

# Transfer of risk in the newsvendor model with discrete demand

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## Abstract

In this paper we consider the transfer of risk in a newsvendor model with discrete demand. We view the newsvendor model as a leader/follower problem where the manufacturer (leader) decides the wholesale price and the retailer (follower) decides the quantity ordered. Taking a Pareto-optimal contract as a starting point, the manufacturer wishes to design a real option contract to enhance profits. A new real option contract is said to be feasible if both parties' expected profit is at least as great as in the original contract. When demand is discrete, there are usually infinite feasible contracts that yield maximum expected profits to the manufacturer. In the paper we show that either all, some or none of these real option contracts offer an improved position for the retailer.

Keywords: newsvendor model, discrete demand, real options, transfer of risk

## 1 Introduction

The one-period newsvendor model has attracted increasing interest in the past two decades. The model is simple, yet it exhibits interesting features that provide insights relevant to real-world problems. As we show in the literature review below, many scientific papers have been written on variations of this model. Most of these focus on the case where demand has a continuous distribution, whereas the discrete case has been studied less thoroughly. We provide a systematic treatment of the discrete case, leading to explicit formulas on how to maximize expected values.

We consider how to improve standard newsvendor contract channel coordination where the retailer takes all the risk by transforming it into a real option contract where the retailer and

manufacturer share the risk to maximize supply chain profit. Intuitively, the introduction of a real option contract should reduce the variance for the retailer compared with a standard newsvendor contract where the retailer bears all the risk. However, that turns out to be false in general, and we provide an explicit example to study this in more detail. We assume here that the manufacturer and the retailer are risk neutral in the sense that they prefer contracts with strictly higher expected profits even when they lead to higher variance. If two contracts offer the same expected profit, however, we suggest that both parties prefer the contract with the smallest variance.

We view the newsvendor model as a Stackelberg game. Both parties have full information on demand and want to maximize expected profit. At the first stage, we consider a classical newsvendor contract, i.e., without real options. The manufacturer decides the wholesale price and the retailer decides the order quantities. As both parties have full information, the manufacturer can compute a value  $W^*$  for the wholesale price, leaving her with maximum expected profit. When the wholesale price  $W^*$  is known, the retailer computes the corresponding order quantity  $q^*$ , providing him with optimal expected profit. The final result of this game is Pareto-optimal, and we refer to this contract as Contract O (original contract).

The basic problem with the game above is that the order quantity  $q^*$  is, in general, different from the optimal order quantity for the supply chain. Real options is one tool that can be used to enhance expected profits for the supply chain. At the second stage of our game, the manufacturer wishes to design a real option contract to enhance her expected profit. In doing so, we assume that she can only offer the retailer a contract where his expected profit is at least as high as in Contract O. Such alternative contracts are said to be feasible. At the second stage of the game, the manufacturer should seek feasible contracts to maximize expected profit. When the distribution is discrete, there are usually many feasible contracts maximizing expected profit, i.e., the solution is not unique. Faced with many different alternatives leading to the same expected profit, we assume that the manufacturer prefers the one with minimum variance. This contract, referred to as Contract A, is unique and is always obtained at values where the retailer has the same expected profit as in the original contract. At this stage, two different cases can occur:

Case 1. Contract A leaves the retailer with the same expected profit as in Contract O and decreases the variance. With our preference structure the new contract will be accepted, and the final result is Pareto-optimal.

Case 2. Contract A leaves the retailer with the same expected profit as in Contract O but increases the variance. With our preference structure the new contract will be rejected. Contract A is not Pareto-optimal. If this happens, the manufacturer must seek alternatives.

To examine further alternatives in Case 2, the manufacturer should consider Contract B, which is the contract with maximum expected profit and maximum variance for the manufacturer. This contract, too, is unique. We show that the sum of the standard deviation of the retailer's profit and the standard deviation of the manufacturer's profit is constant as long as the order quantities are fixed. Higher variance for the manufacturer is then equivalent to lower variance for the retailer. Again two cases can occur.

Case 2a. Contract B decreases the variance compared with Contract O. This contract will therefore be acceptable to the retailer. When this happens, several other contracts will be acceptable as well, and the manufacturer can minimize her variance at Contract C where the retailer has the same expected profit and the same variance as in Contract O. We examine the expected shortfall (i.e., expected profit in the  $r\%$  worst cases) in a contract of this type, and argue that a real option contract can protect the retailer against huge losses when demand is low. When this happens, Contract C will be accepted and is Pareto-optimal.

Case 2b. Contract B increases the variance compared with Contract O. When this happens, no Pareto-optimal contract can be obtained. It seems clear that the manufacturer has an incentive to give up expected profit to compensate the retailer for increased risk, but it is not clear how this can be done. With our preference structure, the retailer is risk neutral in the sense that he only evaluates risk for cases leading to the same expected profit. To be able to discuss a trade-off between expectation and variance, we must equip the retailer with some sort of utility function describing the trade-off. While this is certainly an interesting question, it leads to several complications. The classical closed-form solutions no longer apply, and most cases can only be handled by a numerical approach. Such questions are hence outside the scope of the

present paper.

At several different points in this paper, the optimal contract for the manufacturer is obtained for cases where the retailer is indifferent between two strategies, one of which is not optimal for the manufacturer. The manufacturer must then take some action to prevent the retailer from applying the suboptimal strategy. This can be done in several different ways. One option is to write the contract in such a way that it forces the retailer to apply the strategy that is optimal for the manufacturer; another approach is that the manufacturer gives up a marginal amount of profit to offer the retailer a marginally better contract if the retailer chooses the preferred strategy. In such degenerate cases, however, the retailer can credibly threaten to choose the suboptimal strategy. That, in turn, could lead to game-theoretic issues where the retailer bargains to obtain a bigger share of the profits. Although these are certainly interesting issues for future research, we do not enter into these discussions in this paper. Instead, we tacitly assume that some marginal actions are taken to prevent suboptimal choices in degenerate cases. The paper is organized as follows. In Section 2 we review the literature to explain how our theory fits with the existing literature in the field. In Section 3 we describe the basic properties of the models. To make the paper available to a wider audience, we include a comprehensive discussion of the relevant formulas. In Section 4 we examine a series of examples to prove the existence of the various cases discussed above. To our knowledge, these issues are new in the sense that they have not been discussed previously in the literature. Section 5 includes a short discussion relating the results to the continuous case. We offer some concluding remarks in Section 6.

## 2 Literature review

This paper relates to two areas of research: studies that focus on supply chain coordination and those that examine the behavior of risk-averse and/or risk-seeking supply chain agents.

In the standard newsvendor contract (Lariviere and Porteus (2001)), all members of the supply chain focus on maximizing their own profit. This leads to double marginalization (Cachón (2003)) and an inefficient supply chain. To improve efficiency, several types of coordination mechanisms have been introduced to help actors in a supply chain to achieve the maximum pos-

sible expected profit for the whole supply chain. For risk-neutral agents, the following supply chain contracts have been shown to coordinate the supply chain and obtain the maximum possible expected profit for the supply chain: the real option contract (Rudi and Pyke (2002)), the revenue-sharing contract (Cachón and Larivière (2005), Koulamas (2006), Lin and Hong (2009)), the buyback contract (Pasternack (1985)), the quantity-flexibility contract (Tsay (1999)), and the sales-rebate contract (Taylor (2002)). Rudi and Pyke (2002) discuss the increase in expected profit that can be achieved by a real option contract over the standard newsvendor contract. An excellent review of risk-neutral supply chain coordination contracts, which includes buyback contracts, revenue-sharing contracts, quantity-flexibility contracts, sales-rebate contracts, and quantity-discount contracts, is given by Cachón (2002).

Sweitzer and Cachón (2002) seek to describe and explain managers' newsvendor decisions. Based on two empirical experiments, they suggest that managers' newsvendor decisions are consistent with a preference to reduce ex post inventory error. With this result in mind they also propose techniques that improve decision making.

Real option contracts have been studied in many different settings and include cases with a single supplier as well as those with multiple suppliers. A good overview of the literature related to procurement using option contracts can be found in Fu et al. (2010) and Martínez-de-Albeniz and Simchi-Levi (2009). Fu et al. (2010) have also summarized previous work in a lucid table.

Option-based contracts are useful for reducing the financial impact and inventory risks at product launch for products with a short product life cycle and long production lead time. They are common in such industries as textiles, plastics, and semiconductor manufacturing (Martínez-de-Albeniz and Simchi-Levi (2009)).

We now consider work that takes into account different risk preferences for the individual supply chain decision makers. First, we review papers dealing with newsvendor models that consider different types of risk preferences. Then, we review work dealing with decision making by risk-averse agents in a supply chain with different types of coordination contracts.

Work that extends the standard newsvendor problem with different objectives and utility functions is well summarized in Khouja (1999). Some of the first papers to consider risk preferences in the standard newsvendor model are Lau (1980), Anvari (1987), and Chung (1990). Lau (1980) studied the newsvendor problem of maximizing expected utility, maximizing mean–variance, and maximizing the probability of achieving a budgeted profit. These objectives are rejected by Anvari (1987), who employed the capital asset pricing model to analyze the one-period newsvendor problem. Chung (1990) sharpens the optimal condition presented in Anvari (1987) and provides a simpler method for finding the optimal solution. Eeckhoudt et al. (1995) and Keren and Pliskin (2006) consider a risk-averse newsvendor with a second order opportunity. They find that a risk-averse newsvendor with a concave utility function orders less than a newsvendor who is risk neutral. Eeckhoudt et al. (1995) further show that the optimal order quantity for a risk-averse newsvendor decreases as risk aversion increases. Keren and Pliskin (2006) show that the increase in variance of the newsvendor’s demand may increase or decrease the optimal order quantity for a newsvendor with an increasing differentiable utility function and a uniformly distributed demand. Wang and Webster (2009a) consider a newsvendor with a piecewise-linear loss-aversion utility function and find that if shortage cost is not negligible the newsvendor may order more than a risk-neutral newsvendor. Further, they find that the optimal order quantity may increase in wholesale price and decrease in retail price. Van Mieghem (2007) study how risk and network structure can fundamentally change resource allocation in newsvendor networks. He finds that in contrast to single-resource settings, risk-averse newsvendors may invest more in networks than risk-neutral newsvendors.

Under a mean–variance (MV) framework, Choi et al. (2008) and Wu et al. (2009) report that, with a large stockout cost, the risk-averse newsvendor may order more than the risk-neutral order quantity. Choi et al. (2008) report that with a stockout cost the newsvendor can face both an increasing and a decreasing variance of profit with an increase of order quantity. They further report that with a large enough stockout cost the optimal order quantity for the risk-seeking newsvendor becomes less than the optimal risk-neutral order quantity. Their study also shows that by sacrificing a small amount of expected profit, the risk-averse newsvendor can enjoy a much larger reduction of the variance in profit while the risk-seeking newsvendor can enjoy a much larger increase in the variance of profit. Yang et al. (2008) study a newsvendor model

with capacity constraint under a downside-risk measure and under a Conditional Value-at-Risk (CVaR) risk measure. With a downside-risk measure, the retailer optimizes expected profit, while with the CVaR measure, the retailer only optimizes the part lower than a specific factor  $\alpha$ . The optimal order quantity is increasing in  $\alpha$  and in capacity. Intuitively, to measure the degree of risk aversion, the retailer needs to forecast only one parameter with the CVaR measure to get the optimal decision. Chen and Parlar (2007) study a risk-averse newsvendor who, in order to maximize an expected utility function, has the opportunity to buy a put option to reduce losses resulting from low demand. If the newsvendor pays the option writer the expected option payoff plus a risk premium, he will in return receive the strike price for each unit that the demand falls below the strike quantity. They show that for any given order quantity, the newsvendor's expected profit with the option is independent of the option parameters (strike price/quantity) but it differs from the expected profit without the option from the amount of the risk premium. A suboptimal solution approach in the context of expected utility maximization is proposed. They show, however, that the approach simplifies the computations, provides managerial insights, and does not result in a significant deterioration of expected utility. Under fairly general conditions within the expected utility theory (EUT) framework, a risk-averse newsvendor will order less as the selling price gets larger (Wang and Webster (2009b)). This is in general an unwanted behavior, and Wang and Webster's (2009b) main purpose is to show when, how, and why this limitation of EUT is manifested in newsvendor decision making.

The previous cited literature considers either work within supply chain coordination contracts or work within risk-averse/risk-seeking supply chain agents. We now review the literature that considers both supply chain coordination contracts and risk-averse and risk-seeking agents. The most common formulations proposed to model risk in this literature are the MV formulation and its variants (Lau and Lau (1999), Tsay (2002), Choi et al. (2008a), Choi et al. (2008b)), the downside-risk formulation (Gan et al. (2005)), and the CVaR formulation (Yang et al. (2009)). Lau and Lau (1999) discuss how a manufacturer could establish her pricing and return policy by properly considering not only the manufacturing cost and retail sale price, but also her own risk attitude and that of the retailer, as well as the demand uncertainty. Tsay (2002) analyzes how sensitivity to risk (in the context of MV preferences) affects both sides of the manufacturer–retailer relationship under various scenarios of strategic power, and how

these dynamics are altered by a return policy. Choi et al. (2008a) focus on MV analysis of various scenarios of the supply chains with the wholesale price fixed and the return price as the sole decision parameter in the return policy. Choi et al. (2008b) propose to use an MV formulation to quantify risk proneness and risk aversion in the supply chain. They study how the manufacturer can set the wholesale price to achieve channel coordination in a buyback contract where the return price is a fixed portion of the wholesale price. They show that in some cases, supply chain coordination is not achievable because of the problem of “matching” the risk preferences of the individual supply chain agents. However, as they also mentioned in their paper, a buyback contract where the return price is independent of the wholesale price could more easily achieve channel coordination. Gan et al. (2005) study a supply chain with a risk-neutral supplier and a downside-risk-averse retailer. They design a risk-sharing contract that offers the desired downside protection to the retailer, provides respective reservation profits to the agents and accomplishes channel coordination (according to the definition in Gan et al. (2004)). The contract consists of an initial contract (which could be a buyback contract, a revenue-sharing contract, or a wholesale price contract) and a clause that ensures that a limited portion of the unsold items is fully refundable. This ensures that the retailer orders the optimal supply chain quantity even if the downside risk to the retailer is higher than a level he is otherwise willing to take. Yang et al. (2009) investigate supply chain coordination with a risk-neutral supplier and a risk-averse retailer where the retailer optimizes his CVaR performance measure under commonly used supply contracts such as revenue-sharing, buyback, two-part tariff, and quantity-flexibility contracts.

Gan et al. (2004) consider the coordination of risk-averse agents in an MV framework. Their approach is similar to ours in that they focus on Pareto-optimal contracts, but they go considerably deeper in discussing more subtle game-theoretic issues. Our approach is different in that we consider a Stackelberg game, focus on real options as a coordinating tool, and study the case with discrete demand. In the discrete case, there is usually an infinite number of feasible contracts leading to maximum expected profit for the manufacturer, and this nonuniqueness leads to a number of issues that do not appear in the continuous case.



### 3 Basic properties of the models

In this section, we survey the basic formulas related to the newsvendor model and the real option model. On the technical side, we often need the function  $(x)^+$ , which is defined as follows

$$(x)^+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

#### 3.1 The newsvendor model with real options

In the classical newsvendor model, a retailer plans to sell a commodity in a market with uncertain demand  $D$ . The retailer orders a number of units of the commodity from a manufacturer, and hopes to sell enough of these units to make a profit. We assume that the manufacturer faces a fixed manufacturing cost  $M$  and decides on the wholesale price  $W$ . We further assume that the retailer faces an exogenously given retail price  $R$ , and decides the order quantity  $q$ . Unsold items can be salvaged at the exogenously given and constant salvage value  $S$ . We also assume that both parties are fully informed, i.e., that they know the distribution of  $D$ .

##### 3.1.1 Profit functions

The retailer's profit is denoted by  $\Pi_r(q)$ . Profits in the newsvendor model can be rewritten in several different ways. For the analysis in this paper, it is convenient to express everything in terms of the random variable  $\min[D, q]$ . Using the relation  $(q - D)^+ = q - \min[D, q]$ , we get

$$\Pi_r(q) = R \min[D, q] + S(q - D)^+ - Wq \tag{1}$$

$$= R \min[D, q] + S(q - \min[D, q]) - Wq \tag{2}$$

$$= (R - S) \min[D, q] - (W - S)q \tag{3}$$

The manufacturer's profit is denoted by  $\Pi_m(q)$ . In the classical newsvendor model, the manufacturer has a constant profit, given by the expression

$$\Pi_m(q) = (W - M)q \quad (4)$$

### 3.1.2 Real options

In a real option contract, the retailer can buy options that allow him to purchase a good at a future date. The price of one option is  $c$ . Each option offers the right but not the obligation to buy one unit of the good at a fixed price  $x$ . The order to purchase options is submitted before the actual demand is realized. If the retailer buys  $q$  options, his profit  $\hat{\Pi}_r(q, c, x)$  is given by

$$\hat{\Pi}_r(q, c, x) = (R - x) \min[D, q] - cq \quad (5)$$

In this contract, some of the risk is transferred to the manufacturer. The manufacturer's profit is now a random variable  $\hat{\Pi}_m(q, c, x)$  given by

$$\hat{\Pi}_m(q, c, x) = x \min[D, q] + S(q - D)^+ + (c - M)q \quad (6)$$

$$= x \min[D, q] + S(q - \min[D, q]) + (c - M)q \quad (7)$$

$$= (x - S) \min[D, q] - (M - S - c)q \quad (8)$$

It is important to notice that the supply chain profit  $\hat{\Pi}_{\text{chain}}(q, c, x)$  only depends on the pair  $c, x$  through  $q$ . This is seen as follows

$$\hat{\Pi}_{\text{chain}}(q, c, x) = (x - S) \min[D, q] - (M - S - c)q + (R - x) \min[D, q] - cq \quad (9)$$

$$= (R - S) \min[D, q] - (M - S)q \quad (10)$$

Note, however, that different choices of  $c, x$  will lead the retailer to order a different number of options  $q$ . If the supply chain is organized as a single unit, the administration can instruct the retailer to order a specific quantity  $q$ , regardless of profit. In that case channel coordination can be obtained via  $W = M$ , and no options are needed to optimize profits. In the following, we consider the more important case where the manufacturer and the retailer act independently to maximize their profits. We assume that  $R, S, W$ , and  $M$  are given such that  $R > W > M > S$ . The manufacturer offers options at prices  $c, x$ , and the retailer orders a quantity  $q$  to maximize

his expected profits.

The retailer's optimization problem is simple, given  $c, x$  find  $q = \hat{q}$  such that the expected profit

$$E[\hat{\Pi}_r(q, c, x)] = (R - x)E[\min[D, q]] - cq \quad (11)$$

is as large as possible. Note that  $\hat{q} = \hat{q}(c, x)$ . This makes the manufacturer's optimization problem more complicated, as she wants to choose  $c, x$  to maximize her expected profit

$$E[\hat{\Pi}_m(\hat{q}, c, x)] = (x - S)E[\min[D, \hat{q}]] - (M - S - c)\hat{q} \quad (12)$$

Alternatively a newsvendor may wish to maximize the probability of a target profit and/or target revenue, see, e.g., Yang et al. (2011). That approach leads to different optimization problems for both parties and will not be discussed in our paper.

### 3.2 Discrete distributions and the first stage of the game

At the first stage of the game, the retailer is faced with a standard newsvendor contract, and expected profits are maximized at  $q = q^*$ , which maximizes

$$E[\Pi_r(q)] = (R - S)E[\min[D, q]] - (W - S)q \quad (13)$$

We want to consider the case where  $D$  has a discrete distribution with values  $d_1, d_2, \dots, d_n$  and probabilities  $p_1, p_2, \dots, p_n$ . When  $R, S$ , and  $W$  are given, the optimal order quantity  $q^*$  can be obtained as follows.

#### Proposition 1

Let  $1 \leq k \leq n$  be the smallest integer s.t.

$$\sum_{i=1}^k p_i \geq \frac{R - W}{R - S}$$

Maximum expected profit in (13) is always achieved using  $q = d_k$ . In the degenerate case where

$$\sum_{i=1}^k p_i = \frac{R - W}{R - S}$$

the expected value is constant if  $q \in [d_k, d_{k+1}]$ . In all other cases the optimal order quantity is unique.

Proof

See the appendix. □

Because we have assumed that  $R > W > M > S$ ,  $\frac{R-W}{R-S} < 1$ , and the procedure above always produces the solution to the problem. Note that if  $D$  has a continuous distribution, it can be approximated by a discrete distribution such that  $\sum_{i=1}^k p_i \approx \int_{-\infty}^d f_X(u) du = F_X(d)$ . Passing to the limit, the maximum is therefore obtained at  $d = F_X^{-1}(\frac{R-W}{R-S})$ , which coincides with the well-known formula for the continuous case.

When both parties are fully informed, the manufacturer can anticipate how much the retailer will order, and can choose  $W$  to maximize her expected profit. The function  $W \mapsto E[\Pi_m(q)]$  is quite complicated in the discrete case. As we see from the plot in Figure 3, this function is in general nonmonotone and discontinuous. Several numerical optimizing tools will fail to find the maximum of such functions, so some care must be taken to find the optimal value  $W^*$ . As this is a function of only one variable, numerical solutions can nevertheless be found without much difficulty.

### 3.3 Introducing real options to enhance profits

It is easy to see that with a properly designed real option contract, the manufacturer can always extract all profit from the retailer, i.e., it is possible to write a contract where the manufacturer gets a profit arbitrarily close to the maximum profit for the supply chain. In this paper, we henceforward consider the more interesting case where the retailer already has a newsvendor contract as defined in the previous subsection. The issue for the manufacturer is then to design a real option contract where both parties are better off. In the real option case, the manufacturer chooses  $c, x$  to maximize expected profit. A contract is said to be feasible if both parties have at least as much profit as in the original newsvendor contract. Feasible pairs  $c, x$  are hence constrained by

$$E[\hat{\Pi}_r(\hat{q}, c, x)] \geq E[\Pi_r(q^*)] \tag{14}$$

and

$$\mathbb{E}[\hat{\Pi}_m(\hat{q}, c, x)] \geq (W - M)q^* \quad (15)$$

$\mathbb{E}[\Pi_r(q^*)]$  and  $\mathbb{E}[\Pi_m(q^*)]$  are the minimum values that the manufacturer and retailer must have in order to take part in the contract. These minimum values are a proxy for bargaining power and can also be calculated based on an outside opportunity profit for the firms (Cachón et al. (2005)). The sum of the minimum requirements has to be less than the total expected profit for the supply chain, and the feasible range for  $c$  and  $x$  decreases as this sum increases.

If  $c + x > R$ , the retailer cannot make a positive profit, so we can assume  $c + x \leq R$ . The set  $c + x \leq R$  can be partitioned into sectors  $S_k, k = 1, \dots, n$  defined as follows

$$S_k = \left\{ (c, x) \mid (R - x) \sum_{i=k+1}^n p_i \leq c \leq (R - x) \sum_{i=k}^n p_i \right\} \quad (16)$$

**Proposition 2**

*In sector  $S_k$  the retailer obtains maximal expected profit ordering  $\hat{q} = d_k$  options. At the lower border he obtains the same expected profit using  $q = d_{k+1}$ , and at the upper border he obtains the same expected profit using  $q = d_{k-1}$ .*

Proof

See the appendix. □

By Proposition 2 the manufacturer faces a constant order  $q = d_k$  in sector  $S_k$ . After some reordering of terms, the constraints (14) and (15) take the explicit forms

$$c \leq \frac{1}{d_k} \left( (R - x) \left( \sum_{i=1}^{k-1} d_i p_i + d_k \sum_{i=k}^n p_i \right) - \mathbb{E}[\Pi_r(q^*)] \right) \quad (17)$$

$$c \geq \frac{1}{d_k} \left( (W - M)q^* - (x - S) \left( \sum_{i=1}^{k-1} d_i p_i + d_k \sum_{i=k}^n p_i \right) \right) + (M - S) \quad (18)$$

Notice that the slopes of the borders in (17) and (18) are equal, and hence the borders are parallel. The set of feasible pairs for the manufacturer is the union of the sectors  $S_k$  constrained

by (17) and (18). Note that some of the sectors may be empty, as the border in (17) may be below the border in (18).

The feasible set for the manufacturer is in general quite complicated. An example is the black shaded area in Figure 1. In Figure 1 we used the values  $R = 10, S = 1, M = 4$ , and  $W = 8.2$ .

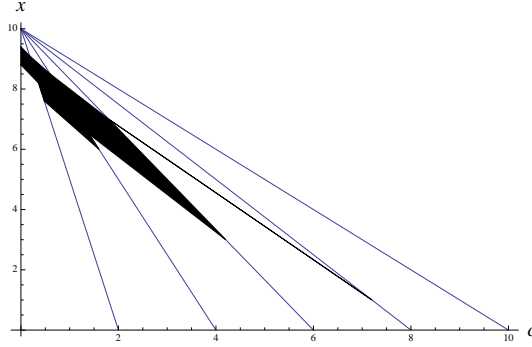


Figure 1: The feasible set with a uniform demand on  $\{10, 20, 30, 40, 50\}$

Nevertheless, the solution to the manufacturer's optimization problem turns out to be very simple. The following result is crucial in this respect.

### Proposition 3

*The manufacturer's expected profit is always maximal in the same sector where the expected profit for the supply chain is maximized.*

### Proof

See the appendix.

□

Proposition 3 greatly simplifies the search for optimal strategies. To find the optimal strategies for the manufacturer, we first consider the unconstrained problem: find  $q$  such that the expected supply chain profit

$$E[\Pi_{\text{chain}}(q)] = (R - S)E[\min[D, q]] - (M - S)q \quad (19)$$

is maximal. It follows directly from Proposition 1 that this maximum is obtained at  $q = d_k$ , where  $k$  is found by the condition

$$\sum_{i=1}^{k-1} p_i < \frac{R-M}{R-S} \leq \sum_{i=1}^k p_i \quad (20)$$

By our assumption,  $0 < \frac{R-M}{R-S} < 1$ , so such  $k$  always exists. By Proposition 3 we can restrict the problem to the subset of  $S_k$  constrained by (17) and (18); see the shaded area in Figure 2 for an example.

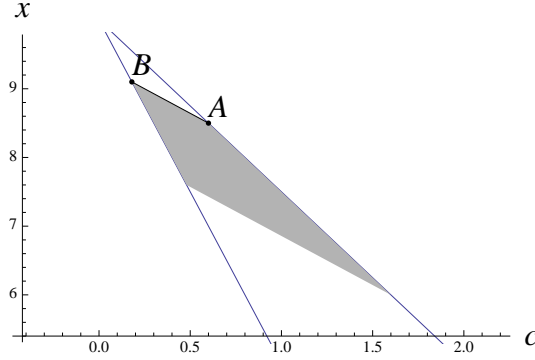


Figure 2: The feasible subset of the optimal sector  $S_4$  of Figure 1

Note that the manufacturer's expected profit is constant along lines parallel to the borders of (17) and (18), and increases when the line moves upwards. (17) invokes the upper border, hence the manufacturer obtains maximal expected profits at the points in sector  $S_k$  where

$$c = \frac{1}{d_k} \left( (R-x) \left( \sum_{i=1}^{k-1} d_i p_i + d_k \sum_{i=k}^n p_i \right) - E[\Pi_r(q^*)] \right) \quad (21)$$

For more information about how the wholesale price affects supply chain performance, see Lariviere and Porteus (2001). They show that as relative variability of demand decreases, the retailer's price sensitivity decreases, wholesale price increases, the decentralized system becomes more efficient, and the manufacturer's share of realized profit increases. They explore factors that may lead the manufacturer to set a wholesale price below that which would maximize her profit, concentrating on retailer participation in forecasting and retailer power.

### 3.4 Transfer of risk

We have seen that the manufacturer has maximum expected profit along the nonempty intersection of (21) with  $S_k$ . That does not mean that all these points are equally favorable to the manufacturer. In a real option contract some of the risk is transferred to the manufacturer, and

this risk varies along (21); see the line between A and B in Figure 2 above. From (6) we get

$$\text{Var}[\hat{\Pi}_m(q, c, x)] = (x - S)^2 \text{Var}[\min[D, q]] \quad (22)$$

Clearly, all feasible points satisfy  $x > S$ . The variance hence decreases as we move down (21). Note, however, that there is a problem at point A in Figure 2. At that particular point the retailer is indifferent between any order in the interval  $[d_{k-1}, d_k]$ , and except for the very unlikely case where the supply chain expected profit is also constant on  $[d_{k-1}, d_k]$ , the manufacturer runs the risk of considerably less expected profit at the border. To avoid the risk associated with ties, the manufacturer could offer a contract slightly below the border to obtain minimum variance. Alternatively, the contract could specify that the retailer orders the supply chain optimal quantity  $d_k$ , which should be acceptable because this is also one of the optimal strategies for the retailer. In that case, minimum variance is obtained at the intersection between (21) and the upper border of  $S_k$ , i.e., point A in Figure 2.

The retailer's variance is given by the expression

$$\text{Var}[\hat{\Pi}_r(q, c, x)] = (R - x)^2 \text{Var}[\min[D, q]] \quad (23)$$

and hence the retailer's variance decreases as we move up (21). Let  $\text{SD}_r, \text{SD}_m, \text{SD}_{\text{chain}}$  denote the standard deviation in the real option contract of the retailer, manufacturer, and supply chain, respectively. If  $X \geq S$ , it follows trivially from (22) and (23) that

$$\text{SD}_r + \text{SD}_m = \text{SD}_{\text{chain}} \quad (24)$$

Because  $\text{SD}_{\text{chain}} = (R - S)\text{SD}[\min[D, q^*]]$ , which is independent of  $x$  and  $c$ , (24) shows how the risk is shared between the two companies.

Intuitively, the introduction of options should reduce the variance for the retailer compared with a standard newsvendor contract where the retailer bears all the risk. Generally, supply chain coordination contracts are introduced to induce the retailer to order more than in the simple wholesale contract. To achieve this goal, the manufacturer has to reduce the retailer's risk of



carrying leftover stock. For the real option contract, this means that the option price has to be set below  $W - S$ . By reducing  $c$ , the manufacturer shifts some risk from the retailer to herself. In return, because of the increased order size, the manufacturer can achieve an increase in her expected profit. Note that when the retailer increases his order size, the variance of expected supply chain profit will in general also increase. If the manufacturer takes all of the expected profit generated by the increase in the order quantity, it would be reasonable to believe that the manufacturer's variance would be similarly increased. That, however, turns out to be false in general.

It can be shown that the variation of the retailer's expected profit may increase, compared with the newsvendor contract, even if his expected profit does not. This can happen if the relative increase in variance from the newsvendor contract is very large. If  $SD_{\text{chain}}$  is much larger than the standard deviation that the retailer faces in the newsvendor contract, however, it may happen that a 100% share of the original standard deviation is smaller than the retailer's share of the total risk  $SD_{\text{chain}}$ . Lau et al. (1999) discuss how expected profit may increase from bearing risk in a buyback contract. They show, among other results, that by setting the return price in the buyback contract as close as possible to the wholesale cost, the manufacturer can shift as much risk (and hence expected profit) as possible to herself. For a wholesale contract with stockout cost, Choi et al. (2008), report that the variance of the retailer may both increase and decrease as  $q$  increases.

To study the risk in more detail, we consider the extreme points along (21). The intersection of (21) with the upper border of  $S_k$  we call A, and the intersection of (21) with the lower border of  $S_k$  we call B. For an example see Figure 2.

To avoid problems associated with ties, we assume that the retailer orders  $q = d_k$  at both points. In general three main cases can occur:

- The retailer's variance is lower than the variance in the newsvendor contract in both A and B.
- The retailer's variance is lower than the variance in the newsvendor contract in B but

higher than that in A.

- The retailer's variance is higher than the variance in the newsvendor contract in both A and B.

These are the cases discussed in the introduction.

## 4 Examples

To demonstrate the various effects we claimed in Section 3.4, we now consider some particular examples. In all these examples, we consider demand  $D$  with  $n = 5$  different values, using the fixed values

$$R = 10, M = 4, S = 1$$

.

As noted by Szegö (2005), the variance can be adopted as a measure of risk only if the relevant distribution is symmetric about the mean. To offer a systematic treatment of the examples below, we therefore consider a symmetric demand  $D$  given by

$$(d_1, d_2, \dots, d_5) = (30, 33, 33 + a, 33 + 2a, 33 + 2a + 3), \quad (25)$$

where  $a > 0$  is a number that we vary in the examples, with fixed probabilities

$$(p_1, p_2, \dots, p_5) = (0.06, 0.29, 0.30, 0.29, 0.06) \quad (26)$$

When we increase  $a$ , we increase the gaps between points 2 and 3 and points 3 and 4 as follows:



Our basic framework is a setting where the manufacturer chooses  $c$  and  $x$  to optimize her performance. We make the additional assumption that the manufacturer can choose  $W = W^*$  in the newsvendor contract to optimize expected profits. Only then can the two contracts be compared on an equal basis.

## 4.1 Existence of a Pareto-optimal Contract A

In this example we consider the case where  $a = 10$ , i.e.,

$$(d_1, d_2, \dots, d_5) = (30, 33, 43, 53, 56)$$

We show that this distribution leads to a Pareto-optimal Contract A as explained in the introduction. In the newsvendor contract the expected profit for the manufacturer is a function of  $W$ . This function is shown in Figure 3.

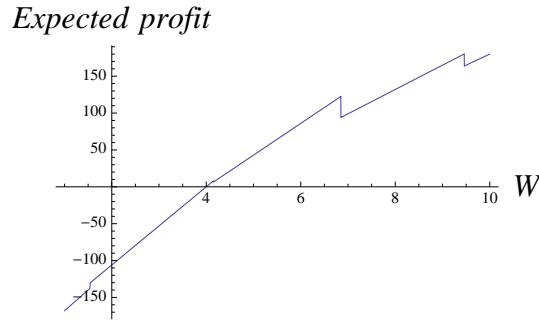


Figure 3: Expected profit for the manufacturer in the newsvendor contract

Expected profit is maximized at  $W^* = 9.46$ . Note, however, that the function is discontinuous at that point. If  $W = 9.46$ , the retailer is indifferent between  $q = 30$  and  $q = 33$ . The  $q = 33$  case provides maximum expected profit for the manufacturer, while her profit when  $q = 30$  is smaller. As mentioned in the introduction we do not wish to enter into a discussion of degenerate cases, but simply assume that some marginal action is taken to prevent the retailer from ordering  $q = 30$  units.

In this case the optimal sector is  $S_4$ , and the feasible subset of this sector is shown in Figure 4.

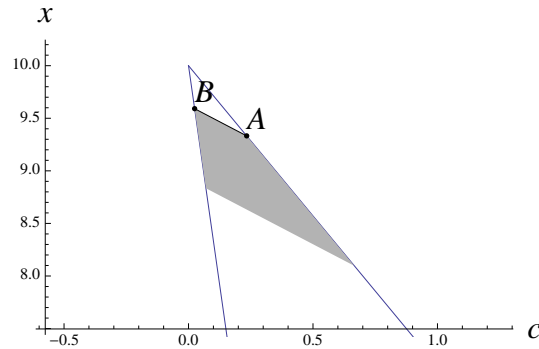


Figure 4: Critical points from Example 4.1

The optimal allocation for the manufacturer is point A in Figure 4. The manufacturer has maximum expected profit along the line between A and B, but A is the preferred choice as this is the point with the smallest variance. The relevant values for the original newsvendor contract and the real option contract at A are shown in the following tables.

Newsvendor contract: $W^* = 9.46, q^* = 33$			Real option contract: $c = 0.23, x = 9.33, q^* = 53$		
	Manufacturer	Retailer		Manufacturer	Retailer
Expected profit	180.2	16.2	Expected profit	210.2	16.2
Standard deviation	0	6.41	Standard deviation	71.7	5.75

As we can see from the tables, the retailer has the same expected profit in both contracts. As the variance is smaller in the real option contract, the retailer should prefer this contract to the newsvendor contract.

## 4.2 Existence of Pareto-optimal Contract C

In this example we consider the case  $a = 20$ , i.e.,

$$(d_1, d_2, \dots, d_5) = (30, 33, 53, 73, 76)$$

We show that this distribution leads to the Pareto-optimal Contract C as explained in the introduction. We compute  $W^* = 9.46$ , and the optimal sector is again  $S_4$ . The feasible subset of  $S_4$  is shown in Figure 5.

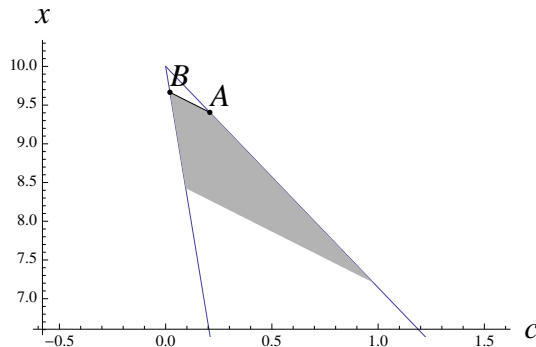


Figure 5: Critical points from Example 4.2

The data for the real option contract at A are shown in the following tables.

Newsvendor contract: $W^* = 9.46, q^* = 33$			Real option contract: $c = 0.21, x = 9.41, q^* = 73$		
	Manufacturer	Retailer		Manufacturer	Retailer
Expected profit	180.2	16.2	Expected profit	240.2	16.2
Standard deviation	0	6.41	Standard deviation	142.6	10.1

From these tables we see that a problem has emerged. The retailer has the same expected profit in both contracts, but the variance is larger in the real option contract. There is no good reason why the retailer should accept the real option contract in this case, and we must therefore search for alternative contracts. If we examine point B in Figure 5, we get the following results.

Newsvendor contract: $W^* = 9.46, q^* = 33$			Real option contract: $c = 0.02, x = 9.67, q^* = 73$		
	Manufacturer	Retailer		Manufacturer	Retailer
Expected profit	180.2	16.2	Expected profit	240.2	16.2
Standard deviation	0	6.41	Standard deviation	147.0	5.67

We see that at point B, the manufacturer still obtains maximum expected profit. The variance of her profit is larger than in A, but in this case the retailer faces a smaller variance in the real option contract. The retailer should prefer the real option contract to the newsvendor contract in this case. The manufacturer can still improve her position from point B, however. There must be a point C along the line between A and B such that the retailer has the same variance as in the newsvendor contract. This point is easily computed and is shown in Figure 6.

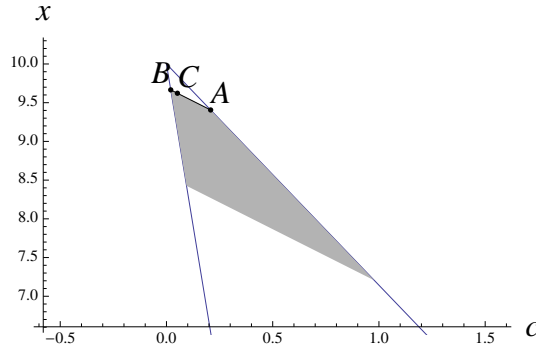


Figure 6: Critical points from Example 4.2

The data for the real option contract at C are given as follows.

Newsvendor contract: $W^* = 9.46, q^* = 33$			Real option contract: $c = 0.05, x = 9.62, q^* = 73$		
	Manufacturer	Retailer		Manufacturer	Retailer
Expected profit	180.2	16.2	Expected profit	240.2	16.2
Standard deviation	0	6.41	Standard deviation	146.2	6.41

As we can see from these tables, the retailer has the same expected profit and the same variance in both contracts. Why should he prefer one contract to the other? The variance is a crude measure of risk and does not contain all relevant information. To examine the risk at a deeper level, we have computed expected shortfall curves for the two contracts. The expected shortfall is defined as follows.

*Expected profit in the  $r\%$  worst cases*

Figure 7 shows the shortfall curves for the two different contracts using the data from point C above. The shortfall curve for the newsvendor contract is the thin line, while the thick line shows corresponding values for the real options contract.

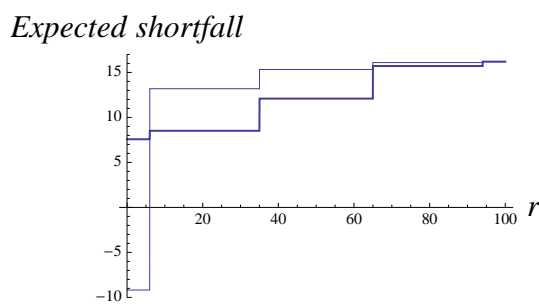


Figure 7: Shortfall curves

We see that the shortfall curve for the real option contract is better for small values of  $r$ . In practice the real option contract protects against a large loss if demand turns out to be 30 units. If the retailer wants to be protected against such losses, he would prefer such a contract to the newsvendor contract even though the expected profits and variances are equal in the two contracts.

### 4.3 Empty Pareto-set at the frontier

In this example we consider the case  $a = 30$ , i.e.,

$$(d_1, d_2, \dots, d_5) = (30, 33, 63, 93, 96)$$

We show that there are no Pareto-optimal solutions at the frontier of this distribution, i.e., Case 2b in the introduction. Here again  $W^* = 9.46$ . The optimal sector is  $S_4$ , and the feasible subset

of  $S_4$  is shown in Figure 8.

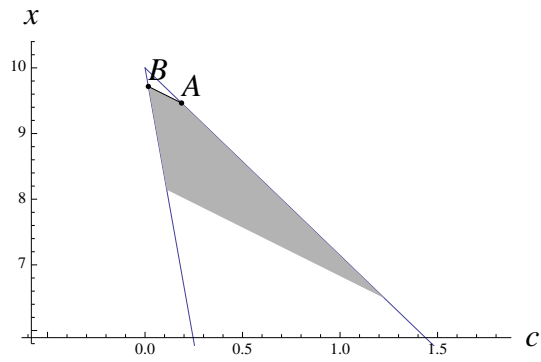


Figure 8: Critical points from Example 4.3

The data for the real option contracts at A and B are shown in the following tables.

Point A:

Newsvendor contract: $W^* = 9.46, q^* = 33$			Real option contract: $c = 0.19, x = 9.46, q^* = 93$		
	Manufacturer	Retailer		Manufacturer	Retailer
Expected profit	180.2	16.2	Expected profit	270.2	16.2
Standard deviation	0	6.41	Standard deviation	214.4	13.55

Point B:

Newsvendor contract: $W^* = 9.46, q^* = 33$			Real option contract: $c = 0.017, x = 9.72, q^* = 93$		
	Manufacturer	Retailer		Manufacturer	Retailer
Expected profit	180.2	16.2	Expected profit	270.2	16.2
Standard deviation	0	6.41	Standard deviation	220.7	7.17

From these tables we see that we have the same problem as in Example 2.2. At point A the retailer has the same profit in both contracts, but the variance is larger in the real option case. In this example, however, the basic problem remains if we move to point B. There are no points along the line from A to B that are acceptable to the retailer. To offer an acceptable contract to the retailer, the manufacturer has to provide the retailer with some extra profit, and the question is then how much extra profit the retailer will need. As mentioned in the introduction, it is not clear how this can be done. Further discussion of this effect is left for future research.

#### 4.4 A parametric discussion of the three main cases

In the examples above we proved the existence of the three principal cases discussed in the introduction. An important next step would be to derive the parameter values for which these cases occur. At this stage, we do not know how this can be done. To shed some further light on this problem, we considered distributions of demand shown in Equation (25) with probabilities shown in Equation (26). Varying the gap  $a$  on the interval  $[0, 30]$ , we computed the retailer's standard deviation at the frontier (maximum expected profit for the manufacturer) at point A (maximum standard deviation for the retailer) and at point B (minimum standard deviation for the retailer). The result is shown in Figure 9.

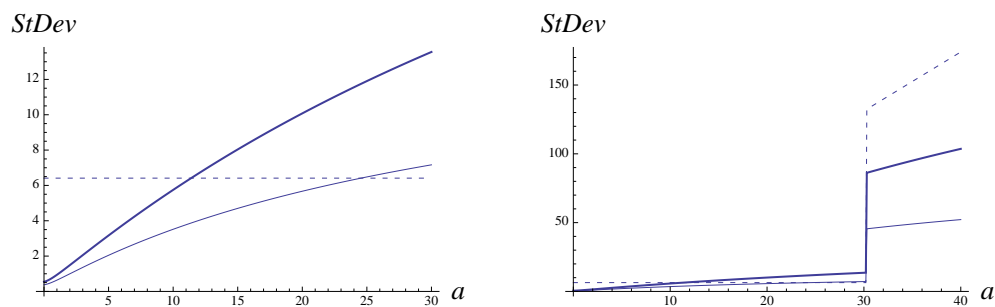


Figure 9: Standard deviation for the retailer at A (thick line) and B (thin line)



The dotted line is the standard deviation in the newsvendor problem, the thick line is the standard deviation at point A, and the third line is the standard deviation at point B. When the gap  $a$  is small, the values at A and B are both below the dotted line. As  $a$  increases, the standard deviation in the real option case increases because the retailer is trading larger volumes in the real option case. When  $a = 11.4$ , the standard deviation at A equals the standard deviation in the newsvendor problem. When  $a = 24.6$  the standard deviation at B equals the standard deviation in the newsvendor problem.

If we increase  $a$  beyond  $a = 30.3$ , the situation is completely reversed, as shown in Figure 9. For small values of  $a$  the manufacturer prefers the retailer to order moderate amounts, i.e.,  $q = d_2$ , in the original newsvendor contract. When  $a$  is sufficiently large, however, the profit from trading large volumes compensates for the risk, and if  $a \geq 30.3$ , the preferred order in the original contract is shifted from  $d_2$  to  $d_3$ . When  $a \geq 30.3$ , the values at A and B are both clearly below the dotted line. In fact they remain in that position no matter how much we increase the size of the gap.

The discussion above shows that even a simple single parameter case leads to several complications, which suggests that general conditions on the parameters may be hard to find.

## 5 Comparison with the continuous case

As all continuous distributions can be approximated arbitrarily well by discrete distributions, and vice versa, most results for the discrete case carry over to the continuous case. Nevertheless, there are some important differences. In this paper we consider contracts on the line segment between the extremes A and B. All contracts along this frontier provide the maximum expected profit to the manufacturer. Among these contracts she prefers that with the smallest variance. At the limit, the feasible set degenerates into a line segment in the continuous case. The line segment between A and B collapses into a single point, in which case there is not too much left for discussion. See Figure 10 for an illustrating example.

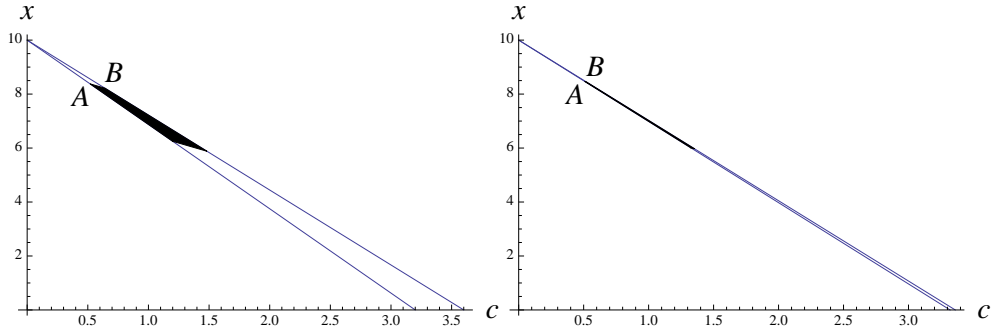


Figure 10: Feasible subsets for a discrete distribution with 25 values (a) and 250 values (b)

### *Existence/nonexistence*

The existence/nonexistence of Pareto-optimal contracts carries over to the continuous case. To see why, we can use approximate delta functions (continuous) to find continuous distributions that exhibit values arbitrary close to those reported in Section 4. That approach, however, does not reflect how continuous distributions are usually applied to newsvendor problems.

In a newsvendor setting, a researcher is often left with a moderately sized (obviously discrete!) set of observations of demand. The exact form of the distribution is usually not known. In such cases, he or she calibrates a parametric expression, very often a normal distribution, to these observations. That is a commonly used approach that often performs well. When the true distribution is discrete, it might be better to work directly with the discrete expressions. The difference is sometimes substantial. If, for example, the true distribution is that in Example 4.3, an approximation with a normal distribution can lead to a loss of up to 20% of expected profit for  $\beta = \frac{W-S}{R-S} \in [0.2, 0.8]$ . A further discussion of discrete distributions related to distribution-free problems, however, is beyond the scope of this paper and is left for future research.

## 6 Concluding remarks

In this paper we have shown how real options can be used to transfer risk (measured in terms of variance) between the manufacturer and the retailer in a newsvendor setting. The major part of the paper has focused on the case in which both parties start out with a Pareto-optimal newsvendor contract, and where the objective of the manufacturer is to design alternative real option contracts where both parties have at least as much expected profit as in the original

contract. Such contracts are said to be feasible.

The main focus of the paper is the discrete demand case. Several interesting issues appear in this case. The set of feasible contracts is, in general, a very complicated nonconvex set. It is therefore surprising to observe that the search for optimal strategies by the manufacturer can always be restricted to a line segment, and along this line segment the sum of the manufacturer's and the retailer's standard deviations is constant.

In the paper we have provided explicit examples showing that Pareto-optimal contracts may or may not exist at the frontier where the manufacturer's expected profit is maximized. On this subset the retailer has exactly the same expected profit as in the original contract, and will only accept the alternative contract if it leads to a reduction in risk. We have proved, however, that the subset of the frontier leading to less risk may be empty.

As there exist distributions where the set of Pareto-optimal contracts is empty, an interesting next step would be to provide necessary and sufficient conditions for this to happen. In this paper we consider these issues given a single parameter family of distributions in which case the Pareto-optimal contracts is empty when the parameter is in a certain interval. Even this simple version leads to complications. The main problem is that the optimal order quantity in the original contract is a discontinuous function of demand. The problem splits into a potentially large number of cases, indicating that simple conditions may be hard to find.

In this paper we have focused on real option contracts. The same set of examples could of course be obtained using buyback contracts. It could be of interest to study these issues using other types of contracts as well. In a follow-up paper, we will show how to construct optimal strategies for a contract mixing a standard newsvendor contract with a real option contract. Mixed contracts involve a number of additional issues, however, and are not discussed here.

## **Acknowledgments**

The authors thank the referees for several useful suggestions to improve the paper.

## 7 Appendix

### 7.1 Proof of Proposition 1

From (3), if  $d_l \leq q \leq d_{l+1}$ , then

$$\mathbb{E}[\Pi_r[q]] = (R - S) \sum_{i=1}^l d_i p_i + (R - S) \sum_{i=l+1}^n q p_i - c q \quad (27)$$

$$= (R - S) \sum_{i=1}^l d_i p_i + \left( (R - S) \sum_{i=l+1}^n p_i - (W - S) \right) q \quad (28)$$

The function in (28) is increasing if

$$(R - S) \sum_{i=l+1}^n p_i \geq (W - S) \quad \Leftrightarrow \quad 1 - \sum_{i=1}^l p_i \geq \frac{W - S}{R - S} \quad \Leftrightarrow \quad \sum_{i=1}^l p_i \leq \frac{R - W}{R - S}$$

If  $p_1 \geq \frac{R - W}{R - S}$ , the function is globally decreasing and a maximum is obtained at  $q = d_1$ . Otherwise, the function increases up to  $d_k$ , and decreases ever after, leading to a global maximum at  $q = d_k$ . If  $\sum_{i=1}^k p_i = \frac{R - W}{R - S}$ , the function is constant on  $[d_k, d_{k+1}]$ . In all other cases the global optimum is unique.  $\square$

### 7.2 Proof of Proposition 2

From (5), if  $d_l \leq q \leq d_{l+1}$ , then

$$\mathbb{E}[\Pi_r[q]] = (R - x) \sum_{i=1}^l d_i p_i + (R - x) \sum_{i=l+1}^n q p_i - c q \quad (29)$$

$$= (R - x) \sum_{i=1}^l d_i p_i + \left( (R - x) \sum_{i=l+1}^n p_i - c \right) q \quad (30)$$

If  $(c, x) \in S_k$ , the function in (30) is increasing in  $q$  if  $l < (k - 1)$  and decreasing in  $q$  if  $l > k$ . If  $l = k - 1$ , the function is constant at the upper border of  $S_k$  and increasing otherwise. If  $l = k$ , the function is constant at the lower border of  $S_k$  and decreasing otherwise. Because  $q \mapsto \mathbb{E}[\Pi_r[q]]$  is a continuous function, this proves that it has a global maximum at  $q = d_k$ .  $\square$

### 7.3 Proof of Proposition 3

Assume that the supply chain has optimal expected profit in sector  $S_k$ . For simplicity we assume that the retailer always orders the optimal quantity for the supply chain  $q^*$  on the borders of  $S_k$ . Outside the closure of  $S_k$ , the retailer will never order the optimal quantity for the supply chain.

If  $(c, x) = (M - S, S)$ , the supply chain will always achieve maximum expected profit and all of this profit is taken by the retailer. Hence we can find a point in  $S_k$  where the retailer has an expected profit larger than or equal to the expected profit he has in the newsvendor contract.

If we let  $(c, x) \in S_k \rightarrow (R, 0)$ , the manufacturer takes all the profit at the limit. Hence we can find a point in  $S_k$  where the retailer's profit is less than or equal to that in the newsvendor contract. Because the retailer's profit is continuous in  $(c, x)$  for all  $(c, x) \in S_k$  and  $S_k$  is connected, we can find a point  $(c^*, x^*)$  in  $S_k$  where the retailer's expected profit is equal to the expected profit  $\Pi_r^*$  in the newsvendor model. Let  $(c, x)$  be any other feasible point outside of  $S_k$ , and let  $q = q(c, x)$  be the optimal order quantity for the retailer for that particular choice of  $(c, x)$ . Because  $(c, x) \notin S_k$ , then

$$\hat{\Pi}_{\text{chain}}(q, c, x) < \hat{\Pi}_{\text{chain}}(q^*, c^*, x^*)$$

Hence

$$\hat{\Pi}_m(q, c, x) + \hat{\Pi}_r(q, c, x) < \hat{\Pi}_m(q^*, c^*, x^*) + \Pi_r^*$$

Because  $(c, x)$  is feasible,  $\hat{\Pi}_r(q, c, x) \geq \Pi_r^*$ , and

$$\hat{\Pi}_m(q, c, x) < \hat{\Pi}_m(q^*, c^*, x^*) + (\Pi_r^* - \hat{\Pi}_r(q, c, x)) \leq \hat{\Pi}_m(q^*, c^*, x^*)$$

That proves that the manufacturer's maximum expected profit cannot be obtained outside of the supply chain optimal sector  $S_k$ .

□

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