MEMORANDUM

No 01/2002

Prices vs quantities: the case of risk averse agents

By
Fridrik M Baldursson and Nils-Henrik M von der Fehr

ISSN: 0801-1117

Department of Economics
University of Oslo

This series is published by the
University of Oslo
Department of Economics

P. O.Box 1095 Blindern
N-0317 OSLO Norway
Telephone: + 47 22855127
Fax: + 47 22855035
Internet: http://www.oekonomi.uio.no/
e-mail: econdep@econ.uio.no

In co-operation with
The Frisch Centre for Economic Research

Gaustadalleén 21
N-0371 OSLO Norway
Telephone: +47 22 95 88 20
Fax: +47 22 95 88 25
Internet: http://www.frisch.uio.no/
e-mail: frisch@frisch.uio.no

List of the latest 10 Memoranda:

No 36	Oddbjørn Raaum, Kjell G. Salvanes and Erik Ø. Sørensen: The neighbourhood is not what it used to be: Has there been equalisation of opportunity across families and communities in Norway? 34 pp.
No 35	Finn R. Førsund and Dag Fjeld Edvardsen: International Benchmarking of Electricity Distribution Utilities. 45 pp.
No 34	Finn R. Førsund and Ove Wolfgang: The Compensation Mechanism in the RAINS Model: The Norwegian Targets for Acidification. 34 pp.
No 33	Christian Brinch, Erik Hernæs and Steinar Strøm: Labour Supply Effects of an Early Retirement Programme. 40 pp.
No 32	Tone Ognedal: Should the standard of evidence be lowered to reduce crime? 33 pp.
No 31	Jon Strand: Public- and private-good values of statistical lives: Results from a combined choice-experiment and contingent-valuation survey. 55 p.
No 30	Ove Wolfgang: Cost-effective Abatement of Ground-level Ozone in Cities and for Larger Regions: Implications of Non-monotonicity. 28 pp.
No 29	Svenn-Erik Mamelund: The Spanish Influenza among Norwegian ethnic minorities 1918-1919. 35 pp.
No 28	Erik Biørn: The Efficiency of Panel Data Estimators: GLS Versus Estimators Which Do Not Depend on Variance Components. 15 pp.
No 27	Qaisar Farooq Akram and Ragnar Nymoen: Employment behaviour in slack and tight labour markets. 34 pp.

A complete list of this memo-series is available in a PDF® format at:
http://www.oekonomi.uio.no/memo/

Prices vs quantities: the case of risk averse agents

Fridrik M Baldursson
University of Iceland (fmbald@hi.is)

Nils-Henrik M von der Fehr
University of Oslo (nhfehr@econ.uio.no)

February 21, 2002

Abstract

We explore the efficacy of price and quantity controls as environmental policy instruments in a stochastic setting in which agents are risk averse. We demonstrate that the assumption of risk aversion may improve the performance of a tax relative to that of a system of tradable quotas, and that restricting quota trade may enhance efficiency even though risk aversion in itself limits volumes of trade. The government may be able to improve the performance of a tradable quota system by judicious choice of distribution and amount of initial quotas and by trading pro-actively in the quota market.

Keywords: regulation, effluent taxes, tradable quotas, uncertainty, risk aversion, environmental management

JEL Classification codes: D81, D9, H23, L51, Q28, Q38

Acknowledgements: We are grateful to Ragnar Arnason and Michael Hoel for insightful comments on an earlier version of the paper.

1 Introduction

It is a well know result that in ideal circumstances environmental goals can be attained at minimum cost by market-based instruments such as taxes on emissions of a pollutant or by issuing transferable emission permits. In particular, for tradable permits in such situations trade will occur until marginal costs are equalised across parties and hence costs of pollution control are minimised. In reality, various market imperfections may cause this result to fail.[1] The purpose of this paper is to study the consequences of a particular type of market imperfection: we explore the effects of uncertainty and risk-aversion on the performance of market-based instruments for environmental regulation.

Our starting point is the observation that when permits are traded in a market, agents are exposed to risk whenever the permit price varies stochastically. This can affect their behaviour if they care about risk;[2] in particular, market participants will try to reduce their exposure to it. As we shall see, the optimal response of agents to risk may depend on their market position. For example, an agent who has to rely on the purchase of permits to cover emissions may, by investing in abatement equipment, reduce his exposure to the random variations in his costs of emissions. Conversely, a potential seller of permits may want to invest less in abatement and hence use more permits himself.

Given that risk affects individual incentives, we would in general expect that market outcomes will be influenced also. Furthermore, such effects will be different in different regulatory regimes. For example, when quotas are tradable, changes in investment incentives under risk may be expected to depend on the initial allocation of permits.[3] It follows that the distribution of permits at the outset will, in general, affect market equilibrium. The outcome may then be expected to be inefficient, one indicator of that being less trade than would be the case were agents risk neutral. Uncertainty and risk aversion may therefore adversely affect the performance of a tradable quota regime relative to regulation by taxes, say. It is conceivable that in

[1] Also, Cole and Grossman (1999) argue that in many cases monitoring and enforcement as well as other institutional and technological costs may cause command-and-control regulation to be more efficient than market based instruments.

[2] Even if agents are risk-neutral they may have an incentive, in an uncertain economic environment, to delay irreversible investment decisions until a more favourable price for quotas is observed. This is a topic that was treated by Chao and Wilson (1993), Dixit and Pindyck (1994), Baldursson and von der Fehr (1998) and Zhao (2000). Saphores and Carr (1999) and Xepapadeas (1999) also use the irreversible investment approach for studying related issues.

[3] We use the terms 'quotas' and 'permits' interchangeably.

such circumstances restricting trade may improve the cost efficiency of quota regulation.

The observation that uncertainty and risk aversion may reduce trade is particularly interesting when the practical experience with quota systems is examined. While some systems, such as the US sulphur dioxide allowance trading scheme, are regarded as successes, others have not been as successful.[4] In particular, low trading levels in some of these programs are an indication that potential gains from trade are not realised.[5] There are several possible explanations for lack of success in a market for a particular pollution permit. All of these are based on market imperfections of one kind or another. Among these are market power in permit and output markets, non-profit maximising behaviour, and transaction costs. Stavins (1995) emphasises the role of transaction costs and shows how they reduce trading levels and raise abatement costs; furthermore, he also shows that the initial allocation of permits can affect equilibrium permit distribution and thus lead to an inefficient outcome.

In this paper we explore another form of transaction costs - namely the risks involved in relying on market-based transactions - by focusing on how risk aversion, in combination with uncertainty, can affect decisions on investment in pollution abatement equipment. Our model is constructed to capture the relevant aspects of a market in which agents must make investment decisions ahead of observing uncertain and dynamic evolution. The model is written without an explicit time dimension, but implicitly there are two time periods: an *ex ante* period before uncertainty is revealed and when investment decisions in capital intensive abatement must be made, and an *ex post* period after the observation of random events and when decisions on abatement involving variable costs only must be made. In tradable quota regimes decisions on quota transactions take place in the *ex post* period (in a later section we extend the model to allow for *ex ante* quota trade also). Uncertainty in our model can, in the most general specification, enter through the number of firms, the amount of pollution or abatement costs. Uncertainty can also be firm specific (idiosyncratic risk) or extraneous (aggregate risk). Firms are assumed to be risk averse, they are infinitesimally small, take all prices as given and consider their own actions and market aggregates only when making decisions. There is an executive authority who sets tax rates, allocates quotas and can enter the quota market to influence the aggregate

[4] See Stavins (1999) for a review and evaluation of the experience with market-based instruments.

[5] Clearly, a tradable permit system can also fail in attaining its primary purpose - that of achieving a given environmental target - for example due to non-compliance and imperfect monitoring (Montero, 1999), but in this paper we focus on the cost efficiency aspect.

amount of emissions if it chooses.

This simple model gives surprisingly rich results.[6] First, we consider a tax regime with a fixed linear tax, and, as may be expected considering Weitzman's seminal paper on 'Prices vs. quantities' (1974), such regulation transfers risk (stemming from the regulation of pollution) from the firms to society at large, since the marginal cost of each firm becomes fixed but aggregate emissions become uncertain. Cost efficiency is achieved since marginal costs are equalised across firms and periods.

We then turn to the study of a quota regime when risk is purely extraneous to firms. We establish the result that for a single firm investment in abatement equipment depends on its initial allocation of quotas and therefore marginal abatement costs will, in general, not be equalised across firms nor time periods (the latter in an expectational sense). In particular, firms that are allocated no quotas will over-invest, in the sense that marginal investment costs are driven up to a level higher than the expected quota price, in order to reduce the risk they are subjected to through the quota market. Conversely, firms that receive all their quotas gratis will under-invest. Costs are therefore minimised neither at the firm level nor across firms. In the special case of constant absolute risk aversion it is possible to establish a monotone relationship between allocated quota and investment. In general, we can show that in a grandfathering regime, in which firms are either given quotas to cover all their emissions (old firms) or none at all (new firms), the volume of trade between firms is less when firms are risk averse than when they are risk neutral.

Since the risk of the market for pollution permits can have such negative effects, it is of interest to consider elimination of firms' exposure to risk by removing the possibility of trade in quotas. Then firms are not faced with uncertainty and will equalise their marginal costs across time periods. In general, marginal costs will, however, not be equalised across firms unless investment costs are linear. Hence, in this special case, restricting quota trade improves cost efficiency.

We also consider the case of risk at the firm level. We assume risk is generated by uncertain emissions of pollutant, but the analysis is basically identical in the case of uncertain abatement costs. Furthermore, by assuming firms are symmetric and risk averse, we can establish a similar result as when risk is extraneous, viz. that there is inefficient over-investment in equilibrium which may be mitigated by increasing the initial amount of quotas. The authority can achieve the desired aggregate emissions by *ex post* purchases

[6] The analytical difficulties encountered are also surprising - we do not get as far as we would like!

in the quota market. Since the uncertainty in this case is at the firm level, restricting quota trade does not shield firms from risk; thus in this case investment is 'too high' once again.

Forward markets for pollution permits are rare.[7] However, since forward trading is a means of controlling risk it is of interest to study the effects of forward markets in our model. We do this in the framework of firm level risk described in the previous paragraph and show that a well-functioning forward market for pollution permits would counteract, but not eliminate, the effects of uncertainty and risk aversion.

In our analysis we generally assume firms are equally risk averse. However, one can argue that firms may have different attitudes towards risk. For example larger firms may have better access to capital markets and be more diversified, and consequently less risk averse, than smaller firms. We briefly consider a case where there are two types of firms: risk averse and risk neutral. We show that allowing for trade between firms with these different characteristics has ambiguous effects on the expected costs of reducing emissions. However, cost efficiency may be enhanced by allocating a larger share of quotas to risk averse firms than to risk neutral ones and then allowing for trade.

2 Modelling framework

Firms are identical, infinitesimally small and are price takers in all markets. The mass of firms is M and firms are indexed by m. Firm m produces $\rho(m)$ units of the pollutant (before any cleaning activities) and aggregate production of the pollutant is given by

$$R = \int_0^M \rho(m)dm. \tag{1}$$

Firms can reduce their emissions by investing in a given technology. The cost of reducing emissions by k equals $f(k)$. We assume that $f(0) = 0$, that f is smooth, increasing and convex and (to rule out corner solutions) that the cost of investment tends to infinity as $k \uparrow k^{\infty}$. If $k(m)$ denotes the investment

[7] A well functioning and liquid spot market is a prerequisite for a successful market in corresponding forward contracts (Radetzki, 1990). Risk aversion provides an incentive for forward trading in itself, yet, to the extent that it is to blame for thin spot markets, it is also an indirect cause of missing forward markets.

by firm m, $m \in [0, M]$, aggregate investment and investment costs equal

$$K = \int_0^M k(m)dm, \tag{2}$$

$$F = \int_0^M f(k(m))dm. \tag{3}$$

Firms can also reduce their emissions by application of a technology involving variable costs only. The abatement cost c is smooth, increasing and convex with $c(0) = 0$ and $c'(a) \uparrow \infty$ as $a \uparrow a^{\infty}$, where $P\{\rho(m) \geq a^{\infty} + k^{\infty}\} = 1$ for all m (so, complete elimination of emissions is ruled out by assumption). If $a(m)$ denotes the abatement undertaken with the variable cost technology by firm m, $m \in [0, M]$, aggregate abatement and abatement costs equal

$$A = \int_0^M a(m)dm, \tag{4}$$

$$C = \int_0^M c(a(m))dm. \tag{5}$$

Emissions at firm m and aggregate emissions are given, respectively, by

$$e(m) = \rho(m) - a(m) - k(m), \tag{6}$$

$$E = \int_0^M e(m)dm = R - A - K. \tag{7}$$

In this environment we shall consider three sources of risk, related to the mass of firms (M), production of the pollutant (ρ) and abatement costs (c), respectively. We assume that uncertainty is revealed only after investment in the k-technology has been made. However, firms can adjust their abatement with the a-technology immediately upon observing the realisation of stochastic parameters.

Due to the up-front investment cost and unspecified market imperfections, firms are risk-averse and maximise expected utility of profits. The concave utility function is denoted by U. Prior to any measures to control pollution firms have an exogenous rate of profit, π.[8]

We consider two types of policies to reduce pollution: a fixed, linear tax t imposed on a unit of pollutant emitted and a quota, i.e., a limit on aggregate

[8]Profits may depend on the number of firms (M) and the production of the pollutant (ρ). Since we are assuming that the number of firms is exogenous (in effect ignoring how environmental policies may affect firms' entry and exit decisions), the essential assumption is that in the relevant range gross profits always exceed the sum of investment and abatement costs.

emissions. The emission limit $\overline{E}$ may be attained by tradable or non-tradable pollution quotas that are distributed in some way initially. The amount of quotas distributed to firm m is denoted by $q(m)$ and the aggregate amount of distributed quotas is

$$Q = \int_0^M q(m)dm. \tag{8}$$

Note that when quotas are tradable Q need not be equal to $\overline{E}$. As will become clear, the policy maker can achieve different outcomes by a judicious, initial choice of quotas, not only at the aggregate level, but also by distributing them in different ways to firms. To achieve the emission target $\overline{E}$ with $Q \neq \overline{E}$ the policy maker must of course act in the *ex post* quota market to purchase/sell the difference $Q - \overline{E}$.

The social damage due to aggregate emissions is given by a convex function D and total expected social cost is given by the expected value of damage and cost of abatement:

$$S = E\left\{D(R - K - A) + C + F\right\}. \tag{9}$$

3 The tax regime

In this section we assume pollution is regulated by a fixed linear tax, t, on emissions. Recall that firms must invest *ex ante*, i.e., before observing the realisation of stochastic variables. Decisions on abatement, on the other hand, are made *ex post* when firms have observed all relevant variables and do not face any uncertainty. The optimisation problem of the firm can therefore be solved in two steps, by first determining the optimal level of abatement, given investment and then, in the second step, one can go on to solve for the right amount of investment.

Given a level of investment k, a firm will minimise the sum of abatement cost and tax payments, $c(a) + t[\rho - a - k]$, and hence select its level of abatement such that

$$c'(a^t) = t. \tag{10}$$

At the investment stage firms maximise expected utility of profits, where profits are given by

$$\Pi^t = \pi - c\left(a^t\right) - f\left(k\right) - t\left[\rho - a^t - k\right]. \tag{11}$$

The first-order condition for this problem is

$$E\left\{U'\left(\Pi^t\right)\left[t-f\left(k^t\right)\right]\right\}=0. \tag{12}$$

Since the marginal profitability of investment, $t-f'\left(k^t\right)$, is deterministic by assumption a firm will choose k^t so as to equate marginal investment cost to the tax rate; that is,

$$f'\left(k^t\right)=0. \tag{13}$$

Thus, we have the following result:

Proposition 1 *When emissions are regulated by a tax, cost efficiency in emission reductions is achieved; in particular,* $a^t(m)\equiv a^t$ *and* $k^t(m)=k^t$ *for all* m *and* $c'\left(a^t\right)\equiv f'\left(k^t\right)=t$.

The proposition implies that internal cost efficiency is attained, since marginal costs are equalised across technologies at individual firms. Furthermore, external cost efficiency is achieved also, since marginal costs are the same at all firms. The reason for both of these results is that the opportunity cost of emissions is determined by the fixed tax. Note, however, that firms are still subject to risk since emissions and profits are uncertain given the investment and abatement choices a^t and k^t, respectively. The particular assumptions about sources of uncertainty are not crucial for the above result.

It should be noted also that cost efficiency of the tax in the presence of risk aversion does not guarantee social optimality. In particular, even if the tax is set optimally aggregate emissions will vary stochastically which may make regulation by taxes unattractive when social damage is taken into account, cf. Weitzman (1974).

4 Quota regimes: pure extraneous risk

In this section we assume that only the mass of firms M is stochastic. For simplicity the production of the pollutant is normalised to unity, i.e., $\rho(m)\equiv 1$ for all m. This set up is meant to capture (in an admittedly crude manner) the dynamic evolution of industries - with entry and exit of firms - to which it may be difficult to fully adjust environmental policies. In practice there may be uncertainty also about production of the pollutant, abatement technologies *et cetera*. As we shall see below, by ignoring stochasticity 'intrinsic' to firms' operations we are able to push to the extreme the differences between how price and quantity controls affect firms' exposure to risk. In particular, the assumption that uncertainty originates from the number of polluters implies that firms will be exposed to risk through the price of quotas only; that is, risk is entirely extraneous, transmitted through market prices. With more general assumptions about sources of uncertainty firms may face risk independently of the choice of policy instrument. We return to the latter possibility in the next section.

4.1 Tradable quotas

In the quota regime aggregate emissions are capped at $\overline{E}$. To ensure a non-vacuous problem we assume that[9]

$$\Pr\left\{M \geq \overline{E}\right\} > 0. \tag{14}$$

When quotas are tradable, a market for them will arise. Denote the market price of a unit of emissions by p. Then, given its investment level, k, a firm will minimise its costs, $c(a) + p[1 - a - k]$, and abatement, a^q, will be determined so that marginal abatement cost equals the opportunity cost of emissions, i.e.,

$$c'(a^q) = p, \tag{15}$$

for all firms. Suppose aggregate investment is K^q. Then, assuming the emission cap is binding, we have

$$A^q = M - \overline{E} - K^q, \tag{16}$$

and therefore, since $A^q = M\ a^q$,

$$p = c'(\frac{M - \overline{E} - K^q}{M}). \tag{17}$$

Now, since M is stochastic, the price of quotas faced *ex ante* by firms is stochastic and therefore their risk aversion comes into play.

Suppose a firm has been allocated a quota to emit q units of pollutant. For a given realisation of the number of firms M, it will abate an amount $a^q = \left[M - \overline{E} - K^q\right] / M$. If the firm decides to invest k, it will therefore have to buy (resp. sell) $1 - a^q - k - q$ quotas in the marketplace. Consequently, the firm will choose k so as to maximise

$$E\left\{U\left(\pi - c(a^q) - f(k) - p\left[1 - a^q - k - q\right]\right)\right\}. \tag{18}$$

[9] Recall that $\rho = 1$ for all firms.

The first-order condition for this problem is given by

$$E\left\{U'\left(\Pi^q\right)\left[p - f'(k^q)\right]\right\} = 0, \tag{19}$$

where

$$\Pi^q = \pi - c(a^q) - f(k^q) - p\left[1 - a^q - k^q - q\right] \tag{20}$$

is the profit of the firm given k^q (net of any costs of obtaining the initial quota allocation q).

The above condition determine the investment at each firm as well as aggregate investment, K^q, and thus market equilibrium is now completely determined. The social cost of emissions is given by

$$S^q = D(\overline{E}) + E\left\{Mc(\frac{M - \overline{E} - \ K^q}{M}) + F^q\right\}. \tag{21}$$

where F^q is the aggregate cost of investment. By inspection of (19), we immediately have:

Proposition 2 *Assume firms are risk averse. Then, for a given emission limit $\overline{E}$, the investment undertaken by any particular firm depends on its initial allocation of quotas.*

Note that this result implies that, for an arbitrary allocation of quotas, investment costs will in general not be minimised across firms:

Corollary 3 *When quotas must be allocated before all polluters have been identified, risk neutrality of polluters is a necessary condition for a system of tradable quotas to minimise the costs of reducing emissions.*

Having established the dependence of investment on the initial quota allocation we next want to consider the particular nature of this relationship.

Intuitively, one might perhaps expect a fairly straightforward relationship between the initial quota allocation and investment in abatement equipment. In particular, a firm that initially has been given very few quotas, and hence needs to cover its emissions by purchasing quotas in the marketplace, reduces its exposure to risk by over-investing in abatement equipment. Correspondingly, a firm with a sufficiently large initial quota holding that will always be selling quotas might be expected to under-invest. More generally, one would expect a negative relationship between initial quota allocation and investment in abatement equipment.

As it turns out, this is not so straightforward. In particular, there is a non-trivial relationship between firm profits and the stochastic quota price. Typically, a firm will be selling quotas when the price is high and buying when the price is low. Consequently, profits will tend to be decreasing in the quota price at low levels of the price but increasing in the price at high levels. It is only when a firm is always either a seller or a buyer (e.g. if it either receives full quota or none) that this relationship is monotone. In these extreme cases we can indeed establish unambiguous results. In the intermediate case, however, we have not been able to prove a general result. As we shall see below, by imposing further restrictions on the utility function a monotone relationship between quota allocation and investment can nevertheless be established.

Lemma 4 *For any distribution of* p*,*

$$\begin{aligned} f'(k^q) &\geq Ep \quad \text{for} \quad q = 0 \\ f'(k^q) &\leq Ep \quad \text{for} \quad q = 1 \end{aligned} \tag{22}$$

and, furthermore, there exists a number q'*,* $0 \leq q' \leq 1$ *such that*

$$f'(k^q) = Ep \quad \text{for } q = q'. \tag{23}$$

Proof. Note that, by the application of the Envelope Theorem, we get

$$\frac{d\Pi^q}{dp} = -[1 - a^q - k^q] + q. \tag{24}$$

Consider the case in which $q = 0$. Then Π^q is a non-increasing function of p by (24). Therefore,

$$\begin{aligned} 0 &= E\{U'(\Pi^q)[p - f']\} \\ &\geq E\left\{1_{\{p<f'\}}U'(\Pi^q_{p=f'})[p - f']\right\} + E\left\{1_{\{p\geq f'\}}U'(\Pi^q_{p=f'})[p - f']\right\} \\ &= U'(\Pi^q_{p=f'})[Ep - f'] \end{aligned} \tag{25}$$

Since $U' > 0$, it follows that $f' \leq Ep$. The second inequality of (22) follows from an analogous argument. The equality (23) follows from (22) and the continuity of f' and of k as a function of q. ■

Remark 1 *In the absence of corner solutions, i.e. if* $\Pr\{0 < a^q + k^q < 1\} > 0$*, the inequalities in (22) hold strictly and in that case* $0 < q' < 1$.

Whether or not firms invest beyond the level at which marginal cost of investment equals the (expected) marginal cost of emissions depends on the amount of allocated quotas. If a firm has not been allocated an emission quota (i.e., $q = 0$), and we are at an interior equilibrium (so that $e = 1 - a - k > 0$ for some realisations of p), then $f'(k) > Ep = Ec'$; that is, the firm over-invests relative to the cost minimising solution. Conversely, if the allocated emission quota is large enough the firm under-invests.

Proposition 5 *There exists a number q^*, $0 < q^* < 1$, such that if, initially, each firm is allocated a quota of q^*, $f'(k^q) = Ec'(a^q)$ for $q = q^*$.*

Proof. Consider the case in which all firms receive identical quotas q and define the function

$$g(q) = f'(k^q) - Ec'(a^q). \tag{26}$$

By Lemma 1, with $p = c'(a^q)$, it follows that

$$g(0) > 0 > g(1). \tag{27}$$

The existence of q^* follows from the continuity of g. ■

Note that when all firms receive the same positive quota the total quota is stochastic. The authority must therefore buy or sell quotas in the secondary market to achieve the desired emissions $\overline{E}$.

From the above analysis a monotone negative relationship between the amount of quota allocated to a firm and its investment might have been expected. However, from the first order condition (19) and application of the Envelope Theorem we get

$$\frac{dk^q}{dq} = \frac{E\left\{U''(\Pi^q)\,p\,[p - f'(k^q)]\right\}}{E\left\{U''(\Pi^q)\,[p - f'(k^q)]^2 - U'(\Pi^q)\,f''(k^q)\right\}}. \tag{28}$$

Clearly, in the above expression the denominator is negative while the numerator can take on both positive and negative values in general. Nevertheless, by making specific assumptions on the utility function we can establish that the numerator is negative and hence investment is reduced by a larger quota allocation.

Let

$$r_A(\Pi) = -\frac{U''(\Pi)}{U'(\Pi)} \tag{29}$$

be the Arrow-Pratt coefficient of absolute risk aversion.

Proposition 6 *Assume $r_A : \mathbb{R} \to \mathbb{R}$ is constant. Then, for all $q \geq 0$,*

$$-1 < \frac{dk}{dq} < 0. \tag{30}$$

Proof. The second-order condition for optimal investment corresponding to (19) may be written

$$\Delta \triangleq E\left\{U''\left(\Pi^q\right)\left[p - f'(k^q)\right]^2 - U'(\Pi^q) f''(k^q)\right\} < 0. \tag{31}$$

Using (19), we have the following comparative statics result:

$$\frac{dk^q}{dq} = \frac{E\left\{U''(\Pi^q) p\left[p - f'(k^q)\right]\right\}}{-\Delta}. \tag{32}$$

Observe that

$$\begin{aligned} E\left\{-U''\left[p - f'\right] f'\right\} &= E\left\{r_A U'\left[p - f'\right] f'\right\} \\ &= r_A E\left\{U'\left[p - f'\right] f'\right\} \\ &= 0 \end{aligned} \tag{33}$$

where the last equality follows from the first-order condition (19). Hence, by (32),

$$\frac{dk}{dq} = \frac{E\left\{U''(\Pi)\left[p - f'(k)\right]^2\right\}}{-\Delta}$$

which, along with (31), implies (30). ■

Combined, Propositions 5 and 6 imply that firms with large initial quota holdings will invest less - and hence emit more - than firms with smaller quotas. In general, if k is decreasing in q then, given an initial allocation of quotas and the corresponding equilibrium, a reallocation of quotas from firms with small quota holdings to firms with large quota holdings - i.e., towards a more uneven allocation - would lead to greater differences in investment. In particular, a marginal reallocation that increases quota holdings of firms that under-invest in the initial equilibrium at the expense of those who initially over-invest will result in increased divergence between marginal investment costs across firms.

Proposition 7 *Assume quotas are distributed such that firms are either given quotas of $q \geq 1$ or none at all. Then in equilibrium the total volume of trade between firms is less when firms are risk averse than when they are risk neutral.*

Proof. Observe that, when firms are risk neutral, the price of quotas is positive with non-zero probability (since $\Pr\left\{M \geq \overline{E}\right\}$). Therefore, in equilibrium, investment, which is identical at all firms, will be positive and there will be trade in quotas. By Lemma 4 firms given quotas of $q \geq 1$ will invest less when risk averse than they would when risk neutral, and, conversely, firms given zero quotas will invest more.

Consider the case in which, due to risk aversion, aggregate investment is higher and, as a consequence, aggregate abatement is lower. Since all firms abate equally, abatement is lower at all firms. It follows that the firms with quotas use more of them for their own production and sell fewer. In the opposite case, in which aggregate investment is lower and, hence, abatement is higher, firms with no quotas will emit less and therefore buy fewer quotas. ■

Remark 2 *Proposition 7 may rephrased by stating that in a grandfathering regime risk aversion reduces trade.*

We conjecture that Proposition 7 hints at a more general result: for any initial allocation of quotas, volumes of trade will be smaller the more risk averse firms are. To prove such a result one would need to show that those selling under risk neutrality under-invest when risk averse and vice versa for those buying. Due to the difficulty of establishing a monotone relationship between quota holdings and investment, and the dependence of the quota price on abatement decisions, we have not been able to get further towards demonstrating such a general result. What we can say, however, is that when firms are sufficiently risk averse trade will be limited; in particular, infinitely risk averse firms would not trade at all.

4.2 Non-tradable quotas

In the case when quotas are non-tradable, firms are not faced with uncertainty. Subject to the allocated quota, each firm will choose its investment and abatement levels so as to minimise total economic costs; that is a firm will solve the minimisation problem

$$\min_{a,k} \left\{ f(k) + c(a) : 1 - a - k \leq \ q \right\}. \tag{34}$$

The first-order condition for this problem (given that the constraint binds) is

$$c'(1 - k - \ q) = f'(k). \tag{35}$$

A system of non-tradable quotas therefore leads to internal cost efficiency (i.e., at the firm level) irrespective of the quota allocation. Clearly, for an arbitrary quota allocation, in general there will not be external cost efficiency; that is, marginal costs will not be equalised across firms. Therefore, aggregate costs of reducing emissions will not be minimised. However, in the case in which the k-technology exhibits constant returns to scale, marginal costs will, in fact, be equalised across firms as well:

Proposition 8 *When there is uncertainty about the number of polluters only and marginal investment costs are constant, a system of non-tradable quotas minimises the aggregate cost of reducing emissions.*

In the constant marginal cost case, limiting tradability of quotas therefore enhances efficiency by eliminating firms' exposure to risk. More generally, there is a trade-off between internal and external cost efficiency. Non-tradability leads to internal cost efficiency while tradability allows for equalisation of marginal abatement (but not investment) costs across firms. Depending on the initial allocation of quotas, either effect may dominate.

5 Quota regimes: firm level risk

In this section we consider risk that originates at the firm level. For simplicity we restrict attention to the case in which the production of the pollutant is stochastic. Results are basically identical in the case in which risk originates from the costs of abatement.

Fix the mass of firms at 1 and assume that the production of the pollutant is stochastic and is observed only after investment in the k-technology has been made. Firms can adjust their abatement with the a-technology immediately upon observing $\{\rho(m) : 0 \leq m \leq 1\}$.

5.1 Tradable quotas

In a tradable quota regime all firms will abate the same amount of pollutant a^q, where $c'(a^q) = p$. Since the mass of firms is normalised to 1, we have

$$a^q = A^q = R - \overline{E} - K^q, \tag{36}$$

and hence the price of quotas is determined by

$$p = c'(R - \overline{E} - K^q). \tag{37}$$

At investment stage the decision problem faced by a firm is to choose k so as to maximise

$$E\left\{U\left(\pi - c(a^q) - f(k) - p\left[\rho - a^q - k - q\right]\right)\right\}. \tag{38}$$

Unlike in the previous section the firm therefore faces two kinds of risk, *viz.* market risk, via aggregate emissions R, and idiosyncratic risk via its own emissions ρ. The first-order condition may be written

$$E\left\{U'\left(\pi - c(a^q) - f(k^q) - p\left[\rho - a^q - k^q - \ q\right]\right)\left[p - f'(k^q)\right]\right\} = 0. \tag{39}$$

To be able to say something about the level of investment in this case we must make some assumptions regarding the form of uncertainty. Let us assume that at firm m, production of the pollutant is given by

$$\rho\left(m\right) = \widehat{R} + \varepsilon\left(m\right), \tag{40}$$

where $\{\varepsilon\left(m\right); 0 \leq m \leq 1\}$is a collection of i.i.d. random variables with mean zero and finite variance independent of $\widehat{R}$, which is a random variable also.[10] Then aggregate pollution production is given by $R = \widehat{R}$. In what follows we identify $\widehat{R}$ with R and drop the hat of the former random variable. Thus pollution production at each firm is composed of an aggregate variable and an idiosyncratic shock. We are able to establish the parallel of Proposition 5 under a simple condition on the distribution of the idiosyncratic shock in (40):

Proposition 9 *Assume U is strictly concave and production of the pollutant is stochastic and given by (40). Furthermore assume that*

$$\Pr\left\{\varepsilon\left(m\right) \geq -L\right\} = 1, \text{ where } L = \inf_{x \geq 0} \frac{c'\left(x\right)}{c''\left(x\right)}. \tag{41}$$

If quotas are tradable and distributed such that all firms are initially allocated an identical amount q, then allocating the entire emission target $\overline{E}$ leads to inefficient over-investment in equilibrium, i.e. $f' > Ep$. Furthermore, there exists a quota $q^* < \overline{E}$, such that if each firm is initially allocated q^*, then $f'\left(k^q\right) = Ec'\left(a^q\right)$ for $q = q^*$.

Proof. Assume the conditions of the proposition hold and that all firms are allocated the same quota, i.e., $q\left(m\right) = q$ for all m. Further, the initial

[10] ε should be thought of as a white noise process (Karlin and Taylor, 1981, p 342).

allocation is such that $Q = \overline{E}$, which implies $q = \overline{E}$ and $\rho - a^q - k - q = \varepsilon$. At the symmetric equilibrium, the first-order condition (39) implies

$$E\left\{U'\left(\pi - c\left(R - \overline{E} - k^q\right) - f\left(k^q\right) - p\varepsilon\right)\left[p - f'\left(k^q\right)\right]\right\} = 0, \tag{42}$$

where $p = c'\left(R - \overline{E} - k^q\right)$. Note that for each fixed value of $\varepsilon > -L$ we have

$$c'(x) + \varepsilon c''(x) < 0 \text{ for all } x \geq 0, \tag{43}$$

and hence $g_\varepsilon(R) = U'\left(\pi - c\left(R - \overline{E} - k^q\right) - f\left(k^q\right) - p\varepsilon\right)$ is increasing in R for each such ε. Note also that for a fixed R, $g_\varepsilon(R)$ is increasing in ε. Now let

$$A = \left\{c'\left(R - \overline{E} - k^q\right) - f'\left(k^q\right) > 0\right\} = \left\{p - f'\left(k^q\right) > 0\right\}, \tag{44}$$

and denote the value of R such that $c'\left(R - \overline{E} - k^q\right) - f'\left(k^q\right) = 0$ by R_0. From (42) we get

$$\begin{aligned} 0 &= E\left\{E\left\{1_A g_\varepsilon(R)\left[p - f'\right]|\varepsilon\right\} + E\left\{1_{A^C} g_\varepsilon(R)\left[p - f'\right]|\varepsilon\right\}\right\} \\ &> E\left\{E\left\{1_A g_\varepsilon(R_0)\left[p - f'\right]|\varepsilon\right\} + E\left\{1_{A^C} g_\varepsilon(R_0)\left[p - f'\right]|\varepsilon\right\}\right\} \\ &= E\left\{E\left\{g_\varepsilon(R_0)\left[p - f'\right]|\varepsilon\right\}\right\} \geq g_{-L}(R_0)\, E\left\{p - f'\right\}. \end{aligned} \tag{45}$$

Consequently, as long as firms are risk averse there is always inefficient overinvestment at the symmetric equilibrium, i.e. $f' > Ec'$. The existence of q^* is shown exactly as in Proposition 5. ■

The condition (41) on the distribution of e is of course restrictive and in certain cases L may be equal to zero. In that case (41) amounts to assuming that $\varepsilon(m) = 0$ and that all firms produce the same amount, R, of pollutant. However, from the argument in the proof of Proposition 9 it seems likely that the condition is in fact overly restrictive and that the results could lend itself to generalisation.

5.2 Forward trading

It is natural to ask whether the introduction of forward markets would change the overinvestment result in Proposition 9. Suppose we are in the same setting as in Section 5.1; that is, we fix the mass of firms at 1 and assume that pollution production at each firm is composed of an aggregate variable R and an idiosyncratic shock $\varepsilon(m)$ as in (40). In addition to investing and abating as before, firms can trade quotas in a forward market at the same time they invest (i.e., before uncertainty is revealed). We denote the quantity of quotas contracted in the forward market by q^f and the forward price by p^f. Abatement a^q and the *ex post* quota price p are determined by (36) and (37) as before and the profit of a firm, after investment, abatement and forward trades is given by

$$\Pi^f = \pi - c(a^q) - f(k) - p\left[\rho - a^q - k - q - q^f\right] - p^f q^f. \tag{46}$$

The first order conditions for maximal expected utility are,

$$\frac{\partial}{\partial k} E\left\{U\left(\Pi^f\right)\right\} = E\left\{U'\left(\Pi^f\right)\left[p - f'\left(k^f\right)\right]\right\} = 0, \tag{47}$$

$$\frac{\partial}{\partial q^f} E\left\{U\left(\Pi^f\right)\right\} = E\left\{U'\left(\Pi^f\right)\left[p - p^f\right]\right\} = 0, \tag{48}$$

which immediately implies that the forward price must equal the marginal cost of investment

$$f'\left(k^f\right) \equiv p^f. \tag{49}$$

As a consequence we see that investment must be the same at all firms, regardless of the amount of quotas q allocated to the firm at the outset.

Consider the symmetric grandfathering case in which all firms are allocated the same quota $\overline{E}$. Since all firms are the same, it is clear that, in equilibrium, there will be no forward trading. There will, however, exist an equilibrium forward price p^{f*}.[11] Note that in this case (46) reduces to

$$\Pi^f = \pi - c(a^q) - f(k) - p\left[\rho - a^q - k - q\right]. \tag{50}$$

Comparing $E\left\{U\left(\Pi^f\right)\right\}$ in the symmetric case to (38) it is clear that Proposition 9 carries through in this case as well and if (41) is satisfied we have

$$f'(k) > Ep^* \tag{51}$$

for all firms in equilibrium. Thus, there is inefficient overinvestment as before and furthermore by (49) we have

$$p^{f*} > Ep^*; \tag{52}$$

that is, the equilibrium forward price is strictly larger than the expected spot price of quotas.

It is clearly difficult to obtain results for the general case in which firms are allocated different quotas. However, it is clear that for allocations that are non-symmetric, but are sufficiently close to the symmetric allocation, there would be trade in the forward market, firms would use a mixture of investment, forward trading and abatement to meet their obligations and the inequality (52) would continue to hold.

[11] At a forward price higher than the equilibrium price all firms would want to sell their quotas in the forward market; at a lower price all would want to buy.

5.3 Non-tradable quotas

In the case when quotas are non-tradable, a firm will choose k so as to maximize

$$E\left\{U\left(\pi - f(k) - c(\rho - k - q)\right)\right\}. \tag{53}$$

The first-order condition for this problem is given by

$$E\left\{U'\left(\pi - c\left(\rho - q - k^n\right) - f\left(k^n\right)\right)\left[c'\left(\rho - q - k^n\right) - f'\left(k^n\right)\right]\right\} = 0. \tag{54}$$

By concavity of the utility function, it is immediate that firms do not minimise expected total cost of investment and abatement; in particular:

Proposition 10 *When there is uncertainty about the amount of the pollutant and quotas are non-tradable, investment cost exceeds expected abatement cost at the margin, i.e.* $f'\left(k^n\right) > E\left\{c'\left(a^n\right)\right\}$.

Clearly, comparing conditions (39) and (54), the equilibrium outcome does depend on whether or not quotas are tradable. In particular, when quotas are not tradable costs of abiding by regulations depend on idiosyncratic shocks only, while when quotas are tradable these costs depend also on aggregate uncertainty at the market level. Unless shocks are perfectly correlated, the market allows for an opportunity for diversification that tends to reduce the risk agents face.

To see this, consider again the case in which firms are allocated symmetric quotas and shocks are of the form given in (40). Suppose the common term R is completely deterministic. Then there is no uncertainty in the aggregate and so the quota market will clear at the same price in all contingencies. Consequently, firms face a fixed opportunity cost of investment and hence all firms will invest so as to equate marginal investment cost to the quota price. Therefore, while non-tradability would lead to inefficient over-investment, tradability would, in this case, guarantee overall cost efficiency.

Note the contrast between this result and the results referred to in Section 4.2. In that case, since uncertainty was purely extraneous, transmitted through market prices, opening the quota market for trade subjects agents to risk that tends to distort their decisions. However, here, where uncertainty is idiosyncratic, trade allows agents to hedge risk through market operations. More generally, there may be some aspects of the market that tend to reduce the risk faced by individual agents, and others that would tend to amplify it. The overall effect cannot be determined on theoretical grounds alone.

5.4 Differing attitudes towards risk

So far we have assumed that firms are equally risk averse. However, one can argue that firms may have different attitudes towards risk. For example, the relative importance (in revenue or cost terms) of activities subject to environmental regulation - and hence the exposure to risk originating from such regulation - may be greater for some firms than others. More generally, some (typically larger) firms will be more diversified than other (typically smaller) firms. Also, some firms have better access to capital markets than others. Whatever the cause, given that risk attitudes differ one may ask how such differences should affect the implementation of environmental policy - in particular the allocation of quotas and their tradability.

For illustration, let there be two groups of firms; risk neutral and risk averse. Risk neutral firms will choose investment such that at the margin investment cost equals the cost of abatement, which, in the tradable quotas case, equals the price of quotas. That is, we have

$$f'(k) = Ep = Ec'(R - \overline{E} - K^q) \tag{55}$$

in the tradable quotas case and

$$f'(k) = Ec'(\rho - q - k) \tag{56}$$

when quotas are non-tradable.

The marginal cost of abatement to which firms are exposed differ between the cases in which quotas are tradable and non-tradable, respectively. When quotas are tradable firms abate the same amount of pollutants and hence face the same abatement cost at the margin. When quotas are non-tradable, however, marginal abatement costs differ as long as investments (or quota allocations) differ. In particular, consider the case in which shocks are perfectly correlated and firms are allocated symmetric quotas such that $q = Q = \overline{E}$. Then, since risk averse firms will invest more than risk neutral firms, they will be exposed both to a lower abatement cost and less risk in the non-tradable quotas case (when marginal cost is determined by the firm's own amount of abatement) than when quotas are tradable (and they face a common marginal cost determined by average abatement).

To see this, note that under the above conditions the first-order conditions (39) and (54) for a risk averse firm in the tradable and non-tradable quota cases, respectively, may be written

$$E\left\{U'\left(\pi - c(\rho - q - K)\ - f(k) + p\left[k - K\right]\right)\left[p - f'(k)\right]\right\} = 0, \tag{57}$$

$$E\left\{U'\left(\pi - c(\rho - q - k) - f(k)\right)\left[c'(\rho - q - k) - f'(k)\right]\right\} = 0. \tag{58}$$

Fixing k, taking the derivative of the left-hand side of (57) with respect to K and applying the condition $p = c'(\rho - q - K)$ we find

$$\frac{d^2 E\{U(\Pi^q)\}}{dKdk} = E\{-U''c''[k-K][p-f'] - U'c''\}. \tag{59}$$

When $k > K$, this expression would typically be negative (U and c both being quadratic is sufficient to ensure this). If so,

$$\begin{aligned} & E\{U'(\pi - c(\rho-q-K) - f(k) + p[k-K])[p-f'(k)]\} \\ > \; & E\{U'(\pi - c(\rho-q-k) - f(k))[c'(\rho-q-k) - f'(k)]\} \end{aligned} \tag{60}$$

and, assuming the objective function is everywhere concave in k, it follows that for the risk averse firms $k^q > k^n$ and $a^q < a^n$ as long as the allocated quotas are the same. Conversely, for the risk neutral firms we have $k^q < k^n$ and $a^q > a^n$.

Allowing for trade between firms with different attitudes towards risk consequently has ambiguous effects on the expected costs of reducing emissions. On the one hand, trade equalises marginal abatement costs by increasing abatement in the low-cost (risk averse) firms and reducing abatement in the high-cost (risk neutral) firms. On the other hand, investment is skewed even further away from optimum, with less investment being undertaken by the low-cost (risk neutral) firms and more by the high-cost (risk averse) firms.

If it is known who are, and who are not, risk averse, cost efficiency may be achieved by allocating a sufficiently large share of quotas to risk averse firms (assuming that such an allocation is possible within the emission limit) and allowing for trade.

6 Conclusion

We have shown in this paper how risk aversion and uncertainty can change the comparison between the commonly considered regulatory instruments of taxes and quotas. In our model the choice of regulation affects firms' exposure to uncertainty. Our omniscient (and benevolent) government can choose to regulate by taxes and let society at large take on the risk through uncertainty in environmental outcomes, or regulate by tradable quotas and transfer risk to firms. Usually an inefficiency arises in the latter case which the authority can counteract in different ways, for example by judicious distribution of initial quotas, separation of pollution target and initial quota allocation and by acting *ex post* in the quota market. The proper action depends on each particular situation. Yet, it is clear that given a certain difference in efficiency between taxes and quotas, as in Weitzman (1974) and the literature on 'prices vs. quantities' (that is, whether to regulate by taxes or by quotas), the introduction of uncertainty and risk aversion will, in general, tilt the balance in direction of regulation by taxes.[12]

The policy implications of our analysis are broadly in line with analyses of other types of marked imperfections. However, there are differences also, one of which concerns the importance of the initial allocation of quotas. Stavins (1995) demonstrates that, in the presence of transaction costs that vary with volumes of trade, overall costs are at their lowest when quotas are allocated in such a way that trade is minimised. A similar result is obtained in cases in which firms have market power, since the incentive to distort price is proportional to trade volumes (Hahn, 1984). This is not so in our model. Given the concavity of preferences, firms lose more from a given purchase than they gain from a correspondingly large sale. Hence, even when, in expected terms, there is no trade at all optimality is not achieved. The optimal *ex ante* allocation is such that trade will indeed take place and, moreover, the government should itself trade actively in the *ex post* market.

References

[1] Baldursson, Fridrik M and Nils-Henrik M von der Fehr (1998): Prices vs. quantities: the irrelevance of irreversibility, Memorandum no. 9, Department of Economics, University of Oslo.

[2] Chao, Hung-po and Robert Wilson (1993): Option value of emission allowances, Journal of Regulatory Economics, 5, 233-49.

[3] Cole, Daniel H and Peter Z Grossman (1999): When is command-and-control efficient? Institutions, technology, and the comparative efficiency of alternative regulatory regimes for environmental protection, Wisconsin Law Review, 1999 (5), 887-938.

[4] Dixit, Avinash and Robert Pindyck (1994): Investment under Uncertainty, Princeton NJ: Princeton University Press.

[5] Hahn, R W (1984): Market power and transferable property rights', Quarterly Journal of Economics, 99, 753-65.

[12] Weitzman's result, obtained in a static model with quadratic cost curves and subsequently refined in a voluminous literature, stated that taxes, compared to quotas, become more attractive as a regulatory tool the greater is the slope of the marginal abatement cost function relative to the marginal environmental damage function.

[6] Karlin, S and H M Taylor (1981): A Second Course in Stochastic Processes, New York: Academic Press.

[7] Montero, Juan-Pablo (1999): Prices vs. quantities with incomplete enforcement, CEPR WP-99009, MIT.

[8] Radetzki, M (1990): A Guide To Primary Commodities in the World Economy, Oxford: Basil Blackwell.

[9] Saphores, Jean-Daniel M and Peter Carr (2000): Pollution reduction, environmental uncertainty and the irreversibility effect, mimeo, Department of Economics/GREEN, Université Laval.

[10] Stavins, Robert N (1995): Transactions costs and tradable permits, Journal of Environmental Economics and Management, 29, 133-48.

[11] Stavins, Robert N (1999): Experience with market-based environmental policy instruments, mimeo, to appear as Chapter 21 of The Handbook of Environmental Economics, Amsterdam: North-Holland/Elsevier Science.

[12] Weitzman, Martin L (1974): Prices vs. quantities, Review of Economic Studies, 41, 477-91.

[13] Xepapadeas, Anastasios (1999): Environmental policy and firm behavior: abatement investment and location decisions under uncertainty and irreversibility, mimeo., Department of Economics, University of Crete.

[14] Zhao, Jinhua (2000): Irreversible abatement investment under cost uncertainties: tradable emission permits and emissions charges, mimeo, Department of Economics, Iowa State University.