Absolute height measurement of specular surfaces with modified active fringe reflection deflectometry

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ABSTRACT

Deflectometric methods have existed for more than a decade for slope measurement of specular freeform surfaces through utilization of the deformation of a sample pattern after reflection from a test surface. Usually, these approaches require two-directional fringe patterns to be projected on a LCD screen or ground glass and require slope integration, which leads to some complexity for the whole measuring process.

This paper proposes a new mathematical measurement model for measuring topography information of freeform specular surfaces, which integrates a virtual reference specular surface into the method of active fringe reflection delfectometry and presents a straight-forward relation between height and phase. This method only requires one direction of horizontal or vertical sinusoidal fringe patterns to be projected on a LCD screen, resulting in a significant reduction in capture time over established method. Assuming the whole system has been pre-calibrated, during the measurement process, the fringe patterns are captured separately via the virtual reference and detected freeform surfaces by a CCD camera. The reference phase can be solved according to spatial geometrical relation between LCD screen and CCD camera. The captured phases can be unwrapped with a heterodyne technique and optimum frequency selection method. Based on this calculated unwrapped-phase and that proposed mathematical model, absolute height of the inspected surface can be computed. Simulated and experimental results show that this methodology can conveniently calculate topography information for freeform and structured specular surfaces without integration and reconstruction processes.

Keywords: absolute surface measurement, active fringe reflection deflectometry, freeform specular measurement

1 Introduction

Rapid and precise form measurement for freeform and structured specular surfaces is a common task in optics, aerospace and MEMS/NEMS fields. For precisely characterising these components, slope and height measurement via optical methods have widely been investigated within recent years.

It is known that stereo of phase measuring deflectometry (PMD) proposed by Knauer et al. [1] can measure the three dimensional shape of a freeform specular object. This method is to obtain slope information for a smooth object surface, and then to integrate these slope data into surface topography via some integration algorithms[2, 3]. For this approach, topography accuracy of one point is related to adjacent points due to integration operation. And regularity of surface has to be carefully considered.[4, 5]

Another method, reflection grating photogrammetry (RGP) based on ray intersection was put forward by Petz and Tutsch[6], which can measure non-continuous specular surfaces. The tested points for this method are independent from the others since it employs ray intersection instead of integration. On the basis of this method, Yong L.X et al.[7] published fringe reflection photogrammetry (FRP) method, which imports constraint bundle adjustment into RGP. Though absolute coordinates of non-continuous specular surfaces can be obtained with these two methods, they do not give a direct expression between height and phase information and needs spatial geometry computation.

Later, Hong W.Z[8] proposed an expression between captured phase and height information for an measured surface, while this method requires an ultra-precision machined reference surface to be put parallel to LCD screen and did not exactly state height-phase relation. In addition, all these methods require x and y direction fringe patterns to be projected on a LCD screen to extract the coded phase information.

Importing a virtual reference mirror in the model of FRP[7], a straight-forward height-phase mathematical model is put forward in this paper. When the whole system is calibrated, this proposed method needs only one direction fringe patterns and the height information can be directly calculated. An elaborate description will be stated in section 2. Simulation and experimental results are drawn in section 3. Section 4 gives a conclusion.

2 Principle of proposed method

Figure 1 shows arrangement of the modified active fringe reflection photogrammetry[6, 7], where phase-shifting sinusoidal fringe patterns in only horizontal (or vertical direction) are displayed on a liquid crystal display (LCD) screen, virtual reference mirror locates at front of LCD monitor with a space of , LCD screen shifts along its axis with Δd distance and the detected specular surface locates in the range of field depth of CCD camera. At each position for this LCD screen, sinusoidal patterns are respectively observed by a CCD camera via a reference mirror and the under-test freeform specular surface.

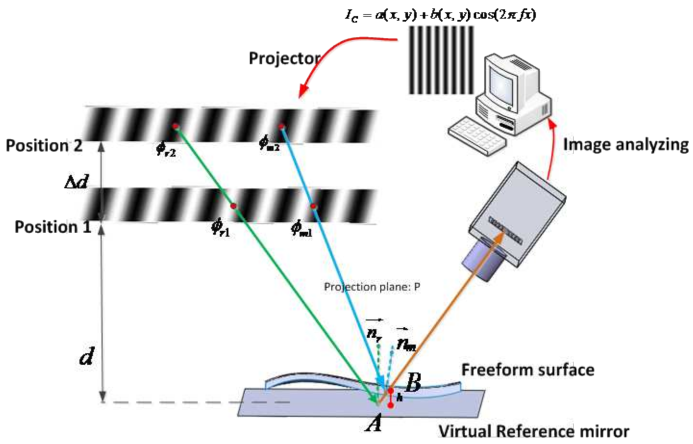

Figure 1 Schematic setup of modified FRP

If a pinhole projection is assumed for the imaging system, based on ray tracing technique of CCD camera, Figure 1 depicts two rays of light, which are extracted from LCD screen and are reflected into the CCD camera via detected surface and reference mirror. These two different incident rays correspond to a same reflection light. Assuming the whole system has been pre-calibrated, the reference phase ϕ_{r1}, ϕ_{r2} can be solved according to spatial geometrical relation between LCD screen and CCD camera and the captured phase ϕ_{m1}, ϕ_{m2} can be unwrapped with a heterodyne technique and optimum frequency selection method.

To simplify this presentation, a single pixel point is considered. Figure 2shows an orthographic view projected along the direction of sinusoidal fringe patterns. Distance between LCD locating at position 1and reference mirror is d. The shifting distance is Δd. Relative to the reference surface, the absolute distance of specular surface is defined as h. Vectors $\vec{n}_{r(p)}$ and $\vec{n}_{m(p)}$ are the projected normal vectors of $\vec{n}_r$ and $\vec{n}_m$. θ is the angle between projected incident line l_r and normal vector of reference mirror, and $\theta + \varphi$ corresponds to an angle between projected incident ray l_m and normal vector of reference surface. The period of sinusoidal fringe pattern is q.

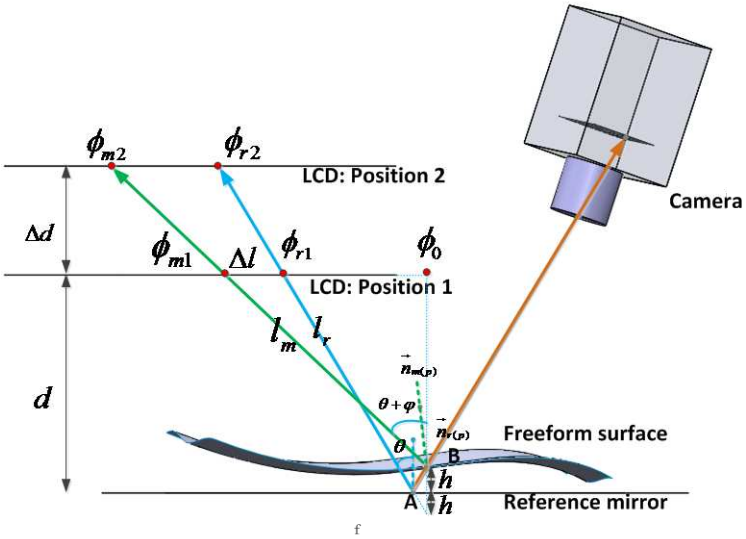

Figure 2 Projection diagram of modified FRP

From its geometrical relation, the following expression can be deduced.

$$\begin{cases} (\phi_{r1}-\phi_{r2})q/2\pi = \Delta d \tan\theta \\ (\phi_{m1}-\phi_{m2})q/2\pi = \Delta d \tan(\theta+\varphi) \end{cases} \tag{1}$$

$$\begin{cases} (\phi_{r1}-\phi_{r2})q/2\pi = \Delta d \tan\theta \\ (\phi_{m1}-\phi_{m2})q/2\pi = \Delta d \tan(\theta+\varphi) \end{cases} \tag{2}$$

$$\begin{cases} (d+h)\tan\theta + \Delta l = (d-h)\tan(\theta+\varphi) \\ (\phi_{r1}-\phi_{m1})q/2\pi = \Delta l \end{cases} \tag{3}$$

From the above formulas, a height expression can be represented as

$$h = \frac{\Delta d(\phi_{m1}-\phi_{r1}) - d[(\phi_{r1}-\phi_{r2})-(\phi_{m1}-\phi_{m2})]}{(\phi_{m1}-\phi_{m2})+(\phi_{r1}-\phi_{r2})} \tag{4}$$

Equation 3 shows that height value can be directly calculated when captured phase, reference phase, distance from LCD to virtual reference surface and shifting distance are known. Furthermore, if the system has been calibrated, the reference mirror then can be put virtually at any position and reference phase can be solved previously according to spatial geometrical relation, which will be depicted below. In addition, Equation 3 also depicts only one direction captured and reference phase are needed. Therefore, once shifting distance and coded phase are measured, the absolute height data can be calculated.

3 Analytical description of reference phase distribution

Figure **3**shows the detailed geometrical arrangement for analysing reference phase distribution in the CCD imaging frame $\{C\}$. Supposing the system has been calibrated and the transformation matrix from LCD frames $\{L\}$ to CCD frame

$\{C\}$ to be T, which is expressed with homogeneous coordinates and can be divided into rotating matrix ${}^{C}_{L}\mathbf{R}$ and translating vector T_1. The virtual reference mirror $\{V\}$ is located at the distance of d from LCD screen frame $\{L\}$. Assuming CCD internal parameter matrix is expressed as A and we define x and y axis of reference mirror coincide with the axis of LCD screen. Then transformation matrix from reference surface frame $\{V\}$ to camera frame $\{C\}$ as $T_r = \begin{bmatrix} {}^{C}_{L}\mathbf{R} & T[0 \quad 0 \quad d \quad 1]' \\ \mathbf{0}_{1\times 3} & \end{bmatrix}$.

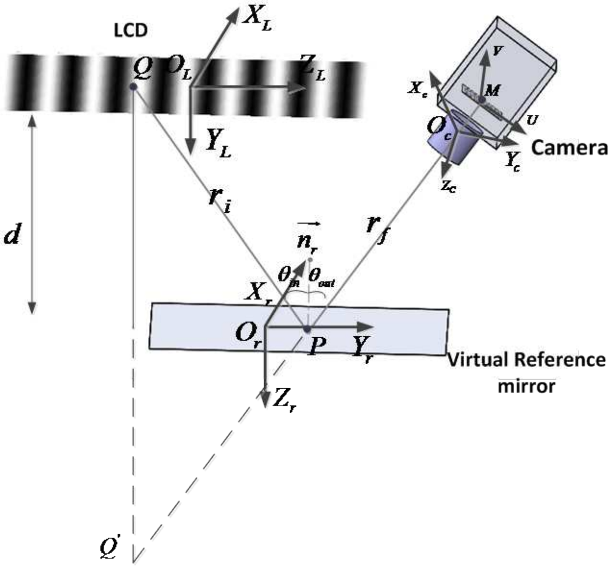

Figure 3 Geometrical analysis of virtual reference phase

To simplify this description, a single pixel point M in CCD pixel plane is considered, whose physical coordinates in frame $\{C\}$ is defined as C_M [1]. Tracing this pixel point M, the reflected ray r_f going through CCD optical centre O_c and point M intersects at the point P with the reference mirror. Based on reflection principle, the incident ray r_i will intersect with LCD screen at point Q. $\vec{n}_r$ is normal vector of point P with respect to frame $\{V\}$,which equals $[0 \quad 0 \quad -1]'$, and coordinates of point M in the virtual reference frame $\{V\}$ can be expressed as

$$\begin{bmatrix} V_M \\ 1 \end{bmatrix} = T_r' \begin{bmatrix} C_M \\ 1 \end{bmatrix} \tag{5}$$

From geometry (Figure 3), intersection point P in the virtual reference frame $\{V\}$ can be calculated using following equation:

$$V_M - V_{O_c} = k_1 (V_{O_c} - V_P) \tag{6}$$

$$V_P = V_{O_c} - \frac{V_M - V_{O_c}}{k_1} = (1 + \frac{1}{k_1}) V_{O_c} - \frac{1}{k_1} V_M \text{ , } k_1 = \frac{V_M(z) - V_{O_c}(z)}{V_M(z)} \tag{7}$$

[1] Throughout this paper, X_P is position of point P with respect to frame $\{X\}$. $X_P(z)$ represents coordinate of point P along z axis in frame $\{X\}$. ${}^{X}_{W}\mathbf{R}$ is the rotation matrix rotating vectors from frame $\{W\}$ to $\{X\}$. $\mathbf{I}_n$ is the $n \times n$ identity matrix, and $\mathbf{0}_{m\times n}$ is the $m \times n$ matrix of zeros.

Likewise, mirrored point $Q^{'}$ can be solved:

$$V_M - V_{O_c} = k_2(V_{O_c} - V_{Q^{'}}) \text{, where } V_{Q^{'}}(z) = d \tag{8}$$

$$V_{Q^{'}} = V_{O_c} - \frac{V_M - V_{O_c}}{k_2} = (1+\frac{1}{k_2})V_{O_c} \quad \frac{1}{k_2}V_M \text{, where } k_2 = \frac{V_M(z) - V_{O_c}(z)}{V_{O_c}(z) - d} \tag{9}$$

Based on principle of mirror image, coordinates of source point Q in frame $\{V\}$ can be deduced as:

$$V_Q = (I_3 - 2V_{\mathbf{n}}V_{\mathbf{n}}^T)V_{Q^{'}} \tag{10}$$

Then the captured reference phase of pixel point M can be computed with this expression $\phi = \frac{L_Q}{q}$, where

$L_Q = \begin{bmatrix} \mathbf{I}_{1\times3} & d \cdot \vec{n_r} \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix} V_Q$, q is the frequency of fringe pattern along x or y direction in frame $\{L\}$.

Unite the equations above, the reference phase can be computed using the following equation

$$\begin{cases} L_Q = \begin{bmatrix} \mathbf{I}_{1\times3} & d \cdot V_{\mathbf{n}} \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix} \begin{bmatrix} I_3 - 2V_{\mathbf{n}}V_{\mathbf{n}}^T & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & 1 \end{bmatrix} \begin{bmatrix} (1+\frac{1}{k_2})V_{O_c} - \frac{1}{k_2}V_M \\ 1 \end{bmatrix} \\ \begin{bmatrix} V_M \\ 1 \end{bmatrix} = \begin{bmatrix} {}^C_L\mathbf{R} & T[0 \quad 0 \quad d \quad 1]' \\ \mathbf{0}_{1\times3} & \end{bmatrix}^{'} \begin{bmatrix} C_M \\ 1 \end{bmatrix} \\ \begin{bmatrix} V_{O_c} \\ 1 \end{bmatrix} = \begin{bmatrix} {}^C_L\mathbf{R} & T[0 \quad 0 \quad d \quad 1]' \\ \mathbf{0}_{1\times3} & \end{bmatrix}^{'} \begin{bmatrix} \mathbf{0}_{3\times1} \\ 1 \end{bmatrix} \\ \phi = \frac{L_Q}{q} \end{cases} \text{, where } k_2 = \frac{V_M(z) - V_{O_c}(z)}{V_{O_c}(z) - d} \tag{11}$$

Equation (11) shows that once camera internal parameters and geometrical relation between LCD screen and CCD camera are known, the reference phase can be directly resolved.

4 Simulation and Experiments

This modified active reflection deflectometry is simulated based on light tracing and topography of a detected specular surface is measured to verify the theoretical analysis. In the simulation, a quadric surface $z = \frac{x^2}{400} \quad \frac{y^2}{600} \quad \frac{xy}{500}$ with measurement range [-25, 25,-25, 25] is generated to be a tested surface shown in Figure 4. For simplicity, the simulated surface is placed to be parallel to LCD screen and has a distance of 185mm from LCD monitor, which leads that only part of the position can be imaged into the camera. And the virtual reference surface is placed at the same position of detected specimen. During the simulation, horizontal fringe patterns with 23.76mm period are projected on the LCD monitor and random noises of 1% pixel position is added on the recorded fringe patterns. The shifting distance of LCD is set as 50mm. The transformation matrix between CCD camera and LCD screen is chosen as

$T = \begin{bmatrix} 0.8052 & -0.0048 & 0.593 & -175.2687 \\ 0..019 & 0.9996 & -0.0177 & -5.0766 \\ -0.5927 & 0.0255 & 0.805 & 166.6307 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. This matrix is based on a practical geometrical relation between CCD and LCD.

The main point of this CCD camera is $[501.0226 \quad 517.5536]$, normalized focal is $[2290.7 \quad 2291.2]$, and the distortion coefficient is $\left[-0.2532 \quad 0.0081 \quad 6.12\times10^{-4} \quad 3.55\times10^{-4} \quad 0 \right]$. Then, part of that quadric surface can be reconstructed according to the modified active reflection deflectometry method and reconstruction error is also shown in Figure **5**, which verifies its implementation.

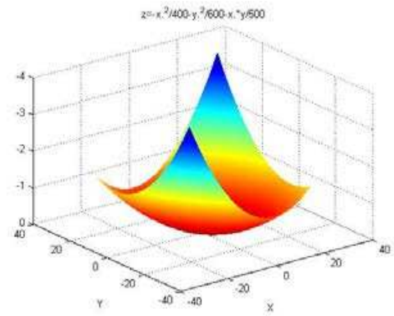

Figure 4 Simulated ideal surface

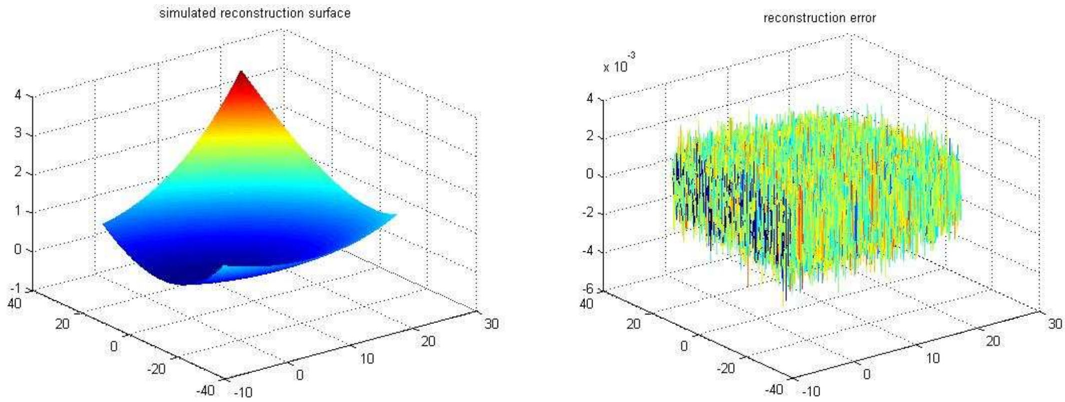

Figure 5 Reconstruction of simulated surface and reconstruction error

The experimental result of testing a concave surface is presented. Figure **6** shows its setup. The diameter of this mirror is 76.2mm and 50.8mm is for its aperture. The LCD screen is a 17-inch transistor liquid crystal display (TFT-LCD) configured at a resolution of 1280×1024 pixels and a square pixel of 0.264mm by side. Vertical sinusoidal fringe patterns of 23.76mm, 25.344mm and 31.68mm are generated on the LCD screen, which obeys the rule of optimum frequency selection. Four-step phase shifting algorithm is as well employed to each period fringe patterns. The under-test surface is located at the focal plane of CCD camera but is set at a random position. The virtual reference surface is placed parallel to LCD screen with 185mm away. The shifting distance of LCD screen is 51.9mm. One of the deformed fringe patterns captured by CCD camera is shown in Figure **6**. Choosing 60% of the reconstructed data, reconstructed surface using the proposed method is shown in Figure **7**(left) and right picture of Figure **7** gives a view of error distribution between measurement results and theoretical surface model, which is in the range of -0.2-0.2mm. The experimental work verifies the feasibility of proposed method, though accuracy is still needed to improve.

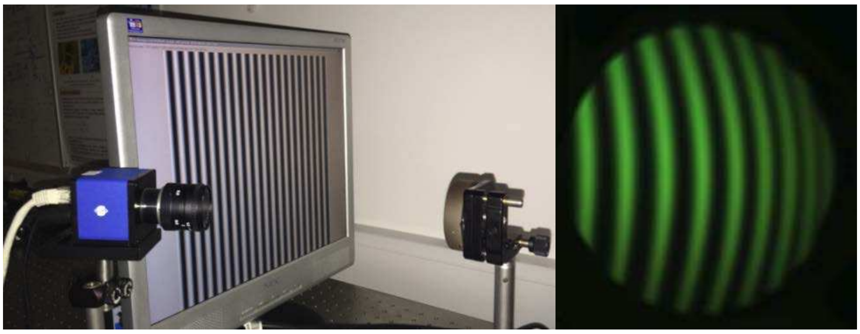

Figure 6 Experimental setup (left) and obtained vertical frigne pattern(right)

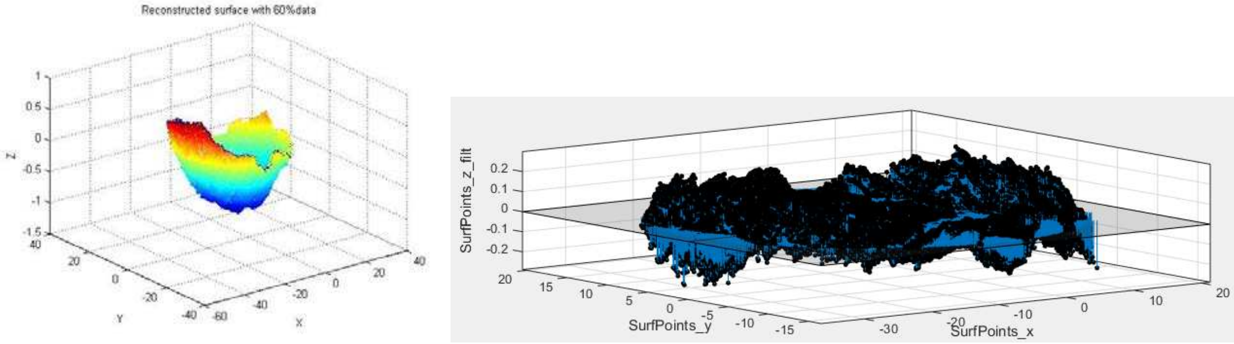

Figure 7 Reconstructed surface (left) and error distribution (right)

5 Conclusions and Discussion

In this paper, a modified method based on active fringe reflection deflectometry is presented to measure absolute topography value of specular surfaces. Compared with previous approach, this method only requires one directional fringe patterns to be projected on the LCD screen by importing a virtual reference surface into the measurement process. This work as well gives a direct relationship between captured phase and height information and tells a mathematical expression of reference phase distribution in CCD camera pixel plane. Simulation results show that this method can get a good result and experimental work verifies its implementation, though experimental test reveals that unavoidable measurement errors of a real measurement system limits the accuracy for this method. As regards future studies in this direction, combining this method with surface reconstruction algorithms, decreasing system errors should be devoted to improve measurement accuracy.

6 Acknowledge

The author gratefully acknowledges the supervisors and other staffs of the Centre for Precision Technologies in Huddersfield University of UK and Hebei Science and Industry University of China.

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