A new equity condition for infinite utility streams and the possibility of being Paretian*

Geir B. Asheim
Department of Economics, University of Oslo
P.O. Box 1095 Blindern, NO-0317 Oslo, Norway

Bertil Tungodden
Department of Economics, Norwegian School of Economics and Business Administration
Hellev. 30, NO-5045 Bergen, Norway

April 21, 2005

Discussion Paper 9/2005

1. Introduction

This paper investigates the properties of a new equity condition for infinite utility streams. The condition, which was introduced in our companion paper Asheim and Tungodden (2004b), is referred to as "Hammond Equity for the Future", and it captures the following ethical intuition: A sacrifice by the present generation leading to a uniform gain for all future generations cannot lead to a less desirable utility stream if the present remains better off than the future.

In the terminology of Suzumura and Shinotsuka (2003), this new equity condition is a *consequentialist* condition, in the sense that it expresses preference for a more egalitarian distribution of utilities among generations. In contrast, the "Weak Anonymity" condition, which often has been invoked to insure equal treatment of generations (by requiring that any finite permutation of utilities should not change the social evaluation of the stream), is a purely *procedural* equity condition. As we discuss in Asheim and Tungodden (2004b), however, "Hammond Equity for the Future" is a very weak consequentialist condition. Under certain consistency requirements on the social preferences, it is not only weaker than the ordinary "Hammond Equity" condition, but it

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* We thank Aanund Hylland and the participants of the 2005 IEA Roundtable Meeting on Intergenerational Equity in Hakone for discussions and comments.

is also implied by other consequentialist equity conditions like the "Pigou-Dalton principle of transfers" and the "Lorenz domination principle".

From Koopmans (1960), Diamond (1965), and later contributions (e.g., Svensson, 1980; Shinotsuka, 1998; Basu and Mitra, 2003a; Fleurbaey and Michel, 2003; Sakai, 2003; Xu, 2005) we know that it is problematic in the context of infinite utility streams to combine procedural *equity* conditions with conditions ensuring the *efficiency* of a socially preferred utility stream. In particular, Diamond (1965) shows that the "Weak Anonymity" condition cannot be combined with the "Strong Pareto" condition when the social preferences are complete, transitive and continuous in the sup norm topology (a result that he attributes to M.E. Yaari). This impossibility result has subsequently been strengthened by showing that the inconsistency remains even if "Strong Pareto" is weakened to "Weak Pareto" (Fleurbaey and Michel, 2003), if "Strong Pareto" is weakened to "Sensitivity for the present" (Sakai, 2003), and if numerical representability is substituted for the assumption that the social preferences are complete, transitive and continuous in the sup norm topology (Basu and Mitra, 2003a).

Suzumura and Shinotsuka (2003) and Sakai (2004) show that the same kind of impossibility results can be established when consequentialist equity conditions are combined with "Strong Pareto". In particular, Suzumura and Shinotsuka (2003) establish that the "Lorenz Domination principle" is not compatible with "Strong Pareto" when social preferences are upper semi-continuous in the sup norm topology.

The investigations by Suzumura and Shinotsuka (2003) and Sakai (2004) motivate doing a similar analysis for our "Hammond Equity for the future" condition. Since "Hammond Equity for the future" is a weak condition when compared to other consequentialist equity conditions, it is of interest to establish whether it to a greater extent can be combined with Paretian conditions. We show in this paper that, unfortunately, this is not the case: The "Hammond Equity for the future" is not compatible with "Strong Pareto" when social preferences are upper semi-continuous in the sup norm topology. As for the corresponding result by Suzumura and Shinotsuka (2003), no consistency requirements (like completeness and transitivity) on the social preferences are required for this result. However, if we impose that the social preferences are complete, transitive and continuous in the sup norm topology, and satisfy an "Independent future" condition, then "Hammond Equity for the future" cannot even be combined with the "Weak Pareto" condition. These are discouraging results, given the weakness of "Hammond Equity for the future" and its possible ethical appeal.

Our paper is organized as follows. In Section 2 we present the setting, and state the conditions that we will return to in later sections. In Section 3 we show under what circumstances "Hammond Equity for the future" is implied by other consequentialist equity conditions. In Section 4 we show that "Hammond Equity for the future" cannot be combined with "Strong Pareto" when preferences are upper semi-continuous in the sup norm topology, while in Section 5 we report on the inconsistency with "Weak Pareto" and "Sensitivity for the present" under additional conditions. Results relating to the "Pigou-Dalton" and "Lorenz domination" conditions are reported as corollaries. Finally, in Section 6 we discuss what these negative results entail for the usefulness of "Hammond Equity for the future" and other consequentialist equity conditions as ethical guidelines for intergenerational equity.

2. Framework and conditions

Let $\Re$ be the set of real numbers and $\aleph$ the set of positive integers. The set of infinite utility streams is $X = Y^{\aleph}$, where $Y = \Re$ or $Y = [0, 1]$. Denote by ${}_1\mathbf{u} = (u_1, u_2, \ldots, u_t, \ldots)$ an element of X, where u_t is the utility of generation t, and denote by ${}_1\mathbf{u}_T = (u_1, u_2, \ldots, u_T)$ and ${}_{T+1}\mathbf{u} = (u_{T+1}, u_{T+2}, \ldots)$ the T–head and T–tail of the utility streams respectively. Write ${}_{\text{con}}w = (w, w, \ldots)$ for a stream with a constant level of utility equal to $w \in Y$. Throughout this paper we assume at least ordinally measurable level comparable utilities; i.e., what Blackorby et al. (1984) refer to as "level-plus comparability".

For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, ${}_1\mathbf{u} \geq {}_1\mathbf{v}$ if and only if $u_t \geq v_t$ for all $t \in \aleph$; ${}_1\mathbf{u} > {}_1\mathbf{v}$ if and only if ${}_1\mathbf{u} \geq {}_1\mathbf{v}$ and ${}_1\mathbf{u} \neq {}_1\mathbf{v}$; and ${}_1\mathbf{u} >> {}_1\mathbf{v}$ if and only if $u_t > v_t$ for all $t \in \aleph$.

Social preferences are a binary relation R on X, where for any ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, ${}_1\mathbf{u}\ R\ {}_1\mathbf{v}$ entails that ${}_1\mathbf{u}$ is deemed socially at least as good as ${}_1\mathbf{v}$. Denote by I and P the symmetric and asymmetric parts of R; i.e., ${}_1\mathbf{u}\ I\ {}_1\mathbf{v}$ is equivalent to ${}_1\mathbf{u}\ R\ {}_1\mathbf{v}$ and ${}_1\mathbf{v}\ R\ {}_1\mathbf{u}$ and entails that ${}_1\mathbf{u}$ is deemed socially indifferent to ${}_1\mathbf{v}$, while ${}_1\mathbf{u}\ P\ {}_1\mathbf{v}$ is equivalent to ${}_1\mathbf{u}\ R\ {}_1\mathbf{v}$ and $\neg {}_1\mathbf{v}\ R\ {}_1\mathbf{u}$ and entails that ${}_1\mathbf{u}$ is deemed socially preferable to ${}_1\mathbf{v}$. We will consider different sets of conditions of R.

First we consider two *consistency* conditions.

Condition O (*Order*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, ${}_1\mathbf{u}\ R\ {}_1\mathbf{v}$ or ${}_1\mathbf{v}\ R\ {}_1\mathbf{u}$. For all ${}_1\mathbf{u}$, ${}_1\mathbf{v}$, ${}_1\mathbf{w} \in X$, ${}_1\mathbf{u}\ R\ {}_1\mathbf{v}$ and ${}_1\mathbf{v}\ R\ {}_1\mathbf{w}$ imply ${}_1\mathbf{u}\ R\ {}_1\mathbf{w}$.

Condition QT (*Quasi-transitivity*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v}$, ${}_1\mathbf{w} \in X$, ${}_1\mathbf{u}\ P\ {}_1\mathbf{v}$ and ${}_1\mathbf{v}\ P\ {}_1\mathbf{w}$ imply ${}_1\mathbf{u}\ P\ {}_1\mathbf{w}$.

Condition **O** implies condition **QT**, while the converse does not hold.

Consider next two *continuity* conditions (relative to the sup norm topology).

Condition C (*Continuity*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if $\lim_{n\to\infty} \sup_t |u_t^n - u_t| = 0$ and, for all n, $\neg {}_1\mathbf{v}\ P\ {}_1\mathbf{u}^n$ (resp. $\neg {}_1\mathbf{u}^n\ P\ {}_1\mathbf{v}$), then $\neg {}_1\mathbf{v}\ P\ {}_1\mathbf{u}$ (resp. $\neg {}_1\mathbf{u}\ P\ {}_1\mathbf{v}$).

Condition USC (*Upper semi-continuity*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if $\lim_{n\to\infty} \sup_t |u_t^n - u_t| = 0$ and, for all n, $\neg {}_1\mathbf{v}\ P\ {}_1\mathbf{u}^n$, then $\neg {}_1\mathbf{v}\ P\ {}_1\mathbf{u}$.

Condition **C** implies condition **USC**, while the converse does not hold.

Consider next four *efficiency* conditions, where the condition **SNP** has been analyzed by Sakai (2003), while the condition **SN** is used by Asheim and Tungodden (2004b).

Condition SP (*Strong Pareto*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u} > {}_1\mathbf{v}$, then ${}_1\mathbf{u}\ P\ {}_1\mathbf{v}$.

Condition WP (*Weak Pareto*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u} \geq {}_1\mathbf{v}$, then ${}_1\mathbf{u}\ R\ {}_1\mathbf{v}$, and if ${}_1\mathbf{u} >> {}_1\mathbf{v}$, then ${}_1\mathbf{u}\ P\ {}_1\mathbf{v}$.

Condition SNP (*Sensitivity for the present*) For all ${}_1\mathbf{w} \in X$, there exist ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, and $T \in \aleph$ such that $({}_1\mathbf{u}_T, {}_{T+1}\mathbf{w})\ P\ ({}_1\mathbf{v}_T, {}_{T+1}\mathbf{w})$.

Condition SN (*Sensitivity*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u} >> {}_1\mathbf{v}$ and there exists $T \geq 1$ such that $u_t = w$ and $v_t = x$ for all $t \geq T$, then ${}_1\mathbf{u}\ P\ {}_1\mathbf{v}$.

Condition **SP** implies conditions **WP** (if R is reflexive) and **SNP**, while the converse does not hold. Moreover, each of conditions **SP** and **WP** implies condition **SN**, while the converse does not hold.

We then turn to four consequentialist *equity* conditions. The two first require only, as we assume throughout this paper, at least ordinally measurable level comparable utilities. For complete social preferences these conditions coincide with those suggested by Hammond (1976) and Asheim and Tungodden (2004b), respectively.

Condition HE (*Hammond Equity*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that there exists a pair r and s such that $u_r > v_r > v_s > u_s$ and $v_t = u_t$ for $t \neq r, s$, then $\neg {}_1\mathbf{u}\ P\ {}_1\mathbf{v}$.

Condition HEF (*Hammond Equity for the future*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that $u_1 > v_1 > x > w$ and $u_t = w$ and $v_t = x$ for all $t > 1$, then $\neg {}_1\mathbf{u}\ P\ {}_1\mathbf{v}$.

The two next equity conditions require, in addition, that utilities are at least cardinally measurable and unit comparable. Such consequentialist equity conditions have been used in the context of infinite streams by, e.g., Birchenhall and Grout (1979), Asheim (1991), and Fleurbaey and Michel (2001), as well as Suzumura and Shinotsuka (2003) and Sakai (2004). The former of the two conditions below is in the exact form suggested by Suzumura and Shinotsuka (2003).

Condition WLD (*Weak Lorenz Domination*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that there exist $T > 1$ such that ${}_1\mathbf{v}_\mathrm{T}$ Lorenz dominates ${}_1\mathbf{u}_\mathrm{T}$ and ${}_{T+1}\mathbf{u} = {}_{T+1}\mathbf{v}$, then $\neg {}_1\mathbf{u}\ P\ {}_1\mathbf{v}$.

Condition WPD (*Weak Pigou-Dalton*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that there exist a positive number ε and a pair r and s such that $u_r - \varepsilon = v_r \geq v_s = u_s + \varepsilon$ and $v_t = u_t$ for $t \neq r, s$, then $\neg {}_1\mathbf{u}\ P\ {}_1\mathbf{v}$.

Condition **WLD** implies condition **WPD**, while the converse does not hold. The implications between condition **HEF**, on the one hand, and the three other equity conditions, on the other hand, will be treated in the next section.

We end this section by stating the following two conditions. The first of these is implied by Koopmans' (1960) postulates 3b and 4, and means that a decision concerning only generations from the second period on can be made as if the present time (period 1) was actually at period 2; i.e., as if generations $\{1, 2, \ldots\}$ would have taken the place of generations $\{2, 3, \ldots\}$. It is stated by this name, but in a slightly stronger form, by Fleurbaey and Michel (2003).

Condition IF (*Independent future*) For all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$ with $u_1 = v_1$, ${}_1\mathbf{u}\ R\ {}_1\mathbf{v}$ if and only if ${}_2\mathbf{u}\ R\ {}_2\mathbf{v}$.

The second is applicable when $Y = [0, 1]$ and coincides with Koopmans' (1960) postulate 5.

Condition ES (*Extreme streams*) For all ${}_1\mathbf{u} \in X$, ${}_{\mathrm{con}}1\ R\ {}_1\mathbf{u}\ R\ {}_{\mathrm{con}}0$.

Condition **WP** implies condition **ES**, while the converse is not true.

3. Hammond Equity for the future

For streams where well-being is constant from the second period on, condition **HEF** states the following: If the present is better off than the future and a sacrifice now leads to a uniform gain for all future generations, then such a transfer from the present to the future cannot lead to stream that is less desirable in social evaluation, as long as the present remains better off than the future.

To appreciate the weakness of condition **HEF**, consider the following result.

Proposition 1 Let $Y = \Re$. If **QT** and **SN** hold, then each of **HE** and **WLD** implies **HEF**. If **O** and **SN** hold, then **WPD** implies **HEF**.

Proof. Assume $u'' > u' > w' > w''$. We must show under the given conditions that each of **HE**, **WLD**, and **WPD** implies $\neg(u'', {}_{\text{con}}w'')\ P\ (u', {}_{\text{con}}w')$.

Since $u'' > u' > w' > w''$, there exists an integer n and utilities $v, x \in Y$ satisfying $u'' > u' > v \geq w' > x > w''$ and $u'' - v = n(x - w'')$.

If **HE** holds, then $\neg(u'', {}_{\text{con}}w'')\ P\ (v, x, {}_{\text{con}}w'')$, and by **SN**, $(u', {}_{\text{con}}w')\ P\ (v, x, {}_{\text{con}}w'')$. By **QT**, $\neg(u'', {}_{\text{con}}w'')\ P\ (u', {}_{\text{con}}w')$.

Consider next **WLD** and **WPD**. Let ${}_1\mathbf{u}^0 = (u'', {}_{\text{con}}w'')$, and define, for $i \in \{1, \ldots, n\}$, ${}_1\mathbf{u}^i$ inductively as follows:

$$u_t^i = u_t^{i-1} - (x - w'') \qquad \text{for } t = 1$$

$$u_t^i = x \qquad \text{for } t = 1 + i$$

$$u_t^i = u_t^{i-1} \qquad \text{for } t \neq 1, 1 + i.$$

If **WLD** holds, then $\neg\ {}_1\mathbf{u}^0\ P\ {}_1\mathbf{u}^n$, and by **SN**, $(u', {}_{\text{con}}w')\ P\ {}_1\mathbf{u}^n$. By **QT**, $\neg(u'', {}_{\text{con}}w'')\ P\ (u', {}_{\text{con}}w')$ since ${}_1\mathbf{u}^0 = (u'', {}_{\text{con}}w'')$.

If **WPD** holds, then by **O**, for $i \in \{1, \ldots, n\}$, ${}_1\mathbf{u}^i\ R\ {}_1\mathbf{u}^{i-1}$, and by **SN**, $(u', {}_{\text{con}}w')\ P\ {}_1\mathbf{u}^n$. By **O**, $(u', {}_{\text{con}}w')\ P\ (u'', {}_{\text{con}}w'')$ since ${}_1\mathbf{u}^0 = (u'', {}_{\text{con}}w'')$. Hence, $\neg(u'', {}_{\text{con}}w'')\ P\ (u', {}_{\text{con}}w')$. □

Note that condition **HEF** involves a comparison between a sacrifice by a single generation and a uniform gain for each member of an infinite set of generations that are worse off. Hence, contrary to the standard Hammond Equity condition, the weakly welfare increasing transfer from the better-off present to the worse-off future specified in condition **HEF** always increases the total amount of utility along a stream, if utilities are made (at least) cardinally measurable and fully comparable. This entails that condition **HEF** is implied by both the "Pigou-Dalton principle of transfers" and the "Lorenz domination principle", independently of what specific cardinal utility scale is imposed (provided that the consistency conditions specified in Proposition 1 are satisfied). Hence, "Hammond Equity for the future" can be endorsed both from an egalitarian and utilitarian point of view. In particular, condition **HEF** is much weaker and more compelling than the standard Hammond Equity condition.

The binary relation R should, in the context of the present paper, be interpreted as the social preferences of generations $\{1, 2, \ldots\}$. However, since the cardinality of infinite streams is the same independently of when they start, the binary relation R allows us to pose the following question: For any ${}_1\mathbf{u} = (u_1, u_2, \ldots) \in X$, and times r and s, does ${}_1\mathbf{v}\ R\ {}_1\mathbf{w}$ or ${}_1\mathbf{w}\ R\ {}_1\mathbf{v}$ hold, where, for all $t \in \aleph$, $v_t = u_{t+r-1}$ and $w_t = u_{t+s-1}$. I.e., for a given ${}_1\mathbf{u} \in X$, if generations $\{1, 2, \ldots\}$ could have taken the place of generations $\{r, r+1, \ldots\}$ or $\{s,$

s+1, …}, one can ask how generations {1, 2, …} would rank these alternatives in social evaluation. Hence, we can determine whether, along ${}_1\mathbf{u} \in X$, the stream starting at time r is socially preferred (in the evaluation of generations {1, 2, …}) to the stream starting at time s (and vice versa), and we write ${}_r\mathbf{u}\ R\ {}_s\mathbf{u}$ (and ${}_s\mathbf{u}\ R\ {}_r\mathbf{u}$) if it is.

By invoking **HEF** in addition to **O**, **C**, **SN**, **IF**, and **ES**, and assuming that $Y = [0, 1]$, it turns out that, along any ${}_1\mathbf{u} \in X$, the stream starting at an earlier time r will never be socially preferred to the stream starting at a later time s.

Lemma 1 Let $Y = [0, 1]$. If **O**, **C**, **SN**, **HEF**, **IF**, and **ES** hold, then any ${}_1\mathbf{u} \in X$, ${}_s\mathbf{u}\ R\ {}_r\mathbf{u}$ if $s > r$.

Proof. This is contained in Asheim and Tungodden (2004b, Proposition 3). □

In Asheim and Tungodden (2004b) we discuss and interpret this result.

4. Strong Pareto

It is straightforward to show that **HEF** is in direct conflict with **SP** under **USC**. Hence, there are no strongly Paretian and upper semi-continuous social preferences that satisfy our new equity condition.

Proposition 2 Let $Y = \Re$. There are no social preferences satisfying **USC**, **SP**, and **HEF**.

Proof. Let $u'' > u^1 = u^2 = \cdots = u^n = \cdots = u' > w^1 > w^2 > \cdots > w^n > \cdots > w' = w''$ and assume that $\lim_{n\to\infty} w^n = w'$. Then, for all n, $\neg(u'', {}_{\text{con}}w'')\ P\ (u^n, {}_{\text{con}}w^n)$ by **HEF** and $(u'', {}_{\text{con}}w'')\ P\ (u', {}_{\text{con}}w')$ by **SP**. This contradicts **USC** since $(u^n, {}_{\text{con}}w^n)$ converges to $(u', {}_{\text{con}}w')$ in the sup norm topology. □

Note that no consistency conditions (like completeness and transitivity) on the social preferences are required for this result, which is a variant of Proposition 7 of Asheim and Tungodden (2004b). Our previous result uses a different continuity condition which is stronger than USC if preferences are complete.

Compared to the result reported by Diamond (1965)—that conditions **C** and **SP** are inconsistent with "Weak Anonymity" (under the additional assumptions of completeness and transitivity)—we will claim that it is equally worrying that conditions **C** and **SP** are inconsistent with assigning priority to an infinite number of worst off generations in comparisons where the assignment of such priority only reduces the well-being of the better-off present generation, as expressed by condition **HEF**. In this respect, note that **HEF** neither implies nor is implied by "Weak Anonymity", and thus Proposition 2 is different from impossibility results based on "Weak Anonymity" as a procedural equity condition.

Since **SP** implies **SN**, we obtain the following corollary by combining Propositions 1 and 2.

Corollary 1 Let $Y = \mathfrak{R}$. If **QT** holds, then there are no social preferences satisfying **USC**, **SP**, and **HE**; or **USC**, **SP**, and **WLD**. If **O** holds, then there are no social preferences satisfying **USC**, **SP**, and **WPD**.

It should be remarked that the results of Corollary 1 can be strengthened; in particular, it follows from Theorem 3 of Suzumura and Shinotsuka (2003) that condition **QT** is not needed for showing that there are no social preferences satisfying **USC**, **SP**, and **WLD**. Moreover, both Suzumura and Shinotsuka (2003, Theorem 1) and Sakai (2004, Theorem 2) show that only condition **QT** is needed for **USC** and **SP** to be incompatible with a strengthened version of **WPD** (namely, for all ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in X$, if ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that there exist a positive number ε and a pair r and s such that $u_r - \varepsilon = v_r \geq v_s = u_s + \varepsilon$ and $v_t = u_t$ for $t \neq r, s$, then ${}_1\mathbf{v}\ P\ {}_1\mathbf{u}$).

5. Weaker Paretian conditions

We now show that **HEF** is even in conflict with **WP**, provided that the social preferences satisfy conditions **O**, **C**, and **IF**. Hence, there are no weakly Paretian, complete, transitive and continuous social preferences that satisfy both "Independent future" and our new equity condition.

Proposition 3 Let $Y = [0, 1]$. If **O** and **IF** hold, then there are no social preferences satisfying **C**, **WP**, and **HEF**.

Proof. Assume that **O**, **C**, **WP**, **HEF**, and **IF** hold. Let ${}_1\mathbf{u} \in X$ be strictly decreasing; i.e., for all $t \in \aleph$, $u_t > u_{t+1}$. Then it follows directly from **WP** that ${}_1\mathbf{u}\ P\ {}_2\mathbf{u}$ (where we recall from Section 3 that, by notational convention, the comparison relates to two paths starting at time 1, so that **WP** applies). Since **WP** implies **SN** and **ES**, this contradicts Lemma 1, which entails that ${}_2\mathbf{u}\ R\ {}_1\mathbf{u}$ under **O**, **C**, **SN**, **HEF**, **IF**, and **ES**. □

Since **O** implies **QT** and **WP** implies **SN**, we obtain the following corollary by combining Propositions 1 (which holds also if $Y = [0, 1]$) and 3.

Corollary 2 Let $Y = [0, 1]$. If **O** and **IF** hold, then there are no social preferences satisfying **C**, **WP**, and **HE**; or **C**, **WP**, and **WLD**; or **C**, **WP**, and **WPD**.

Moreover, as the following proposition establishes, **HEF** is also in conflict with **SNP** and **SN**, provided that the social preferences satisfy conditions **O**, **C**, **IF**, and **ES**.

Proposition 4 Let $Y = [0, 1]$. If **O**, **IF**, and **ES** hold, then there are no social preferences satisfying **C**, **SNP**, **SN** and **HEF**.

Proof. Assume that **O**, **C**, **SN**, **HEF**, **IF**, and **ES** hold. Then it follows from **O**, **ES** and Lemma 1 that, for all ${}_1\mathbf{u}, {}_1\mathbf{v} \in X$, and, for all $T \in \aleph$, $({}_1\mathbf{u}_T, {}_{\text{con}}0)\ I\ ({}_1\mathbf{v}_T, {}_{\text{con}}0)$. This contradicts **SNP**. □

Since **O** implies **QT**, we obtain the following corollary by combining Propositions 1 and 4.

Corollary 3 Let $Y = [0, 1]$. If **O**, **IF**, and **ES** hold, then there are no social preferences satisfying **C**, **SNP**, **SN**, and **HE**; or **C**, **SNP**, **SN**, and **WLD**; or **C**, **SNP**, **SN**, and **WPD**.

We end this section by reproducing an observation contained in Asheim and Tungodden (2004b, Proposition 3), namely that **HEF** is *not* in conflict with **SN** even if the social preferences satisfy conditions **O**, **C**, **IF**, and **ES**.

Proposition 5 Let $Y = [0, 1]$. If **O**, **IF**, and **ES** hold, then there exist social preferences satisfying **C**, **SN**, and **HEF**.

Proof. Consider the social preferences having the following numerical representation

$$W({}_1\mathbf{u}) = \lambda \limsup_{t\to\infty} u_t + (1-\lambda) \liminf_{t\to\infty} u_t, \quad \text{where } 0 \le \lambda \le 1.$$

It can be verified that these social preferences satisfy **O**, **C**, **SN**, **HEF**, **IF**, and **ES**. □

6. Concluding remarks

Condition **HEF** assigns priority to an infinite number of worst off generations in comparisons where the assignment of such priority only reduces the well-being of the better-off present generation. We consider this to be a compelling consequentialist equity condition. In particular, as discussed in Section 3, the condition can be endorsed both from an egalitarian and utilitarian point of view. It is therefore discouraging that condition **HEF** to such a large extent limits the possibility of being Paretian (cf. Propositions 2, 3, and 4). In principle, there are two ways out of the ethical dilemma that these results pose.

One possibility is to drop continuity. In line with earlier literature, the analysis indicates that the continuity condition is not an innocent technical assumption; rather, the condition has significant normative implications in the social evaluation of infinite utility streams (e.g., in the words of Svensson, 1980, p. 1234, "the continuity requirement is a value judgment"). By employing social preferences over infinite utility streams defined by Basu and Mitra (2003b), Asheim and Tungodden (2004a), and Bossert et al. (2005) (and, if necessary, invoking Szpilrajn's, 1930, Lemma to complete the preferences), we can establish the existence of two kinds of social preferences that satisfy **O**, **SP**, **HEF**, and **IF**: One is classical utilitarian, the other is egalitarian and based on leximin. Such preferences are appealing, since they satisfy "Weak Anonymity" as well as the four consequentialist equity axioms listed in Section 2. On the other hand, they are all insensitive toward the information provided by either interpersonal level comparability or interpersonal unit comparability. Classical utilitarianism makes no use of interpersonal level comparability (even if utilities are level comparable), while leximin makes no use of interpersonal unit comparability (even if utilities are unit comparable).

Another possibility is to weaken the Paretian requirement to condition **SN**. Then, as reported in Proposition 5, there are social preferences satisfying **O**, **C**, **HEF**, **IF**, and **ES**. However, the social preferences used in the constructive proof of Proposition 5 are unappealing, since they entail invariance for the well-being during any finite part of the stream. In particular, such social preferences do not satisfy Chichilnisky's (1996) "No dictatorship of the future" condition. However, there are more attractive alternatives. Conditions **O**, **C**, **SN**, **HEF**, **IF**, and **ES** imply insensitivity for the interests of the present *only* when the present utility exceeds the stationary equivalent of the utility stream. The conditions do not preclude a trade-off between the interests of the present and future otherwise. Therefore, there exist social preferences satisfying conditions **O**, **C**, **SN**, **HEF**, **IF**, and **ES** that are consistent with both of Chichilnisky's (1996) no-dictatorship conditions ("No dictatorship of the present" and "No dictatorship of the future"), and make use of both interpersonal level comparability and interpersonal unit comparability of (at least) cardinally measurable fully comparable utilities. These possibilities are discussed in greater detail in Asheim and Tungodden (2004b).

Thus, it is our view that the impossibility results reported in the present paper should not be used to rule out "Hammond Equity for the future" and other consequentialist equity conditions as ethical guidelines for intergenerational equity. They do, however, show that consequentialist equity conditions seriously restrict the set of possible intergenerational social preferences.

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