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Assessing the Role of Longitudinal Variability of Vertical Track Stiffness in the Long-Term Deterioration

Assessing the role of longitudinal variability of vertical track stiffness in the long-term deterioration

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ABSTRACT: The longitudinal variation of support stiffness is believed to be a key driver for the differential settlement of the ballast. This paper presents an innovative approach to correlate these two variables. In order to overcome the lack of input data, a statistical model is applied and the spatial correlation of the input stiffness data is controlled to generate naturally occurring variations. A large set of simulations is run and the relationship between deterioration rate and support properties is then analysed using log-linear multiple regression models. The results show that it is not only its mean value that has an impact in the long term settlement behaviour, but it is also its longitudinal variability. Also, main results suggest that the speed effect might be more crucial in localized corrective maintenance needs than in preventive maintenance needs.

1 INTRODUCTION

The performance of the railway system in terms of dynamic loading is depending mainly on the track support conditions. Usually, the track stiffness is used as the main parameter to describe the support conditions and track stability (Sussmann et al., 2001). It is defined as the ratio of the load applied to the rail over the vertical rail deflection (Dahlberg, 2010). Ideally that parameter is constant, but in reality this condition is very unlikely to happen. Some reasons for that are the non-uniformly compacted ballast layer, some local drainage problems or even the presence of voids. Therefore, there is a non-uniform track loading and a non-uniform track deterioration, generally known as differential settlement, leading to a general increment of maintenance needs and costs.

Despite the major role played in the system long-term behaviour, it is very difficult to derive a measure of the actual variability of the track stiffness along the railway. There are many techniques to experimentally acquire those values, for example using the Falling Weight Deflectometer (FWD) equipment or the Swedish Rolling Stiffness Measurement Vehicle (RSDV) measuring train (Berggren et al., 2010). In the first case, it is possible to estimate directly the stiffness of the support underlying the sleeper, while in the latter one the measurements are taken at rail level and include the rail-pad layer and rail bending stiffness. Thus, in the first case the methodologies are usually rather costly and the data acquired may not be long enough to be statistically representative, while in the second case it still poses a problem to back filter the actual support stiffness estimate. The lack of reliable data makes it near-impossible to derive a clear correlation between the physical properties of the railway system and its long-term behaviour.

In the past, several authors investigated the role of spatially varying track stiffness on both the contact forces and the track deterioration (e.g. (Lopez Pita et al., 2004, Li and Berggren, 2010, Frohling, 1997, Dahlberg, 2010, Frohling et al., 1996)). Nevertheless, it seems there is a gap in the literature regarding a mathematical relationship between these two variables. Therefore, the main aim of the present study is to assess the role of longitudinal variability of the vertical track stiffness in the long-term behaviour of the track degradation.

The proposal of this research is to statistically generate significant input data sets for track stiffness out of a known finite set of measurement values and then use a vehicle-track interaction model to predict track force in the time domain and derive long-term settlement of the track. The novelty of this approach resides both in the control of the spatial correlations of the input stiffness data to generate naturally occurring variations and in the attempt at producing a correlation between track settlement and track stiffness. The focus on spatial correlations is in line with current research in statistical modelling of rail track irregularities (Andrade and Teixeira, 2015).

In Section 2, the statistical approach used to create the new sets of track stiffness data which can appropriately reproduce the statistical spatial properties (i.e. auto-correlation) of the measured ones is presented. Section 3 describes the vertical model of vehicle/track interaction system and the iterative process used to evaluate the long-term track behaviour. The main results in terms of correlation between deterioration rate and support properties are presented in Section 4. The influence of vehicle speed is also discussed. Finally, in Section 5 conclusions are drawn and future works are discussed.

2 STATISTICAL REPRESENTATION OF THE TRACK STIFFNESS

Two sets of sleeper support stiffness data have been analysed, whose main characteristics are reported in Table 1. The data was measured in the U.K. using the FWD equipment.

Table 1. Main characteristics of the measured sites.

SITE	Number of measured sleepers	Support stiffness mean value [kN/mm/sleeper end]	Support stiffness SD [kN/mm/sleeper end]	Minimum value kN/mm/sleeper end]	Maximum value kN/mm/sleeper end]	KS test p-value
A	155	84.6	14.4 (Var[Kz]=208)	44.4	143.8	0.32
B	80	110.4	16.2 (Var[Kz]=262)	59.8	157.9	0.90

The distribution curves are shown in Figure 2, assuming a normal distribution of the support stiffness. This hypothesis has been validated performing the Kolmogorov-Smirnov (K-S) goodness-of-fit test and checking that the *p*-value of each set of data (Table 1) is not less than the 10% significance level (Dodge, 2008).

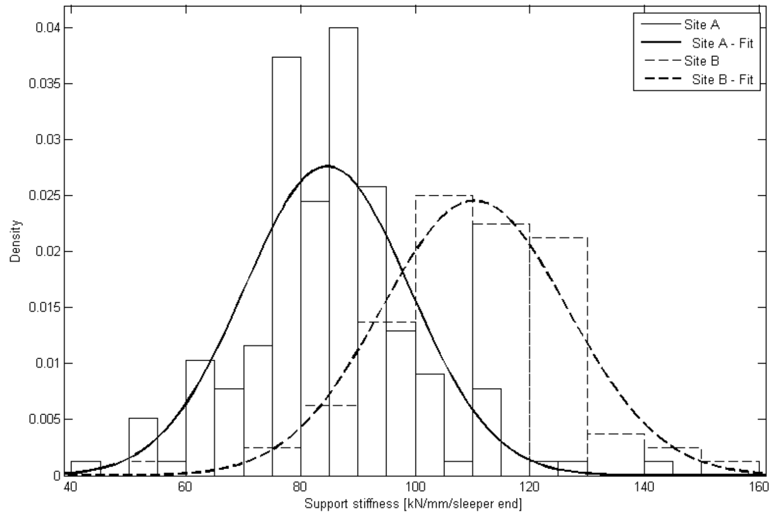

Figure 1. Distribution curves for the two sites considered.

Such distribution function in theory allows generating any number of data sets with the correct mean value and distribution. However, it does not ensure that the correct spatial distribution of stiffness, i.e. the correct variation between one sleeper and the next, is achieved along the track.

In order to reproduce the spatial properties of the measured data, the ARIMA modelling approach has been used in the present study (Cryer and Chan, 2008).

In a general way, the ARIMA model distinguishes three components: a mean component (μ) and/or a weighted sum of neighbouring values and/or a weighted sum of neighbouring error values (e_t).

From a mathematical point of view, a time series $\{Y_t\}$ is said to follow an Integrated Autoregressive Moving Average (ARIMA) model if the dth difference $W_t = \nabla^d Y_t$ is a stationary Autoregressive Moving Average (ARMA) process. If $\{W_t\}$ follows an ARMA (p, q) model, then $\{Y_t\}$ is an ARIMA (p, d, q) process. For practical purposes, values for d are usually assumed to be equal to d=1 or at most d=2. For instance, for a stationary ARIMA (p, 0, q) model with d=0 and with a mean equal to μ:

$$W_t = \mu + \varphi_1 W_{t-1} + \varphi_2 W_{t-2} + \cdots + \varphi_p W_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} \quad (1)$$

Different model specifications can be compared based on the Akaike's Information Criterion (AIC) (Akaike, 1973). This criterion conducts model selection based on the one with minimum value for the AIC:

$$\text{AIC} = -2\log(L^*) + 2k \quad (2)$$

Where L* is the maximum likelihood and k is the number of parameters (k=p+q+1).

Table 2 provides the best ARIMA models with estimated values for the associated parameters, with the respective standard deviations in parenthesis.

Table 2. ARIMA models with estimated values for each site analysed.

Site	Model	φ_1	φ_2	φ_3	φ_4	φ_5	θ_1	σ_e^2	AIC
A	ARIMA (5, 1, 0)	-0.5683 (0.0809)	-0.4721 (0.0905)	-0.4753 (0.0908)	-0.3511 (0.0898)	-0.0438 (0.0838)	-	160	1232
B	ARIMA (1, 1, 1)	0.2621 (0.1211)	-	-	-	-	-0.9524 (0.0407)	243	666

There is no consistent ARIMA model for both sites analysed, i.e. no model specification (p,d,q) can describe the track stiffness regardless of the site. This is because it is not possible to represent the localised characteristics in a general way. For instance, for Site A ARIMA(5,1,0) is considered the best ARIMA model, whereas for Site B the ARIMA(1,1,1).

It is worth noting that the variance of the error term reduces in all cases. For example for Site A the uncontrolled variance of the support stiffness varies from 208 (Table 1) to 160 (Table 2), representing a reduction of the error term of circa 23%. For Site B, the relative reduction is 8%.

3 VEHICLE-TRACK INTERACTION SYSTEM

The model used in the present study to calculate the vehicle/track interaction is shown in Figure 2.

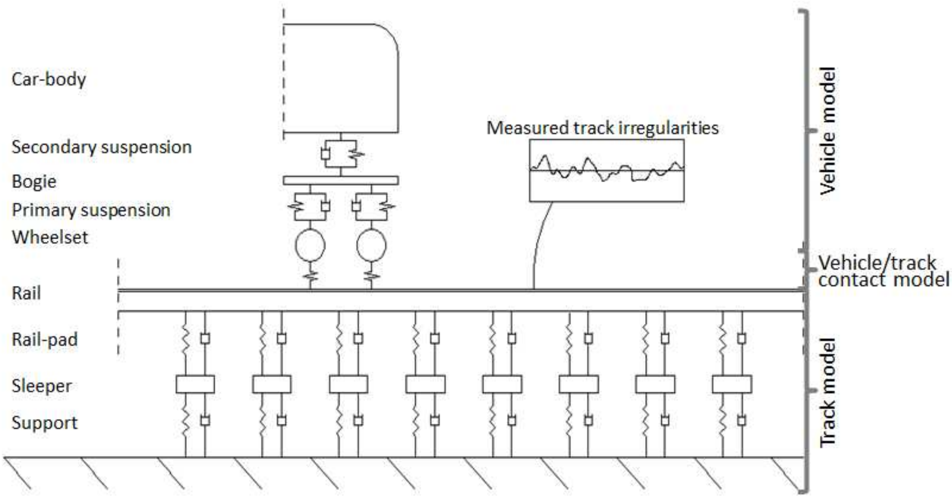

Figure 2. Vertical vehicle/track interaction model.

The vehicle is described as a 5 degree of freedom (DOF) model, including half car-body, one bogie and two wheelsets. Linearized primary and secondary suspensions are considered. The track consists of two-layer ballasted track, including the rail-pad layer, the sleeper mass and the support layer. The rail is modelled as a Timoshenko beam. Four beam elements are considered within each sleeper-spacing in order to achieve a good resolution of results (Grossoni et al., 2015). These two sub-systems are coupled together through non-linear Hertzian contact (Zhai et al., 2009).

In particular, the vehicle represent a typical freight wagon characterised by:

- *Axle load:* 22.5 t, corresponding to circa 110 kN per wheel;
- *Suspensions:* primary and secondary suspensions, including linearised stiffness (respectively 13 and 6.2 MN/m) and linearized damping (respectively 90 and 100 kNs/m).

The main track parameters used in the model are:

- *Rail section*: 60E1;
- *Rail pad vertical dynamic stiffness*: 270 MN/m (medium-hard rail pad);
- *Vertical support stiffness*: as calculated in the previous paragraph;
- *Sleeper mass*: 308 kg (typical concrete sleeper);
- *Sleeper spacing*: 0.65 m.

Due to time constraints the vertical irregularities are described using the measured data corresponding at each site, evolving with track settlement but non-correlated to the input track stiffness in each running cases.

The vertical model described is then used within an iterative process (Figure 3) to calculate the track long-term behaviour.

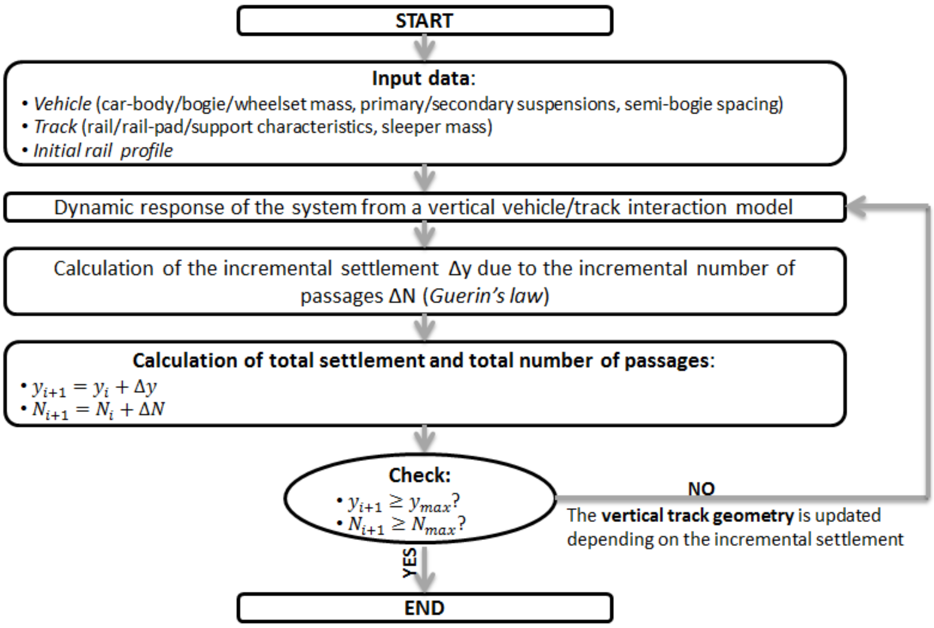

Figure 3. Flow chart of the iterative process used to calculate the track long-term behaviour.

In particular, after the initialization with the vehicle, track and vertical rail profile data, the dynamic response of the vehicle/track interaction system is calculated in terms of contact forces and displacements. The track settlement law is therefore applied and the incremental settlement Δy due to the incremental traffic ΔN is calculated. The total settlement is evaluated as the sum of the total settlement of previous iteration and the incremental settlement of the current one, as the plastic deformation is irrecoverable and the plastic deformations increase monotonically (Suiker and de Borst, 2003). Finally, a check in terms of maximum settlement and maximum traffic is performed. In case it is not satisfied, the process continues updating the vertical track geometry depending on the incremental settlement.

In the present study, the Guerin's law (Guerin, 1996) is adopted and the incremental settlement Δy is calculated as:

$$\frac{\Delta y}{\Delta N} = \alpha \cdot \delta y_{ball,max}{}^{\beta} \tag{3}$$

Where $\delta y_{ball,max}$ is the maximum elastic ballast deformation and α and β two coefficients depending on the soil type. In the present study, they are assumed fixed and respectively equal to 9.67e-06 and 1.46.

4 RESULTS

The simulations for both sites A and B have been analysed together in order to estimate a general law for the ballast deterioration rate. Four speed values (80/120/140/180 km/h) have been considered and the total number of simulations is 137.

Two dependent variables have been investigated: the deterioration rate of the SD of the ballast layer settlement (β_{SD_b}) and the deterioration rate of the maximum of the ballast layer settlement (β_{max_b}). These two dependent variables were computed based on the evolution of the settlement of the ballast layer with accumulated tonnage. For example, in case of deterioration rate of the SD of the ballast layer settlement β_{SD_b}, firstly the SD of the signal along the track per each set of accumulated passage (ΔN) is calculated. A linear regression is then estimated and the deterioration rate is equal to the slope of the best fit line. The flow chart of the methodology is shown in Figure 4.

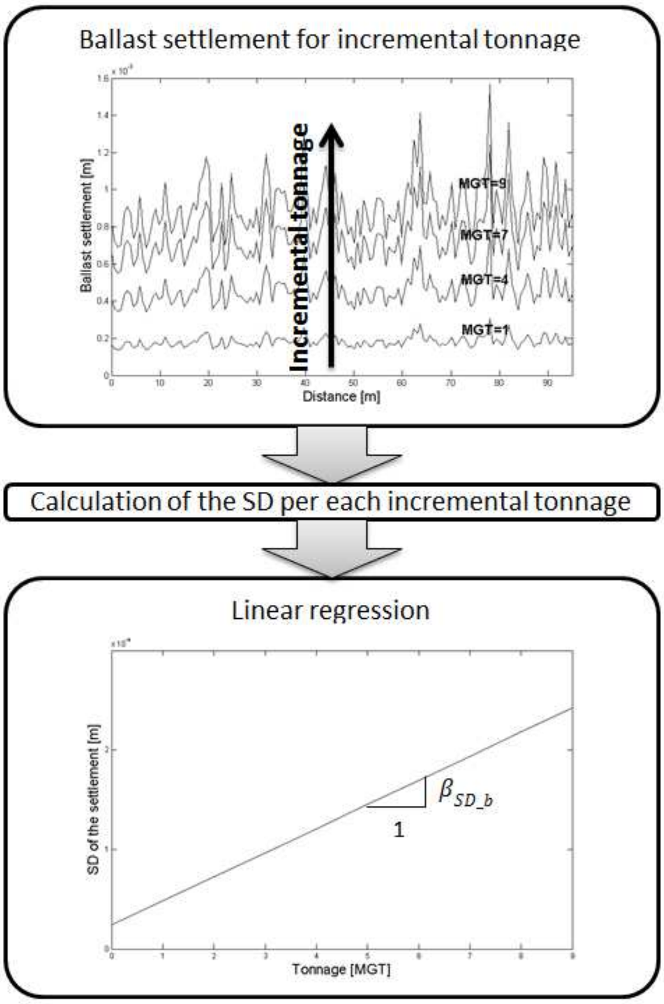

Figure 4. Methodology for the calculation of the deterioration rate.

Similar methodology is adopted to calculate the deterioration rate of the maximum of the ballast layer settlement β_{max_b}, but, instead of computing the SD, the maximum value of the absolute signal is calculated per each accumulated passage.

The following step consists on relating both the degradation rates with some explaining variables. These are the mean of the vertical stiffness (μ_{K_z}), the standard deviation of the vertical stiffness (σ_{K_z}) and the travelling speed (S). Main statistics of dependent and independent variables are presented in Table 3.

Table 3. Main statistics of the dependent and independent variables.

Variables		Main statistics			
		Mean	SD	Min	Max
Dependent	β_{SD_b}	4.2	1.9	2.3	9.3
	β_{max_b}	18.5	11.6	5.9	57.7

Independent	μ_{K_z}	104.1	22.4	56.2	148.3
	σ_{K_z}	20.0	8.4	13.4	49.3
	S	127.7	35.5	80	180

Three log-linear models (hereafter called M1, M2 and M3) have been estimated to assess the contribution of each explaining variable in the variability of the dependent variables β_{SD_b} and β_{max_b} . The general expression of the models is as follows:

$$\beta_{SD\_b} = \exp(\hat{\lambda}_0 + \hat{\lambda}_1 \cdot \mu_{K_z} + \hat{\lambda}_2 \cdot \sigma_{K_z} + \hat{\lambda}_3 \cdot S) \quad (4)$$

$$\beta_{max\_b} = \exp(\hat{\lambda}_0 + \hat{\lambda}_1 \cdot \mu_{K_z} + \hat{\lambda}_2 \cdot \sigma_{K_z} + \hat{\lambda}_3 \cdot S) \quad (5)$$

Where $\hat{\lambda}_i$ are the estimated coefficients reported in Table 4.

In particular, the models explored add sequentially the explaining variables: M1 takes in account only the mean of the vertical stiffness and is characterised by two coefficients, M2 the mean and the SD of the vertical stiffness with three coefficients and M3 the vertical stiffness characteristics (mean and SD) as well as the train speed with four coefficients.

Table 4. Estimated logarithmic regression parameters.

Dependent variables	Model	Independent Variables	$\hat{\lambda}_i$	Std. Error	t-value	R^2
$\boldsymbol{\beta_{SD\_b}}$	M1	(intercept)	2.9064	0.0807	36.02	0.74
		μ_{K_z}	-0.0150	0.0008	-19.83	
	M2	(intercept)	2.6765	0.0416	64.35	0.94
		μ_{K_z}	-0.0168	0.0004	-43.60	
		σ_{K_z}	0.0209	0.0010	20.36	
	M3	(intercept)	2.4993	0.0469	53.27	0.95
		μ_{K_z}	-0.0168	0.0003	-48.86	
		σ_{K_z}	0.0211	0.0009	23.16	
		S	0.0013	0.0002	6.11	
$\boldsymbol{\beta_{max\_b}}$	M1	(intercept)	4.6440	0.1555	29.87	0.53
		μ_{K_z}	-0.0181	0.0015	-12.42	
	M2	(intercept)	4.4089	0.1438	30.65	0.63
		μ_{K_z}	-0.0200	0.0013	-14.96	
		σ_{K_z}	0.0214	0.0035	6.02	
	M3	(intercept)	3.7067	0.1550	23.92	0.74
		μ_{K_z}	-0.0197	0.0011	-17.35	
		σ_{K_z}	0.0221	0.0030	7.36	
		S	0.0051	0.0007	7.33	

All p-values for the t-statistic test are lower than 10^{-8} meaning that the effect of each independent variable is statistically significant (Table 4).

For both the variables analysed, the mean of the vertical stiffness exhibits a negative coefficient, i.e. as the mean of the vertical stiffness increases, the deterioration rate tends to reduce, as expected. The SD of the vertical stiffness, on the contrary, exhibits a positive coefficient, suggesting that increasing that variable leads to increasing the settlement rates. This fact has major impact in the way the vertical stiffness is perceived as the main quality indicator. In other words, it is not only its mean value that has an impact in the long term settlement behaviour, but it is also its longitudinal variability, quantified here using the SD. For higher variability of the vertical stiffness, the deterioration rates will then be higher.

From M1 models, it is shown that the mean of the vertical stiffness explains circa 74% of the variability of the deterioration rate of SD and circa 53% of the variability of the deterioration rate of the maximum settlement. In model M2 and model M3, on the other hand, the SD of the vertical stiffness and the speed, which are added as explaining variables, explain an additional

20% (from 74% to 94%) and 1% (from 94% to 95%) of the total variability of the deterioration rate of SD, and an additional 10% (from 53% to 63%) and 11% (from 63% to 74%) of the total variability of the deterioration rate of maximum settlement. Regarding the coefficients $\hat{\lambda}_2$ and $\hat{\lambda}_3$, it is possible to conclude that effect of the SD of the vertical stiffness is similar for both deterioration rates, whereas the effect of the speed is higher for the deterioration rate β_{max_b} than to the deterioration rate β_{SD_b}. This suggests that speed effect might be more crucial in localized corrective maintenance needs than in preventive maintenance needs.

Figure 5 presents the predicted values for the deterioration rates using Eqs. 4 and 5 and the estimated parameters in Table 4 for different mean values of the vertical stiffness, varying the SD of vertical stiffness and fixing the speed (Figure 5(a,c)), and varying the speed values and fixing the SD of vertical stiffness (Figure 5(b,d)).

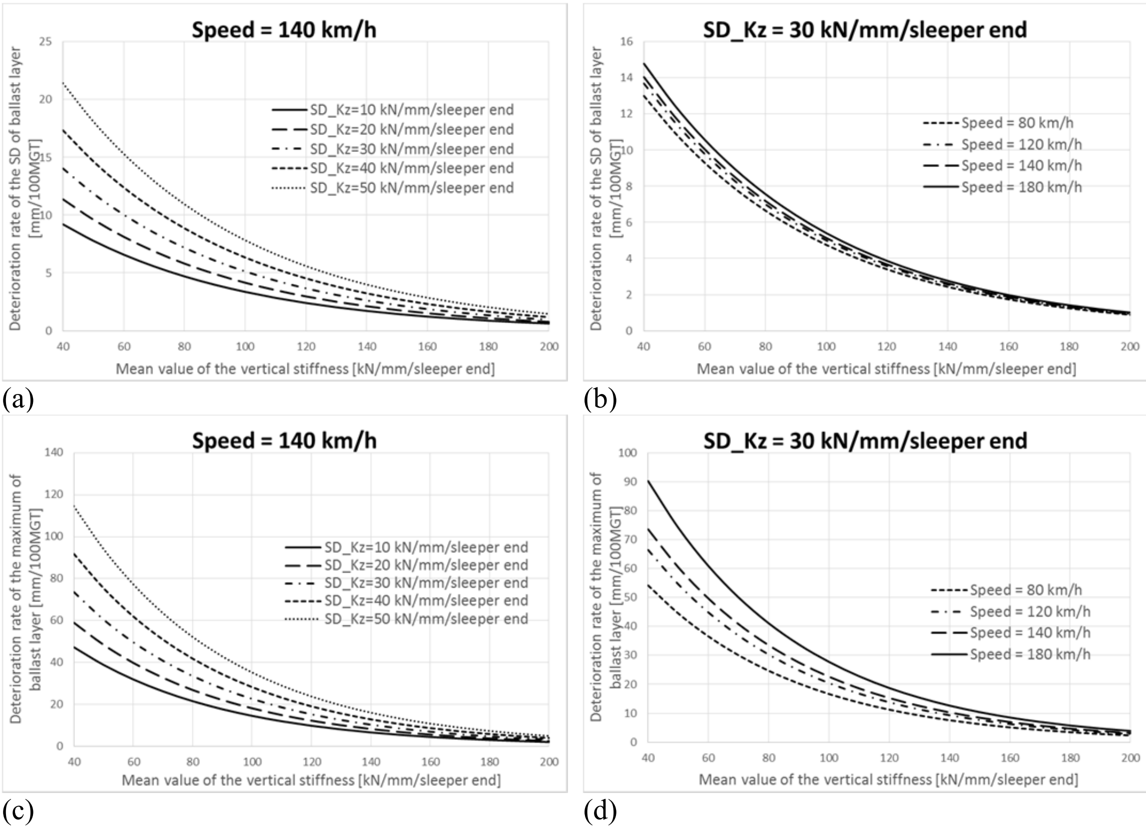

Figure 5. Deterioration rate of the SD of ballast layer versus mean value of the vertical stiffness (a) varying the SD of the support stiffness and (b) varying the speed and deterioration rate of the maximum of ballast layer versus mean value of the vertical stiffness (a) varying the SD of the support stiffness and (b) varying the speed.

It shows that, for the same speed and mean of the vertical stiffness (i.e. ceteris paribus), an increase of 10 kN/mm in the standard deviation of the vertical stiffness corresponds to a relative increase in the deterioration rates of 21.1% ($\hat{\lambda}_2 \times 10 = 0.211$) for the β_{SD_b} and of 22.2% for the β_{max_b} . It also shows that, for the same SD_kz and mean of the vertical stiffness (i.e. ceteris paribus), an increase of 40 km/h in the speed corresponds to a relative increase in the deterioration rates of 5.2% ($\hat{\lambda}_3 \times 40 = 0.052$) for the β_{SD_b} and of 20.4% for the β_{max_b}. Therefore, it is possible to draw the same conclusions of Table 4.

5 CONCLUSIONS AND FURTHER WORK

This paper presents a statistical approach to correlate track stiffness properties to track deterioration. Firstly, innovative techniques are established to create larger sets of data which can effectively reproduce the spatial correlations of measured track stiffness. These sets are used to enable stochastic analysis of the vehicle-track interaction and track degradation.

The main results drawn are that not only mean value of track stiffness influences drastically the degradation rate of ballast, but also there is an important relationship between the SD of track stiffness and the degradation rate of ballast. The influence of speed has been also explored and what emerged is that this parameter is mostly relevant for local defects deterioration rather than general track quality.

Possible impacts of this study can be for quality control of new installations (e.g. ground preparation continuous modulus testing). Also, the influence of support stiffness is known to be particularly important in transition zones, but it should equally be taken into account for plain line design/maintenance.

Planned further works include a better understanding of track stiffness characteristics based on larger and more representative set of measurements. The influence of long-term effects on support stiffness should be also addressed through a continuous monitoring of the site over time. Another research question is about how the results presented in the present paper are related to the rail geometry, that is how the vertical irregularities are related to the influence of track stiffness. Finally, the effect of unsprung mass should be assessed, even if it is believed to be more related to local defects, such as joints or S&Cs.

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