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A novel approach for ANFIS modelling based on Grey system theory for thermal error compensation

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***Abstract*— The fast and accurate modelling of thermal errors in machining is an important aspect for the implementation of thermal error compensation. This paper presents a novel modelling approach for thermal error compensation on CNC machine tools. The method combines the Adaptive Neuro Fuzzy Inference System (ANFIS) and Grey system theory to predict thermal errors in machining. Instead of following a traditional approach, which utilises original data patterns to construct the ANFIS model, this paper proposes to exploit Accumulation Generation Operation (AGO) to simplify the modelling procedures. AGO, a basis of the Grey system theory, is used to uncover a development tendency so that the features and laws of integration hidden in the chaotic raw data can be sufficiently revealed. AGO properties make it easier for the proposed model to design and predict. According to the simulation results, the proposed model demonstrates stronger prediction power than standard ANFIS model only with minimum number of training samples.**

Keywords*— *Adaptive Neuro Fuzzy Inference System; Grey system theory; thermal errors.

I. INTRODUCTION

High value manufacturing requires machine tools that can produce consistently high accuracy parts. Deformations due to the changes in the temperature of the machine tool structure create relative displacement between the tool tip and the workpiece during the machining process, which affects the dimensional accuracy of manufactured parts; these are known as the thermal errors [1], which have been reported to be approximately 70% of the total positioning error of the CNC machine tool [2]. Artificial Intelligence models have been shown to be efficient in thermal error modelling [3-5], since those methods are able to learn complex nonlinear relations and to treat imprecise data. However, these models are based on the input-output data patterns of the system under consideration. The size of the input-output data set is very crucial when the generation of data is a costly affair (machine downtime). For instance, the process of obtaining such data can take several hours for internal heating tests and many days or more for the environmental tests [6]. This is unacceptable in many production environments. Therefore, success in obtaining a reliable and robust model depends heavily on the choice of system variables involved as well as the available data set and the domain used for training purposes.

The Adaptive Neuro Fuzzy Inference System (ANFIS) has become an attractive powerful modelling technique, combining well established learning laws of Artificial Neural Networks (ANNs) and the linguistic transparency of fuzzy logic theory [7]. In the ANFIS models, different methods can be used to train the model for optimisation of the fuzzy rules [8]. A sufficient number of data samples should be used to obtain an accurate model. There is no formula to estimate the number of data sample needed to train the ANFIS network. The number can vary greatly depending on the complexity of the system under consideration [9]. However, many ANFIS networks have been trained successfully with small amounts of training data [10] [11] [9]. Buragohain and Mahanta [9] have proposed an ANFIS based modelling method where the number of data samples employed for training was minimized by application of an engineering statistical technique called full factorial design. Furthermore in [10] [11] they have applied another method called V-Fold technique. Although, their techniques were able to construct an ANFIS model with a small number of training samples (as few as 7), they still used all the experimental samples in order to select the optimal ones. Data transformation can also change the smoothness and comparability of the data. For instance, Huang and Chi Chu [12] have proposed a data transformation technique to simplify the fuzzy modelling procedures. The transformation method allows the whole raw data to be mapped to another domain such that there is no need to adjust the membership functions, and the fuzzification process is simply taking place on the fixed ones. Shmilovici and Aguilar-Martin [13] have also utilised Box-Cox transform to improve the quality of the fuzzy model, before parameter optimization occurs. Therefore, optimisation in the number of training patterns and data domain used for training are of prime concern in the field of fuzzy modelling.

The Grey systems theory, introduced by Deng in the early 1980s [14], is a methodology that focuses on solving problems involving incomplete information or small samples. The technique works on uncertain systems with partially known information by generating, mining, and extracting useful information from available data. Its most significant advantage is that it needs a small number of experimental data for

accurate prediction. Furthermore, assumptions regarding the statistical distribution of data are not needed when the Grey theory is used [15]. Grey theory considers that although the objective system appears complex, with a small amount of data, it always has some internal laws governing the existence of the system and its operation [15]. The Accumulated Generating Operation (AGO) is the most important characteristic of the Grey system theory, and its benefit is to increase the linear characteristics and reduce the randomness of the samples. As a result of accumulation, one can potentially uncover a development tendency existing in the process of Grey accumulation so that the changing trend becomes more apparent and laws of integration hidden in the raw data can be sufficiently revealed [15]. Nowadays, it is combined with intelligent computational techniques such as neural networks [3, 16], genetic algorithm [17], and fuzzy logic [18]. These successful applications inspire us to explore Grey system characteristics to systematically address ANFIS modelling.

To supplement the ANFIS model, we use the AGO to increase the linear characteristics and reduce the randomness from the measuring samples. This simple but effective technique allows us to build the thermal model under the condition of small training data. In short, the proposed model incorporates the AGO method into the ANFIS model to improve its prediction accuracy and robustness with minimal efforts. The experimental results show that the proposed model has excellent performances in terms of the accuracy of its predictive ability and reduction of machine downtime when compared against traditional and other self-learning techniques.

II. MATERIAL AND METHODS

A. Accumulation Generation Operation (AGO)

Accumulation generation is a technique used to uncover a development tendency existing in the process of accumulating Grey quantities so that the features and laws of integration hidden in the raw data can be discovered [19]. The dynamic characteristic of the proposed model results from the accumulation generation operation (AGO). The technique transforms the original data to first order 1-AGO data, which has a manageable approach to reduce the randomness of the samples, making it easier for the proposed model to be designed and to predict. The procedure of AGO is summarised as follows:

- Step 1: Consider the original series as

$$X^{(0)} = x^{(0)}(1), x^{(0)}(2), \dots\ x^{(0)}(k-1), x^{(0)}(k). \qquad (1)$$

- Step 2: from the original series, selecting the first value as the first value of the new series, selecting the first value plus the second one of the original series as the second value of the new series, selecting the sum of the first three values of the original series as the third value of the new series, and so on, as follows:

$$X^{(1)} = x^{(1)}(1), x^{(1)}(2), \dots x^{(1)}(n-1), x^{(1)}(n). \qquad (2)$$

By so doing, we obtain the new 1-AGO series $X^{(1)}$ of the original data $X^{(0)}$, which has more regular series for the benefit of modelling instead of modelling with original data.

Inverse accumulating generators operation (IAGO) can be applied to obtain the original series, selecting the first value as the first value of the new series, selecting the second value minus the first one of the original series as the second entry of the new series, selecting the third value minus the second one of the original series as the third value of the new series, and so on. The mathematical expression is as the following.

$$X^{(0)} = x^{(1)}(k) - x^{(1)}(k-1),$$
$$\text{Where} \quad k = 2,3,\dots,n. \quad x^{(0)}(1) = x^{(1)}(1) \qquad (3)$$

Therefore, by applying AGO transformation, the following important advantages can be obtained: (i) removing extreme fluctuation and noise so that the new series is more stable for modelling, (ii) the new series has a linear characteristic which makes it easier to model instead of modelling with the original data, (iii) and it has the characteristic of determining realistic governing laws from the available data [15]. The emphasis is to discover the true properties of the system under the condition of small training data.

B. Adaptive Neuro-Fuzzy Inference System (ANFIS)

The Adaptive Neuro Fuzzy Inference System (ANFIS), was first introduced by Jang [7]. According to Jang, the ANFIS is a neural network that is functionally the same as a Takagi-Sugeno type inference model. The ANFIS is a hybrid intelligent system that takes advantages of both ANN and fuzzy logic theory in a single system. By employing the ANN technique to update the parameters of the Takagi-Sugeno type inference model, the ANFIS is given the ability to learn from training data, the same as ANN. The solutions mapped out onto a Fuzzy Inference System (FIS) can therefore be described in linguistic terms. In order to explain the concept of ANFIS structure, five distinct layers are used to describe the structure of an ANFIS model. The first layer in the ANFIS structure is the fuzzification layer; the second layer performs the rule base layer; the third layer performs the normalization of membership functions (MFs); the fourth and fifth layers are the defuzzification and summation layers, respectively. More information about the ANFIS structure is given in [7]. Fig. 1 shows basic structure of the ANFIS with two inputs.

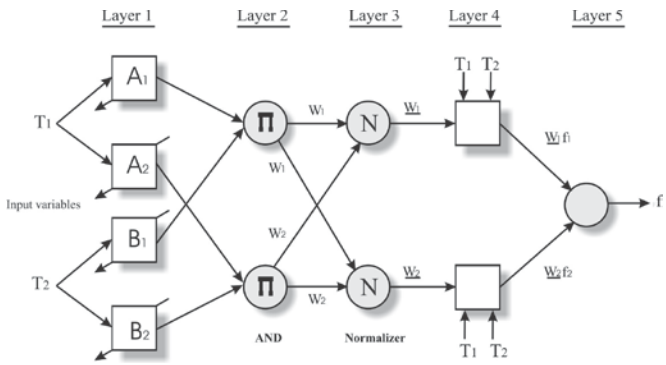

Fig. 1. Basic structure of ANFIS model.

ANFIS model design consists of two sections: constructing and training. Construction involves selecting the input variables, input space partitioning, choosing the number/type of MFs for inputs, generating fuzzy rules, premise and conclusion parts of fuzzy rules and selecting initial parameters

for MFs. Training data patterns should first be generated to build an ANFIS model. These data patterns consist of ANFIS model inputs and the desired output. However, the size of the input-output data pattern is very crucial when the generation of data is a costly affair (machine downtime). Furthermore, success in obtaining a reliable and robust model depends heavily on the choice of the domain used for construction and training purposes. To simplify the fuzzy modelling, we present a new scheme which maps the raw training data from their domain into another domain and in turn to simplify the fuzzification process. In this paper, we will use the AGO method to transfer the given data patterns to another domain as discussed earlier, which has linear characters. So, system behaviours and their hidden laws of evolution and motion of events can be accurately described. As a result, the most important characteristic of Grey theory can be implemented into ANFIS modelling.

Construction of the ANFIS model requires the division of the input-output data into rule patches. This can be achieved by using a number of methods such as grid partitioning, subtractive clustering method and fuzzy c-means (FCM) [20]. According to Jang [7], grid partition is only suitable for problems with a small number of input variables (e.g. fewer than 6). A model with three inputs with three fuzzy sets per input produces a complete rule set of 27 rules, whereas a model with six inputs requires 729 (3^6) rules. Clearly standard ANFIS models are practically limited to low dimensional modelling. It is important to note that an effective partition of the input space can decrease the number of rules and thus increase the speed in both learning and application phases. In order to obtain a small number of fuzzy rules, a fuzzy rule generation technique that integrates ANFIS with FCM clustering will be applied in this paper, where the FCM is used to systematically create the fuzzy MFs and fuzzy rules base for ANFIS. In addition, it also helps to determine the initial parameters of the fuzzy model. This is important because an initial value, which is very close to the final value, will eventually result in the quick convergence of the model towards its final value during the training process [21].

In order to maximise the model performance, a learning procedure is followed to refine the model parameters. In the training section, the membership function parameters are able to change through the learning process. The adjustment of these parameters is assisted by a supervised learning of the input-output dataset that are given to the model as training data. Different learning techniques can be used, such as a hybrid-learning algorithm combining the least squares method, and the gradient descent method is adopted to solve this training problem.

III. Results and Discussion

To verify the applicability of the proposed model, an example simulating thermal error compensation (by the same authors) in [4] is investigated. The experiments were performed on a small vertical milling centre (VMC). Three noncontact displacement transducers (NCDTs) were used to measure the drift of the tool in the X, Y and Z axes. The thermal data were measured using 58 temperature sensors placed in strips at the carrier and spindle boss surfaces. Another eleven ambient temperature sensors were placed around the machine to pick up the ambient temperature. A general overview of the experimental setup is shown in Fig. 2.

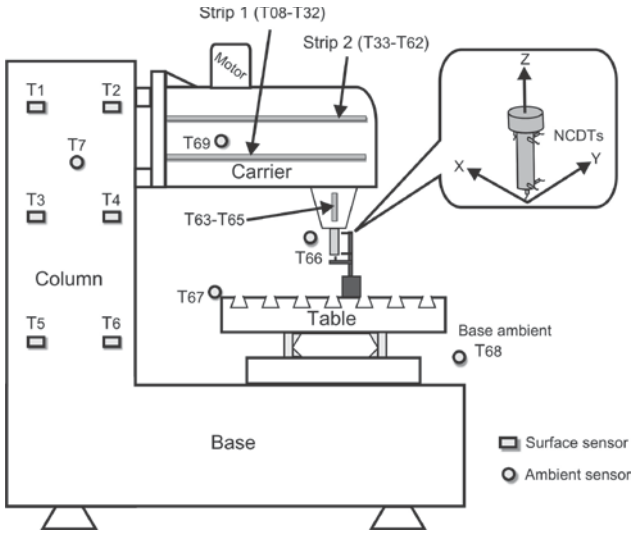

Fig. 2. A general overview of the experimental setup.

The machine was examined by running the spindle at a speed of 4000 rpm for 180 minutes to excite the thermal behaviour. It was then stopped for 120 minutes for cooling. The temperature sensors on the machine tool and the thermal drift of the spindle were measured every 10 seconds simultaneously. The maximum drift of the X-axis is 2 μm, the Y-axis is 30 μm, and the Z-axis is 10 μm. In this paper, the X-axis thermal drift is much smaller than that of Y-axis and Z-axis due to mechanical symmetry and therefore can be ignored; only the Y-axis and Z-axis will be considered. The thermal drift of the Y-axis was investigated as an example for the modelling, and error compensation.

The representative temperature sensors for modelling were selected from each group (Surface sensors and ambient sensors) according to their influence coefficient value using Grey model GM(0, N), more details about this model is given in our work [4]. The representative thermal sensors T2, T11, T44, T64, T65 and T67, which are located on the column, carrier, spindle boss, and base, are selected as the thermal key sensors for modelling.

Normally, an ANFIS model can be directly constructed from the given data patterns, which involves all operation conditions if possible. To obtain a satisfactory performance, both the structure and parameter identifications of the ANFIS model are indispensable. A total of 1795 samples (approximately: 300 minutes) were obtained from the previous experiment. The experimental samples are divided into two separated sets: the training set and the testing set. The training set is used to calibrate/train the model using a FCM and ANFIS algorithm, and the testing set is used to verify the accuracy and the effectiveness of the trained model. Among these samples, only 10 samples were used for calibration, while 1785 samples were used for testing purposes. The AGO was used to transform these samples to another domain as discussed in Section II. The 10 samples were chosen at the beginning of the

test (TABLE I and Fig. 3); six temperature sensors are used as inputs and the Y-axis displacement as output.

TABLE I. The training data from first 10 readings.

No	T2	T11	T44	T64	T65	T67	Output
0	0	0	0	0	0	0	0
1	0.101	1.082	0.755	1.250	1.292	0.069	4.984
2	0.202	2.164	1.510	2.501	2.585	0.138	9.972
3	0.303	3.247	2.265	3.753	3.879	0.208	14.96
4	0.405	4.331	3.021	5.005	5.173	0.278	19.96
5	0.506	5.414	3.777	6.258	6.468	0.347	24.96
6	0.608	6.498	4.534	7.511	7.763	0.417	29.97
7	0.710	7.583	5.290	8.764	9.059	0.487	34.98
8	0.812	8.667	6.047	10.01	10.35	0.557	39.99
9	0.914	9.752	6.805	11.27	11.65	0.627	45.01
10	1.016	10.83	7.562	12.52	12.94	0.697	50.04

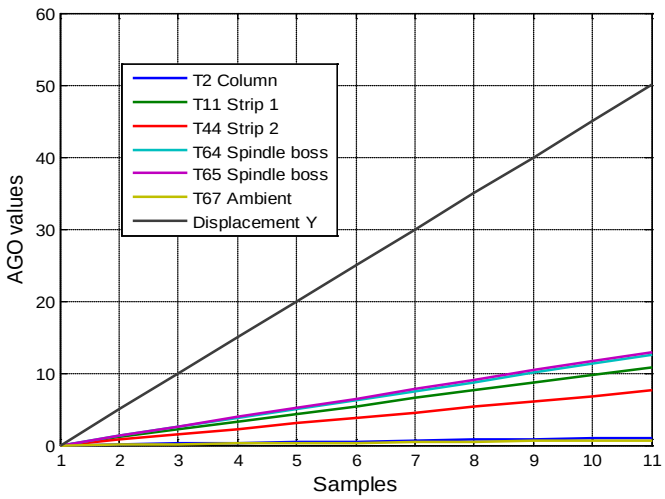

Fig. 3. The training data from first 10 readings in AGO domain.

Hence, six temperature sensors were selected as input for the model and the thermal drift in Y direction was chosen as a target variable. The MATLAB function (genfis3) was used to generate the initial fuzzy model by using fuzzy c-means (FCM) clustering with extracting a set of rules that models the training data behaviour. By doing so, the FCM clustering function is automatically evoked to determine the number of rules and MFs for the FIS model. The Gaussian functions are used to describe the membership degree of these inputs, due to their advantages of being smooth and nonzero at each point; this type of MF has been used successfully in our previous work [4]. After setting the initial parameter values in the ANFIS model, the input membership functions are adjusted using a hybrid learning scheme. In Fig. 4, an example of MFs for one input before and after learning is presented. From Fig. 4 we observe that MFs being initialized with FCM change slightly even after training. It reveals the fact that the initial MFs are quite adaptive to the characteristics of the model and thus speed up the convergence.

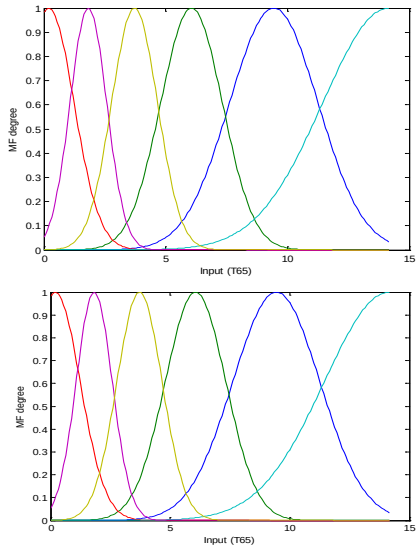

Fig. 4. Membership functions obtained through ANFIS and FCM clustering.

The performance of the model was computed using three performance criteria, including Root-Mean-Square Error (RMSE), Nash-Sutcliffe efficiency coefficient (E), correlation coefficient (R) and also residual value. The equations of first two are defined as:

$$RMSE = \sqrt{\frac{\sum_{k=1}^{n}(Z-P)^2}{n}} \quad (4)$$

$$E = 1 - \frac{\sum(Z-P)^2}{\sum(Z-\bar{Z})^2} \quad (5)$$

Where,

Z: is thermal drift.
P: is the predicted thermal drift.
$\bar{Z}$: is average of the thermal drift.
n: is the number of measured data.

Next, different ANFIS models were evaluated using Root-Mean-Square Error (RMSE), in order to measure the deviation between the measured and predicted values. In Fig. 5 the RMSE of ANFIS model for the training data is plotted versus the epochs. It is observed that after 5 epochs were used, the performance does not improve any further. Before generating the final model, it is essential to obtain the optimum number of clusters. For this purpose, several ANFIS models can be constructed with a different number of clusters. The optimum size of the model structure was determined, and the results are summarised in TABLE II. It was found that the ANFIS model with 6 clusters exhibited the lowest error RMSE=1.4805 for testing dataset. The corresponding rules of the optimum model are provided in TABLE III.

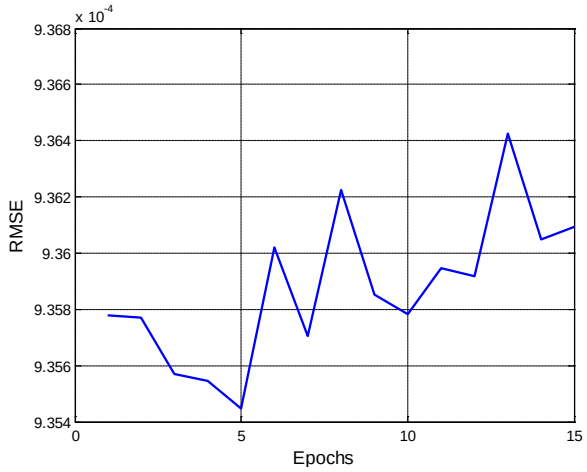

Fig. 5. RMSE of the ANFIS model during training process.

TABLE II. Performance of ANFIS models-based FCM clustering.

Model	Number of clusters	Convergence epochs	RMSE of the training data	RMSE of the testing data
1	2	1	10.3861×10^{-006}	1.9220
2	3	3	4.45636×10^{-005}	1.4843
3	4	4	1.55172×10^{-005}	1.4866
4	5	5	1.1104×10^{-005}	1.4854
5	6	5	1.10424×10^{-005}	1.4805

TABLE III. Linguistic rules.

Linguistic rules
1. If (T2 is T2cluster1) and (T11 is T11cluster1) and (T44 is T44cluster1) and (T64 is T64cluster1) and (T65 is T65cluster1) and (T67 is T67cluster1) then (displacement is displacement1cluster1)
2. If (T2 is T2cluster2) and (T11 is T11cluster2) and (T44 is T44cluster2) and (T64 is T64cluster2) and (T65 is T65cluster2) and (T67 is T67cluster2) then (displacement is displacement1cluster2)
3. If (T2 is T2cluster3) and (T11 is T11cluster3) and (T44 is T44cluster3) and (T64 is T64cluster3) and (T65 is T65cluster3) and (T67 is T67cluster3) then (displacement is displacement1cluster3)
4. If (T2 is T2cluster4) and (T11 is T11cluster4) and (T44 is T44cluster4) and (T64 is T64cluster4) and (T65 is T65cluster4) and (T67 is T67cluster4) then (displacement is displacement1cluster4)
5. If (T2 is T2cluster5) and (T11 is T11cluster5) and (T44 is T44cluster5) and (T64 is T64cluster5) and (T65 is T65cluster5) and (T67 is T67cluster5) then (displacement is displacement1cluster5)
6. If (T2 is T2cluster6) and (T11 is T11cluster6) and (T44 is T44cluster6) and (T64 is T64cluster6) and (T65 is T65cluster6) and (T67 is T67cluster6) then (displacement is displacement1cluster6)

After finishing the clustering and training process, the proposed ANFIS model can predict the thermal error from a relatively small training sample as shown in Fig. 6. Although the correlation coefficient between measured values and predicted values was closed to 1 (98%), the result is not as good as required in terms of accuracy, especially in the cooling down cycle (the maximum residual value is approximately ±4 μm). It is anticipated that further improvement in accuracy could be achieved by including cooling cycle data as part of the training data. Fig. 7 shows the output results of the simulation. The correlation coefficient is 99% and the maximum residual value is approximately ±2 μm. In practice, the training data could be also obtained by carrying out a short heating and cooling test before the stage of a manufacturing process.

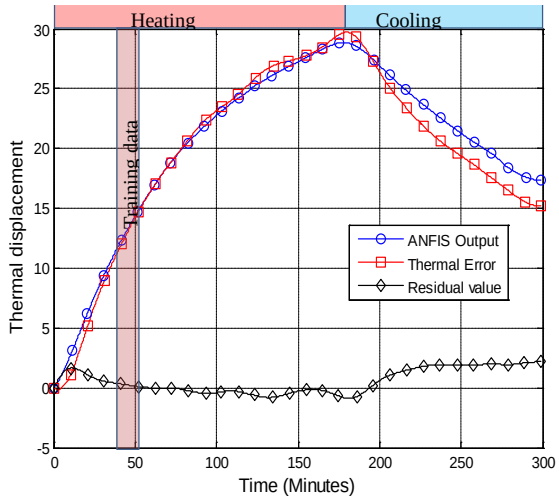

Fig. 6. ANFIS model output vs the actual thermal drift (4000 rpm, 5h test).

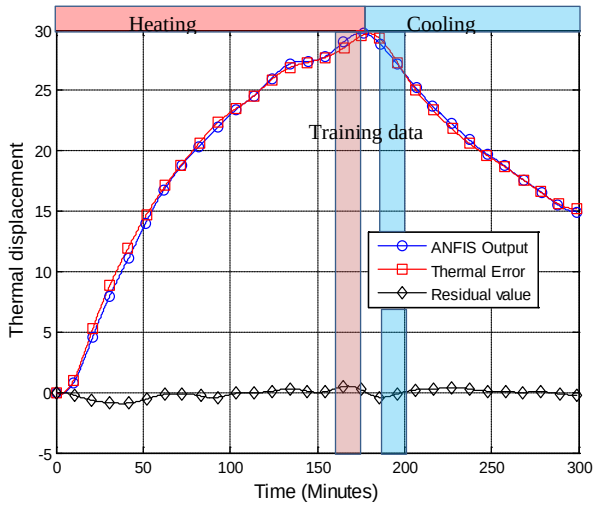

Fig. 7. ANFIS model output vs the actual thermal drift (4000 rpm, 5h).

Furthermore, different ANFIS models can be constructed using only 8 samples at any time of the heating cycle. TABLE IV illustrates 5 models that were constructed at different times during the test. From these results, it can be observed that all the models have promising values during the testing stage. Thus, the proposed model is a powerful and precise predictor of the thermal errors of the machine tool but requiring less training data and converging epochs.

TABLE IV. The characteristics of the Grey-ANFIS models.

Model	**8 samples after**	**Testing stage**			
		RMSE	***R***	***E***	***Residual***
Model-1	30 min	1.1885	0.9971	0.9743	2.5 μm
Model-2	50 min	1.1848	0.9979	0.9774	2.2 μm
Model-3	70 min	1.0989	0.9969	0.9806	2.5 μm
Model-4	90 min	1.1321	0.9959	0.9812	2.4 μm
Model-5	110 min	1.1988	0.9941	0.9705	2.3 μm

For the purpose of comparison, another test was carried out on the same machine in an operational cycle as follows: It was allowed to run at spindle speed 4000 rpm for 120 minutes, and then paused for 60 minutes before running for another 120 minutes; and then stopped for 180 minutes. Here the standard ANFIS model is derived by using the first 1080 samples (first 3 hours; 120 minutes heating and 60 minutes cooling) for training purpose. As earlier, T2, T11, T44, T64, T65 and T67 sensors were selected as inputs. The number of the membership function is three for each input and in total 729 rules can be obtained to define their relationship with thermal displacement. After setting the initial parameter values in the standard ANFIS models, the input membership functions are adjusted using a hybrid learning scheme. We next use 6 training data at the beginning of the test and another 6 samples from the cooling cycle (1 minute heating and 1 minute cooling) to construct another ANFIS model based on AGO transform. Predictive results using both models are presented in Table 6, where the two models are examined by the same testing dataset. A simulation test on the standard ANFIS model and the proposed ANFIS model are shown in Fig. 8 and Fig. 9 respectively. It is observed that the proposed model with only 12 training data samples has small residual value than the standard ANFIS model using 3 hours test (1080 training samples). Furthermore, the standard ANFIS model established by only 12 training data set has high residual value due to complexity of the network, as a result of large number of parameters, which have to be trained. According to evaluation criteria values in TABLE V, it is very clear that the proposed model has a smaller RMSE, residual value (±2 µm), higher efficiency E, and fewer rules contrasting with the standard ANFIS model. Therefore, the proposed ANFIS model is an excellent modelling choice for predicting the thermal error of the machine tools with the benefit of a small amount of training samples.

TABLE V. Performance calculation of the used models.

Models		**Standard ANFIS model**	**Proposed ANFIS model**
Number of training data		1080	12
Convergence epochs		300	3
Number of rules		729	5
Performance indices	***E***	0.86	0.98
	RMSE	3.16	1.11
	Residual	±6 µm	±2 µm

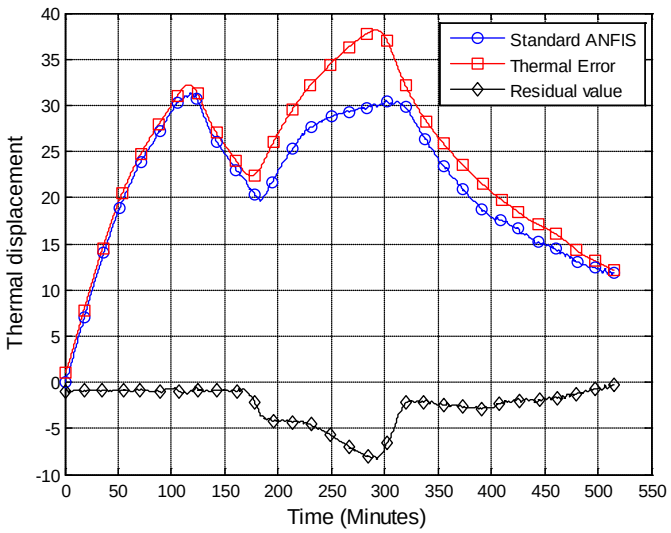

Fig. 8. Standard ANFIS model output vs the actual thermal drift (4000 rpm, 8h).

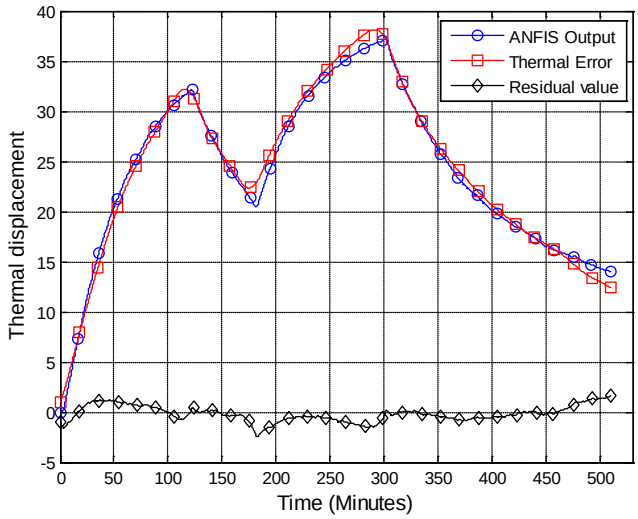

Fig. 9. Proposed ANFIS model output vs the actual thermal drift (4000 rpm, 8h).

Consequently, this paper develops a simple, less computationally intensive and low-cost approach with a high adaptation rate based on ANFIS model and Grey system theory to predict the thermal error compensation on CNC machine tools. The results obtained from the proposed model exhibit better performance than conventional ANFIS model in [4], with far fewer training samples.

During manufacturing processes, the temperature signals are collected in real time and the errors are estimated with the ANFIS model. The calculated compensation values will be used to modify the axis positions to maintain the end of the tool at the datum position. An example of such a model for a CNC machine tool is given by White et al. [22].

IV. Conclusion

In this contribution, we successfully used an ANFIS model and Grey system theory to predict thermal errors of a small

vertical milling centre with a limited amount of data for calibrate the model. The small amount of training data is a key performance variable since it directly impacts the amount of non-production testing time on the machine. To supplement the ANFIS model, we have used the AGO to increase the linear characteristics and reduce the randomness from the measuring samples. This simple but effective technique allows us to build the thermal model with a minimum amount of temperature and displacement data in a very short time scale. As a result of the proposed method, the initial ANFIS model can be sufficiently well defined to the point that it might only need a small number of training iterations. Thus, the proposed ANFIS model does not require time-consuming iterative learning procedure or prohibitive downtime required to conduct the tests. The proposed model not only preserves a fast learning characteristic but also has an excellent prediction capability. Simulation results show that the thermal error in the Y direction can be significantly reduced to less than ±2 μm using testing dataset. The work presented here is to provide the reader a novel direction to ANFIS modelling.

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