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Discussion paper

Do all-equity firms destroy value by holding cash?

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Abstract

Empirical evidence shows that as of 2006, nearly every fifth large U.S. public corporation was all-equity financed and that the corresponding average cash holding were nearly twice as high as of the average U.S. firm. This paper therefore presents a simple real-options model to characterize the value of cash for all-equity financed firms and analyze its impact on a firm's investment decision. The model shows that precautionary saving may lead to a delay in investment policy compared to the benchmark of full external financing. This is because saving is an option to invest at a lower price in the future and this option has an additional time value, thereby delaying optimal investment. In the context of growth options and external financing frictions cash has extra value but this value is mostly negatively related to volatility. Testing empirically whether all-equity firms destroy value by holding that much cash, I show that on average the market values cash approximately at par. Moreover, cash is rather valued at a premium if the presence of growth opportunities is being controlled for.

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1 Introduction

As of 2006, approximately 20% of all large U.S. publicly listed COMPUSTAT firms were all-equity financed.[1] Although facing no bankruptcy risk, average cash holdings of all-equity firms were equal to a huge 46% of total assets. This is roughly twice as a high as for levered and unlevered firms on average. Overall, the fraction of all-equity firms has varied between around 25% in the 1950s and roughly 5% in the 1970s. Since then, the number of firms opting to have no leverage in their capital structure has steadily increased. On the other hand, relative cash levels were flat at 25% of total assets between the 1950s and 1970 before surging to levels as high as nearly 50% of total assets. Both trends are visualized in Figure [1].

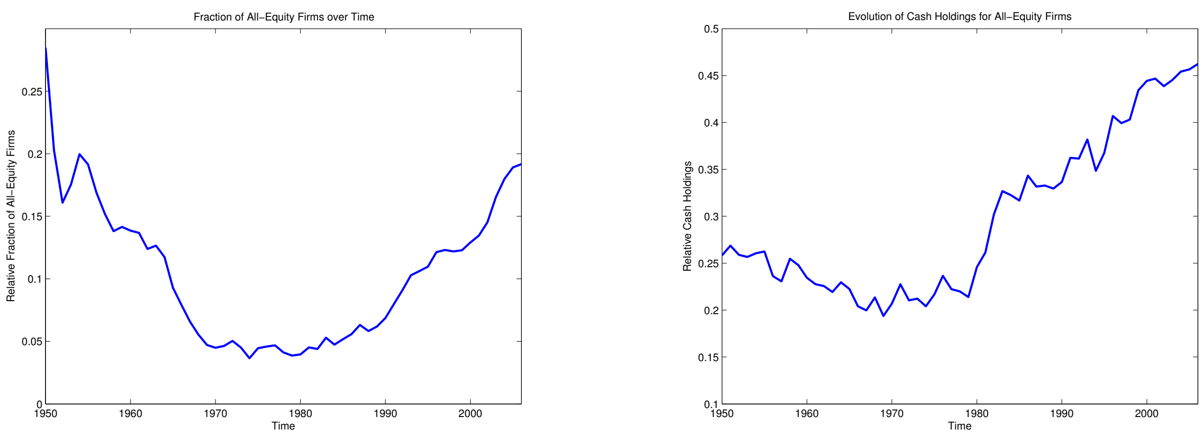

Figure 1: Evolution of All-Equity Firms and Corresponding Cash Holdings

Investigating why firms choose no to include debt in their capital structure, Strebulaev and Yang (2006) find that the choice can neither be explained by firm size nor industry. While it is true on average that all-equity firms are smaller than their levered counterparts, it turns out that in 13% of the cases, all-equity financed dividend paying firms are larger than the 75^{th} percentile of their industry. Similarly, Strebulaev and Yang (2006) also show that while traditional high growth industries such as technology and healthcare have a larger fraction of all-equity firms, the median

[1] The definition of large U.S. publicly listed firms follows Strebulaev and Yang (2006).

industry still has 7.5% of zero levered firms.

The objective of this paper is to investigate both theoretically and empirically whether, conditional on being all-equity financed, all-equity firms destroy value by saving cash. Using a simple real-options model, I first derive the value of cash in case a firm can increase production capacity. If the firm does not have internal funds, it needs to access external capital markets to finance the expansion which comes at a cost due to informational asymmetries between the firm and outside investors. On the other hand, saving also reduces firm value as management might waste resources which in turn induces costly monitoring activities by shareholders. The firm therefore has to trade off costs of external finance against agency costs of free cash flow to optimally exercise its option and maximize firm value.

The model quantifies the value of internal funds and shows that even in absence of bankruptcy a firm might value one dollar of additional funds at more than its notional amount. Furthermore, I show that volatility has an ambiguous and mostly negative effect on the value of internal funds. This is because cash derives its value by possibly avoiding costs of external finance when exercising the option. However, in the majority of cases costs of external finance lose their relative importance when volatility is increased. Focusing on the relation between cash holdings and investment policy, the paper also shows that precautionary saving might lead to a delay in investment policy compared to the case in which the project was completely externally financed. Thus, because saving is an option to pay lower financing costs in the future, a value maximizing firm will wait longer to invest than if it did not save at all.

The model solves for optimal saving policy and also describes the marginal value of internal funds depending on different allocations of cash and cash flow. The marginal value of cash is influenced by costs of external finance, agency costs of free cash flow, the level of the firm's cash account and current cash flow. Firms value cash most when financing costs are high, agency costs small and the probability of exercising the option is (relatively) high. Alternatively, regressions based on simulated data also reveal that cash is valued at a premium in the context of growth options and that the premium is higher if firms just started to retain funds.

The empirical section combines the estimation equation implied from the theoretic model with an approach by Fama and French (1998) which has been used to analyze the impact of taxes and financing decisions on firm value. By doing so, I propose an extension to the existing literature centering around Pinkowitz et al. (2006) and show that on average, all-equity firms do not destroy value by holding that much cash. Moreover, the presence of growth opportunities further increases the value of cash.

This paper mainly relates to both theoretical and empirical literature dealing with the value of cash or financial flexibility. In a related paper, Gamba and Triantis (2008) determine optimal capital structure of a firm which can invest in profitable growth opportunities. They use a neoclassical model in which the firm is partly financed with equity and debt and can decide whether it retains earnings, pays a dividend or pays down debt. Saving cash serves two functions. First, it allows the firm to avoid defaulting in low profitability states as the cash on hand decreases its net debt exposure. Second, by making external financing costly it allows the firm to prevent additional financing costs when growth opportunities are exercised in high profitability states. They find that the value of financial flexibility can be quite large in the presence of profitable growth opportunities or when the firm is exposed to negative income shocks. Also, Asvanunt et al. (2007) look at optimal investment policy when external financing is costly. Focusing on a levered firm they show how optimal investment policies differ depending on whether firm or equity value is maximized. Allowing the firm to save cash, they show that riskier firms have higher optimal cash balances, thereby confirming recent empirical work by Acharya et al. (2009) who show that empirically there is a positive relation between cash holdings and credit spreads.

The theoretic part in this paper differs from Gamba and Triantis (2008) and Asvanunt et al. (2007) on several fronts. First, this work focuses on whether cash has any significant economic value in case it does not serve as a liquidity buffer reducing bankruptcy risk. Doing so, I am able to provide new insights regarding the valuation of cash and the impact of cash holdings on corporate investment policy. Specifically, it is shown that the value of cash is mostly negatively related to volatility. The intuition is that cash only has value in good states of nature, i.e. when the firm is doing well. Saving essentially reduces the future strike price of the capacity expansion option such that, compared to the case of full external financing, the growth option is more in the money. As it turns out, for most parameter values an increase in volatility is less valuable in case the firm saves or, put differently, when the option is more in the money. Given that a substantial number of firms is all-equity financed and has huge cash holdings, this result is also of practical relevance. On the other hand, the paper also shows that optimal saving policy leads to a delay in the firm's investment policy. Again, because saving provides the firm with an option to exercise the project at a lower strike price in the future, a value maximizing firm is willing to invest later even if it faces the same current cost of external financing. Second, contrary to both papers I assume quadratic agency costs of free cash flow to account for the fact that managers want to engage in empire building in the case of high retention which in turn induces costly monitoring activities by shareholders. The assumption of convex agency costs considers the fact that monitoring gets increasingly expensive the more funds are retained within the firm. Besides, it also implies an interior solution for optimal corporate saving policy for which I derive a concrete expression. Third, I am able to derive a closed-form solution for a simplified version of the model which is useful for better understanding the negative relation between the value of cash with respect to volatility. Finally, this work also includes an empirical section which tests the main implications of the model.

Other related work includes Boyle and Guthrie (2003) who analyze a firm's dynamic investment decision when the firm is allowed to save cash to relax an exogenously given financing constraint resulting from asymmetric information. They show that due to the possibility of future earnings shocks, a firm may be willing to exercise its growth option prior to the benchmark case established by an otherwise unconstrained firm. This setup is different from my model, where the firm starts as a constrained firm and can reduce its dependence on external capital markets by engaging in precautionary saving.[2] Finally, using a representative agent framework, Eisfeldt and Rampini (2009) study level and dynamics of the value of aggregate liquidity when external shocks occur. They find that aggregate value is highest when investment opportunities are abundant but levels of current cash flow are low.

Early empirical literature on the value of cash includes Opler et al. (1999) who analyze a static trade-off model of optimal corporate cash holdings. They find that small firms with growth opportunities tend to hold more cash than the average firm. Focusing on the cash-flow sensitivity of cash, Almeida et al. (2004) show that firms save operating cash flow if they are financially constrained. This is tested by regressing the change of cash holdings on operating cash flow, market-to-book ratio and size. Han and Qiu (2007) extend the model of Almeida et al. (2004) by not allowing the firm to hedge future cash flow risk. They are able to show that an increase in volatility of cash flow leads to higher contemporary saving decisions. However, Riddick and Whited (2009) question the results found in Almeida et al. (2004) and argue that the correlation is mainly due to measurement error in the market-to-book ratio which acts as a proxy for marginal q.[3] Erickson and Whited (2000) and Riddick and Whited (2009) show that when using higher-order moments, they are able to increase estimator precision and mitigate the measurement error problem. Doing so, it turns out that the coefficient on operating cash flow turns negative, indicating that there is a negative relationship between operating performance and saving. This can be explained by the

[2] Hirth and Uhrig-Homburg (2010) extend their work and introduce financing costs into the original model. However, similar to the underlying paper by Boyle and Guthrie the focus is not on the value of cash. Another difference concerns the general setup of the model which will be discussed in section [2].

[3] Modern q theory as introduced by Lucas and Prescott (1971) argues that marginal adjustment costs of investing have to be equal to the shadow value of capital, termed as marginal q. This shadow value measures the firm's expectation of the marginal gain from investing. Measurement error stems from the fact that empirical tests have to rely on average q, or market-to-book ratio, as an input in the regression models. However, average q relates the valuation of the firm's existing capital to its replacement costs and is equal to marginal q only under very restrictive assumptions. Further details on the q theory of investment can be found in Hayashi (1982) and Hennessy (2004).

fact that if firms receive positive (negative) shocks to their cash flow, they tend to invest more (less) thereby reducing (adding) funds from (to) their cash account. Finally, Bates et al. (2009) document a huge increase in average cash holdings, specifically for non-dividend paying and riskier firms. Using variants of the regression setup proposed by Opler et al. (1999) they find that this is mostly due to changing firm characteristics.

Focusing on the market value of cash, Pinkowitz and Williamson (2004) and Pinkowitz et al. (2006) rely on an estimation approach initially proposed by Fama and French (1998). They regress firm value on different accounting variables including the level or changes in the cash account to infer the marginal contribution of the individual regressors. This approach has also been employed by Drobetz et al. (2009) who estimate the value of cash in the context of informational asymmetries. An alternative but similar way has been suggested by Faulkender and Wang (2006) who regress excess stock return on cash and different control variables to get an estimate of the marginal value of cash. This paper contributes to the empirical literature by suggesting a simple, yet important extension to the approach proposed by Pinkowitz et al. (2006). Doing so, I am able to show that on average all-equity firms do not destroy value by holding that much cash.

The paper proceeds as follows. In Section 2, I first introduce the model and the corresponding valuation equations. Besides, I derive a closed-form solution for a simplified version of the model and perform comparative statics. Section 3 analyzes the impact of saving on a firm's investment decision and computes the value of internal funds. I also run regressions based on simulated data to obtain an alternative estimate of the value of cash. Section 4 empirically estimates the value of cash. Section 5 concludes.

2 The Model

As Modigliani and Miller (1958, 1961) have shown, in a frictionless world financing, payout and investment policy are independent of each other. To make internal financing matter, I therefore introduce two frictions, namely costs of external finance and agency costs of free cash flow.

The value of internal funds is derived within a simple real-options framework by calculating the value to internally finance an investment. Similar to Dixit and Pindyck (1991) and McDonald and Siegel (1986), I model a firm which has the option to increase production capacity. Departing from traditional real-option models, I focus on the question of how the expansion is financed. If at the time of the capacity expansion the firm has sufficient cash available, it will be able to internally finance the investment at its true investment costs. However, if it turns out that liquid funds are insufficient - which might be the case if the company has paid out part of its cash as dividends to its shareholders - the firm additionally incurs costs of external finance for raising the missing amount. Thus, retaining cash within the firm allows the firm to exercise its expansion option at a lower strike price. However, there exists a trade-off as holding cash is also costly due to the existence of agency conflicts between management and shareholders.

To explore whether there is a value of internal funds, I proceed as follows. I first derive firm and option value for a firm following an optimal retention policy. To quantify the gain from saving, this value is then compared to the case when the firm has not engaged in precautionary saving.

2.1 Basic Setup

Consider a firm which produces a single product and operates at some initial capacity level K_0.[4] The cash flow produced by the firm is risky and follows a Geometric Brownian Motion

$$dx = \mu x dt + \sigma x dW^Q \tag{1}$$

[4] For simplicity, the initial capacity level is normalized to 1.

where dW^Q is a standard Brownian motion under the risk neutral measure Q and μ and σ are mean and volatility of the growth rate of x. I further assume that there exists a traded asset being perfectly correlated with the firm's cash flow which has the following dynamics $dX = rXdt + \sigma X dW^Q$ where $r > \mu$ and $\delta \equiv r - \mu$.

The firm is all-equity financed such that all earnings accrue to shareholders either via dividend payments or via capital gains. If the firm retains its earnings, it can put the money on the cash account where it earns a riskless return r. However, following Jensen (1986) saving cash is costly as management might be more likely to engage in value-destroying "empire building" when cash reserves are abundant. Shareholders therefore would want to monitor the firm, which comes at a cost. I follow Eisfeldt and Rampini (2009) in assuming that only the fraction of the operating cash flow which is retained within the firm is subject to quadratic agency costs. The main intiution underlying this argument is that liquid funds can then be allocated to a financial intermediary such that each period only the retained fraction of earnings has to be monitored. Letting C denote the cash account, α the retained fraction of cash flow and combining above, we get that

$$dC = \left\{ \alpha x - \frac{\phi}{2}(\alpha x)^2 + rC \right\} dt \tag{2}$$

where ϕ is a parameter capturing the severance of agency costs of free cash flow.[5] To ensure that firm value is maximized, I allow the firm to choose an optimal retention policy by treating α as a stochastic optimal control variable.[6]

The explicit treatment of agency costs of free cash flow marks a sharp distinction to the models of Asvanunt et al. (2007) and Gamba and Triantis (2008) as saving becomes increasingly expensive the higher the fraction of retained earnings. Specifically, Gamba and Triantis assume that there

[5] Note that taxation is not included in this model. While there is a tax disadvantage of keeping cash within the firm, it is also true that at investor level, dividends are usually taxed at a higher rate than capital gains. A meaningful calculation would therefore require to specify the tax burden at the investor level. To abstract from these practical complexities, this paper focuses on agency costs of free cash flow as the opposing friction.

[6] For more details see Proposition [1].

is a tax disadvantage of keeping the cash within the firm resulting in a linear treatment of agency costs. Asvanunt et al. assume that the return on the cash account is lower than the risk-free rate r, i.e. $r_x < r$. On the other hand, quadratic agency costs capture the intuition that if the firm is to receive a positive cash flow shock, management is more likely to deduct part of the cash flow and use it for empire building activities. To prevent management from doing so, shareholders thus have to incur higher monitoring costs. Besides, the assumption also enables me to derive an analytical expression for optimal corporate saving and dividend policy.

It is important to notice that the setup is also different from Boyle and Guthrie (2003) who assume distinct dynamics for operating profits and cash account. This is due to the fact that they investigate the possibility of future financing shortfalls and its implications for optimal exercise policy compared to an otherwise unconstrained firm.[7] In this model, the focus is on another aspect. Starting with a firm which has to finance the whole project externally, I analyze how much value the firm would add by not paying out dividends and instead optimally saving part of the cash flow to reduce future financing needs.

The firm has the option to increase capacity to a higher level K_1 by paying the necessary investment costs IC. However, if it lacks internal funds it has to raise all or part of the missing amount externally. External financing comes at a cost. Specifically, I consider the following general cost function $e(C_t)$ which equals

$$e(C_t) = \begin{cases} \gamma_0 + \gamma_1 \left(IC - C_t \right) + \gamma_2 \left(IC - C_t \right)^2 & \text{when } C_t < IC, \\ 0 & \text{else} \end{cases} \tag{3}$$

The specification of this function has been taken and adapted from Hennessy and Whited (2007)

[7]Boyle and Guthrie assume that prior to exercising the growth option the firm consists of assets in place G and the cash account X. Assets in place generate an income stream equal to $\nu G dt + \phi G dZ$ which directly affects the cash account whose dynamics are given by $dX = rXdt + \nu G dt + \phi G dZ$.

who structurally estimate external financing costs.[8] Total costs of capacity expansion are therefore given by the sum of investment costs and costs of external finance.

Total firm value depends on both state variables x_t and C_t and is finally given by the sum of expected dividend payments and expected capital gains which include the cash retained within the firm and the capital gain due to potential capacity expansion.

Proposition 1 *Total firm value, denoted by $V(x, C)$ is a function of both state variables x and C and has to satisfy the following Hamilton-Jacobi-Bellman (HJB) equation under the risk-neutral measure Q*

$$rV = \max_{\alpha} \left\{ (1-\alpha)x + (r-\delta)xV_x + (\alpha x - \frac{\phi}{2}(\alpha x)^2 + rC)V_C + 1/2\sigma^2 x^2 V_{xx} \right\} \tag{4}$$

where the first order condition implies that

$$\alpha^* = \frac{V_C - 1}{\phi x V_C} \tag{5}$$

with the additional requirement that $\alpha^* \in [0, 1]$.

Proof: See Appendix.

One can see that optimal corporate saving policy depends on different factors. When agency costs of cash flow are zero, i.e. $\phi = 0$, there will be a bang bang type of solution. As long as the marginal value of cash exceeds one, the firm would want to retain all earnings. When there is no value premium of cash, it would instead pay out all proceeds as a dividend. The introduction of quadratic agency costs of free cash flow implies that there will be some allocation of x and C such that it will be optimal to save a fraction of current earnings. Along with general intuition, there is a positive relation between the severity of agency costs of free cash flow and implied dividend

[8] External financing costs thus capture in a reduced form both costs stemming from informational asymmetries as well as transaction costs.

payout ratio.

In order to determine total firm value one has to solve the HJB-equation with respect to the following boundary conditions.

$$
\begin{aligned}
V(0, C_t) &= C_t \\
V(x^*, C_\tau) &= \tfrac{K_1 x^*}{\delta} + C_\tau - IC - e(C_\tau) \\
V_x(x^*, C_\tau) &= \tfrac{K_1}{\delta}
\end{aligned} \tag{6}
$$

where x^* is the investment threshold of the capacity expansion option and C_τ denotes the amount of cash available at the time the option is exercised. The first condition states that if the value of the cash flow hits zero, the firm is liquidated and is only worth the value of the cash account, C_t. The second condition implies that at the time of exercising the option the firm receives the payoff of the capacity expansion, pays corresponding investment and financing costs and retains a corporate cash account equal to C_τ. The last condition is the traditional smooth-pasting condition ensuring optimal exercise policy. The value-matching condition reflects the fact that after exercising the option all future earnings are paid out as dividends such that $V(x_\tau, C_\tau) = E_\tau^Q \left[\int_\tau^\infty e^{-r(t-\tau)} K_1 x_t dt \right] + C_\tau$.

2.2 The Value of Internal Funds

The value of internal funds is derived by comparing total firm value under optimal saving policy to the case when all earnings are paid out as dividends. As such it quantifies the maximum increase in firm value by optimally trading off costs of external finance against agency costs of free cash flow.

Definition 1 *The value of internal funds is defined as the change in total firm value due the fact* *that the firm follows an optimal saving policy. Specifically, it is given by*

$$R(x, C) \equiv V(x, C) - V^B(x, C) \tag{7}$$

where C is the shortcut for $C(x, \alpha)$ and $V^B(x, C)$ denotes firm value under the benchmark case of complete external financing.

For the benchmark case, denoted by $V^B(x, C)$, I assume that all earnings are paid out as a dividend. It is derived as follows.

Proposition 2 *The benchmark case is assumed to be an all-equity firm which pays out all earnings as a dividend to its shareholders and which finances the project completely externally. Total firm value, denoted as $V^B(x, C)$ satisfies the following PDE*

$$rV^B = x + (r - \delta)xV_x^B + rCV_C^B + 1/2\sigma^2 x^2 V_{xx}^B \tag{8}$$

and is given by

$$V^B(x, C) = C + \frac{K_0 x}{\delta} + Bx^{\beta_1} \tag{9}$$

where $B = \left(\frac{(K_1 - K_0)x_2^*}{\delta} - IC - e(0)\right)\left(\frac{1}{x_2^*}\right)^{\beta_1}$ with the corresponding optimal trigger level x_2^*.

Proof: See Appendix.

While the benchmark case has a closed-form solution, it turns out that equation [4] can not be solved analytically if subject to the boundary conditions given in equation [6]. I therefore choose to solve $V(x, C)$ numerically by resorting to finite difference methods, i.e. Crank Nicholson Scheme. The PDE is solved on a grid with nodes $(x_j, C_i) : j = 1, ..., M, i = 1, ..., N$ where $x_j = jdx$ and $dC = C_i - C_{i-1}$. Partial derivatives are approximated by

$$
\begin{aligned}
V_x &= \frac{1}{2}\left(\frac{V_{i-1,j+1}-V_{i-1,j-1}}{2dx}+\frac{V_{i,j+1}-V_{i,j-1}}{2dx}\right)\\
V_{xx} &= \frac{1}{2}\left(\frac{V_{i-1,j+1}-2V_{i-1,j}+V_{i-1,j-1}}{(dx)^2}+\frac{V_{i,j+1}-2V_{i,j}+V_{i,j-1}}{(dx)^2}\right)\\
V_C &= \frac{V_{i,j}-V_{i-1,j}}{dC}
\end{aligned}
\tag{10}
$$

which implies that the resulting difference equation at node (x_j, C_i) can be formulated as

$$
-a_jV_{i-1,j-1}-(b_j-d_{i,j})V_{i-1,j}-c_jV_{i-1,j+1}=a_jV_{i,j-1}+(b_j+d_{i,j})V_{i,j}+c_jV_{i,j+1}+e_j \tag{11}
$$

where

$$
\begin{aligned}
a_j &= \frac{\sigma^2 j^2-\mu j}{4}\\
d_{i,j} &= \frac{\alpha j dx-\phi/2(\alpha j dx)^2+ridc}{dc}\\
b_j &= -\frac{\sigma^2 j^2+r}{2}\\
c_j &= \frac{\sigma^2 j^2+\mu j}{4}\\
e_j &= (1-\alpha)jdx
\end{aligned}
\tag{12}
$$

Further details regarding the numerical solution can be found in the Appendix.

The value of internal funds introduced by Definition [1] gives an absolute answer to the value of cash but it can not be used to judge whether the amount gained or lost due to not paying out dividends is economically significant. I therefore compare the value of internal funds to the initial value of the capacity expansion option for the benchmark firm.

Definition 2 *The relative gain from saving is defined by comparing the value of internal funds to the value of the capacity expansion option of the benchmark case. Specifically, it is defined as*

$$S(x, C) \equiv \frac{V(x, C) - V^B(x, C)}{Bx^{\beta_1}} \tag{13}$$

By construction $S(x, C)$ captures the gain from saving by comparing the value of internal funds to the value of the initial growth option and it quantifies by how much the firm can relatively increase the value of its growth option if it follows an optimal saving policy. Before I proceed to the numerical implementation, I present a simplified version of the model which allows for a closed form solution.

2.3 Implications from a Simplified Model

This section introduces a simplified model to determine the maximum attainable value of internal funds. This is a hypothetical value as it does not consider the attainability of the solution and agency costs of free cash flow. Nevertheless, it is very useful as its closed-form solution allows me to perform comparative statics which then can be compared against the dynamics of the true value.

I therefore consider the following fictitious example. Consider the case of a firm which has an initial cash balance C_0 greater than the necessary investment costs. Let's further assume that the firm is not subject to agency costs of free cash flow. The investment environment described in the previous section is still valid and the firm therefore plans to increase its capacity level. However, the firm needs to decide whether it wants to pay out all its cash holdings and future earnings as dividends or not. By doing so it would have to finance the project completely externally and investment costs would increase to $IC + e(0)$. The firm therefore calculates the value of internal funds and assesses the relative gain from saving.

Proposition 3 *The hypothetical value of internal funds, $R^h(x)$, is only a function of x and is given by*

$$R^h(x, C) = R^h(x) = x^{\beta_1} (A - B) \tag{14}$$

where $A = \left(\frac{(K_1 - K_0)x_1^*}{\delta} - IC \right) \left(\frac{1}{x_1^*} \right)^{\beta_1}$ *with the corresponding optimal trigger level x_1^*.*

Proof: See Appendix.

Depending on individual firm characteristics, such as assumed factor of capacity expansion, costs of external finance, drift rate, volatility and risk-free rate, $R^h(x)$ will take on different values. The advantage of calculating this hypothetical value is that the closed form solution helps to quickly assess whether saving is potentially valuable and it allows for a better understanding of how individual parameters affect the value of internal funds.

Doing so, it can be shown that the magnitude of the increase in production capacity, does not affect the relative gain from saving. This fact is useful also in that it will make the numerical results presented in the following section more robust as the relative gain from saving is unrelated to the increase in capacity level.

Proposition 4 *The relative gain from saving is unrelated to the magnitude of increase in capacity. That is,*

$$\frac{\partial S^h(x)}{\partial \Delta K} = 0 \tag{15}$$

where $\Delta K = K_1 - K_0$.

Proof: See Appendix.

A final interesting feature to notice is that the hypothetical value of internal funds is ambiguously related to volatility. This is because cash derives its value by possibly avoiding costs of external finance when exercising the option. However, for most cases costs of external finance lose their relative importance when volatility is increased which induces a negative relation between value of internal funds and volatility.

Proposition 5 *The effect of volatility on the hypothetical value of internal funds is ambiguous. Specifically, it depends on the costs of external finance and on the level of volatility. The expression for the partial derivative of the value of internal funds with respect to volatility is given by*

$$\frac{\partial R^h(x)}{\partial \sigma} = Ax^{\beta_1}\frac{\partial \beta_1}{\partial \sigma}\left\{\log\left(\frac{x}{x^*}\right) - (1+\gamma)^{1-\beta_1}\log\left(\frac{x}{x^*(1+\gamma)}\right)\right\} \tag{16}$$

Proof: See Appendix.

The intuition for this result is as follows. It is well known that an increase in volatility leads to an increase in the value of a call option on the firm's assets, thus the capacity expansion option is positively related to an increase in uncertainty. This holds true for both the option value under complete external and internal financing. However, for most parameter values the increase is more pronounced in case the project is financed completely externally, thereby implying the (mostly) negative relation between volatility and the value of internal funds. In other words, cash reduces the future strike price and therefore implies that the option is more in the money. However, loosely speaking, this also enlarges the area of downside risk which is why an increase in volatility is not necessarily value enhancing. In that sense, the result can be seen as the opposite of the asset substitution problem.

3 Numerical Analysis

The objective of this section is to assess both the value of cash and how cash holdings and investments are interrelated. The analysis starts by investigating how saving and cash holdings affect corporate investment decisions. I then analyze the dynamics of optimal saving and payout policy, compute the value of internal funds and analyze its relation with respect to volatility. As a next step, I run regressions based on simulated data to obtain an alternative estimate of the value of cash. The section concludes with a variety of robustness checks.

Similar to many other financing and investment models, the problem studied in this paper does not have a closed form solution. I therefore solve the model using numerical techniques and illustrate results using a simple example. For this purpose, the risk-free rate is set to 6%, the drift rate μ to 1%, cash flow volatility to 19% and the agency cost parameter ϕ is set equal to 0.05. These values are similar to both existing papers and empirical observations.[9] Assuming a starting value of the cash flow process of 1, i.e. $x_0 = 1$, it follows that the initial fundamental value of the firm equals 20. In order to make the growth option economically relevant, I set the costs of the expansion option equal to 10 and assume that production can be increased by 50 percent, i.e. $\Delta K = 0.$. Finally, the cost of external finance is taken from Hennessy and Whited (2007) and is set to their estimate for small firms to capture the effect of external financing constraints. Specifically, the variable cost component γ_1 is assumed to be 12% whereas the quadratic cost component γ_2 equals 0.04%. To investigate the impact of firm size, robustness checks will set μ equal to 4% thereby implying a higher initial fundamental valuation and consequently lower relative costs of investment. Also, external financing costs will be decreased and the magnitude of the expansion option will be reduced.

[9] The risk-free rate is obtained using Datastream's historical monthly Fed Funds data available since 1955 whereas the volatility parameter is similar to Boyle and Guthrie (2003) and Mauer and Triantis (1994). The agency cost parameter is taken from Eisfeldt and Rampini (2009).

3.1 Cash Holdings and Investment

Ever since Fazzari et al. (1988) there has been an intensive discussion on the impact of financial constraints on corporate investment decisions. In a recent paper, Denis and Sibilkov (2010) show that financially constrained firms benefit from cash holdings as it enables them to pursue value increasing investment projects. To compare the impact of optimal corporate saving on the investment policy of the firm, I therefore compare option trigger levels under complete external financing to the case of optimal corporate saving policy. This is visualized in Figure [2].

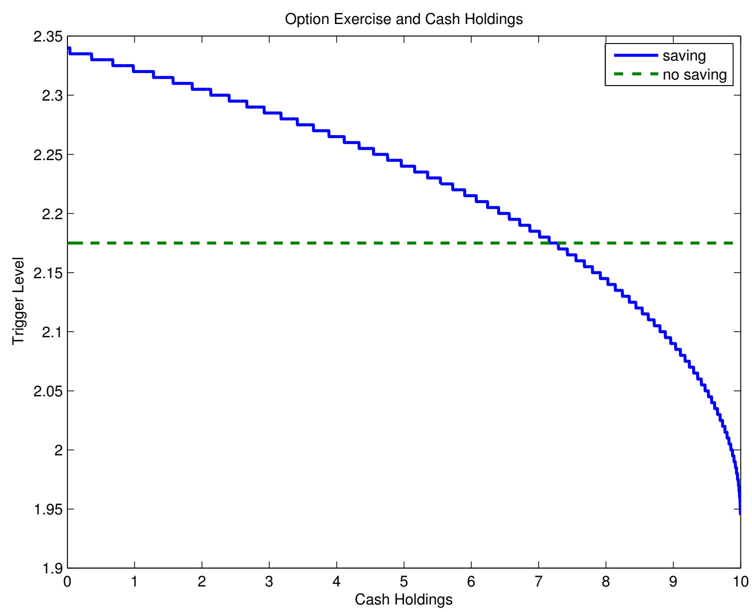

Figure 2: Relationship between the level of the cash account and option trigger levels.

Unsurprisingly, if the firm only has an (explicit) capacity expansion option but no freedom regarding the choice of the corresponding financing strategy, the optimal trigger level does not depend on the level of cash which is depicted by the green dashed line. However, if the firm has the additional (implicit) option to choose a corresponding optimal financing strategy, the relation between trigger level and cash holdings becomes nonlinear as shown by the blue solid line. Most interestingly, it can be seen that if the firm has no cash savings, a firm with both an explicit expansion option and an implicit financing option would increase capacity later than if it chooses to completely externally finance the project although they face precisely the same costs of external finance at this point. This is due to the fact that under optimal corporate saving policy, there is an additional value of not exercising the option immediately as the firm possesses a second option to increase its capacity at a lower strike price. Only if the level of cash is sufficiently high, then it would make sense for the firm to exercise its option prior to the case of complete external financing. This result is important as it implies that a firm which saves cash to reduce future financing needs might invest later than if it financed the project externally.

This simple example illustrates another difficulty in accurately distinguishing between constrained and unconstrained firms and subsequently make predictions regarding their investment behavior. Using debt as a criterion to classify the degree of financial constraints, both cases presented in Figure [2] would be classified as unconstrained. Payout or saving policy alone is also not useful in predicting investment decisions because its implications depend on the level of cash holdings relative to total investment costs. In case the firm never saved and followed a full payout policy - as illustrated by the green line - investment would never depend on the amount of cash the firm has whereas under optimal corporate saving the effect is ambiguous. As long as the level of cash is sufficiently low, the firm would wait even longer whereas at some point, investment would strongly depend on the amount of internal funds.

Using Monte Carlo Simulation, one can then look at how much cash a firm would save until it exercises the option. The left panel in Figure [3] displays the distribution of the firm's cash holdings prior to exercising the growth option. On average, the firm will have saved 8.65 units of cash when it exercises the option and we can further see that in the majority of cases it will have more than half of the necessary investment available as internal funds. Alternatively, one can look at the actual exercise threshold under optimal corporate saving policy. It turns out that on average, the firm will exercise the option if the cash flow equals 2.09. Focusing on the implied distribution of exercise thresholds shown in the right panel in Figure [3], it turns out that the distribution is skewed to the left, meaning that there is a high chance that the firm will be able to internally finance the investment, thereby demonstrating the importance of internal funds.

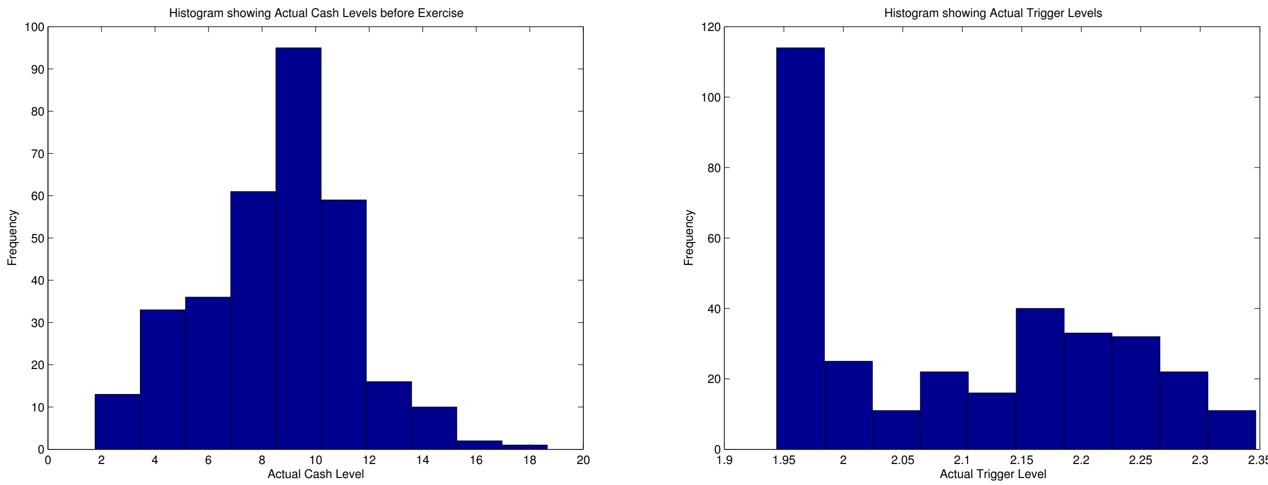

Figure 3: Implied Distribution for Cash Holdings and Exercise Levels.

3.2 Optimal Saving Policy and the Value of Cash

As a next step, we can now investigate by how much the value of the initial growth option increases if the firm follows an optimal corporate saving policy. The value maximizing policy leads to an increase of $S(x_0, C_0)$ to 6.6 %. In other words, if the firm starts with no cash at hand and if the starting cash flow is equal to one, then the firm is able to increase the value of the capacity expansion option by approximately 7 percent. Figure [4] shows the relative gain from saving across different allocations of x, holding the initial cash level constant at zero. If x approximately equals 0.3, the relative gain from saving is highest at approximately 9%. However, if x approaches zero, the relative gain from saving also drops off precipitously as the probability that the option will ever get exercised is very low. On the other hand, increasing x above 0.3 also decreases the relative gain from saving as the firm approaches the exercise threshold and therefore is left with less time to build up the necessary cash reserves.

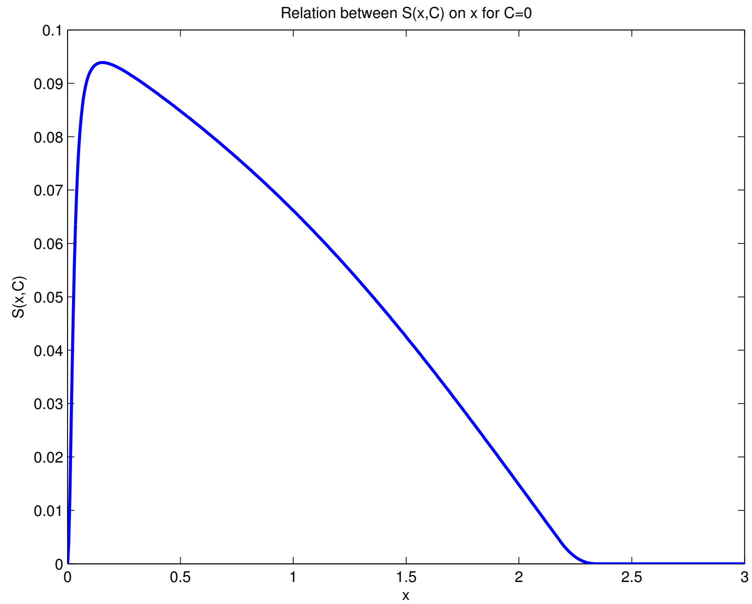

Figure 4: The influence of x on the value of S(x,0).

Clearly, the results depend strongly on the magnitude of the agency costs, captured by the cost parameter ϕ. For example, if one changes the value of ϕ to 0.025, then the relative gain from saving evaluated at x_0 and C_0 increases to 8.55%. Even more so, if the firm does not suffer from agency costs of free cash flow, then it would be able to increase the value of the capacity expansion option by 11.45% if it retains funds within the firm. Given the different magnitude of the value added by saving, it is not surprising that optimal corporate saving policy varies strongly across different allocations of cash and cash flow, even when holding the agency cost parameter fixed. Recalling from the previous section that the saving rate α is determined optimally by setting

$$\alpha^* = \frac{V_C - 1}{\phi x V_C}$$

one can see in Figure [5] that for values of x close to zero the firm chooses to pay out most funds as dividends. The reason is that agency costs of free cash flow dominate as the probability of exercising the option is low. However, for slightly higher values of x it is optimal to retain some fraction of the cash flow in order to reduce future financing costs. Moreover, by simultaneously increasing C we can see that the optimal retention ratio increases to as much as 100%. One the other hand, one can also observe that although firms build up cash reserves in order to reduce potential investment costs, it is optimal to retain less than 100% of the cash flow in most cases, thereby implying that it is optimal for all-equity firms to simultaneously hold cash and pay dividends.

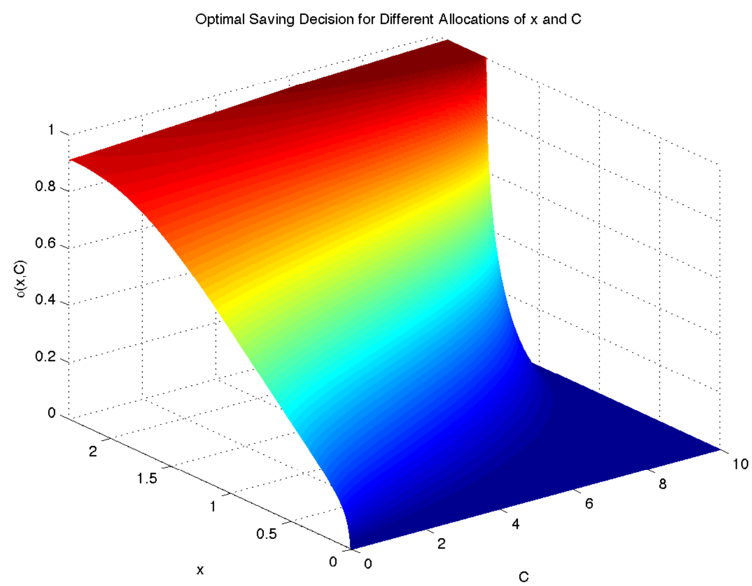

Figure 5: Optimal saving policy for different allocations of x and C.

In sum, optimal corporate saving policy is driven by the severance of agency costs of free cash flow, current operating cash flow and the marginal value of cash. It is thus natural to next investigate how the marginal value of cash differs across various allocations of cash and cash flow. Figure [6] shows that for low levels of cash and cash flow the marginal value of internal funds equals one. This can be explained by the fact that the probability of exercising the option is very low and thus agency costs of free cash flow play a dominant role. Increasing the cash balance even leaves the marginal value of internal funds unchanged for a wider range of values of x. In other words, if the firm already has a lot of cash and current cash flow is low then it does not value an extra dollar at a premium to its notional amount. On the other hand, increasing x quickly leads to an increase in V_C above one. Thus, the more likely it is that the option gets exercised the more value the firm places on internal funds. Depending on the allocation of x and C in the state space, the value premium of cash converges to approximately 12%. This corresponds to the case when the option would get exercised immediately such that the marginal value of cash is equal to the marginal costs of external finance.

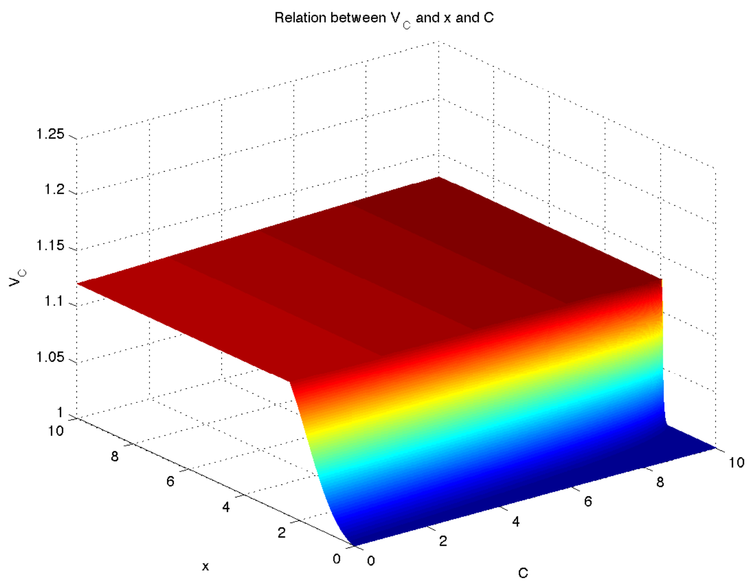

Figure 6: The marginal value of cash

Finally, I assess the relation of the value of internal funds with respect to volatility. The simplified model of the previous section has shown that the effect of volatility on the hypothetical value of internal funds is ambiguous which is illustrated in the left panel of Figure [7]. Performing a similar analysis for the full model, it turns out that this relation is rather robust with respect to the simplifying assumptions as one can see in the right panel of Figure [7]. The intuition is that cash derives its value by possibly avoiding costs of external finance. While the value of the expansion option is positively related to volatility, both for the case of external and internal financing, the increase is more pronounced for the case when the project is completely externally financed. In other words, cash reduces the future strike price and therefore implies that the option is more in the money. However, loosely speaking, this also enlarges the area of downside risk which is why an increase in volatility is not necessarily value enhancing. Or put more simply, external financing costs lose their relative importance for moderate to high levels of volatility, thereby implying the negative relation between the value of internal funds and uncertainty.

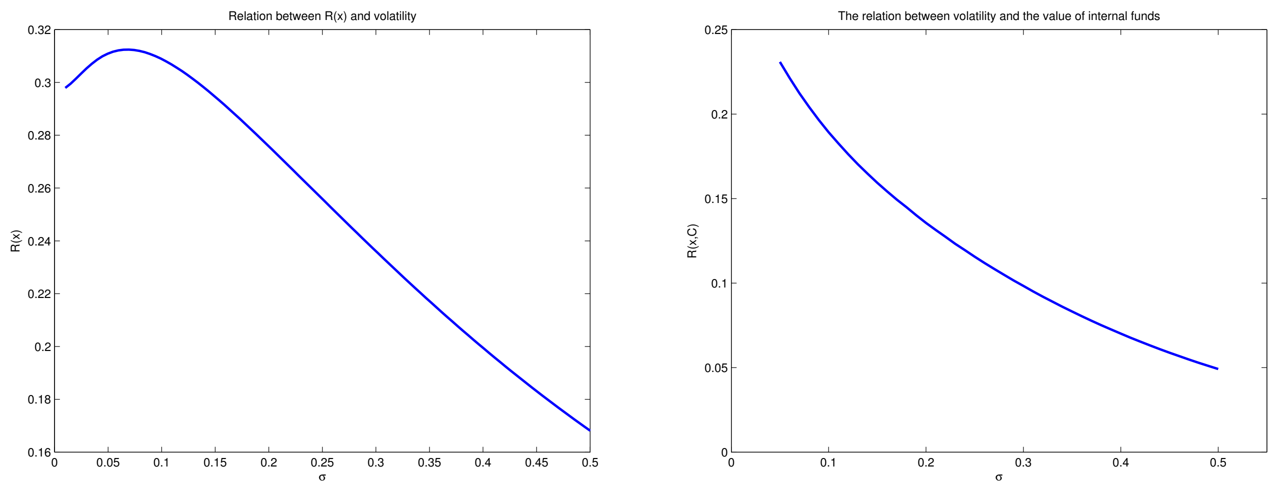

Figure 7: The impact of volatility on the value of internal funds for different payout policies

While the relation between the value of internal funds and volatility can be explained using its definition as the difference between the two capacity expansion options, one might wonder whether this also holds true for the marginal value of cash. I therefore also show how V_C changes with different levels of volatility. The relationship is again highly nonlinear and similar to the previous figures. Figure [8] plots the marginal value of cash as a function of σ for the case when cash flow is held constant at x_0 and the level of cash equals 0.1, 1, 5 and 7.5 respectively. It is interesting to notice that for moderate levels of volatility, i.e. 25 to 50 percent, the marginal value of cash is decreasing across low to medium endowments of cash whereas the decrease only starts for levels of 40% and higher in case cash holdings equal 7.5 units.

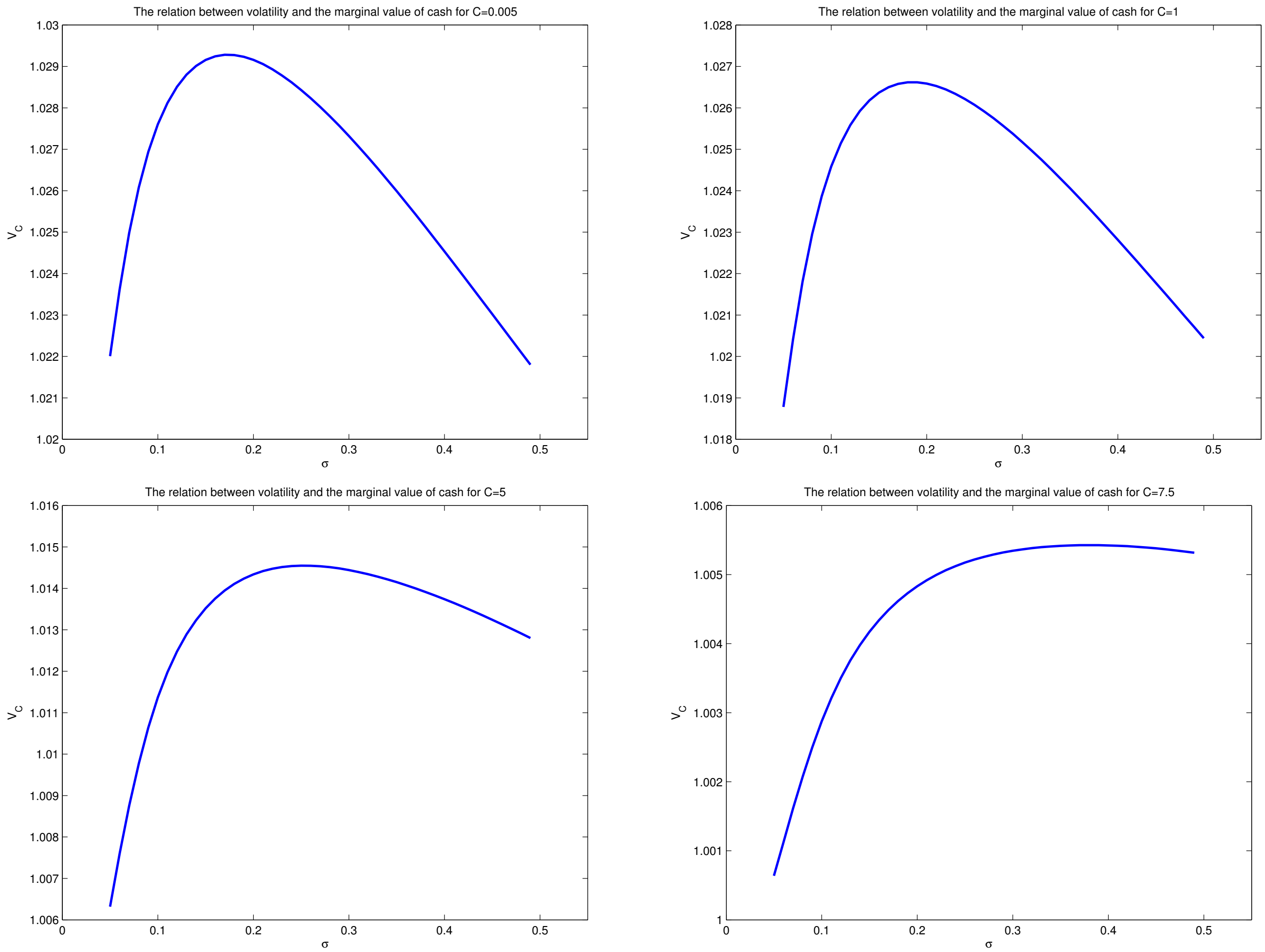

Figure 8: The marginal value cash and volatility for different levels of cash.

The analysis has shown that cash is valuable in the context of growth opportunities and that the actual value depends on the specific combination between cash holdings and cash flow. In general, if there is a realistic probability that the growth option will be exercised, the firm will optimally retain some fraction of its cash flow to save for future investment outlays. At the same time, it is still optimal to pay out the remaining fraction as dividends, thereby showing that simultaneous dividend payments and cash holdings are the outcome of a value maximizing strategy. Most interestingly, it turns out that saving has an ambiguous effect on the investment policy of the firm. If the firm starts out with very low cash holdings, then it will actually wait longer to exercise its growth option than if it funded the project completely externally.

3.3 Simulated Data and Regression Analysis

Another possibility to assess the value of cash is to use regression analysis. This section therefore relates observable model implied variables to overall firm value in order to obtain an estimate of the shadow value of cash. Specifically, the model has shown that total firm value is given by

$$V(x_t, C_t) = K_0 x_t + C_t + G(x_t, C_t) + E\left[\int_{t+1}^{\infty} e^{-r(s-t)} K_0 x_s dt\right] \tag{17}$$

where $K_0 x_t$ denotes the level of current cash flow and the term $E[\int_{t+1}^{\infty} e^{-r(s-t)} K_0 x_s dt]$ represents the discounted value of all future cash flows. Total firm value thus consists of the cash flow generated today conditional on the asset capacity K_0, the amount of cash the firm has retained, the value of the growth option and the expected value of all future discounted cash flows. Because the true value of the growth option can not be observed and can only be backed out by the identity given in equation [17], I replace $G(x_t, C_t)$ with the closed form approximation provided in Section [2] to arrive at the following testable equation

$$V(x_t, C_t) = a + \beta_1 K_0 x_t + \beta_2 C_t + \beta_3 G(x_t) + \beta_4 E\left[\int_{t+1}^{\infty} e^{-r(s-t)} K_0 x_s dt\right] + \epsilon \tag{18}$$

The marginal value of cash is thus assessed by calculating the partial derivative of $V(x_t, C_t)$ with respect to C_t, i.e.

$$\frac{\partial V_t(x_t, C_t)}{\partial C_t} = \beta_2 \tag{19}$$

Using Monte Carlo Simulation I therefore calculate firm value, cash holdings, fundamental firm value and compute the approximated value of the growth option contingent on the realization of the cash flow process. Furthermore, I set the length of the time step dt equal to 1/250, the time horizon equal to 20 years and set the number of replications equal to $1,000$.[10] Table [1] shows corresponding results when equation [17] is estimated for the entire set of firms which have not yet exercised the growth option and when the time to save is limited to four or eight years. To reduce the impact of the initial starting condition, I drop the first 100 realizations of each replication.

Table 1: The Simulated Value of Cash

	(1) All Coefficient	(2) If Time < 4 years Coefficient	(3) If Time < 8 Years Coefficient
Growth Options	0.943***	0.909***	0.874***
Fundamental Value at t+1	0.897***	0.918***	0.848***
Cash Holdings	1.012***	1.032***	1.036***
Cash Flow	3.082*	2.726*	4.179**

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

It can be seen that when the analysis is made for the entire simulated dataset, the average value premium of cash equals 1.2 percent. Thus, even without conditioning on whether some firms already have enough cash to internally finance the investment, cash is valued at a slight premium to its notional amount. However, if the analysis is restricted to firms during the time of building up the cash reserves, then the estimated value of cash more than doubles to 3.2 and 3.6 percent respectively. In other words, by excluding firms who already saved a lot of cash but still have not exercised their growth options, we can see that the marginal value of cash increases substantially.

To make sure that results are not driven by specific parameter values, the following section will perform various robustness checks by varying the profitability of the firm and its financing and investment costs.

3.4 Robustness Checks

I will now investigate the robustness of the results with respect to different parameter values. Specifically, I will analyze three different scenarios. Under the first one, I reduce the importance

[10] Note that results are robust to setting the time step equal to monthly or quarterly data.

of external financing costs by setting the value equal to Hennessy and Whited (2007) estimate for large firms, i.e. $\gamma_1 = 5.3\%$ and $\gamma_2 = 0.02\%$. Secondly, I decrease the impact of the capacity expansion option and set it equal to 15%. Finally, I assume that the the firm is rather profitable and set μ equal to 4%. This also allows me to investigate the effect of lower investment costs as under the new assumption the initial fundamental value of the firm equals 50 which reduces the relative importance of the investment costs to roughly 20%. Finally, results will be presented for the interaction between cash holdings and investment policy, saving policy and the value of internal funds and its relationship with respect to volatility.

Figure [9] displays the relation between the level of the cash account and option trigger levels in the left panel and the relative gain from saving in the right panel for all analyzed cases. The top graphs correspond to the case of low financing costs, the ones in the middle to a lower capacity expansion option and the two figures on the bottom relate to higher profitability. Focusing first on the relation between exercise policy and cash holdings, one can see that for all cases a saving firm would exercise the growth option later than if it finances the growth option completely externally. Thus, saving has its own time value which implies an optimal delay in the firm's investment policy. Most interestingly, in the case of high profitability the firm would almost always wait longer to exercise the growth option and only if it has retained more than 90% of the investment costs, it would exercise the option earlier than under the case of complete external financing. The graphs in the right panel of Figure [9] show that the relative gain from saving is reduced in case of lower financing costs or profitability. This is because holding all other parameter values constant, the relative effect of external financing costs is reduced if profitability is higher or financing costs are low. However, one can also see that if the magnitude of the capacity expansion option is reduced to 15%, the maximum relative gain from saving is still equal to roughly 9%, as predicted in Section [2].

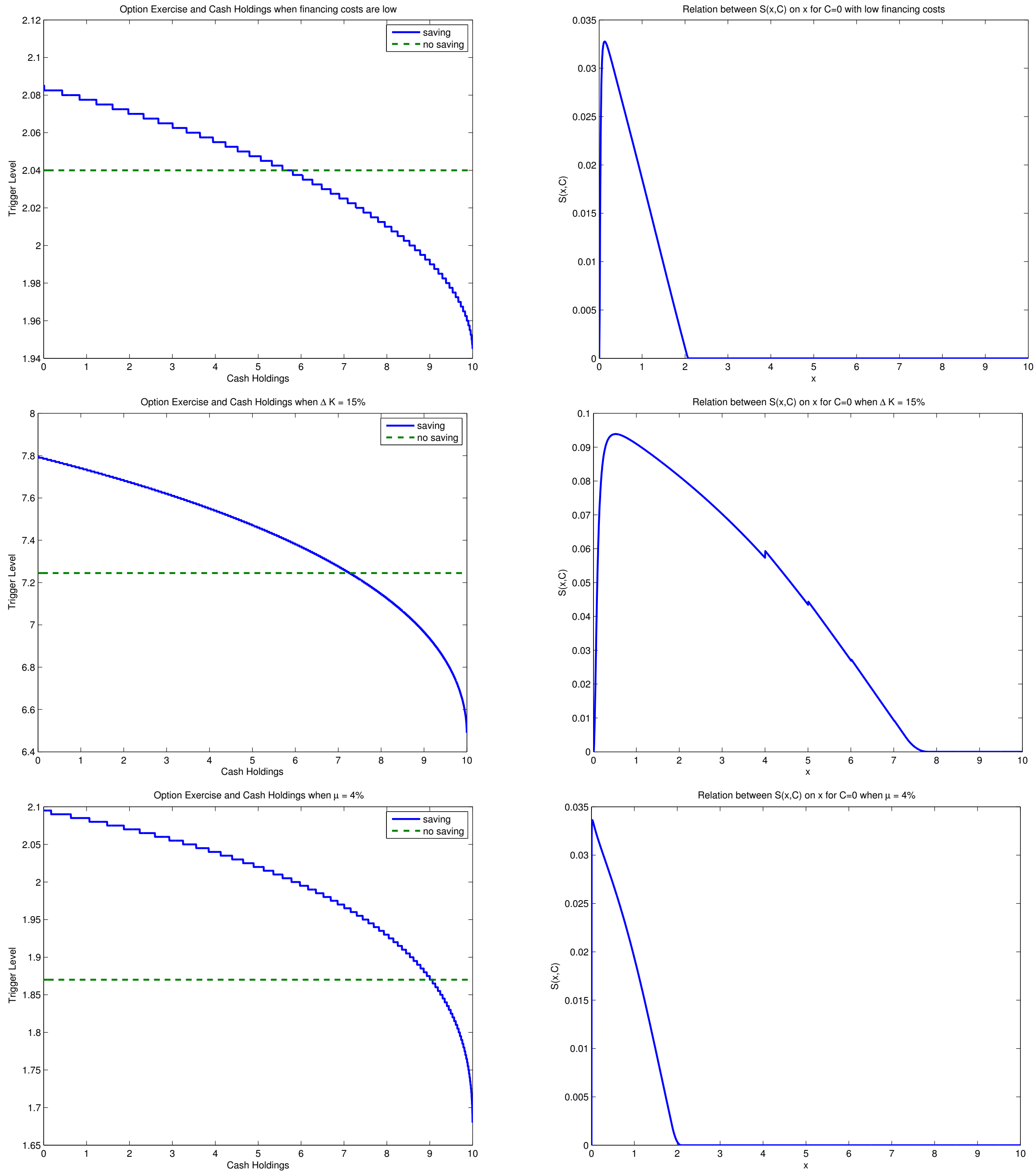

Figure 9: The impact of volatility on the value of internal funds for different payout policies

Finally, one can analyze the effect on volatility on the value of cash. The left panel in Figure [10] shows the relation between the value of internal funds $R(x, C)$ and volatility, as defined in the real-options model. It can be seen that for low financing costs and high profitability, the relation is similar to the baseline case. When the magnitude of the capacity expansion option is reduced, volatility only has a negative impact on the value of internal funds if it is already higher than approximately 30 or 40 percent. Thus it seems that, technically speaking, if the potential payoff from exercising is less convex then increasing volatility has a more positive impact in case the option is more in the money, i.e. in case the firm engages in precautionary saving.

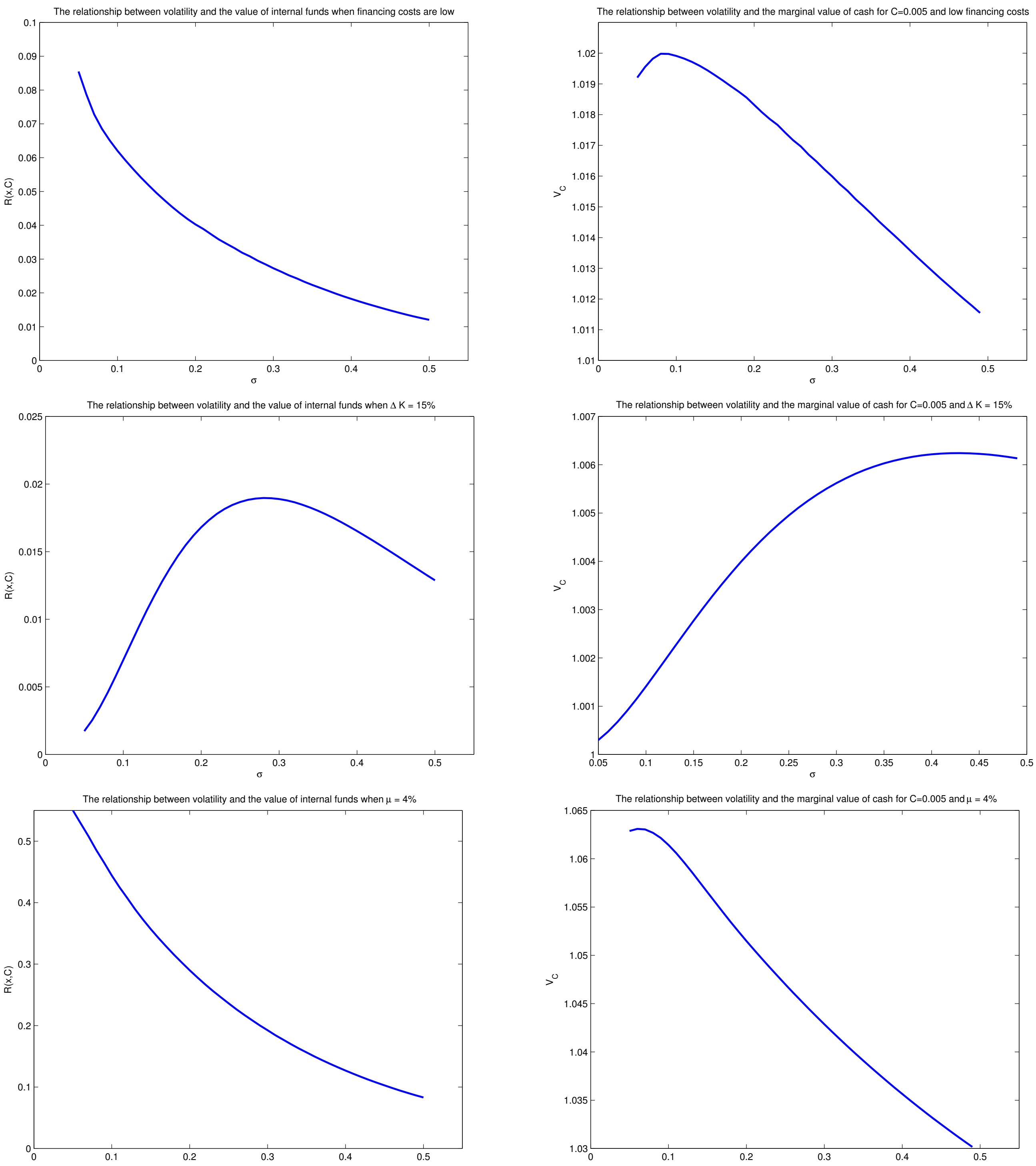

Figure 10: Robustness Check: The impact of volatility on the value of internal funds

4 Empirical Analysis

The model presented in the previous section has shown that in the context of growth opportunities cash can be valued at a premium to its notional amount if external financing costs and agency costs of free cash flow are traded-off optimally. The objective of this section is to combine the model implied valuation equation with existing empirical literature in order to assess whether all-equity firms add or destroy value by holding that much cash.

It turns out that the formulation presented in equation [17] is similar to existing empirical literature on the value of cash which has been influenced by Fama and French (1998) who analyze the impact of taxes and financing decisions on firm value. They estimate cross-sectional regressions to analyze how firm value is related to leverage, dividends and other firm characteristics. The regression specification in Fama and French (1998) captures the intuition of this model that firm value consists of a fundamental part relating to the book value of assets and cash flow plus it also controls for growth opportunities and tax effects. Specifically, their setup is given by

$$\begin{aligned} \frac{V_t - A_t}{A_t} &= a + \beta_1 \frac{E_t}{A_t} + \beta_2 \frac{RD_t}{A_t} + \beta_3 \frac{I_t}{A_t} + \beta_4 \frac{D_t}{A_t} + \beta_5 \frac{dA_t}{A_t} + \beta_6 \frac{dA_{t+1}}{A_t} \\ &+ \beta_7 \frac{dY_t}{A_t} + \beta_8 \frac{dY_{t+1}}{A_t} + \beta_9 \frac{dV_{t+1}}{A_t} + \epsilon_t \end{aligned} \tag{20}$$

where V_t is total market value of a firm, A_t is book value of assets, $V_t - A_t$ is interpreted as the spread of value over cost, E_t stands for earnings, RD_t is R&D expenditures, I_t is interest expenses, D_t is total dividends paid, dA_t is the lagged change of total assets, i.e. $dA_t = A_t - A_{t-1}$ and dA_{t+1} equals the corresponding lead change in A_t, i.e. $dA_{t+1} = A_{t+1} - A_t$. The vector Y_t is introduced to facilitate notation and it consists of earnings, research and development expenditures, interest payments and dividends with dY_t being the lagged change in variable Y_t whereas dY_{t+1} equals the lead change in in variable Y_t.[11] To avoid that results are dominated by the largest firms in the sample, all variables are scaled by the book value of assets.[12]

The Fama and French (1998) approach is similar to the general valuation equation implied from the real-options model as it relates firm value to the value of cash-flow generating assets in place, measured by A_t and E_t and it uses leads of the different variables to account for market expectations and future cash flows. Research and development expenses are included because accounting requirements imply that total assets might be understated for firms with growth opportunities, i.e. for firms with high R&D expenses. Combining the Fama and French (1998) setup with the setup used for the regressions based on simulated data, i.e. equation [18] and following Pinkowitz et al. (2006) who recognize that the book value of assets can be split up into a cash and non-cash component, I propose the following regression specification.

$$\begin{aligned} \frac{V_t - NA_t}{A_t} &= a + \beta_1 \frac{C_t}{A_t} + \beta_2 \frac{E_t}{A_t} + \beta_3 \frac{RD_t}{A_t} + \beta_4 \frac{D_t}{A_t} + \beta_5 \frac{dC_t}{A_t} + \beta_6 \frac{dC_{t+1}}{A_t} \\ &+ \beta_7 \frac{dNA_t}{A_t} + \beta_8 \frac{dNA_{t+1}}{A_t} + \beta_{11} \frac{dY_t}{A_t} + \beta_{10} \frac{dY_{t+1}}{A_t} + \epsilon_t \end{aligned} \tag{21}$$

Specifically, I regress the spread between the market value of the firm and its non-cash assets on the current level of the firm's cash account and its earnings which relate to the current income stream $K_0 x_t$ generated under asset capacity K_0 in the real-options model. Research and development expenses are included to proxy for growth opportunities $G(x_t, C_t)$ and leads of the different variables are used to account for market expectations and future cash flows. The coefficient on dividend payments can be interpreted as a proxy for the presence of agency costs of free cash flow or tax

[11]Specifically, Y_t is defined as $Y_t \equiv [E_t,\ RD_t,\ I_t,\ D_t]$

[12]Besides, this also helps to mitigate heteroskedasticity in the error terms.

distortions.[13] To avoid that large firms dominate the sample and to mitigate heteroskedasticity in the error terms, I follow Fama and French (1998) and scale all variables by total assets.

It is important to notice that this specification is an extension to Pinkowitz et al. (2006) as it allows for the separate inclusion of the level of cash while still accounting for the level of non-cash assets NA_t in the estimation setup. The inclusion is implied by the real-options model which has shown that non-cash assets, as proxied for by the existing level of production capacity, are an important source of firm value. Besides, the approach suggested in this paper includes both the level and changes of cash simultaneously, much like Fama and French (1998) considered both the level and changes of assets. As I will argue in this paper, the consideration of both the level of cash and non-cash assets has important consequences when estimating the marginal value of cash.

For what follows, I first describe the data underlying the econometric analyses and give brief summary statistics. Next, I estimate the value of cash for the entire sample of all-equity firms to assess whether on average all-equity firms add or destroy value by holding cash. Then, I look at the value of cash in the context of growth opportunities before concluding the section with a variety of robustness checks.

4.1 Data and Summary Statistics

The study uses accounting data from COMPUSTAT and includes firm year observations from 1950 to 2006. To ensure comparability of the results to Pinkowitz and Williamson (2004) industrial annual files are used. As usual, financial firms and utilities are deleted from the sample.[14] All variables are calculated similarly to Fama and French (1998) and Pinkowitz and Williamson (2004). Finally, fiscal year-end values are used.

[13] Note that this analysis concerns all-equity firms which is why interest payments do not enter the estimation equation.

[14] Specifically, a firm is omitted if its primary SIC is between 4900 and 4999 or between 6000 and 6999.

Total firm value is given by calculating the sum of market value of equity at fiscal year-end and book value of debt, i.e. ($V = D54 \times D199 + D34 + D9$).[15] Earnings correspond to earnings before extraordinary items, interest, deferred tax credits and investment tax credits, i.e. ($E = D18 + D15 + D50 + D51$). Cash is equivalent to cash plus marketable securities ($C = D1$). Dividends are measured as common dividends paid, i.e. $D = D21$. Net assets equal the difference between total assets and cash, i.e. $NA = (D6 - D1)$. Research and development expenses finally correspond to data item number $D46$ and the entry is set to zero if missing. All-equity financed firms are defined following Strebulaev and Yang (2006) by requiring that the the sum of both short-term debt ($D34$) and long-term debt ($D9$) equals zero. As the regression specification includes both leads and lags, this implies that a firm is only included in the sample if it is classified as an all-equity firm for three consecutive years. Finally, in order to reduce the influence of outliers trim the sample by dropping 1 percent of the observations in each tail of the explanatory variables.[16]

Table [6] displays summary statistics for for selected variables. It can be seen that the average firm with zero debt has a value-to-asset ratio of approximately 1.92. This is substantially more than the average value-to-asset ratio for the entire sample including levered firms, which is around 1.28. Besides, all-equity firms have cash holdings equal to 34 percent of their assets and they pay out approximately 2 percent of total assets as dividends. This compares to 11.5 percent relative cash holdings and 1 percent dividends when using the entire sample of levered and unlevered firms. Interestingly, all-equity firms are slightly less profitable with a return on assets of 3.4 percent compared to 4.2 percent return on assets for the entire sample. However, all-equity firms invest approximately 5 percent of their assets into research and development whereas the corresponding entry for the entire sample equals only 2.3 percent. The table for the entire sample can be found in the Appendix.

[15] The numbers correspond to Compustat data item numbers.

[16] Fama and French (1998) explain that scaling by assets leads to potentially huge outliers if the asset value is close to zero. Given that all-equity firms are on average smaller than their levered counterparts, I therefore trim 1% of the observations in each tail of the explanatory variable. Because trimming is done with respect to the full sample this reduces the total sample by less than 2%.

Table 2: Summary Statistics for All-Equity Firms.

Variable	Mean	Std. Dev.	Min.	Max.
Value-to-Assets Ratio	1.917	1.684	0.272	14.796
Relative Cash Holdings	0.337	0.201	0	0.862
Relative Noncash Assets	0.663	0.201	0.138	1
Relative Earnings	0.034	0.163	-1.891	0.263
Relative Research and Development Expenses	0.047	0.077	0	0.501
Relative Dividends	0.018	0.025	0	0.096
N		6429		

Summing up, all-equity firms are valued at higher multiples and invest more heavily into research and development than the average firm of the entire sample which also includes levered firms. Confirming Strebulaev and Yang (2006), relative cash holdings are huge and make up roughly one third of total assets. Using the Fama and French 12 Industry Classification scheme, one can further see that each single industry holds roughly at least as much cash as the average cash holdings reported in Bates et al. (2009) for both levered and unlevered firms. It is therefore only natural to ask whether all-equity firms destroy or add value by holding that much cash.

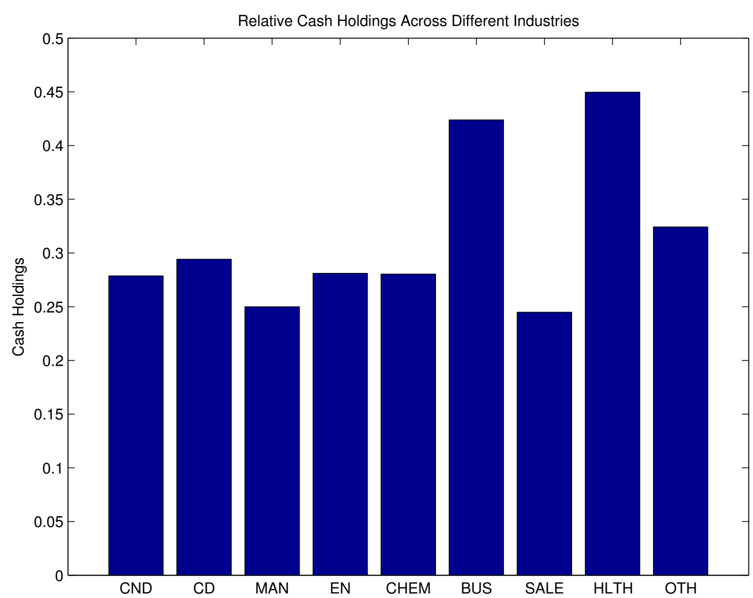

Figure 11: Average Cash Holdings of Different Industries as classified according to the Fama and French 12 Industry Classification Scheme.

4.2 Results

This section presents results for the value of cash using equation [21] and compares it to those obtained when following the Pinkowitz et al. (2006) approach. Regressions will be estimated accounting for firm fixed effects and by including time dummies. Additionally, standard errors are estimated according to Discroll and Kraay (1998) to account for possible cross-sectional interdependence among the error terms.[17] Due to the introduction of lead and lag variables in the regression setup, the value of cash is given by

$$\frac{\partial\left(\frac{V_t - NA_t}{A_t}\right)}{\partial\left(\frac{C_t}{A_t}\right)} = \hat{\beta}_1 + \hat{\beta}_5 - \hat{\beta}_6 \tag{22}$$

When interpreting results, I will further impose the null hypothesis that the true value is equal to one, i.e. that cash is valued at par without frictions. The analysis proceeds as follows. First, I estimate the value of cash for the entire sample of all equity firms. Then I will proxy for growth opportunities before finally displaying results regarding the sensitivity of cash with respect to volatility.

Table [3] displays results for the value of cash and compares it to the two approaches proposed by Pinkowitz et al. (2006). Again, the extension considers the fact that the setup proposed in this paper also includes the level of non-cash assets and that the level and changes of cash are included simultaneously whereas Pinkowitz et al. (2006) consider them separately. Hence, the labels "level regression" and "changes regression" in their paper.[18] It can be seen that when the value of cash is estimated according to the framework proposed in this paper, cash is valued at par in the cross-section. Specifically, by plugging the coefficients of C_t, dC_t and dC_{t+1} into equation [22] the estimated value is equal to 1.003 which is also statistically insignificantly different from 1. In other words, on average all-equity firms do not destroy value by holding that much cash.

[17] As a robustness check, results will also be presented when estimation is done according to the Fama-MacBeth approach

[18] More details on the approach of Pinkowitz et al. (2006) are provided in the Appendix.

Table 3: The Value of Cash

	(1) Kisser (2010)		(2) PSW (2006) - Level		(3) PSW (2006) - Changes	
	Coefficient	T-Statistics	Coefficient	T-Statistics	Coefficient	T-Statistics
C_t	1.836***	6.95	0.495*	2.37		
E_t	0.597	1.13	1.843***	3.89	0.635	1.25
RD_t	3.760**	3.29	4.404***	3.66	3.752**	3.28
D_t	4.663	1.46	2.632	0.79	4.791	1.51
dC_t	1.016***	8.27			1.200***	10.49
dC_{t+1}	1.849***	12.11			1.695***	13.92
dNA_t	1.396***	13.36	1.043***	9.56	1.267***	11.57
dNA_{t+1}	1.456***	10.65	1.048***	7.87	1.566***	10.09
dE_t	0.020	0.13	0.227	1.40	-0.015	-0.10
dE_{t+1}	0.649*	2.22	1.562***	5.57	0.620*	2.10
dRD_t	-0.462	-0.73	-0.236	-0.36	-0.339	-0.50
dRD_{t+1}	3.224**	3.22	4.542***	4.15	3.281***	3.31
dD_t	9.945*	2.48	9.619*	2.41	10.459**	2.59
dD_{t+1}	8.674**	2.62	10.124**	3.15	9.289**	2.84
dV_{t+1}	-0.277***	-8.95	-0.235***	-8.36	-0.276***	-9.05
Observations	6429		6429		6429	
R^2	0.302		0.243		0.285	

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

However, we can also see that the interpretation would be different if estimated according to Pinkowitz et al. (2006). Specifically, if the level regression was employed one would conclude that firms actually destroy value as the coefficient of the cash variable equals 0.495 whereas under the changes regression the result depends on whether the coefficient of dC_t or the difference between dC_t and dC_{t+1} is the basis of the interpretation.

While this result is interesting per se, the ultimate interest lies in analyzing the value of cash in the context of growth options and with respect to volatility. Traditionally, the literature on financing and investment decisions uses the ratio between the market value of a firm's physical assets and its replacement costs as a proxy for the value of growth opportunities. Examples include Hayashi (1982), Fazzari et al. (1988), Erickson and Whited (2000) and Hennessy (2004). However, given that the market value enters the numerator of the dependent variable, one has to search for another proxy for the presence of growth opportunities. Potential candidates employed in the finance and accounting literature include research and development (R&D) expenses and capital expenditures (CAPEX). See for example Stowe and Xing (2006), Pinkowitz and Williamson (2004), Goyal et al. (2002), Lang et al. (1996), Gaver and Gaver (1993) , Skinner (1993) and Smith and Watts (1992) among others. Given the relatively small sample size for all-equity firms and the fact that for a substantial fraction R&D Expenses equal zero, I use CAPEX for the subsequent analysis. Specifically, Goyal et al. (2002) propose to use the ratio between CAPEX and the book value of its assets to control for the presence of growth opportunities. I therefore extend equation [21] by including an interaction term between relative cash holdings and capital expenditures, i.e.

$$\begin{aligned}\frac{V_t - NA_t}{A_t} &= a + \beta_1 \frac{C_t}{A_t} + \beta_2 \frac{C_t}{A_t} \frac{CAPEX_t}{A_t} + \beta_3 \frac{E_t}{A_t} + \beta_4 \frac{RD_t}{A_t} + \beta_5 \frac{D_t}{A_t} + \beta_6 \frac{dC_t}{A_t} + \beta_7 \frac{dC_{t+1}}{A_t} \\ &+ \beta_8 \frac{dNA_t}{A_t} + \beta_9 \frac{dNA_{t+1}}{A_t} + \beta_{10} \frac{dY_t}{A_t} + \beta_{11} \frac{dY_{t+1}}{A_t} + \epsilon_t\end{aligned} \tag{23}$$

Table [4] displays results when including the interaction between cash holdings and capital expenditures. It can be seen that the coefficient on the interaction term is positive and statistically different from zero. In other words, if firms invest into capital expenditures and have positive cash holdings then this has a positive impact on firm value. Clearly, in a frictionless world the coefficient should be equal to zero. Moreover, this effect is not limited to the estimation approach suggested in this paper but also manifests itself in the level regression approach suggested by Pinkowitz et al. (2006). While the effect is also positive in the changes regression, it is statistically insignificant.

We have thus seen that cash has additional value if one controls for the presence of growth opportunities. As a last step, I analyze whether this value is negatively related to volatility as it was implied by the real-options model introduced in the Section [2]. I follow Han and Qiu (2007) and Minton and Schrand (1999) who calculate historic cash flow volatility by computing the coefficient of variation of operating cash flow. This approach has been also suggested by Albrecht and Richardson (1990) and Michelson et al. (1995).[19] More specifically, I calculate operating cash

[19] The intuition for using the coefficient of variation is to have a unitless measure of variation.

Table 4: The Value of Cash in the Context of Growth Options

	(1) Kisser (2010)		(2) PSW (2006) - Level		(3) PSW (2006) - Changes	
	Coefficient	T-Statistics	Coefficient	T-Statistics	Coefficient	T-Statistics
C_t	1.646***	5.95	0.321	1.37		
C_t X $Capex_t$	4.650***	4.05	3.610**	2.86		
E_t	0.434	0.81	1.778***	3.71	0.531	1.04
RD_t	3.603**	3.03	4.328***	3.39	3.596**	3.03
D_t	4.336	1.26	2.296	0.65	4.360	1.24
dC_t	1.094***	8.12			1.075***	6.25
dC_{t+1}	1.886***	12.23			1.736***	13.42
dNA_t	1.303***	10.78	0.942***	7.46	1.272***	10.49
dNA_{t+1}	1.456***	8.75	1.024***	6.47	1.610***	9.05
dE_t	0.247	1.80	0.440**	3.19	0.170	1.29
dE_{t+1}	0.715*	2.21	1.671***	5.35	0.697*	2.18
dRD_t	-0.601	-1.00	-0.381	-0.59	-0.323	-0.53
dRD_{t+1}	3.232**	2.98	4.623***	3.97	3.246**	3.02
dD_t	9.833*	2.18	9.428*	2.08	10.451*	2.28
dD_{t+1}	8.188*	2.45	9.823**	3.03	8.813**	2.64
dV_{t+1}	-0.285***	-9.02	-0.243***	-8.20	-0.283***	-8.82
dC_t X $Capex_t$					2.494	1.43
Observations	6030		6030		6030	
R^2	0.317		0.257		0.300	

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

flow by using quarterly Compustat data and computing it as sales (D2) less cost of goods sold (D30) minus selling, general and administrative expenses (D1) less the change in net working capital which in turn is given by the sum of non-missing amounts for accounts receivable (D37), inventory (D38) and other current assets (D39) less the sum of non-missing amounts for accounts payable (D46), income taxes payable (D47) and other current liabilities (D48). Initial volatility is calculated using quarterly observations from 1962 until 1989. A company is excluded from the sample if it has less than 15 quarterly observations available. Each year the estimation window is extended by one year while the initial observation period is held fixed at 1962. To check whether the negative relation between the value of cash and volatility holds, I estimate the following regression.

$$\begin{aligned} \frac{V_t - NA_t}{A_t} &= \alpha + \beta_1 \frac{C_t}{A_t} + \beta_2 \frac{C_t}{A_t}\frac{CAPEX_t}{A_t} + \beta_3 \frac{C_t}{A_t}\frac{CAPEX_t}{A_t}\sigma_t + \beta_4 \frac{C_t}{A_t}\frac{CAPEX_t}{A_t}\sigma_t^2 \\ &+ \ ... + \beta_8 \frac{dC_t}{A_t} + \beta_9 \frac{dC_{t+1}}{A_t} + ... + \epsilon_t \end{aligned} \tag{24}$$

where the coefficients β_3 and β_4 capture the effect of volatility and variance on the value of cash in the context of growth opportunities.

Table 5: The Value of Cash, Growth Options and Volatility

	(1) Kisser (2010)		(2) PSW (2006) - Level		(3) PSW (2006) - Changes	
	Coefficient	T-Statistics	Coefficient	T-Statistics	Coefficient	T-Statistics
C_t	3.781***	5.81	2.504***	4.66		
C_t X $Capex_t$	5.137	0.84	1.463	0.24		
C_t X $Capex_t$ X σ	-1.318	-1.37	-1.476	-1.66		
C_t X $Capex_t$ X σ^2	0.017	1.53	0.019+	1.81		
E_t	-0.164	-0.06	4.177	1.67	3.239	1.33
RD_t	3.261	0.68	7.540	1.61	9.177+	1.92
D_t	17.331	1.60	16.239	1.32	29.115*	2.42
dC_t	2.371*	2.24			1.905	1.32
dC_{t+1}	2.961***	4.18				
dNA_t	4.023***	6.61	2.658***	4.66	3.296***	5.13
dNA_{t+1}	0.939	1.58	-0.854	-1.21	-0.178	-0.25
dE_t	0.811	1.02	1.419	1.63	0.842	0.92
dE_{t+1}	1.622+	1.82	3.586***	4.98	3.541***	5.01
dRD_t	-1.963	-0.57	-2.620	-0.85	-4.452	-1.43
dRD_{t+1}	-8.734***	-3.94	-5.914*	-2.42	-6.120*	-2.08
dD_t	-9.330	-0.81	-5.102	-0.49	-2.778	-0.25
dD_{t+1}	29.060	1.12	35.674+	1.81	26.939	1.55
dV_{t+1}	-0.241+	-2.00	-0.121	-0.89	-0.156	-1.26
dC_t X $Capex_t$					8.431	0.85
dC_t X $Capex_t$ X σ					1.976	0.55
dC_t X $Capex_t$ X σ^2					-0.015	-0.29
Observations	312		312		312	
R^2	0.429		0.334		0.324	

+ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table [5] shows that while cash flow volatility has a negative impact on the value of cash, the coefficient is not statistically different from zero. Interestingly, one can see that while the coefficients on volatility and variance are insignificant, they still increase the R squared substantially from 32 to 43 percent. However, it should be noted that the sample size is very small due to the requirement that firms are all-equity financed for the entire period over which cash flow volatility is calculated.

4.3 Robustness Checks

The Appendix presents results with respect to two different robustness checks. First, I present results in case the regressions are estimated according to the Fama MacBeth approach. Second, I estimate how the value of cash changes if estimated over a more recent time period.

The results presented in this paper are based on estimating a fixed effects model while standard errors are calculated according to Discroll and Kraay (1998) which are robust to any form of cross-sectional dependance of the error terms. As a robustness check, I also present results according to the Fama MacBeth approach. Following Petersen (2009) who also emphasizes the need to employ Fama MacBeth approach for a correctly specified model, I use the demeaned value of the individual variables as independent and dependent variables in the regression setup. Figure [7] displays corresponding results in the Appendix. Again, it turns out that by controlling for net assets, the value of cash for all-equity firms is not statistically different from one. On the other hand, the interaction term between is statistically indifferent from zero in all three regressions. Further unreported results show that that volatility does not impact the value of cash under the Fama MacBeth approach.

Finally, I briefly check whether results are robust to the specified time period and therefore estimate the value of cash for all-equity firms since 1990. The value of cash is approximately equal to one whereas the interaction term between cash and capital expenditures is significantly positive. Full results are shown in Table [8] in the Appendix.

5 Conclusion

As of 2006, roughly 20% of all large U.S. public corporations were all-equity financed. On top of that, average cash holdings were nearly twice as high as in the case of both levered and unlevered firms. This paper thus addresses the question whether all-equity financed firms destroy or add value by hoarding cash.

I therefore propose a simple real-options model which derives the value of internal funds within a capacity expansion problem. Departing from traditional real option models, the paper focuses on the question of how the costs of the investment are actually financed and thereby derives the optimal trade-off between costs of external finance and agency costs of free cash flow. Doing so, it is shown that cash is valued at a premium to its notional amount and that there is a negative relation between the value of cash and volatility. This is because cash derives its value by reducing future financing costs and thereby essentially reduces the exercise price of the capacity expansion option. As it turns out, this option loses importance in case of higher volatility. The paper also shows that if a firm engages in precautionary saving it might wait longer to exercise its growth option than if it funded the project completely externally. Again, because saving is nothing else than an additional option to exercise the same project at a lower price, it has its own value of waiting which influences the investment decision. Also, it turns out that simultaneously holding cash and paying dividends can be the result of a value maximizing strategy.

The empirical section combines the model implied estimation equation with an empirical approach introduced by Pinkowitz et al. (2006). By doing so, I propose a simple, yet important extension to the existing literature and show that on average all-equity firms do not destroy value by holding that much cash. Moreover, cash is rather valued at a premium if the presence of growth opportunities is controlled for.

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A Theory Part

A.1 Proof of Proposition [1]

[Proof] Using the fact that $\mu = r - \delta$ we can write that $dx = (r-\delta)xdt + \sigma x dW^Q$. Let's suppose we construct a risk-free portfolio by holding θ_1 units of the firm and shorting θ_2 units of the traded asset. The long position of the portfolio entitles us to an instantaneous dividend payment $\theta_1(1-\alpha)x$. The value of the portfolio P is given by $(\theta_1 V - \theta_2 X)$ and it follows that the total return from holding the portfolio over a short time interval dt equals

$$dP = \theta_1\left((1-\alpha)xdt + dV\right) - \theta_2 dX \tag{25}$$

Applying Ito's Lemma leaves us with

$$dP = \theta_1\left((1-\alpha)xdt + V_x dx + V_C dC + \frac{1}{2}\sigma^2 x^2 V_{xx} dt\right) - \theta_2 dX \tag{26}$$

For $\theta_1 = 1$, it immediately follows that θ_2 equals $\left(\frac{V_x x}{X}\right)$ which then implies that $dP = rPdt$. Combining above and using the fact that $P = (V - \frac{xV_x}{X}X)$ we obtain that

$$rV = (1-\alpha)x + (r-\delta)xV_x + (\alpha x - \frac{\phi}{2}(\alpha x)^2 + rC)V_C + 1/2\sigma^2 x^2 V_{xx} \tag{27}$$

The only step missing is to treat α as a stochastic optimal control by imposing that

$$rV = \max_{\alpha}\left\{(1-\alpha)x + (r-\delta)xV_x + (\alpha x - \frac{\phi}{2}(\alpha x)^2 + rC)V_C + 1/2\sigma^2 x^2 V_{xx}\right\} \tag{28}$$

Taking the FOC with respect to α implies that

$$\alpha^* = \frac{V_C - 1}{\phi x V_C} \tag{29}$$

with the additional requirement that $\alpha^* \in [0,1]$.

A.2 Proof of Proposition [2]

[Proof] By assumption α is set to 0 such that the PDE in equation [4] simplifies to

$$rV^B = x + (r-\delta)xV_x^B + rCV_C^B + 1/2\sigma^2x^2V_{xx}^B \tag{30}$$

which has to be solved with respect to

$$\begin{gathered} V^B(0, C_t) = C_t \\ V^B(x^*, C_\tau) = \tfrac{K_1 x^*}{\delta} + C_\tau - IC - e(IC) \\ V_x^B(x^*, C_\tau) = \tfrac{K_1}{\delta} \end{gathered} \tag{31}$$

Assuming that $V^B(x, C) = \nu C + Bx^\beta + \gamma x$ and solving the PDE with respect to the boundary conditions implies that

$$x_2^* = \frac{\beta_1}{(\beta_1 - 1)(K_1 - K_0)}\delta(IC + e(IC)) \tag{32}$$

where β_1 is the positive root of the fundamental quadratic

$$\frac{1}{2}\beta(\beta-1) + \mu\beta - r = 0 \tag{33}$$

It follows that $V^B(x, C) = \frac{K_0 x}{\delta} + Bx^{\beta_1} + C$ where $B = \left(\frac{(K_1-K_0)x_2^*}{\delta} - IC - e(IC)\right)\left(\frac{1}{x_2^*}\right)^{\beta_1}$ and τ equals the exercise time of the option which is formally defined as $\{\tau := \inf\{u \geqslant 0 : x_u = x_2^*\}\}$.

A.3 Proof of Proposition [3]

[Proof] By assumption we have that $C \geq \mathrm{IC}$ and $\alpha = 0$. It suffices to solve the PDE given in [8] with respect to

$$\begin{aligned} V'(0, C_t) &= C_t \\ V'(x^*, C_\tau) &= \tfrac{K_1 x^*}{\delta} + C_\tau - IC \\ V'_x(x^*, C_\tau) &= \tfrac{K_1}{\delta} \end{aligned} \tag{34}$$

Assuming that $V'(x, C) = \nu C + A x^\beta + \gamma x$ and solving the PDE with respect to the boundary conditions implies that

$$x^* = \frac{\beta_1}{(\beta_1 - 1)(K_1 - K_0)} \delta IC \tag{35}$$

where β_1 is the positive root of the fundamental quadratic

$$\frac{1}{2}\beta(\beta - 1) + \mu\beta - r = 0 \tag{36}$$

It follows that $V'(x, C) = \frac{K_0 x}{\delta} + A x^{\beta_1} + C$ where $A = \left(\frac{(K_1 - K_0) x^*}{\delta} - IC \right) \left(\frac{1}{x^*} \right)^{\beta_1}$ and τ equals the exercise time of the option which is formally defined as $\{\tau := \inf \{u \geqslant 0 : x_u = x^*\}\}$.

On the other hand, if the firm decides to pay out the initial cash balance and all future earnings as dividends, then the dynamics of the cash account are given by the following equation

$$dC = (rC - C)dt \tag{37}$$

Using similar arguments as when deriving the PDE in equation [4] we obtain that

$$rV'' = x + C + (r-\delta)xV''_x + (rC - C)V''_C + 1/2\sigma^2 x^2 V''_{xx} \tag{38}$$

Because of the full payout assumption it follows that total costs of exercising the option are given by $IC + e(IC)$. Assuming that the solution is given by $V''(x,C) = \nu C + Bx^\beta + \gamma x$ it directly follows that $V''(x,C) = V^B(x,C)$ such that the solution is given by

$$R^h(x,C) = x^{\beta_1}(A - B) \tag{39}$$

A.4 Proof of Proposition [4]

[Proof] Notice that by definition we have that $S^h(x) = \frac{Ax^{\beta_1}}{Bx^{\beta_1}} - 1$. Noting that $x^* = \frac{\beta_1}{(\beta_1-1)(K_1-K_0)} IC\delta$, defining $\Delta K = K_1 - K_0$ and taking the derivative of the whole expression with respect to ΔK we obtain the desired result.

A.5 Proof of Proposition [5]

[Proof] Concerning the partial derivative of any growth option with respect to volatility, it is sufficient to observe that

$$\frac{\partial Ax^{\beta_1}}{\partial \sigma} = Ax^{\beta_1} \log\left(\frac{x}{x^*}\right) \frac{\partial \beta_1}{\partial \sigma} \tag{40}$$

as $\frac{\partial Ax^{\beta_1}}{\partial x^*}\frac{\partial x^*}{\partial \beta_1}$ equals zero. Given that the positive solution to the fundamental quadratic is characterized by the same parameters for both the constrained and unconstrained firm, we only need to know that $\frac{\partial \beta_1}{\partial \sigma} < 0$. Further details can be found in Dixit & Pindyck Dixit and Pindyck (1991).

Applying above to $\frac{\partial R^h(x)}{\partial \sigma}$ we get that

$$\frac{\partial R^h(x)}{\partial \sigma} = \frac{\partial \beta_1}{\partial \sigma}\left\{Ax^{\beta_1}\log\left(\frac{x}{x^*}\right) - Bx^{\beta_1}\log\left(\frac{x}{x_2^*}\right)\right\} \tag{41}$$

Using the fact that $x_2^* = x^*(1+\gamma)$ where $\gamma = (\gamma_1 + \gamma_2 IC)$ and that $Bx^{\beta_1} = Ax^{\beta_1}(1+\gamma)^{1-\beta_1}$, we can rewrite the equation as

$$\frac{\partial R^h(x)}{\partial \sigma} = \frac{\partial \beta_1}{\partial \sigma}\left\{Ax^{\beta_1}\log\left(\frac{x}{x^*}\right) - Ax^{\beta_1}(1+\gamma)^{1-\beta_1}\log\left(\frac{x}{x^*(1+\gamma)}\right)\right\} \tag{42}$$

which again can be rewritten as

$$\frac{\partial R^h(x)}{\partial \sigma} = Ax^{\beta_1}\frac{\partial \beta_1}{\partial \sigma}\left\{\log\left(\frac{x}{x^*}\right) - (1+\gamma)^{1-\beta_1}\log\left(\frac{x}{x^*(1+\gamma)}\right)\right\} \tag{43}$$

Due to the fact that $x < x^* < x^*(1+\gamma)$ we know that $\log\left(\frac{x}{x^*}\right) > \log\left(\frac{x}{x^*(1+\gamma)}\right)$. The question whether the expression in the bracket is positive or negative will depend on $(1+\gamma)^{1-\beta_1}$ which will lie between 0 and 1 for different values of γ and β_1.

B Numerical Solution

Equation [11] is defined for $2 \leq j \leq M$ and $2 \leq i \leq N$. One alternative would be to solve the PDE by using the boundary conditions defined in [6]. However, this is computationally demanding as one has to consider the entire grid. To overcome this drawback, I will divide the state space of C into two different regimes. We know that if the firm has sufficient cash available, i.e. $C \geq IC$, it will not retain its earnings as it only incurs costs of holding cash within the firm. Therefore, for $C \geq IC$ we will have that $\alpha = 0$. It is also known that because $C \geq IC$ the firm will not need to access costly external capital markets and as a consequence it will be able to finance the investment at its true costs IC. However, when $\alpha = 0$ and $C \geq IC$ the value of an option to invest can be derived analytically as there is no contradiction between the boundary conditions anymore.

Proposition 6 *For $C \geq IC$ and $\alpha = 0$, total firm value, denoted by $V'(x, C)$ has a closed firm solution which is given by*

$$V'(x, C) = \frac{K_0 x}{\delta} + Ax^{\beta_1} + C \tag{44}$$

where $A = \left(\frac{\Delta K x^*}{\delta} - IC\right)\left(\frac{1}{x^*}\right)^{\beta_1}$ with the corresponding optimal trigger level x^*.

Proof: See Appendix.

Thus, as long as $x < x^*$ we know that for $C \geq IC$ value-matching and smooth-pasting conditions are given by

$$\begin{aligned} V(x^*, C_\tau) &= V'(x^*, C_\tau) \\ V_x(x^*, C_\tau) &= V'_x(x^*, C_\tau) \end{aligned} \tag{45}$$

This has the advantage that the grid for C has an upper limit equal to the value of IC which drastically decreases computational requirements.

While for the case when $C \geq IC$, optimal trigger level was independent of the two state variables, exercise policy when allowing the firm to retain cash depends on the level of C which in turn is affected by the retention rate α and operating profit x. The optimal exercise point will depend on the level of cash the firm has available which in turn will be affected by the firms operating profit and its retention rate.

C Empirical Part

C.1 The Pinkowitz et al. (2006) Approach

Pinkowitz et al. (2006) propose the following specification to test for the value of cash

$$\begin{aligned}\frac{V_t}{A_t} &= \alpha + \beta_1\frac{dC_t}{A_t} + \beta_2\frac{dC_{t+1}}{A_t} + \beta_3\frac{dNA_t}{A_t} + \beta_4\frac{dNA_{t+1}}{A_t} + \beta_5\frac{Y_t}{A_t} \\ &+ \beta_6\frac{dY_t}{A_t} + \beta_7\frac{dY_{t+1}}{A_t} + \beta_8\frac{dV_{t+1}}{A_t} + \epsilon_t \end{aligned} \tag{46}$$

where for simplicity I introduce the vector Y_t to summarize earnings, interest payments, dividends and research and development expenses, i.e.

$$Y_t \equiv [E_t,\ RD_t,\ I_t,\ D_t] \tag{47}$$

with dY_t and dY_{t+1} corresponding to the lagged and lead changes in the underlying variables. Because the levels of cash C_t and non-cash assets NA_t do not explicitly appear in the equation, it must be implicitly assumed that they are included in the constant.[20] Pinkowitz et al. (2006) argue that the coefficient β_1 thus captures the market value of cash for a one dollar increase in cash-holdings compared to the previous period.

Alternatively, Pinkowitz et al. (2006) propose a level regression which includes the level of cash C_t instead of the lagged and lead changes dC_t and dC_{t+1}.

$$\begin{aligned}\frac{V_t}{A_t} &= \alpha + \beta_1\frac{C_t}{A_t} + \beta_2\frac{dNA_t}{A_t} + \beta_3\frac{dNA_{t+1}}{A_t} + \beta_4\frac{Y_t}{A_t} + \beta_5\frac{dY_t}{A_t} \\ &+ \beta_6\frac{dY_{t+1}}{A_t} + \beta_7\frac{dV_{t+1}}{A_t} + \epsilon_t \end{aligned} \tag{48}$$

[20] Note that $\frac{NA_t + C_t}{A_t} = 1$.

Given that non-cash assets do not appear in the equation, the coefficient on C_t does not necessarily capture the value of cash. This might be the reason why it was only suggested as an alternative to the changes regression.

C.2 Results

Table 6: Summary Statistics for Levered and Unlvered Firms. All variables are scaled by the volume of total assets.

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**
Value-to-Assets Ratio	1.282	1.073	0.268	15.04
Relative Cash Holdings	0.115	0.137	0	0.862
Relative Noncash Assets	0.885	0.137	0.138	1
Relative Earnings	0.042	0.133	-2.422	0.265
Relative Research and Development Expenses	0.023	0.051	0	0.565
Relative Dividends	0.01	0.015	0	0.096
N		124905		

Table 7: Robustness: The Value of Cash (FMB)

	(1) Kisser (2010) Coefficient	(2) PSW (2006) - Level Coefficient	(3) PSW (2006) - Changes Coefficient	(4) GO: Kisser (2010) Coefficient	(5) GO: PSW (2006) - Level Coefficient	(6) GO: PSW (2006) - Changes Coefficient
C_t	2.542***	2.588***		1.710***	1.134***	
E_t	4.750***	9.159***	10.678***	4.222***	8.489***	9.872***
RD_t	-1.939	2.770	6.268***	-2.378*	2.725**	6.617***
D_t	0.597	6.394**	9.174***	5.155*	11.486***	13.507***
dC_t	-1.044		3.514***	-2.856**		1.140
dC_{t+1}	-0.069		1.321	-1.422		0.026
dNA_t	-1.590**	1.462**	1.573*	-1.759**	0.887	1.060
dNA_{t+1}	-0.169	0.510	0.639	-0.270	0.096	0.860
dE_t	-2.834*	-5.793***	-6.489***	-2.466*	-5.542***	-7.000***
dE_{t+1}	2.484*	1.615	1.894	3.227**	3.379***	3.591***
dRD_t	15.520	-5.014	-12.918	17.681	-5.670	-20.883
dRD_{t+1}	21.438*	9.415	6.334	14.307	6.321	5.687
dD_t	64.024***	33.272***	34.863**	60.208***	35.774**	40.990***
dD_{t+1}	45.497***	28.509**	29.570*	32.546***	8.678	6.979
dV_{t+1}	0.059	0.065	-0.183	0.333	0.267	0.175
C_t X $Capex_t$				-0.353	-2.323	
dC_t X $Capex_t$						-1.873
Observations	6429	6429	6429	6030	6030	6030
R^2	0.898	0.975	0.954	0.864	0.773	0.939

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: Robustness: The Value of Cash since 1990

	(1) Kisser (2010) Coefficient	(2) PSW (2006) - Level Coefficient	(3) PSW (2006) - Changes Coefficient	(4) GO: Kisser (2010) Coefficient	(5) GO: PSW (2006) - Level Coefficient	(6) GO: PSW (2006) - Changes Coefficient
C_t	1.950***	0.490		1.542***	0.152	
E_t	0.822	1.908***	0.851*	0.754	1.909***	0.819
RD_t	4.770***	5.509***	4.727***	4.481**	5.255***	4.559***
D_t	-3.211	-5.511	-2.953	-2.201	-4.690	-2.964
dC_t	0.965***		1.165***	1.043***		1.049***
dC_{t+1}	1.873***		1.705***	1.785***		1.663***
dNA_t	1.414***	1.148***	1.279***	1.253***	0.977***	1.228***
dNA_{t+1}	1.536***	1.246***	1.677***	1.463***	1.160***	1.613***
dE_t	-0.000	0.173	-0.026	0.227	0.375**	0.161
dE_{t+1}	0.835**	1.644***	0.809**	0.896**	1.715***	0.876**
dRD_t	-0.161	-0.022	-0.019	-0.256	-0.176	0.051
dRD_{t+1}	4.115***	5.454***	4.142***	4.076***	5.414***	4.120***
dD_t	5.847	4.955	5.709	6.710	5.735	7.038
dD_{t+1}	7.663	9.367*	9.126*	8.397	10.033*	9.070*
dV_{t+1}	-0.279***	-0.235***	-0.277***	-0.279***	-0.238***	-0.276***
C_t X $Capex_t$				5.711***	5.157***	
dC_t X $Capex_t$						1.957
Observations	3533	3533	3533	3409	3409	3409
R^2						

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$