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# AN OPERATIONAL INDICATOR FOR NETWORK MOBILITY USING FUZZY LOGIC

by

Rawia Ahmed EL-Rashidy\* and Susan M. Grant-Mullert†

## ABSTRACT

This paper proposes a fuzzy logic model for assessing the mobility of road transport networks from a network perspective. Two mobility attributes are introduced to account for the physical connectivity and road transport network level of service. The relative importance of the two mobility attributes has been established through the fuzzy inference reasoning procedure that was implemented to estimate a single mobility indicator. The advantage of quantifying two mobility attributes is that it improves the ability of the mobility indicator developed to assess the level of mobility under different types of disruptive events.

A case study of real traffic data from seven British cities shows a strong correlation between the proposed mobility indicator and the Geo distance per minute, demonstrating the applicability of the proposed fuzzy logic model. The second case study of a synthetic road transport network for Delft city illustrates the ability of the proposed network mobility indicator to reflect variation in the demand side (i.e. departure rate) and supply side (i.e. network capacity and link closure). Overall, the proposed mobility indicator offers a new tool for decision makers in understanding the dynamic nature of mobility under various disruptive events.

**Keywords:** Mobility, transport network, fuzzy logic; indicators; physical connectivity.

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# 1 Introduction

Mobility is essential to economic growth and social activities, including commuting, manufacturing and supplying energy (Rodrigue et al., 2009). Higher mobility (or in other words, a better ability of the network to deliver an improved service) is a very important issue for decision makers and operators as it relates to the main function of the road transport network. Consequently, an assessment of road transport network mobility is essential in order to evaluate the impact of disruptive events on network functionality and to investigate the influence of different policies and technologies on the level of mobility. Disruptive events may be classified as manmade or climate change related events, the scale of which will also have an impact on road transport network mobility. For example, a small accident may lead to the closure of one lane of a local road or a major accident may cause the closure of a motorway for several hours, with cascading effects on the entire network. Climate change related events (e.g. floods, inclement weather and heavy snowfall) have seen significant increase with resulting impacts on the road transport network. As an example, at the European level, the financial cost of network interruption from extreme weather is estimated to be in excess of €15 billion (FEHRL, 2004) whereas, in the USA, the estimated network repair costs due to snow and ice is 5 bn US\$ (Enei et al., 2011).

Mobility could have two dimensions (Berdica, 2002). Firstly, mobility as “the ability of people and goods to move from one place (origin) to another (destination) by use of an acceptable level of transport service” - commonly measured by vehicle kilometres and evaluated through surveys (Litman, 2008). Secondly, from the road transport network prospective, mobility is defined as the ability of a road transport network to provide access to jobs, education, health service, shopping, etc., therefore travellers are able to reach their destinations at an acceptable level of service (Kaparias and Bell, 2011, Hyder, 2010). Therefore, mobility is a measure of the performance of the transport system in connecting spatially separated sites, which is normally identified by system indicators such as travel time and speed. However, here the mobility concept is used as a key performance indicator to measure the functionality of the road network under a disruptive event, as in the second case above. It is therefore used to reflect the ability of a network to offer users a certain level of service in terms of movement.

The main objective of this study is to develop a single mobility indicator based on two mobility attributes using the fuzzy logic approach. Two case studies are considered to validate the technique: the first case based on real traffic data between seven British cities and the second case study concerned with a synthetic road transport network for Delft city.

## **2 Mobility Assessment**

As with many transport concepts, there are no universally agreed indicators to assess road transport network mobility from a network prospective. According to the National Research Council (2002), mobility assessment should take into account system performance indicators such as time and costs of travel. They propose that the mobility level is inversely proportional to variations in travel time and cost, whereas, Zhang et al. (2009) suggested that travel time and average trip length are two key indicators to evaluate system mobility. The study (Zhang et al., 2009) developed a performance index to evaluate the mobility of an intermodal system, measured by the ratio of travel speed to the free flow speed weighted by truck miles travelled. However the performance index could be adapted to measure road transport mobility by considering total traffic flow rather than average daily truck volume. In line with this approach, Wang and Jim (2006) used the average travel time per mile as a mobility indicator, where the distance is the Geo distance rather than actual distance travelled. The use of the Geo distance rather than travel distance could lead to an overestimation of mobility as the Geo mileage is generally shorter than the actual travel distance between two locations.

Cianfano et al. (2008) suggested a number of indicators based on link travel time and speed to evaluate road network mobility. Specifically, they (Cianfano et al., 2008) introduced a vehicle speed indicator,  $VSI$ , measuring the variation in speed compared to free flow conditions. A value of  $VSI$  of 1 would indicate that vehicles are experiencing a travel speed across the network equal to the free flow speed (i.e. the average free flow speed of the network). Under extreme conditions  $VSI = 0$  indicates a fully congested road network. Cianfano et al., (2008) also proposed a mobility indicator based on travel time. According to Lomax and Schrank (2005), transport performance measures based on travel time fulfil a range of mobility purposes. However, other researchers (Zhang et al., 2009, Cianfano et al., 2008) have used simple and

applicable indicators that could be easily implemented at a real-life network scale. They only considered the impact of traffic flow conditions (presented as the variation in travel speed compared with free flow speed) and took into account the impact of unconnected zones. If some links are not available (e.g. closed due to an incident) they are omitted from the indicator calculations, producing misleading values.

Murray-Tuite (2006) proposed a number of indicators to estimate mobility under disruptive events, some of which were scenario based measures such as the time needed to vacate a town's population and the capability of emergency vehicles (ambulance, police) to pass from one zone through to another. Murray-Tuite (2006) also suggested that the average queue time per vehicle, the queue length on the link and finally, the amount of time that a link can offer average speeds lower than its nominal speed limit could also be considered as mobility indicators.

Chen and Tang (2011) introduced the notion of link mobility reliability, calculated using a statistical method based on historical data i.e. speed data for 3 months derived from floating cars. They also investigated the possible influencing factors on mobility reliability. Their results showed that the mobility reliability of an urban road network is correlated with network saturation (volume/capacity ratio) and road network density.

At the operational level, TAC (2006) carried out a survey including Canadian provincial and territorial jurisdictions regarding current practices in performance measurement for road networks related to six outcomes including mobility. The study found that average speed and traffic volume are widely used as measures of mobility. The study also found that the concepts of accessibility and mobility are used interchangeably in practice, which could conflict with academic practice, where accessibility and mobility are very different concepts. For example, Gutiérrez (2009), emphasised that the mobility concept relates to the actual movements of passengers or goods over space, whereas accessibility refers to a feature of either locations or individuals (the facility to reach a destination). In other words, accessibility could be defined as the potential opportunities for interaction (Hansen, 1959) that are not only influenced by the quality of the road transport network, but also by the quality of the land-use system (Straatemeier, 2008). Widespread communication technologies could play a crucial role in virtual accessibility (Janelle and Hodge, 2000).

A number of further mobility indicators have been reported, namely, origin-destination travel times, total travel time, average travel time from a facility to a destination, delay per vehicle mile travelled, lost time due to congestion and volume/capacity ratio (TAC 2006). Meanwhile, Hyder (2010) suggested three indicators to measure the mobility of the road transport network, namely, maximum volume/capacity ratio, maximum intersection delay and minimum speed. The study (Hyder, 2010) used linguistic expressions to evaluate the indicators (as shown in Table 1) and suggested that mobility is gauged by the lowest value of these indicators.

However none of this existing research has considered the impact of the road transport network infrastructure, such as road density, on network mobility. Therefore, the research presented here considers the impact of network infrastructure and network configuration using graph theory measures alongside traffic conditions indicators, as discussed above. The use of the network configuration and traffic flow conditions will reflect the impact of different kinds of disruptive events. For example, in case of a flood, some parts of the network could become totally disconnected whilst other parts of the network could benefit from lower network loading. Therefore the impact of such an event could be masked if the mobility indicator only considers traffic conditions. In the case of adverse weather conditions the overall network capacity could decrease (Enei et al., 2011) leading to congested conditions, but not necessarily affecting travel distance. Consequently, the consideration of both attributes i.e. physical connectivity and traffic conditions, is necessary to cover both cases. In section 3 below, mobility attributes are introduced.

### **3 Mobility Modelling of Road Transport Networks**

In the research here, the mobility concept is treated as a performance measure expressing the level of road transport network functionality under a disruptive event. Therefore, mobility is used as a concept to reflect the ability of a network to offer its users a certain level of service in terms of movement. To obtain a single mobility indicator a number of mobility attributes are used to capture a range of mobility issues, as outlined above.

### 3.1 Mobility Attributes

Based on the definition of mobility (i.e. the ability of the road transport network to move road users from one place to another with an acceptable level of service), two attributes are proposed. Firstly, an attribute is used to evaluate physical connectivity, i.e. the ability of road transport to offer a route to connect two zones. The second attribute is implemented as a measure of the road transport network level of service, based on traffic conditions. Figure 1 shows a schematic diagram of the mobility attributes and the various factors affecting them. In the following sub-sections both attributes are presented and a justification for their selection is provided.

#### 3.1.1 Physical Connectivity

The physical connectivity (i.e. existence of a path between OD pairs), is a key factor on the level of network mobility. For example, the unavailability of a certain route may lead to unsatisfied demand, economic loss or safety concerns arising from the disconnection of a group of travellers who are then effectively trapped.

Physical connectivity can be measured by a number of indicators based on graph theory, as shown in Levinson (2012). The influence of network configuration on connectivity could be studied by calculating the gamma index ( $\gamma$ ). The  $\gamma$  index is measured as the percentage of the actual number of links to the maximum number of possible links (Rodrigue et al., 2009). The  $\gamma$  index is a useful measure of the relative connectivity of the entire network, as a transport network with a higher gamma index has a lower travel cost under the same demand (Scott et al., 2006). However,  $\gamma$  is not able to reflect the zone to zone level of connectivity and its impact on overall connectivity. Road density also has drawbacks in similarity to the  $\gamma$  index. The detour index (also referred to as the circuitry measure) is defined as the ratio of the network distance to the Euclidean distance, or Geo distance, and is another graph theory measure that is widely used to investigate the impacts of network structure. According to Rodrigue et al. (2009), the detour index is a measure of the ability of road transport to overcome distance or the friction of space. Meanwhile, Parthasarathi and Levinson (2010) concluded that the network detour index measures the inefficiency of the transport network from a travellers' point of view.

In the research here a physical connectivity attribute,  $PCA$ , is developed based on the detour index but modified to consider zone to zone connectivity (see Eq.1 below).

$$PCA_{ij}(r) = \frac{GD_{ij}}{TD_{ij}(r)} \quad (1)$$

where  $GD_{ij}$  is the Geo distance between zone  $i$  and zone  $j$ .  $TD_{ij}$  is the actual travel distance between zone  $i$  and zone  $j$  using route  $r$ . The value of  $PCA_{ij}(r)$  varies from 1 (representing 100% physical connectivity), to zero (where there is no connectivity). In the case of a high impact disaster the degree of connectivity would intuitively be expected to be zero. In such a case, the actual travel distance,  $TD_{ij}(r)$ , may be mathematically assumed to be infinity to express the unsatisfied demand and, accordingly, the value of  $PCA_{ij}(r)$  becomes zero.

To explain the importance of physical connectivity (represented by  $PCA$ ), 9 routes listed in Table 2 with very similar free flow travel speeds were investigated to eliminate the impact of traffic conditions on mobility. The data for the 7 routes was obtained using google map, i.e. travel distance ( $TD$ ), free flow travel time ( $FFTT$ ), as shown in Figure 2 for the Leeds to Birmingham route. The free flow travel and actual travel speeds, ( $FFTS$  and  $TS$ ) were calculated based on the traffic from the google map website (maps.google.co.uk). The  $GD_{ij}$  between each OD pair was calculated using the Euclidean distance based on Pythagorean theorem (i.e.  $GD_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ ) where  $x$  and  $y$  are the National Grid Coordinates obtained using a “gazetteer” query that allows search for and download particular records from the Ordnance Survey's 1:50,000 Landranger series maps‡.

The  $PCA$  was then calculated for each route using Eq. (1) with  $GD_{ij}$  and  $TD_{ij}$ . Furthermore, the mobility indicator developed by Wang and Jim (2006) (average travel time per mile of Geo distance, i.e.  $TT_{ij}/GD_{ij}$ ) was also calculated for free flow conditions and under different traffic conditions. For compatibility, an inverse of the indicator developed by Wang and Jim (2006) should be considered for comparisons with the  $PCA$ . For example, the higher the Geo distance per minute ( $GDpM$ ), the more

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miles are travelled in a minute, hence a higher mobility level. The trend for *PCA* in comparison with *GDpM* and the free flow Geo distance per minute (*FFGDpM*) can then be calculated, as shown in Figure 3.

The coefficient of determination  $R^2$  was used to reflect the correlation between *PCA* and *FFGDpM*. A very high correlation ( $R^2 = 0.99$ ) between *PCA* and *FFGDpM* is shown in Figure 3(a), highlighting the importance of *PCA* in estimating the mobility level in the case of the free flow conditions.  $R^2$  decreases to 0.8, however, in the case of traffic flow with a lower travel speed. The travel speeds presented in Table 2 are close to the free flow speeds and, consequently, the correlation is still relatively high. As traffic speed decreases, the correlation is expected to be weaker. These findings indicate that *PCA* is insufficient to assess the level of mobility under different traffic flow conditions. As a result, the impact of traffic conditions should also be taken into account, as explained below.

### 3.1.2 Traffic Condition Attribute

A wide range of mobility attributes have been developed that are based on traffic conditions, as discussed in section 1.3. Some of these are defined using link data, such as *VSI*, while others are based at zone level such as the performance index (*PI*) and road transport network mobility (*M*). As physical connectivity is calculated at zone level, the variation in travel speed between each OD pair can be adopted to indicate the level of service, given it is widely accepted as a mobility attribute (TAC, 2006). The travel speed between each OD pair ( $TS_{ij}$ ) can then be calculated using Eq. (2) and the traffic condition attribute (*TCA*) is obtained using Eq. (3) below.

$$TS_{ij}(r) = \frac{TD_{ij}(r)}{TT_{ij}(r)} \quad (2)$$

$$TCA(r) = \frac{TS_{ij}(r)}{FFTS} \quad (3)$$

where  $TS_{ij}$  is the travel speed between zone  $i$  and zone  $j$  for a route  $r$ ,  $TT_{ij}$  is the actual travel time between zone  $i$  and zone  $j$  for a route  $r$  and *FFTS* is the free flow travel speed in the network considered. For example, in the case of motorways, *FFTS* could be taken as 70 mi/hr. The value of *TCA* varies between 1 and zero. A value of  $TCA = 1$  indicates that vehicles have a travel speed across the network equal to the

free flow speed (i.e. the average free flow speed of the network). Under extreme conditions  $TCA = 0$ , indicating a fully congested road network.

A number of routes with a very high  $PCA$  ( $\approx 0.80$ ) are presented in Table 3 to show the impact of  $TCA$  in the case of high physical connectivity. A very high correlation was found between  $TCA$  and  $GDpM$  in the case of routes with very high  $PCA$ , as shown in Figure 4(a). A low correlation was, however, obtained between  $TCA$  and  $GDpM$  in the case of routes presented in Table 2 ( $R^2 = 0.0061$ ; see Figure 4(b)). Consequently, it could be concluded that the combined impact of both  $PCA$  and  $TCA$  on mobility is not linear and requires a flexible approach that has the ability to estimate the impact of each attribute according to its level.

### **3.2 Mobility Indicator Using Fuzzy Logic Approach**

The fuzzy logic approach has a wide range of applications in different disciplines e.g. transport, engineering, economics, environmental, social, medical and management fields due to its ability to model the dynamics of a complex nonlinear system that cannot be mathematically modelled (Bianchi and Gaudenzi, 2013; Ross, 2010). Furthermore, the fuzzy logic approach has the ability to interpolate the inherent vagueness of the human mind and to determine a course of action, when the existing circumstances are not clear (Zadeh, 1965). In other words, it can deal with the uncertainty arising when the boundaries of a class of objects are not sharply defined (Nguyen and Walker, 1997).

In environmental applications, Camastra et al. (2015) proposed a fuzzy decision system for genetically modified plant environmental risk assessment using Mamdani inference and Liu and Lai (2009) developed an integrated decision-support framework for environmental impact assessment considering air, water, soil, noise, solid waste, terrestrial, aquatic, economics, society and culture. In transport fields, fuzzy logic applications could be categorized into two main areas, namely soft and hard applications. Hard applications refer to the use of fuzzy logic in hardware design, for example, a fuzzy controller for a traffic junction (e.g. Bi et al., 2014), ramp metering and variable speed limit control (e.g. Pham et al. 2014; Ghods et al. 2007). Soft applications refer to the use of fuzzy logic in modelling the uncertainty associated with various parameters such as travel demand. According to Kalic´ and Teodorovic (2003), the fuzzy logic technique is successfully used in transport modelling including

route choice, trip generation, trip distribution, model split and traffic assignment. For example, Sabounchi et al. (2014) used the fuzzy logic approach to model the impact of users' perceptions on the travel mode selection, whereas Foulds et al. (2013) developed a fuzzy set O–D estimation model. Furthermore, Errampalli et al. (2012) introduced a microscopic traffic simulation model based on the fuzzy logic approach to model traveller behavior on the urban road network.

However, like any other approach, the fuzzy logic technique has its own merits and drawbacks. Davarynejad and Vrancken (2009) and Ross (2010) highlighted a number of these merits and drawbacks based on a comprehensive review. For example, it is a simple method as it uses an easy modelling language and is a powerful tool due to its ability to model experience and knowledge of human operator. It has also the ability to deal with imprecise information. The criticism by Davarynejad and Vrancken (2009) of the fuzzy logic approach focused on its application in hardware, for example, its limited use in traffic control signal or isolated ramp metering rather than traffic control due to the complexity of describing large-scale applications using quantitative information. Fuzzy systems are also limited to the problem solver knowledge, as expressed linguistically, which is of a shallow and meager nature (Ross, 2010). Furthermore, fuzzy models can sometimes be difficult to develop and need numerous simulations before they can be used (Velasquez and Hester, 2011).

In this research, a fuzzy logic approach has been implemented to scale both attributes and combine their impact to measure the mobility level. The flexibility of fuzzy logic approach has allowed the developed model to be adapted to different scenarios as different relative importance of each attribute can be allocated. This has been achieved through using fuzzy reasoning, (the process of deriving conclusions from a set of IF–THEN fuzzy rules). The fuzzy logic approach includes four main steps, namely fuzzification, fuzzy rule base, fuzzy interference engine and defuzzification. The first step, fuzzification, converts *PCA* and *TCA* crisp values to degrees of membership by means of a lookup to one or more of several membership functions. In the fuzzy rule base, all possible fuzzy relationships between *PCA* and *TCA* form the input whilst the output for the mobility indicator *MI* is then found using an 'IF–THEN' format. The fuzzy interference engine collects all the fuzzy rules in the fuzzy rule base and learns how to transform a set of inputs to related outputs. The final step, defuzzification, converts the resulting fuzzy outputs from the fuzzy interference engine

to a crisp number representing the mobility indicator  $MI$ . A brief introduction on the implementation of these steps to estimate a single mobility indicator  $MI$  from the proposed two attributes,  $PCA$  and  $TCA$  is described below.

### 3.2.1 Fuzzy Membership of Mobility Attributes

The relative importance of both  $PCA$  and  $TCA$  has been established through the definition of membership functions as inputs to the fuzzy inference reasoning procedure. In the proposed method, both  $PCA$  and  $TCA$  are expressed by fuzzy sets labelled using gradual linguistic terms, i.e. the crisp values of  $PCA$  and  $TCA$  are converted to fuzzy values, for example high, medium and low. Each attribute is divided into a number of fuzzy subsets and represented by membership grade functions ( $\mu$ ). Various membership functions have been proposed in the literature (Ross, 2010), for example triangular, trapezoid, Gaussian distribution and sigmoid functions. However, the triangular and trapezoid membership functions were adopted to fuzzify different assessed levels of the mobility attributes and indicator as they are by far the most common forms encountered in practice. They also have the benefit of simplicity for grade membership calculations (Ross, 2005, Torlak et al., 2011, El-Rashidy and Grant-Muller, 2014). Other membership functions may also be used, however, previous research (Shepard, 2005) indicated that real world systems are relatively insensitive to the shape of the membership function. Membership functions were also recently determined using optimization procedures, provided that a comprehensive database is available (Jiang et al., 2008). The fuzzy triangular and trapezoidal membership grade functions for each attribute ( $PCA$ ,  $TCA$  and  $MI$ ), are presented in Figure 5. Five assessment levels i.e. very low, low, medium, high and very high were proposed to model  $PCA$ ,  $TCA$  and  $MI$ , where each level is defined by a fuzzy function having membership grades varying from 0 to 1. A value of 1.0 means a 100% membership whereas a value of 0 represents non-membership (e.g.  $\mu_{low}(PCA) = 1$  if  $PCA \leq 0.25$  as shown in Figure 5). The membership grade function adopted can be adjusted or re-scaled to reflect real life conditions and expert opinion.

### 3.2.2 Fuzzy inference system and fuzzy rule base

A fuzzy inference system (FIS) is concerned with developing explicit rules in the form of IF-Then statements. These rules convert implicit knowledge and expertise of the particular application then build a block of rules determining the decision outputs. The

FIS adopted here is based on Mamdani and Assilian (1975) as it is the most common in practice and literature due to its simplicity (Ross 2010).

Generally, there are  $m^n$  fuzzy rules where  $m$  is the number of subsets used to define the ' $n$ ' input parameters. As the number of subsets  $m$  used for either  $PCA$  or  $TCA$  is 5, the total number of fuzzy rules is 25. The fuzzy base rules have been identified through analysis of the relationship between  $PCA$ ,  $TCA$  and  $GDpM$  for 110 routes. These rules could be modified to include expert opinion or new data sets. These fuzzy base rules have the following descriptive form:

R<sup>1</sup>    **IF**  $PCA$  is Very Low and  $TCA$  is Very Low **Then**  $MI$  is Very Low

R<sup>2</sup>    **IF**  $PCA$  is Very Low and  $TCA$  is Low **Then**  $MI$  is Very Low

...    ... ..    .....

R<sup>25</sup>    **IF**  $PCA$  is Very High and  $TCA$  is Very High **Then**  $MI$  is Very High

The Mamdani method has several functions that qualify as fuzzy intersection, referred to in the literature as t-norms as introduced by Menger (1942), (quoted in Ross 2010). T-norms are used for the connectives of inputs; for example 'min' or 'product' operator. The 'product' t-norm was chosen for the fuzzy inference rules determined here as it makes the output sensitive to every input, whereas, only one input controls the conclusion in case of the 'min' t-norm operator. The 'product' t-norm inference formula adopted in the current formulation is given by:

$$\mu_c(MI) = \mu_A(PCA)\mu_B(TCA) \tag{4}$$

where  $A$ ,  $B$  and  $C$  are fuzzy subsets.

### 3.2.3 Defuzzification of mobility indicator

Defuzzification is the inverse process of fuzzification, whereby the calculated fuzzy values of the mobility indicator are converted to crisp values. There are a number of defuzzification techniques, such as the max membership principle, centroid method (centre of area or centre of gravity) and weighted average method. For more details of these techniques and their uses, see Ross (2010). Here the centroid method, that calculates the centre of gravity for the area under the curve, was used as it allows for

an accumulating effect for each assessment level on the calculated *MI* (Ross, 2010). It is also the most prevalent and appealing technique (Ross 2010).

Figure 6 shows a surface plot representation of all these rules using the 'product' t-norm operator and the centroid method. This figure reflects the importance of both *PCA* and *TCA* on the mobility indicator *MI*, as high mobility can only be achieved when both *PCA* and *TCA* are high. The maximum values of *PCA* or *TCA* could only, however, achieve a medium to low mobility level on their own. The above rules are only used for demonstration purposes of the effective application of fuzzy logic in determining the mobility indicator. However, the validity of these rules were studied using data from a real life case study, as presented in Section 4. Following the fuzzification of the two input parameters using the membership functions shown in Figure 5, the applicable rules were activated and the results generated.

### 3.2.4 Numerical example illustrating FL processes

In this section a numerical example is used to demonstrate the main steps of the fuzzy logic approach in combining the two attributes to estimate the mobility indicator. The route between Birmingham and London was chosen for this purpose. The full details of the route are presented in Tables 4 and 5 (route 3 between the two cities) where  $PCA = 0.71$  and  $TCA = 0.58$ . Based on Figure 7, defuzzification of  $PCA = 0.71$  gives a membership grade of the very high and high subsets of 0.55 and 0.40, respectively. Similarly defuzzification of  $TCA = 0.58$  provides a membership grade of the high and medium subsets of 0.53 and 0.47, respectively. Consequently, four If-Then rules were activated, as listed in Figure 7. These four rules identify the mobility level to be members of the high and medium subsets. For each rule, the compatibility of the rule was calculated using the 'product' t-norm, for example for rule 1, the compatibility level for the mobility high subset is  $0.53 \times 0.40 = 0.21$ . For each rule, a trapezoid conclusion was truncated based on the rule compatibility value. The truncated membership functions for each rule were then aggregated using the 'min' operator. The centre of gravity technique was then employed to defuzzify the aggregated membership function obtained and the value of the mobility indicator was calculated, as presented in Figure 7.

The fuzzy logic toolbox Graphical User Interface (GUI) in MATLAB environment was used to build the FIS described and to model *MI* from the two attributes *PCA* and *TCA*.

To test the validity of the proposed model a number of scenarios of real transport networks were studied, as presented in more detail in Section 4 below.

### 3.3 NETWORK MOBILITY INDICATOR

Despite the importance of an OD based mobility indicator, a network wide indicator could be needed to assess the level of mobility under different conditions. To evaluate network mobility, the network mobility indicator (*NMI*) was estimated from the mobility indicator *MI* obtained from the fuzzy logic inference system described above. Each  $MI_{ij}$  is aggregated based on the level of demand between each OD pair, as presented in Eq. (5) below:

$$NMI = \frac{\sum_{i \neq j} MI_{ij} d_{ij}}{\sum_{i \neq j} d_{ij}} \quad (5)$$

$d_{ij}$  is the demand between zone  $i$  and zone  $j$ .

## 4 CASE STUDY 1

Different routes between 7 British cities, namely London, Bath, Leeds, Birmingham, Bradford, Brighton and Manchester were chosen to show the applicability of the proposed technique. For each OD pair (e.g. Brighton and Manchester), various alternative routes available in Google maps in both directions were considered. For example, Figure 8 shows different routes from Bath, Birmingham, Bradford, Leeds, Brighton and Manchester to London. For each route, the travel distance in addition to the free flow travel time is shown in Figure 8. The travel time for each route was obtained from the google maps website based on the traffic conditions at the time of data collection (between 8:00am and 10:00am on 10 March 2014). Table 3 presents the routes' characteristics including travel distance, time and speed, in addition to the free flow time and speed. Table 4 shows a numerical example of the calculated values of *PCA*, *TCA* and *GDpM* for the routes presented in Table 3, in addition to the estimated values of *MI* produced using the FIS rules presented in Section 3.2.2. Figure 9 shows the correlation between *MI* and *GDpM*. The high value of  $R^2$  (=0.9) between *MI* and *GDpM* shows the efficiency of the proposed mobility fuzzy model in estimating *MI* values for different routes using both attributes, *PCA* and *TCA*.

To check the validity of the technique on a wider scale, all the routes between the seven cities (110 routes) were used. Figure 10 shows the correlation between the mobility indicator and travel distance per minute for all the routes between the seven cities: Figure 10(a) for free flow conditions and Figure 10(b) with current traffic conditions. Figure 10(a) shows a high correlation between the mobility level under free flow conditions  $FFMI$  and  $FFGDpM$  ( $R^2= 0.90$ ) whereas Figure 10(b) shows a high correlation under different traffic flow conditions. These findings further support the successful application of the proposed technique.

## 5 Case study 2

Case study 1 (explained above) was used to show the validity of the proposed technique in a real life application. However, there is still a need to check the variation of  $MI$  under different scenarios. To achieve this, a synthetic road transport network for Delft city was employed to illustrate the mobility of the road network under different scenarios using the proposed methodology. Delft is a city and municipality in the province of South Holland in the Netherlands. The total population is 98675 with a density of 4,324.1 per km<sup>2</sup> (Statistics Netherlands, 2012). In general, cars are widely used in the Netherlands and people use this mode for almost half their trips (Statistics Netherlands, 2012). The synthetic Delft road network model is made available with OmniTrans software (Ver. 6.022). The network is only a representation and may deviate from the real network for the city of Delft. The Delft case study was chosen due to the availability of the data needed to illustrate the methodology. However, the focus of the research is the methodology itself rather than the empirical findings and the method should be applicable to any road transport network.

The Delft road transport network consists of 25 zones; two of which are under development (24 & 25), and 1142 links; 483 links are two-way whilst 176 are one-way including connectors and different road types (as shown in Figure 11).

A dynamic assignment model (Madam), available in the four steps transport modelling software OmniTrans, was implemented to investigate the ability of  $MI$  to respond to variations in demand i.e. applying different departure rates every 5 minutes. The Madam model uses turning movements (proportions) calculated for each node in the network and created using static assignment for route choice, which was carried out



prior to the Madam model. The main drawback of this approach is that modelling route choice in such a way leads to fixed routes during dynamic simulation time. Consequently, *PCA* does not change in response to demand variations. However, the traffic data obtained from the simulation was based on static assignment as opposed to 'real-world' observations. This approach cannot capture the full effects of unexpected link closures or increases in demand as it is not able to capture queuing, imperfect information, etc. To obtain more realistic impact results, two issues should be considered; traveller behaviour (e.g. the proportion of travellers who will change their route with congestion or the closure of a link) and the availability of an en-route choice model implemented within the traffic assignment software. However, the main aim of the analysis reported here is to investigate the ability of the attributes to reflect the importance of traffic conditions.

## 5.1 DEMAND VARIATION SCENARIO

Different departure rates every 5 minutes were used to investigate the impact of demand variations on the network mobility indicator estimated by the FIS proposed. 15 minute aggregated travel data (i.e. travel time and distance between each OD in the network) were obtained. A computer programme was developed using MATLAB (R2011a) to calculate *PCA* and *TCA* (Eqs. 1, 2 and 3) for each OD pair (i.e. 484 routes for each 15 minutes time step; in total 9 time periods from 7:00pm to 9:00pm) and *MI* was then estimated using the FIS proposed. The network mobility indicator, *NMI*, was calculated using Eq. (5). Similar to the real life case study, a very high correlation was achieved between *NMI* and *GDpM* for the 9 time periods, as presented in Figure 12.

Figure 13 presents the variations in *PCA*, *TCA* and *NMI* different departure rates. *PCA* does not show any change with demand variations as route choice does not change within the Madam model in OmniTrans (as explained earlier). Consequently, the network mobility indicator *NMI* shows the same trend as *TCA*. Figure 13 also demonstrates that the proposed *NMI* decreases as the departure rate increases, reflecting the ability of the network to accommodate the increase in demand. However as the departure rate decreases, for example between 7:30 and 8:15, *NMI*, is seen to increase.

## 5.2 Disruptive Event

The road transport network may be exposed to a wide range of disruption, which varies in type, magnitude and consequences. Disruptive events can be classified as manmade (i.e. a traffic accident) or natural events such as climate change related events (e.g. floods and extreme weather conditions). In this section, an accident impact will be modelled using a single link closure, whereas a natural event impact is simulated using network wide capacity reductions, as explained below.

### 5.2.1 Link Closure

A number of links were selected to investigate the ability of the proposed attributes to reflect the impact of link closure on mobility. 10 link closure scenarios were carried out using a static assignment model for the morning peak for the purposes of illustration, though many more links could be considered if needed. In each scenario, only one link was blocked, e.g. closed due to a road accident or roadwork (see Figure 14 for link closure locations). Both attributes, the physical connectivity attribute (*PCA*) and traffic condition attribute (*TCA*), were calculated based on the zone level data output. Figure 15 and Table 6 show the results for *PCA*, *TCA* and *NMI* due to the 10 link closures. The impact of link closure on both attributes, *PCA* and *TCA*, is seen to vary from one link to another. For example links 1 and 5 have the greatest impact on *PCA* as the closure of these links leads to a 5% decrease in *PCA* when compared with full network operation. The closure of links 3, 4, 6 and 7 has the highest impact on *TCA* as each link closure leads to a 10% reduction in *TCA* in comparison to full network operation. The highest aggregated impact of a link closure, measured by the corresponding decrease in *NMI*, occurs with the closure of links 2, 3, 4, 6 and 7.

### 5.2.2 Impact of a Network Wide Disruptive Event

Overall network capacity could be reduced in real life due to the effect of network wide events such as heavy rain or snowfall. The levels of reduction in network capacity and speed were assumed based on evidence in the literature (Enei et al., 2011; Pisano and Goodwin, 2004; Koetse and Rietveld, 2009). The main aim of this analysis was to examine the ability of *NMI* to capture the impact of a reduction in network capacity under similar variations in demand. This group of scenarios involved a reduction in capacity of 5%, 10% and 15% in order to model the impact of a weather related event.

Figure 16 shows the variations in the network mobility indicator, *NMI*, for the reduced network capacity and similar variations in the departure rate as illustrated in Figure 13. From Figure 16, *NMI* shows variations during the modelling period (7:00-9:00) for reduced capacity compared with the full network capacity. In general, the largest reduction in the level of network mobility occurs with a 15% capacity reduction under different departure rates. It is worth noting that the response rate in terms of improvement in mobility associated with a decrease in the departure rate is dependent on network capacity. For example, when the reduction in network capacity is 15%, network mobility does not improve much with varying departure rates in comparison with lower reductions in network capacity.

### **5.3 CONCLUSIONS**

A fuzzy model incorporating two mobility attributes, namely a physical connectivity attribute and traffic condition attribute, has been proposed to obtain a single mobility indicator. The merit of using both attributes is to allow the inclusion of different types of disruptive events and their impacts on network mobility. This is in contrast to the case of a single mobility attribute that may refer to the level of mobility without providing insight into the cause. Furthermore, the fuzzy inference reasoning procedure was able to accommodate the relative importance of each attribute under different conditions compared with alternatives such as the use of fixed weights for each attribute. For example, under a free flow condition, the technique was able to estimate the level of mobility that is more influenced by the physical connectivity than the traffic condition.

The applicability of the mobility fuzzy model is confirmed by comparing the proposed mobility indicator by the Geo distance per minute for two case studies. The two case studies showed that the mobility is highly affected by the traffic condition in case of high physical connectivity, i.e. the travel distance is very close from the Geo distance between two zones. Furthermore, the importance of considering both attributes is emphasised by the second case study of the synthetic road transport network for Delft city, e.g. individual link closures could have different impacts on either attribute. For example, a link closure could lead to detours decreasing the physical connectivity attribute causing longer travel distances among some zones. Therefore, the network loading is reassigned, leading to improved flow in some parts of the network.

The synthetic road transport network for Delft city demonstrated that the network mobility indicator changes with the demand variation; as the departure rate increases, the network mobility indicator decreases. Furthermore, the network mobility indicator varies with the supply side variations (i.e. network capacity reduction and link closure). Together these findings indicate that the mobility indicator behaves in an intuitively correct manner. The network mobility could be used by policy makers, local road authorities or strategic Highway Agencies to evaluate the overall effectiveness of particular policies or, for example, to assess the implementation of new technologies.

Although the proposed approach has been demonstrated by two case studies, further investigation is needed in the future, including the involvement of expert opinions and the use of other datasets to improve fuzzy rules. Furthermore, type-2 fuzzy logic could be implemented to improve the fuzzy inference system and compared with type-1 fuzzy logic outcome used in this paper.

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Mobility Indicator	Low	Medium	High
Maximum volume/capacity	>75%	50-75%	<50%
Maximum intersection delay	>300 seconds	60-300 seconds	<60 seconds
Minimum speed	<25 kph	25-50 kph	>50 kph

Table 1 Linguistic expressions and corresponding values of mobility indicators (Hyder 2010).

Route	<i>GD</i> (mi)	<i>TD</i> (mi)	<i>FFTS</i> (mi/hr)	<i>TS</i> (mi/hr)	<i>PCA</i>	<i>FFGDpM</i> (mi/min)	<i>GDpM</i> (mi/min)
Bradford-Birmingham	88.46	128	57.31	51.2	0.69	0.66	0.59
Brighton-Birmingham	133.01	208	57.78	52.88	0.64	0.62	0.56
Leeds-Birmingham	90.48	133	57.83	53.56	0.68	0.66	0.61
Brighton-Bradford	210.64	272	57.87	54.95	0.77	0.75	0.71
Leeds-London	166	195	57.64	48.95	0.86	0.82	0.69
London-Manchester	160.05	200	57.42	50.21	0.80	0.77	0.67
Brighton-Manchester	199.48	266	57.82	54.85	0.75	0.72	0.69
London-Bradford	168.23	203	57.7	50.33	0.83	0.80	0.70
Bath-Manchester	142.69	181	57.46	51.96	0.79	0.75	0.68

Table 2 *GD*, traffic information, *PCA*, *FFGDpM* and *GDpM* for different routes.

	<i>GD</i> (mi)	<i>TD</i> (mi)	<i>FFTS</i> (mi/hr)	<i>TS</i> (mi/hr)	<i>PCA</i>	<i>GDpM</i> (mi/min)	<i>TCA</i>
Brighton- Bath	101.99	127	43.05	35.61	0.80	0.48	0.51
Leeds- Bath	168.029	209	49.37	43.09	0.80	0.58	0.62
London-Manchester	160.06	200	57.42	50.21	0.80	0.67	0.72
Leeds-Bradford	8.62	10.8	25.92	20.90	0.80	0.28	0.30
Leeds-London	165.99	208	56.73	49.33	0.80	0.66	0.70

Table 3 *GD*, traffic information, *PCA*, *GDpM* and *TCA* for different routes.



$i \backslash j$		London												
		$GD_{ij}$ (mi)	Route 1				Route 2				Route 3			
			$TD_{ij}$ (mi)	$TT_{ij}$ (min)	$FFTT_{ij}$ (min)	$TS_{ij}$ (mi/hr)	$TD_{ij}$ (mi)	$TT_{ij}$ (min)	$FFTT_{ij}$ (min)	$TS_{ij}$ (mi/hr)	$TD_{ij}$ (mi)	$TT_{ij}$ (min)	$FFTT_{ij}$ (min)	$TS_{ij}$ (mi/hr)
Bath	96.23	116	154	130	45.19	122	174	149	42.41	-*	-*	-*	-*	
Birmingham	98.48	118	162	127	43.70	139	204	157	40.88	152	204	164	47.35	
Bradford	168.23	203	261	212	46.67	212	283	222	43.04	216	287	228	45.16	
Brighton	45.70	53.3	127	87	25.18	63.2	130	94	29.17	-*	-*	-*	-*	
Leeds	166.00	195	239	203	48.95	195.	250	150	46.80	225	253	229	53.36	
Manchester	160.10	200	242	211	49.59	202.	258	223	46.98	209	240	214	52.25	

-\* indicates no third route between the two cities at the time of data collection (between 8:00am and 10:00am on 10 March 2014)

Table 4 Different routes to London City with their traffic performance measures.

$i \backslash j$		London											
		Route 1				Route 2				Route 3			
		$PCA_{ij}$	$TCA_{ij}$	$MI_{ij}$	$GDpM_{ij}$	$PCA_{ij}$	$TCA_{ij}$	$MI_{ij}$	$GDpM_{ij}$	$PCA_{ij}$	$TCA_{ij}$	$MI_{ij}$	$GDpM_{ij}$
Bath	0.83	0.65	0.63	0.62	0.79	0.60	0.58	0.55	_*	_*	_*	_*	
Birmingham	0.83	0.62	0.60	0.61	0.78	0.69	0.75	0.63	0.71	0.58	0.57	0.48	
Bradford	0.83	0.67	0.70	0.64	0.83	0.61	0.59	0.59	0.79	0.63	0.61	0.59	
Brighton	0.86	0.36	0.38	0.36	0.72	0.42	0.47	0.35	_*	_*	_*	_*	
Leeds	0.85	0.7	0.77	0.69	0.85	0.67	0.70	0.66	0.74	0.76	0.84	0.66	
Manchester	0.80	0.71	0.79	0.66	0.79	0.67	0.70	0.62	0.77	0.75	0.85	0.67	

\_\* indicates no third route between the two cities at the time of data collection (between 8:00am and 10:00am on 10 March 2014)

Table 5  $PCA$ ,  $TCA$ ,  $MI$  and  $GDpM$  values for routes presented in Table 4.

	PCA	TCA	NMI
Full Network	0.76	0.65	0.61
Link 1	0.71	0.58	0.54
Link 2	0.72	0.56	0.53
Link 3	0.75	0.55	0.53
Link 4	0.75	0.55	0.53
Link 5	0.71	0.61	0.56
Link 6	0.75	0.55	0.53
Link 7	0.75	0.55	0.53
Link 8	0.74	0.60	0.57
Link 9	0.74	0.56	0.55
Link 10	0.75	0.59	0.57

Table 6 *PCA*, *TCA* and *NMI* variations arising from individual link closure.

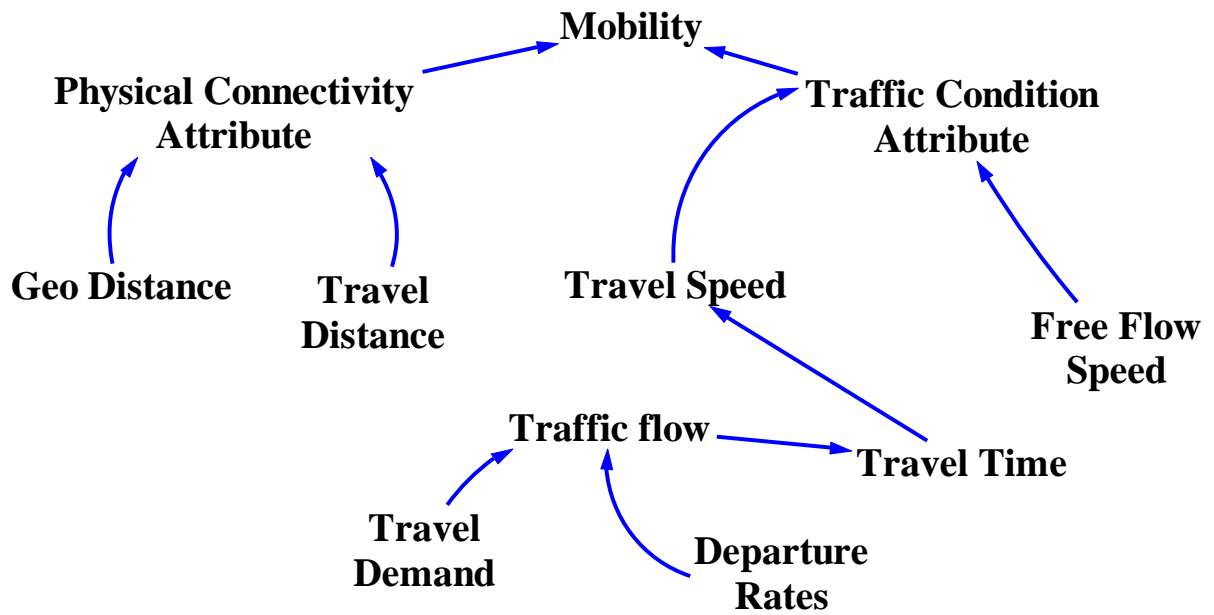
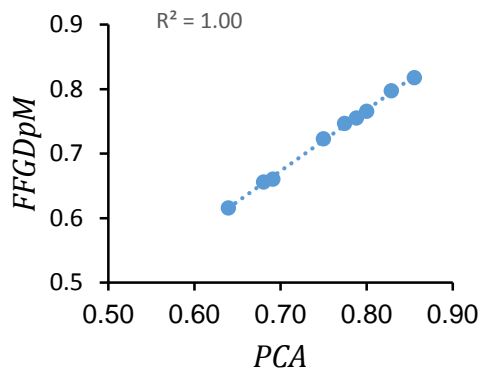


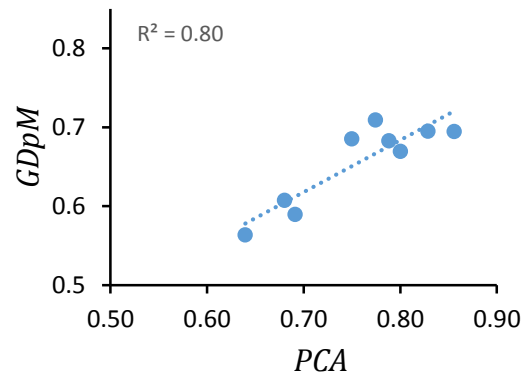
Figure 1 Conceptual Framework for the Proposed Mobility Model.



Figure 2 Routes between Leeds and Birmingham (Google. 2014).

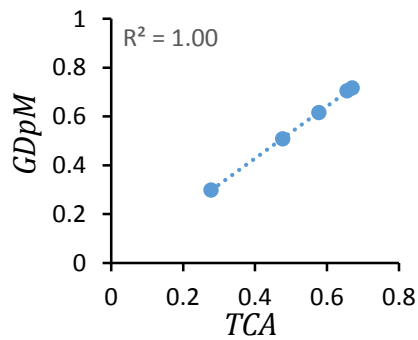


(a) *PCA* and *FFGDpM*

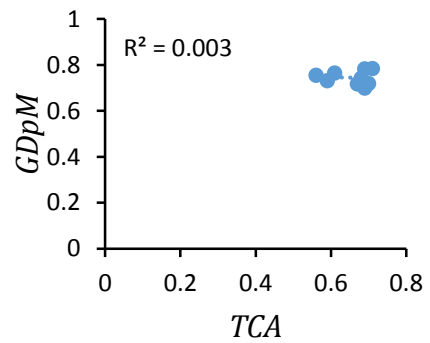


(b) *PCA* and *GDpM*

Figure 3 Relationship between *PCA* and *GDpM*, *FFGDpM*.



(a) *TCA* and *GDpM* for routes in Table 3



(b) *TCA* and *GDpM* for routes in Table 2

Figure 4 Correlation between *TCA* and *GDpM* for routes presented in Table 3 and Table 2.

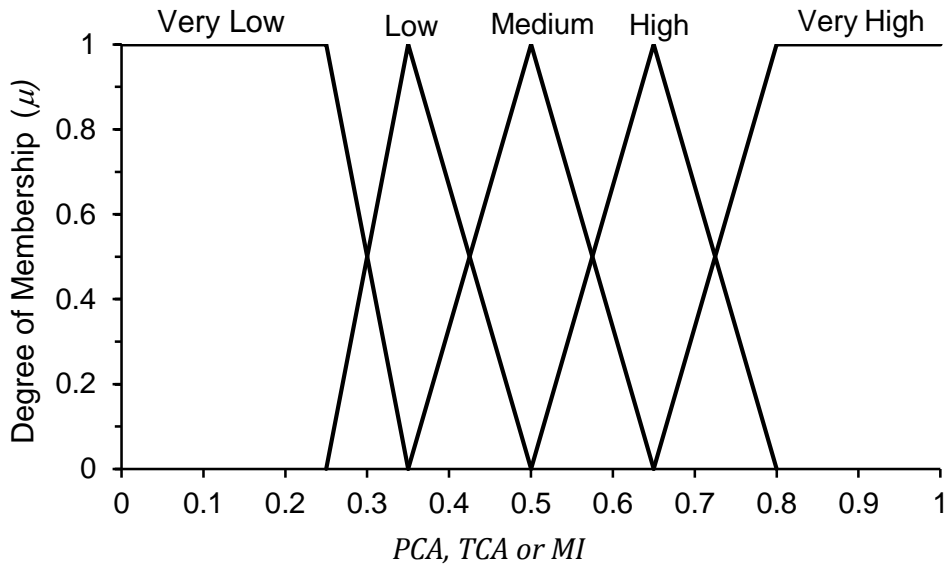


Figure 5 Triangular and trapezoidal membership functions for *PCA*, *TCA* and *MI*.

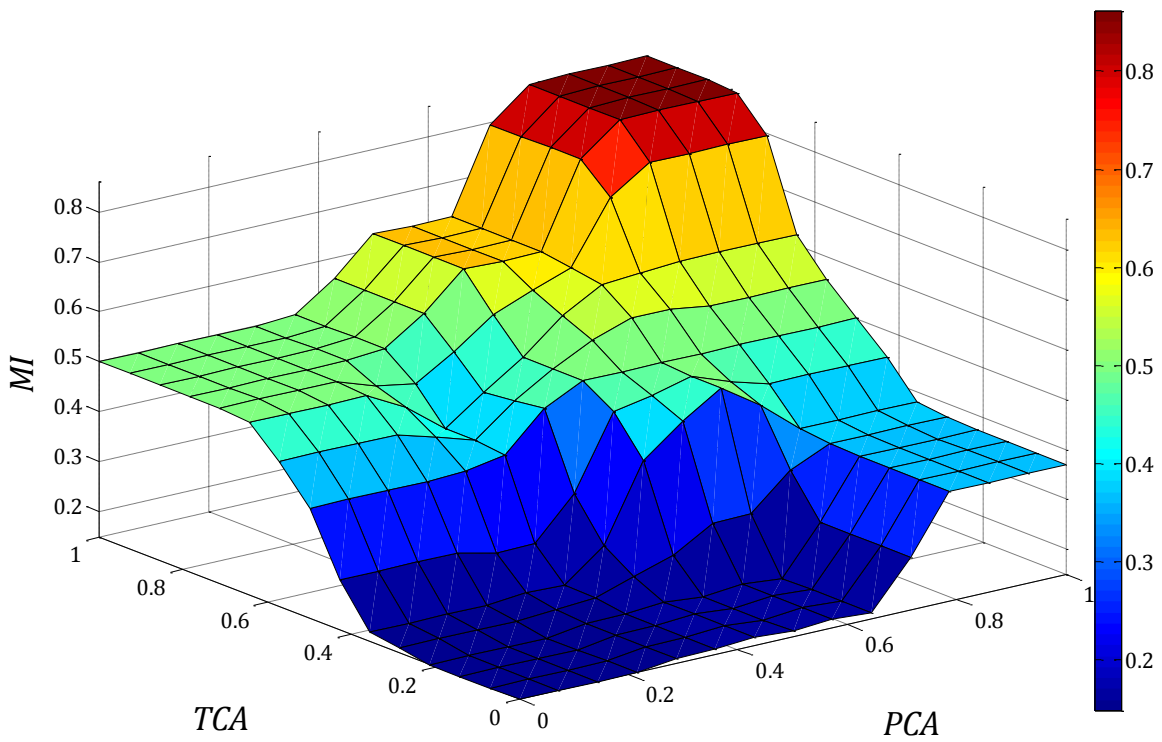


Figure 6 Surface plot of *PCA*, *TCA* and *MI*.

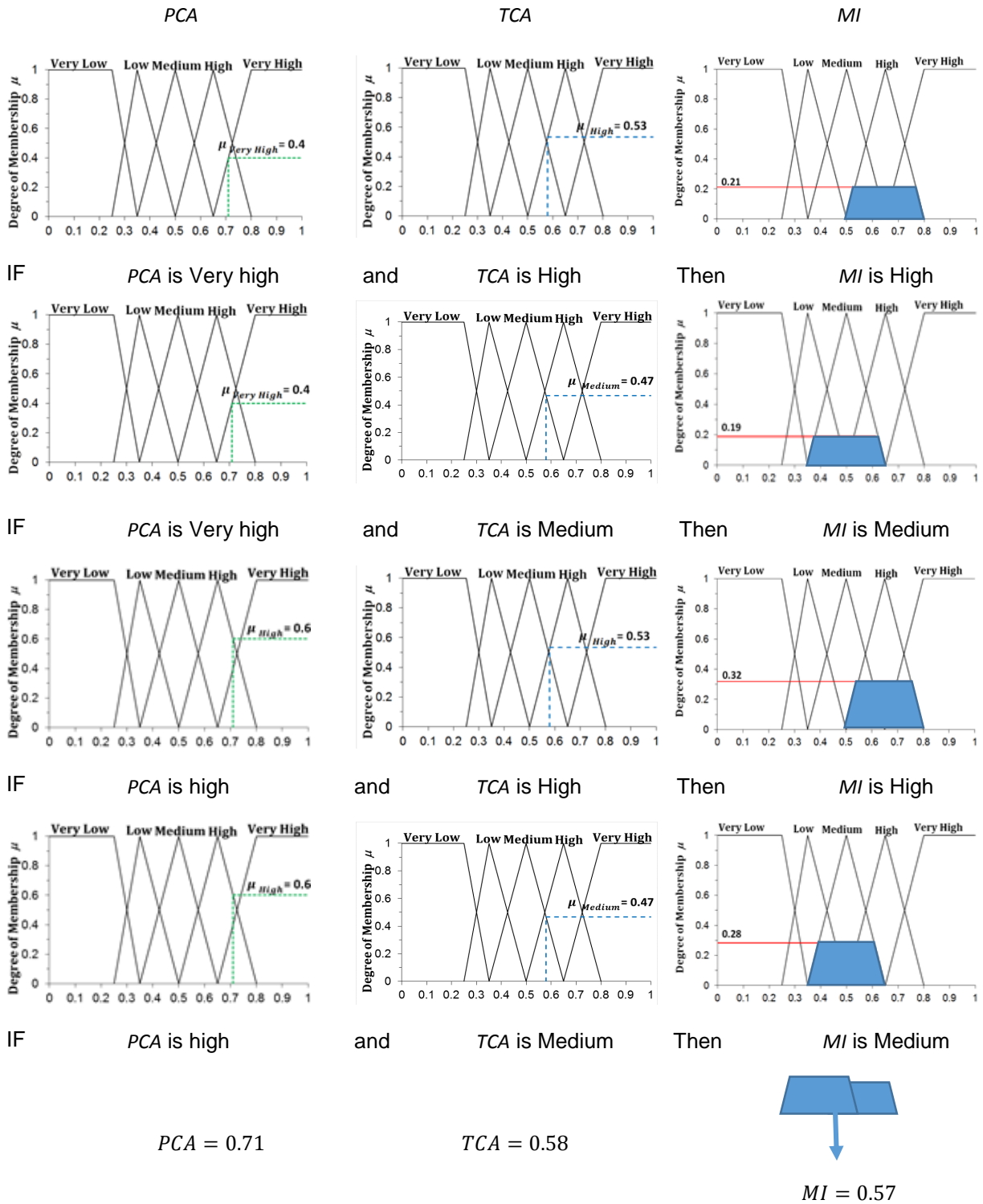


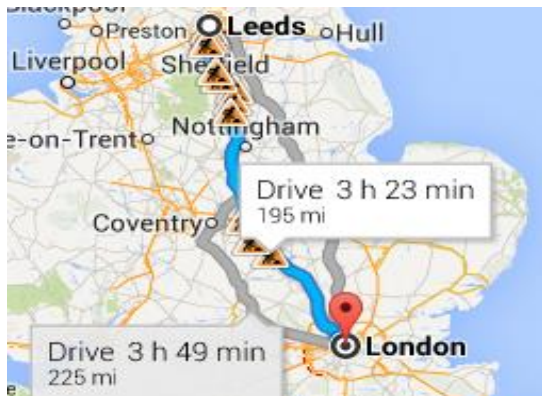
Figure 7 Graphical representation of fuzzy reasoning.



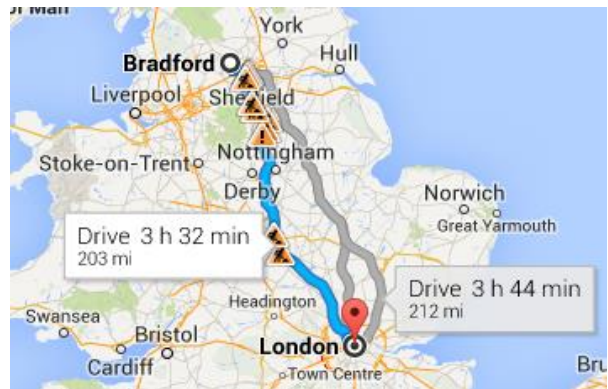
(a) Bath-London routes



(b) Birmingham-London routes



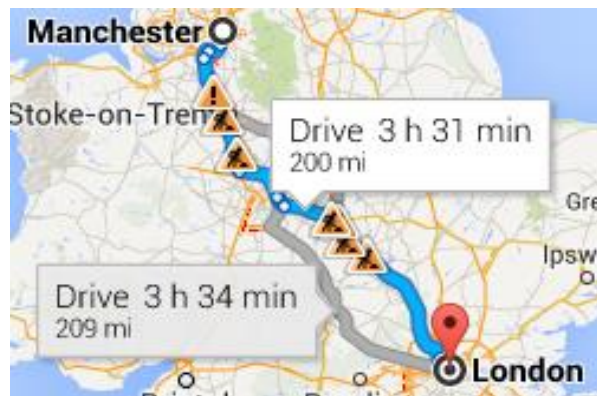
(c) Leeds-London routes



(d) Bradford-London routes



(e) Brighton-London routes



(f) Manchester-London routes

Figure 8 Route maps with travel distance and free flow travel time (Google. 2014).



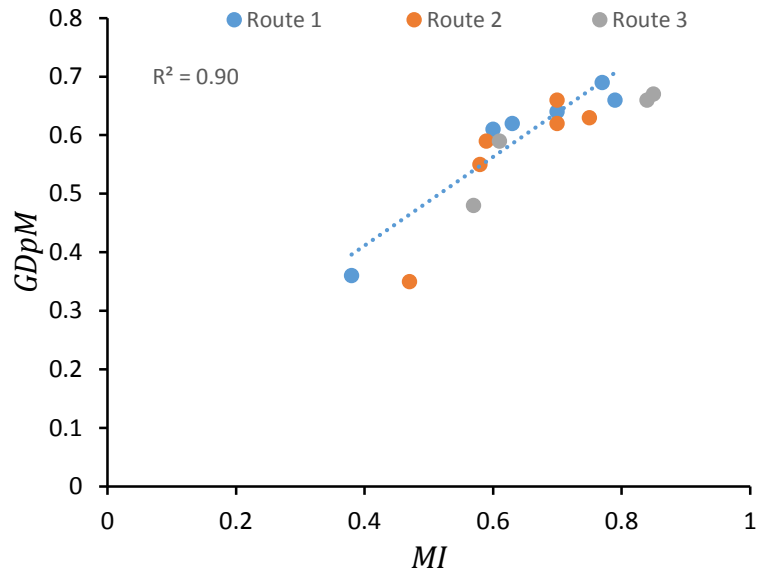


Figure 9 Correlation between  $MI$  and  $GDpM$  for routes shown in Table 3.

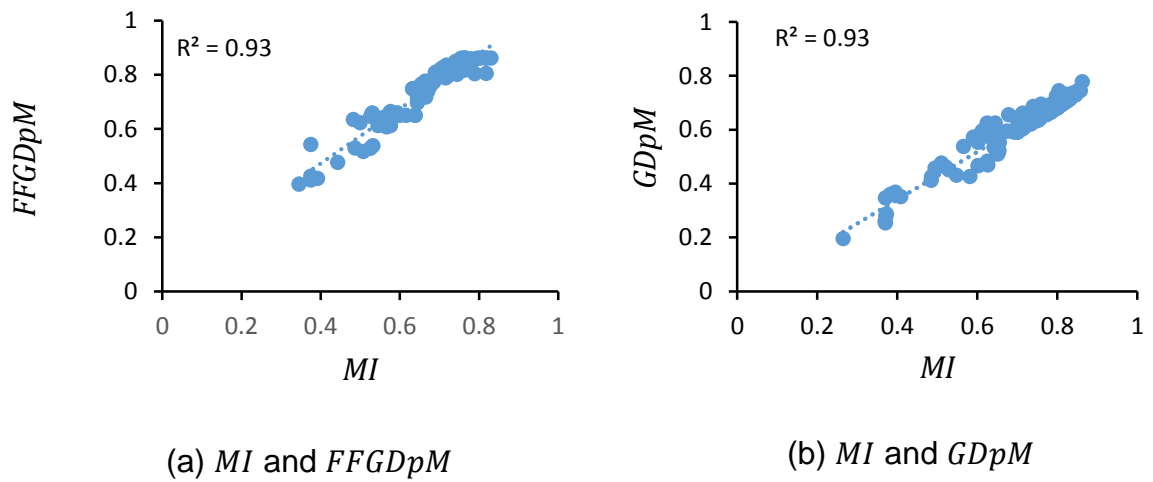


Figure 10 Correlation of  $MI$ ,  $FFGDpM$  and  $GDpM$  for the 110 routes between the seven British cities.

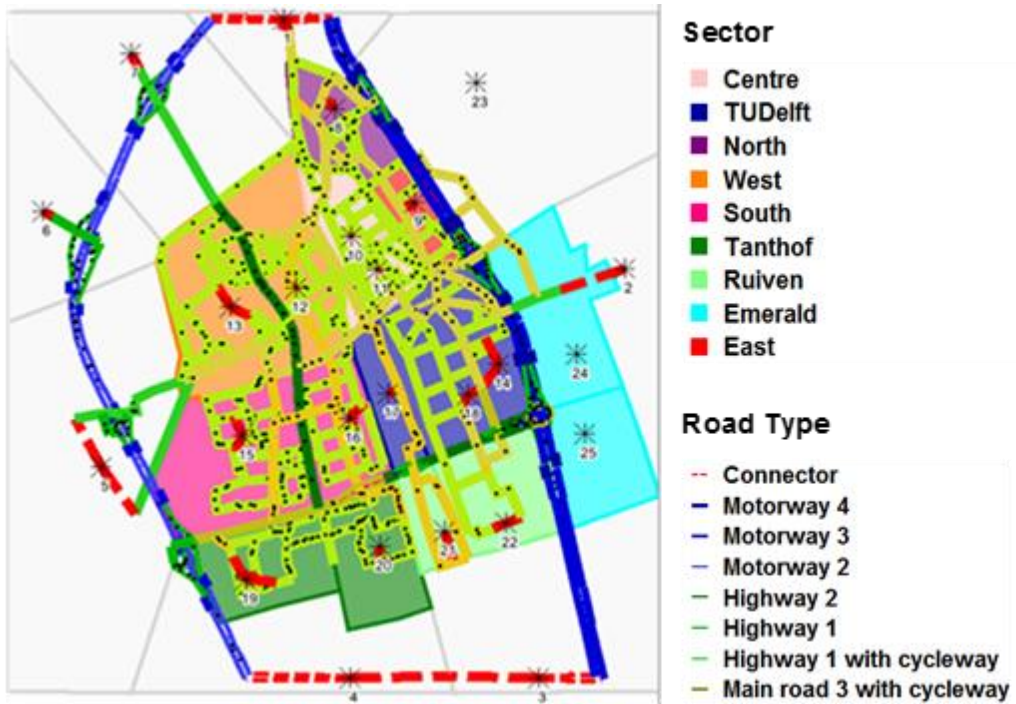


Figure 11 Delft Road Transport Network.

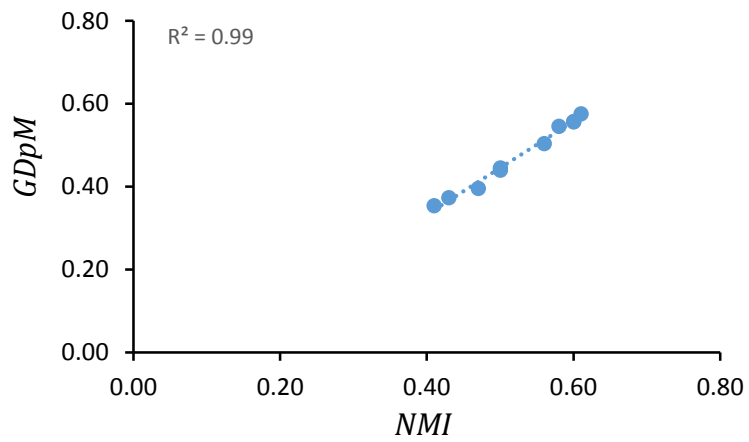


Figure 12 Correlation between  $NMI$  and  $GDpM$ .

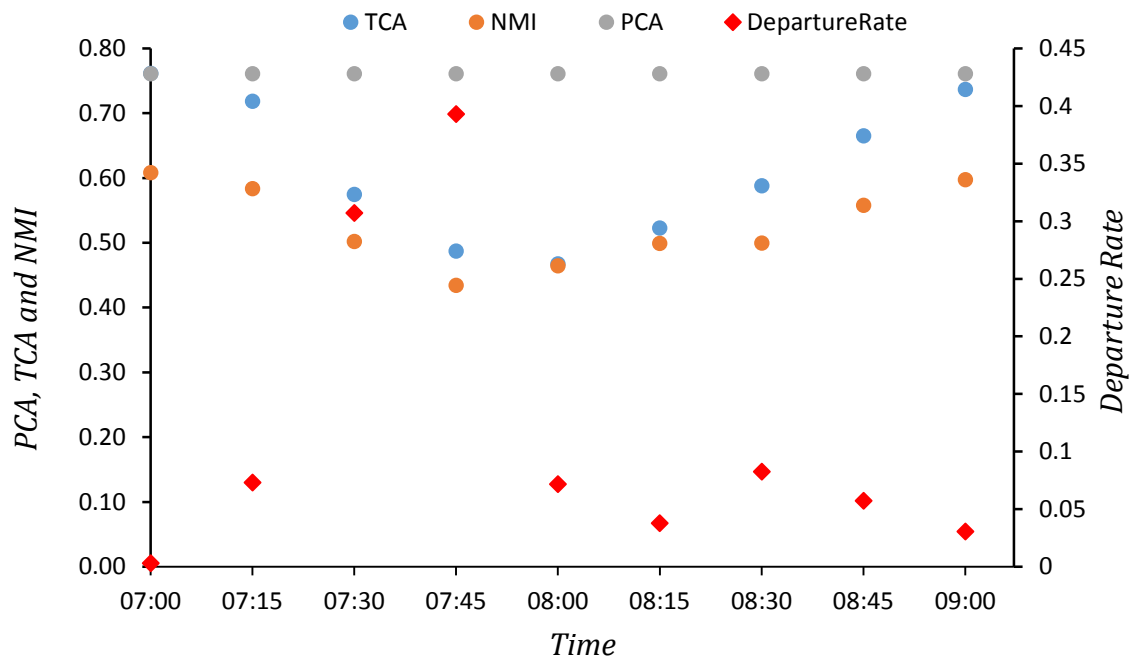


Figure 13 *PCA*, *TCA* and *NMI* variations under different departure rates against time.

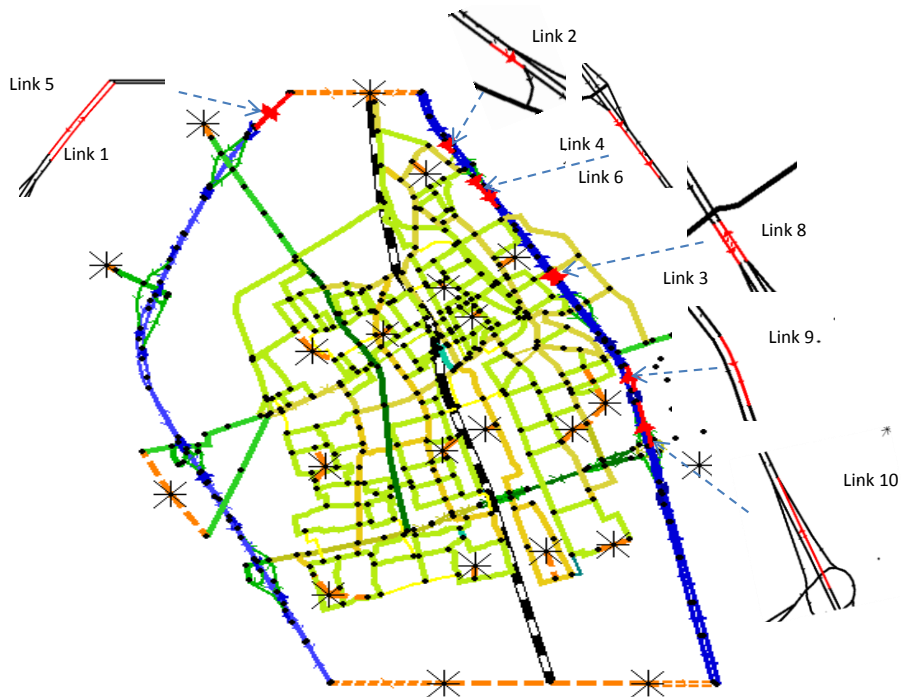


Figure 14 Link closure locations for different scenarios.

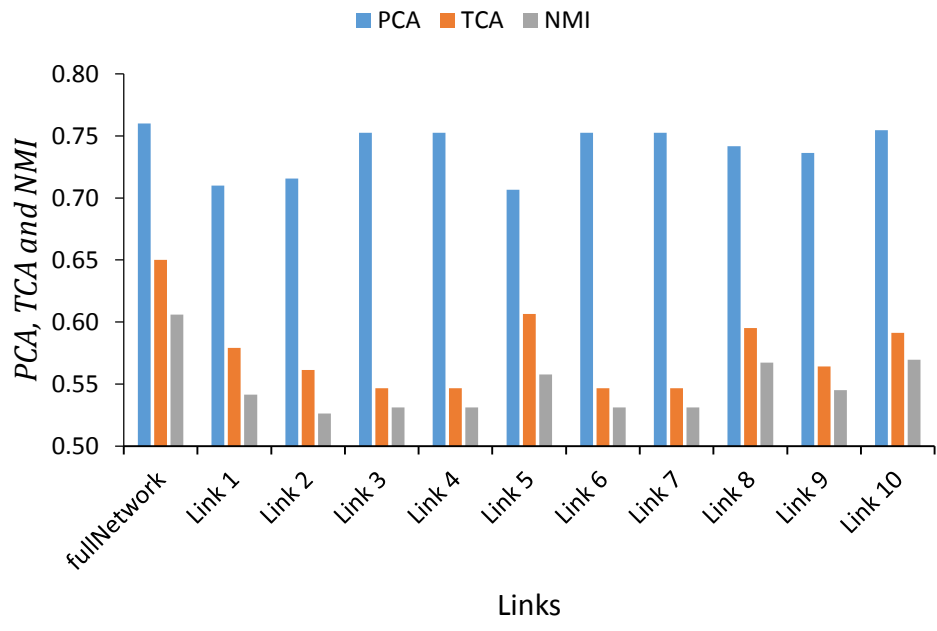


Figure 15 *PCA*, *TCA* and *NMI* variations due to link closure.

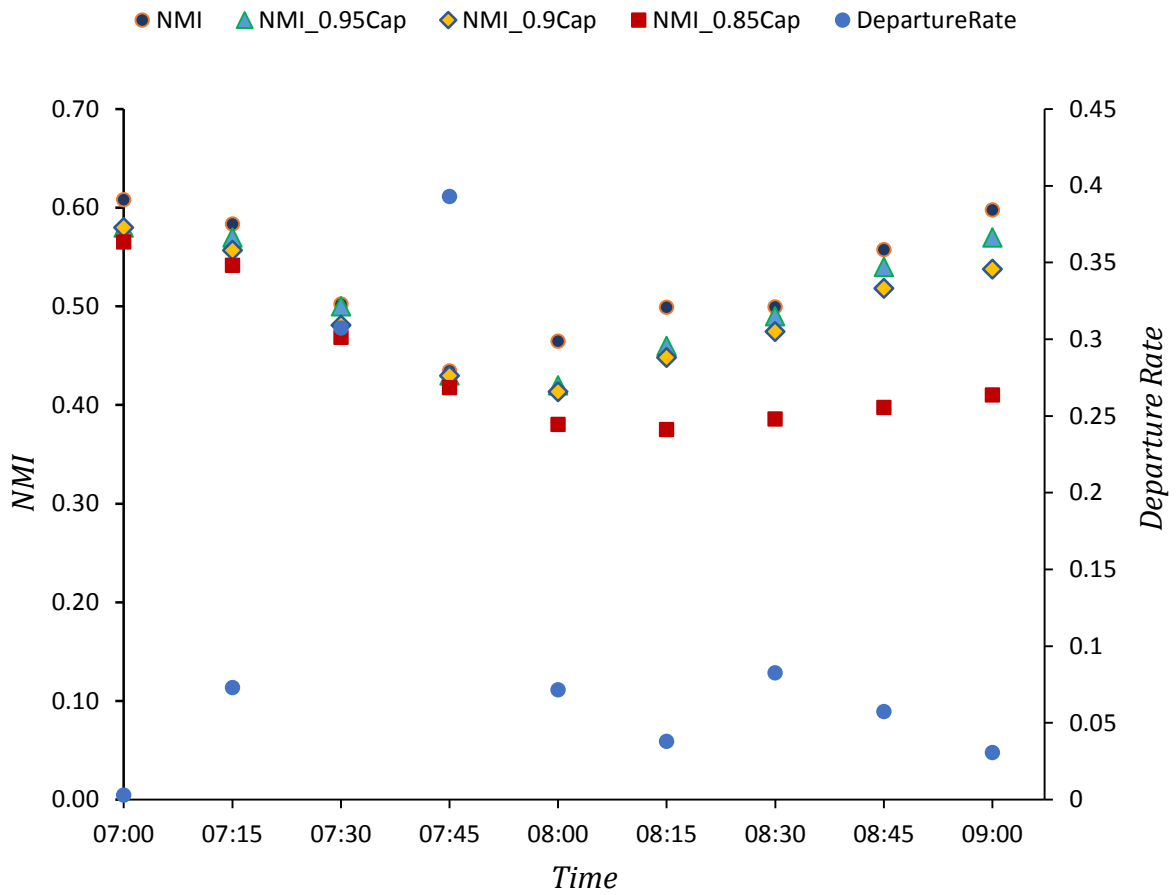


Figure 16 Variation in network mobility indicator against time for different levels of network capacity.