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Stochastic Partial Differential Equations: Approximations and Applications

ANDREA BARTH

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It is a mistake to think you can solve any major problems just with potatoes.

Douglas Adams
English humorist and science fiction novelist (1952 – 2001)

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1

Introduction

This thesis deals with the numerical approximation of stochastic differential equations driven by square integrable martingales, and which take values in an appropriate Hilbert space. We also develop some applications to mathematical finance models.

This introduction contains a brief exposition of some methods for numerical approximation of ordinary and partial differential equations, followed by an introduction to numerical methods for stochastic differential equations. The approximation theories for these two fields combine, together with the theory of Hilbert–space valued stochastic processes, to produce the current ways of estimating solutions to stochastic partial differential equations. In order to provide a self contained summary of the topic in hand, we also include a presentation of Hilbert–space valued stochastic differential equations and their integration theory.

In this work we combined Galerkin approximations with stochastic partial differential equations driven by square–integrable martingales. Numerical methods and approximation schemes for Hilbert–space valued stochastic differential equations have been studied, among others, in [2, 31, 32, 38, 42, 49, 56, 78, 105, 115] and the references therein. This list does not claim to be exhaustive. Most of these articles consider space–time white noise as the driving process.

Finally we give a short overview of the papers contained in this thesis, indicating the main results.

1.1 Numerical approximation of differential equations

In the theory of ordinary differential equations (ODE's) and partial differential equations (PDE's) different techniques to approximate equations have been studied thoroughly, were emphasis is given to stability, consistency and convergence.

A discretization method transforms finding a solution to a differential equation into a finite dimensional problem. Arguably, the most popular numerical methods for the approximation of the solution of a PDE are: finite difference methods (FDM's), finite element methods (FEM's) and spectral methods. FDM's use fixed, equidistant grid points to convert differential operators into discrete ones using neighboring points. They evaluate candidates for a solution at these points. FDM's can be subdivided in explicit and implicit schemes. The former are in general less stable, but simpler to implement. Among the implicit ones, Crank–Nicolson schemes are known to be unconditionally stable. As a drawback, at every step in time a system of equations has to be solved numerically. Due to the fact that most FDM's are defined over an equidistant mesh, they may not be suitable for some problems. FEM's and spectral methods approximate the candidate functions in terms of linear combinations of linearly independent test (or basis) functions. This approach does not require equidistant discretizations. Moreover these methods are known to perform much better in terms of stability and convergence, especially if the solution lacks smoothness. However, implementation is a more daunting task than in the case of FDM's. As an example, the backward difference for the first order derivative of some suitable function u over the discretization $a = x_0 < x_1 < \ldots < x_N = b$ with $x_i = i\Delta x$ and $[a, b] \subset \mathbb{R}$ reads

$$\left(\frac{\partial u}{\partial x}\right)_i \approx \frac{u_i - u_{i-1}}{\Delta x}.$$

In contrast, to use a FEM for the same example one chooses a grid (not necessary equidistant) and a family of finite element spaces. Each finite element space has a basis of piecewise polynomial shape functions. The FEM on an equidistant grid with piecewise linear shape functions coincides with the FDM. In this sense FEM's represent a generalization of FDM's.

The literature on numerical methods for PDE's is vast. The FDM was introduced by Thom (see [108]) in the 1920's as *the method of square* to solve nonlinear hydrodynamic equations. Soon after, the fundamental work of Courant, Friedrichs and Lewy (1928) ([35]) on the solution of physical problems with finite differences appeared. It had a great impact on the further development of numerical methods and their stability analysis for the approximation of partial differential equations. FDM's for elliptic problems were first studied in the 1930 work of Gerschgorin ([53]). During the 1950's the work on time dependent problems picked up and grew rapidly with the introduction of computers. The first to study parabolic problems was, to the author's knowledge, John (1952) in [71]. Furthermore the work of Friedrichs, Lax and Wendroff should be mentioned. For a general introduction to finite differences we refer the reader to [45, 96].

The origins of the FEM can be traced back to the 1956 paper of Turner, Clough, Martin and Topp ([111]). However, the variational formulation of a boundary value problem originated much earlier. The work of Lord Raleigh ([95]), Ritz ([98]) and Galerkin ([50]) should be mentioned here. In the 1970's FEM's were applied to the approximation of the solution of partial differential equations. See, among others, the works of Fujita & Mizutani ([47]), Ushijima ([112]) and Zlámal ([120], [119],...) on parabolic PDE's, and those of Ciarlet ([33],...) and Nedoma ([84]). For an overview on FEM we refer the reader to [34, 23] or the recent works [64, 109].

For time–dependent, initial–boundary value problems, approximation can be done independently for space and time. For the FDM the same technique (probably with different accuracies) is used in both time and space; whereas for a FEM the equation is stated in its weak form, which is then discretized in a finite dimensional space. For the approximation in time either a FEM or a finite difference method can be introduced. Different stability results can be reached with varying methods, an Euler scheme is conditionally stable while a Crank–Nicolson scheme performs with unconditional stability. However, only for certain PDE's the best possible order of convergence can be achieved with a FEM. Finite element methods for most parabolic equations converge with optimal rate due to the smoothing effect of the second order differential operator. This is the case even if the initial data is not smooth or if noise is added at certain points in time. For first order hyperbolic equations the situation is quite different. Optimal convergence with a Galerkin approximation is only attainable if the solution is smooth enough. To address this issue, an additional diffusion term is added in the discretization of the variational form, which leads to Petrov–Galerkin methods.

The list of methods provided above is not comprehensive. However, the methods mentioned are among the most common. Collocation methods should not go unmentioned, as well as spectral methods. In the latter one uses globally defined functions rather than piecewise polynomials as approximating functions.

1.2 Numerical approximation of stochastic differential equations

Stochastic differential equations play a central role in financial mathematics. The benchmark model for stock prices is a geometric Brownian motion and numerous models for fixed income markets are based on Ornstein–Uhlenbeck processes. The quest for stable numerical methods arises somewhat naturally.

The approximation of stochastic differential equations (SDE's) is a fairly well studied topic (see [60, 65]). One should mention here especially the contributions of Talay ([93, 86, 106],...) and Platen ([74, 89],...). We give a brief introduction to approximation of stochastic differential equations, where we restrict ourselves to the Brownian case. Fur further reading on the topic, we refer to [66, 74, 24, 78] and the references therein.

The Euler-Maruyama approximation is one of the simpler discetization schemes for an Itô–Process. Consider the real–valued process $X(t)$ with $t \in [0, T]$ satisfying the stochastic differential equation

$$dX(t) = a(X(t))dt + b(X(t))dW(t)$$

or in integral form

$$X(t) = X(0) + \int_0^t a(X(s))ds + \int_0^t b(X(s))dW(s). \tag{1.2.1}$$

We introduce the (equidistant) time discretization via $0 = t_0 < t_1 < \ldots < t_n = T$, where k denotes the length of each interval $[t_i, t_{i+1}]$ for $i = 0, \ldots, n-1$. The Euler-Maruyama scheme then reads

$$X^{n+1} = X^n + a(X^n)k + b(X^n)\Delta W$$

where $\Delta W = W(t_{n+1}) - W(t_n)$ denotes the Brownian increment on $[t_n, t_{n+1}]$.

It is shown in [74] that this scheme has convergence order of 0.5, in other words of $\sqrt{k}$. The Euler-Maruyama scheme gives adequate numerical results if the drift and diffusion coefficients are nearly constant, or if the noise is additive and a and b are smooth enough. In general, however, the Euler–Maruyama scheme does not produce satisfactory results and higher order schemes, as the Milstein scheme, should be used.

The Milstein scheme has convergence order of 1 (or k). For Equation (1.2.1) it reads

$$\begin{aligned} X(t) = X(0) &+ \int_0^t a\big(X(0) + \int_0^s a(X(r))dr + \int_0^s b(X(r))dW(r)\big)\big)ds \\ &+ \int_0^t b\big(X(0) + \int_0^s a(X(r))dr + \int_0^s b(X(r))dW(r)\big)dW(s) \\ &= X(0) + a(X(0))\int_0^t ds + b(X(0))\int_0^t ds \\ &+ \int_0^t a\big(\int_0^s a(X(r))dr + \int_0^s b(X(r))dW(r)\big)ds \\ &+ \int_0^t b\big(\int_0^s a(X(r))dr + \int_0^s b(X(r))dW(r)\big)dW(s). \end{aligned}$$

We further approximate X by a recursive procedure à la Picard–Lindelöf. The approximation of the dynamics above is given by

$$X^{n+1} = X^n + a(X^n)k + b(X^n)k + a(\hat{X}^n)k + b(\hat{X}^n)\Delta W$$

with

$$\hat{X}^n = a(X^n)k + b(X^n)\Delta W.$$

Higher order schemes can be derived in the same manner.

1.3 Stochastic Integration with respect to Hilbert–space valued processes

There are various approaches to the theory of stochastic partial differential equations (SPDE) and their analysis. Some of them are the *mild solution approach* (see [37, 88]), the *variational approach* (see [85, 77, 100, 57, 58]) and the *martingale measure approach* (see [114]). We mainly use the first approach, which is equivalent to the semigroup approach in PDE theory. We define SPDE's in integral form, which requires integration theory for Hilbert–space valued Itô integrals. Below we give an introduction to Hilbert–space valued Lévy processes, since they play a central role in many applications, and they are not as broadly known as their Brownian counterpart. However, for more generality, we discuss the integration theory under the broader scope of Hilbert–space valued square integrable martingales. Most of this section is based on [88]. For a further introduction to Hilbert–space valued stochastic processes we refer the reader to [37, 30, 92, 28]. From this point on $(U, (\cdot,\cdot)_U)$ is a Hilbert space with corresponding Borel sigma algebra $\mathcal{U}$, and $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ a filtered probability space satisfying the usual conditions.

Definition 1.3.1. A stochastic process $(L = L(t), t \geq 0)$ with values in U is called Lévy-Process if the following conditions are satisfied

i The $(U,\mathcal{U})$-valued random variables $L(t) - L(s)$ and $L(v) - L(u)$ for all $0 \leq u < v < s < t < +\infty$ are independent

- ii L has stationary increments, i.e. the law $\mathcal{L}(L(t)-L(s))$ of $L(t)-L(s)$ depends only on $t-s$ for $t>s$
- iii $L(0)=0$
- iv L is stochastically continuous, i.e. for every $\epsilon>0$ and $0\leq s<t$: $\lim_{s\to t} P(\|L(t)-L(s)\|_U>\epsilon)=0$.

If we denote by μ_t the law of $L(t)$, the family $\{\mu_t\}$ of measures form a convolution semigroup of measures. Note that every Lévy-Process is also a Markov-Process.

Theorem 1.3.2. *Every Lévy process has a càdlàg modification.*

Definition 1.3.3. Let $L(t-):=\lim_{s\uparrow t} L(s)$, then for $t\geq 0$ $\Delta L(t):=L(t)-L(t-)$ is called the process of the jumps of L.

Lévy processes can be built from Poisson and Wiener processes, so we mention some basic properties of Poisson processes below. For a further discussion of Wiener processes in Hilbert spaces we refer the reader to [37].

For $a\in[0,+\infty)$, $\mathcal{P}(a)$ denotes the Poisson distribution with parameter a, that is $\mathcal{P}(a)(k)=(\frac{a^k}{k!})\exp(-a)$ for $k=0,1,....$

Definition 1.3.4. A Poisson process with intensity a is a Lévy process $\pi=(\pi(t),t\geq 0)$ such that $\pi(t)$ has the Poisson distribution $\mathcal{P}(at)$, for every $t\geq 0$.

Remark. The value $\pi(t)$ is the number of events that have occurred before or at time t.

A Poisson process can be expressed as

$$\pi(t)=\begin{cases}0 & \text{if } t<Z_1\\ k & \text{if } t\in[Z_1+\cdots+Z_k, Z_1+\cdots+Z_{k+1}),\end{cases}$$

where $\{Z_n\}$ is a sequence of independent, exponentially distributed random variables with parameter a. Vice versa, for a given Poisson process, there exists a sequence $\{Z_n\}$ of independent, exponentially distributed random variables such that the formula above holds. It can be shown that if π is a Poisson process then it has only jump size 1 and conversely any $\mathbb{Z}_+$–valued Lévy process with only jump size 1 is a Poisson process.

Since (see Theorem 1.3.9) all Lévy processes are built out of a deterministic part, a Wiener process, a compound Poisson process and a compensated compound Poisson process, we require the definition of a compound Poisson process in a Hilbert space.

Definition 1.3.5. A *compound Poisson process* with Lévy measure (jump intensity measure) ν is a càdlàg Lévy process L satisfying

$$P(L(t)\in\Gamma)=\mathrm{e}^{-\nu(U)t}\sum_{k=0}^{\infty}\frac{t^k}{k!}\nu^{*k}(\Gamma),\ \forall t\geq 0.$$

Here $\Gamma\in\mathcal{B}(U)$ and ν is a finite measure on the Hilbert space U, such that $\nu(0)=0$ ($\nu^0=\delta_0$).

For applications it is of considerable interest to construct a compound Poisson process, when ν is given.

Theorem 1.3.6. *Let ν be a finite measure with support on $U \setminus \{0\}$ and $a = \nu(U)$. If then $Z_1, Z_2, \ldots$ are U–valued, iid random variables with law equal to $a^{-1}\nu$ and $(\Pi(t), t \geq 0)$ is a Poisson process with intensity a independent of $Z_1, Z_2, \ldots$. Then*

$$L(t) = \sum_{j=1}^{\Pi(t)} Z_j \tag{1.3.1}$$

is a compound Poisson process with jump measure ν.

The converse of the theorem above is true as well. Thus, for a given compound Poisson process we can always find a sequence of random variables $\{Z_i\}$ and a Poisson process Π as to express L as in equation (1.3.1). This yields a method to simulate a compound poisson process.

Definition 1.3.7. The Poisson random measure corresponding to L is

$$\pi([0,t],\Gamma) := \#\{s \leq t : \Delta L(s) \in \Gamma\} \qquad \Gamma \in \mathcal{B}(U \setminus \{0\}),$$

and the compensated Poisson random measure is

$$\hat{\pi}([0,t],\Gamma) := \pi([0,t],\Gamma) - t\nu(\Gamma), \qquad t \geq 0,\ \Gamma \in \mathcal{B}(U \setminus \{0\}).$$

One can show that $(\pi([0,t],\Gamma), t \geq 0)$ is a Poisson process with intensity $\nu(\Gamma)$ and that the process $(\hat{\pi}([0,t],\Gamma), t \geq 0)$ is a martingale with respect to the filtration generated by $\pi([0,s],\Gamma)$ for $0 \leq s < t$ and $\Gamma \in \mathcal{B}(U \setminus \{0\})$.

Proposition 1.3.8. *Let L be a compound Poisson process with jump intensity measure ν. If L is integrable, i.e. $\int_U |y|_U \nu(dy) < \infty$, its mean is given by*

$$\mathbb{E}L(t) = t \int_U y\nu(dy).$$

The compensated compound Poisson process $\hat{L}(t) := L(t) - \mathbb{E}L(t)$, $t \geq 0$, is square integrable if $\int_U |y|_U^2 \nu(dy) < \infty$, and then the variance is given by

$$\mathbb{E}|\hat{L}(t)|_U^2 = t \int_U |y|_U^2 \nu(dy),$$

and for all $x, \tilde{x} \in U$

$$\mathbb{E}(\hat{L}(t), x)_U (\hat{L}(t), \tilde{x})_U = t \int_U (x,y)_U (\tilde{x},y)_U \nu(dy).$$

With the above results in hand we can derive the Lévy–Khinchin–Formula, which provides some insight on the construction of Lévy processes.

Assume L is a càdlàg Lévy process on U. We define the Lévy process with jump size 1, which is as stated before also a Poisson process, as

$$\pi_A(t) := \sum_{s \le t} \chi_A(\Delta L(s)),$$

here $t \geq 0$ and A is a Borel set separated from 0. The expectation of this process is

$$\mathbb{E}\pi_A(t) = t\,\mathbb{E}\pi_A(1) = t\nu(A)$$

where ν is a finite measure also separated from 0. We define the Lévy process L_A as follows:

$$L_A(t) := \sum_{s \le t} \chi_A(\Delta L(s))\Delta L(s).$$

Theorem 1.3.9. *If ν is a jump measure corresponding to a Lévy process then*

$$\int_U (|y|_U^2 \wedge 1)\nu(dy) < \infty. \tag{1.3.2}$$

Every Lévy process has the following representation:

$$L(t) = at + W(t) + \sum_{k=1}^{\infty}(L_{A_k}(t) - t\int_{A_k} y\nu(dy)) + L_{A_0}(t) \tag{1.3.3}$$

where $A_0 := \{x : |x|_U \geq r_0\}$, $A_k := \{x : r_k \leq |x|_U < r_{k-1}\}$ and r_k is a sequence decreasing to 0. All processes here are independent and the series converges uniformly on each bounded subinterval of $[0, \infty)$ $P - a.s.$.

We can write the decomposition in a slightly different form:

$$L(t) = at + W(t) + \sum_{n=1}^{\infty} L_n(t) + L_0(t), \qquad t \geq 0, \tag{1.3.4}$$

here

$$L_n(t) := L_{A_n}(t) - t\int_{A_n} y\nu(dy)$$

are independent compensated compound Poisson processes, each having jump intensity measure $\chi_{\{r_{n+1} \leq |y|_U < r_n\}}(y)\nu(dy)$. L_0 is a compound Poisson process with jump intensity measure $\chi_{\{|y|_U \geq r_0\}}(y)\nu(dy)$. From the previous theorem we can directly derive the Lévy-Khinchin formula:

Theorem 1.3.10. *Let $L_1^+(U)$ be the space of all nuclear, positive semi-definite operators on U, and ν a non-negative measure concentrated on $U \setminus \{0\}$ satisfying the integrability condition in the decomposition. If $a \in U$ and $Q \in L_1^+(U)$, then there is a convolution semigroup of measures $\{\mu_t\}$ such that*

$$\int_u e^{i(x,y)_U}\mu_t(dy) = e^{-t\psi(x)}, \tag{1.3.5}$$

where

$$\psi(x) = -i(a,x)_U + \frac{1}{2}(Qx,x)_U + \int_U (1 - e^{i(x,y)_U} + \chi_{\{|y|_U<1\}}(y) i(x,y)_U)\nu(dy). \qquad (1.3.6)$$

The reverse implication is true as well.

Definition 1.3.11. The measure ν above is called the Lévy measure or the jump intensity measure of L or equivalent $\{\mu_t\}$.

Suppose that L is a centered, integrable Lévy process, then L is a martingale with respect to $\mathcal{F}_t$. If we compute the covariance of a square integrable Lévy process, we get:
There exists a symmetric, non-negative and nuclear operator Q, such that, for all $t, s \geq 0$ and $x, y \in U$, we get for the covariance

$$\mathbb{E}(L(t),x)_U(L(s),y)_U = t \wedge s(Qx,y)_U,$$

and if we assume $0 \leq s < t$

$$\mathbb{E}(L(t),x)_U(L(s),y)_U = (Q(s)x,y)_U.$$

As for the variance of $L(t)$, we have

$$\mathbb{E}\|L(t)\|_U^2 = t \operatorname{Tr}(Q).$$

Since Q does not change with time it can be seen as the spatial covariance of L.

Theorem 1.3.12. *A Lévy process on a Hilbert space is square integrable if and only if*

$$\int_U |y|_U^2 \nu(dy) < \infty, \qquad (1.3.7)$$

additionally L admits the Lévy–Khinchin–Representation. If Q_0 and Q_1 are respectively the covariance operators of the Wiener part and of the Jump part, then the covariance of L is given by $Q_0 + Q_1$.

Remark. If L is a centered, square integrable Lévy-process with covariance operator Q, then the processes $|L(t)|_U^2 - t\operatorname{Tr}(Q)$ and $L(t) \otimes L(t) - tQ$ are U-valued and $L_1(U)$-valued martingales.

We can generalize these results to square integrable martingales. To this end we denote the space of all càdlàg square integrable martingales on U with respect to $\mathcal{F}_t$ by $\mathcal{M}^2(U)$.

Definition 1.3.13. Let $M, N \in \mathcal{M}^2(U)$, we define $\langle M, N\rangle$ as the unique predictable process for which

$$(M_t, N_t)_U - \langle M, N\rangle_t \qquad (1.3.8)$$

is a martingale.

By the Doob-Meyer Decomposition the angle bracket process always exists. Expression (1.3.8) is the generalization of $|L(t)|_U^2 - t\operatorname{Tr}(Q)$ for the case of Lévy processes. We introduce the operator angle bracket process subsequently to the following Lemma.

Lemma 1.3.14. *If $M, N \in \mathcal{M}^2(U)$, then there exists a predictable process $(q(s), s \geq 0)$ such that*

$$\langle M, N\rangle_t = \int_0^t q(s) d(\langle M, M\rangle_s + \langle N, N\rangle_s).$$

Theorem 1.3.15. *Let $M \in \mathcal{M}^2(U)$, then there exists a unique, right continuous process $(\langle\langle M, M\rangle\rangle_t, t \geq 0)$ taking values in $L_1^+(U)$ such that $\langle\langle M, M\rangle\rangle_0 = 0$ and the process $(M(t) \otimes M(t) - \langle\langle M, M\rangle\rangle_t, t \geq 0)$ is an $L_1(U)$–valued martingale. Moreover, there exists a predictable $L_1^+(U)$-valued process $(Q_t, t \geq 0)$ with*

$$\langle\langle M, M\rangle\rangle_t = \int_0^t Q_s d\langle M, M\rangle_s.$$

The process Q is called the martingale covariance of M.

Consider the following class of martingales

$$\mathcal{C} := \{M \in \mathcal{M}^2(U) : \exists Q \in L_1^+ \text{ such that } \forall t \geq s \geq 0,\ \langle\langle M, M\rangle\rangle_t - \langle\langle M, M\rangle\rangle_s \leq (t-s)Q\}.$$

Since $Q \in L_1^+(U)$ there exists an orthonormal basis $\{e_n\}$ of U consisting of eigenvectors of Q. This yields the representation $Qe_n = \gamma_n e_n$, where γ_n is the eigenvalue corresponding to e_n. The square root of Q is defined as

$$Q^{\frac{1}{2}} x := \sum_n (x, e_n)_U \gamma_n^{\frac{1}{2}} e_n, \qquad x \in U.$$

$Q^{-\frac{1}{2}}$ is the pseudo inverse to $Q^{\frac{1}{2}}$. Let us denote by $(\mathcal{H}, (\cdot,\cdot)_{\mathcal{H}})$ the Hilbert space defined by $\mathcal{H} = Q^{\frac{1}{2}}(U)$ endowed with the inner product $(x, y)_{\mathcal{H}} = (Q^{-\frac{1}{2}} x, Q^{-\frac{1}{2}} y)_U$ where $x, y \in \mathcal{H}$.

Let $L_{HS}(\mathcal{H}, H)$ refer to the space of all Hilbert-Schmidt operators from $\mathcal{H}$ to H. We denote by $\hat{\mathcal{S}}(U, H)$ the class of all simple processes with values in $L(U, H)$. For $\varphi \in \hat{\mathcal{S}}(U, H)$ we define for all $t \geq 0$

$$I_t(\varphi) := \sum_{j=0}^{m-1} \chi_{B_j} \varphi_j (M(t_{j+1} \wedge t) - M(t_j \wedge t)).$$

For nonnegative numbers $t_i < t_{i+1}$ for $i = 1, \ldots, m-1$ and $B_i \in \mathcal{F}_{t_i}$. We can prove for simple processes the Itô–Isometry

$$\begin{aligned}\mathbb{E}\|I_t(\varphi)\|_H^2 &= \mathbb{E}\int_0^t \|\varphi(s) Q_s^{1/2}\|^2_{L_{HS}(U,H)} d\langle M, M\rangle_s \\ &=: \|\varphi\|_t^2.\end{aligned}$$

The completion of $\hat{\mathcal{S}}$ with respect to $\|\cdot\|_T$ is denoted by $\mathcal{L}_T^2(H)$ for $T < \infty$. We can extend I_t to a continuous linear operator from $(\mathcal{L}_T^2(H), \|\cdot\|_T)$ to $L^2(\Omega.\mathcal{F}, \mathbb{P}; H)$ and for $\varphi \in \mathcal{L}_T^2(H)$ $\mathbb{E}\|I_T(\varphi)\|_H^2 = \|\varphi\|_T^2$. Furthermore, $(I_t(\varphi), t \in [0, T])$ is a square integrable and mean–square continuous H–valued martingale.

If $M \in \mathcal{C}$ and let $\mathcal{F}_{[0,T]}$ denotes the σ–field of predictable sets in $\Omega \times [0, T]$, then we have the following Proposition

Proposition 1.3.16. *Let $\mathbb{L}^2_{\mathcal{H},T}(H) = L^2(\Omega \times [0,T], \mathcal{F}_{[0,T]}, \mathbb{P}dt; L_{HS}(\mathcal{H},H)) \subseteq \mathcal{L}^2_T$ be the space of integrands, then for every $X \in \mathbb{L}^2_{\mathcal{H},T}(H)$*

$$\mathbb{E}\left|\int_0^t X(s)\,dM(s)\right|_H^2 \leq \mathbb{E}\int_0^t \|X(s)\|^2_{L_{HS}(\mathcal{H},H)}\,ds. \tag{1.3.9}$$

Integration with respect to a general Lévy processes $L(t) = mt + M(t) + P(t)$ where P is a (not necessarily square integrable) compound Poisson process is also possible. For further details we refer to [88], Chapter 8.6.2.

1.4 An overview

In this section we give an overview of the papers that constitute the main body of this work. Even though the first and fourth papers could have been grouped together, we opted for a chronologically consistent ordering. The reasoning for this being that we believe this provides a clearer view of how one paper motivated the subsequent one. We conclude this section with a (non–comprehensive) list of further questions.

1.4.1 Hedging of spatial temperature risk with market-traded futures

The aim of this paper is to define a synthetic temperature futures, i.e. a temperature derivative composed of market–traded futures. This is done to provide (temperature) risk–hedging tools for places where a market product is not readily available. The structure of a synthetic temperature futures is given in Proposition 2.3.1. Here, a linear system containing as data the underlying temperatures that generate currently traded assets is solved. The solution to this system provides the best hedge in the mean–square sense. In the Appendix we explicitly calculate an example using some popular temperature indices.

The second contribution of this paper is an algorithm that provides a fast way to simulate temperature over a domain given temperature data on some fixed measuring stations. The approximation of the driving random field cannot be done via finite difference approximation, since the problem does not allow for too many nodal points of the approximation. Instead, a spline method is appropriate here.

1.4.2 A Finite Element Method for martingale–driven Stochastic Partial Differential Equations

The main motivation behind this paper was the simulation of stochastic heat equations. This was of interest to us since temperature is better described by a diffusion process. In order to have a more general result we directly focus on stochastic partial differential equations driven by square–integrable martingales. To approximate first parabolic and then first order hyperbolic equations we use a Galerkin approximation in space and a backward Euler approximation in time. We derive error estimates in mean–square sense for: the semidiscrete parabolic case (Theorem 3.3.2); the corresponding fully discrete case (Theorem 3.3.4); the semidiscrete first order hyperbolic case (Theorem 3.4.1); the corresponding fully discrete case (Theorem 3.4.2). It should be noted that while the convergence results for the parabolic case are optimal (in the space approximation), such is not the case in the hyperbolic setting.

1.4.3 Almost sure convergence of a Galerkin–Milstein Approximation for Stochastic Partial Differential Equations

In a certain sense this paper extends the results of Section 3. We use a Galerkin approach in space and a Crank–Nicolson Milstein scheme in time to approximate the infinite dimensional Zakai equation. The latter is related to the parabolic case mentioned above. We prove almost sure convergence of the semidiscrete approximation in Theorem 4.4.1 and of the fully discrete approximation in Theorem 4.5.1. Mean–square convergence is then a direct corollary. The Crank–Nicolson Milstein scheme converges faster than the previously used Euler–Maruyama scheme, but of course it carries heavier computational weight.

1.4.4 Forward dynamics in energy markets – an infinite dimensional framework

In this paper we present an infinite dimensional model for forwards in energy markets. We work within the Heath–Jarrow–Morton paradigm, in similar fashion to what is done in interest rate theory. We define the notion of a *flow forward*, which captures the fact that electricity is to be provided over a future time interval. Two infinite dimensional approaches are presented. The first one derives the price of a flow forward from the forward price; whereas the second one deals with flow forwards directly. For the no–arbitrage condition of the direct approach we give a novel characterization of the structure of the admissible forward prices.

As is the case for the fixed income theory, we deal with Hilbert–space valued first order hyperbolic equations. Making use of the results contained in Section 3, we simulate as an example a standard model for electricity forwards. To this end we employ a Petrov–Galerkin method and consider a correlated NIG–Field as the source of randomness.

1.4.5 Notes on numerical aspects for Finite Element Methods for Stochastic Partial Differential Equations

We were motivated to write this set of notes by the obstacles we had to overcome when implementing the Finite Element Methods described above. For implementations one has to deal with finite dimensional realizations of the driving noise. We use the eigenfunctions of the covariance operator as a basis for the Hilbert space where the noise is defined. Since the sequence of the corresponding eigenvalues decreases to 0, it is natural to truncate the sequence of eigenfunctions. We provide a result on the convergence of the finite dimensional realization of the infinite dimensional random field in Lemma 6.2.1.

To showcase the methodology, we provide simulations on both parabolic and first order hyperbolic problems. We give explicit details of the Finite Element Method, including the shape functions.

1.4.6 Further questions

As it seems to always be the case with scientific research, more problems surfaced during this work than were solved. We mention a few here.

The approach we chose via pathwise approximations is not the only possible choice. There is an error we make when calculating the mean and the variance from simulations by "approximating" the probability space. One could think of methods that directly approximate the law of the solution to the SPDE. Depending on the number of simulations and the size of the problem computational time grows quite rapidly.

Another question is the approximation of the covariance operator. In our simulations we approximate the eigenvalues and eigenfunctions of the covariance operator by a finite dimensional matrix. Investigations on this error are not included here. One could think of more sophisticated ways to truncate the sequence of eigenvalues and eigenfunctions. This should be done for every kernel function separately.

In the second paper we calculate our estimates for functions in $\dot{H}^r$, this could be generalized to a larger class of functions, since there are examples where the solution to the SPDE is not in $\dot{H}^r$. It should be noted that one derives the same convergence results.

As briefly mentioned in the second paper, the Galerkin approximations might not result in stable simulations. A further investigations in other methods for first–order hyperbolic stochastic differential equations would be interesting.

A very challenging question is the solution of non–linear stochastic partial differential equations. Solution theory is already very limited and in general non-existent . Numerical approximations could provide, as they do in PDE theory, some insight on the behavior of solutions.

Bibliography

[1] P. Alaton, B. Djehiche, and D. Stillberger, *On modelling and pricing weather derivatives*, Applied Mathematical Finance, 9 (2002), pp. 1–20.

[2] E. J. Allen, S. J. Novosel, and Z. Zhang, *Finite element and difference approximation of some linear stochastic partial differential equations*, Stochastics and Stochastics Reports, 64 (1998), pp. 117–142.

[3] J. H. Argyris and S. Kelsey, *Energy theorems and structural analysis*, Aircraft Eingeneering, 26 (1960).

[4] N. Audet, P. Heiskanen, J. Keppo, and I. Vehviläinen, *Modelling of electricity forward curve dynamics in the nordic market*, Modelling Prices in Competitive Markets, (2004), pp. 252–265.

[5] P. Auscher and P. Tchamitchian, *Square roots of elliptic second order divergence operators on strongly Lipschitz domains: L^2 theory*, J. Anal. Math., 90 (2003), pp. 1–12.

[6] G. Barbançon, *Théorème de newton pour les fonctions de classe C^r*, Annales scientifiques de l'É.N.S, 4^e série, 5 (1972), pp. 435–457.

[7] A. Barth, *A finite element method for martingale–driven stochastic partial differential equations.* preprint, January 2009. submitted.

[8] A. Barth and F. E. Benth, *Forward dynamics in energy markets – an infinite dimensional framework.* preprint, 2009. submitted.

[9] A. Barth and A. Lang, *Almost sure convergence of a Galerkin–Milstein approximation for stochastic partial differential equations.* preprint, 2009. submitted.

[10] ——, *Notes on numerical aspects of finite element methods for stochastic partial differential equations.* preprint, 2009. submitted.

[11] F. E. Benth, l. Cartea, and R. Kiesel, *Pricing forward contracts in power markets by the certainty equivalence principle: Explaining the sign of the market risk premium*, Journal of Banking & Finance, 32 (2008), pp. 2006–2021.

[12] F. E. Benth, J. Kallsen, and T. Meyer-Brandis, *A non-gaussian ornstein-uhlenbeck process for electricity spot price modeling and derivatives pricing*, Applied Mathematical Finance, 14 (2007), pp. 153–169.

[13] F. E. Benth and S. Koekebakker, *Stochastic modeling of financial electricity contracts*, Energy Economics, 30 (2008), pp. 1116–1157.

[14] F. E. Benth and T. Meyer-Brandis, *The information premium for non-storable commodities.* To appear in J. Energy Markets, 2008.

[15] F. E. Benth and J. Saltyte-Benth, *Stochastic modelling of temperature variations with a view towards weather derivatives*, Applied Mathematical Finance, 12 (2005), pp. 53–85.

[16] ——, *The volatility of temperature and pricing of weather derivatives*, Quantitative Finance, 7 (2007), pp. 553–561.

[17] F. E. Benth, J. Saltyte-Benth, and S. Koekebakker, *Putting a price on temperature*, Scandinavian Journal of Statistics, 34 (2007), pp. 746–767.

[18] ——, *Stochastic modelling of electricity and related markets*, World Scientific, Advanced Series on Statistical Science and Applied Probability - Vol. 11, 2008.

[19] C. Bernhardt, C. Klüppelberg, and T. Meyer-Brandis, *Estimating high quantiles for electricity prices by stable linear models*, Journal of Energy Markets, 1 (2008), pp. 3–19.

[20] H. Bessaih and H. Schurz, *Upper bounds on the rate of convergence of truncated stochastic infinite-dimensional differential systems with H-regular noise.*, J. Comput. Appl. Math., 208 (2007), pp. 354–361.

[21] T. Björk and A. Gombani, *Minimal realizations of interest rate models*, Finance and Stochastics, 3 (1999), pp. 413–432.

[22] T. Björk, Y. Kabanov, and W. Rungaldier, *Bond market structure in the presence of marked point process*, Math. Finance, (1997).

[23] S. Brenner and L. Scott, *The mathematical theory of finite element methods*, Springer, 1994.

[24] D. Brigo and F. Mercurio, *Interest Rate Models - Theory and Practice: With Smile, Inflation and Credit (Springer Finance)*, Springer, 2006.

[25] D. C. Brody, J. Syroka, and M. Zervos, *Dynamical pricing of weather derivatives*, Quantitative Finance, 2 (2002), pp. 189–198.

[26] S. D. Campbell and F. X. Diebold, *Weather forecasting for weather derivatives*, Journal of the American Statistical Association, 100 (2000), pp. 6–16.

[27] T. Caraballo and P. E. Kloeden, *The pathwise numerical approximation of stationary solutions of semilinear stochastic evolution equations*, Appl. Math. Optimization, 54 (2006), pp. 401–415.

[28] R. A. Carmona and B. L. Rozovskii, *Stochastic Partial Differential Equations: Six Perspectives (Mathematical Surveys and Monographs)*, American Mathematical Society, 1999.

[29] R. A. Carmona and M. R. Tehranchi, *Interest rate models: an infinite dimensional stochastic analysis perspective.*, Springer Finance. Berlin: Springer, 2006.

[30] P.-L. Chow, *Stochastic partial differential equations*, Chapman & Hall/CRC Applied Mathematics and Nonlinear Science Series. Boca Raton, FL: Chapman & Hall/CRC, 2007.

[31] P.-L. Chow and J.-L. Jiang, *Almost sure convergence of some approximate solutions for random parabolic equations.* Random partial differential equations, Proc. Conf., Oberwolfach/Ger. 1989, ISNM 102, 45–54., 1991.

[32] P.-L. Chow, J. L. Jiang, and J. L. Menaldi, *Pathwise convergence of approximate solutions to Zakai's equation in a bounded domain.* Da Prato, G. (ed.) et al., Stochastic partial differential equations and applications. Proceedings of the third meeting on stochastic partial differential equations and applications held at Villa Madruzzo, Trento, Italy, January 1990. Harlow: Longman Scientific & Technical. Pitman Res. Notes Math. Ser. 268, 111–123, 1992.

[33] P. G. Ciarlet, *The Finite Elements Method for Elliptic Problems*, North-Holland, 1977.

[34] ——, *The finite element method for elliptic problems*, North–Holland, Amsterdam, New York, Oxford, 1978.

[35] R. Courant, K. Friedrichs, and H. Lewy, *Über die partiellen differenzengleichungen der mathematischen physik*, Math. Ann., 100 (1928), pp. 32–74.

[36] D. Crisan, J. G. Gaines, and T. Lyons, *Convergence of a branching particle method to the solution of the Zakai equation*, SIAM Journal on Applied Mathematics, 58 (1998), pp. 1568–1590.

[37] G. Da Prato and J. Zabczyk, *Stochastic equations in infinite dimensions*, Encyclopedia of Mathematics and Its Applications. 44. Cambridge: Cambridge University Press, 1992.

[38] A. M. Davie and J. G. Gaines, *Convergence of numerical schemes for the solution of parabolic stochastic partial differential equations*, Mathematics of Computation, 70 (2001), pp. 121–134.

[39] M. Davis, *Pricing weather derivatives by marginal value*, Quantitative Finance, 1 (2001), pp. 305–308.

[40] J. E. Dendy, *Two methods of Galerkin type achieving optimum L^2 rates of convergence for first order hyperbolics*, Siam journal of num. anal., 11 (1974), pp. 637–653.

[41] F. Dornier and M. Queruel, *Caution to the wind*, Energy Power Risk Manag. Weather risk special report, (2000), pp. 30–32.

[42] Q. Du and T. Zhang, *Numerical approximation of some linear stochastic partial differential equations driven by special additive noises*, SIAM J. Numer. Anal., 40 (2002), pp. 1421–1445.

[43] I. Ekeland and E. Taflin, *A theory of bond portfolios.*, Ann. Appl. Probab., 15 (2005), pp. 1260–1305.

[44] K.-J. Engel and R. Nagel, *One-parameter semigroups for linear evolution equations.*, Graduate Texts in Mathematics. 194. Berlin: Springer, 2000.

[45] G. E. Forsythe and W. R. Wasow, *Finite-difference methods for partial differential equations*, Wiley, New York, 1967.

[46] D. Frestad, F. E. Benth, and S. Koekebakker, *Modeling the term structure dynamics in the nordic electricity swap market.* To appear in Energy Journal, 2009.

[47] H. Fujita and A. Mizutani, *On the finite element method for parabolic equations, i: Approximation of holomorphic semi-groups*, J. Math. Soc. Japan, 28 (1976), pp. 93–107.

[48] H. Fujita, N. Saito, and T. Suzuki, *Operator theory and numerical methods*, Studies in Mathematics and its Applications. 30. Amsterdam: North-Holland/ Elsevier, 2001.

[49] J. Gaines, *Numerical experiments with s(p)de's*, Stochastic Partial Differential Equations, 216 (1995), pp. 55–71.

[50] B. G. Galerkin, *Rods and plates. series occuring in various questions concerning the elastic equilibrium of rods and plates*, Vestnik Insz., 19 (1915), pp. 897–908.

[51] H. Geman, *Insurance and weather derivatives.*, RISK Books, 1999.

[52] H. Geman and M. P. Leonardi, *Alternative approaches to weather derivatives valuation*, Manag. Finance, 31 (2005), pp. 46–72.

[53] S. Geschgorin, *Fehlerabschätzungen für das differenzenverfahren zur lösung partieller differentialgleichungen*, Z. Angew. Math. Mech., 10 (1930), pp. 373–382.

[54] T. Gneiting, *Radial positive definite functions generated by Euclid's hat*, Journal of Multivariate Analysis, 69 (1999), pp. 88–119.

[55] C. Grossmann and H.-G. Roos, *Numerics of partial differential equations. (Numerik partieller Differentialgleichungen.)*, Teubner Studienbücher: Mathematik. Stuttgart: B. G. Teubner, 1992.

[56] I. Gyöngy, *Lattice approximations for stochastic quasi-linear parabolic partial differential equations driven by space-time white noise*, Potential Analysis, 11 (1999), pp. 1–37.

[57] I. Gyöngy and N. V. Krylov, *On stochastic equations with respect to semimartingales: I*, Stochastics, 4 (1980), pp. 1–21.

[58] ——, *On stochastic equations with respect to semimartingales: II. Itô formula in Banach spaces.*, Stochastics, 6 (1981), pp. 153–173.

[59] I. Gyöngy and D. Nualart, *Implicit scheme for quasi-linear parabolic partial differential equations perturbed by space-time white noise*, Stochastic Processes and their Applications, 58 (1995), pp. 57–72.

[60] E. Hausenblas, *Error analysis for approximation of stochastic differential equations driven by Poisson random measures*, SIAM J. Numer. Anal., 40 (2002), pp. 87–113.

[61] E. Hausenblas and J. Seidler, *A note on maximal inequality for stochastic convolutions*, Czech. Math. J., 51 (2001), pp. 785–790.

[62] ——, *Stochastic convolutions driven by martingales: Maximal inequalities and exponential integrability*, Stochastic Anal. Appl., 26 (2008), pp. 98–119.

[63] D. Heath, R. Jarrow, and A. Morton, *Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation*, Econometrica, 60 (1992), pp. 77–105.

[64] J. Hesthaven and T. Warburton, *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications*, Springer–Verlag, New York, 2008.

[65] D. J. Higham, *Mean-square and asymptotic stability of the stochastic theta method*, SIAM J. Numer. Anal., 38 (2000), pp. 753–769.

[66] ——, *An algorithmic introduction to numerical simulation of stochastic differential equations.*, SIAM Rev., 43 (2001), pp. 525–546.

[67] J. Hoffmann-Jørgensen and G. Pisier, *The law of large numbers and the central limit theorem in banach spaces*, Ann. Probability, 4 (1976), pp. 587–599.

[68] R. H. W. Hoppe, *Crank-Nicolsen-Galerkin approximation for Maxwell's equations*, Math. Meth. in the Appl. Sci., 4 (1982), pp. 123–130.

[69] J. Jakubowski and J. Zabczyk, *Exponential moments for HJM models with jumps*, Finance and Stochastics, 11 (2007), pp. 429–445.

[70] S. Jewson and A. Brix, *Weather Derivative Valuation*, Cambridge University Press, Apr. 2005.

[71] F. John, *On integration of parabolic equations by difference methods*, Comm. Pure Appl. Math., 5 (1952), pp. 155–211.

[72] S. Kantorovitz, *On Liu's analyticity criterion for semigroups.*, Semigroup Forum, 53 (1996), pp. 262–265.

[73] T. Kato, *Perturbation theory for linear operators. Corr. printing of the 2nd ed.*, Grundlehren der mathematischen Wissenschaften, 132. Berlin – Heidelberg – New York: Springer, 1980.

[74] P. E. Kloeden and E. Platen, *Numerical solution of stochastic differential equations*, Applications of Mathematics. 23. Berlin: Springer, 1992.

[75] P. E. Kloeden and S. Shott, *Linear-implicit strong schemes for Itô-Galerkin approximations of stochastic PDEs*, J. Appl. Math. Stochastic Anal., 14 (2001), pp. 47–53.

[76] S. Koekebakker and F. Ollmar, *Forward curve dynamics in the nordic electricity market*, Managerial Finance, 31 (2005), pp. 74–95.

[77] N. V. Krylov and B. L. Rozovskii, *Stochastic evolution equations*, Current problems in mathematics, 14 (1979), pp. 71–147.

[78] A. Lang, P.-L. Chow, and J. Potthoff, *Almost sure convergence of a semidiscrete Milstein scheme for SPDE's of Zakai type.* submitted, April 2009.

[79] S. Larsson and V. Thomee, *Partial Differential Equations with Numerical Methods*, Texts in Applied Mathematics, Springer, May 2003.

[80] C. Marinelli, *Local well-posedness of Musiela's SPDE with Lévy noise*, 2007.

[81] G. D. Masi, T. Björk, W. Runggaldier, and Y. Kabanov, *Towards a general theory of bond markets*, Finance and Stochastics, 1 (1997), pp. 141–174.

[82] R. J. Melosh, *Structural analysis of solids*, J. ASCE Struct. Div., ST4-89 (1963), p. 205223.

[83] M. Musiela, *Stochastic PDEs and the term structure model*, Journees Internationales de France IGRAFFI. La Baule, 69 (1993).

[84] J. Nedoma, *The finite elements solution to elliptic and parabolic equations using simplicial isoparametric elements*, R.A.I.R.O., 13 (1979), pp. 257–289.

[85] E. Pardoux, *Équations aux dérivées partielles stochastiques de type monotone*, Séminaire sur les Équations aux Dérivées Partielles (1974–1975), 3 (1975).

[86] E. Pardoux and D. Talay, *Approximation and simulation of solutions of SDE*, Acta Applicandae Math., 3 (1985), pp. 23–47.

[87] A. Pazy, *Semigroups of linear operators and applications to partial differential equations*, Applied Mathematical Sciences, 44. New York – Berlin – Heidelberg – Tokyo: Springer, 1983.

[88] S. Peszat and J. Zabczyk, *Stochastic partial differential equations with Lévy noise. An evolution equation approach*, Encyclopedia of Mathematics and Its Applications 113. Cambridge: Cambridge University Press, 2007.

[89] E. Platen, *An introduction to numerical methods for stochastic differential equations*, Acta Numer., 8 (1999), pp. 197–246.

[90] E. Platen and J. West, *A fair pricing approach to weather derivatives*, Financial Engineering and the Japanese Markets, 11 (2005), pp. 23–53.

[91] M. Powell, *The theory of radial basis function approximation*, Advances in Numerical Analysis II: Wavelets, Subdivision, Algorithms, and Radial Basis Functions, (1992), pp. 105–210.

[92] C. Prévôt and M. Röckner, *A concise course on stochastic partial differential equations*, Lecture Notes in Mathematics 1905. Berlin: Springer, 2007.

[93] P. Protter and D. Talay, *The Euler scheme for Lvy driven stochastic differential equations*, Annals of Probability, 25 (1997), pp. 393–423.

[94] A. Quarteroni and A. Valli, *Numerical approximation of partial differential equations*, Springer Series in Computational Mathematics. 23. Berlin: Springer, 1994.

[95] L. Rayleigh, *Theory of sound*, vol. I, II, Macmillan, London, 1894, 1896.

[96] R. D. Richtmyer and K. W. Morton, *Difference methods for initial-value problems*, Interscience Publishers, London, 1967.

[97] F. Riesz and B. Sz.-Nagy, *Functional analysis*, Dover Books on Advanced Mathematics, Dover Publications Inc., New York, 1990. Translated from the second French edition by Leo F. Boron, Reprint of the 1955 original.

[98] W. Ritz, *Über eine neue methode zur lösung gewisser variationsprobleme in der mathematischen physik*, J. Reine Angew. Math., 135 (1908), pp. 1–61.

[99] G. Rossmanith, *Das temperaturmodell von Benth und Saltyte-Benth und dessen anwendung auf standorte in deutschland*, master thesis, Munich University, Germany, 2007. (in German).

[100] B. L. Rozovskii, *Stochastic evolution equations*, Mathematics and its applications, 35 (1990).

[101] T. H. Rydberg, *The normal inverse gaussian Lévy process: simulation and approximation*, Stochastic Models, 13 (1997), pp. 887–910.

[102] J. Saltyte-Benth, F. E. Benth, and P. Jalinskas, *A spatial-temporal model for temperature with seasonal variance*, Journal of Applied Statistics, 34 (2007), pp. 823–841.

[103] W. Schoutens, *Levy Processes in Finance: Pricing Financial Derivatives*, Wiley Series in Probability and Statistics, Wiley, May 2003.

[104] E. S. Schwartz, *The stochastic behavior of commodity prices: Implications for valuation and hedging*, Journal of Finance, 52 (1997), pp. 923–73.

[105] T. Shardlow, *Numerical methods for stochastic parabolic PDEs*, Numer. Funct. Anal. Optimization, 1–2 (1999), pp. 121–145.

[106] D. Talay, *Simulation and numerical analysis of stochastic differential systems: a review*, in Probabilistic Methods in Applied Physics, P. Kre and W. Wedig, eds., vol. 451 of Lecture Notes in Physics, Springer-Verlag, 1995, ch. 3, pp. 54–96.

[107] L. Tao and H. Yong, *A generalization of discrete Gronwall inequality and its application to weakly singular Volterra integral equation of the second kind*, J. Math. Anal. Appl., 282 (2003), pp. 56–62.

[108] A. Thom and C. Apelt, *Field computations in Engeneering and Physics*, London: D. Van Nostrand, 1961.

[109] V. Thomée, *Galerkin finite element methods for parabolic problems. 2nd revised and expanded ed.*, Springer Series in Computational Mathematics 25. Berlin: Springer, 2006.

[110] L. Tubaro, *An estimate of Burkholder type for stochastic processes defined by the stochastic integral*, Stochastic Anal. Appl., 2 (1984), pp. 187–192.

[111] M. J. Turner, R. W. Clough, H. C. Martin, and L. J. Topp, *Stiffness and deflection analysis of complex structures*, Journal of Aeronautical Sciences, 23 (1956), p. 805824.

[112] T. Ushijima, *On the finite element approximation of parabolic equations-consistency, boundedness and convergence*, Mem. Numer. Math., 2 (1975), pp. 21–33.

[113] R. Wait and A. Mitchell, *Finite Element Analysis and Applications*, Wiley, 1985.

[114] J. Walsh, *An introduction to stochastic partial differential equations*, École d'Été de Probabilités de Saint Flour XIV - 1984, (1986), pp. 265–439.

[115] Y. Yan, *The finite element method for a linear stochastic parabolic partial fifferential equation driven by additive noise*, 2003.

[116] H. Yoo, *Semi-discretization of stochastic partial differential equations on $\mathbb{R}$ by a finite-difference method*, Mathematics of Computation, 69 (2000), pp. 653–666.

[117] M. Zakai, *On the optimal filtering of diffusion processes*, Z. Wahrsch. verw. Geb., 11 (1969), pp. 230–243.

[118] A. Zapranis and A. Alexandridis, *Modelling the temperature time-dependent speed of mean reversion in the context of weather derivatives pricing*, Applied Mathematical Finance, 15 (2008), pp. 355–386.

[119] M. Zlámal, *Finite elements methods for non-linear parabolic equations*, R.A.I.R.O., 11 (1974), pp. 93–107.

[120] ——, *Finite elements methods for parabolic equations*, Mathematics of Computation, 28 (1974), pp. 393–409.