H_v \, [\gamma \cdot n] \, \xi^\dagger M_L \right\} \ . \tag{6.20}$$

Relating this to the form factors in Eq. (5.21) we obtain

$$\zeta^{(v)} \ = \ \frac{1}{4} m^2 \, G_H \, G_A \, F \sqrt{\frac{M_H}{E}} \ , \tag{6.21}$$

where the hadronic quantity F is obtained from the loop calculations in fig. 2,

$$F \ = \ \frac{N_c}{16\pi} \ + \ \frac{3 \, f_\pi^2}{8m^2 \, \rho}(1 - g_A) \ - \ \frac{(24 - 7\pi)}{768 \, m^4} \, \langle \frac{\alpha_s}{\pi} G^2 \rangle \ \ \simeq 0.09. \tag{6.22}$$

Then we obtain the coupling constant

$$G_A \ = \ \frac{4\zeta^{(v)}}{m^2 \, G_H \, F} \sqrt{\frac{E}{M_H}} \ , \tag{6.23}$$

where $\zeta^{(v)}$ is numerically known to be $\simeq \ 0.3$ for $B \to \pi$ [2], but is expected to be larger for $D \to P$ [28].

Chapter 7

Adding vector mesons to the χQM's

7.1 Including soft vector mesons

The heavy light chiral quark model can be extended to include the light vector mesons (ρ, ω, K^*). This model is used to calculate the decays of D mesons to light vector mesons in Paper 3. We start with the χQM and add in terms for the vector meson mass, and the interaction terms between the quarks and vector mesons. The vector mesons are in a matrix V_μ analogous to the Π matrix that contains the light pseudoscalar mesons (π,K, η). The matrix V_μ is given by

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\phi \end{pmatrix} . \tag{7.1}$$

There is a similar matrix A_μ that contains the axial vector mesons. The vector models starts with the chiral quark model Lagrangian and includes a mass term for the vector mesons and an interaction term,

$$\mathcal{L}_{V\chi QM} = \mathcal{L}_{\chi QM} + \mathcal{L}_{Int} + \mathcal{L}_{mass}. \tag{7.2}$$

The interaction between quarks and the vector and axial mesons is given by

$$\mathcal{L}_{Int} = \overline{\chi}[h_V \gamma^\mu V_\mu + h_A \gamma^\mu \gamma_5 A_\mu]\chi. \tag{7.3}$$

The fields χ are the rotated light quark fields defined in equation (6.3). The vector meson interaction term is

$$\mathcal{L}_{IV} = h_V \overline{\chi}\gamma^\mu V_\mu \chi. \tag{7.4}$$

The mass term is

$$\mathcal{L}_{mass} = \overline{m}_V^2 Tr[V_\mu V^\mu]. \tag{7.5}$$

The strength of the coupling between vector mesons and the rotated quark fields, given by h_V in equation (7.4), can be determined by calculating the vector current within the model. This will include the bare quark loop and the diagrams with two soft gluons emitted as shown in figure 7.1. The first term in figure 7.1 represents the contribution to the current from the bare quark loop,

$$J^\mu = -iN_c \int \frac{d^4}{(2\pi)^4} Tr[(\gamma^\mu L \Lambda^n) iS(p)(h_v \gamma^\nu V_\nu) iS(p)]\,. \tag{7.6}$$

Where Λ^n is defined by

$$\Lambda^n = \xi^\dagger \lambda^n \xi\,, \tag{7.7}$$

and λ^n is the $SU(3)$ group generator. The coupling to the weak current is $\Lambda^n \gamma^\mu L$.

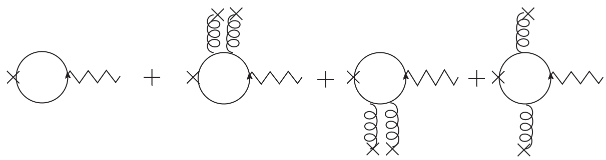

Figure 7.1: The four terms that contribute to the vector current.

The bare quark loop includes divergent integrals I_1 and I_2 which can be evaluated in terms of the quark condensate, gluon condensate and the constituent quark mass. Summing the contribution from all the loops in figure 7.1 gives the relation for h_V,

$$m_V f_V = \frac{1}{2} h_V \left(-\frac{\langle \bar{q}q \rangle}{m} + f_\pi^2 - \frac{1}{8m^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right) \tag{7.8}$$

in terms of measurable quantities and the quark condensate, and the gluon condensate.

The calculation from the current using the mass and decay constant from the ρ meson gives the value

$$h_V = 7.1. \tag{7.9}$$

Deandrea et al. [9], have a coupling constant of $h_V = 5.8$ obtained from setting $h_V = m_V/f_\pi$.

7.2 Including hard vector mesons

The LEχQM can be extended to include energetic vector mesons. This model is used to calculate the semileptonic form factors for $D \to V$ transitions in Paper 2. The hard vector mesons are in a matrix V_n^μ, analogous to the V^μ matrix that contains the low energy vector mesons,

$$V_n^\mu \;=\; \begin{pmatrix} \frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & \rho_n^+ & K_n^{*+} \\ \rho_n^- & -\frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & K_n^{*0} \\ K_n^{*-} & \bar{K}_n^{*0} & -\Phi_n \end{pmatrix}\,. \tag{7.10}$$

Here ρ_n^0, ρ_n^+, K_n^{*+} etc. are the (reduced) vector meson fields corresponding to energetic light vector mesons with momentum $\sim En^\mu$. The vector mesons V_n^μ couple to the hard light quarks, q_n and the soft light quarks χ, with the interaction

$$\mathcal{L}_{LEV} \;\sim\; \bar{\chi}\,(\sigma \cdot F_n)\, q_n \,+\, h.c.\ , \tag{7.11}$$

where

$$F_n^{\mu\nu} = \partial^\mu V_n^\nu - \partial^\nu V_n^\mu + [V_n^\mu, V_n^\nu]. \tag{7.12}$$

Here, we have used a derivative coupling, similar to the case for the energetic meson coupling in Eq. (6.18). We can write the interaction term

$$\mathcal{L}_{LEV} \;=\; G_V\, E\bar{\chi}\,(\gamma\cdot n\gamma\cdot Z_n)\, q_n \;+\; h.c.\;, \tag{7.13}$$

where

$$Z_n^\mu = V_n^\mu \left(\xi\, R + \xi^\dagger\, L\right). \tag{7.14}$$

The coupling G_V is determined by the experimental value for the $D \to \rho$ for D-meson decays or from $B \to \rho$ for B meson decays. The hard vector field Z_n in Eq. (7.14) contains the soft pion fields ξ. In our case where no extra soft pions are going out, we set $\xi \to 1$, and for the momentum space $V_n \to k_M \sqrt{E}(\epsilon_V^*)$, with the isospin factor $k_M = 1/\sqrt{2}$ for ρ^0 and $k_M = 1$ for charged ρ's. For the D-meson with spin-parity 0^- we have $H_v^{(+)} \to P_+(v)(-i\gamma_5)\sqrt{M_H}$. Using this, the involved traces are calculated, and we obtain $J^\mu_{tot}(H \to V_n)$ for the $\overline{D} \to V$ transition. The current for a heavy meson to hard vector meson transition is [28],

$$J^\mu_{tot}(H_v \to V_n) = -2i\sqrt{\frac{E}{M_H}}\;\mathrm{Tr}\left\{\gamma^\mu L H_v \left(\zeta_\perp \gamma\cdot n \;-\; \frac{m_V}{m}\zeta_{||}\right)\,\sigma\cdot F_n\,\xi^\dagger\;[\gamma\cdot n]\right\}\;, \tag{7.15}$$

where the tensor F_n is given by Eq. (7.12) and V_n is a hard vector meson as in Eq. (7.10).

Relating this current to the LEET form factors in Eq. (5.22) we obtain

$$\zeta_\perp \;=\; \frac{m^2}{4}\, G_H\, G_V\, F\,\sqrt{\frac{M_H}{E}}, \tag{7.16}$$

where the factor F, which comes from the loop integrals is the same as the F for the hard meson model in (6.22). Using the equations (6.22), and (7.16), gives [25]

$$G_V \;=\; \frac{2\,\zeta_\perp}{m\, G_H\, F}\sqrt{\frac{E}{M_H}}\;, \tag{7.17}$$

where $\zeta_\perp$ for $D \to \rho$ is known to be $\simeq 0.59$ from CLEO data [10]. Using this data gives a value for $G_V = 10.9$, which is used in the calculations of heavy meson decays.

Chapter 8

Summary of the papers

8.1 Paper 1: On the color suppressed contribution to $\bar{B}_d^0 \to \pi^0\pi^0$

We show that the measured branching fraction of the decay mode $B \to \pi^0\pi^0$ can be accommodated within the standard model by including the nonfactorizable contributions. The $B \to \pi^+\pi^-$ and $B \to \pi^0\pi^0$ decay modes are dynamically different. The $B \to \pi^+\pi^-$ is dominated by the factorizable currents, while the $B \to \pi^0\pi^0$ has a large non-factorizable contribution due to gluon interactions. We use the Large Energy Chiral Quark Model to calculate the non-factorizable decays modes. This allows us to include the contribution of the gluon condensate to the $B \to \pi^0\pi^0$ mode.

8.2 Paper 2: Form factors for semileptonic D decays

We calculate the form factors for the $D \to P$ and $D \to V$ currents. We extend the chiral quark model to include vector mesons VχQM, and high energy vector mesons LEVχQM. A fit to the $D \to V$ vector form factors determines the coupling constants h_V and G_V for the VχQM and the LEVχQM. We average over the values for the $V(0)$ and $A_0(0)$ form factors to determine best fit values of the LEET form factors where we obtain $\zeta \simeq 0.5$, $\zeta_1 \simeq 0.3$, $\zeta_\perp \simeq 0.6$, $\zeta_{||} \simeq 0.7$ and $\zeta_\perp^{(a)} \simeq 0.7$.

8.3 Paper 3: D to V η,η' decays including gluon fusion

In this paper, the $D \to V\eta, \eta'$ decay modes are calculated using the vector chiral quark model described in section VII. This allows us to include the effects of gluon fusion on the production of η'. In this decay, two soft gluons are emitted by the current loops, which then fuse to form an η'. We calculate a value for the $D_s \to \rho\,\eta'$ rate which is higher than the $D_s \to \rho\,\eta$ rate which is consistent with the experimental results. We show that the production of η' by gluon fusion can account for the measured branching fractions for these modes that are higher than would be expected from using flavor symmetry or chiral perturbation theory alone. Previous calculations gave a value for $D_s \to \rho\,\eta'$ that was $\sim 1/3$ of the value for the $D_s \to \rho\,\eta$ branching fraction.

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Color suppressed contribution to $\bar{B}_d^0 \to \pi^0\pi^0$

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The decay modes of the type $B \to \pi\pi$ are dynamically different. For the case $\bar{B}_d^0 \to \pi^+\pi^-$ there is a substantial factorized contribution which dominates. In contrast, the decay mode $\bar{B}_d^0 \to \pi^0\pi^0$ has a small factorized contribution, being proportional to a small Wilson coefficient combination. However, for the decay mode $\bar{B}_d^0 \to \pi^0\pi^0$ there is a sizeable nonfactorizable (color suppressed) contribution due to soft (long distance) interactions, which dominate the amplitude. We estimate the branching ratio for the mode $\bar{B}_d^0 \to \pi^0\pi^0$ in the heavy quark limit for the b quark. In order to estimate color suppressed contributions we treat the energetic light (u, d, s) quark within a variant of Large Energy Effective Theory combined with a recent extension of chiral quark models in terms of model- dependent gluon condensates. We find that our calculated color suppressed amplitude is suppressed by a factor of order Λ_{QCD}/m_b with respect to the factorizable amplitude, as it should according to QCD-factorization. Further, for reasonable values of the constituent quark mass and the gluon condensate, the calculated nonfactorizable amplitude for $\bar{B}_d^0 \to \pi^0\pi^0$ can easily accommodate the experimental value. Unfortunately, the color suppressed amplitude is very sensitive to the values of these model-dependent parameters. Therefore fine-tuning is necessary in order to obtain an amplitude compatible with the experimental result for $\bar{B}_d^0 \to \pi^0\pi^0$. A possible link to the triangle anomaly is discussed.

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I. INTRODUCTION

Because of numerous experimental results coming from *BaBar* and Belle, there is presently great interest in decays of B mesons. LHC will also provide us with more data for such processes. B decays of the type $B \to \pi\pi$ and $B \to K\pi$, where the energy release is big compared to the light meson masses (heavy to light transitions), has been treated within *QCD-factorization* [1] and *Soft Collinear Effective Theory* (SCET) [2]. In the high energy limit, the amplitudes for such decay modes factorize into products of two matrix elements of weak currents, and some nonfactorizable corrections of order α_s can be calculated perturbatively. However, there are additional contributions of order Λ_{QCD}/m_b which cannot be reliably calculated within perturbative theory [1]. The so-called pQCD-model and QCD sum rules have also been used for B-meson decays [3,4].

For decay modes which are of the heavy to heavy type, involving b and c quarks, the decay amplitudes have been described within *Heavy Quark Effective Field Theory* (HQEFT) [5]. Some transitions of *heavy to heavy* type in the heavy quark limits $(1/m_b) \to 0$ like $B - \bar{B}$ mixing [6] has been studied within *Heavy-Light Chiral Perturbation Theory* (HLχPT) [7]. Furthermore, other transitions which are formally *heavy to heavy* in the heavy quark limits $(1/m_b) \to 0$ *and* $(1/m_c) \to 0$, like the Isgur-Wise function [8] for $B \to D$, have been studied within HLχPT [7]. The cases $\bar{B} \to D\bar{D}$ [9] and $B \to D^*\gamma$ [10,11] have also been studied within such a framework, even if the energy release in these processes is above the chiral symmetry breaking scale. Still this framework gives amplitudes of the right order of magnitude. The calculation of such transitions have in addition been supplemented with calculations within a *Heavy-Light Chiral Quark Model* (HLχQM) to determine quantities which are not determined within HLχPT itself [9,11,12].

As pointed out in a series of papers [9,11–13], there are processes which have factorized amplitudes multiplied by a very small Wilson coefficient combination, such that nonfactorized amplitudes are expected to dominate. Examples are $\bar{B}_{d,s}^0 \to D^0\bar{D}^0$ [9], $\bar{B}^0 \to D^0\eta'$ [12] and $\bar{B}_d^0 \to D^0\pi^0$. The latter process $\bar{B}_d^0 \to D^0\pi^0$ was considered recently [13,14]. In that case a heavy b quark decaying to a light, but energetic quark was involved. Then the light energetic quark might be described by an effective theory. The first version of such a framework was *Large Energy Effective Theory* (LEET) [15,16]. The HQEFT covers processes where the heavy quarks carry the main part of the momentum in each hadron. To describe processes where energetic light quarks emerge from decays of heavy b quarks, LEET was introduced [15] and used to study the current for $B \to \pi$ [16].

The idea was that LEET should do for energetic light quarks what HQEFT did for heavy quarks. In HQEFT one splits off the heavy motion from the full heavy quark field, thus obtaining a reduced field depending on the velocity of the heavy quark. Similarly, in LEET one splits off the large energy from the full field of the energetic light quark, thus obtaining an effective description for a reduced light quark which depends on a lightlike four vector. It was later shown that LEET in its initial formulation was incomplete and did not fully reproduce QCD physics [17]. Then LEET was further developed to be fully consistent with QCD and became the Soft Collinear Effective Theory (SCET) [2].

In the present paper we consider decay modes of the type $B \to \pi\pi$. The decay mode $\bar{B}_d^0 \to \pi^- \pi^+$ has a substantial factorized amplitude, given by the current matrix element for $\bar{B}_d^0 \to \pi^+$ transition times the matrix element of the weak current for the outgoing π^-, which is proportional to the pion decay constant f_π. The relevant Wilson coefficient is also the maximum possible, namely, of order 1 times the relevant Cabibbo-Kobayashi-Maskawa (CKM) quark mixing factors and the Fermi coupling constant. This is in contrast to the process $\bar{B}_d^0 \to \pi^0\pi^0$ which is color suppressed. As said above, decays of the type $B \to 2\pi$ have been extensively studied within QCD-factorization, SCET, and QCD sum rule methods [18]. In spite of tremendous efforts it has not been possible to obtain an amplitude compatible with the experimental result $\bar{B}_d^0 \to \pi^0\pi^0$. The purpose of this paper is study this decay mode within an alternative model-dependent framework.

First we point out that the factorized contribution to the decay mode $\bar{B}_d^0 \to \pi^0\pi^0$, which is given by the $B \to \pi$ transition amplitude times the decay constant of the π^0 meson, is almost zero because it is proportional to a very small Wilson coefficient combination. For the dominant nonfactorizable (color suppressed) amplitude for $\bar{B}_d^0 \to \pi^0\pi^0$ we will, as mentioned above, use a model named *Large Energy Chiral Quark Model* (LEχQM) recently constructed and used to handle the process $\bar{B}_d^0 \to \pi^0 D^0$ [13,14]. Here a variant of LEET was combined with ideas from previous chiral quark model (χQM) calculations similarly to what has been done for other nonleptonic decays [6,11,12,19,20].

A priori it might look strange to use the framework of chiral quark models when the energy release is big compared to the chiral symmetry breaking scale Λ_χ. The point is that the motion of the heavy quark or energetic light quark can be split off, and the various versions of heavy-light or large energy chiral quark models and a corresponding chiral perturbation theory (χPT) can be used to describe the redundant strong interactions corresponding to momenta of order 1 GeV and below.

It might be argued that we should have used the full SCET theory as the basis our new model. However, the purpose of our paper is to estimate, in analogy with previous papers [6,11,12,19–23], the effects of soft-gluon emission in terms of gluon condensates, where transverse quark momenta and collinear gluons will not play an essential role. In any case this construction [13] will be a model. Therefore it suffices for our purpose to use the more simple formulation of LEET. We will combine LEET with chiral quark models (χQM) [21,24–27], containing only soft gluons making condensates. In LEχQM [13] an energetic quark is bound to a soft quark with an *a priori* unknown coupling, as proposed in [21]. The unknown coupling is determined by calculating the known $B \to \pi$ current matrix element within the model [13]. This fixes the unknown coupling because the matrix element of this current is known [16]. Then, in the next step, we use this coupling to calculate the nonfactorized (color suppressed) amplitude contribution to $\bar{B}_d^0 \to \pi^0\pi^0$ in terms of the lowest dimension gluon condensate, as have been done for other nonleptonic decays [6,11,12,19,20]. After the quarks have been integrated out, we obtain an effective theory containing soft light mesons as in HLχPT, but also fields describing energetic light mesons. A similar idea with a combination of SCET with HLχPT is considered in [28]. The LEχQM was constructed in analogy with the previous *Heavy-Light Chiral Quark Model* (HLχQM) [20] and may be considered to be an extension of that model.

One might think that to be completely consistent, we should also have calculated the Wilson coefficients within a relevant large energy framework. For this purpose the use of LEET would be dubious because it is an incomplete theory as mentioned above. However, as we will see below, the main uncertainty in our final amplitude will be due to uncertainty in our model-dependent gluon condensate due to emission of soft gluons. Therefore the Wilson coefficients calculated within full QCD as in [29] will be appropriate for our purpose.

In the next Sec. II we present the weak four quark Lagrangian and its factorized and nonfactorizable matrix elements. In Sec. III we present our version of LEET, and in Sec. IV we present the new model LEχQM to include energetic light quarks and mesons. In Sec. V we calculate the nonfactorizable matrix elements due to soft gluons expressed through the (model-dependent) quark condensate. In Sec. VI we give the results and conclusion.

II. THE EFFECTIVE LAGRANGIAN AT QUARK LEVEL

We will study decays of $\bar{B}_d^0$ generated by the weak quark process $b \to u\bar{u}d$. We restrict ourselves to processes where the b quark decays. This means the quark level processes $b \to du\bar{u}$. Processes where the anti- b quark decays proceed analogously. The effective weak Lagrangian at quark level is [29] (neglecting penguin operators)

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{\rm ub} V_{\rm ud}^* [c_A Q_A + c_B Q_B], \qquad (1)$$

where the subscript L denotes the left-handed fields: $q_L \equiv Lq$, where $L \equiv (1 - \gamma_5)/2$ is the left-handed projector in Dirac-space. The local operator products $Q_{A,B}$ are defined as

$$Q_A = 4\bar{u}_L \gamma_\mu b_L \bar{d}_L \gamma^\mu u_L; \qquad Q_B = 4\bar{u}_L \gamma_\mu u_L \bar{d}_L \gamma^\mu b_L. \qquad (2)$$

In these operators summation over color is implied. In Eq. (1), c_A and c_B are Wilson coefficients. At tree level

$c_A = 1$ and $c_B = 0$. At one loop level, a contribution to c_B is also generated, and c_A is slightly increased. These effects are handled in terms of the *Renormalization Group Equations* (RGE) [29], and the coefficients can be calculated at for instance $\mu = m_b$ or $\mu = 1$ GeV. Using the color matrix identity

$$2t^a_{in}t^a_{lj} = \delta_{ij}\delta_{ln} - \frac{1}{N_c}\delta_{in}\delta_{lj},$$

and Fierz rearrangement, the amplitudes for the processes $\bar{B}^0 \to \pi^+\pi^-$ may be written as

$$\mathcal{M}_{\pi^+\pi^-} = 4\frac{G_F}{\sqrt{2}}V_{\rm ub}V^*_{\rm ud}\Big[\Big(c_A + \frac{1}{N_{\rm c}}c_B\Big)\langle\pi^-|\bar{d}_L\gamma_\mu u_L|0\rangle \times\langle\pi^+|\bar{u}_L\gamma^\mu b_L|\bar{B}^0\rangle + 2c_B\langle\pi^+\pi^-|\bar{d}_L\gamma_\mu t^a u_L\bar{u}_L\gamma^\mu t^a b_L|B^0\rangle\Big], \quad (3)$$

and for $\bar{B}^0 \to \pi^0\pi^0$

$$\mathcal{M}_{\pi^0\pi^0} = 4\frac{G_F}{\sqrt{2}}V_{\rm ub}V^*_{\rm ud}\Big[\Big(c_B + \frac{1}{N_{\rm c}}c_A\Big)\langle\pi^0|\bar{u}_L\gamma_\mu u_L|0\rangle \times\langle\pi^0|\bar{d}_L\gamma^\mu b_L|\bar{B}^0\rangle + 2c_A\langle\pi^0\pi^0|\bar{d}_L\gamma_\mu t^a b_L\bar{u}_L\gamma^\mu t^a u_L|B^0\rangle\Big]. \quad (4)$$

Here the terms proportional to $2c_A$ and $2c_B$ with color matrices inside the matrix elements are the genuinely nonfactorizable contributions.

Since c_A is of order one and c_B of order $-1/3$ [12,13], we refer to the coefficients

$$c_f \equiv \Big(c_A + \frac{1}{N_{\rm c}}c_B\Big) \simeq 1.1; \qquad c_{nf} \equiv \Big(c_B + \frac{1}{N_{\rm c}}c_A\Big) \simeq 0, \quad (5)$$

as favorable (c_f) and nonfavorable (c_{nf}) coefficients, respectively. Thus, the decay mode $\bar{B}^0_d \to \pi^+\pi^-$ has a sizeable factorized amplitude proportional to c_f. In contrast, the decay mode $\bar{B}^0_d \to \pi^0\pi^0$ has a factorized amplitude proportional to the nonfavorable coefficient c_{nf} which is close to zero. In this case we expect the nonfactorizable term (involving color matrices) proportional to $2c_A$ to be dominant, i.e. the last line of Eq. (4) dominates. A substantial part of this paper is dedicated to the calculation of this nonfactorizable contribution to the $\bar{B}^0_d \to \pi^0\pi^0$ decay amplitude.

Thus the main task of this paper will be to calculate the matrix element of the operator Q_C consisting of the product of two colored currents occurring in the last line of Eq. (4):

$$Q_C = (\bar{d}_L\gamma_\mu t^a b_L)(\bar{u}_L\gamma^\mu t^a u_L) \quad (6)$$

for the color suppressed process $\bar{B}^0_d \to \pi^0\pi^0$. This matrix element will be estimated in Sec. V where we use the LEχQM to estimate nonfactorizable amplitudes in terms of emission of soft gluons making gluon condensates.

III. AN ENERGETIC LIGHT QUARK EFFECTIVE DESCRIPTION (LEETδ)

An energetic light quark might, similarly to a heavy quark, carry practically all the energy E of the meson it is a part of. The difference is that now the mass of the energetic quark is close to zero compared to the heavy quark mass m_Q and E, which are assumed to be of the same order of magnitude. We assume that the energetic light quark is emerging from the decay of a heavy quark Q with momentum $p_Q = m_Q v + k$. The heavy quark is described by the HQEFT Lagrangian for the reduced quark field Q_v [5]:

$$\mathcal{L}_{\rm HQET} = \bar{Q}_v(iv\cdot D)Q_v + \mathcal{O}(1/m_Q), \quad (7)$$

where Q_v is the reduced heavy quark field (often named h_v in the literature), v its four velocity and m_Q the mass of the heavy quark.

The momentum of the light energetic light quark q can be written

$$p^\mu_q = En^\mu + k^\mu, \qquad |k^\mu| \ll |En^\mu|, \qquad m_q \ll E, \quad (8)$$

where E, which is of order m_Q, is the energy of the energetic light quark, m_q is the light quark mass. Further, n is the lightlike four vector which might be chosen to have the space part along the z-axis, $n^\mu = (1; 0, 0, 1)$, in the frame of the heavy quark where $v = (1, \underline{0})$. Then $(v\cdot n) = 1$ and $n^2 = 0$. Inserting this in the regular quark propagator, in the limit where the approximations in (8) are valid, we obtain the propagator

$$S(p_q) = \frac{\gamma\cdot p_q + m_q}{p^2_q - m^2_q} \to \frac{\gamma\cdot n}{2n\cdot k}. \quad (9)$$

This propagator is the starting point for the Large Effective Theory (LEET) constructed in Ref. [16].

Unfortunately, the combination of LEET with χQM will lead to infrared divergent loop integrals for $n^2 = 0$ (see Sec. IV). Therefore, the formalism was modified [13,14] and instead of $n^2 = 0$, we use $n^2 = \delta^2$, with $\delta = \nu/E$ where $\nu \sim \Lambda_{\rm QCD}$, such that $\delta \ll 1$. An expansion in δ will then within our model be equivalent to an expansion in $\Lambda_{\rm QCD}/m_b$.

In the following we describe the modified LEET [16] where we keep $\delta \neq 0$ with $\delta \ll 1$. We call this construction LEETδ [13] and define the *almost* lightlike vectors

$$n = (1, 0, 0, +\eta), ; \qquad \tilde{n} = (1, 0, 0, -\eta), \quad (10)$$

where $\eta = \sqrt{1-\delta^2}$. This means that

$$n^\mu + \tilde{n}^\mu = 2v^\mu, \qquad n^2 = \tilde{n}^2 = \delta^2, \qquad v\cdot n = v\cdot\tilde{n} = 1, \qquad n\cdot\tilde{n} = 2 - \delta^2. \quad (11)$$

In the following we use the projection operators given by

$$\mathcal{P}_+ = \frac{1}{N^2}\gamma\cdot n(\gamma\cdot\tilde{n}+\delta),$$
$$\mathcal{P}_- = \frac{1}{N^2}(\gamma\cdot\tilde{n}-\delta)\gamma\cdot n, \tag{12}$$

where $N=\sqrt{2n\cdot\tilde{n}}=2+\mathcal{O}(\delta^2)$. One factors out the main energy dependence, just as was analogously done in HQEFT, and define the projected reduced quark fields [16]

$$q_\pm(x) = e^{iEn\cdot x}\mathcal{P}_\pm q(x),$$
$$q(x) = e^{-iEn\cdot x}[q_+(x)+q_-(x)]. \tag{13}$$

As in [16], the field q_- was eliminated and one obtained for $q_+\equiv q_n$ the effective Lagrangian [13]:

$$\mathcal{L}_{\mathrm{LEET}\delta} = \bar{q}_n\Big(\frac{\gamma\cdot\tilde{n}+\delta}{N}\Big)(in\cdot D)q_n + \frac{1}{E}\bar{q}_nXq_n+\mathcal{O}(E^{-2}), \tag{14}$$

which (for $\delta=0$) is the first part of the SCET Lagrangian. The operator X is given in [13]. Equation (14) yields the LEETδ quark propagator

$$S_n(k)=\mathcal{P}_+\Big[\frac{\gamma\cdot\tilde{n}+\delta}{N}(n\cdot k)\Big]^{-1}=\frac{\gamma\cdot n}{N(n\cdot k)}, \tag{15}$$

which reduces to (9) in the limit $\delta\to 0$. In addition, for a light energetic quark, the propagator within SCET [2] will for small transverse quark momenta $p_\perp\to 0$ coincide with Eq. (15).

Based on LEET, it was found [16] in the formal limits $M_H\to\infty$ and $E\to\infty$, that a heavy $H=(B,D)$ meson decaying by the weak hadronic vector current V^μ to a light pseudoscalar meson is described by a matrix element $\langle P|V^\mu|H\rangle$ of the form

$$\langle P|V^\mu|H\rangle = 2E[\zeta^{(v)}(M_H,E)n^\mu+\zeta_1^{(v)}(M_H,E)v^\mu], \tag{16}$$

where

$$\zeta^{(v)}=C\frac{\sqrt{M_H}}{E^2},\qquad C\sim(\Lambda_{\mathrm{QCD}})^{3/2},\qquad \frac{\zeta_1^{(v)}}{\zeta^{(v)}}\sim\frac{1}{E}. \tag{17}$$

This behavior is consistent with the energetic quark having x close to 1, where x is the quark momentum fraction of the outgoing pion [16].

IV. EXTENDED CHIRAL QUARK MODEL FOR HEAVY AND ENERGETIC LIGHT QUARKS (LEχQM)

The chiral quark model (χQM) [24,25] and the Heavy-Light Chiral Quark Model (HLχQM) [20], include meson-quark couplings and thereby allow us to calculate amplitudes and chiral Lagrangians for processes involving heavy quarks and low-energy light quarks. In this section we will extend these models to include also hard, energetic light quarks.

For the pure light and soft sector the χQM Lagrangian can be written as [19,24]

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{\chi}[\gamma\cdot(iD+\mathcal{V})+\gamma\cdot\mathcal{A}-m]\chi, \tag{18}$$

where m is the constituent mass term being due to chiral symmetry breaking. The small current mass term is neglected here. Here we have introduced the flavor rotated fields $\chi_{L,R}$:

$$\chi_L=\xi^\dagger q_L,\qquad \chi_R=\xi q_R, \tag{19}$$

where q is the light quark flavor triplet and

$$\xi=\exp\{i\Pi/f\},$$
$$\Pi=\begin{pmatrix}\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0\\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}}\end{pmatrix}. \tag{20}$$

Further, $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are vector and axial vector fields, given by

$$\mathcal{V}_\mu\equiv\frac{i}{2}(\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger),$$
$$\mathcal{A}_\mu\equiv-\frac{i}{2}(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger). \tag{21}$$

To couple the heavy quarks to mesons there are additional meson-quark couplings within HLχQM [20]:

$$\mathcal{L}_{\mathrm{int}}=-G_H[\bar{\chi}_a\bar{H}_v^aQ_v+\bar{Q}_vH_v^a\chi_a], \tag{22}$$

where Q_v is the (reduced) heavy quark field and H is the heavy $(0^-,1^-)$ meson field(s)

$$H_v^{(+)}=P_+(v)(\gamma\cdot P^*-i\gamma_5P_5), \tag{23}$$

P^*_μ being the 1^- and P_5 the 0^- fields, and $P_+(v)=(1+\gamma\cdot v)/2$. The quark-meson coupling G_H is determined within the HLχQM to be [20]

$$G_H^2=\frac{2m}{f_\pi^2}\rho,\qquad \rho=\frac{(1+3g_A)}{4(1+\frac{m^2N_c}{8\pi f_\pi^2}-\frac{\eta_H}{2m^2f_\pi^2}\langle\frac{\alpha_s}{\pi}G^2\rangle)}, \tag{24}$$

where $\eta_H=(8-\pi)/64$. The quantity ρ is of order one.

For hard light quarks and chiral quarks coupling to a hard light meson multiplet field M, one extends the ideas of χQM and HLχQM, and assume that the energetic light mesons couple to light quarks with a derivative coupling to an axial current [13]:

$$\mathcal{L}_{\mathrm{int}q}\sim\bar{q}\gamma_\mu\gamma_5(i\partial^\mu M)q. \tag{25}$$

One combines LEETδ with the χQM and assume that the ingoing light quark and the outgoing meson are energetic and have the behavior $\exp(\pm iEn\cdot x)$ as in (13). To describe (outgoing) light energetic mesons, we use an octet 3×3 matrix field $M=\exp(+iEn\cdot x)M_n$ of the same form as Π in (20):

$$M_n = \begin{pmatrix} \frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & \pi_n^+ & K_n^+ \\ \pi_n^- & -\frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & K_n^0 \\ K_n^- & \bar{K}_n^0 & -\frac{2\eta_n}{\sqrt{6}} \end{pmatrix}, \quad (26)$$

where π_n^0, π_n^+, K_n^+ etc. are the energetic light meson fields with momentum $\sim En^\mu$.

Combining (25) with the use of the rotated soft quark fields in (19) and using $\partial^\mu \to iEn^\mu$ one arrives at the ansatz for the LEχQM interaction Lagrangian:

$$\mathcal{L}_{\text{int}q\delta} = G_A E\bar{\chi}(\gamma \cdot n) Z q_n + \text{H.c.}, \quad (27)$$

where q_n represents an energetic light quark having momentum fraction close to 1 and χ represents a soft quark (see Eq. (19)). Further, the coupling G_A is determined by physical requirements [13,16], and

$$Z = \xi M_R R - \xi^\dagger M_L L. \quad (28)$$

Here M_L and M_R are both equal to M_n, but they have formally different transformation properties. This is analogous to the use of quark mass matrices $\mathcal{M}_q$ and $\mathcal{M}_q^\dagger$ in standard *Chiral Perturbation Theory* (χPT). They are in practice equal, but have formally different transformation properties.

The axial vector coupling introduces a factor $\gamma \cdot n$ to the vertex (see (27)), which simplifies the Dirac algebra within the loop integrals. In order to calculate the nonfactorizable contribution, one must first find a value for the large energy light quark bosonization coupling G_A. This was done [13] by requiring that our model should be consistent with the Eqs. (16) and (17). Applying the Feynman rules of LEχQM [13] we obtain the following bosonized current (before soft-gluon emission forming a condensate is taken into account):

$$J_0^\mu(H_{v_b} \to M_n) = -N_c \int đk \, \text{Tr}\{\gamma^\mu L i S_v(k)[-iG_H H_{v_b}^{(+)}] \times iS_\chi(k)[iEG_A \gamma \cdot nZ] iS_n(k)\}, \quad (29)$$

where $đk \equiv d^D k/(2\pi)^D$ (D being the dimension of space-time), and

$$S_v(k) = \frac{P_+(v)}{v \cdot k}, \qquad S_\chi(k) = \frac{(\gamma \cdot k + m)}{k^2 - m^2}, \qquad S_n(k) = \frac{\gamma \cdot n}{Nn \cdot k}, \quad (30)$$

are the propagators for heavy quarks described by (18), for light constituent quarks, and (14) for light energetic quarks. The presence of the left projection operator L in Z ensures that we only get contributions from the left-handed part of the interaction in (27), that is, $Z \longrightarrow -\xi^\dagger M_L L$. The contribution in (29) corresponding to the $B \to \pi$ current is illustrated by the lower part of the diagram in Fig. 1.

Loop diagrams within LEχQM depend on momentum integrals of the form

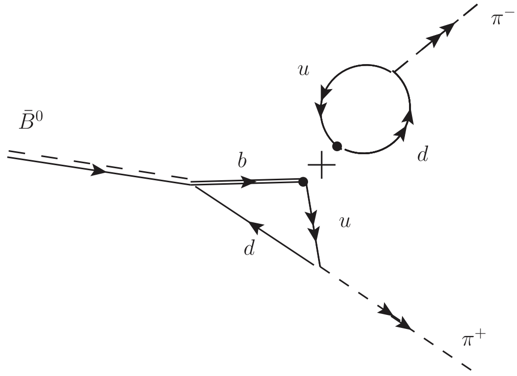

FIG. 1. The factorized contribution to the $B^0 \to \pi^+\pi^-$ decay, as described in combined HLχQM and LEχQM. Double lines, single lines and the single line with two arrows are representing heavy quarks, light soft quarks and light energetic quarks, respectively. Heavy mesons are represented by a single line combined with a parallel dashed line, and a light energetic pion is represented by a dashed line with a double arrow.

$$K_{rst} = \int \frac{đk}{(v \cdot k)^r (k \cdot n)^s (k^2 - m^2)^t}, \quad (31)$$

$$K_{rst}^\mu = \int \frac{đk k^\mu}{(v \cdot k)^r (k \cdot n)^s (k^2 - m^2)^t} = K_{rst}^{(v)} v^\mu + K_{rst}^{(n)} n^\mu. \quad (32)$$

These integrals have the important property that $K_{rst}^{(n)}$ dominates over $K_{rst}^{(v)}$ and K_{rst} with one power of $1/\delta$. In the present model, we choose $\nu = m$ which is of order Λ_{QCD}. Thus the constituent light quark mass m is the equivalent of Λ_{QCD} within our model. Some details of the calculation of the $B \to \pi$ is given in Ref. [13].

To calculate emission of soft gluons we have used the framework of Novikov *et al.* [30]. In that framework the ordinary vertex containing the gluon field A_μ^a will be replaced by the soft-gluon version containing the soft-gluon field tensor $G_{\mu\nu}^a$:

$$ig_s t^a \Gamma^\mu A_\mu^a \to -\frac{1}{2} g_s t^a \Gamma^\mu G_{\mu\nu}^a \frac{\partial}{\partial k_\nu} \ldots|_{k=0} \quad , \quad (33)$$

where k is the momentum of the soft-gluon. (Using this framework one has to be careful with the momentum routing because the gauge where $x^\mu A_\mu^a = 0$ has been used.) Here $\Gamma^\mu = \gamma^\mu$, v^μ, or $n^\mu(\gamma \cdot \tilde{n} + \delta)/N$ for a light soft quark, heavy quark, or light energetic quark, respectively. Our loop integrals are *a priori* depending on the gluon momenta $k_{1,2}$ which are sitting in some propagators. These gluon momenta disappear after having used the procedure in (33). (Note that the derivative has to be taken with respect to the whole loop integral).

Emision from the heavy quark or light energetic quark are expected to be suppressed. This will be realized in most cases because the gluon tensor is antisymmetric, and therefore such contributions are often proportional to

$$G^a_{\mu\nu}v^\mu v^\nu = 0, \quad \text{or} \quad G^a_{\mu\nu}n^\mu n^\nu = 0. \tag{34}$$

However, there are also contributions proportional to

$$G^a_{\mu\nu}v^\mu n^\nu \neq 0, \tag{35}$$

analogous to what happens in some diagrams for the Isgur-Wise diagram where there are two different velocities v_b and v_c [31]. Such contributions appear within our calculation when two soft gluons are emitted from the heavy quark line.

Using the prescription [19,20,25,30]

$$g_s^2 G^a_{\mu\nu}G^a_{\rho\lambda} \to 4\pi^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{1}{12}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho}), \tag{36}$$

for the gluon condensate one obtains the leading bosonized current [13]

$$J^\mu_{\text{tot}}(H\to M) = -i\frac{G_H G_A}{2}m^2 F\,\text{Tr}\{\gamma^\mu L H_v^{(+)}[\gamma\cdot n]\xi^\dagger M_L\}, \tag{37}$$

where the quantity F obtained from loop integration is *a priori* containing a linearly divergent integral, which is related to the axial coupling $g_{\mathcal{A}}$, and can be traded for $g_{\mathcal{A}}$. One obtains [13] for the quantity F:

$$F = \frac{3f_\pi^2}{8m^2\rho}(1-g_A) + \frac{N_c}{16\pi} - \frac{(24-7\pi)}{768m^4}\langle\frac{\alpha_s}{\pi}G^2\rangle. \tag{38}$$

Note that F is dimensionless. The parameter ρ is given in (24). Numerically, it was found [13] that $F \simeq 0.08$.

In order to obtain the HLχPT Lagrangian terms $\text{Tr}(\bar{H}^a H^b v_\mu \mathcal{V}^\mu_{ba})$ and $\text{Tr}(\bar{H}^a H^b \gamma_\mu\gamma_5 \mathcal{A}^\mu_{ba})$, having coefficients $+1$ and $-g_{\mathcal{A}}$ respectively, one calculates quark loops with attached heavy meson fields and vector and axial vector fields $\mathcal{V}^\mu$ or $\mathcal{A}^\mu$. Then logarithmic and linearly divergent integrals obtained within the loop diagrams are identified with physical quantities or quantities of the model [19,20,24,25].

In order to fix G_A in (27), we compare (16) with (37). In our case where no extra soft pions are going out, we put $\xi\to 1$, and for the momentum space $M_L \to k_M\sqrt{E}$, with the isospin factor $k_M = 1/\sqrt{2}$ for π^0 (while $k_M = 1$ for charged pions). Moreover for the B meson with spin-parity 0^- we have $H_v^{(+)} \to P_+(v)(-i\gamma_5)\sqrt{M_H}$. Using this, the involved traces are easily calculated, and we obtain $J^\mu_{\text{tot}}(H\to M)$ for the case $\bar{B}^0_d \to \pi^+$:

$$J^\mu_{\text{totx}}(\bar{B}^0_d\to\pi^+) = \frac{G_H G_A}{2}(\sqrt{M_H E})m^2 F n^\mu. \tag{39}$$

Using the Eqs. (16), (38), and (39), one obtains [13]

$$G_A = \frac{4\zeta^{(v)}}{m^2 G_H F}\sqrt{\frac{E}{M_H}}, \tag{40}$$

where $\zeta^{(v)}$ is numerically known [32]. Within our model, the analogue of Λ_{QCD} is the constituent light quark mass m. To see the behavior of G_A in terms of the energy E, the quantity C in (17) is written as $C \equiv \hat{c}m^{3/2}$, which gives

$$G_A = \left(\frac{4\hat{c}f_\pi}{mF\sqrt{2\rho}}\right)\frac{1}{E^{3/2}}, \tag{41}$$

which explicitly displays the behavior $G_A \sim E^{-3/2}$. In terms of the number N_c of colors, $f_\pi \sim \sqrt{N_c}$ and $F \sim N_c$ which gives the behavior $G_A \sim 1/\sqrt{N_c}$, i.e. the same behavior as the coupling G_H in (22).

The bosonized current in (37) can now be written as

$$J^\mu_{\text{tot}}(H\to M) = -2i\zeta^{(v)}\sqrt{\frac{E}{M_H}}\text{Tr}\{\gamma^\mu L H_v^{(+)}[\gamma\cdot n]\xi^\dagger M_L\}. \tag{42}$$

V. NONFACTORIZABLE PROCESSES IN LEχQM

In this section we calculate the nonfactorizable contribution to $\bar{B}^0_d \to \pi^0\pi^0$ in Eq. (4). This will be formulated as a quasifactorized product of two colored currents, as illustrated in Fig. 2. Then the nonfactorized aspects enters through color correlation between the two parts, using Eq. (36). Such a calculation within HLχQM and HLχPT is done previously [9] for $\bar{B}^0_{d,s} \to D^0\bar{D}^0$. Here we will use the colored current for $B\to\pi$, within the LEχQM presented in the preceding section; see the diagram in Fig. 2. Using the G_A value from the preceding section, we may now calculate the nonfactorizable contribution to the process by adding one soft-gluon to each loop. Then we

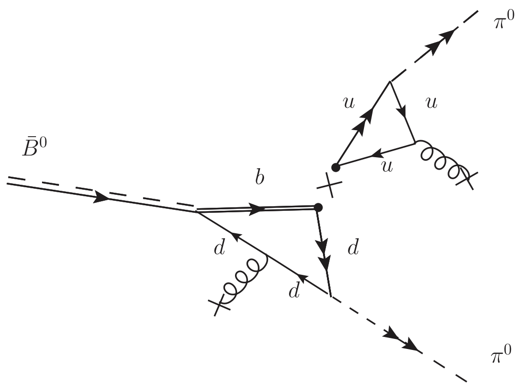

FIG. 2. Nonfactorizable contribution containing large energy light fermions and mesons. There is also a corresponding diagram where the outgoing antiquark $\bar{u}$ is hard.

calculate the decay width for $\bar{B}_d^0 \to \pi^0\pi^0$ from this nonfactorizable amplitude, and compare our results with a experiment.

For a low-energy quark interacting with one soft-gluon, one might in simple cases use the effective propagator [19,33]

$$S_1^G(k) = \frac{g_s}{4} t^a G^a_{\mu\nu} \frac{(2m\sigma^{\mu\nu} + \{\sigma^{\mu\nu}, \gamma \cdot k\})}{(k^2 - m^2)^2}, \qquad (43)$$

where $\{a, b\} \equiv ab + ba$ denotes the anticommutator. This expression is consistent with the prescription in (33), and can be used for the diagram in Fig. 2.

Then one gets [13] the following contribution to the bosonized colored $B \to \pi$ current, shown in the lower part of the diagram in Fig. 2:

$$J^\mu_{1G}(H \to M)^a = -\int d\!\!{}^-k\, \mathrm{Tr}\{\gamma^\mu L t^a i S_v(k)[-iG_H H_v^{(+)}] \times i S_1^G(k)[iEG_A \gamma \cdot n Z] i S_n(k)\}, \qquad (44)$$

where a is a color octet index. Once more, we deal with the momentum integrals of the types in (31) and (32). Taking the color trace, rewriting (44), we obtain a contribution of the form

$$J^\mu_{1G}(H_b \to M)^a = g_s G^a_{\alpha\beta} T^{\mu;\alpha\beta}(H_b \to M), \qquad (45)$$

where the contribution from the lower part of the diagram in Fig. 2 alone is to leading order in δ:

$$T^{\mu;\alpha\beta}(H_b \to M) = \frac{G_H G_A}{128\pi} \epsilon^{\sigma\alpha\beta\lambda} n_\sigma \mathrm{Tr}(\gamma^\mu L H_v^{(+)} \gamma_\lambda \xi^\dagger M_L), \qquad (46)$$

where $E \cdot \delta = m$ has been explicitly used.

There are also other diagrams not shown. In one case the gluon is emitted from the energetic quark. This diagram is zero due to (34). Furthermore, there is a diagram not shown where the gluon is emitted from the heavy quark which contains a nonzero part due to (35). This gives an additional contribution to the colored $B \to \pi$ current which is nonzero. However, this one will be projected out because it should be proportional to the Levi-Civita tensor to give a nonzero result for the $\bar{B}_d^0 \to \pi^0\pi^0$ amplitude as a whole, as will be seen from Eq. (49) below.

The colored current for an outgoing π^0 should now be calculated in the LEχQM (see the upper part of the diagram in Fig. 2), and we find

$$J^\mu_{1G}(M_{\tilde{n}})^a = -\int d\!\!{}^-k\, \mathrm{Tr}\{\gamma^\mu L t^a i S_1^G(k)[iEG_A \gamma \cdot \tilde{n} Z] i S_{\tilde{n}}(k)\}. \qquad (47)$$

This colored π^0 current has the general form

$$J^\mu_{1G}(M_{\tilde{n}})^a = g_s G^a_{\alpha\beta} T^{\mu;\alpha\beta}(M_{\tilde{n}}), \qquad (48)$$

where the tensor T is given by

$$T^{\mu;\alpha\beta}(M_{\tilde{n}}) = 2\left(-\frac{G_A E}{4}\right) Y \tilde{n}_\sigma \epsilon^{\sigma\alpha\beta\mu} \mathrm{Tr}[\lambda^X M_{\tilde{n}}], \qquad (49)$$

where the λ^X within the trace is the appropriate Gell-Mann SU(3) flavor matrix. For an outgoing hard π^0 this trace has the value $\sqrt{E/2}$ when going to the momentum space. The explicit factor 2 in front of this expression comes from the corresponding diagram, where in the upper part of the diagram the antiquark could be hard and the quark could be soft and emit a soft-gluon. The factor Y contains the result of loop momentum integration. The relevant loop integral is now

$$K^\mu_{012} = \int \frac{d\!\!{}^-k k^\mu}{(k \cdot n)(k^2 - m^2)^2} = \frac{I_2}{\delta^2} n^\mu, \qquad (50)$$

which gives

$$Y = -iI_2 = \frac{f_\pi^2}{4m^2 N_c} \lambda \equiv Y_\lambda \equiv \frac{1}{4m^2 N_c}\left(f_\pi^2 - \frac{1}{24m^2}\langle \frac{\alpha_s}{\pi} G^2 \rangle\right). \qquad (51)$$

Here the parameter λ is of order 10^{-2} to 10^{-1} and very sensitive to small variations in the model-dependent parameters m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$.

It is easily seen that the experimental value of the $\bar{B}_d^0 \to \pi^0\pi^0$ amplitude can be accommodated for a constituent mass m around 220 Mev and a value for $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4}$ around 315 MeV. These values are of the same order as used in previous articles [6,9,11–13,20–22]. But in contrast to these previous cases the present amplitude for $\bar{B}_d^0 \to \pi^0\pi^0$ is very sensitive to variations of the model-dependent parameters m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. Or more specific, the colored current $J^\mu_{1G}(M_{\tilde{n}})^a$ in (47)–(49) is very sensitive to these parameters. In other words, Y_λ has to be fine-tuned in order to produce the experimental result.

In a recent paper [31] an extra mass parameter was introduced in the propagator of heavy quarks. One might do the same for propagator of the light energetic quark, and use

$$S_n = \frac{\gamma \cdot n}{N(n \cdot k + \Delta_n)}. \qquad (52)$$

This would also bring this propagator more in harmony with the SCET propagator if $\Delta_n \sim p_\perp^2/E$. This will to first order in Δ_n give an extra contribution in the loop integral obtained from the diagram in Fig. 2. However, also taking into account the corresponding diagram where the light antiquark is the energetic one, this first order term in Δ_n cancels. But there will be terms of second order in Δ_n, which are of order δ^2. Such contributions have to be considered together with higher order (in δ) terms obtained from the interaction given by the operator X in (14).

One should note that the colored current given by (48) and (49) is determined by a triangle diagram. Thus one might speculate if it can in some way be related to the triangle anomaly. Namely, the diagram in Fig. 2 would have, for standard full propagators, the mathematical properties of the diagram relevant for the triangle anomaly. Using dimensional regularization in this case, with dimension $D = 4 - 2\epsilon$, the loop integration gives an divergent result $\sim I_2 \sim 1/\epsilon$ while the corresponding Dirac trace is $\sim\epsilon$. Thereby one obtains a finite expression for the triangle diagram in that case. However, in the present case we have replaced one of the standard (full) quark propagators by the SCET-like propagator $S_{\tilde{n}}$. Then the trace will not be $\sim\epsilon$ while the corresponding loop integral is still divergent $\sim 1/\epsilon$. This means that the diagram is in total divergent. Within our various chiral quark models including heavy quarks and light energetic quarks, the naive dimensional regularization (NDR) has been used, and divergent integrals have been identified with physical parameters [6,9,11–13,20–22,31]. Using other schemes additional finite terms of type ϵ/ϵ might appear [19], and some parameters might have to be redefined.

We also note that the description of the anomaly is rather tricky when going from the low-energy process $\pi^0 \to 2\gamma$ to higher energies where some cancellations occur [34,35]. In [34] the high energy processes $Z \to \pi^0\gamma$ and $\gamma^* \to \pi^0\gamma$ was studied. (Here the high energy virtual photon γ^* is coming from an energetic e^+e^- pair). In this case a part of the amplitude corresponding to low-energy decay $\pi^0 \to 2\gamma$ is cancelled. But there is a remaining *anomaly tail* relevant for some high energy processes [34,35]. Trying to adapt such a description in our case, the tensor T in (49) for an outgoing π^0 and soft-gluon would be replaced by

$$T^{\mu;\alpha\beta}(An) = \frac{I_{An}}{4\pi^2 f_\pi \sqrt{2}} p^\pi_\sigma \epsilon^{\sigma\alpha\beta\mu}, \tag{53}$$

where we have taken into account that couplings and color traces are different from the calculations in [34,35]. The quantity I_{An} is an integral given by

$$I_{An} = \int_0^1 \frac{x dx}{\eta x(1-x) - 1}, \tag{54}$$

where $\eta \equiv p_\pi^2/m^2$. Using, as before, m as a constituent mass and $p^\pi = E\tilde{n}$ would give $\eta = 1$ leading to $I_{An} \simeq 0.6$. However, as the anomaly tail is of perturbative character [34,35] one might think that it is more relevant to use masses closer to the current masses of order 5 to 10 MeV. In this case one has an asymptotic behavior $I_{An} \simeq ln(\eta)/\eta$, and this would give values for I_{An} of order 10^{-2}.

Now we use (36) and also include the Fermi coupling the Cabibbo-Kobayashi-Maskawa matrix elements, and the coefficient $2c_A$ for the nonfactorizable contributions to the amplitude, where c_A is the Wilson coefficient for the $\mathcal{O}_A$ local operator. Using Eqs. (45) and (47) we find the effective Lagrangian at mesonic level for the nonfactorizable contribution to $\bar{B}_d^0 \to \pi^0\pi^0$:

$$\mathcal{L}^{\mathrm{LE}\chi\mathrm{QM}}_{\mathrm{Non.fact.}} = \frac{4\pi^2 c_A}{3}\left(4\frac{G_F}{\sqrt{2}} V_{\mathrm{ub}} V^*_{\mathrm{ud}}\right)\langle\frac{\alpha_s}{\pi} G^2\rangle S(H_b \to M_n M_{\tilde{n}}), \tag{55}$$

where $S(H_b \to M_n M_{\tilde{n}})$ is the tensor product

$$S(H_b \to M_n M_{\tilde{n}}) \equiv T^{\mu;\alpha\beta}(H_b \to M_n) T_{\mu;\alpha\beta}(M_{\tilde{n}}). \tag{56}$$

Using Eqs. (46) and (49), and $n \cdot \tilde{n} \simeq 2$, we find the amplitude expressed entirely by known parameters, we find an explicit expression for $S(H_b \to MM_{\tilde{n}})$ in the case $\bar{B}_d^0 \to \pi^0\pi^0$:

$$S(\bar{B}_d^0 \to \pi^0\pi^0) = 6(\frac{1}{\sqrt{2}})^2 \frac{G_A^2 G_H}{128\pi} Y E^2 \sqrt{M_B}. \tag{57}$$

We will now compare this nonfactorizable amplitude for $\bar{B}_d^0 \to \pi^0\pi^0$ with the factorized amplitude which dominates $\bar{B}_d^0 \to \pi^+\pi^-$:

$$\mathcal{M}_{\pi^+\pi^-} = \left(4\frac{G_F}{\sqrt{2}} V_{\mathrm{ub}} V^*_{\mathrm{ud}}\right) \cdot c_f \cdot \left(\frac{1}{2} J_\mu(\pi^-)\right) \cdot \left(\frac{1}{2} J^\mu(\bar{B}_d^0 \to \pi^+)\right), \tag{58}$$

where

$$J_\mu(\pi^-) = f_\pi E \tilde{n}_\mu,\; J^\mu(\bar{B}_d^0 \to \pi^+) = 2En^\mu \zeta^{(v)}. \tag{59}$$

The form factor $\zeta^{(v)}$ is defined in (16) and (17).

Using the Eqs. (40) and (55)–(59), we find the following ratio between the nonfactorized for $\bar{B}_d^0 \to \pi^0\pi^0$ and the factorized amplitudes $\bar{B}_d^0 \to \pi^+\pi^-$ is

$$r \equiv \frac{\mathcal{M}(\bar{B}_d^0 \to \pi^0\pi^0)_{\mathrm{Non-Fact}}}{\mathcal{M}(\bar{B}_d^0 \to \pi^+\pi^-)_{\mathrm{Fact}}} = \frac{c_A}{c_f}\frac{\kappa}{N_c}\frac{E\zeta^{(v)}}{\sqrt{mM_B}}, \tag{60}$$

where κ is a model-dependent hadronic factor

$$\kappa = \frac{\pi N_c \langle\frac{\alpha_s}{\pi}G^2\rangle Y}{2F^2 m^4 \sqrt{2\rho}}. \tag{61}$$

It will be interesting how the ratio r scales with energy E. Using the scaling behavior for $\zeta^{(v)}$ with $C = \hat{c} m^{3/2}$ in (17) we find for the ratio r:

$$r \simeq \frac{c_A}{c_f}\frac{\kappa\hat{c}}{N_c}\frac{m}{E}. \tag{62}$$

Our calculations show that the ratio r of the amplitudes are suppressed by $1/N_c$, as it should. The ratio is also scaling like m/E. Because $E \simeq m_b/2$ and m is the equivalent of Λ_{QCD} in our model, we have found that the nonfactorized amplitude is suppressed by $\Lambda_{\mathrm{QCD}}/m_b$ as required by the analysis in Ref. [1].

Concerning numerical predictions from our model, we have to stick to Eq. (60). The measured branching ratios for $\bar{B}_d^0 \to \pi^-\pi^+$ and $\bar{B}_d^0 \to \pi^0\pi^0$ are $(5.13 \pm 0.24) \times 10^{-6}$ and $(1.62 \pm 0.31) \times 10^{-6}$, respectively [36]. In order to predict the experimental value solely with the mechanism considered in this section, we should have $r \simeq 0.56 \pm 0.11$.

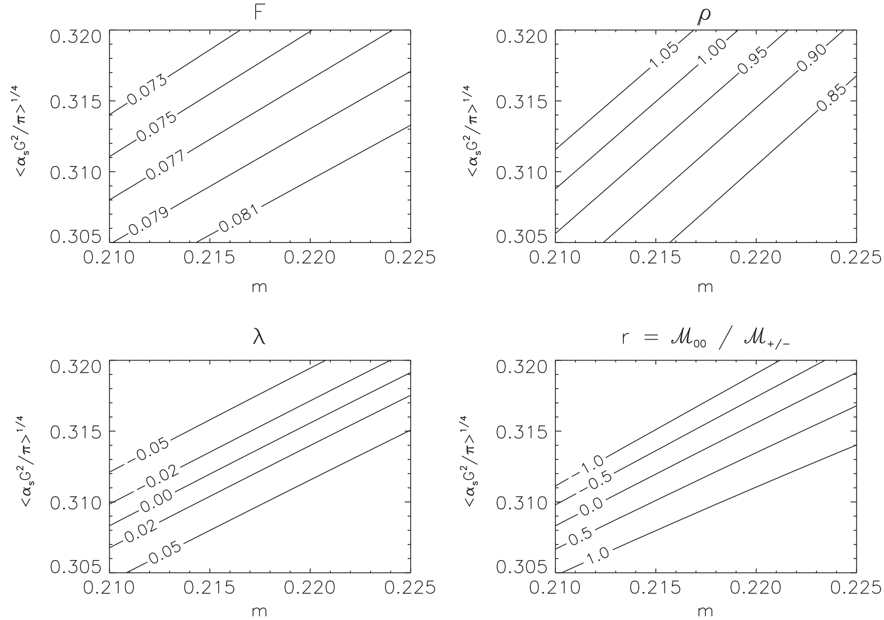

FIG. 3. Plots for the quantities F, ρ, λ and r in terms of m and $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4}$. We observe that for reasonable values of these parameters the ratio r can take a wide range of values such that fine-tuning is required to reproduce the experimental value.

Numerically, we use $\zeta^{(v)} \simeq 1/3$ [32]. In previous papers on the heavy-light chiral quark model constituent masses $m \sim 220$ MeV and $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4} \sim 315$ MeV has been used. From the plot of r in Fig. 3, we observe that the experimental value of r can easily be accommodated by values of such orders. The bad news is that in our case the value of Y_λ and thereby κ and r is very sensitive to the explicit choice of m and $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4}$. Thus fine-tuning has to be used.

We also find that the perturbative anomaly tail will numerically reproduce the amplitude for $I_{An} \simeq 3.2 \times 10^{-2}$, corresponding to a quark mass $m_0 \simeq 11$ MeV, i.e. of same order of magnitude as typical current quark masses. Using a hybrid description with a quark model with constituent quark masses for the colored $\bar{B}^0 \to \pi^0$ current in (44)–(46), and the anomaly tail description [34,35] for the colored π^0 current in (47)–(49), is not preferable. Also, such a hybrid description also fails to show the behavior Λ_{QCD}/m_b required by QCD-factorization. Still it might be interesting that we can numerically match the colored current for outgoing π^0 with the anomaly tail description.

Note that there are also mesonic loop contributions similar to those contributing to processes of the type $B \to D\bar{D}$ and $B \to \gamma D$ [9,11]. For those processes intermediate $D^*(1^-)$ mesons contributed. In the present case the analogous contributions would involve energetic vector

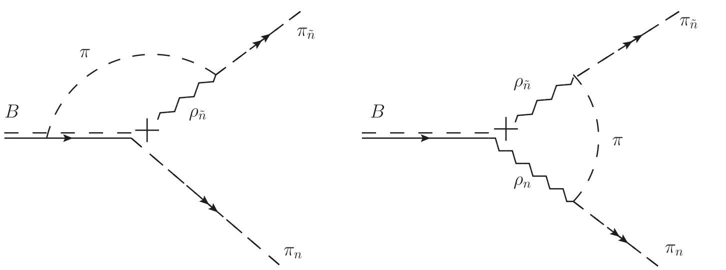

FIG. 4. Meson loops for $\bar{B}^0_d \to \pi\pi$. The zigzag lines represent energetic ρ mesons. The dashed lines with double arrows are energetic light mesons and the dashed line with no arrow is a soft pion.

mesons ρ_n, and we would need the amplitudes for $B \to \rho_n \rho_{\tilde{n}}$. Such loops are shown in Fig. 4. The diagram to the right would be calculable within an extended theory involving energetic vector mesons. Unfortunately while the diagram to the left would be dubious because typical loop momenta would significantly exceed 1 GeV, and would require insertion of *ad hoc* form factors or should be handled within dispersion relation techniques. Both diagrams would of course require knowledge of the $\rho_n \pi_n \pi$ coupling in Fig. 4. In any case such calculations are beyond the scope of this paper.

VI. CONCLUSION

We have pointed out that the factorized amplitude for process $\bar{B}_d^0 \to \pi^0\pi^0$ is proportional to a Wilson coefficient combination close to zero. Thus the nonfactorizable contributions dominate the amplitude for this decay mode. To handle the nonfactorizable contributions we have extended previous chiral quark models for the pure light quark case [24] used in [19,23,25], and the heavy-light case [20] used in [6,9,11,12,21,22], to include also energetic light quarks.

We have found that within our model we can account for the amplitude needed to explain the experimental branching ratio for $\bar{B}_d^0 \to \pi^0\pi^0$ [36]. In addition, the ratio r between the nonfactorizable and factorized amplitude scales as $\Lambda_{\rm QCD}/m_b$ in agreement with QCD-factorization [1]. However, the bad news is that the calculated amplitude is very sensitive to our model-dependent parameters, i.e. the constituent quark mass m, and the gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. Anyway, final state interactions should be present [37].

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Form factors for semileptonic D decays

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We study transition form factors for decays of D mesons. That is, we consider matrix elements of the weak left-handed quark current for the transitions $D \to P$ and $D \to V$, where P and, V are light pseudoscalar or vector mesons, respectively. Our motivation to perform the present study of these form factors is future calculations of nonleptonic decay amplitudes. We consider the transition form factors within a class of chiral quark models. Especially, we study how the large energy effective theory limit works for D-meson decays. In this paper, we extend previous work on the case $B \to \pi$ to the case $D \to P = \pi, K$. Further, we extend our previous model based on the large energy effective theory to the entirely new case $D \to V = \rho, K^*, \ldots$ To determine some of the parameters in our model, we use existing data and results based on some other methods like lattice calculations, light-cone sum rules, and heavy-light chiral perturbation theory. We also obtain some new predictions for relations between form factors.

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I. INTRODUCTION

In the present paper, we study transition form factors of D-meson decays, i.e., the $D \to P = \pi, K, \ldots$ and $D \to V = \rho, K^*, \ldots$ transition form factors within extended chiral quark models. Knowledge of the semileptonic form factors is, of course, necessary to calculate factorizable contributions to the nonleptonic decays of mesons. Further, knowledge about these form factors might determine or at least restrict some parameters of our models and thereby indirectly be of importance for our (model dependent) calculations for nonleptonic decays. We are, of course, aware of the technical challenges when calculating nonleptonic decays of D mesons [1], and we will come back to this in a future publication.

The $D \to P$ and $D \to V$ transition form factors have been calculated by various methods. These have their strength in different regions of the momentum transfer q squared, from q^2 near zero for light-cone sum rules (LCSR) [2–7] to $q^2 = q^2_{\rm max}$ for the heavy-light chiral perturbation theory (HLχPT) [8]. For earlier work, see, for instance, Refs. [9,10]. In the region $q^2 \to 0$ where the momentum of the outgoing meson is high, one might study form factors within the large energy effective theory (LEET), invented in Ref. [11] and further elaborated in Ref. [12]. This theory was later developed into the soft collinear effective theory (SCET) [13].

In the region of large momentum transfer ($q^2 \to q^2_{\rm max}$), lattice QCD has been used [14–17]. Form factors have been calculated [8,18–20] within HLχPT, which is based on the heavy quark effective theory (HQEFT). Calculations within HLχPT have also been supplemented [21] by calculations within the heavy-light chiral quark model (HLχQM) [21–25]. Within the heavy quark symmetry, there are corrections of the order $\mathcal{O}(1/m_c)$, which will be larger in the D sector than in the B sector. In any case, the form factors are influenced by nearby meson poles. Heavy ($H = B,\ D$) to light ($P = \pi,\ K,\ \eta$) transitions have also been treated in a mesonic picture [26] and in relativistic quark models [27–29].

Our intention is to find how well chiral quark models describe the form factors. Namely, in the next step, we want to calculate nonfactorizable contributions to nonleptonic decays of D mesons. Then we ought to know how well the chiral quark models work in various energy regions, and specifically we need to know the various form factors within the LEET. Some form factors are relatively well known. But for some cases, we perform additional model-dependent studies. Therefore, these models will be briefly presented. Compared to previous work, we will, in this paper, also include light vectors $V = (\rho, \omega, K^*)$. The transitions $H \to P$ and $H \to V$ are illustrated in Fig. 1.

II. DECOMPOSITION OF SEMILEPTONIC FORM FACTORS

For an heavy pseudoscalar meson $H = B,\ D$ decaying into a light pseudoscalar meson P, the vector current $J^\mu_V(H \to P)$ depends on the involved momenta p_H and p. This current can be decomposed into two form factors. There are two commonly used decompositions,

$$J^\mu_V(H \to P) = F_+(q^2)(p_H + p)^\mu + F_-(q^2)(p_H - p)^\mu \tag{1}$$

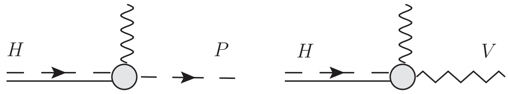

FIG. 1. Diagrams for $H \to P$ and $H \to V$ transitions at the mesonic level. The vertical line denotes a virtual electroweak boson (W, Z, γ).

and

$$J_V^\mu(H \to P) = F_1(q^2)\left[(p_H + p)^\mu - \frac{(M_H^2 - m_P^2)}{q^2} q^\mu\right] + \frac{M_H^2 - m_P^2}{q^2} F_0(q^2) q^\mu, \quad (2)$$

where $q = p_H - p$ is the momentum transfer and M_H and m_P are the masses of the heavy and light mesons, respectively. The relations between the form factors in Eqs. (1) and (2) are

$$F_1 = F_+; \qquad F_0 = F_+ + \frac{q^2}{M_H^2 - m_P^2} F_-. \quad (3)$$

The heavy to light transitions $H \to V$, where $H = (B, D)$ and $V = (\rho, K^*, \omega, \phi)$, with mass m_V, can proceed through both vector and axial currents. These can be decomposed into (in total) four form factors. The vector current depends on only one form factor $V(q^2)$, and is commonly parametrized as

$$J_V^\mu(H \to V) = \langle V(p,\varepsilon)|\bar{q}\gamma^\mu Q|H(p_H)\rangle = \frac{2V(q^2)}{M_H + m_V} \varepsilon^{\mu\nu\rho\sigma}(\varepsilon_V^*)_\nu p_\rho (p_H)_\sigma, \quad (4)$$

where ε_V^* is the polarization vector for the outgoing vector meson V. The axial current includes three form factors, A_0, A_1, and A_2, and is written as

$$J_A^\mu(H \to V) = \langle V(p,\varepsilon)|\bar{q}\gamma^\mu\gamma_5 Q|H\rangle = (M_H + m_V)\left(\varepsilon_V^{*\mu} - \frac{(\varepsilon_V^* \cdot q)}{q^2} q^\mu\right) A_1(q^2) - \left((p + p_H)^\mu - \frac{M_H^2 - m_V^2}{q^2} q^\mu\right) \frac{(\varepsilon_V^* \cdot q)}{M_H + m_V} A_2(q^2) + \frac{2m_V(\varepsilon_V^* \cdot q)}{q^2} q^\mu A_0(q^2). \quad (5)$$

For the light leptons ($l = \mu$, e), the amplitudes for $D \to Vl\nu$ are dominated by the form factors $V(q^2)$, $A_1(q^2)$, and $A_2(q^2)$. The vector form factor $V(q^2)$ is dominated by vector resonances, while the $A_1(q^2)$ and $A_2(q^2)$ are dominated by axial resonances, and the $A_0(q^2)$ form factor is dominated by the pseudoscalar resonances. Bećirević and Kaidalov [30] proposed a double pole form for the $F_+(q^2)$ function. This includes the pole at a heavy vector meson H^* for the first pole and a term that includes contributions for higher mass resonances in an effective pole. The form factors, $F = F_+$, V, A_0, etc., can be written in the generic form

$$F(q^2) = \frac{F(0)}{[1 - \frac{q^2}{m_{\text{pole}}^2}][1 - \frac{\alpha q^2}{m_{\text{pole}}^2}]}, \quad (6)$$

where the parameter α parametrizes the contribution of the higher mass resonances into an effective pole.

III. ASYMPTOTIC BEHAVIOR OF FORM FACTORS

The HQET and LEET give constraints on the structure of the form factors. From the HQET one can estimate the behavior of the form factors in the limit of zero recoil (see Ref. [21] and references therein):

$$F_+ \sim \sqrt{M_H}; \qquad F_- \sim \frac{1}{\sqrt{M_H}}. \quad (7)$$

The form factors in the LEET limit, with $p_H^\mu = M_H v^\mu$ and $p = En^\mu$, can be parametrized as [12]

$$\langle P|\bar{q}\gamma^\mu Q_v|H\rangle = 2E(\zeta n^\mu + \zeta_1 v^\mu). \quad (8)$$

The 4-vectors v, n are given by $v = (1; \vec{0})$ and $n = (1; 0, 0, 1)$ in the rest frame of the decaying heavy meson. Here, the ζ should scale with energy E as [12]

$$\zeta \equiv \zeta(M_H, E) = C\frac{\sqrt{M_H}}{E^2}, \qquad C \sim (\Lambda_{\text{QCD}})^{3/2}, \qquad \frac{\zeta_1}{\zeta} \sim \frac{1}{E}. \quad (9)$$

In the limit $M_H \to \infty$ and $E \to \infty$, the ratio $\zeta_1/\zeta \to 0$. An explicit relation between ζ_1 and ζ will be given later in Sec. VI. The LEET may be used to estimate form factors at large recoil, where the momentum carried by the electroweak bosons (W, Z, γ) is at a minimum, that is, for $q^2 \to 0$. Using Eq. (9) for small q^2, i.e., for $E \simeq M_H/2$, one obtains the behavior [31]

$$F_+ \sim F_0 \sim \frac{1}{M_H^{3/2}}. \quad (10)$$

We will need the following relations between the various form factors and the quantities ζ_i of the LEET formalism:

$$F_1 = F_+ = \zeta + \frac{E}{M_H}\zeta_1; \qquad F_- = -\zeta + \frac{E}{M_H}\zeta_1. \tag{11}$$

It should be noted that in Ref. [12] ζ_1 is neglected because it is suppressed by $1/E$, as seen in Eq. (9) and later in Eq. (64).

For transitions $H(0^-) \to V(1^-)$, one obtains in the LEET limit ($M_H \to \infty$ and $E \to \infty$) for the vector current:

$$\langle V|\bar{q}\gamma^\mu Q_v|H\rangle = 2iE\zeta_\perp \varepsilon^{\mu\nu\rho\sigma} v_\nu n_\rho (\varepsilon_V^*)_\sigma. \tag{12}$$

Here, the form factor $\zeta_\perp$ scales in the same way as ζ in Eq. (9) but with a different factor C:

$$\zeta_\perp = C_\perp \frac{\sqrt{M_H}}{E^2}. \tag{13}$$

For the axial current, the corresponding matrix element should have the form

$$\langle V|\bar{q}\gamma^\mu\gamma_5 Q_v|H\rangle = 2E\zeta_\perp^{(a)}[\varepsilon_V^{*\mu} - (\varepsilon_V^* \cdot v)n^\mu] + 2m_V\zeta_\parallel(\varepsilon_V^* \cdot v)n^\mu. \tag{14}$$

Here, the form factor $\zeta_\perp^{(a)}$ is equal to $\zeta_\perp$ to leading order, and $\zeta_\perp^{(a)}$ and $\zeta_\parallel$ scale in the same manner as $\zeta_\perp$ and ζ.

We will need the relations between the various form factors V, A_0, A_1, and A_2 and the quantities ζ_i in the LEET case [12],

$$V = \left(1 + \frac{m_V}{M_H}\right)\zeta_\perp; \quad A_0 = \frac{m_V}{M_H}\zeta_\perp^{(a)} + \left(1 - \frac{m_V^2}{M_H E}\right)\zeta_\parallel,$$
$$A_1 = \left(\frac{2E}{M_H + m_V}\right)\zeta_\perp^{(a)}; \quad A_2 = \left(1 + \frac{m_V}{M_H}\right)\left[\zeta_\perp^{(a)} - \frac{m_V}{E}\zeta_\parallel\right], \tag{15}$$

which should be valid in the $q^2 \to 0$ limit. These form factors are plotted in Sec. VII.

IV. HEAVY-LIGHT CHIRAL PERTURBATION THEORY

The HLχPT is based on the HQEFT, where, to lowest (zero) order in $1/m_Q$, the 0^- and the 1^- heavy mesons are degenerate and described by a field H_v,

$$H_v = P_+(v)(\gamma \cdot P^* - i\gamma_5 P_5), \tag{16}$$

where $P_+(v) = (1 + \gamma \cdot v)/2$ is a projection operator and v is the velocity of the heavy quark. Further, P_μ^* is the 1^- field, and P_5 is the 0^- part of the heavy meson field. These mesonic fields enter the Lagrangian of the HLχPT,

$$\mathcal{L}_{\mathrm{HL}\chi\mathrm{PT}} = -\mathrm{Tr}(\bar{H}_v i v_\mu \partial^\mu H_v) + \mathrm{Tr}(\bar{H}_v{}^a H_v^b v_\mu \mathcal{V}_{ba}^\mu) - g_A \mathrm{Tr}(\bar{H}_v{}^a H_v^b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu), \tag{17}$$

where a, b are $SU(3)$ flavor indices and $g_A = 0.59$ is the axial coupling. Further, $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are vector and axial vector fields, for pseudoscalar mesons given by

$$\mathcal{V}_\mu \equiv \frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \qquad \mathcal{A}_\mu \equiv -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \tag{18}$$

where

$$\xi = \exp\{i\Pi/f\},$$
$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \tag{19}$$

where $\eta \equiv \eta_8$. To calculate the form factors for the η and η', we use the η_8, η_0 basis,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}. \tag{20}$$

Here, we use the value of $\theta = 13.7^\circ$ from Ref. [32].

Based on the symmetry of the HQEFT, the bosonized current for decay of a system with one heavy quark and one light quark ($Q_v\bar{q}$) forming H_v is [8,33]

$$\bar{q}_L \gamma^\mu Q_v \longrightarrow \frac{\alpha_H}{2}\mathrm{Tr}[\xi^\dagger \gamma^\mu L H_v], \tag{21}$$

where Q_v is a reduced heavy quark field that is described in Sec. V, v is its velocity, and H_v is the corresponding heavy meson field. This bosonized current is compared with the matrix elements defining the meson decay constants f_H (where $H = B, D$). These currents are the same when QCD corrections below m_Q are neglected (see Refs. [25,34]). The $H \to P$ form factors obtained from HLχPT are illustrated in Fig. 2. Using the double pole parametrization, form factors were calculated in Ref. [19]:

$$F_+(q^2_{\max}) = \frac{\alpha_H}{2\sqrt{M_H}f} g_A \frac{M_H}{m_P + \Delta_{H^*}} + \frac{\tilde{\alpha}}{2\sqrt{M_H}f}\tilde{g}\frac{M_H}{m_P + \Delta H'^*}. \tag{22}$$

Here, α_H is defined,

$$\alpha_H = f_H\sqrt{M_H}. \tag{23}$$

The term involving $\tilde{\alpha}$ and $\tilde{g}$ is the contribution from the higher resonances. (In Ref. [21], the higher resonance term was not included. Instead, some nonpole terms were included). One can also include light vectors with an effective coupling to heavy mesons, given by Ref. [20],

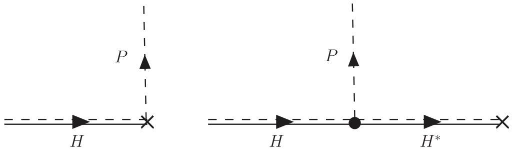

FIG. 2. Contributions to F_+ within the HLχPT. The single pole term is shown on the right.

$$\mathcal{L}_{\mathrm{HHV}} = i\frac{g_V}{\sqrt{2}}\lambda \mathrm{Tr}(\bar{H}_v H_v \sigma_{\mu\nu} F_V^{\mu\nu}), \tag{24}$$

where the coupling $g_V \simeq 5.9$ and

$$F_V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu + [V^\mu, V^\nu]. \tag{25}$$

This term will give a dominating pole term in the $D \to V$ form factor similar to the one for $D \to P$ above. From Eq. (24), one obtains [20]

$$V(q^2_{\max}) = -\frac{\alpha_H}{2\sqrt{M_H} f}\frac{g_V\lambda}{\sqrt{2}}\frac{M_H}{m_V + \Delta_{H^*}} + \frac{\tilde{\alpha}}{2\sqrt{M_H} f}\tilde{\lambda}\frac{M_H}{m_V + \Delta H'^*}, \tag{26}$$

where the second term is coming from higher resonances. It might also be calculated in the HLχQM following closely the calculation for $D^* \to D\gamma$ [35]. The coupling $\tilde{\lambda}$ is a corresponding term for higher resonances.

V. VARIOUS CHIRAL QUARK MODELS

Calculating the matrix elements of quark currents, we have used chiral quark models. Within such models, one splits the various quark fields into different categories according to their relevant energy and mass scale.

In Sec. V A, we consider the ordinary soft quark fields q and the related soft flavor rotated fields χ (representing soft constituent light quarks) at energies ranging from the constituent quark mass $m \sim 220$ MeV up to the chiral symmetry breaking scale Λ_χ of order 1 GeV. These are the quarks of the chiral quark model (χQM) [25,36–39]. where light quarks couple to light mesons.

In Sec. V B, we also indicate how the quark fields chiral quark model of Sec. V A might be connected to light vectors $V = \rho, K^*, \ldots$, in a model we call VχQM to be described in Sec. V B. In Sec. V C, we describe the HLχQM [21–25] based on the HQEFT [34]. Here, the motion of the heavy quark with mass m_Q ($= m_b$ or m_c) with momentum p_Q is split in the leading term $m_Q v$, where v is the velocity of the heavy quark, and the motion for the reduced quark field Q_v is corresponding to momenta k of order a few hundred MeV such that $p_Q = m_Q v + k$. The reduced heavy quark field Q_v (also called h_v in the literature) is together with a quark field of χQM coupled to heavy meson fields H_v.

In Sec. V D we describe the large energy chiral quark model (LEχQM) based on the LEET [11,12] and invented in Ref. [40], and later used in Ref. [41]. Here, the motion of the energetic light quark with energy E and 4-momentum $p_q = En + k$ (where n is a lightlike vector) is split off, and the reduced energetic quark fields q_n have momenta k analogous to the reduced heavy quark fields. Here, the reduced energetic quark fields q_n combine with the ordinary χQM to make energetic light pseudoscalar meson fields M_n. In the second part of Sec. V D, we describe how this LEχQM can be extended to light energetic vectors V_n^μ. This is an invention that is new in this paper.

A. χQM for low-energy light quarks

For the pure light sector, the chiral quark model gives the interactions between light quarks and light pseudoscalar mesons. The χQM Lagrangian can be written as

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{q}(i\gamma^\mu D_\mu - \mathcal{M}_q)q - m(\bar{q}_R\Sigma^\dagger q_L + \bar{q}_L\Sigma q_R), \tag{27}$$

where q is the light quark flavor triplet, $\mathcal{M}_q$ is the current mass matrix, and $\Sigma = \xi \cdot \xi$ contains the light pseudoscalar mesons. (The current mass term $\mathcal{M}_q$ will often be neglected). The covariant derivative D_μ contains soft gluons, which might form gluon condensates within the model. The quantity m is interpreted as the constituent light quark mass appearing after the spontaneous symmetry breaking $SU(3)_L \times SU(3)_R \to SU(3)_V$. The Lagrangian (27) can be transformed into a useful version in terms of the flavor-rotated fields $\chi_{L,R}$:

$$\chi_L = \xi^\dagger q_L, \qquad \chi_R = \xi q_R. \tag{28}$$

The Lagrangian in Eq. (27) is then rewritten in the form

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{\chi}[\gamma\cdot(iD + \mathcal{V}) + \gamma\cdot\mathcal{A}\gamma_5 - m]\chi - \bar{\chi}\tilde{M}_q\chi, \tag{29}$$

where the fields $\mathcal{V}$ and $\mathcal{A}$ are given in Eq. (18) and where the term including the current mass matrix $\mathcal{M}_q$ is given by

$$\tilde{M}_q = \tilde{M}_q^V + \tilde{M}_q^A\gamma_5, \tag{30}$$

where

$$\tilde{M}_q^V = \frac{1}{2}(\xi\mathcal{M}_q\xi + \xi^\dagger\mathcal{M}_q^\dagger\xi^\dagger) \quad \text{and}$$
$$\tilde{M}_q^A = \frac{1}{2}(\xi\mathcal{M}_q\xi - \xi^\dagger\mathcal{M}_q^\dagger\xi^\dagger). \tag{31}$$

This term has to be taken into account when calculating $SU(3)$-breaking effects.

B. χQM including light vector mesons (VχQM)

The VχQM adds light vector mesons to the χQM. The vector meson fields V_μ are given as Π in Eq. (19) with pseudoscalars $P = (\pi, K, \eta)$ replaced by vectors $V = (\rho, \omega, K^*, \phi)$:

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\phi \end{pmatrix}. \quad (32)$$

These fields are coupled to the light quark fields by the interaction Lagrangian,

$$\mathcal{L}_{IV} = h_V \bar{\chi}\gamma^\mu V_\mu \chi. \quad (33)$$

The coupling constant h_V can be determined from the left-handed current for $vac \to V$, and we find the $SU(3)$ octet current

$$J^a_\mu(vac \to V) = \frac{1}{2} m_V f_V \mathrm{Tr}[\Lambda^a V_\mu], \quad (34)$$

where the quantity Λ^a is given by $\Lambda^a = \xi\lambda^a\xi^\dagger$, and λ^a is the relevant $SU(3)$ flavor matrix. For the currents, we obtain

$$m_V f_V = \frac{1}{2} h_V \left(-\frac{\langle \bar{q}q \rangle}{m} + f_\pi^2 - \frac{1}{8m^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right), \quad (35)$$

which can be used to determine h_V. We find, by using $f_\rho \simeq 216$ MeV, that $h_V \simeq 7$ for standard values of m, $\langle \bar{q}q \rangle$, and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ [21,25,35].

C. HLχQM

The HLχQM adds heavy meson and heavy quark fields to the χQM. The (reduced) heavy quark field Q_v is related to the full-field $Q(x)$ as

$$Q_v(x) = P_+ e^{-im_Q v\cdot x} Q(x), \quad (36)$$

where $P_\pm$ are projection operators $P_\pm = (1 \pm \gamma \cdot v)/2$. The heavy quark propagator (corresponding the reduced field Q_v) is $S_v(p) = P_+/(v \cdot p)$. The Lagrangian for the reduced heavy quark fields is

$$\mathcal{L}_{\mathrm{HQEFT}} = \bar{Q}_v iv \cdot D Q_v + \mathcal{O}(m_Q^{-1}), \quad (37)$$

where D_μ is the covariant derivative containing the gluon fields.

To couple the heavy quarks to light pseudoscalar mesons, there are additional meson-quark couplings within the HLχQM [21],

$$\mathcal{L}_{\mathrm{int}} = -G_H [\bar{\chi}_a \bar{H}^a_v Q_v + \bar{Q}_v H^a_v \chi_a], \quad (38)$$

where a is an $SU(3)$ flavor index and Q_v is the reduced heavy quark field in Eq. (36). The quark-meson coupling G_H is determined within the HLχQM to be [21]

$$G_H^2 = \frac{2m}{f_\pi^2} \rho, \quad (39)$$

where ρ is a hadronic quantity of order 1 [21].

The VχQM can be combined with the HLχQM to give a reasonable description of the weak current for D-meson decays $D \to V$ [20]. A coupling of V^μ to heavy mesons might be given by Eq. (17) with $\mathcal{V}^\mu \to h_V V^\mu$ or by the tensor coupling in Eq. (24). In Ref. [20] the factor λ is found to be $\lambda = -0.53\ \mathrm{GeV}^{-1}$. It might also be calculated in the HLχQM following closely the calculation for $D^* \to D\gamma$ [35]. Using the results of Ref. [35], we obtain

$$\lambda = -\frac{\sqrt{2} h_V \beta}{4 g_V}, \quad (40)$$

where β is defined in Ref. [35]. The value of β obtained there gives $\lambda \simeq -0.4\ \mathrm{GeV}^{-1}$, in agreement with the value $\lambda \simeq -0.41\ \mathrm{GeV}^{-1}$ in Ref. [8].

The current $J^\mu(H \to V)$, obtained from a quark loop diagram such as in Fig. 4, has the form

$$J^\mu_{\mathrm{tot}}(H_v \to V) = \mathrm{Tr}\{\xi^\dagger \gamma^\mu L H_v [A\gamma \cdot V + Bv \cdot V]\}, \quad (41)$$

where A and B are hadronic parameters containing the couplings G_H and h_V, gluon condensates, and the constituent quark mass. This expression is analogous to Eq. (28) in Ref. [21] for the case $H \to P$. However, the $D \to V$ form factor will be dominated by the pole term shown on the right in Fig. 3, and we will not go further into the detailed structure of the nonleading terms A and B.

D. LEχQM

The LEχQM adds high-energy light mesons and quarks to the χQM. Unfortunately, the combination of the standard version of the LEET [11,12] with the χQM will lead to infrared-divergent loop integrals for $n^2 = 0$. Therefore, the following formalism is modified and instead of $n^2 = 0$: we use $n^2 = \delta^2$, with $\delta = \nu/E$, where $\nu \sim \Lambda_{\mathrm{QCD}}$, such that $\delta \ll 1$. In the following, we derive a modified LEET in which we keep $\delta \neq 0$ with $\delta \ll 1$. We call this construction LEETδ [40] and define the *almost* lightlike vectors

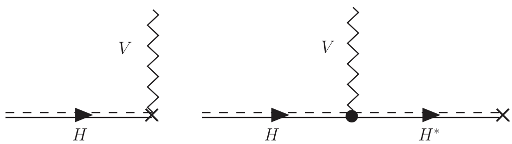

FIG. 3. Contributions to $H \to V$ form factors within the HLχPT. The single pole term is shown on the right.

$$n = (1, 0, 0, +\eta); \qquad \tilde{n} = (1, 0, 0, -\eta), \tag{42}$$

where $\eta = \sqrt{1-\delta^2}$. This gives

$$n^\mu + \tilde{n}^\mu = 2v^\mu, \qquad n^2 = \tilde{n}^2 = \delta^2,$$
$$v \cdot n = v \cdot \tilde{n} = 1, \qquad n \cdot \tilde{n} = 2 - \delta^2. \tag{43}$$

For the LEET, the reduced quark field is defined by

$$q_n(x) = e^{-iEn\cdot x}\mathcal{P}_+ q(x), \tag{44}$$

corresponding to Eq. (36) and where the projection operators are

$$\mathcal{P}_+ = \frac{1}{N^2}\gamma \cdot n(\gamma \cdot \tilde{n} + \delta), \qquad \mathcal{P}_- = \frac{1}{N^2}(\gamma \cdot \tilde{n} - \delta)\gamma \cdot n, \tag{45}$$

where $N^2 = n \cdot \tilde{n}$. The LEETδ Lagrangian corresponding to the HQEFT Lagrangian in Eq. (37) is [40]

$$\mathcal{L}_{\mathrm{LEET}\delta} = \bar{q}_n\left(\frac{\gamma \cdot \tilde{n} + \delta}{N}\right)(in \cdot D)q_n + \mathcal{O}(E^{-1}). \tag{46}$$

For $\delta \to 0$, this is the first part of the SCET Lagrangian. The quark propagator is

$$S_n(k) = \frac{\gamma \cdot n}{N(n \cdot k)}, \tag{47}$$

which reduces to the LEET propagator in the limit $\delta \to 0$ (which also means $N \to 2$). For further details, we refer to Ref. [40].

The term $\mathcal{O}(E^{-1})$ in Eq. (46) contains a term originating from the current mass m_q for the light energetic quark(s). We have found that a further development beyond Ref. [40] gives the $SU(3)$-breaking current mass term m_q:

$$\Delta\mathcal{L}_{\mathrm{LEET}\delta}(m_q) = \frac{m_q}{E}\bar{q}_n\left(i\tilde{n} \cdot D - \frac{m_q}{2}\gamma \cdot \tilde{n}\right)q_n. \tag{48}$$

For hard light quarks and chiral quarks coupling to a hard light meson multiplet field M, the χQM and HLχQM were extended [40], and it was assumed that the energetic light mesons couple to light quarks with a derivative coupling to an axial current,

$$\mathcal{L}_{\mathrm{int}q} \sim \bar{q}\gamma_\mu\gamma_5(i\partial^\mu M)q. \tag{49}$$

The outgoing light energetic mesons are described by an octet 3×3 matrix field $M = \exp\,(+iEn \cdot x)M_n$, where M_n has the same form as Π in Eq. (19):

$$M_n = \begin{pmatrix} \frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & \pi_n^+ & K_n^+ \\ \pi_n^- & -\frac{\pi_n^0}{\sqrt{2}} + \frac{\eta_n}{\sqrt{6}} & K_n^0 \\ K_n^- & \bar{K}_n^0 & -\frac{2\eta_n}{\sqrt{6}} \end{pmatrix}, \tag{50}$$

where π_n^0, π_n^+, K_n^+, etc., are the fields for the hard mesons. Furthermore, q_n is related to M_n in the same manner as Q_v is related to H_v.

Combining the interaction (49) with the rotated soft quark fields in Eq. (28), and using $\partial^\mu \to iEn^\mu$, yields the LEχQM interaction Lagrangian [40]

$$\mathcal{L}_{\mathrm{LE}\chi\mathrm{QM}} = G_A E\bar{\chi}(\gamma \cdot n)Z_n q_n + \mathrm{H.c.} \tag{51}$$

Here, q_n is the reduced field corresponding to an energetic light quark having a momentum fraction close to 1 [see Eq. (46)], and χ represents a soft quark [see Eq. (28)]. Further, G_A is an unknown coupling to be determined by relating a current calculation to measured data. Further,

$$Z_n = \xi M_R R - \xi^\dagger M_L L. \tag{52}$$

Here, M_L and M_R are both equal to M_n, but they have formally different transformation properties. This is in analogy with chiral perturbation theory, in which the quark mass matrices $\mathcal{M}_q$ and its Hermitian conjugate $\mathcal{M}_q^\dagger$ are equal but have formally different transformation properties under $SU(3)_L \times SU(3)_R$). Equation (51) for the LEχQM is the analog of Eq. (38) in the HLχQM case.

Calculating the matrix elements of quark currents for the $H_v \to M_n$ transition in the LEχQM, we obtain an expression for the form factor ζ in terms of model parameters [40],

$$\zeta = \frac{1}{4}m^2 G_H G_A F\sqrt{\frac{M_H}{E}}, \tag{53}$$

where the quantity F coming from loop integration in Fig. 4 (with soft gluons forming gluon condensates added) is [40]

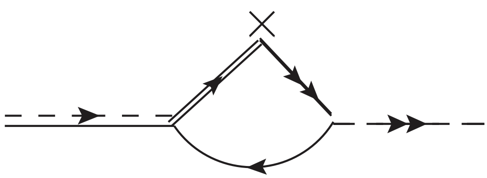

FIG. 4. Current matrix element in the LEχQM. The double dashed line is the (external) heavy meson H_v, and the dashed line with two arrows is the external energetic light meson. The internal lines are double for heavy quark Q_v, single with two arrows for the energetic light quark q_n, and with one arrow for the soft light quark χ.

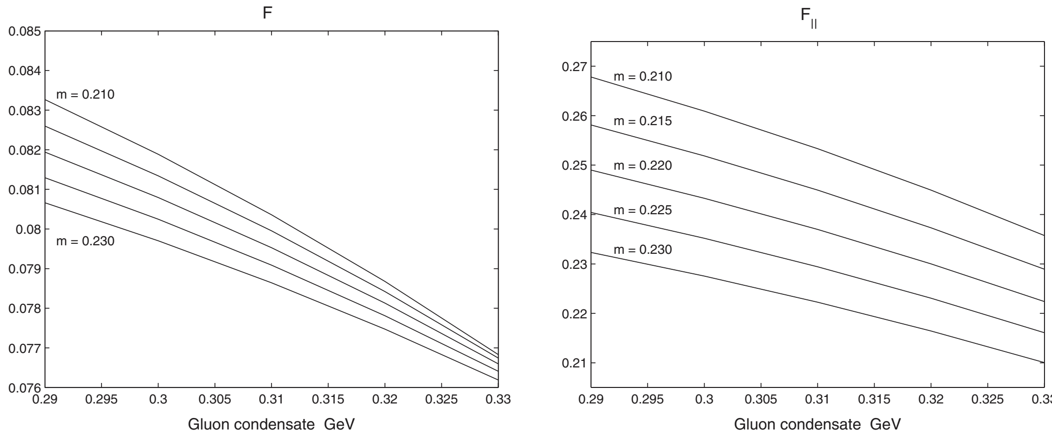

FIG. 5. F and $F_{\parallel}$ as a function of $\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/4}$ for values of the constituent quark mass from $m = 0.210$ to $m = 0.230$ GeV.

$$F = \frac{N_c}{16\pi} + \frac{3f_\pi^2}{8m^2\rho}(1-g_A) - \frac{(24-7\pi)}{768m^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle, \quad (54)$$

which is numerically $F \simeq 0.08$. In Fig. 5, the quantity F is plotted as function of the quark condensate for typical values of the constituent quark mass. We obtain the expression for the coupling constant

$$G_A = \frac{4\zeta}{m^2 G_H F}\sqrt{\frac{E}{M_H}}, \quad (55)$$

where ζ is numerically known [2,7,42] to be $\simeq$0.3 for the transition $B \to \pi$ but is larger, $\zeta \simeq 0.6$ for $D \to \pi$ [6]. We can use the data for ζ in the $D \to \pi$ and $D \to K$ transitions to determine the value of the coupling G_A. We then use this to calculate the ζ form factors for the transitions $D \to \eta$ and $D \to \eta'$.

Within our model, the constituent light quark mass m is the analog of $\Lambda_{\rm QCD}$. To see the behavior of G_A in terms of the energy E, we therefore write C in Eq. (9) as $C \equiv \hat{c}m^{\frac{3}{2}}$ and obtain

$$G_A = \left(\frac{4\hat{c}f_\pi}{mF\sqrt{2\rho}}\right)\frac{1}{E^{\frac{3}{2}}}, \quad (56)$$

which explicitly displays the behavior $G_A \sim E^{-3/2}$. In terms of the number N_c of colors, $f_\pi \sim \sqrt{N_c}$ and $F \sim N_c$, which gives the behavior $G_A \sim 1/\sqrt{N_c}$, i.e., the same behavior as for the coupling G_H in Eq. (38). The coupling G_A is an auxiliary quantity that can be used in place of the quantity ζ.

In this paper, we will extend the LEχQM further to include energetic vector mesons, V_n^μ in analogy with M_n in Eq. (50). In this model, we will use a derivative coupling, as was used for the coupling of light energetic mesons to quarks through an axial vector field in Eq. (49). This is in analogy with light mesons coupling to quarks in Eq. (29). We will therefore begin from the ansatz with the tensor field $F_V^{\mu\nu}$ in Eq. (25) [49]:

$$\mathcal{L}_{\rm LE\chi V} \sim \bar{\chi}\sigma \cdot F_V \chi. \quad (57)$$

It was found in Ref. [40] that derivative coupling gave the best description of the $H \to P$ high-energy current. Using $V \to \exp\,(iEn \cdot x)V_n$, we obtain the interaction (remember that $\partial^\mu V_\mu = 0$ implies $n \cdot V_n = 0$)

$$\mathcal{L}_{\rm LE\chi V} = EG_V\bar{\chi}(\gamma \cdot n\gamma \cdot Z_n)q_n + \text{H.c.}, \quad (58)$$

where

$$Z_n^\mu = V_n^\mu(\xi R + \xi^\dagger L) \quad (59)$$

and

$$V_n^\mu = \begin{pmatrix} \frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & \rho_n^+ & K_n^{*+} \\ \rho_n^- & -\frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & K_n^{*0} \\ K_n^{*-} & \bar{K}_n^{*0} & -\Phi_n \end{pmatrix}^\mu. \quad (60)$$

Here, ρ_n^0, ρ_n^+, K_n^{*+}, etc., are the (reduced) vector meson fields corresponding to energetic light vector mesons. The coupling G_V is determined by the experimental value for the form factors for $B \to \rho$ (for B decays) or the $D \to \rho$ (for D decays) at $q^2 = 0$, obtained by considering experiment and lattice calculations when available or LCSR calculations.

In our case, where no extra soft pions are going out, we set $\xi \to 1$, and for the momentum space, we set $V_n^\mu \to k_M\sqrt{E}(\varepsilon_V^*)^\mu$. The isospin factor is $k_M = 1/\sqrt{2}$ for ρ^0 and $k_M = 1$ for charged ρ's. For the D meson with spin parity 0^-, we have $H_v^{(+)} \to P_+(v)(-i\gamma_5)\sqrt{M_H}$. Then, the involved traces are calculated, and we obtain $J^\mu_{\rm tot}(H_v \to V_n)$ for the $H_v \to V_n$ transition.

From the current calculation, we obtain a relation between the coupling G_V and the form factor $\zeta_\perp$. The formula relating $\zeta_\perp$ and G_V will be similar to that relating ζ and G_A,

$$\zeta_\perp = \frac{1}{4} m^2 G_H G_V F \sqrt{\frac{M_H}{E}}, \tag{61}$$

that is obtained by the replacing $\zeta \to \zeta_\perp$ and $G_A \to G_V$ in Eqs. (55) and (56). The loop integration is the same for both cases; therefore, the loop factor will also be F in this case as in Ref. [40]. Here, $\zeta_\perp$ is numerically known for $B \to \rho$, where it is $\zeta_\perp \simeq 0.3$ [3,7], and for $D \to \rho$, it is $\zeta_\perp \simeq 0.59$ from CLEO data [46].

VI. RESULTS FROM THE LEχQM

Within the LEχQM, and in the limit $\zeta_1/\zeta \sim \delta \to 0$, the bosonized current for the $H_v \to M_n$ transition can be written as

$$J^\mu_{\rm tot}(H_v \to M_n) = -2i\zeta \sqrt{\frac{E}{M_H}} \mathrm{Tr}\{\gamma^\mu L H_v [\gamma\cdot n] \xi^\dagger M_L\}. \tag{62}$$

Simlarly, in the LEχQM, the bosonized current for the vector case $H_v \to V_n$ can be written as

$$J^\mu_{\rm tot}(H_v \to V_n) = -2i\sqrt{\frac{E}{M_H}} \mathrm{Tr}\left\{\gamma^\mu L H_v \left(\zeta_\perp \gamma\cdot n - \frac{m_V}{m}\zeta_\parallel\right) \sigma\cdot F_n \xi^\dagger [\gamma\cdot n]\right\}, \tag{63}$$

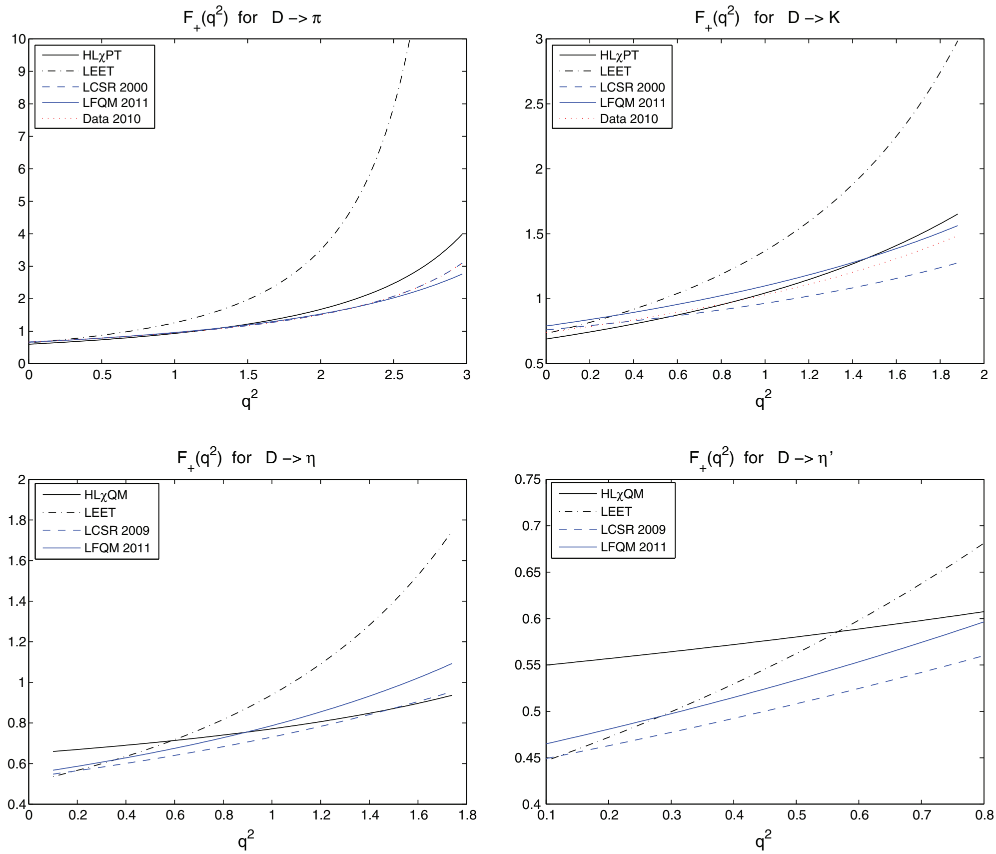

FIG. 6 (color online). $D \to P$ form factors F_+ comparing frameworks used: the HLχPT is from Ref. [19], LCSR 2000 is from Ref. [4], LCSR 2009 is from Ref. [43], the LEET is from Ref. [12], the LFQM from Ref. [44], and the "Data" are from Ref. [45].

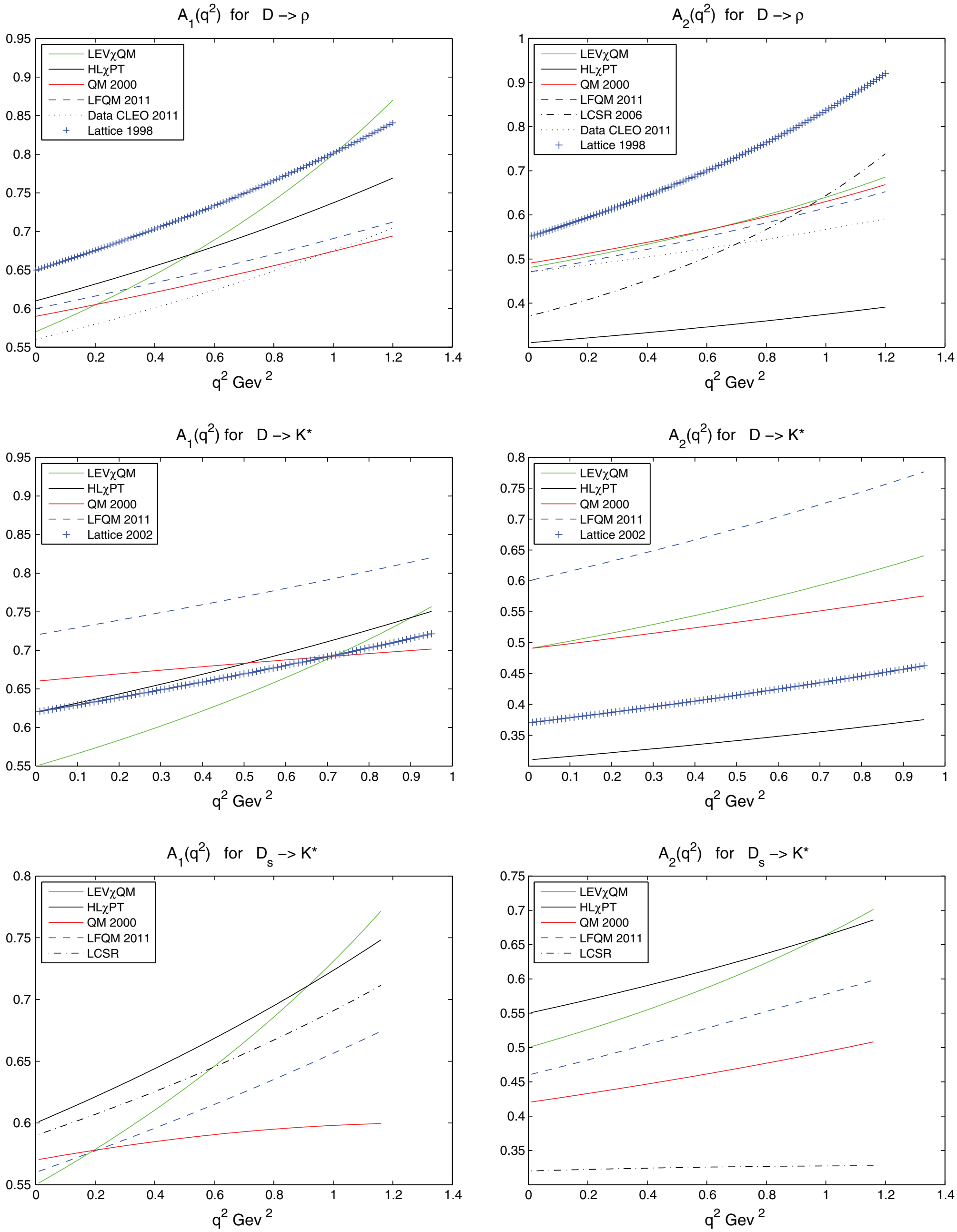

FIG. 7 (color online). $D \to V$ form factors A_1 and A_2: Data CLEO are from Ref. [46], LCSR 2006 is from Ref. [47], the LFQM is from Ref. [44], Lattice 1998 is from Ref. [14], Lattice 2002 is from Ref. [15], the HLχPT is from Ref. [20], and the QM 2000 is from Ref. [48]. The LEχQM is from the calculation in this paper.

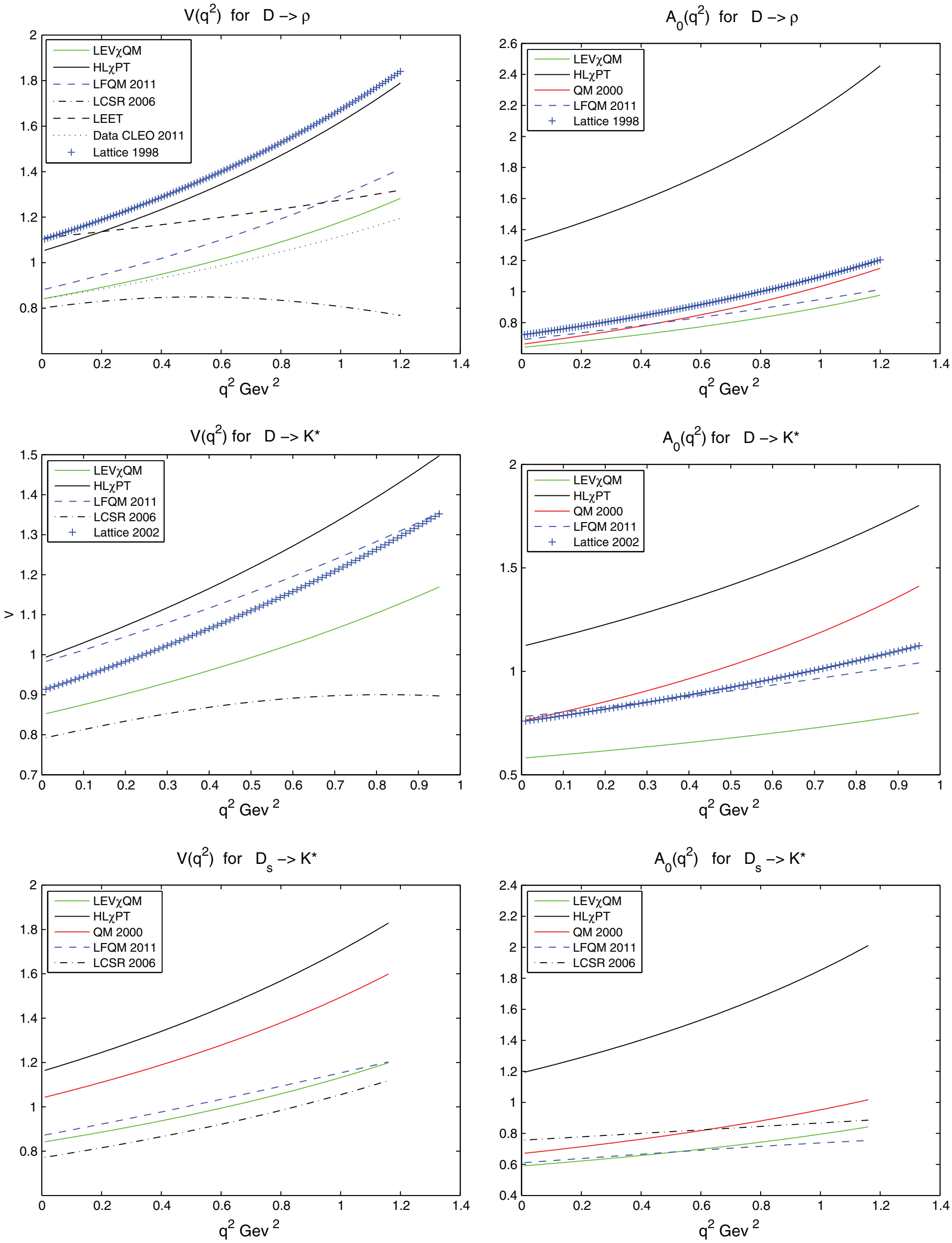

FIG. 8 (color online). $D \to V$ form factors A_0 and V: Data CLEO are from Ref. [46], LCSR 2006 is from Ref. [47], the LFQM is from Ref. [44], Lattice 1998 is from Ref. [14], Lattice 2002 is from Ref. [15], the HLχPT is from Ref. [20], and the QM 2000 is from Ref. [48]. The LEχQM is from the calculation in this paper.

where the tensor F_n is given by Eq. (25) with V_n given as in Eq. (60). Here, we assume that $\delta = m/E \ll 1$, which implies that $\zeta_\perp^{(a)} \to \zeta_\perp$.

We find the following new predictions within the LEχQM:

$$\zeta_1 = \frac{mF_\parallel}{EF}\zeta, \qquad \zeta_\perp^{(a)} = \zeta_\perp + \frac{m}{E}\zeta_\parallel, \qquad \zeta_\parallel = \frac{mF_\parallel}{m_V F}\zeta_\perp, \tag{64}$$

where

$$F_\parallel = \frac{N_c}{16\pi} + \frac{3f_\pi^2}{8m^2\rho}(1-g_A) + \frac{f_\pi^2}{2m^2}\ell + \frac{1}{48m^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(\frac{7\pi}{16}-2\right) \tag{65}$$

is a loop function analogous to F in Eq. (54), which arises from the loop integrals of the current in Eq. (63), and plotted in Fig. 5, right. Here, the appearance of $\ell = \ln(2/\delta)$ is due to the infrared behavior of some of the loop integrals. With the model parameters, we find $F_\parallel \simeq 0.24 \simeq 3F$.

The values for F and $F_\parallel$ are obtained with the simplified LEET propagator in Eq. (47). For the $B \to D$ case, an extra Δ of order 20 MeV was used in the heavy quark propagator [50]. A similar assumption (which is closer to the SCET propagator) might be used here, leading to modified values of F and $F_\parallel$. However, as we are already considering model-dependent predictions, we do not go into further details here. We observe that, although $\zeta_1/\zeta \sim \frac{m}{E}$ as it should, the numerical suppression is not strong because $F_\parallel \simeq 3F$, and $\delta = \frac{m}{E} \simeq 0.24$ is not as small for D-meson decays as it is for B decays (where $\delta \simeq 0.08$).

TABLE I. Form factors for $D \to P$ at $q^2 = 0$. The values for $F_+(0)$ are taken from data when available and from sum rules for $D \to \eta$, η'. The values for $F_+(0)_\chi$ are determined using the LEχQM.

Decay	$F_+(0)$	$F_+(0)_\chi$	ζ	ζ_1
$D \to \pi$	0.67	0.96	0.65	0.46
$D \to K$	0.74	1.06	0.65	0.44
$D \to \eta$	0.55	0.66	0.65	0.34
$D \to \eta'$	0.45	0.55	0.60	0.37

So far, we have considered the $SU(3)$ limit $m_q \to 0$. One may also calculate $SU(3)$ corrections from the mass correction Lagrangian in Eq. (48), for hard outgoing s quarks. We find that the first-order term does not contribute within the LEχQM. The second-order term in Eq. (48) contributes and gives terms suppressed by $m_s^2/(mE)$ compared to terms already calculated. These will therefore be discarded in this work. For decaying B_s and D_s, there will be first-order m_s corrections from the ordinary light sector χQM, through mass terms in Eq. (31). However, these corrections must be considered together with meson loops. Some of these loops might be calculated as in chiral perturbation theory, while others are formally suppressed and problematic to handle within our formalism. Therefore, we do not go further into these details.

VII. PLOTTING THE FORM FACTORS

In this section, we plot transition form factors for $D \to P$ in Fig. 6 and $D \to V$ in Figs. 7 and 8 as a function of the squared momentum transfer q^2. We have plotted the curves from experimental data [45,46], lattice gauge calculations [14,15], LCSRs [2–7], and the light front quark model (LFQM) [44]. The plots do not include error bars because these would make them difficult to read. For plots based on the LEET, $q^2 = 0$ is the reference point that is determined by data, and the shape is determined by a single pole.

To obtain the curves for our LEχQM for a generic form factor $F(q^2)$ (F_+, V, A_i), we use data (CLEO) for $D \to \pi$ and $D \to \rho$ for the $F(0)$'s. We then combine these $F(0)$'s with the theoretical relations in Eqs. (11), (15), and (64) to find the best numerical fit for the ζ_i's (see Tables I and II). Using the relations (15) and (64), we will obtain a reasonable overall fit for the following ζ's:

$$\zeta \simeq 0.6, \quad \zeta_1 \simeq 0.4, \quad \zeta_\perp \simeq 0.6, \quad \zeta_\parallel \simeq 0.5, \quad \zeta_\perp^{(a)} \simeq 0.7. \tag{66}$$

We have then plugged these values for the ζ_i's back in Eqs. (11) and (15) to produce values $F(0)_\chi$ for our model. We then use the single pole assumption in Eq. (13) to produce the curves for $F(q^2)_\chi$. As a biproduct, we predict the curves for other cases with no data (say with K or K^* in the final state) in the $SU(3)$ limit.

For the HLχPT, the no-recoil point $[q^2 = (q^2)_{\max}]$ is the reference point for plots that is determined by Eqs. (22) and

TABLE II. Form factors for $D \to V$ at $q^2 = 0$. The values for $V(0)$ and $A_0(0)$ are taken from LCSRs for $D_s \to K^*$ and lattice calculations for $D \to \rho$, K^*. The fitted values for $V(0)_\chi$, $A_0(0)_\chi$, $A_1(0)_\chi$, and $A_2(0)_\chi$ are determined from the ζ's, which are calculated using the LEχQM. $V(0)_\chi$ for $D \to \rho$ is the input value from CLEO data.

Decay	$V(0)$	$V(0)_\chi$	$A_0(0)$	$A_0(0)_\chi$	$\zeta_\perp$	$\zeta_\perp^{(a)}$	$\zeta_\parallel$	$A_1(0)$	$A_1(0)_\chi$	$A_2(0)$	$A_2(0)_\chi$
$D \to \rho$	0.84	0.84	0.65	0.64	0.59	0.69	0.50	0.56	0.58	0.47	0.48
$D \to K^*$	0.91	0.87	0.76	0.64	0.58	0.68	0.50	0.62	0.57	0.37	0.43
$D_s \to K^*$	0.77	0.86	0.76	0.64	0.58	0.67	0.43	0.59	0.55	0.32	0.44

(26). The plots for $D \to P$ with $P = \pi$, K, η are different because of the different masses. However, we have not explicitly calculated $SU(3)$-breaking effects, and Eq. (66) should be valid in the $SU(3)$ limit $m_s \to 0$. This means the plots for $D \to \pi$ and $D \to \rho$ are the most relevant. The other plots are included for comparison. According to our model [see Eq. (48)], $SU(3)$ corrections due to hard s quarks (as in $D \to K$ and $D \to K^*$ transitions) should be small, while $SU(3)$ corrections due to soft s quarks (as in decays of D_s) should be larger, as pointed out at the end of Sec. VI.

VIII. CONCLUSIONS

We have collected present information on various form factors for the transitions $D \to P$ and $D \to V$ ($P =$ pseudoscalar, $V =$ vector) obtained from various methods and sources such as data, lattice gauge theory, LCSRs, etc. From the plots, we have as far as possible determined the values of relevant form factors at $q^2 = 0$ and then extracted values for the LEET form factors ζ_i. The LEχQM gives relations between the ζ_i's. We have previously found [40] $\zeta_1/\zeta \sim m/E$. Here, we have in addition found relations between the ζ's and have shown that $\zeta_\perp^{(a)} \to \zeta_\perp$ for $m/E \to 0$ as it should.

We observe that the curves for the form factor F_+ for the case $D \to \pi$ show a remarkable agreement for $q^2 \to 0$ (for the LEET, this is done by construction). This is in contrast to the values of $V(0)$ for which the plots show a large variation among the various methods used. This makes $\zeta_\perp$ uncertain. However, $\zeta_\perp$ is also related to $A_0(0)$ such that we obtain a reasonable fit using Eqs. (13) and (66). We observe what we expected, namely, that the LEET and LEχQM work best for q^2 close to zero, while the HLχQM (eventually supplemented by the HLχPM) works best close to the no-recoil point.

The LEχQM gives a good fit to the V and A_0 form factors for the $D \to \rho$ and $D_s \to K^*$ transitions. However, for the $D \to K^*$ transition, the V and A_0 curves lie below the curves for the lattice data. For the $D \to K^*$ transition calculation, the hard quark in the loop is an s quark. We did not include the correction, which is on the order of the mass of the s quark, m_s. This is a source of small error for this transition. We observe that the LEχQM values for the axial form factor A_1, being transverse to the momentum [see Eqs. (5) and (15)], do not match well for any of the transitions.

The LEET form factors ζ and $\zeta_\perp$, together with data for the $D \to \pi$ and $D \to \rho$ transitions, will determine the coupling constants G_A and G_V, which may be used in the calculation of nonfactorizable (color suppressed) nonleptonic D-meson decays, in the same manner as has previously been done for $K \to \pi\pi$ [39,51], $D \to K^0\bar{K}^0$ [52], $B \to D\bar{D}$ [53,54], $B \to D\pi$ [40], and $B \to \pi^0\pi^0$ [41]. Then nonleptonic decay amplitudes can be written in terms of the LEET form factors ζ_i, both for the factorized and the color-suppressed cases. We are, of course, aware that the LEET expansion might have relatively large corrections beyond the order considered here. Still, we think that our results will be helpful for further studies of nonfactorizable nonleptonic decay amplitudes for D mesons.

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