APPLICATION OF DYNAMIC FACTOR MODELLING TO FINANCIAL CONTAGION

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Abstract

Contagion has been described as the spread of idiosyncratic shocks from one market to another in times of financial turmoil. In this work, contagion has been modelled using a global factor to capture the general market movements and idiosyncratic shocks are used to capture co-movements and volatility spill-over between markets. Many previous studies have used pre-specified turmoil and calm periods to understand when contagion occurs. We introduce time-varying parameters which model the volatility spillover from one country to another. This approach avoids the need to pre-specify particular types of periods using external information. Efficient Bayesian inference can be made using the Kalman filter in a forward filtering and backward sampling algorithm. The model is applied to market indices for Greece and Spain to understand the effect of contagion during the European sovereign debt crisis 2007-2013 (Euro crisis) and examine the volatility spillover between Greece and Spain. Similarly, the volatility spillover from Hong Kong to Singapore during the Asian financial crisis 1997-1998 has also been studied.

After a review of the research work in the financial contagion area and of the definitions used, we have specified a model based on the work by Dungey et al. (2005) and include a world factor. Time varying parameters are introduced and Bayesian inference and MCMC simulations are used to estimate the parameters. This is followed by work using the Normal Mixture model based on the paper by Kim et al. (1998) where we realised that the volatility parameters results depended

on the value of the ‘mixture offset’ parameter. We propose method to overcome the problem of setting the parameter value.

In the final chapter, a stochastic volatility model with with heavy tails for the innovations in the volatility spillover is used and results from simulated cases and the market data for the Asian financial crisis and Euro crisis are summarised. Briefly, the Asian financial crisis periods are identified clearly and agree with results in other published work. For the Euro crisis, the periods of volatility spillover (or financial contagion) are identified too, but for smaller periods of time.

We conclude with a summary and and outline of further work.

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Chapter 1

Introduction

This section covers a description of financial contagion, a review of definitions of financial contagion, a description of two recent financial crises, an overview of some statistical methods used in financial contagion studies and a description of the dynamic factor model.

1.1 Financial contagion

Financial contagion is the spread of adverse financial or economic conditions from one country to another. This is the commonly understood meaning and in a general sense, the contagion effects can be seen in increased volatility in the equity or bond markets. There is no clear agreement how the contagion effects are defined or measured (Forbes & Rigobon 2000). There are different types of financial crises. For example a currency crisis caused by an exchange rate pegged to an internationally accepted strong currency. If the economic fundamentals do not justify the exchange rate, a speculative attack on the currency will cause a drain on the resources of a country in trying to defend the exchange rate. Another type of financial crisis results from high national deficits and the govenment is forced to borrow at a higher interest rate, reduce state sector spending and reduce investment which eventually leads to an economic downturn. Another example of a financial crisis is when a sudden rise in commodity prices (for example oil or fuel prices) leads to an economic downturn in a number of industries and may spread to other countries. A financial crisis in the banking sector caused by liquidity problems or a poorly performing loan portfolio can spread to housing and manufacturing industries through reduction in funds available for investment in these sectors. The cause and spread of the financial crisis is difficult to identify clearly. Financial contagion is generally interpreted as the spread of financial problems from one country to another or from one asset class to another. Given the various types of financial crises, it is understandable that different measures have been used to identify the spread of contagion effects for different cases. Generally, spread of volatility and changes in correlation have been used to study contagion effects.

The US depression of 1930s, the stock market crash 1987, US subprime lending crisis 2007-2008 and the sovereign debt and banking problems 2008-2013 in the Eurozone area are some examples of financial crises where the impact of economic problems have spread to other countries. The spread of the crises is through channels like trade, financial investments and services. Trade channels may be the physical export or import of goods and also through companies with a global presence. Financial channels would be banking, investments in equity markets, in bonds and in options and insurance markets. Funding from a common lender to multiple countries and the withdrawal or reduction of the borrowing facilities from several countries which have similar economic conditions can have detrimental effect in a number of countries at the same time (Dornbusch et al. 2000). A downturn in one market in the service sector (e.g. tourism) can lead to unemployment in related industries in other countries and lead to the movement of employees between countries. The transmission rates for different channels are different. Trade and service sector changes take place over a longer period, possibly months. However, for the integrated financial sector, the changes to the equity markets take place over a short period of time.

The prospects or fear of an adverse economic outcome in one region leads to movements of financial assets out of that region. Sales of assets in a falling market causes further reduction in prices and results in sharp declines in asset prices. If a country has previously attracted capital investment, then a loss of confidence in that country will lead to sudden reversal of capital flows and volatile markets (often referred to as flight to quality, the move to less risky assets). Herd behaviour as discussed by Pericoli & Sbracia (2003*a*) in uncertain times when investors follow the actions of others (believing the other investors have better information or from the fear of holding a position contrary to the other investors) causes further movement of assets out of the country, possibly weakening the currency. Investors may also believe that other countries with similar economic fundamentals would face similar economic problems and may withdraw assets from that country.

A number of well known crises have had major economic impact across multiple countries. Pericoli & Sbracia (2003*b*) have reviewed the issues that have been discussed in contagion literature over the previous thirty years. Financial contagion studies have addressed asset classes like currency, equity markets and property. The triggers and spread of a financial crisis appear to be different for each crisis.

Bernanke (1994) concludes that the return to the gold standard after the war and the resulting monetary contraction (the currency had to be backed by gold reserves) caused the US Depression in the 1930s. This was followed by lower consumer expenditure and concerns over employment prospects. Through trade links and bank failures the crisis spread world wide and the impact persisted into the 1930s. For the Stock Market crash in 1987, there was a period of rising equity prices in the preceding months and fears of an overvalued market (and possible threats to oil supply routes). This resulted in a lack of confidence in the markets and then the sudden fall propagated by automated algorithm trading, systems unable to cope with the volume of trades and asset sales in a falling market. The market crash spread from the USA to European and Far Eastern markets. The UK Crisis in 1992 is an example of a currency crisis where an attempt was made by the UK government to maintain the Sterling exchange rates within a narrow band of rates against the European currencies in the European Exchange Rate Mechanism (ERM) with the aim of exchange rate stability and the reduction in the rate of inflation (effectively following the German Mark with low interest rates and low inflation). When interest rates rose in Germany after the reunification, UK interest rates were kept low to protect the private housing sector interest payments on mortgages. The market believed that Sterling was overvalued in the ERM and Sterling came under speculative attack. In an attempt to defend the exchange rate within the ERM, the government used reserves to defend sterling and raised overnight interest rates to stem the outflow of Sterling. The interest rates could not be raised long term because the interest rates would damage the industrial and housing sectors. In the end, the UK had to give up the attempts to maintain an exchange rate in ERM band of exchange rates. France continued to peg the exchange rate and went through a slow economic recovery. The UK economy performed better and recovered more quickly (Mishkin 1999, Masson 1995). This is an example of a trigger (German reunification) and related economic problems spreading to neighbouring countries.

The 2007-08 Financial Market Crisis (US subprime lending) was caused by mortgage lending to subprime borrowers and the collapse in the value of the mortgage backed securities. Credit default derivatives led to margin calls which resulted in liquidity problems and financial institution failures. The financial crises became a global crisis resulting in the failure of large financial institutions and the need for state intervention to support failing banks.

The European sovereign debt crisis 2007-13, resulted from some Eurozone countries running unsustainable deficits (for example, mounting Greek sovereign debt) or through banking sector problems (Spanish bank mortgage lending to property developers, Cypriot banks lending to Greece). In an attempt to support the countries in difficulty, a Eurozone bailout fund was created. The uncertainty over the progress of the bailout negotiations for Greece and the ability of the government to implement austerity measures continued to make the European markets volatile for a long period. The market volatility spread to Spain, Ireland and Portugal where investors feared similar financial difficulties.

A common theme running through these crises is debt (for example, private sector borrowing to finance property purchases or state sector borrowing) and the failure to service the debt and repay the loans when there is an economic downturn. This failure weakens the financial institutions like banks and puts at risk the public savings. This leads to the reduciton in investment in manufacturing and service sectors and leads to reduced consumption. The spread of uncertainty (volatility) may be between asset classes and between countries. In recent years, the use of electronic trading, integrated markets and derivative based trading has increased the scale and speed of the spread of contagion to other countries. The financial crises around pegged exchange rates is explained differently - an overvalued asset (the currency) is sold by speculators (speculative attack on the currency) until the country cannot defend the currency. The currency is then devalued and the speculators profit from the situation. It could be argued that the one factor to trigger the crises is the perceived end of the growth cycle and a downturn in the economy in a country with a pegged exchange rate.

1.2 Contagion definitions

A number of definitions are used to identify financial contagion (Pericoli & Sbracia 2003*b*). One group of definitions is based on correlation changes. There is normal level of 'comovement' (correlation) in equity markets and a change in the correlation between two markets is used to indicate the spread of contagion. Correlation change is then interpreted as the propagation of a country specific shock (an adverse outcome in one country) to one or more countries. Correlation change definitions are used to study the contagion spread between equity markets. Another group of definitions is based on 'equilibrium shift'. This assumes that the economy is in one equilibrium state and moves to another equilibrium state under contagion conditions. The equilibrium shift definition is used to study the probability of a move from one pegged exchange rate to another pegged exchange rate (generally a devaluation) given that a currency crisis in a neighbouring country has resulted in a currency devaluation. Financial contagion studies use appropriate definitions to study the contagion effects for specific asset classes like equity, bonds and currency exchange rates.

In financial markets, there is a normal level of correlation between markets in related countries. During periods of uncertainty, the volatility in one market may increase. For the 'volatility spillover' definition, contagion is used to mean the volatility spillover from one country to another, additional to the normal level of comovement (Corsetti et al. 2001). Forbes & Rigobon (2002) describe contagion as a 'significant change in market comovement after a shock to one country'. Polson & Scott (2012) use the definition 'excess correlation in the residuals from a factor model incorporating global and regional market risk factors'. Given that volatility is a measure of uncertainty, the volatility spillover definition can be interpreted as 'the spread of uncertainty across international financial markets' (Pericoli & Sbracia 2003*b*). 'Contagion is a significant increase in comovements of prices across markets, conditional on a crisis occurring in one market or group of markets' (Pericoli & Sbracia 2003*b*). This definition also refers to an increase in comovements compared to a normal level. The *significant increase* is interpreted along the same lines as excessive comovement.

'Contagion occurs when cross-country comovements of asset prices cannot be explained by fundamentals'. Pericoli & Sbracia (2003*b*) use this definition if the econmomy can be assumed to be in multiple equilibrium states and moves from one equilibrium state to another (where the shift cannot be explained by economic criteria). This definition can also be used for the volatility spillover from one country to another (the volatility change in the affected country cannot be explained by the fundamental economic factors in that country).

Contagion is also defined as 'shocks that result in discontinuity in the data generating process'. This approach is used in multiple equlibria case and the definition is used to interpret the switch from one equilibrium state to another. For a stock market crisis, this can also be interpreted as a "sharp fall in the stock market index, with an upsurge in the volatility of asset prices". For a banking crisis, this can be used used to study the change in " the ratio of non-performing assets to total assets" (Pericoli & Sbracia 2003*b*).

'Contagion is a significant increase in the probability of a crisis in one country, conditional on a crisis occurring in another country' (Pericoli & Sbracia 2003*b*). This definition has been applied in the study of currency crises based on multiple equilibrium states. Eichengreen et al. (1996) study currency crises and use the definition 'probability of a crisis in a country at a point in time is correlated with the incidence of crises in other countries'. Investors assume an equilibrium state based on economic fundamentals for the pegged currency and attempt a speculative attack on a pegged currency rate if the fundamental economic factors change.

‘Shift contagion occurs when the transmission channel intensifies or, more generally, the transmission channel changes after a shock in one market’. Pericoli & Sbracia (2003*b*) indicate that this can be used to measure jumps between multiple equilibria or for discontinuity in economic variables, using the ‘transmission channel’ as an example.

In order to identify a crisis, some event such as an increase in volatility or the volatility exceeding a threshold level, is required to determine the crisis period. This is subjective and leads to loss of information (Eichengreen et al. 1996). The ‘change in the correlation’ definitions also require the prior identification of calm and crises periods. This method too becomes subjective and relies on an event or change in volatility to determine tranquil and calm periods in the markets.

The World Bank uses a ‘Broad definition’ for shock transmission across countries, a ‘Restrictive definition’ for transmission of shocks to other countries or change the cross-country correlation beyond any fundamental link among the countries and effectively measuring excess co-movement, a ‘Very Restrictive definition’ to measure a change in the transmission mechanism and also to mean an increase in the cross-country correlations during crisis periods compared to tranquil periods (Billio & Pelizzon 2003).

We attempt to move away from the prior need to identify crisis periods. The approach used in this study is to examine the development of time varying parameters that describe the volatility of the returns process and the volatility spillover. This approach would then identify the changes in the model parameters over time (say, changes in variances, covariances or volatility spillover) without the need for prior identification of the volatile and tranquil periods.

1.3 Stylised facts

We outline some commonly known results relating to the spread of contagion, the decline of equity indices and the changes in volatility derived from financial contagion studies. Negative adverse shocks spread from larger to smaller regions, the reverse impact is not significant (Corsetti et al. 2001). This is explained by the fact that trade, services and financial investments of a small country would have a smaller impact on a larger country and would be difficult to detect. The decline of an index is sharper compared to a similar rise in the market. This is referred to as the gain/loss asymmetry (Cont & Tankov 2003). This is partly explained by herd behavior in turbulent times and the sale of assets in a falling market as the market participants try to reduce the possibility of financial losses. The increase in the volatility of returns is greater after a fall in the market compared to the change in the volatility after a similar rise in the market (Cont & Tankov 2003). During financial crises, stock market returns have heavy tails, that is, the frequency of extreme values is higher than would be predicted by a Normal distribution.

Polson & Scott (2012) explain contagion shock clustering as "time series clustering" meaning that large shocks today correlate with large shocks tomorrow, "cross-sectional clustering" meaning that large shocks in one country correlate with large shocks in another country and "directional clustering" meaning that shocks show a directional (positive or negative) bias in country level returns. The results are expressed in terms of variances, covariances and correlation of returns.

Bailouts have a positive impact on the receiving country with a possibility of negative impact on the funding nations. This is a reasonable conclusion given that the funding countries are acting as lenders to a country already in difficulty. In the case of the Euro crisis, with Germany being the largest of the fund providers, there was a fall in equity markets in Germany when the first Greek bailout was agreed.

Market integration, automated or algorithm based trading, complex derivatives and leveraged trading are important factors which have changed the speed and extent of the spread of contagion on equity and currency markets.

1.4 Two cases of financial crises

Two financial crisis which have been discussed widely in recent financial contagion studies are outlined below. The data from these two crisis periods have been used in this study.

1.4.1 European sovereign debt crisis 2007-2013

After the setting up of the common currency, each Eurozone country continued to regulate its own financial sectors (Lane 2012) . The countries were set a state sector deficit limit as a proportion of the GDP and the total national debt not to exceed a proportion of the GDP. A no bailout condition meant that there could be sovereign default if a country failed to meet its loan repayments. During periods of economic downturn, the state sector deficit limits would be breached if the GDP declined and the state sector deficit remained the same. As the countries retained banking supervision powers, failures of the bank and any state support could result in the country breaching the Eurozone financial constraints. On the other hand, being part of the Eruozone meant the banks could borrow from external sources within the Eurozone without the exchange rate risks. In a period of low interest rates (2000-2007), the high level of bank lending resulted in consumption growth and increase in lending to the building industry. During that period, Greece, Spain and Portugal were running high current account deficits.

There were private capital inflows into Greece over that period. Merler & Pisani-Ferry (2012) point out that the banks in those countries were holding a large portfolio of sovereign debt (bonds) and a sovereign debt crisis would result in a banking crisis. The start of the European sovereign debt crisis 2007-2013 (also referred to as the Euro crisis in this document) can be linked to the global financial crisis 2007-2008 which was triggered by the subprime lending in USA leading to a period of recession and banking sector crisis in the European countries. Reduction in funding to (or even the withdrawal from) the perceived high risk countries resulted in financial difficulties, the need for sovereign guarantees and then the need for bailout assistance from external sources (ECB, IMF, World Bank) to the countries in difficulty. This assistance came with reform conditions which made the situation more difficult in the short term. In Spain, in 2009 the property prices declined leading to banking sector defaults and a reduction in the GDP resulted in a higher current account deficit (reduced tax revenues, increased social spending as the unemployment levels rose). In Greece, the revision of previous years current account deficits to much higher levels and the forecast deficit for the following year to much higher levels than permitted by the Eurozone rules marked the beginning of the crisis for Greece. There was a period of large private capital outflows from Greece. Merler & Pisani-Ferry (2012) point out that current account deficit alone is not a good indicator, private capital flows have to be considered. Ireland and Portugal suffered similarly and sovereign debt spreads rose compared to the German yields.

In the periods following that, the Euro crisis has been characterised by discussions about bailout funds, conditions attached to the bailout funds, market anticipation of the bailout levels and the actual announcement of the bailout agreements (following discussions in Eurozone funding countries and the IMF) which resulted in volatile markets in the European countries. Possible Greek exit from the Eurozone and growing resistance in Greece to the austerity conditions and external monitoring which was part of the bailout funding conditions continued to cause financial market turmoil in Greece which spread to the other European countries.

The plots in Figure 1.4.1 show the decline in the equity indices and the volatility in the daily log returns for Greece, Spain and the Eurostoxx index during the financial crisis in the Eurozone countries.

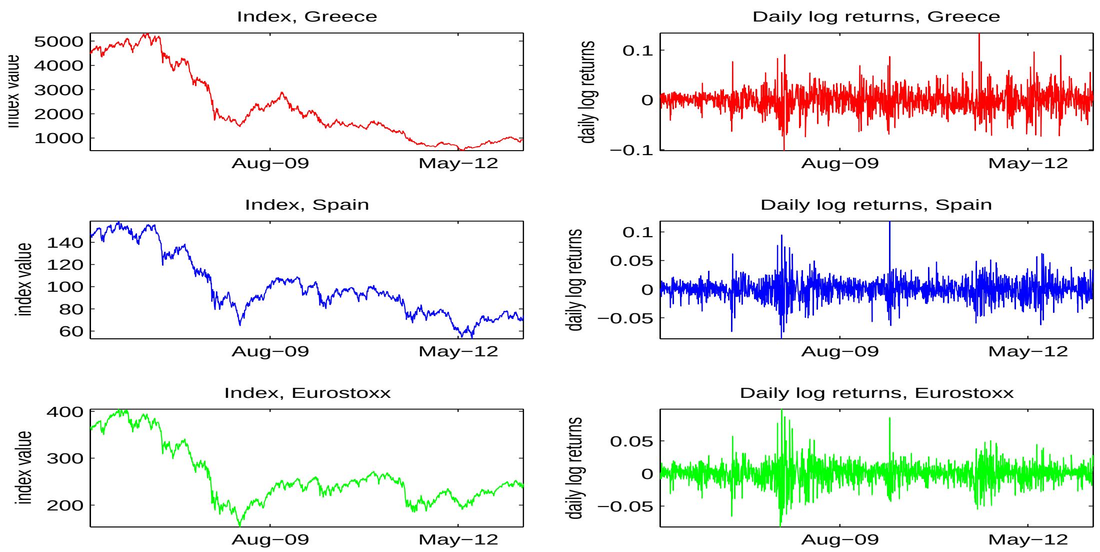

Figure 1.4.1: Stock market indices and daily log returns for Greece, Spain and Eurostoxx 2007-2013.

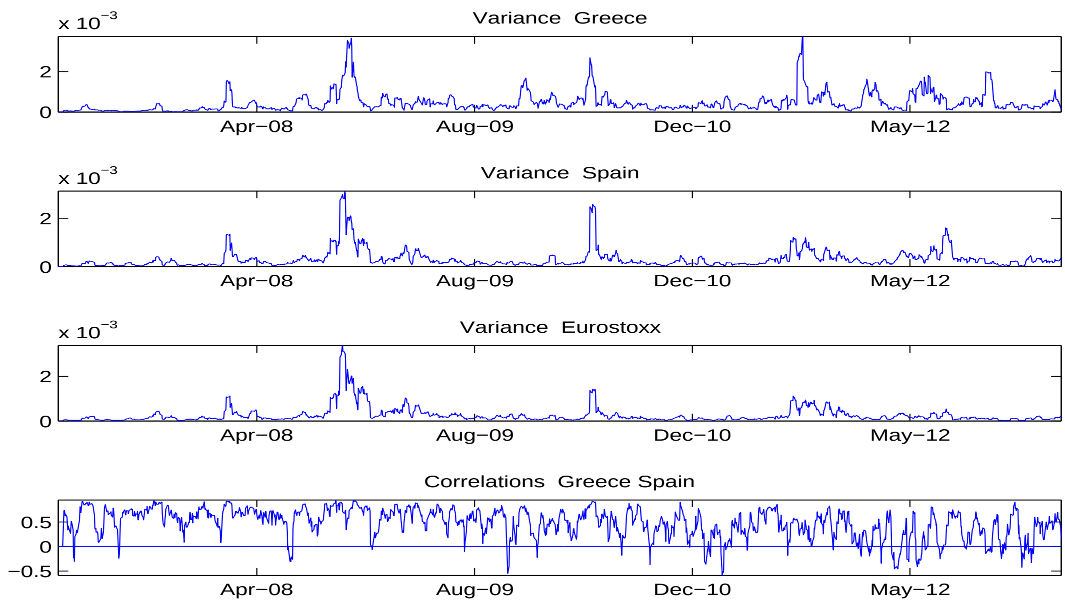

Figure 1.4.2: Variances using daily log returns for Greece, Spain and Eurostoxx (2007-2013). Correlations Greek - Spain using daily data, calculated using 10 day intervals.

The variances and correlations are calculated using the daily log returns over the previous 10 days. For Greece, Spain and the Eurostoxx returns, the variance (volatility) is high during the crisis periods. The Greece-Spain correlations are also high around the crisis periods.

1.4.2 Asian financial crisis (1997-1998)

In 1997, a number of Far East countries with pegged currency exchange rates which were not consistent with fundamental economic criteria came under speculative attack and were forced to abandon the pegged exchange rates. This was followed by a period of volatile stock markets and economic downturn spreading to a number of countries in the region. It resulted in IMF intervention and measures to reduce the state deficits (Jang & Sul 2002). There are conflicting beliefs regarding the cause of the crisis. Corsetti et al. (1999) believe that the cause was the 'moral hazard' in lending to high risk projects with the knowledge that the governments would support the companies in case of failure. In supporting the companies to repay the loans denominated in foreign currencies, the currency reserves of the countries were depleted, government current account deficits increased, capital investment levels dropped, the currencies weakened and the governments could not sustain the pegged exchange rates. An alternative view is that the underlying problems can be explained by the high level of borrowings by banks (loans denominated in foreign currencies without hedging the currency risks) and the banks lending to local companies for projects that were not profitable. In an economic downturn, non performing loans caused financial difficulties in a country and the problems then spread to the countries in the region.

In July 1997, Thailand was forced to abandon the Baht pegged exchange rate, the equity market declined and Thailand turned to the IMF for bailout assistance. In September, similar events happened in Indonesia and was followed by Korea in November 1997. This could also be explained as the loss of investor confidence in Thailand that resulted in pulling out investments in the other South East Asian countries and triggered a volatile period in the markets. The crisis also spread to Hong Kong, Singapore, Japan and other countries in the region. This crisis has been studied as a change in comovements between the countries (Jang & Sul 2002), who also use the Granger causality test to conclude that the crisis spread between the economically integrated Far East countries. Studies by Malliaris & Urrutia (1992) detect correlation changes during the crisis period compared to the period before and after the crisis. Baig & Goldfajn (1998) control for news in origin country and affected country using daily news reports and create dummy variables to represent the good and bad news for each country. They use Vector Autoregression (VAR) method to look at the comovements in the markets during the crisis period and find evidence of financial contagion in the currency and equity markets. If the markets are correlated and there is sharp change in one market followed by an expected level of change in another market, it is seen as markets responding to each other. If the correlations change significantly, it is interpreted as contagion. Baig & Goldfajn (1998) investigate the correlation changes using exchange rates, equity markets and sovereign risk spreads. They detect a higher level of correlations during the crisis period using the sovereign risk spreads (compared to a control group of European countries) and leads them to conclude that there was contagion, possibly explained as the foreign investors treating all the countries in the crisis region as having similar risks. The general conclusion in their work is that in crisis periods, market participants tend to move together across the related countries and shocks in one market are transmitted to other countries as evidenced by correlation increase during crisis periods.

The plots in Figure 1.4.3 show the decline in the equity indices (left hand plot) and the volatile periods using the daily log returns for Hong Kong, Singapore and Japan (right hand plot) during the Asian financial crisis 1997-1998.

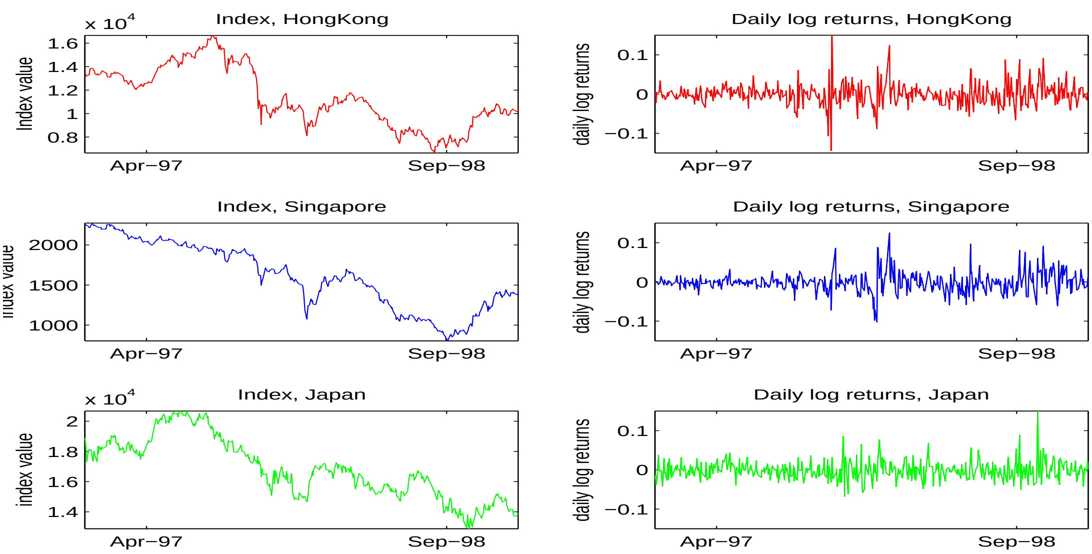

Figure 1.4.3: Stock market indices and daily log returns for Hong Kong, Singapore and Japan 1997-1998.

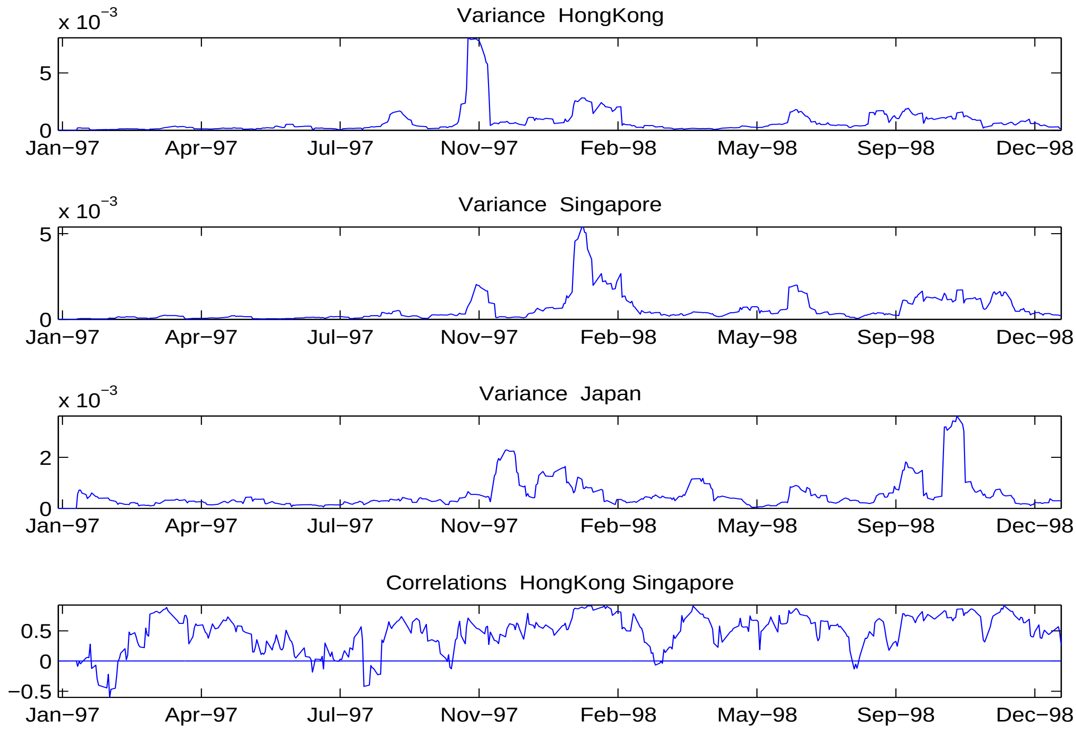

Figure 1.4.4: Variances using daily log returns for Hong Kong, Singapore and Japan (1997-1998). Correlations Hong Kong and Singapore using daily data, calculated using 10 day intervals.

The variances and correlations are calculated over a 10 day period. The results show the high volatility during the crisis periods from November 1997 to February 1998 for Hong Kong and Singapore. There was also a volatile period in the Japanese markets in the last quarter of 1998. The Hong Kong and Singapore markets are highly correlated except for a few small periods when they show low or negative correlations.

1.4.3 Contagion summary

There is normally an economic event to trigger a financial crisis in a country. Through trade and financial channels, the negative impact on the financial markets and increased volatility spread to related countries. As the markets are largely integrated, there is a normal level of correlation (comovement) under normal circumstances. In times of crisis in one country, there is volatility spillover to other countries and the correlations change. The aim of the contagion studies is to detect changes in the correlations or to detect volatility spillover from one country to another.

Intervention (say by the IMF, World Bank, ECB) provides short term liquidity assistance and longer term funding as bailout. The bailout conditions and monitoring the implementation of economic changes gives some confidence to the markets and withdrawal of assets (capital outflow) may be at a slower pace and more orderly. Strict financial constraints imposed by the bailout funds on the government spending may lead to public sector unemployment and reduction in social care programs. Similarly, strict capital requirements on banks could reduce corporate lending and investment in industries

The next section outlines the statistical methods used to study financial contagion.

1.5 Statistical analysis of contagion

In the statistical studies of financial contagion, the main methods that have been used are Regression analysis (using predefined volatile periods or using extreme values of the returns), Markov switching models (jumps from one equilibrium state to another) and Jump Diffusion models (propagation of price changes). The methods and applications are outlined below.

1.5.1 Regression and Correlation models

In the early work, correlation changes for predefined calm and volatile periods were used to study contagion effects (King & Wadhwani 1990). However, Forbes & Rigobon (2002) show that cross market correlations increase when the variance in one market increases even when there is no change in the relationship between the two markets. In crisis periods, when volatility is high, correlation estimates tend to increase and are biased upwards. After allowing for a correction for this bias and using the corrected correlation tests Forbes & Rigobon (2002) do not find evidence of contagion in certain cases. The tests rely on the 'conditional correlation', meaning that the correlation periods need prior identification using other information. Dungey et al. (2005) use a latent factor model of contagion and show that the correlation tests used in a number of contagion studies are similar. Factor models are used with common shocks and idiosyncratic shocks to explain the asset returns. The models can be extended by including lagged variables and including autoregressive dynamics for the variance and covariance terms. In an evaluation of contagion tests, Dungey et al. (2005) show that the Dungey test and Forbes Rigobon (adjusted) correlation test are similar. And they also show that the 'Exchange Market Pressure' method used by Eichengreen et al. (1996) can be interpreted as identifying a change in the correlation. For methods that rely on threshold values (Eichengreen et al. (1996) 'Exchange Market Pressure', Bae et al. (2003) 'Co-exceedances') there is loss of information and the sample size may be small.

Eichengreen et al. (1996) use a binary probit model to look at the probability of a currency crisis in a country given that there is a currency crisis in another country. They use market variables like changes in exchange rate and interest rate to calculate the 'exchange market pressure' (EMP) index which is used to indicate a crisis if the EMP values exceed a threshold. The crisis indicator is modeled against variables like the presence of crisis in another country, international reserves, domestic credit, inflation, output, employment and current account surplus to calculate the parameter to indicate the transmission of the currency crisis from one country to another. The tests are based on the significance of the transmission parameter.

Bae et al. (2003) use a different approach to measuring financial contagion, using the count of the extreme returns (returns below the 5th quantile or above 95th quantile) and then using 'co-exceedances' (extreme returns in multiple countries at the same time). Using the co-exceedances, the contagion effect is analysed by seeing how markets move together in crisis periods. Their argument is that a large number of days with small shocks means that a few days with large shocks are hidden in a correlation measure. Their data shows co-exceedances for extreme negative values are more likely than co-exceedances for positive values.

They use the count of the co-exceedances in a multinomial logistic model (positive and negative extreme values are analysed separately) to estimate the probability of co-exceedances. Part of the extreme returns that are not explained by a region's own covariates (interest rates, exchange rates, market volatility) but are explained by the extreme returns in another region is termed as contagion. They conclude that exchange rate changes are significant but interest rate changes less so. Extreme returns in Asia can be used to explain extreme returns in S. America. Like other methods that use dummy variables (based on exceeding a certain level), this method depends on the selected threshold value and suffers from information loss.

Favero & Giavazzi (2002) consider how the propagation of financial shocks is modified during periods of crisis and conclude that the use of full information models (which avoids the problems of having small samples for high volatility observations exceeding a threshold) is more efficient in detecting non linearities (sudden increase or decrease in correlations compared to normal times) in the transmission process. They use local and global information shocks in a VAR model for interest rate spreads. They conclude that country specific shocks change the correlations between countries (including a change in the sign). Similarly, international shocks can also change the correlations between other countries.

1.5.2 Markov Switching models

When contagion is defined as discontinuity in the process generating the data, Markov Switching models can be used to model jumps between multiple equilibrium states. The economy switches from one equilibrium state to another state during a crisis period. The number of equilibrium states (regimes) has to be determined exogenously and usually a fixed small number of states is used. This approach may not be useful in explaining the fundamental economic changes that lead to the financial contagion.

Jeanne & Masson (2000) use Markov Switching models to study currency devaluation probability. They present arguments for the existence of multiple equilibria. The Markov Switching model represents transition between multiple equilibrium states. The net benefit of maintaining the currency peg is based on economic fundamentals (variables) and the probability of devaluation. At a given benefit threshold, the decision maker will devalue. The economic variables are treated as exogenous variables. In the 2 state case, the Markov process transition probabilities are estimated using economic fundamentals and investor expectations of devaluation. The Hamilton (1994) EM algorithm is used for the maximum likelihood estimates for the parameters.

Billio & Caporin (2005) describe contagion (not interdependence) to mean that the transmission mechanism is discontinuous. In a multivariate GARCH model, they allow the parameters and the dynamic conditional covariance matrix to depend on a state which follows a latent Markov process (thus allowing for discontinuities in the correlations and in the parameters). A modified Hamilton filter is used to update the correlation matrix and includes steps to update the transition matrix.

Based on the earlier work of Hamilton who outlined the Markov Switching model, Fratzscher (2003) attempts to explain discontinuities (or abrupt changes) in currency exchange rates. The exchange rate is expressed as a function of lagged variables and the variance conditional on the Markov state. The lagged variables include real and financial variables to take into account trade and financial links with other countries. A Markov Switching VAR model is estimated using the EM algorithm. The model is used to study a currency crisis. Using Markov Switching models, Fratzscher (2003) concludes that there is evidence of financial contagion in the Southeast Asian markets.

1.5.3 Jump Diffusion Model

Aït-Sahalia et al. (2015) observe that large price drops affect other markets (asset returns across regions) and large price changes happen in succession (clustering) which is unlikely under Standard Brownian Motion. The aim is to model this propagation of price changes in the markets. They use a 'Hawkes Jump Diffusion' model (mutually exciting jump process with continuous Brownian Motion component). The jump intensity follows a stochastic process. When a jump occurs, the intensity of the jumps increases, but is mean reverting to a steady state. The parameters are estimated using the Generalised Method of Moments. The model is applied to market returns and they find evidence of self-excitation in the US market and cross-excitation from US to UK market (but the reverse is less pronounced).

1.5.4 Summary

The regression and correlation methods have relied on prior identification of crisis and tranquil periods using subjective criteria or knowledge of other financial events at the time. When using dummy variables, there is loss of information and reduced sample size. The Markov Switching models explain the timing of the jumps between multiple equilibria but do not help to explain the role of the fundamental economic factors in the spread of the contagion. The Jump Diffusion model allows for the economic outcomes in tranquil periods and also allows for sudden changes during volatile periods.

1.6 Factor model and data

The Factor Model, where observed variables (e.g. asset returns) are expressed as a function of other variables (e.g. output, interest rates etc, called common factors) and an error term (idiosyncratic shock), was outlined by Sargent et al. (1977) and Engle et al. (1990). It was initially used for Asset Pricing and portfolio management in order to understand the relationships between economic variables. For these models, there is evidence that variances and covariances are time varying (Ng et al. 1992) hence it is necessary to model the parameters as time varying and to allow for cross correlation in the idiosyncratic shocks.

The dynamic factor model with N asset returns y_{it} for $i = 1, \ldots, N$ is written as

$$y_{i,t} = \beta_{i,t} f_t + \epsilon_{i,t},$$

where $y_{i,t}$ (N x 1) is the return for asset i at time t, β_t (N x K) is the factor loading at time t and $\beta_{i,t}$ is the row of factor loadings for asset i, f_t (K x 1) are the factors and $\epsilon_{i,t}$ (N x 1) are the error terms. In the analysis of economic time series, the factors used may be global or country specific factors and may include lagged variables. Depending on the covariance dynamics for the error term, the model can be evaluated using Maximum Likelihood or Least Squares methods or under the GARCH framework (Santos & Moura 2014). More complicated models for which analytical methods are difficult to apply can be estimated using Bayesian inference and MCMC simulations.

Data

Equity market indices reflect the general level of economic activity and confidence in the future performance of all the business sectors combined. The pricing of equity takes into account the income (profits, dividends and the confidence level associated with the income stream), the discount rate and the inflation rate. Any fear of changes in these factors leads to an adjustment of equity prices. Rebalancing of equity portfolios, moves to other asset classes and possible movements of investments to other countries are also reflected in the equity markets. Hence the equity index for each country has been used as a factor in the contagion model.

For the Asian financial crisis 1997-1998, Hong Kong (Hang Seng), Singapore (STI), Japan (Nikkei) daily close index values have been used from the Yahoo Finance data. The indices have been converted to a common currency (USD) and then the daily log returns have been calculated. For the European sovereign debt crisis 2007-2013, Greek TM, Spain 30 and Eurostoxx TM indices (all in Euros) have been used from the Eurostoxx data to calculate the daily log returns. When one country has a holiday, the returns from the other 2 countries are omitted.

1.7 Constant parameter results

Asian financial crisis 1997-1998

The model described in Chapter 2 is used with constant volatility parameters for the Asian financial crisis and the results are shown below. Three periods are analysed, namely, the full period of the data (7 Jan 1997 - 30 Dec 1998), selected tranquil period (7 Jan 1997 - 24 Mar 1997) and selected volatile period (9 Oct 1997 - 30 Dec 1997, Hong Kong dollar under speculative attack). The volatility spillover parameter γ is described in more detail in Chapter 2.

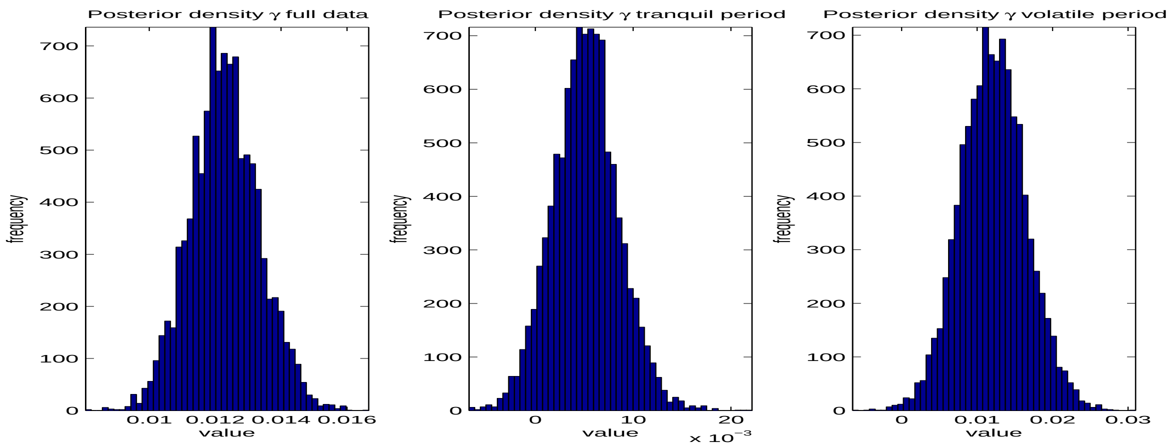

Figure 1.7.1: Posterior density for the volatility spillover parameter γ, all data, tranquil period and volatile period.

Period	γ Posterior mean	95 % credible interval
Full data	0.0123	(0.0103, 0.0144)
Tranquil period	0.0051	(-0.0013, 0.0120)
Volatile period	0.0121	(0.0037, 0.0210)

Table 1: Posterior mean and 95% credible interval for γ.

The results in Table 1 show the posterior mean and 95% credible interval for the spillover parameter γ. When all the data is used, the results show a positive value for the volatility spillover. When just the tranquil period is used the results include zero in the credible interval (indicating no contagion effect). For the volatile period, γ shows a positive value and indicates contagion effect. This shows the need for prior identification of volatile periods if constant parameters are used to study contagion periods.

European sovereign debt crisis 2007-2013

Similarly, the sampler with constant volatility parameters is used for the Euro crisis data and the results are shown below. Three periods are analysed, namely, the full period of the data (2 Jan 2007 - 23 Apr 2013), selected tranquil period (2 Jan 2007 - 31 Jul 2007) and selected volatile period (11 Sept 2009 - 4 Feb 2010, Greek bailout by ECB/IMF).

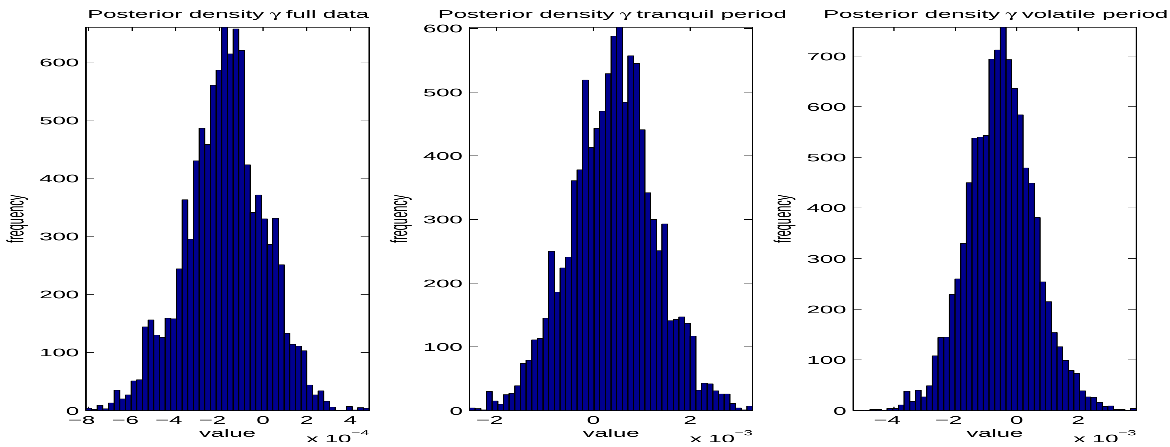

Figure 1.7.2: Posterior density for γ, all data, tranquil period and volatile period.)

Period	γ Posterior mean	95 % credible interval
Full data	-0.000175	(-0.000546, 0.000173)
Tranquil period	0.000401	(-0.001300, 0.002100)
Volatile period	-0.000509	(-0.002700, 0.001700)

Table 2: Posterior mean and 95% credible interval for γ.

The results in Table 2 show the posterior mean and 95% credible interval for the spillover parameter γ. For all the 3 selected periods, the credible interval includes zero (indicating no contagion effect). This is a limitation of using constant volatility parameters, even when the selected intervals are used if the contagion effect is short lived (over a few days at a time).

The two examples above show that by using constant parameter values, if a long period of market returns is analysed as a single period, the contagion periods are not identified. By using external events to identify high volatility periods, the constant parameter model shows higher values of the volatility spillover parameter but can miss short lived volatility spillover (contagion) periods and is subjective in the selection of high volatility periods.

1.8 Outline of following chapters

The objective is to model the volatility spillover from one country to another using time varying parameters. A global factor is used to explain the general market movements between two countries and the volatility spillover from one country to another. Chapter 2 looks at the time varying parameter model, Chapter 3 looks at the difficulty of setting a mixture offset value when a Normal Mixture is used to express the model in linear State Space form and Chapter 4 looks at the Stochastic Volatility model with normal and heavy tailed distributions for the innovations in the volatility spillover. Bayesian inference and MCMC simulations are used to estimate the parameters. A number of tests are carried out using simulated data and then results from the the Asian financial crisis and Euro crisis are described in Chapter 4. The work is summarised at the end with an outline of further work.

Chapter 2

Factor models and financial contagion

Economic factors can be used to explain the behaviour of asset returns. The Dynamic Linear Model which is used to express financial returns as a function of global and local economic factors is introduced with the dynamics of the covariance used to model volatility spillover. The model is also referred to as a 'factor model' (Harvey et al. 1992) or as a 'latent factor model' or 'dynamic linear model'. Dungey et al. (2005) use a factor model with constant parameters and a decomposition of the covariance to test the significance of the volatility spillover parameter. We add time varying parameters which follow an AR(1) process and use Bayesian inference and MCMC methods to identify the credible intervals for the spillover parameter. The sampler is tested using simulated data and then results for the Asian financial crisis and Euro crisis are shown at the end.

2.1 Factor model for financial contagion

According to the Asset Pricing Model (Fama & French 2004), asset returns depend on global factors (diversifiable risk) and idiosyncratic factors (non-diversifiable risk). The returns on an asset are expressed as a function of returns on a set of other assets (may include lagged values of the assets). The Asset Pricing Model is used as a dynamic factor model by Dungey et al. (2005) for a 2 country example,

$$y_{1,t} = f_t\theta_1 + \delta_1\nu_{1,t}, \tag{2.1.1}$$

$$y_{2,t} = f_t\theta_2 + \delta_2\nu_{2,t}, \tag{2.1.2}$$

where $y_{1,t}$ and $y_{2,t}$ are the log returns for countries 1 and 2 at time $t = 1, \ldots, T$; f_t is a global factor which affects the returns on all assets; θ_1, θ_2 are the factor loadings; $\nu_{1,t}, \nu_{2,t}$ are error terms (idiosyncratic shocks); $\nu_{i,t} \sim (0, 1)$, representing zero mean and unit variance without specifying the distribution, for $i = 1, 2$; δ_1, δ_2 are the loadings which determine the contribution of the idiosyncratic shocks. The error terms and the global returns are uncorrelated,

$$\mathrm{E}[\nu_{i,t}, \nu_{j,t}] = 0, \qquad \forall i \neq j,$$

$$\mathrm{E}[\nu_{i,t}, f_t] = 0, \qquad \forall i.$$

In finance literature, volatility of an asset is measured either by using the standard deviation or the variance of the returns for the asset. It is a measure of the uncertainty of the returns for an asset. Uncertain periods in the financial markets are associated with crisis periods (or periods of high risk from the investment perspective). In crisis periods, the volatility of asset returns spreads from one asset to another (or from one country to another or from one region to another). This spread of volatility from one country to another is termed volatility spillover. In equations 2.1.1 and 2.1.2, the returns $y_{1,t}$ and $y_{2,t}$ are independently distributed and there is no volatility spillover from country 1 to country 2.

The variance of the returns for country 2 and the covariance of the returns for the 2 countries are,

$$\mathrm{Var}(y_2) = \theta_2^2\sigma_f^2 + \delta_2^2,$$
$$\mathrm{Cov}(y_1, y_2) = \theta_1\theta_2\sigma_f^2,$$

where σ_f^2 is the variance of the global returns f_t.

In equation (2.1.1), $\nu_{1,t}$ represents the idiosyncratic shock which results in the volatility of country 1 with a loading δ_1. To model the spread of volatility from country 1 to country 2, the same idiosyncratic shock $\nu_{1,t}$ adds to the shock in the returns for country 2 with a loading γ. In times of crisis, the returns for the 2 countries are written as,

$$y_{1,t} = f_t\theta_1 + \delta_1\nu_{1,t}, \tag{2.1.3}$$
$$y_{2,t} = f_t\theta_2 + \gamma\nu_{1,t} + \delta_2\nu_{2,t}, \tag{2.1.4}$$

where γ is the volatility spillover parameter and the volatility spillover from country 1 to country 2 is represented by $\gamma\nu_{1,t}$. The variance of the returns for country 2 and the covariance of the returns for the 2 countries are now given by,

$$\mathrm{Var}(y_2) = \theta_2^2\sigma_f^2 + \delta_2^2 + \gamma^2,$$
$$\mathrm{Cov}(y_1, y_2) = \theta_1\theta_2\sigma_f^2 + \gamma\delta_1.$$

When the spillover from country 1 to country 2 is included in the model, the change in the covariance of y_1 and y_2 is $\gamma\delta_1$. As $\delta_1 > 0$ by definition, a value of $\gamma > 0$ is interpreted as an increase in covariance when there is volatility spillover. A value of $\gamma < 0$ is seen as a decrease in covariance between the returns for the 2 countries. In both these cases, the change in the correlation arising from the volatility spillover is used to conclude the presence of contagion effects. A value of $\gamma = 0$ is then interpreted as no evidence of volatility spillover (or contagion) between the 2 countries.

The change in the variance of y_2 between the contagion and no contagion cases is γ^2. Dungey et al. (2005) break down the total variance of y_2 into contribution by the global factor, the idiosyncratic factor and volatility spillover as

$$\frac{\theta_2^2\sigma_f^2}{\theta_2^2\sigma_f^2+\delta_2^2+\gamma^2},\ \frac{\delta_2^2}{\theta_2^2\sigma_f^2+\delta_2^2+\gamma^2},\ \frac{\gamma^2}{\theta_2^2\sigma_f^2+\delta_2^2+\gamma^2}.$$

These 3 terms can be used to measure the relative strength of the contagion effect.

The Dungey et al. (2005) model can be extended to include lagged values of the returns, lagged values of the world factor and autoregressive error terms. Following on from the method outlined by them, the approach has been used in a number of volatility and contagion studies. A few of the studies have been summarised below and show the areas that the model has been applied to and how it can be improved. At the end, we summarise how we have used the model to include time varying parameters.

Bekaert et al. (2009) use factor model with world and regional shocks to model the returns and examine the effect of world and country-industry factors. They recognise the need for time varying parameters and re-estimate the parameters at six month intervals. They conclude that country factors are more relevant than industry factors and that there is an upward trend in return correlations in the European stock markets because of increasing economic and financial integration.

Caporin et al. (2013) use Bayesian quantile regression methods and credit default swaps (CDS) and bond price data and conclude that risk spillover is not affected by the size of the shock. They notice a change in the intensity of the propagation of shocks in the 2003-2006 pre-crisis period compared to the 2008-2011 global financial crisis period. The increase in the correlations arises from 'larger shocks and heteroskedasticity in the data, not from similar shocks propagated with higher intensity'.

Pesaran & Pick (2007) use a contagion model to 'distinguish between contagion and interdependence'. They examine correlations in tranquil and crisis periods and use higher correlations during crises periods to indicate contagion. They highlight that the correlations methods used in the literature rely on identification of crisis and non crisis periods which would need sufficiently long periods to estimate the parameters reliably. They also identify the need to include country specific regressors and the need to use high frequency data (daily data).

Dungey & Martin (2007) aim to model financial crises across asset classes and countries by using equity and currency markets and include common and idiosyncratic factors. They use the term spillover to mean the transmission of shocks and contagion to mean the transmission of unexpected shocks (by which they mean the residual transmission after allowing for all other sources of shocks). For the currency returns, they model the spillover from a world factor, asset market factor and country factor and contagion from an idiosyncratic shock. The model also allows for a country specific shock applied to all assets for the country. They conculde that equity market shocks are significant for 3-4 days and dissipate after that.

Markwat et al. (2009) model the domino effect from local to global crashes and model the probability of crash (for example a global crash), given a regional crash. Their method relies on prior identification of periods with and without crashes. They say 'interdependence exists at all times, contagion is a form of dependence that does not exist during tranquil periods'. They define extreme drops in the returns to indicate a crash in a country or a region and a global crash if the extreme returns are for a group of large markets. They use a regression model to estimate the probability of a global crash. They find evidence of domino effect, as local crashes precede a global crash.

The Dungey et al. (2005) model has constant parameters and uses significance tests for the presence of contagion. This approach needs to identify crisis and non crisis periods using external information and is subjective. It would also be difficult to identify short lived contagion periods.

We use country equity index returns and a regional index returns (or the index returns of a large country in the region) in our study. Our approach has modified the Dungey et al. (2005) model to include time varying loadings for the global factor which explains the changes which affect all the countries in a region and to use time varying parameters for the spillover effect and idiosyncratic volatility. Using time varying parameters avoids the need for prior identification of volatile and tranquil periods by using external events to determine the periods. From other financial market studies, it is known that the relationships to the global factor and the volatility spillover change over time. By using time varying parameters, short periods of contagion would be identified more easily, something that could be missed if the tranquil and crisis periods are defined using external events. The dynamics of the load factors and spillover parameter have been added to represent the model in state space form. Using time varying parameters also permits Bayesian inference with MCMC simulations to sample the parameters and determine the credible intervals used to identify non-zero (or large) values of the spillover parameter. And by using Bayesian methods, more complex dynamics and relationships can be modeled more easily.

The objective of using time varying parameters is to overcome the limitations of a constant parameter model.

2.2 Factor Model with time varying parameters

Changing the model in 2.1.3 and 2.1.4 to include time varying factor loadings, time varying loadings for the idiosyncratic shocks and the volatility spillover, the 2 country volatility spillover model in state space form is written as

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} f_t & 0 & 1 & 0 \\ 0 & f_t & 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_{1,t} \\ \theta_{2,t} \\ \theta_{3,t} \\ \theta_{4,t} \end{pmatrix} + \begin{pmatrix} \delta_{1,t} & 0 \\ \gamma_t & \delta_{2,t} \end{pmatrix} \begin{pmatrix} \nu_{1,t} \\ \nu_{2,t} \end{pmatrix}, \tag{2.2.1}$$

$$\begin{pmatrix} \theta_{1,t} \\ \theta_{2,t} \\ \theta_{3,t} \\ \theta_{4,t} \end{pmatrix} = \begin{pmatrix} \theta_{1,t-1} \\ \theta_{2,t-1} \\ \theta_{3,t-1} \\ \theta_{4,t-1} \end{pmatrix} + \begin{pmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \\ w_{4,t} \end{pmatrix} \tag{2.2.2}$$

where $y_{1,t}, y_{2,t}$ are the daily log returns for country 1 and country 2, a global factor f_t (observed data) is used to explain the market movements affecting both countries. An intercept has also been added (we do not assume that the intercept is zero). The spillover parameter γ_t is a time varying parameter. The volatility spillover from country 1 to country 2 is then represented by $\gamma_t \nu_{1,t}$; $\theta_{1,t}$ and $\theta_{2,t}$ are the regression coefficients for the global factor f_t; $\theta_{3,t}$ and $\theta_{4,t}$ are the regression intercept terms and $\delta^2_{1,t}$ and $\delta^2_{2,t}$ are the conditional volatility parameters for countries 1 and 2. The error terms $\nu_{1,t}$ and $\nu_{2,t}$ are independently distributed as $\mathrm{N}(0, 1)$. Log returns $y_{i,t}$ and global parameter f_t are observations at times $t = 1, \ldots, T$ and $i = 1, 2$. Equation 2.2.2 is the state evolution equation for the regression parameters and $\theta_{i,t}$ are independently distributed for $i = 1, \ldots, 4$ and conditionally independent. The conditional covariance matrix W of θ is defined to be a diagonal matrix with elements $w_{i,i}^2$ for $i = 1, \ldots, 4$. And conditional on $\theta_t = (\theta_{1,t}, \theta_{2,t}, \theta_{3,t}, \theta_{4,t})'$, the covariance matrix for the returns $y_{1,t}$ and $y_{2,t}$ is

$$V_t = \begin{pmatrix} \delta_{1,t}^2 & \delta_{1,t}\gamma_t \\ \delta_{1,t}\gamma_t & \delta_{2,t}^2 + \gamma_t^2 \end{pmatrix}.$$

The volatility spillover parameter γ_t and the variance parameters $\delta_{1,t}, \delta_{2,t}$ are time varying parameters. An AR(1) process is used for the log volatility and for the spillover parameter γ_t with normal error terms. Heavy tailed distributions are introduced later for errors in the AR(1) process that is used for the volatility spillover parameter γ_t.

For the volatility parameters, the notation is changed to that which is commonly used for Stochastic Volatility models

$$\delta_{1,t} = \exp\left(\frac{h_{1,t}}{2}\right),$$
$$\delta_{2,t} = \exp\left(\frac{h_{2,t}}{2}\right).$$

Hence

$$h_{1,t} = \log \delta_{1,t}^2,$$
$$h_{2,t} = \log \delta_{2,t}^2.$$

The likelihood function is

$$p(\tilde{y}_t | h_{1,t}, h_{2,t}, \theta, \gamma) \propto |V_t|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\tilde{y}_t' V_t^{-1} \tilde{y}_t)\right), \tag{2.2.3}$$

where

$$\tilde{y}_t = \begin{pmatrix} y_{1,t} - f_t\theta_{1,t} - \theta_{3,t} \\ y_{2,t} - f_t\theta_{2,t} - \theta_{4,t} \end{pmatrix}$$

and the covariance matrix is

$$V_t = \begin{pmatrix} e^{h_{1,t}} & \gamma_t e^{h_{1,t}/2} \\ \gamma_t e^{h_{1,t}/2} & \gamma_t^2 + e^{h_{2,t}} \end{pmatrix}. \tag{2.2.4}$$

Prior distributions

The aim of selecting an appropriate prior distribution for a parameter is to select a distribution so that the 'true' value of the parameter is within the values permitted for the distribution. If possible, a conjugate (or conditionally conjugate) density is used to enable easier sampling from the posterior density. A non informative prior or an improper prior is used if it leads to a proper posterior density for the parameter to be sampled. The objective is to ensure that the posterior density is not strongly influenced by the prior density, hence allowing the posterior density to depend more clearly on the information in the data (Andrew Gelman & Rubin 2004). The parameters (often called the hyper parameters) for the prior distribution can be set as constants so the the prior is non informative or the parameters can be estimated from the data or prior distributions can also be used for the hyper parameters and this allows more flexibility.

Regression parameters have Normally distributed errors, $\theta_{i,t}|\theta_{i,t-1} \sim \mathrm{N}(\theta_{i,t-1}, w_{i,i}^2)$ and the prior density for $w_{i,i}^2$ is $\mathrm{IG}(0.001, 0.001)$ so that the posterior density depends largely on the data. For the volatility parameter $h_{1,t}$ and $h_{2,t}$, a conditional Normal (as an AR(1) process) is used but the posterior distribution cannot be recognised as a standard distribution. For the volatility parameter γ an AR(1) process $\gamma_t = \mu_3 + \phi_3(\gamma_{t-1} - \mu_3) + \sigma_{\eta 3}\eta_t$ with Normally distributed error terms is used and standard results for the Kalman Filter with Normal errors can be used. For the AR(1) process parameters, the priors used by Kim et al. (1998) have been used (an improper prior for μ with proper posterior density; a Beta prior for ϕ with parameters such that the prior mean is around 0.86 which is close to the volatilty persistence value found in financial data and an IG(5, 0.05) for the variance parameter σ_η^2).

2.3 Sampler

The regression parameters θ, the volatility parameters $h_{1,t}$ and $h_{2,t},$ for $t = 1, \ldots, T$, the volatility spillover $\gamma_t,$ for $t = 1, \ldots, T$ and the covariance matrix W are updated using filtering methods, full conditional distributions or posterior densities.

Updating θ

We use the Forward Backward Sampling method (Carter & Kohn 1994) to sample the values of θ_t. Backward Sampling uses Bayes theorem to update the distribution of θ_{t-1} conditional on θ_t as,

$$p(\theta_{t-1}|\theta_t) \propto p(\theta_t|\text{backward distribution})p(\theta_{t-1}|\text{forward distribution})$$

where the first of the two terms on the right is the likelihood of θ_t and the second term is the prior density of θ_{t-1}. The aim is to derive the forward distribution at time T, using all the information up to time T. Then sample θ_T and update the distributions for $t = T-1, T-2, \ldots, 1$ which means that all the distributions have been derived using all the available information up to time T

To derive the forward distributions, conditional on the world factor f_t, the model in equations (2.2.1) and (2.2.2) which is in linear State Space form for θ with normally distributed errors is used. The Kalman Filter prediction and updating steps (Meinhold & Singpurwalla 1983) are used to derive the forward distributions for θ_t for $t = 1, \ldots, T$. The distribution at time T is the result of using all the observations $y_{1,t}$ and $y_{2,t}$ for $t = 1, \ldots, T$. For the Backward Sampling, we use the decomposition

$$p(\theta_1, \theta_2, \ldots, \theta_T|y_1, y_2, \ldots, y_t = \prod_{i=1}^{T} p(\theta_{T-i+1}|\theta_{T-i+2}, \ldots, \theta_T, y_1, y_2, \ldots, y_T).$$

Backward Sampling starts by sampling θ_T and the distribution for θ_{T-1} is updated conditional on the sampled value of θ_T. Then θ_{T-1} is sampled from the updated density at time $T-1$ and the process is repeated to have a sample θ_t for $t = 1, \ldots, T$.

Denoting the forward distribution results from the Kalman Filter step as $\theta_{t-1}|y_{1:t-1} \sim \mathrm{N}(\hat{\theta}_{t-1}, \hat{\Sigma}_{t-1})$, the backward distribution for θ_{t-1} is given by

$$\begin{aligned}(\theta_{t-1}|\theta_{t:T}, y) &\sim \mathrm{N}(\mu_{t-1}, C_{t-1}),\\ \mu_{t-1} &= (W^{-1} + \hat{\Sigma}_{t-1}^{-1})^{-1}(W^{-1}\theta_t + \hat{\Sigma}_{t-1}^{-1}\hat{\theta}_{t-1}),\\ C_{t-1} &= (W^{-1} + \hat{\Sigma}_{t-1}^{-1})^{-1}\end{aligned}$$

where W is the covariance matrix for the state equation for θ and $\theta_{t:T}$ are the backward samples $\{\theta_t, \theta_{t+1}, \ldots, \theta_T\}$. The process of Forward filtering using the Kalman Filter and Backward sampling is referred to as 'Forward Filtering Backward Sampling (FFBS)'.

Updating W

Using a conjugate Inverse Gamma prior for $w_{i,i}^2$,

$$w_{i,i}^2 \sim \mathrm{IG}(\alpha, \beta),$$

the full conditional density for updating $w_{i,i}^2$ given $\theta_{i,t}, i = 1, \ldots, 4, t = 1, \ldots, T$ is

$$w_{i,i}^2|\theta_{it} \sim \mathrm{IG}(A, B)$$

where

$$A = \left(\alpha + \frac{T-1}{2}\right)$$

and

$$B = \left(\beta + \frac{\sum_{t=2}^{t=T}(\theta_{i,t} - \theta_{i,t-1})^2}{2}\right).$$

To make the prior for $w_{i,i}^2$ non informative, the parameters for the prior distribution of $w_{i,i}^2$ are set to $\alpha = 0.001$ and $\beta = 0.001$.

Updating volatility parameter h_1

From equation 2.2.3 the joint density for $\tilde{y}_t = (\tilde{y}_{1,t}, \tilde{y}_{2,t})'$ is

$$p(\tilde{y}_t|h_{1,t}, h_{2,t}, \theta, \gamma) \propto |V_t|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\tilde{y}_t^{'} V_t^{-1} \tilde{y}_t)\right),$$

and using an AR(1) process for $h_{1,t}$

$$h_{1,t} = \mu_1 + \phi_1(h_{1,t-1} - \mu_1) + \sigma_{\eta_1}\eta,$$

the density for $h_{1,t}$, using the results from Kim et al. (1998), is

$$h_{1,t}|h_{1,t-1}, h_{1,t+1} \sim \mathrm{N}(\mu_1 + \frac{\phi_1(h_{1,t-1} - \mu_1) + (h_{1,t+1} - \mu)}{1 + \phi_1^2}, \frac{\sigma_{\eta_1}^2}{1 + \phi_1^2}),$$

where μ_1 is the long term mean for h_1, ϕ_1 is the volatility persistence parameter for h_1, $\sigma_{\eta_1}^2$ is the variance for h_1. The posterior density for $h_{1,t}$ is

$$p(h_{1,t}|\tilde{y}_t, \theta, \mu_1, \phi_1, \sigma_{\eta_1}^2, h_{2,t}) \propto p(h_{1,t}|h_{1,t-1}, h_{1,t+1}, \mu_1, \phi_1, \sigma_{\eta_1})\, p(\tilde{y}_t|h_{1,t}, h_{2,t}, \theta, \gamma).$$

As V_t has terms in $h_{1,t}$, it is not possible to recognise this as a standard density for $h_{1,t}$. A symmetric random walk is used to propose $h_{1,t}^{'}$ and an Adaptive Metropolis-Hastings (Griffin & Stephens (2013), Haario et al. (2001)) step is used to obtain an acceptance rate of around 20%. The proposed $h^{'}_{1,t}$ is

$$h^{'}_{1,t} = h_{1,t} + \mathrm{N}(0, \sigma^2_{h_{1,t}}),$$

and the M-H ratio is

$$a = \min\left(\frac{p(h^{'}_{1,t}|\tilde{y}_t)p(\tilde{y}_t|h^{'}_{1,t})q(h^{'}_{1,t}, h_{1,t})}{p(h_{1,t}|\tilde{y}_t)p(\tilde{y}_t|h_{1,t})q(h_{1,t}, h^{'}_{1,t})}\right).$$

A symmetric random walk is used to propose $h^{'}_{1,t}$ and hence the q terms cancel out in the M-H ratio. The proposed move is accepted with probability a. An Adaptive M-H step is used to update $\sigma^2_{h_{1,t}}$, the variance term for the random walk.

The parameters $\phi_1, \mu_1, \sigma^2_{\eta_1}$ for the AR(1) process for h_1 are updated as below, using the results from Kim et al. (1998)

Updating μ_1

Using a diffuse prior for μ_1, so that $\mu_1 \propto 1$ and the full conditional density is

$$p(\mu_1|h_{1,1}, \ldots, h_{1,T}) \sim \mathrm{N}(\hat{\mu_1}, \sigma^2_{\mu_1}),$$

where

$$\hat{\mu_1} = \sigma^2_{\mu_1}\left(\frac{(1-\phi_1^2)}{\sigma^2_{\eta_1}}h_{1,1} + \frac{(1-\phi_1)}{\sigma^2_{\eta_1}}\sum_{t=1}^{T-1}(h_{1,t+1} - \phi_1 h_{1,t})\right)$$

and

$$\sigma^2_{\mu_1} = \sigma^2_{\eta_1}\left((T-1)(1-\phi_1)^2 + (1-\phi_1^2)\right)^{-1}.$$

Updating ϕ_1

Using $\phi_1 = 2\phi^* - 1$, and ϕ^* with a Beta(a,b) prior distribution

$$\pi(\phi_1) \propto \left(\frac{1+\phi_1}{2}\right)^{a-1}\left(\frac{1-\phi_1}{2}\right)^{b-1},$$

the posterior density for ϕ_1 is

$$p(\phi_1|h_{1,1}, \ldots, h_{1,T}) \propto \pi(\phi_1) f(h_1|\mu_1, \phi_1, \sigma^2_{\eta_1})$$

and the log of the posterior density is

$$\log f(h_1|\mu_1, \phi_1, \sigma^2_{\eta_1}) \propto -(h_1-\mu_1)^2(1-\phi_1^2) + \frac{1}{2}\log(1-\phi_1^2) - \sum_{t=1}^{T-1} \frac{((h_{1,t+1}-\mu_1) - \phi_1(h_{1,t}-\mu_1))^2}{2\sigma^2_{\eta_1}}.$$

The range of ϕ_1 is $-1 < \phi_1 < 1$ and the parameters for the prior density are $a = 20$, $b = 1.5$ which gives a prior mean of 0.86. The parameter is sampled from a Truncated Normal density to ensure that $-1 < \phi_1 < 1$ and Adaptive M-H is used to get acceptance rates of around 20% for the proposed values.

Updating $\sigma^2_{\eta_1}$

Using an Inverse Gamma prior density $\sigma^2_{\eta_1} \sim \text{IG}\left(\frac{\sigma_r}{2}, \frac{\text{S}_\sigma}{2}\right)$, the full conditional distribution is $\sigma^2_{\eta_1}|h_1, \phi_1, \mu_1 \sim \text{IG}(A, B)$ where the parameters A and B are

$$A = \frac{T + \sigma_r}{2}$$

and

$$B = \frac{\text{S}_\sigma + (h_{1,1}-\mu_1)^2(1-\phi_1^2) + \sum_{t=1}^{T-1}((h_{1,t+1}-\mu_1) - \phi_1(h_{1,t}-\mu_1))^2}{2}.$$

The parameters for the prior density are $\sigma_r = 5, \text{S}_\sigma = 0.01\sigma_r$.

Updating $h_{2,t}$ and AR(1) parameters for $h_{2,t}$

For the parameter h_2, an AR(1) process has been used. The sampling for h_2 and the AR parameters $\mu_2, \phi_2, \sigma^2_{\eta_2}$ is similar to that for h_1 and the AR parameters $\mu_1, \phi_1, \sigma^2_{\eta_1}$. One at a time sampling is used for h_2. A simple random walk is used to propose $h'_{2,t}$ with an Adaptive Metropolis-Hastings step to obtain 20% acceptance rates.

Updating γ and AR(1) parameters for γ

Rearranging equation 2.2.1,

$$\nu_{1,t} = \frac{y_{1,t} - f_t\theta_{1,t} - \theta_{3,t}}{\text{e}^{h_{1,t}/2}}.$$

Substituting $\nu_{1,t}$, in 2.2.2,

$$y_{2,t} = f_{2,t}\theta_{2,t} + \theta_{4,t} + \left(\frac{y_{1,t} - f_t\theta_{1,t} - \theta_{3,t}}{e^{h_{1,t}/2}}\right)\gamma_t + e^{h_{2,t}/2}\nu_{2,t}, \tag{2.3.1}$$

and using an AR(1) process for γ,

$$\gamma_t = \mu_3 + \phi_3(\gamma_{t-1} - \mu_3) + \sigma_{\eta_3}\eta_t, \tag{2.3.2}$$

$$\eta_t \sim \mathrm{N}(0, 1),$$

means that equations (2.3.1) and (2.3.2), given the parameters θ, $h_{1,t}, h_{2,t}$ and f_t, and $y_{1,t}$, are in linear State Space form for γ with normally distributed errors. Hence we can use the FFBS method to sample γ_t in a similar way to the steps described for θ_t above. The sampling for the parameters μ_3, ϕ_3 and $\sigma^2_{\eta_3}$ for the AR(1) process for γ is similar to that for the AR(1) parameters for h_1.

2.4 Results

Simulated data with known parameters is used to understand how the sampler performs. The sampler described in section 2.3 is written in Matlab and 25000 iterations are used as burn in period, followed by a further 25000 iterations and the samples are thinned to 1 in 5. Following that, the Asian financial crisis and Euro crisis data are analysed and the results shown below.

2.4.1 Simulated data

Parameter	Initial value	(μ, σ^2)
θ_1	0.7	$(0, 10^{-4})$
θ_2	0.5	$(0, 10^{-4})$
θ_3	0.0	$(0, 10^{-5})$
θ_4	0.0	$(0, 10^{-5})$

Parameter	μ	ϕ	σ_η^2
h_1	-5	0.95	0.02^2
h_2	-5	0.95	0.02^2
γ	10^{-4}	0.60	0.01^2

Table 3: Parameters used to simulate the data

Using the initial value of θ_i as in Table 3 above, θ_{it} is generated as $\theta_{it} = \theta_{i,t-1} + \mathrm{N}(\mu, \sigma^2)$, for $i = 1, \ldots, 4$, $t = 2, \ldots, T$. The initial value of the parameter h_1 is sampled from the steady state distribution $\mathrm{N}(\mu, \frac{\sigma_\eta^2}{1-\phi^2})$ and then the AR(1) process $h_{1,t} = \mu + \phi(h_{1,t-1} - \mu) + \mathrm{N}(0, \sigma_\eta^2)$, for $t = 2, \ldots, T$, is used to generate the values of $h_{1,t}$. The parameters $h_{2,t}$ and γ_t are generated similarly using the values in Table 3 above. The log returns for world factor f_t are generated independently from $\mathrm{N}(0, 0.016^2)$. The log returns $y_{1,t}$ and $y_{2,t}$ are then simulated using equation 2.2.1.

2.4.2 Results, simulated data

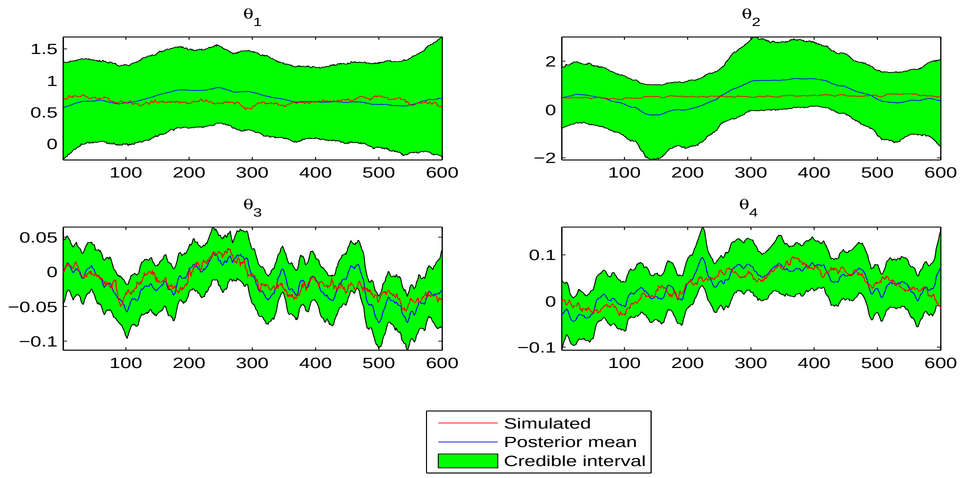

Figure 2.4.1: Simulated value, posterior mean and 95 % credible interval for the regression parameter θ.

The posterior means for the regression parameters closely follow the simulated values and the 95% credible interval for the posterior mean includes the simulated values.

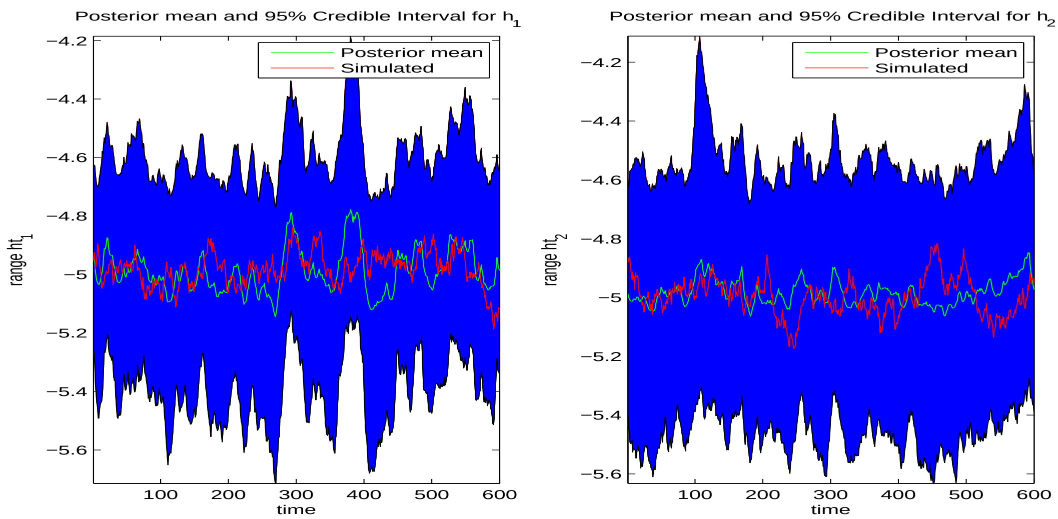

Figure 2.4.2: Simulated value, posterior mean and 95 % credible interval for h_1, h_2.

The posterior mean for the volatility parameters h_1 and h_2 are close to the simulated values. The simulated values are within the 95% credible interval.

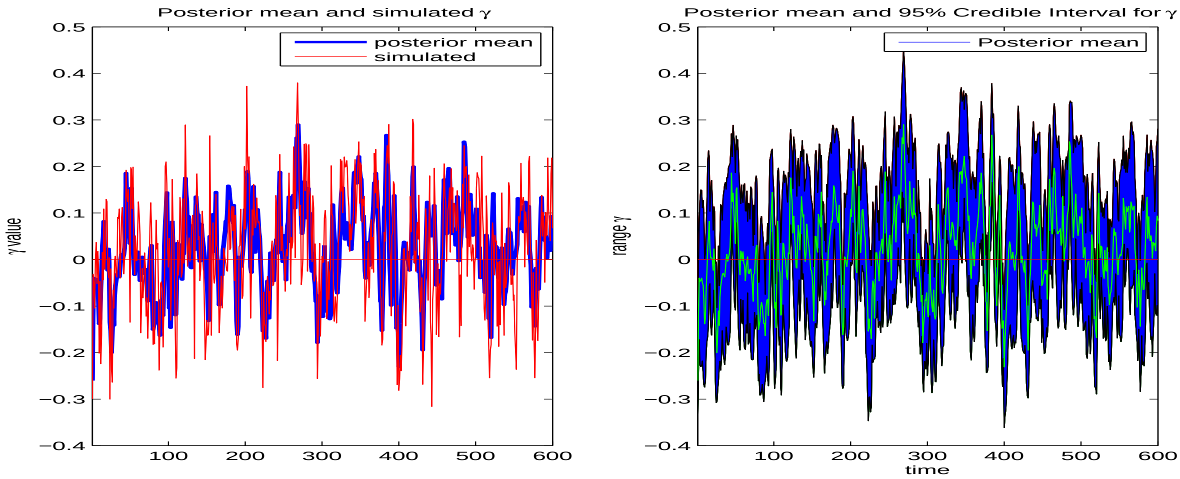

Figure 2.4.3: Posterior mean and simulated γ (left plot), posterior mean and 95 % credible interval for γ (right plot).

The sampler estimates the spillover parameter quite well, the posterior mean generally follows the simulated values (left hand side plot). The 95% credible interval and the posterior mean are shown in the right hand side plot. The simulated line is not shown in the plot to keep the plot readable. Using other plots not shown here, it has been checked that the simulated values are within the credible interval (which shows that the sampler performs well for the spillover parameter).

AR Parameter results

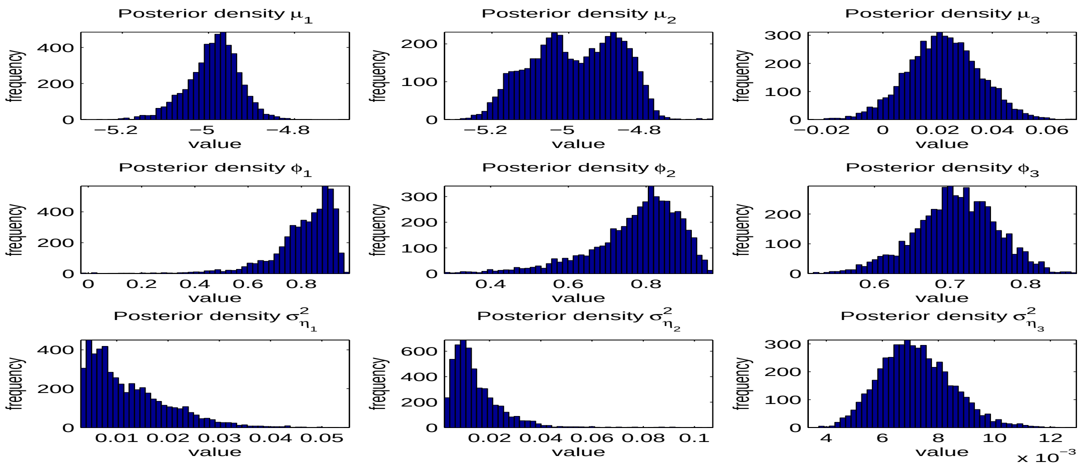

Figure 2.4.4: 95 % Posterior distributions for the AR(1) parameters $(\mu, \phi, \sigma_\eta^2)$ for h_1 (left column), h_2 (middle column) and γ (right column).

The trace plots for the parameters show that the values converge quite quickly and a burn in period of 25000 iterations is sufficient. The posterior densities are shown in the above plots.

Parameter	Simulated value	Posterior mean	95% Credible Interval
μ_1	-5	-4.9884	(-5.1183, -4.8816)
ϕ_1	0.9500	0.8074	(0.4692, 0.9411)
$\sigma^2_{\eta_1}$	0.0004	0.0126	(0.0037, 0.0319)
μ_2	-5	-4.9823	(-5.1888, -4.7974)
ϕ_2	0.9500	0.7731	(0.4680, 0.9322)
$\sigma^2_{\eta_2}$	0.0004	0.0150	(0.0037, 0.0403)
μ_3	0.0001	0.0221	(-0.0043, 0.0482)
ϕ_3	0.6000	0.5919	(0.5919, 0.8076)
$\sigma^2_{\eta_3}$	0.0100	0.0072	(0.0049, 0.0099)

Table 4: Simulated value, posterior mean and 95% credible interval, AR(1) parameters μ, ϕ and σ^2_η for h_1, h_2 and γ.

The values for μ_1, μ_2, μ_3 and ϕ_1, ϕ_2, ϕ_3 are close to the parameters used to generate the simulated data or the 95% credible interval for the posterior mean includes the values used to generate the simulate the data. The results of the variance parameters $\sigma^2_{\eta_1}, \sigma^2_{\eta_2}$ for the AR processes for h_1 and h_2, are higher than the values used to simulate the data. This could be explained by the fact that the regression parameters θ are also simulated with error terms and the sampler would not be able to accurately estimate the variances for h_1, h_2 and θ. The variance $\sigma^2_{\eta_3}$ for the innovations of the spillover parameter is estimated better, the posterior mean is closer to the simulated value.

Simulated spillover parameter and credible interval

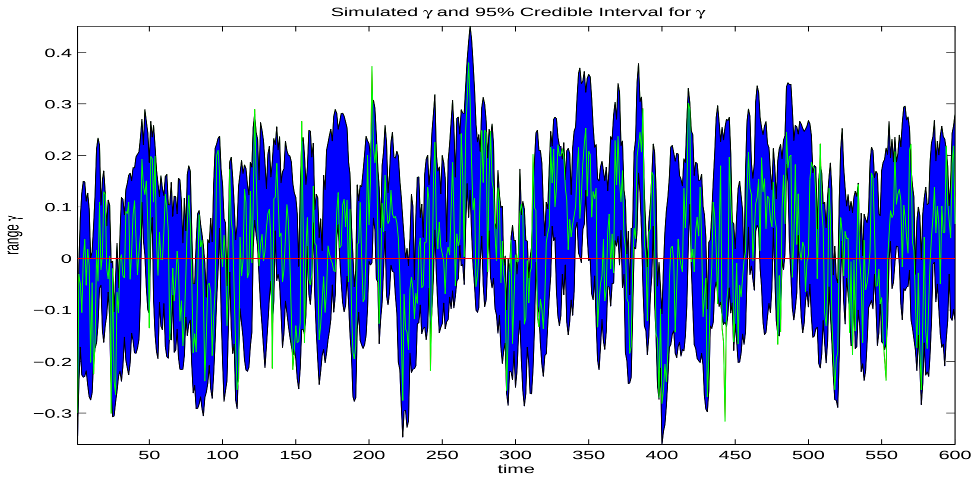

Figure 2.4.5: Simulated γ and 95 % credible interval

One of the main parameters of interest is the volatility spillover parameter. From the simulated data, we know the nonzero values γ_t. But the 95% credible interval includes zero except for a few periods in the above plot and hence we cannot conclude that γ_t is nonzero for the other periods from the above results. Hence, when the results for spillover parameter clearly show nonzero values, we can conclude volatility spillover or contagion effect. In this sense, this is a conservative result.

2.4.3 Results, Asian financial crisis 1997-1998

The results below are from running the MCMC simulations for 50000 iterations (the first 25000 used as a burn in period) and thinning the samples to 1 in 5.

Overall Covariance

Using equation (2.2.1) and treating the global returns f_t as a variable, θ as a known parameter, the overall covariance between the countries (Hong Kong and

Singapore) is

$$\begin{aligned}\mathrm{Cov}(y_{1,t}, y_{2,t}) &= \mathrm{E}[y_{1,t}y_{2,t}] - \mathrm{E}[y_{1,t}]\mathrm{E}[y_{2,t}] \\ &= \theta_{1,t}\theta_{2,t}\mathrm{Var}(f_t) + \delta_{1,t}\gamma_t \\ &= \theta_{1,t}\theta_{2,t}\mathrm{Var}(f_t) + \mathrm{e}^{h_{1,t}/2}\gamma_t \qquad (2.4.1)\end{aligned}$$

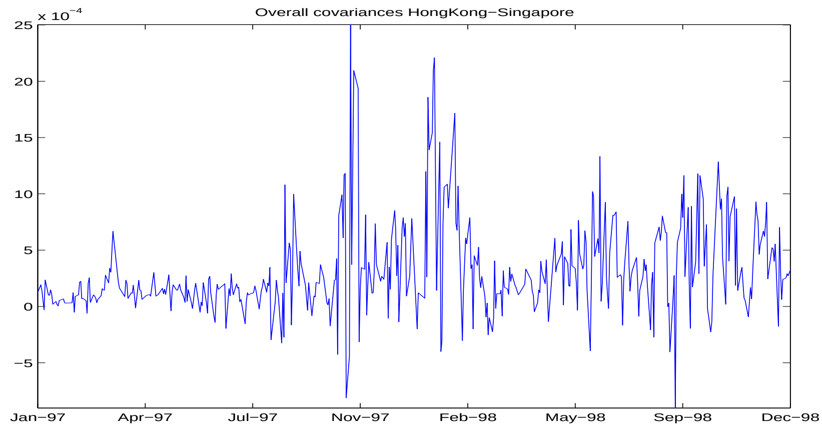

Figure 2.4.6: Overall covariances Hong Kong - Singapore 1997-1998.

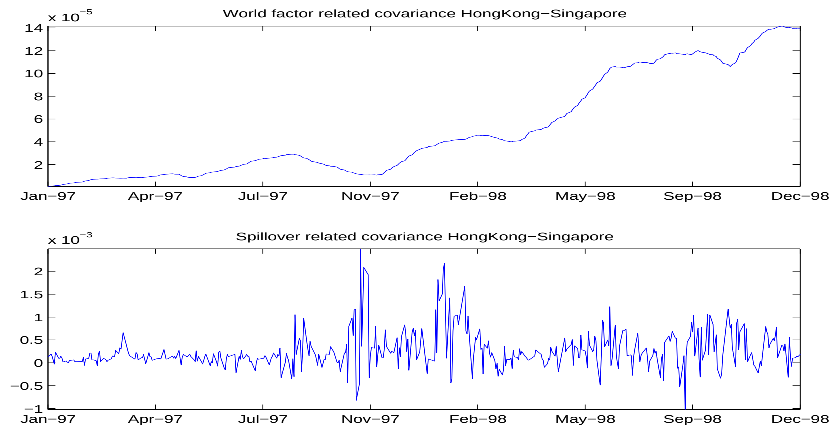

Figure 2.4.7: Covariance decomposition, world factor and spillover related covariance for Asian financial crisis.

Looking at the covariance decomposition (the two terms in equation (2.4.1)) as shown in Figure (2.4.7), the world factor contribution is very small compared to the volatility spillover part. The overall covariance results in Figure (2.4.6) show the sharp increase in covariances for the Asian financial crisis periods in Hong Kong during October 1997 and January 1998. This period of crisis is described in more detail with the volatility spillover (credible interval) results below. Pericoli & Sbracia (2003*b*) list some of the contagion definitions in use and one of them 'contagion is the significant increase in comovements of prices and quantities across markets.... ' could be used to conclude contagion effect shown by the increase in the covariances above.

Baur & Fry (2009) use equity returns to study the Asian financial crisis of 1997–1998 and find that 'interdependencies are substantially more important than contagion' meaning that the markets are highly correlated and excess correlation is difficult to detect. However, they find short periods of positive and negative results. They conclude that 'in comparison to other Asian crisis countries, Hong Kong is the main driver of contagion in the crisis'.

There is also another period of high covariances in the above results. The events related to these periods are described in the volatility spillover results below.

Volatility spillover

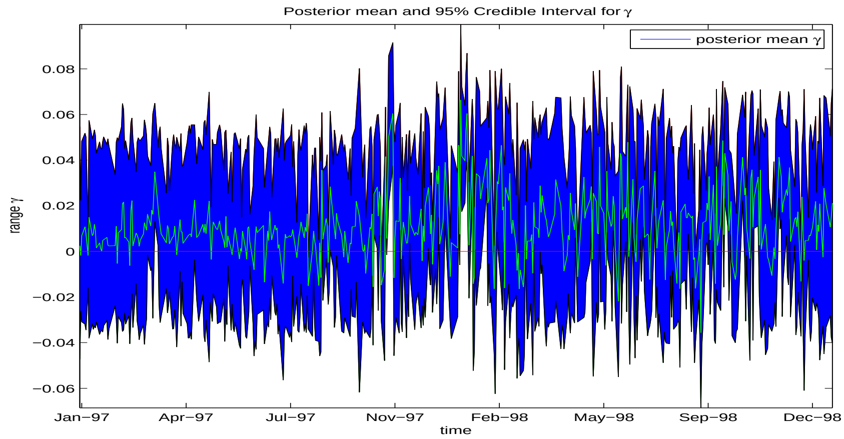

Figure 2.4.8: 95 % Credible interval and posterior mean γ, volatility spillover Asian financial crisis

The plot shows the posterior mean and the 95% credible interval for the volatility spillover parameter. Generally the credible interval includes zero and we cannot conclude that financial contagion from Hong Kong to Singapore took place. However, there are 2 specific periods when the volatility spillover is non zero. The Hong Kong dollar was pegged to the US dollar and came under speculative attack in October 1997. The stock markets were volatile and the overnight interest rates were raised to defend the Hong Kong dollar. At the time of the Asian financial crisis, the Hong Kong Monetary Authority had to intervene in the stock markets and purchased large quantity of equities to halt the decline in the prices. The volatility spilled over into neighbouring countries and the effect shows in the above plot around October 1997. The results are shown in more detail in the plots below (with the dates and events at the time) and we can conclude that there is evidence of volatility spillover or contagion from Hong Kong to Singapore at certain times.

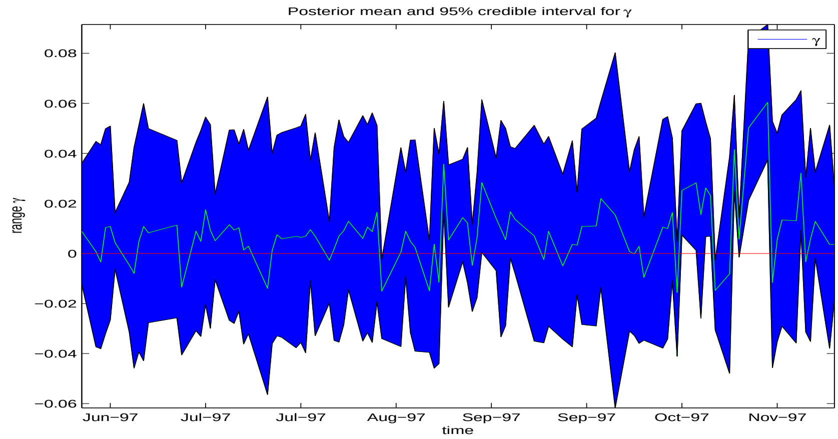

Figure 2.4.9: 95 % Credible interval and posterior mean γ.

Chiang et al. (2007) use correlation analysis and find evidence of financial contagion during October 1997 and January 1998. They also identify contagion between Hong Kong and Singapore during the crisis period and detect four contagion periods. The time varying model we have used also shows contagion effects in October and November 1997 which is similar to the results in Chiang et al. (2007). The data we have used is daily returns and the results show contagion periods from 17 October 1997 to 31 October 1997 (excluding the weekends, period of the Hong Kong stock market crash) followed by a few days in November (4, 11, 20 November 1997) when the Japanese bank Hokkaido Takushoku closed because of bad loans. We could interpret this volatility spillover from Hong Kong to Singapore as the spread of contagion from Japan to Hong Kong and then to Singapore.

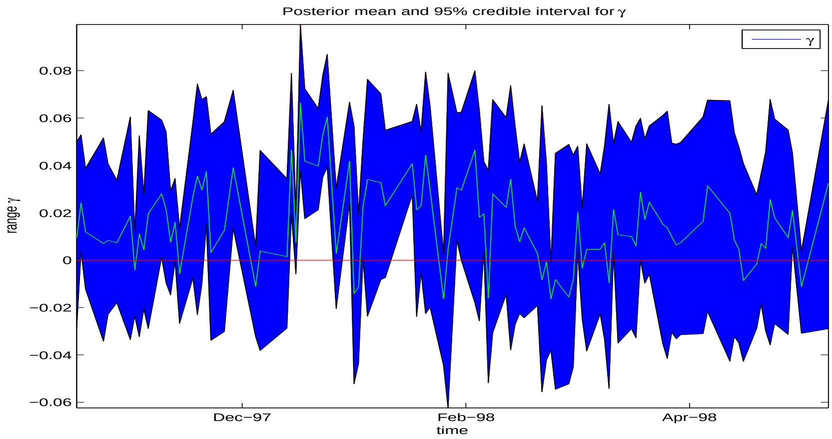

Figure 2.4.10: 95 % Credible interval and posterior mean γ.

This shows the volatile period around January 1998 when Peregrine Investment based in Hong Kong collapsed and the Hong Kong Index (Hang Seng) dropped over 8% in a short period of time. There was a period of rising unemployment and large companies (for example, Cathay Pacific, Peregrine) had to lay off staff (Sek-Hong & Lee 1998). The plot above shows the volatility spillover being significant in December 1997 - February 1998 period. The result for January 1998 is similar to the results in Chiang et al. (2007). Jang & Sul (2002) find evidence of contagion between Hong Kong and Singapore at the same time In more detail, using the time varying parameters model, the days identified as contagion days are December 1997 (3 days), January 1998 (8 days) and February 1998 (3 days). The January dates are around 16 January, when the Japanese government approved a 228 bn US dollar stabilisation package.

There are further periods of volatility spillover in from August 1998 - November 1998, related to serious Japanese economic problems.

2.4.4 Results, European sovereign debt crisis 2007-2013

The results below are shown for 50000 iterations with a 25000 burnin period and samples thinned to 1 in 5. The overall covariances are calculated as in equation (2.4.1).

Overall Covariance

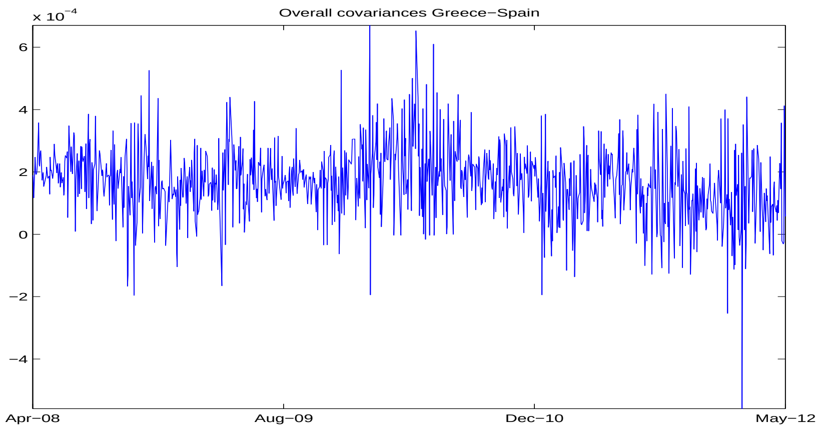

Figure 2.4.11: Overall covariances Greece and Spain.

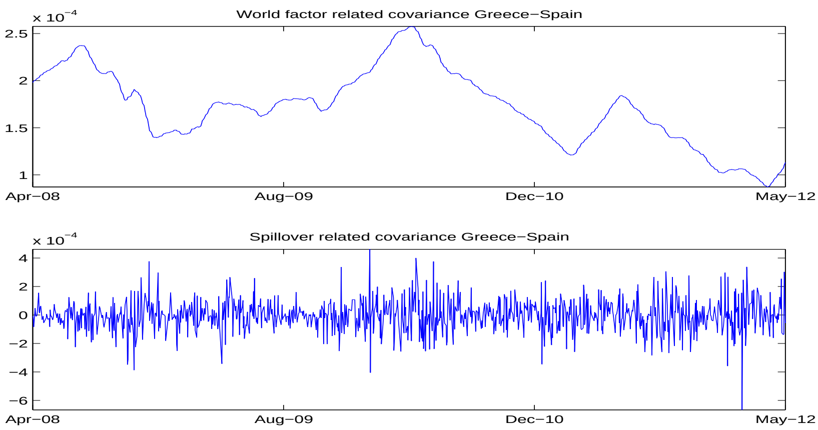

Figure 2.4.12: Covariance decomposition, world factor related and spillover related covariance for Euro crisis.

Looking at the decomposition of the overall covariance (the two terms in equation 2.4.1) as shown in Figure 2.4.12, the world factor covariance is of the same scale as the spillover component. The spillover component is much smaller than the spillover component for the Asian financial crisis (Figure 2.4.7), by a factor of 10. This may explain the difficulty in identifying the spillover non zero values in the later chapters.

Using more detailed plots (not included here), it can be seen that the periods of high covariances (positive or negative) are related to events during the Euro crisis. For example, in October 2008 a general election was planned in Greece, amid allegations of over spending by the previous government; December 2009, Greece admits debts have reached 300 bn Euros which was 113 % of the GDP; February-June 2010, EU tells Greece to make further cuts in spending and emergency loans and 110 bn Euro bailout package for Greece; January-February 2012 S&P downgrades the credit ratings for 9 Eurozone countries, debt write off talks with Greece and lenders demand more austerity measures from Greece; May-June 2012 Spanish banks ask the government for assistance and Spain requests bailout support for 100 bn Euros form the EU.

Volatility spillover

For the Euro crisis, the volatility spillover parameter is shown below. The spillover parameter shows non zero values (using a 95% credible interval) indicating financial contagion. The volatility spillover is over short periods of time and can be seen in the more detailed plots.

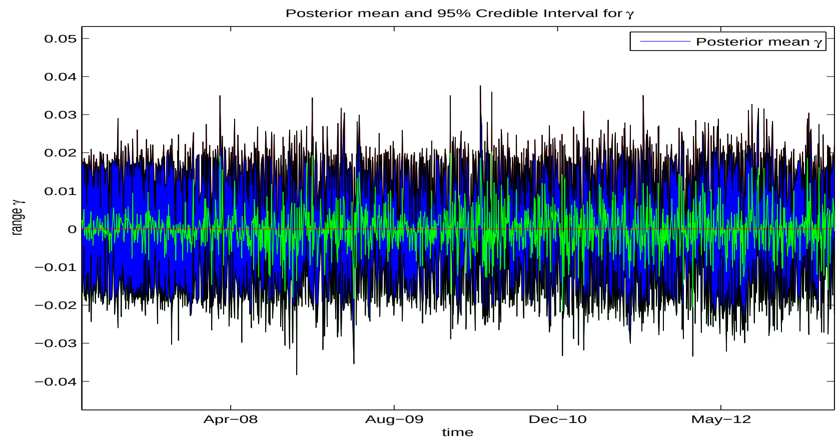

Figure 2.4.13: 95 % Credible interval and posterior mean γ, volatility spillover Euro crisis.

The markets in the Eurozone area are highly integrated and the changes in one market are quickly reflected in the other markets. Additional volatility spillover can be detected for small time intervals. As Forbes & Rigobon (2002) and Missio & Watzka (2011) explain, the short lived spillover is termed contagion - a longer term change in the correlation would be interpreted as a result of a long term change in the interdependence between the two countries. The scale in the above plot does not allow the results to be viewed easily. Below, the volatility spillover is shown over a few days for the volatile periods during the Euro crisis.

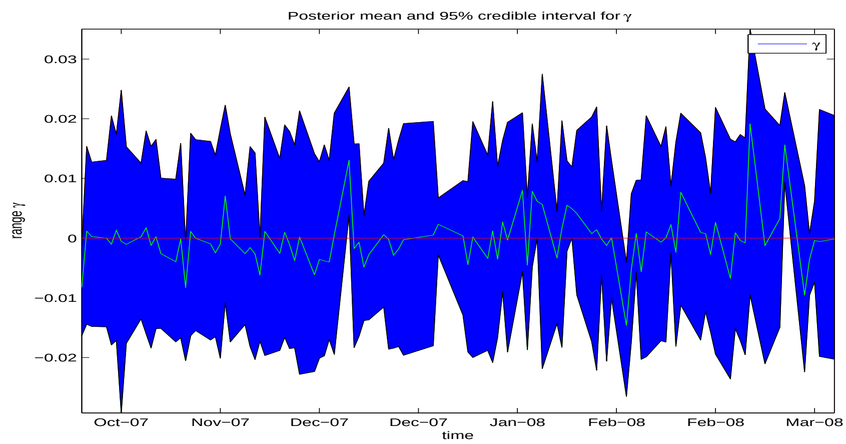

Figure 2.4.14: 95 % Credible interval and posterior mean γ.

At the time of the global financial crisis in 2007-2008, the markets were volatile in all the European countries. Greece experienced more difficulties because of its reliance on shipping and tourism. The government increased spending in an attempt to manage the economic downturn but the sovereign debt increased rapidly as a proportion of the GDP. The may explain the volatility spillover, 7 days in the period October 2007 - March 2008.

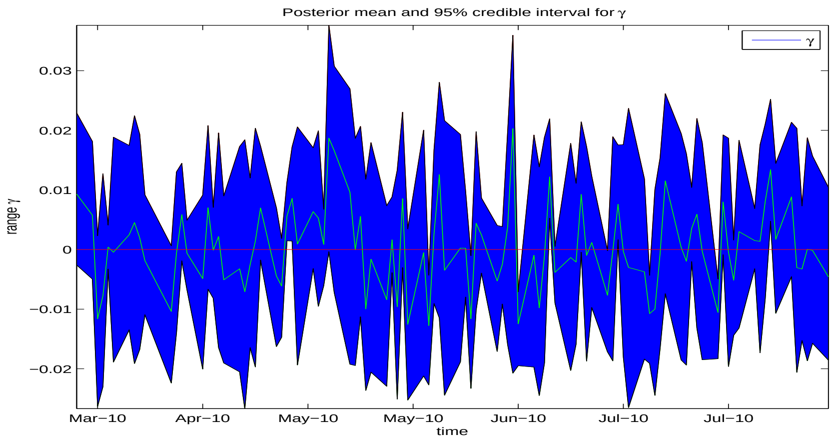

Figure 2.4.15: 95 % Credible interval and posterior mean γ.

April-August 2010 was a turbulent time in Greece. Austerity measures were announced in Greece, there were protests against the austerity measures and an election was announced and Spain had a banking crisis while an IMF bailout was being considered for Spain. Using bond spreads between Greece and the benchmark German yields, Missio & Watzka (2011) use a GARCH and Dynamic Conditional Correlation (DCC) model and conclude there is contagion effect in the summer of 2010 (meaning there is excess correlation between Greece and Spain, beyond the normal level and anything that can be explained by the fundamentals in Spain). Using equity indices and the time varying parameter model with volatility spillover shows similar results to those obtained by Missio & Watzka (2011). In detail, the time varying model shows specific dates as 3, 4, 24, 26 February 2010, 28-29 April 2010, 25 May 2010, 5 days in June 2010, 3 days in July). June 2010 was a turbulent time for the Eurozone countries with Greece, Spain and Portugal experiencing financial difficulties and bank share prices declined amid fears of a Greek default.

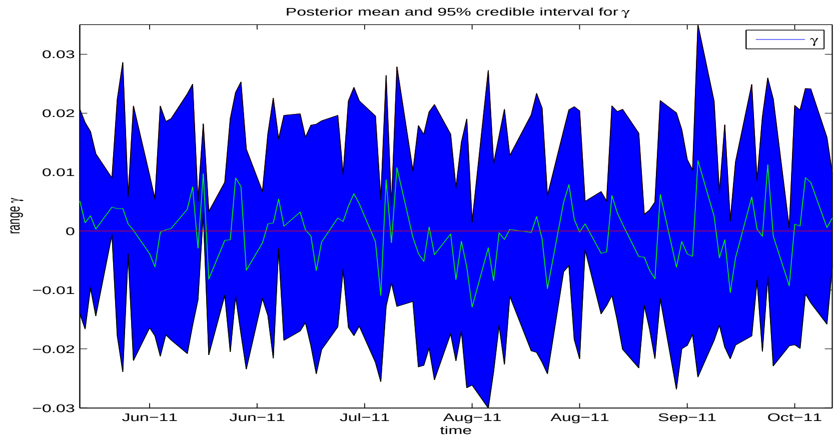

Figure 2.4.16: 95 % Credible interval and posterior mean γ.

In July 2011, all 3 credit rating agencies (S&P, Moody's, Fitch) cut Greek credit rating, 109 billion Euros bailout fund was arranged through the European Financial Stability Fund and in October 2011, 50 % debt write off was agreed for Greece and further austerity measures were announced. There was an announcement of a referendum on the austerity and reform package. This was a turbulent time for the Greek markets. The above results did not identify any significant periods of spillover except for just one day in June 2011. This may be due to the awareness in the financial markets of the of the bailout discussions and of the possible credit rating downgrades before the actual announcements were made.

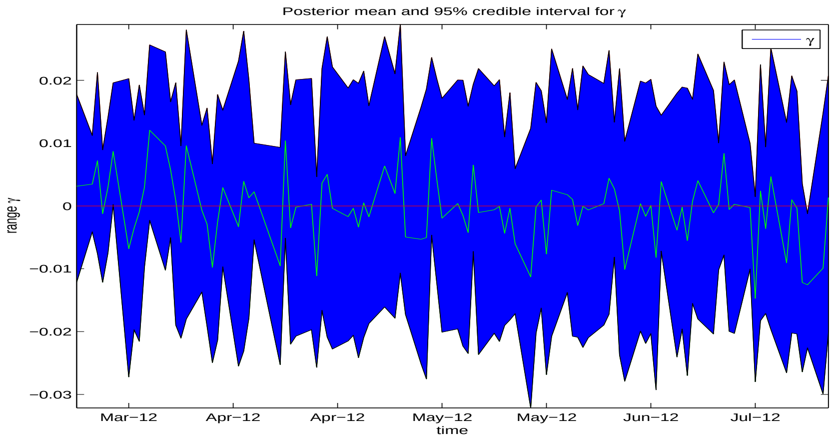

Figure 2.4.17: 95 % Credible interval and posterior mean γ.

February-July 2012 was a period of market volatility after the breakdown in talks between Greece and its creditors (Reuters, 16 June 2012) and fears that Greece could crash out of the Euro (Financial Times, 20 June 2012). Spain and Cyprus both requested financial support from the ECB (ECB announcements, 27 June 2012). There was pressure on the Greek government to deepen the cuts (Financial Times, 12 July 2012) and the ECB suspended Greek bonds as collateral (20 July 2012). The uncertainty caused by these events made the markets volatile and resulted in volatility spillover to other countries. However, the results above detect a few dates when volatility spillover is identified (15 Feb 2012, 9 March 2012, 20 July 2012).

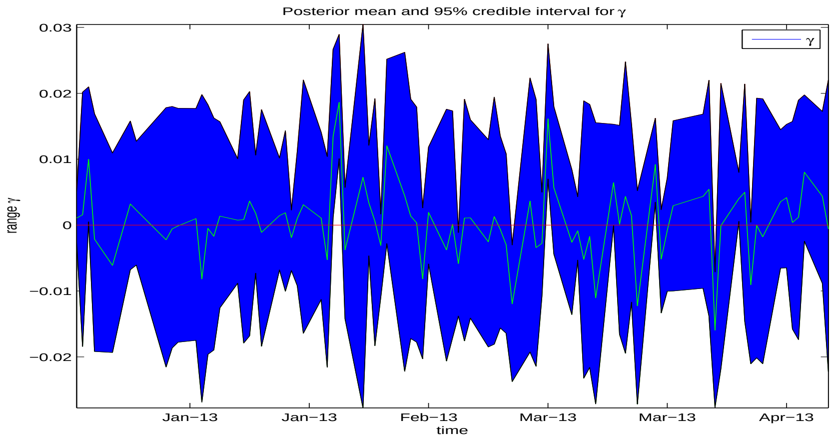

Figure 2.4.18: 95 % Credible interval and posterior mean γ.

In January 2013, the unemployment level in Greece reached 26.8%, the highest rate in the EU and in April 213, youth unemployment climbed to almost 60%. The detailed dates showing volatility spillover are 30,31 January 2013; 20 Feb 2013; 1,7, 25 March 2013 and 4,8 April 2013. This is just a continuation of the uncertain periods in the Greek economy.

2.5 Conclusion

The model estimates the simulated parameters reasonably well. For the Asian financial crisis, the volatility spillover (contagion) is identified quite well (possibly because the events had significant or unexpected economic impact). The volatility spillover for the European sovereign debt crisis is identified for short intervals during some of the volatile periods (possibly because the markets are highly integrated in a single currency area, there were ongoing financial bailout discussions and markets are better informed and factor in the implications over a longer period of time).

Chapter 3

Mixture Offset Parameter

3.1 Introduction

This chapter looks at setting the value of the mixture offset parameter used when a stochastic volatility model with Normal error term is expressed in linear State Space form for the volatility parameter and the error term is approximated by a Normal Mixture. A 'mixture offset' parameter c is used to make the approximation robust for small values of the log returns. When MCMC methods are used for the Normal Mixture model, the posterior results for the volatility parameter depend on the value of the offset parameter used. A method is proposed to standardise the residuals which makes the results less dependent on the value of the offset parameter used. To check the proposed method, simulated data with known parameters have been used. Results from using the method on Eurostoxx daily log returns are shown at the end.

It is known that the volatility of asset price returns varies over time. Over the long term, the volatility of equity returns may appear to be stable but usually there are periods of high volatility and calm market periods when the volatility may be low

(Enders 2004). For the analysis of economic time series, Engle (1982) has used ARCH models. The volatility is modeled as a function of the lagged values of the asset returns. This allows the estimation of the time varying volatility process parameters using least squares regression methods. This was further developed into the GARCH process which treats the variance as a latent variable that is modeled separately as an AR process (Bollerslev 1987, Taylor 2007). Method of Moments and Maximum Likelihood methods have been used to estimate the model parameters. To allow for heavy tailed distributions, this has been extended to model the *log of the volatility* as a separate AR process. This formulation results in the returns process with time varying log volatility as in Jacquier et al. (1995) and Chib et al. (2002). The asset returns may be expressed as function of past returns or other economic variables and the log volatility is modeled as a separate AR process. Expressed in State Space form, a form of the Stochastic Volatility model is written as

$$\begin{aligned} y_t &= e^{h_t/2}\nu_t, \\ h_t &= \mu + \phi(h_{t-1} - \mu) + \sigma_\eta \eta_t, \end{aligned} \tag{3.1.1}$$

where y_t is the log return at time t (may be expressed as percentage log return), ν_t is the error term distributed as $\mathrm{N}(0,1)$, h_t is the log variance which follows an AR(1) process with parameters μ, ϕ, σ_η and the error terms $\eta_t \sim \mathrm{N}(0,1)$. The error terms ν_t and η_t are uncorrelated. This is not in linear State Space form for h_t and does not permit the straightforward use of Forward Filtering Backward Sampling techniques. One at a time sampling would need to be used to sample from the posterior density for the volatility parameters h_t for $t = 1, \ldots, T$, where T is the number of observations. Equation (3.1.1) is written as $\log y_t^2 = h_t + \log \nu_t^2$ and the error term is then approximated by a Normal Mixture distribution (Kim et al. 1998). Conditional on the mixture state, the model is in linear State Space form with Normal errors for the volatility parameter h_t. In order to make the approximation robust for small values of y_t, a small offset parameter c is used and the approximated model is expressed as $\log(y_t^2 + c) = h_t + \log \nu_t^2$. The value used for the offset parameter can play an important part in the final results and if the value is not chosen carefully, it can lead to incorrect results. We propose a method which overcomes the problem of selecting the offset parameter parameter value.

The remainder of this chapter covers the Kim et al. (1998) method to linearise the log volatility model (section 3.2) and an explanation of sampling for the mixture state from the full conditional density. In section 3.3 the use of Forward Filtering Backward Sampling (FFBS) with an additional Metropolis-Hastings (M-H) step to sample the proposed volatility parameter h_t is shown. This is followed by the full conditionals for the AR(1) volatility process parameters. In section 3.4, the difficulty of using an appropriate value of the offset parameter c is highlighted and this is followed by a proposed method to overcome the problem of selecting a value of the offset parameter (section 3.5). Results using simulated data and Eurostoxx daily log returns are discussed at the end.

3.2 Normal Mixture Method

The early work on Bayesian analysis of Stochastic Volatility models was carried out by Jacquier et al. (1995) using rejection sampling. Kim et al. (1998) proposed an improved rejection sampling method using the nonlinear model in (3.1.1) with a bounded function for $p(y_t|h_t)$ to sample h_t from the posterior density. This is a simple approach to implement, but it leads to one at a time sampling for h_t and poor mixing when sampling correlated values of h_t (Chib & Carlin 1999). In order to overcome this, the observations in the SV model in (3.1.1) are transformed (Kim et al. 1998) and the error term is approximated by a Normal Mixture density. From equation (3.1.1),

$$\log y_t^2 = h_t + \log \nu_t^2, \tag{3.2.1}$$

which is now linear in h_t. The error term $\log \nu_t^2$ has a $\log \chi_1^2$ density. This is approximated by a Normal Mixture density. To make the Normal Mixture approximation robust for small or zero values of y_t, a small value offset parameter c is used (Fuller 2009) and equation (3.2.1) is replaced by

$$\log y_t^* = h_t + \log \nu_t^2 \tag{3.2.2}$$

where

$$y_t^* = y_t^2 + c.$$

Writing $z_t = \log \nu_t^2$, the $\log \chi_1^2$ density is approximated as

$$p(z_t) = \sum_{i=1}^{7} q_i f_N(z_t | m_i - 1.2704, v_i^2)$$

where $f_N(x|\mu, \sigma^2)$ is the Normal density with mean μ and variance σ^2. Using numerical methods, Kim et al. (1998) estimate the probabilities q_i, means $(m_i - 1.2704)$ and the variances v_i^2 for the 7 component Mixture Normal to closely approximate the $\log \chi_1^2$ density. Using a mixture state $s_t = i,$ for $i = 1, \ldots, 7$ and $p(s_t = i) = q_i$, the mixture model can also be written as

$$\begin{aligned} z_t | s_t = i &\sim \mathrm{N}(m_i - 1.2704, v_i^2), \\ p(s_t = i) &= q_i. \end{aligned}$$

Using the above Normal Mixture approximation, equation (3.2.2) is written as

$$\begin{aligned} \log y_t^* | s_t = i &\sim \mathrm{N}(h_t + m_i - 1.2704, v_i^2), \\ p(s_t = i) &= q_i. \end{aligned} \tag{3.2.3}$$

Kim et al. (1998) use the Normal Mixture approximation to write the model in linear State Space form and block sample the volatility parameter $h = (h_1, h_2, \ldots, h_T)$ using the 'simulation signal smoother' (De Jong & Shephard 1995). To use the Normal Mixture approximation, Kim et al. (1998) use $c = 0.001$ for the Mixture Offset parameter, but state that 'it is possible to let c depend on the values taken by y_t^2'.

When using this approximation, we found that the value of offset parameter c that is used for the MCMC simulations changes the sampled results quite significantly, especially in a model with many parameters. We propose a method to overcome the problem of selecting the value of the offset parameter c.

3.3 Parameter sampler

The aim is to sample the volatility parameter h_t for $t = 1, \ldots, T$ and to sample the AR(1) parameters $\theta = \{\mu, \phi, \sigma_\eta\}$ using an MCMC sampler.

The joint posterior density for h and θ is given by

$$p(h, \theta | y) = \frac{p(y|h,\theta)p(h|\theta)p(\theta)}{p(y)}.$$

Kim et al. (1998) use an approximate density for $p(y|h,\theta)$ and we denote this by $\tilde{p}(y|h,\theta)$ (the tilde indicates approximate densities using the Normal Mixture),

$$\begin{aligned}\tilde{p}(y_t|h_t,\theta_t) &= \sum_{s_t} \tilde{p}(y_t|h_t,\theta_t,s_t)\tilde{p}(s_t), \\ &\approx p(y_t|h_t,\theta_t).\end{aligned}$$

Here, $\tilde{p}(y_t|h_t, \theta_t, s_t) \sim \mathrm{N}(h_t + m_i - 1.2704, v_i^2)$ and $\tilde{p}(s_t = i)$ for $i = 1, \ldots, 7$ as in the 7 component Normal Mixture calculated by Kim et al. (1998).

The approximate posterior for $h = (h_1, h_2, \ldots, h_T)$ is given by

$$\tilde{p}(h, \theta, s|y) = \prod_{t=1}^{T} \frac{\tilde{p}(y_t|h_t, \theta, s_t)\tilde{p}(s_t)p(h_t|\theta)p(\theta)}{\tilde{p}(y_t)}.$$

The marginal distribution of h_t, summing over $s_t = i,$ for $i = (1, 2, \ldots, 7)$ is

$$\begin{aligned}
\tilde{p}(h_t, \theta|y_t) &= \sum_s \tilde{p}(h_t, \theta, s_t|y_t), \\
&\propto \sum_s \tilde{p}(y_t|h_t, \theta, s_t)\tilde{p}(s)p(h_t|\theta)p(\theta), \\
&\approx p(y_t|h_t, \theta)p(h_t|\theta)p(\theta), \\
&\propto p(h_t, \theta|y_t).
\end{aligned}$$

We use an augmented posterior so that we can block sample the volatility parameter,

$$p_q(h_t, \theta, s_t|y_t) \propto p(y_t|h_t, \theta)p(h_t|\theta)p(\theta)g(s_t|h_t, \theta). \qquad (3.3.1)$$

If $\sum_s g(s|h_t, \theta) = 1$, i.e. g is a probability mass function,

$$\begin{aligned}
p_q(h_t, \theta|y_t) &= \sum_s p_q(h_t, \theta, s_t|y_t) \\
&\propto p(y_t|h_t, \theta)p(h_t|\theta)p(\theta) \\
&= p(h_t, \theta|y_t)
\end{aligned}$$

Define $g(s|h, \theta)$ as

$$g(s|h,\theta) = \frac{\tilde{p}(y|h,\theta,s)\tilde{p}(s)}{\tilde{p}(y|h,\theta)} \qquad (3.3.2)$$
$$\propto \prod_{t=1}^{T} \frac{\tilde{p}(y_t|h_t,\theta_t,s_t)\tilde{p}(s_t)}{\tilde{p}(y_t|h_t,\theta)}$$

which is the full conditional of s using the Kim et al. (1998) approximation.

MCMC sampling

Using a Normal Mixture model in (3.2.3) above and the AR(1) process for h_t from equation 3.1.1

$$\log(y_t^*|s_t = i) = h_t + m_i - 1.2704 + \nu_t, \qquad (3.3.3)$$
$$p(s_t = i) = q_i,$$
$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma_\eta \eta_t$$

where the error term $\nu_t \sim \mathrm{N}(0, v_i^2)$ and $\eta_t \sim \mathrm{N}(0,1)$ and ν_t and η_t are independently distributed. We use the sampling sequence $s|\theta, h, y$, then $h|\theta, s, y$ and $\theta|h, y$. Using the model in (3.3.3) we use the Kalman Filter and Backward Sampling method (also referred to as Forward Filtering Backward Sampling method, FFBS, (Carter & Kohn 1994)) to propose h_t for $t = 1, \ldots, T$. A Metropolis-Hastings step as described below is used to accept the proposed values of h.

With Normally distributed errors for h_t in (3.1.1), appropriate priors are used for the parameters μ and σ_η^2. This results in full conditional distributions in closed form allowing the parameters to be sampled conveniently. For ϕ, the posterior cannot be recognised as a standard density and Adaptive Metropolis-Hastings

(Griffin & Stephens 2013, Haario et al. 2001) sampling is used.

Updating Mixture Indicator s

Using equation (3.3.2), the posterior density for $s = (s_1, s_2, \ldots, s_T)$ is given by

$$g(s|h, \theta, y) = \prod_{t=1}^{T} \frac{\tilde{p}(\log(y_t^*)|h_t, \theta, s_t = i)\tilde{p}(s_t = i)}{\sum_{j=1}^{j=7} \tilde{p}(\log(y_t^*)|h_t, \theta, s_t = j)\tilde{p}(s_t = j)}.$$

FFBS and a Metropolis-Hastings step are used to sample h_t.

The calculation of the mean and variance for the backward distribution ($\mu_{B,t}, \sigma^2_{B,t}$) is explained below. Conditioning on $s_t = i$, $\log(y_t^*)$ is Normally distributed. Given the current values of μ, ϕ, σ^2_η, use the Kalman Filter steps to calculate the distributions for h_t, $t = 1, 2, \ldots, T$. For the Backward Sampling, update the distribution at $t-1$ as below,

$$\sigma^2_{B,t-1}|h^{'}_{t:T}, h = \left(\frac{1}{\sigma^2_{t-1}} + \frac{\phi^2}{\sigma^2_\eta}\right)^{-1},$$

$$\mu_{B,t-1}|h^{'}_{t:T}, h = \left(\frac{1}{\sigma^2_{t-1}} + \frac{\phi^2}{\sigma^2_\eta}\right)^{-1} \left(\frac{\mu_{t-1}}{\sigma^2_{t-1}} + \frac{\phi h_t{'} - \phi\mu(1-\phi)}{\sigma^2_\eta}\right),$$

where μ_{t-1} and σ^2_{t-1} are the forward distribution mean and variance at time $t-1$, $h_t{'}$ is the sample value at time t. B in the subscript indicates the Backward distribution parameter. μ, ϕ, σ^2_η are the current values of the AR process parameters. The updated parameters $\sigma^2_{B,t-1}$ and $\mu_{B,t-1}$ are then used to propose $h^{'}_{t-1}$ at time $t-1$.

A Metropolis-Hastings step is used to sample the proposed $h^{'}_t = (h^{'}_1, h^{'}_2, \ldots, h^{'}_T)$ drawn using the FFBS step above.

The M-H ratio for accepting the proposed $h^{'}$ with probability a is

$$a = \min\left(1, \frac{p_q(h^{'}, \theta, s|y)}{p_q(h, \theta, s|y)} \frac{q(h|h^{'}, \theta, y, s)}{q(h^{'}|h, \theta, y, s)}\right),$$

using equation (3.3.1),

$$= \min\left(1, \frac{p(y|h^{'}, \theta)p(h^{'}|\theta)g(s|h^{'}, \theta, y)}{p(y|h, \theta)p(h|\theta)g(s|h, \theta, y)} \frac{q(h|h^{'}, \theta, y, s)}{q(h^{'}|h, \theta, y, s)}\right).$$

The calculation of the transition probabilities $q(h^{'}|h, \theta, y, s)$ and $q(h|h^{'}, \theta, y, s)$ is explained further. The probability $q(h^{'}|h, \theta, y, s)$ is calculated using the FFBS step (without resampling $h^{'}$) where the distribution of y is calculated given h. Similarly, the probability $q(h|h^{'}, \theta, y, s)$ is calculated using the FFBS step (without resampling h) where the distribution of y is calculated given $h^{'}$.

Updating μ

Using a diffuse prior for μ, $\mu \propto 1$, the full conditional density for μ is

$$p(\mu|h, y) \sim \mathrm{N}(\hat{\mu}, \sigma_\mu^2),$$

where

$$\hat{\mu} = \sigma_\mu^2 \left(\frac{(1-\phi^2)}{\sigma_\eta^2} h_1 + \frac{(1-\phi)}{\sigma_\eta^2} \sum_{t=1}^{t=T-1} (h_{t+1} - \phi h_t)\right),$$

and

$$\sigma_\mu^2 = \sigma_\eta^2 \left((T-1)(1-\phi)^2 + (1-\phi^2)\right)^{-1}.$$

Updating ϕ

Using $\phi = 2\phi^* - 1$, and ϕ^* with a Beta(a,b) prior distribution

$$\pi(\phi) \propto \left(\frac{1+\phi}{2}\right)^{a-1} \left(\frac{1-\phi}{2}\right)^{b-1},$$

the full conditional distribution for ϕ is

$$p(\phi|h,y) \propto \pi(\phi)p(h|\mu,\phi,\sigma_\eta^2)$$

and the log of the full conditional distribution is

$$\begin{aligned}\log f(h|\mu,\phi,\sigma_\eta^2) \propto & -(h_1-\mu)^2(1-\phi^2) \\ & +\frac{1}{2}\log(1-\phi^2) \\ & -\sum_{t=1}^{t=T-1}\frac{((h_{t+1}-\mu)-\phi(h_t-\mu))^2}{2\sigma_\eta^2}.\end{aligned}$$

The range of ϕ is $-1 < \phi < 1$ and the parameters for the prior density are $a = 20$, $b = 1.5$ which gives a prior mean of 0.86. For the results shown below, sampling from a truncated Normal is used to ensure ϕ in the permitted interval and an Adaptive Metropolis-Hastings (Griffin & Stephens 2013, Haario et al. 2001) method is used to get an acceptance rate of about 20% of the proposed values.

Updating σ_η^2

Using a prior density $\sigma_\eta^2 \sim$ Inverse Gamma $\left(\frac{\sigma_r}{2}, \frac{\mathrm{S}_\sigma}{2}\right)$, the posterior density $\sigma_\eta^2|h,\phi,\mu \sim$ Inverse Gamma(a,b) where the parameters a and b are

$$a = \frac{T+\sigma_r}{2}$$

and

$$b = \frac{\mathrm{S}_\sigma + (h_1-\mu)^2(1-\phi^2) + \sum_{t=1}^{t=T-1}((h_{t+1}-\mu)-\phi(h_t-\mu))^2}{2}.$$

The parameters for the prior density are $\sigma_r = 5, \mathrm{S}_\sigma = 0.01\sigma_r$ The AR(1) process parameters μ, ϕ and σ_η are sampled from the correct posterior distributions (as shown above and described in Kim et al. (1998)) and do not directly depend on the approximation introduced by the Normal Mixture.

3.4 Results

3.4.1 Simulated data

Test data is generated using the correct model in (3.1.1) above with known parameters so that the sampled parameter values from the MCMC simulations can be validated against the values used to generate the data. To generate the data, the error terms ν_t and η_t are sampled independently from $\mathrm{N}(0,1)$ and h_t calculated using the AR parameters $\mu = -10, \phi = 0.95$ and $\sigma_\eta^2 = 0.01^2$. To start generating the simulated data, h_0 is sampled from the steady state distribution $\mathrm{N}(\mu, \frac{\sigma_\eta^2}{1-\phi^2})$. 600 observations for the daily log returns are simulated and used for the tests. A second test data set is generated with $\mu = 0, \phi = 1$ giving $\mathrm{E}[h] = 0$ and resulting in y_t samples close to a Standard Normal density .

3.4.2 Results using simulated data

The samples of simulated observations above (2 simulated data sets) have been used for the results in Table 5 below. The effect of the offset parameter can be seen by calculating the sample mean and sample variance of $(\log(y_t^2 + c) - h_t)$, for $t = 1, 2 \ldots, T$. This should result in an approximate sample mean of -1.2704 and variance of 4.93 if the data came from a $\log(\chi_1^2)$ distribution.

$c = 10^{-n}$		
n	Sample mean	Sample variance
2	3.4358	0.0033
3	1.3474	0.0804
4	-0.1277	0.7304
5	-0.8329	2.1094
6	-1.0801	3.2750
7	-1.1551	3.9135
8	-1.1818	4.2797
9	-1.1947	4.5248
10	-1.2012	4.6796

$c = 10^{-n}$		
n	Sample mean	Sample variance
2	-1.1151	2.8937
3	-1.2941	4.0771
4	-1.3585	4.7792
5	-1.3812	5.1255
6	-1.3892	5.2833
7	-1.3915	5.3368
8	-1.3918	5.3453
9	-1.3918	5.3462
10	-1.3918	5.3463

Table 5: Sample mean and variance when h is simulated using $\mu = -10, \phi = 0.95, \sigma_\eta^2 = 0.01^2$ (left hand table) and when h is simulated using $\mu = 0, \phi = 1, \sigma_\eta^2 = 0.01^2$, using different values of the offset parameter c.

As the calculation shows, when the data does not come from a Standard Normal distribution, Table 5 (left hand table), c needs to be near 10^{-9} for the mean and variance of the sample to be close to the $\log \chi_1^2$ mean and variance. When the data is from a Standard Normal distribution, Table 5 (right hand table), c needs to be near 10^{-3} for the sample mean and variance to be near the the $\log \chi_1^2$ mean and variance. In a model with more variables, for example in a regression setting y_t is modelled as a function of other parameters and possibly its own lags and we need to estimate the volatility parameters, the residuals would depend on the MCMC samples of a number of parameters. In this case, it is difficult to set one value of c and expect that the mean and variance of the residuals will be close to the mean and variance of $\log \chi_1^2$ as the values of the other parameters change over the MCMC iterations.

3.4.3 Results using different values of c

Using the KSC Normal Mixture model in equation (3.3.3) and the sampler summarised in section 3.3, MCMC iterations are used for the model parameters using different values of the mixture offset parameter c. From the sample paths for the prameters, it can be seen that the algorithm converges relatively quickly. A number of different starting values of the parameters have been used to check that the method converges to the same values. An initial 20000 iterations are used as a burn in period followed by a further 20000 iterations which are thinned to 1 in 5 to sample the parameters. The parameter samples are used to calculate the Effective Sample Size results using the methods in Sokal (1997) and Gamerman & Lopes (2006).

As the results in Table 6 show, the posterior mean for the AR parameter, μ, is -9.6107 when using the offset parameter $c = 10^{-4}$ and -10.1112 when using $c = 10^{-6}$, compared to the value of $\mu = -10$ used to create the simulated data. When $c = 10^{-6}$ is used, the values of the volatility persistence parameter ϕ and the variance parameter σ_η^2 are estimated slightly better.

	$c = 10^{-4}$	$c = 10^{-6}$
Parameter	Posterior mean (credible interval)	Posterior mean (credible interval)
μ (-10)	-9.6107 (-9.6523, -9.5663)	-10.1112 (-10.2353, -9.9982)
ϕ (0.95)	0.7728 (0.6968, 0.8339)	0.7980 (0.5645, 0.9464)
σ_η^2 (0.01^2)	0.0133 (0.0114, 0.0154)	0.0081 (0.0032, 0.0215)

Table 6: Posterior mean and 95 % credible intervals for the AR(1) parameters μ, ϕ and σ_η^2 when using different values of the offset parameter, residuals not standardised.

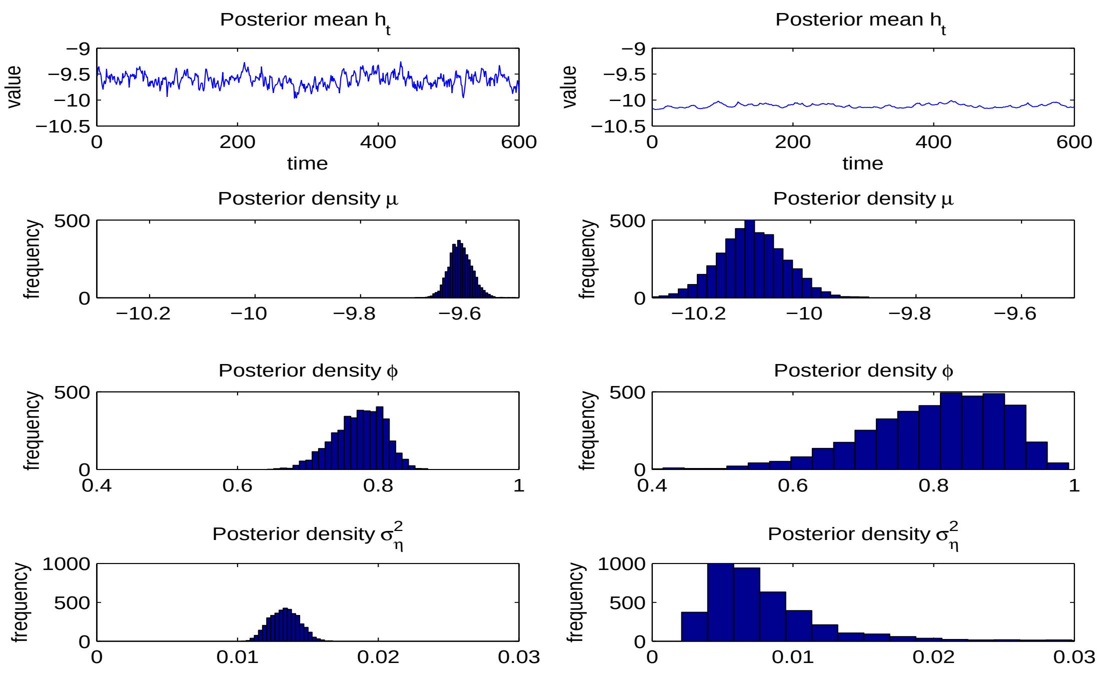

Figure 3.4.1: Volatility parameter h_t and AR(1) parameter μ, ϕ, and σ_η^2 posterior distributions, left column using $c = 10^{-4}$, right column using $c = 10^{-6}$, residuals not standardised.

We can see in the above plots that the value of the offset parameter that is used to linearise the model, changes the sampled results for the volatility parameter

h_t, (Figure 3.4.1 plots, first row) and the AR(1) parameter μ (second row) . The AR(1) persistence parameter ϕ results are less dependent on the offset parameter values. In both cases, the variance parameter results are higher than the simulated value. If we continue to use smaller values of the offset parameter, for example, $c = 10^{-7}$ or smaller, then the volatility parameter is sampled correctly and the results are close to the values used to simulate the data.

Importance Sampling

As an alternative to the M-H step to accept the proposed values, h_t is proposed using FFBS and then an Importance Sampling step is used for the volatility parameter h. Kim et al. (1998), use ratio of the correct density and the approximate density to calculate the weights as

$$
\begin{aligned}
w(\theta, h) &= \log f(\theta, h|y) - \log k(\theta, h|y) \\
&= \text{const} + \log f(y|h) - \log k(y^*|h), \\
f(y|h) &= \prod_{t=1}^{t=T} f_N(y_t|0, h_t), \\
k(y^*|h) &= \prod_{t=1}^{t=T} \sum_{i=1}^{i=7} q_i k_N(y_t^*|h_t + mi - 1.2704, v_i^2), \\
\log f(y|h) &= \sum_{t=1}^{t=T} \left(-\frac{h_t}{2} - \frac{1}{2} y_t^2 \exp(-h_t) \right), \\
\log k(y^*|h) &= \sum_{t=1}^{t=T} \log \left(\sum_{i=1}^{i=7} q_i \frac{1}{v_i} \exp(-\frac{1}{2} \frac{(y_t^* - (h_t + m_i - 1.2704))^2}{v_i^2}) \right).
\end{aligned}
$$

Using the above weights, Importance Sampling is used and the results for the credible interval for h are compared to the M-H sampler.

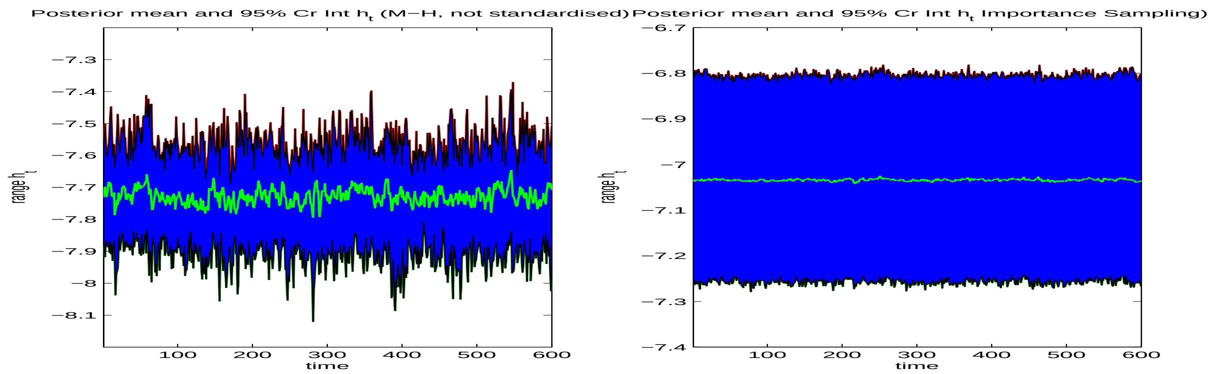

Figure 3.4.2: Volatility parameter posterior mean and credible interval, using $c = 10^{-3}$, residuals not standardised using M-H to sample h_t (left plot), and Importance Sampling (right plot).

The results for the volatility parameter do not improve by using importance sampling (in this case, the h_t was simulated using $\mu = -10$). The results continue to depend on the value of the offset parameter used. To overcome the problem of sampling the volatility parameter, we propose below a method which overcomes the difficulty of selecting an appropriate value of the offset parameter c.

3.5 Standardisation approach

3.5.1 Method

As we see from the results above, the value used for the offset parameter c influences the posterior mean of the volatility parameter. The problem arises from the assumption in the application of the mixture model that the error term for the log returns has a Standard Normal distribution and the square of the error term is then approximated with a Normal Mixture for $\log \chi^2_1$. We propose a method so that the error term in equation (3.1.1) is standardised as

$$\nu_t = \frac{y_t}{e^{h_t/2}},$$
$$\log(\nu_t^2 + c) = \log\left(\frac{y_t^2}{e^{h_t}} + c\right)$$
$$= \log(y_t^2 + ce^{h_t}) - h_t.$$

Rearranging the above and writing the AR(1) process for h_t, the model in becomes

$$\log(y_t^2 + ce^{h_t}) = h_t + \log(\nu_t^2),$$
$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma_\eta \eta_t,$$

where $\nu_t \sim \mathrm{N}(0,1)$ and the Normal Mixture approximation for $\log \chi_1^2$ can be used. Conditioning on the mixture indicator $s_t = i$, this can be expressed with Normal errors and FFBS method described above can be used to sample and accept proposed values of h_t as in section 3.3. For $\log(y_t^2 + ce^{h_t})$ we use $\log(y_t^2 + ce^{h_t^{(n)}})$ where $h_t^{(n)}$ is the value of h at the previous iteration n of the MCMC sampling. Using simulated data, the MCMC model is run without standardising the error term and with standardising the error term. The results are shown in the next section.

3.5.2 Results

Figure 3.5.1 below shows the results obtained for the volatility parameter h_t when the offset parameter $c = 10^{-4}$ is used without standardising and when using the the same value of c with standardising. When the error term is standardised, the volatility parameter results are much closer to the simulated volatility values.

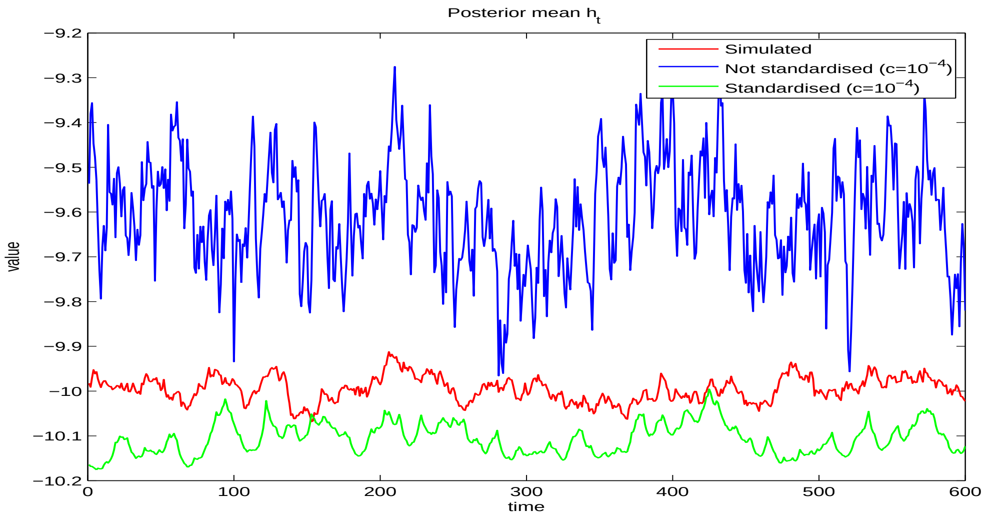

Figure 3.5.1: Posterior mean of the volatility parameter h_t using offset $c = 10^{-4}$ without standardising the residuals, and with standardising the residuals.

The plots below, Figure 3.5.2, show the parameter results with the error term standardised, using $c = 10^{-4}$ (left column) and $c = 10^{-6}$ (right column).

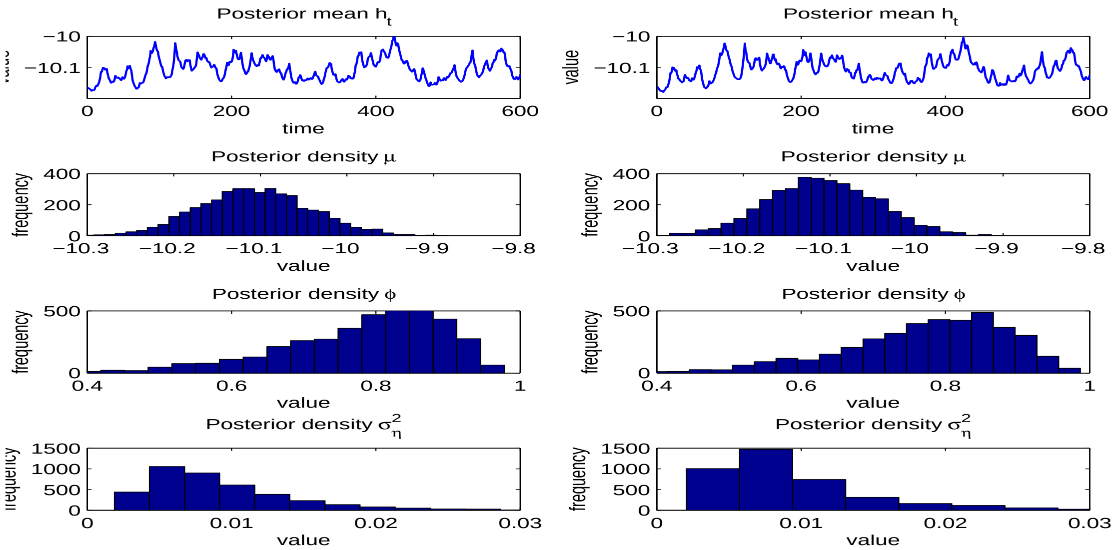

Figure 3.5.2: Volatility parameter h_t and AR(1) parameters μ, ϕ andσ^2_η posterior distributions, left column using $c = 10^{-4}$ and right column using $c = 10^{-6}$, residuals standardised

As the results in the plots in Figure 3.5.2 show, when using the standardised method, the value of the offset parameter that is used has a much smaller effect on sampling the volatility parameter h_t. The results from using 2 different offset values on the simulated data set for the AR(1) process parameters and the volatility parameter h_t are the same (for both cases, μ is close to -10, the value used to simulate the data) as in Table 7.

	$c = 10^{-4}$	$c = 10^{-6}$
Parameter	Posterior mean (credible interval)	Posterior mean (credible interval)
μ (-10)	-10.1064 (-10.2305, -9.9757)	-10.1084 (-10.2334, -9.9829)
ϕ (0.95)	0.7762 (0.4621, 0.9390)	0.7716 (0.5130, 0.9369)
σ_η^2 (0.01^2)	0.0095 (0.0031, 0.0246)	0.0102 (0.0032, 0.0328)

Table 7: Posterior mean and 95 % credible interval for the AR(1) parameters μ, ϕ, σ_η^2 using different values of the offset parameter, residuals standardised.

Mean Square Error of the parameters

The results in the plots in Figures 3.5.1 and 3.5.2 above can also be interpreted using the MSE for the parameters. When error term is not standardised, the MSE for h depends on the mixture offset parameter c as in Table 8 and is high when $c = 10^{-3}$ is used. For the standardised method, Table 9, the MSE for h does not depend on the value of c. For both cases, the MSE for the other parameters does not depend on c and the MSE results are closer to the case when very small values of c are used.

$c = 10^n$				
n	MSE h	MSE ϕ	MSE μ	MSE σ^2_η
-1	47.7370	0.0778	47.7952	0.00007
-2	19.1030	0.2165	19.1506	0.00006
-3	5.1416	0.0814	5.1644	0.00002
-4	0.0164	0.0327	0.1520	0.00002
-5	0.0161	0.0383	0.0172	0.00002
-6	0.0149	0.0337	0.0164	0.00009
-7	0.0151	0.0432	0.0165	0.00014
-9	0.0149	0.0399	0.0166	0.00014
-11	0.0153	0.0474	0.0167	0.00016
-13	0.0162	0.0444	0.0178	0.00013

Table 8: Mean Square Error when posterior mean of the parameter is compared to the simulated value, residuals not standardised.

$c = 10^n$				
n	MSE h	MSE ϕ	MSE μ	MSE σ_η^2
-1	0.0143	0.0390	0.0157	0.00012
-2	0.0158	0.0404	0.0172	0.00013
-3	0.0146	0.0391	0.0169	0.00012
-4	0.0140	0.0459	0.0155	0.00013
-5	0.0160	0.0393	0.0173	0.00013
-6	0.0145	0.0448	0.0158	0.00002
-7	0.0164	0.0432	0.0179	0.00015
-9	0.0157	0.0356	0.0175	0.00016
-11	0.0151	0.0562	0.0170	0.00015
-13	0.0159	0.0482	0.0170	0.00013

Table 9: Mean Square Error, when posterior mean of the parameter is compared to the simulated value, residuals standardised.

3.6 Using Eurostoxx Index

The example below uses Eurostoxx daily log returns calculated from the index closing values from January 2007 to April 2013. The stochastic volatility model is used to sample the volatility and the AR parameters. A value of $c = 10^{-3}$ was used for the mixture offset parameter and the error term was standardised for the MCMC iterations.

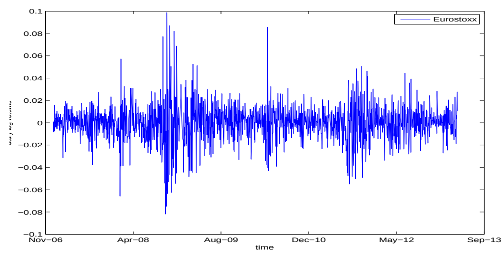

Figure 3.6.1: Eurostoxx daily log returns, European sovereign debt crisis 2007-2013.

3.6.1 Results

The results below show the mean value of the volatility parameter h_t and the 95% credible interval. The other plots show the posterior distributions of the parameters μ, ϕ, σ_η^2.

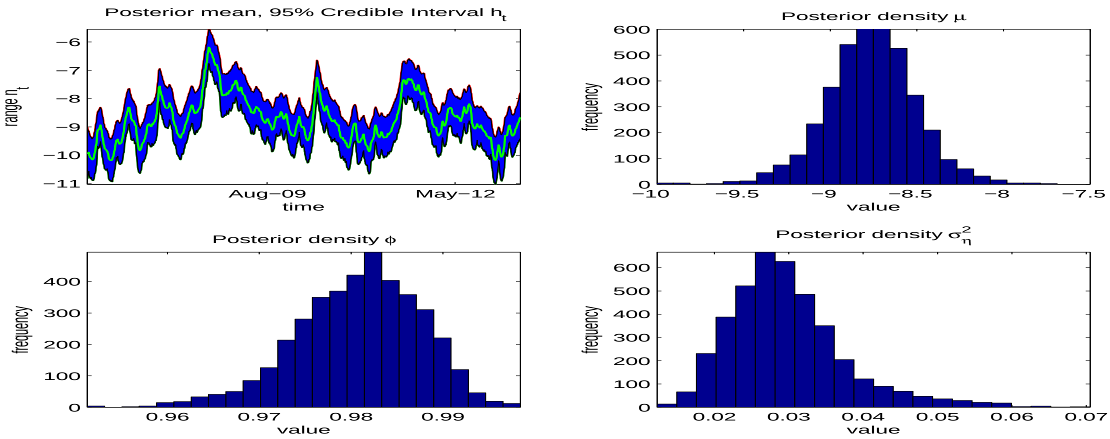

Figure 3.6.2: Volatility parameter with 95% credible interval and posterior distribution for the AR(1) process parameters μ, ϕ and σ_η^2 using $c = 10^{-3}$, residuals standardised.

The volatility parameter shows higher values during the crisis periods which show as volatility clusters on the log returns plot in Figure (3.6.1). The volatility persistence paramter ϕ is high, around 0.98, similar to values seen in other financial studies (Mahieu & Schotman 1998).

3.6.2 Effective Sample Size

The MCMC samples from the Eurostoxx data have been used to calculate the effective sample size (Gamerman & Lopes 2006). In order to check that the burn in period is adequate, the trace plots have been checked to see that the values have converged. To check that the results converge to the same results, different initial values for the volatility parameter (h_t) and the mixture indicator (s_t) have been used (Sokal 1997) to initialise the iterations.

Bartlet's test is used to calculate the standard error at lag k as

$$\mathrm{SE}(r_k) = \sqrt{\frac{1 + 2\sum_{i=1}^{i=k-1} r_i^2}{\mathrm{N}}} \text{ for } k > 1$$

where r_i is the correlation at lag i, N is the number of samples. The standard error is then used to calculate a 95% confidence interval used to determine the number of non-zero correlation coefficients which are used to calculate the Effective Sample Size.

The Effective Sample Size (ESS) is calculated as

$$\mathrm{ESS} = \frac{\mathrm{N}}{1 + 2\sum_{i=1}^{j} r_i}$$

here N is the number of samples from the iterations and r_i is the correlation coefficient at lag i and j is the count of the non-zero correlation coefficients.

For the h_t samples for the Eurostoxx data (using the MCMC results when $c = 10^{-3}$ and y_t is standardised) the effective sample size is around 1700 for h_t (as in the plot below), 300 for ϕ and about 3800 for μ. The correlations are calculated using the Matlab function autocorr.

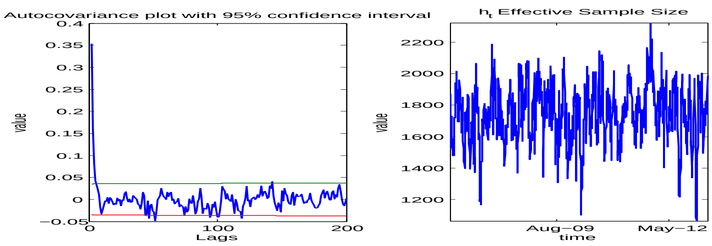

Figure 3.6.3: ACF plot for h_t (left plot) and Effective Sample Size for h_t (right plot), using $c = 10^{-3}$, residuals standardised.

For example, at $t = 100$, the left plot shows the ACF values for the h_t samples. The plot also shows the bounds for 2 standard deviations (approximately 95% confidence interval for the ACFs). The ACFs decay quickly (generally, the values are not significant beyond 6 lags for other values of t).

Effective Sample Size, Eurostoxx daily returns

$c = 10^n$			
n	ESS h	ESS ϕ	ESS μ
-0 (c=1)	57.0	2600	3891
-1	38.3	20.5	14.5
-2	29.6	24.8	21.0
-3	36.4	28.5	16.8
-4	365	124	3932
-5	757	105	3404
-6	1887	271	3908
-7	2006	288	3609
-9	1512	223	3709
-11	1644	279	3985
-13	1491	195	3781

$c = 10^n$			
n	ESS h_t	ESS ϕ	ESS μ
-0 (c=1)	18.5	25.9	34.9
-1	27.8	88.1	3564
-2	266	42.4	3830
-3	1709	321	3864
-4	1490	242	3875
-5	1720	202	3794
-6	1237	177	3837
-7	1772	218	3399
-9	1254	122	3817
-11	945	216	4226
-13	466	167	3926

Table 10: Effective sample size, residuals not standardised (left table), residuals standardised (right table), results for different values of the offset parameter.

For the not standardised case, the ESS values for h_t, ϕ and μ, stabilise as c gets smaller than 10^{-4}. For the not standardised case, when c is large (10^{-2} or 10^{-1}) relative to y, $\log(y_t^2 + c)$ distorts the data and the sampler is not able to estimate the parameters correctly. In this sense, ESS is meaningless, but has been shown for completeness.

Using the standardised method, Table 10 (right table), the ESS results are stable for all the parameters when $c < 10^{-2}$. For $c = 10^{-1}$ and 10^{-2}, the ESS results for ϕ and μ are better than the not standardised case. For the other values of c, the results are similar.

The MCMC results are from running 40000 iterations (first 20000 as a burn in period) and samples are thinned to 1/5 resulting in 4000 samples for each parameter. MATLAB has been used for writing the MCMC sampler.

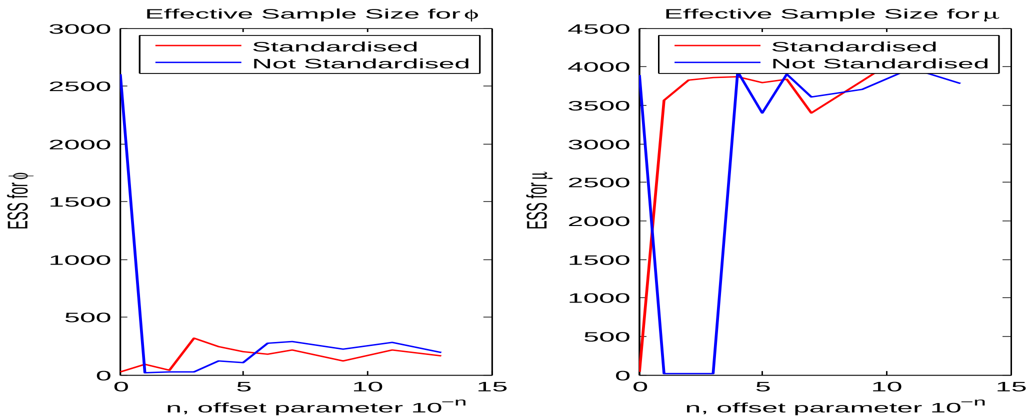

Figure 3.6.4: Effective Sample Size for ϕ (left plot) and μ (right plot) comparing results when residuals are not standardised and when standardised.

In the left plot in Figure (3.6.4), the ESS for ϕ is low for both the standardised and not standardised cases (except when $c = 10^{-0}$ i.e. $c = 1$) which distorts the data. The results for μ, right plot in Figure (3.6.4), shows larger values of the ESS when the standardised method is used. Once much smaller values of c are used, the ESS for μ is similar for both the cases. This indicates that the standardising method performs better without having to rely on setting an appropriate "small" value of the offset parameter.

3.7 Conclusion

When the SV model is expressed in linear State Space form using a Normal Mixture model, the value of the offset parameter used for the MCMC sampling can lead to incorrect results for the volatility parameter. To overcome this, the error term is standardised with an added Metropolis-Hastings step to accept the proposed volatility parameter samples from the Forward Filtering Backward Sampling distributions. This approach gives good results for the volatility parameter using any small value of the offset parameter.

The Effective Sample Size for the volatility parameter and the AR process parameters is higher when the standardised approach is used, indicating that this is a better sampling procedure. The MSE for the volatility parameter also indicates a better result for the volatility parameter when the standardised approach is used.

Chapter 4

Normal Mixture Samplers

4.1 Introduction

In Chapter 2, for the posterior inference, one at a time sampling was used for the volatility parameter h_2. In this chapter, multiple values of the volatility parameter are sampled using the FFBS method with the aim of improving the slow convergence when sampling autocorrelated values. This is achieved by using a Normal Mixture estimate for the error term for the log returns and this allows the returns and the volatility parameter h_2 to be expressed in linear State Space form with Normal errors. As explained in Chapter 3, the error term for state evolution equation is standardised to overcome the problem of setting a value for the mixture offset parameter c when the MCMC sampler is used to infer the parameter values. The standardised method with $c = 10^{-6}$ has been used for the MCMC simulations.

For the innovations in the volatility spillover parameter γ, 3 different prior densities are used. First a Normal prior is used, followed by a Student-t prior and then a Normal-Gamma prior density is used. The objective is to evaluate the use of heavy tailed distributions to identify the extreme values when the spillover has larger jumps.

The 3 samplers are described below. This is followed by the results of using the samplers on simulated data, first using the simulated spillover and daily returns for 3 countries to check how well the parameters are estimated. Then, different levels (known values) of the spillover parameter are used to evaluate the performance of the samplers for estimating the spillover. The evaluation is based on the number of days (using the credible interval) of spillover identified under different conditions and also using the 'root mean squared error' (RMSE) to see how well the spillover parameter is estimated. Then the samplers are used for the Asian financial crisis 1997-1998 and European sovereign debt crisis 2007-2013 and the results are discussed below.

4.2 Sampler

The same volatility spillover model as in section 2.2 is used in this chapter

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} f_t & 0 & 1 & 0 \\ 0 & f_t & 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_{1,t} \\ \theta_{2,t} \\ \theta_{3,t} \\ \theta_{4,t} \end{pmatrix} + \begin{pmatrix} \delta_{1,t} & 0 \\ \gamma_t & \delta_{2,t} \end{pmatrix} \begin{pmatrix} \nu_{1,t} \\ \nu_{2,t} \end{pmatrix}, \tag{4.2.1}$$

$$\begin{pmatrix} \theta_{1,t} \\ \theta_{2,t} \\ \theta_{3,t} \\ \theta_{4,t} \end{pmatrix} = \begin{pmatrix} \theta_{1,t-1} \\ \theta_{2,t-1} \\ \theta_{3,t-1} \\ \theta_{4,t-1} \end{pmatrix} + \begin{pmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \\ w_{4,t} \end{pmatrix}. \tag{4.2.2}$$

The model parameters and prior distributions are the same as in section 2.3 where we used one at a time sampling for the volatility parameter h_{2t}.

Updating volatility parameter h_2

Using equations (4.2.1, 4.2.2), and using $\delta_{1,t} = \exp(\frac{h_{1,t}}{2})$ and $\delta_{2,t} = \exp(\frac{h_{2,t}}{2})$, $y_{2,t}$ can be expressed as

$$y_{2,t} - (f_t\theta_{2,t} + \theta_{4,t}) - \gamma_t \frac{y_{1,t} - f_t\theta_{1,t} - \theta_{3,t}}{\mathrm{e}^{h_{1,t}/2}} = \mathrm{e}^{h_{2,t}/2}\nu_{2,t} \tag{4.2.3}$$

and the state equation for $h_{2,t}$ as $h_{2,t} = \mu_2 + \phi(h_{2,t-1} - \mu_2) + \sigma_{\eta_2}\eta_t$.

This is not in linear State Space form for h_{2t}. We linearise the above by using Normal Mixture density as in Kim et al. (1998). Conditioning on the mixture state, the observation equation has Normally distributed error terms. The evolution equation for the volatility parameter also has Normally distributed errors. This allows the use of the standard results for the Kalman Filter and Backward Sampling (or FFBS) to propose the volatility parameter and then use the Metropolis-Hastings method to sample the proposed values.

The LHS in equation 4.2.3 is the residual and we define the LHS as r_{2t}. Conditional on y_1 and the parameters θ, h_{1t}, γ_t,

$$r_{2,t}|y_1, \theta, \delta_1, \gamma \sim \mathrm{N}(0, h_{2,t}),$$

$$r_{2,t} = \mathrm{e}^{\frac{h_{2,t}}{2}}\nu_{2,t},$$

$$\nu_{2,t} \sim \mathrm{N}(0, 1)$$

and

$$\log(r_{2,t}^2 + c) = h_{2,t} + \log \nu_{2,t}^2$$

where c is a small offset parameter used to make the model robust for small values of $r_{2,t}$. The term $\log \nu_{2,t}^2$ has a $\log \chi_1^2$ density. Kim et al. (1998) approximate the $\log \chi_1^2$ density with a 7 component Normal Mixture density and estimate the parameters $q_i, m_i, v_i^2, 1.2704$ for $i = 1, \ldots, 7$ for the Normal Mixture density using numerical methods so that the Normal Mixture closely approximates the $\log \chi_1^2$ density.

Using the Normal Mixture approximation, and an AR(1) process for h_{2t} with Normal errors

$$\log(r_{2,t}^2 + c)|s_t = i = h_{2,t} + m_i - 1.2704 + \nu_t,$$
$$p(s_t = i) = q_i,$$
$$h_{2,t} = \mu_2 + \phi(h_{2,t-1} - \mu_2) + \sigma_{\eta 2}\eta_t,$$

where the error terms $\nu_t \sim \mathrm{N}(0, v_i^2)$ and $\eta_t \sim \mathrm{N}(0, 1)$ are independently distributed. This is in linear State Space form for $h_{2,t}$.

The State Space equation for $h_{2,t}$ is now in the same form as that for h_t used in the Chapter 3 for the mixture offset parameter. Using the method in Chapter 3, the error term is standardised and Forward Filtering Backward Sampling method is used to propose the values of $h_{2t}, t = 1, \ldots, T$. A Metropolis-Hastings step as described in Section 3.3 is used to accept the proposed values. Conditional on h_2, the posterior inference for the AR(1) parameters for h_2 is the same as shown in Section 2.3.

4.3 Spillover parameter

4.3.1 Normal prior

The spillover parameter γ follows an AR(1) process with the Normal errors. From equations 2.3.1 and 2.3.2,

$$y_{2,t} - f_t\theta_{2,t} - \theta_{4,t} = \left(\frac{y_{1,t} - f_t\theta_{1,t} - \theta_{3,t}}{\mathrm{e}^{h_{1,t}/2}}\right)\gamma_t + \mathrm{e}^{h_{2,t}/2}\nu_{2,t},$$

$$\gamma_t = \mu_3 + \phi_3(\gamma_{t-1} - \mu_3) + \sigma_{\eta_3}\eta_t,$$

$$\eta_t \sim \mathrm{N}(0,1).$$

Given the parameters θ, h_{1t}, h_{2t} and f_t, and the returns $y_{1,t}, y_{2,t}$, these equations are in linear State Space form for γ with normally distributed errors. Hence we can use the FFBS method to sample γ_t. The sampling for the parameters μ_3, ϕ_3 and $\sigma^2_{\eta_3}$ for the AR(1) process for γ is similar to that for the AR(1) parameters for h_1 as described in section 2.3

4.3.2 Student-t prior

In this section, the Student-t distribution is used as a prior density for the innovations in the volatility spillover. The aim is to see whether this helps to identify the extreme values of the spillover. We use the result that the ratio of a variable with a Standard Normal density and the square root of a variable with Gamma density with location and scale parameters both equal to $\frac{\nu}{2}$ results in Student-t distribution with ν degrees of freedom. The parameter ν is inferred from the data.

State Space equations for γ.

From equations (2.2.1, 2.2.2),

$$(y_{2,t} - f_t\theta_{2,t} - \theta_{4,t}) = \left(\frac{y_{1,t} - f_t\theta_{1,t} - \theta_{3,t}}{e^{h_{1,t}/2}}\right)\gamma_t + e^{h_{2,t}/2}\nu_{2,t}. \tag{4.3.1}$$

We use an AR(1) process for γ,

$$\gamma_t = \mu_3 + \phi_3(\gamma_{t-1} - \mu_3) + \sigma_{\eta_3}\frac{\eta_t}{\sqrt{w_t}}, \tag{4.3.2}$$

where $\eta_t \sim \mathrm{N}(0,1)$ and w_t has a Gamma distribution $G(\frac{\nu}{2}, \frac{\nu}{2})$. Conditional on y_1, h_1 and the AR parameters, equations (4.3.1 and 4.3.2) are in linear State Space form for γ. Conditioning on w_t, results in γ with Normally distributed errors with conditional distribution $\gamma_t|\gamma_{t-1}, w_t \sim \mathrm{N}(\mu_3 + \phi_3(\gamma_{t-1} - \mu_3), \sigma^2_\eta/w_t)$. FFBS method as described in section (2.3) is used to sample γ with a Metropolis-Hastings step to accept the proposed values.

Conditional on γ and the AR parameters for γ, updating the model parameters θ, W, h_1, h_2 and the related AR(1) parameters for h_1 and h_2 remains the same as in section (2.3). The changes when using a Student-t prior distribution is used for updating γ and the AR(1) parameters for γ are summarised below.

Backward Sampling

Using the the Forward Filtering Backward Sampling method, the distribution for γ_{t-1} is given by

$$\begin{aligned}
\gamma_{t-1}|\gamma'_{t:T}, \gamma, w_t &\sim \mathrm{N}(\mu_{B,t-1}, \sigma^2_{B,t-1}), \\
\sigma^2_{B,t-1} &= \left(\frac{\phi_3^2}{\sigma^2_{\eta_3}/w_t} + \frac{1}{\sigma^2_{t-1}}\right)^{-1}, \\
\mu_{B,t-1} &= \left(\frac{\phi_3^2}{\sigma^2_{\eta_3}/w_t} + \frac{1}{\sigma^2_{t-1}}\right)^{-1}\left(\frac{\phi_3\gamma'_t - \phi_3\mu_3(1-\phi_3)}{\sigma^2_{\eta_3}/w_t} + \frac{\mu_{t-1}}{\sigma^2_{t-1}}\right),
\end{aligned}$$

where $\gamma_t^{'}$ is the backward sample at time t, w_t is the sampled value at time t, ϕ_3, μ_3 and $\sigma^2_{\eta 3}$ are the AR process parameters and μ_{t-1} and σ^2_{t-1} are the mean and variance for the forward distributions using the Kalman Filter.

AR parameters for γ with Student-t errors

Updating μ_3

Using a diffuse prior for μ_3, so that $\mu_3 \propto 1$ the full conditional density is $\mu_3 \sim \mathrm{N}(\hat{\mu}_3, \sigma^2_{\mu_3})$ where

$$\sigma^2_{\mu_3} = \sigma^2_{\eta 3} \left(\frac{(1-\phi_3^2)}{1/w_1} + \sum_{t=2}^{t} \frac{(1-\phi_3)^2}{1/w_t} \right)^{-1},$$

$$\hat{\mu}_3 = \frac{\sigma^2_{\mu_3}}{\sigma^2_{\eta 3}} \left(\frac{\gamma_1(1-\phi_3^2)}{1/w_1} + (1-\phi_3) \sum_{k=2}^{t} \frac{(\gamma_k - \phi_3 \gamma_{k-1})}{1/w_k} \right).$$

Updating ϕ_3

Using $\phi_3 = 2\phi^* - 1$, and ϕ^* with a Beta(a,b) prior distribution

$$\pi(\phi_3) \propto \left(\frac{1+\phi_3}{2} \right)^{a-1} \left(\frac{1-\phi_3}{2} \right)^{b-1},$$

and posterior density of ϕ_3 given $\gamma_{1:T}$ is

$$p(\phi_3|\gamma_{1:T}) \propto p(\phi_3) p(\gamma_1|\sigma_{\eta 3}, w_1) \prod_{t=2}^{t=T} p(\gamma_t|\gamma_{t-1}, w_{t-1})$$

$$\propto \left(\frac{1+\phi_3}{2} \right)^{\phi_3 - 1} \left(\frac{1-\phi_3}{2} \right)^{\phi_3 - 1} \mathrm{e}^{-\frac{1}{2} \frac{(\gamma_1 - \mu_3)^2}{\sigma^2_{\eta 3}/w_1})}$$

$$\prod_{t=2}^{T} \left(\frac{\sigma^2_{\eta 3}}{w_t} \right)^{-\frac{1}{2}} \mathrm{e}^{-\frac{1}{2} \frac{((\gamma_t - \mu_3) - \phi_3(\gamma_{t-1} - \mu_3))^2}{\sigma^2_{\eta 3}/w_t}}$$

Updating $\sigma^2_{\eta_3}$

Using an Inverse Gamma prior density $\sigma^2_{\eta_3} \sim \text{IG}\left(\frac{\sigma_r}{2}, \frac{\text{S}_\sigma}{2}\right)$, the full conditional distribution is $\sigma^2_{\eta_3}|\gamma, \phi_3, \mu_3$ is

$$\text{IG}\left(\frac{\sigma_r + T}{2}, \frac{S_\sigma}{2} + \frac{1}{2}\frac{(\gamma_1 - \mu_3)^2(1-\phi_3^2)}{1/w_1} + \frac{1}{2}\sum_{t=2}^{T}\frac{((\gamma_t - \mu_3) - \phi_3(\gamma_{t-1} - \mu_3))^2}{1/w_k}\right).$$

The parameters for the prior density are $\sigma_r = 5, \text{S}_\sigma = 0.01\sigma_r$.

Updating w_t

The prior density for w_t is $w_t \sim \text{G}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$ and the full conditional density for w_t give γ_t is

$$\begin{aligned} p(w_t|\gamma_t) &\sim \text{G}(\alpha, \beta) \\ \alpha &= \frac{\nu + 1}{2} \\ \beta &= \frac{1}{2}\left(\nu + \left(\frac{\gamma_t - \mu_3(1-\phi_3) - \phi_3\gamma_{t-1}}{\sigma_\eta}\right)^2\right). \end{aligned}$$

Updating ν degrees of freedom

The prior density for ν is $\nu \sim \text{G}(\alpha, \beta)$ is $(\alpha = 1, \beta = 0.1)$

$$p(\nu) = \frac{\beta^\alpha}{\Gamma(\alpha)}\nu^{\alpha-1}e^{-\beta\nu}.$$

Likelihood for $w_t|\nu \sim \text{G}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$ is

$$p(w_t|\nu) = \frac{(\frac{\nu}{2})^{\frac{\nu}{2}}}{\Gamma(\frac{\nu}{2})}w_t^{(\frac{\nu}{2}-1)}e^{-\frac{\nu}{2}w_t}.$$

Hence the posterior density is

$$p(\nu|w) \propto \frac{\beta^\alpha}{\Gamma(\alpha)}\nu^{\alpha-1}e^{-\beta\nu}\prod_{t=1}^{T}\frac{(\frac{\nu}{2})^{\frac{\nu}{2}}}{\Gamma(\frac{\nu}{2})}w_t^{(\frac{\nu}{2}-1)}e^{-\frac{\nu}{2}w_t}.$$

The posterior density cannot be recognised as a standard density. The log of the posterior density, keeping only the terms in ν is

$$\log(p(\nu|w)) = (\alpha-1)\log(\nu) - \beta\nu + \sum_{t=1}^{T}\left(\frac{\nu}{2}\log(\frac{\nu}{2}) - \log(\Gamma(\frac{\nu}{2})) + (\frac{\nu}{2}-1)\log(w_t) - \frac{\nu}{2}w_t\right).$$

A truncated Normal is used to propose ν (to ensure that $\nu > 0$) and an Adaptive Metropolis-Hastings step is used to obtain acceptance rates of around 20%.

4.3.3 Normal-Gamma prior

In this section, the Normal-Gamma density (Griffin & Brown 2010) is used as a prior for the spillover parameter γ. The aim is to use a heavy tail distribution and see whether this identifies the extreme values of the spillover parameter better compared to the other priors described above. The parameters for the the Gamma distribution are inferred from the data.

The volatility parameter γ follows an AR(1) process as

$$\gamma_t = \mu_3 + \phi_3(\gamma_{t-1} - \mu_3) + \sigma_{\eta 3}\sqrt{w_t}\eta_t,\ \eta_t \sim \mathrm{N}(0,1),$$
$$w_t|\lambda \sim \mathrm{G}(\lambda,\lambda),$$
$$\lambda \sim \mathrm{Exp}(1).$$

Conditional on the AR(1) process parameters and w_t, the distribution for γ_t is

$$\gamma_t|\mu_3,\phi_3,\sigma_{\eta 3},w_t \sim \mathrm{N}\left(\mu_3 + \phi_3(\gamma_{t-1}-\mu_3), w_t\sigma_{\eta 3}^2\right).$$

Updating AR(1) parameters for γ

Conditional on w_t, the variance of the distribution of γ_t changes as below,

$$\gamma_t|w_t, \mu_3, \phi_3, \sigma_{\eta_3} \sim \mathrm{N}(\mu_3 + \phi_3(\gamma_{t-1} - \mu_3), \sigma^2_{\eta_3} w_t).$$

This changes the variance term used in the inference for the AR(1) parameters. Updating the AR(1) process parameters remains similar to that used in section 4.3.2 for the Student-t prior distribution case with a change for the variance term.

Updating w_t

The prior density for w_t is

$$w_t|\lambda \sim \mathrm{G}(\lambda, \lambda)$$

and the parameter λ follows an Exponential distribution

$$\lambda \sim \mathrm{Exp}(1),\ p(\lambda) = \mathrm{e}^{-\lambda},\ \lambda > 0.$$

The posterior density for w_t is

$$\begin{aligned} p(w_t|\gamma_t, \lambda) &\sim p(\gamma_t|w_t, \lambda)p(w_t|\lambda)p(\lambda), \\ &= \frac{1}{\sigma_{\eta_3}\sqrt{w_t}} \mathrm{e}^{\frac{-\frac{1}{2}(\gamma_t - (\mu_3 + \phi_3(\gamma_{t-1} - \mu_3)))^2}{\sigma_{\eta_3} w_t}} \frac{\lambda^\lambda}{\Gamma(\lambda)} w_t^{\lambda-1} \mathrm{e}^{-\lambda w_t} \mathrm{e}^{-\lambda}, \end{aligned}$$

($\lambda, \mu_3, \phi_3, \sigma_{\eta_3}$ are known, keeping only the terms in w_t)

$$\propto w_t^{\lambda - 1 - \frac{1}{2}} \mathrm{e}^{-\frac{1}{2}\left(2\lambda w_t + \frac{(\gamma_t - (\mu_3 + \phi_3(\gamma_{t-1} - \mu_3)))^2}{\sigma_{\eta_3} w_t}\right)}.$$

The posterior density cannot be recognised as standard density. Hence, a symmetric random walk is used to propose w_t and an Adaptive M-H step is used to obtain about 20 % moves.

Updating λ

The posterior density for λ is

$$\begin{aligned} p(\lambda|w) &= p(w|\lambda)p(\lambda) \\ &= \left(\prod_{t=1}^{t=T} \frac{\lambda^\lambda}{\Gamma(\lambda)} w_t^{\lambda-1} \mathrm{e}^{-\lambda w_t}\right) \mathrm{e}^{-\lambda}. \end{aligned}$$

This cannot be recognised as a standard density. A Truncated Normal is used to propose values of λ and an Adaptive M-H step is used to obtain moves of around 20 %.

4.4 Simulated data results

In this section, 2 types of simulated data are used. For one case, the data is simulated using the model, with the spillover γ following an AR(1) process. For the second case, the spillover has known values (a background level as small value, with sudden jumps of different magnitudes). The parameters for simulating the data are based on the results from the Asian financial crisis (with jump sizes based on the results) and another set of data is generated using the parameter results from the Euro crisis results (with jump sizes based on the results). The variances of the other parameters (for example, regression parameters, volatility parameters, the daily returns for the world factor) are increased and the MCMC results are compared to see how well the spillover is identified under 'noise' conditions. Another test is also performed using shorter spillover periods.

4.4.1 Parameter γ simulated using an AR(1) process

The simulated data as used in tests in section 2.4.1 has been used here. The simulated data is used to check the ability of the sampler to infer the true parameter values. The results shown below are for the Normal Mixture with the Normal errors for the spillover innovations (other samplers also give similar results).

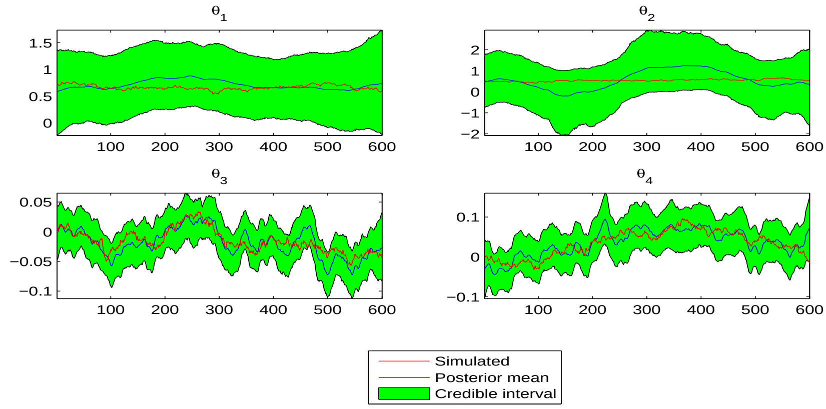

Figure 4.4.1: 95 % Credible interval and posterior mean for regression parameter θ, using simulated data.

The posterior mean for the θ values is close to the simulated values and the simulated value is within the 95% credible interval.

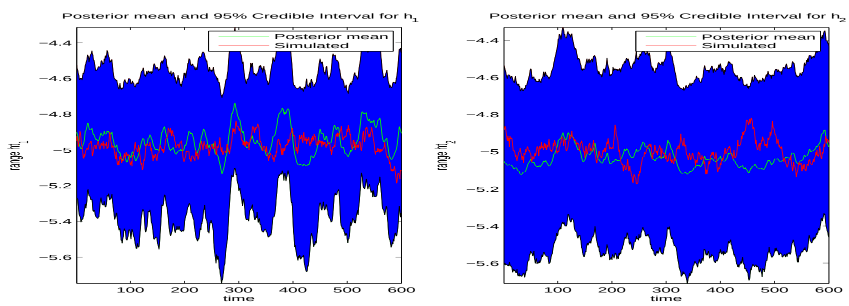

Figure 4.4.2: Posterior mean, simulated and 95 % credible interval for the volatility parameters h_1 and h_2, using simulated data.

The posterior means for the volatility parameters h_1 and h_2 are close to the simulated values. Compared to the one at time sampler in Chapter 2, the 95% credible interval for h_2 is slightly larger, the posterior mean has a greater variance when the Normal Mixture is used.

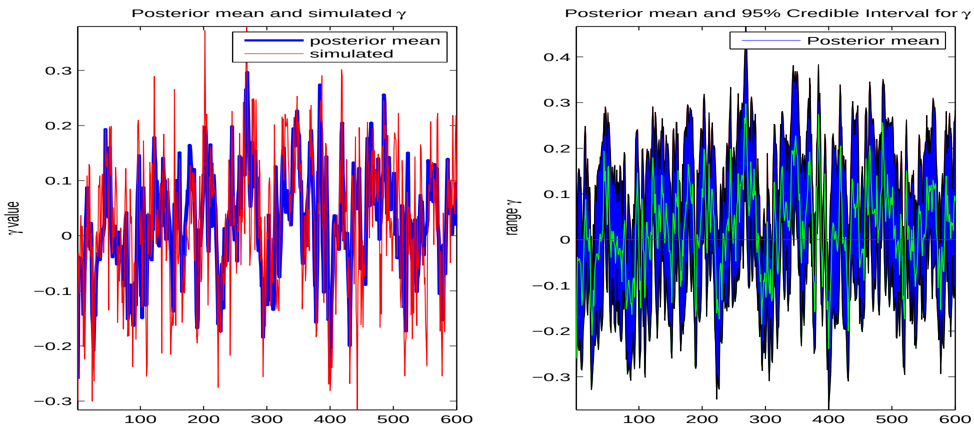

Figure 4.4.3: Volatility spillover γ, simulated and posterior mean (left plot), posterior mean and 95% credible interval (right plot).

The posterior mean for the spillover parameter γ closely follows the simulated values as in the left plot. The 95% credible interval for the posterior mean of the spillover γ shows a few non zero values in right plot. As can be seen on the simulated data plot these are the larger values for the simulated spillover. For the smaller values of the simulated spillover values, the non zero values are not identified.

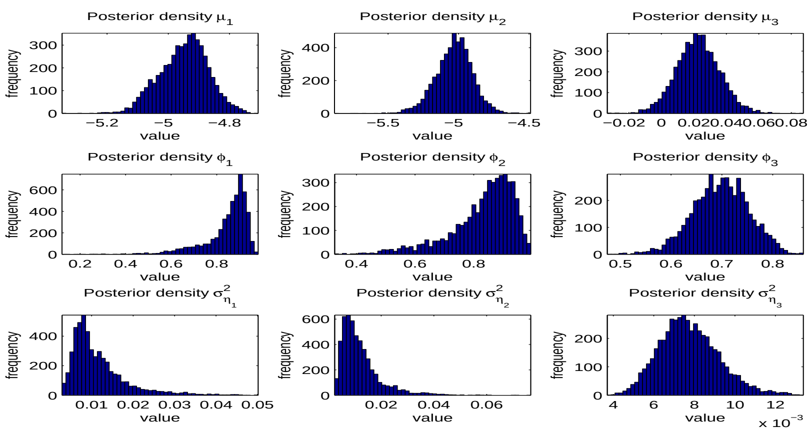

Figure 4.4.4: Posterior distributions for the AR(1) parameters μ, ϕ and σ_η^2.

For h_1, h_2 and γ, the AR(1) parameters μ, ϕ are estimated well as shown by the posterior distribution plots above. The variance results σ_η^2 for h_1 and h_2 are higher than the values used to simulate the data. The credible interval for the variance for the spillover parameter includes the value used to simulate the data.

Parameter	Simulated value	Posterior mean	95% Credible Interval
μ_1	-5	-4.9489	(-5.1051, -4.8009)
ϕ_1	0.9500	0.8449	(0.5752, 0.9486)
$\sigma^2_{\eta_1}$	0.0004	0.0116	(0.0041, 0.0312)
μ_2	-5	-5.0318	(-5.3093, -4.7814)
ϕ_2	0.9500	0.8308	(0.5409, 0.9686)
$\sigma^2_{\eta_2}$	0.0004	0.0121	(0.0038, 0.0341)
μ_3	0.0001	0.0226	(-0.0042, 0.0494)
ϕ_3	0.6000	0.6977	(0.5869, 0.8024)
$\sigma^2_{\eta_3}$	0.0100	0.0078	(0.0052, 0.0110)

Table 11: Simulated mean, posterior mean and 95% credible interval for the AR(1) parameters μ, ϕ and σ^2_η.

These results in Table 11 are similar to the results obtained using one at a time sampling as in Table 4. For this sampler, Normal Mixture approximation has been used for the error term for h_2. AR(1) parameter ϕ_2 is estimated better, the credible interval for μ_2 is slightly larger and the credible interval for $\sigma^2_{\eta_2}$ is slightly smaller.

4.4.2 Simulated data, known γ values

In this section, we evaluate the performance of the 4 different samplers (sampler S1, one at a time sampling with a Normal prior for the spillover parameter γ; sampler S2, Mixture Normal for parameter h_2 with Normal prior for the spillover parameter γ; sampler S3, Mixture Normal for parameter h_2 with Student-t prior for the spillover parameter γ and sampler S4, Mixture Normal for parameter h_2 with a Normal-Gamma prior for the spillover parameter γ) in detecting the volatility spillover under different simulated conditions.

For the simulated data, the spillover parameter γ has low values ± 0.0005 for most of the time as background spillover level and followed by sudden jumps in spillover for 5 or 10 days. Different sizes of jumps based on results from the Asian financial crisis and Euro crisis are used. The variance of the regression parameter θ (which represents the relationship between the country returns and the world factor) is increased in order to test how well the spillover is identified when the relationship with the world factor is volatile. The volatility of the country returns is also increased for some tests. Cases with high volatility for the world factor returns are also used. All the simulated cases have 600 observations. The MCMC results for the 4 samplers are obtained using 25000 iterations (initial 12500 iterations as burn in) and the samples are thinned to 1 in 5.

Two measures are used to see how well the samplers perform. One, to identify when zero is not in the credible interval for γ, thus indicating that a positive or negative value of γ has been identified (that is, there is volatility spillover) and the number of days where the value is identified as non zero are counted. The second aim is to see how well the parameter γ has been estimated for the 4 samplers. RMSE is calculated using the posterior mean of γ and the simulated value of γ. The RMSE is calculated for the whole period (600 days) and also for each period when the spillover level is set to specific values (for example, ± 0.04)).

Denoting the credible interval as CI, the correctly identified spillover days (for spillover level 0.10) are counted as,

$$\begin{aligned} \text{if } 0 \in \text{CI}, \text{ then } d_t^{(0.10)} &= 0, \\ 0 \notin \text{CI}, \text{ then } d_t^{(0.10)} &= 1, \end{aligned}$$

which can be used to see how well the sampler performs in identifying the spillover days for a given level of spillover.

To see how well the posterior mean estimates the simulated value (for example, a spillover level l), when $\gamma_t = l$,

$$\text{rmse}(\gamma_t = l) = \sqrt{\frac{\sum_{t=1}^{n}(\hat{\gamma}_t - l)^2}{n}},$$

where $\hat{\gamma}_t$ is the posterior mean at time t, $\gamma_t = l$ is the simulated value at time t and n is the number of periods when the simulated spillover level is l.

Another approach for checking how well the samplers perform is to count the number of days when the posterior mean of the spillover $\hat{\gamma}_t$ exceeds a given threshold β,

$$\begin{aligned} \alpha_t(\beta) &= 1 \text{ if } |\hat{\gamma}_t| > \beta, \\ &= 0 \text{ if } |\hat{\gamma}_t| < \beta. \end{aligned}$$

This gave very similar results to the RMSE results and only the RMSE results have been used below.

Using the model in section 4.2, and the parameters described below for each case, the daily log returns are simulated. The value of the spillover parameter is set at known levels with jumps as described above. First, the simulated data is based on the parameters results from the Asian financial crisis data. This is used as base case and cases with higher variance of the regression parameters

θ, increased variance of the volatility parameters $h_{1,t}, h_{2,t}$, increased variance of the world factor and then the spillover reduced from 10 days to 5 days are used. Each test is referenced by a letter, for example (K), to group the results from a particular test case.

Parameter	μ	ϕ	σ_η^2
θ_1	0.35	1.0	0.5 x 10^{-3}
θ_2	0.30	1.0	0.5 x 10^{-3}
θ_3	0.0	1.0	5 x 10^{-5}
θ_4	0.0	1.0	5 x 10^{-5}
h_1	-8.0	0.93	0.40^2
h_2	-12.0	0.85	0.14^2

Table 12: Parameter values used to simulate the data (K).

The daily returns for the world factor f_t are simulated from $N(0, 0.02^2)$. In this case, the spillover jump values $(\pm 0.01, \pm 0.02, \pm 0.04, \pm 0.06)$ are used for specific times and a background spillover level of ± 0.0005 at other times. These jump values are similar in size to the results from the Asian financial crisis data. The spillover jumps are used for 10 days for each level of jump (except one test at the end where 5 days are used to see the results using a smaller spillover period).

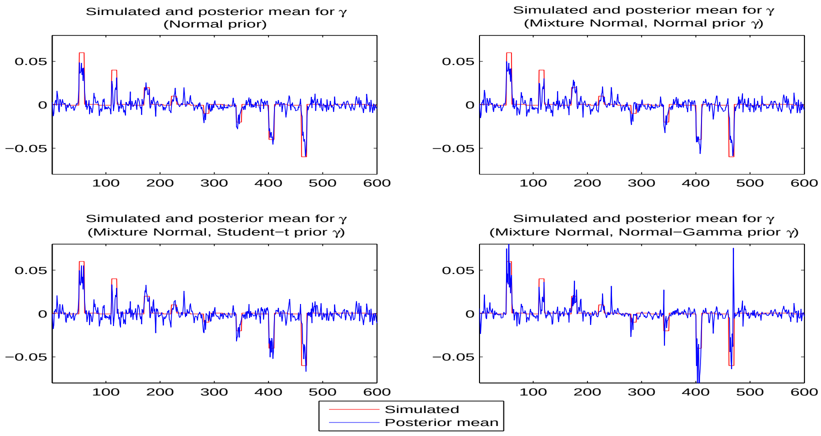

Figure 4.4.5: Simulated and posterior mean for γ using 4 samplers (K).

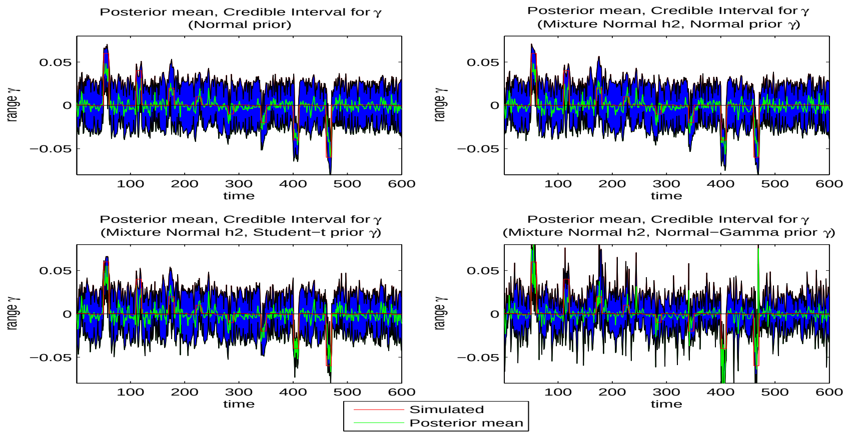

Figure 4.4.6: Posterior mean and 95 % credible interval for γ using 4 samplers (K).

γ	0.01	0.02	0.04	0.06	-0.01	-0.02	-0.04	-0.06
S1	1	2	4	9	2	3	7	7
S2	1	2	4	9	2	3	9	8
S3	1	3	4	9	2	8	9	8
S4	1	5	4	9	2	3	4	4

Table 13: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (K).

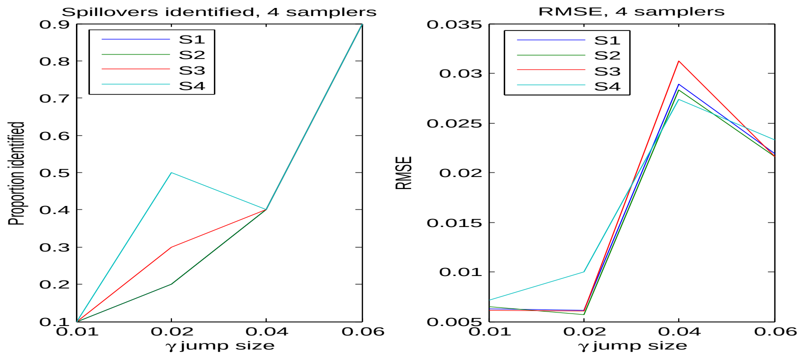

Figure 4.4.7: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (K).

In Figure 4.4.5, the posterior mean of the spillover is shown with the simulated values. The jump values are estimated well by all the 4 samplers. For the larger jumps (± 0.06), the samplers with the normal prior (samplers S1 and S2) underestimate the value. The samplers with the heavy tails (samplers S3 with a Student-t prior, S4 with a Normal-Gamma prior) estimate the values better. In Figure 4.4.6, the 95% credible interval for the posterior mean of the spillover is shown. The aim is to identify the cases when zero is not in the credible interval (that is, non zero values of the spillover). In Table 13, the count of the number of days when the non zero values of the spillover have been identified are shown. The larger spillover values are better identified except for Sampler S4 (for level -0.06).The results show that the smaller spillovers would be difficult to detect. In Figure 4.4.7, the left hand plot shows the proportion of days identified for each spillover level (using only the the positive values of the spillover) and shows that the larger spillovers are better identified compared to the smaller spillovers. The right hand side plot shows the RMSE results. The error is smaller for the smaller spillover values. In this sense, the smaller values are better estimated. But for the smaller values, the non zero values of the spillover are not identified using the credible interval.

In case (M), the variance of the regression parameters is increased (for θ_1, increased from 0.5 x 10^{-3} to 1 x 10^{-3} and similarly for θ_2). This would represent a more rapidly changing relationship with the world factor. The results from the 4 samplers are shown below.

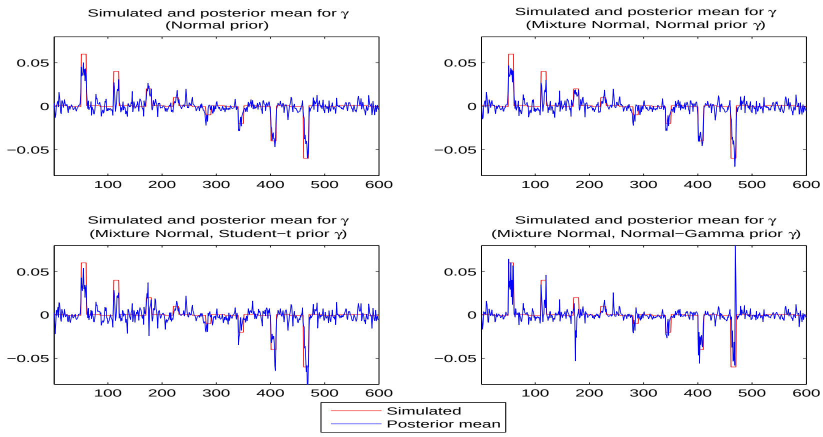

Figure 4.4.8: Simulated and posterior mean for γ using 4 samplers (M).

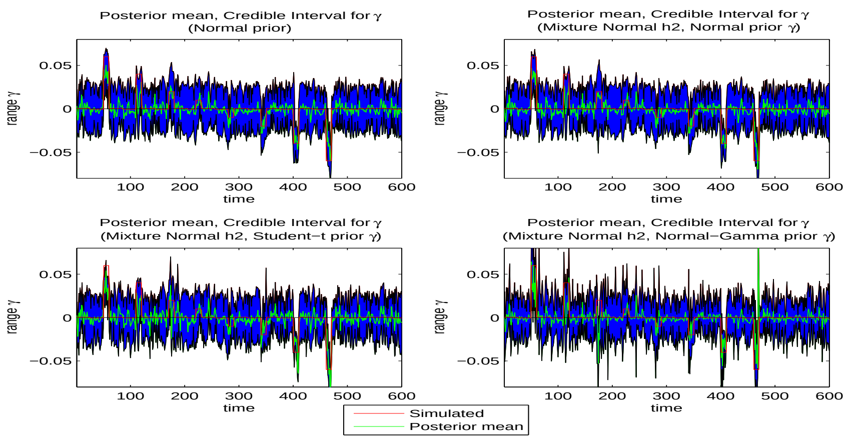

Figure 4.4.9: Posterior mean and 95 % credible interval for γ using 4 samplers (M).

γ	0.01	0.02	0.04	0.06	-0.01	-0.02	-0.04	-0.06
S1	1	2	4	9	2	3	7	7
S2	1	0	3	9	2	3	9	8
S3	1	0	4	9	2	4	10	8
S4	1	0	4	8	2	3	8	5

Table 14: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (M).

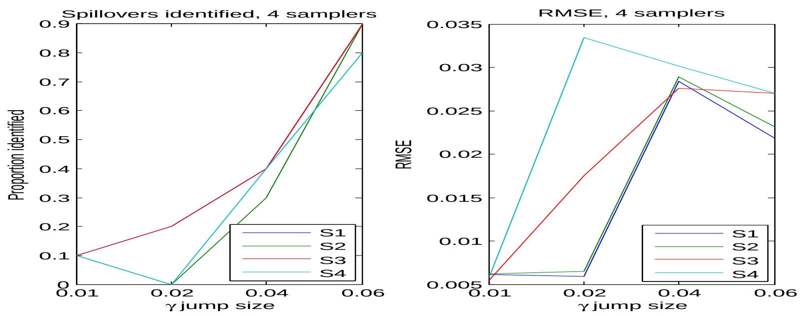

Figure 4.4.10: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (M).

As the results in Figure 4.4.8 show, the posterior mean of the spillover still estimates the simulated values well. In Figure 4.4.9, the credible interval continues to identify the larger values well. Compared to the base test (K), the number of days identified (Table 14) for the smaller spillover value 0.02 are fewer. This can be explained as when the regression parameters have a higher variance, the regression parameters are estimated less well and hence would be difficult to identify the small spillover values. In Figure 4.4.10, the RMSE values are higher for the small jump (0.02) compared to the RMSE values in the base test (K).

In case (O), the variance of the volatility parameters is increased, for h_1 increased from 0.4^2 to 0.6^2 and similarly for h_2 increased from 0.14^2 to 0.2^2. This would represent more volatile markets in the country where the contagion originates and the country where the volatility spills over to. The other parameters remain the same as for the base test (K). The results from the 4 samplers are shown below.

γ	0.01	0.02	0.04	0.06	-0.01	-0.02	-0.04	-0.06
S1	1	0	4	9	2	3	7	7
S2	1	0	3	9	2	3	9	5
S3	1	1	4	9	2	3	9	7
S4	1	1	4	8	2	3	7	6

Table 15: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (O).

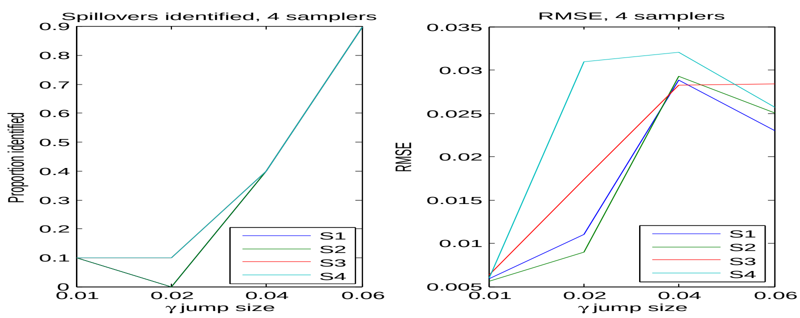

Figure 4.4.11: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (O).

The posterior mean and the credible interval plots are not shown (they look similar to earlier results). Comparing the days identified in Table 15, to the results for the base case (K), the number of days identified for spillover level ± 0.02 is lower. Again, if the volatility parameter has a high variance, the parameters would be estimated less well it would be difficult to detect the spillover (the credible interval would be larger in the presence of other noise).

In case (Q), the variance of the world factor is increased from 0.02^2 to 0.04^2. This would represent more volatile markets in the world factor (or a large country in the region). The other parameters remain the same as for the base test (K). The results from the 4 samplers are shown below.

γ	0.01	0.02	0.04	0.06	-0.01	-0.02	-0.04	-0.06
S1	1	0	4	9	2	3	7	7
S2	1	1	4	9	2	3	7	5
S3	1	2	4	9	2	3	7	5
S4	1	3	3	8	2	4	10	6

Table 16: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (Q).

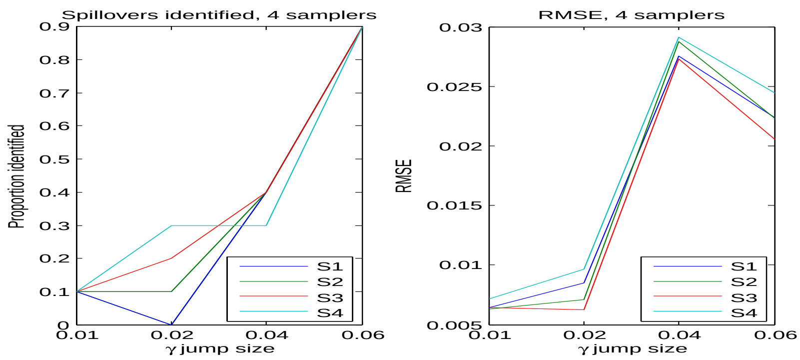

Figure 4.4.12: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (Q).

The posterior mean and the credible interval plots are not shown (they look similar to earlier results). Comparing the days identified in Table 16, to the results for the base case (K), the number of days identified level ± 0.02 is lower. This can also be explained along similar lines as the above results. In the presence of high volatility of the world factor, the regression parameters and the spillover parameter would be estimated less accurately. Is this case, sampler S4 (with the Normal-Gamma prior) performs slightly better.

In case (S), the spillover days are reduced from 10 to 5. This would represent short lived spillover effect. The other parameters remain the same as for the base test (K). The results from the 4 samplers are shown below.

γ	0.01	0.02	0.04	0.06	-0.01	-0.02	-0.04	-0.06
S1	0	1	2	4	2	2	3	2
S2	0	2	2	4	2	2	3	2
S3	0	1	2	5	2	3	3	2
S4	0	2	2	4	2	2	3	2

Table 17: Number of days volatility spillover identified from 5 days using 4 samplers and different jump levels (S).

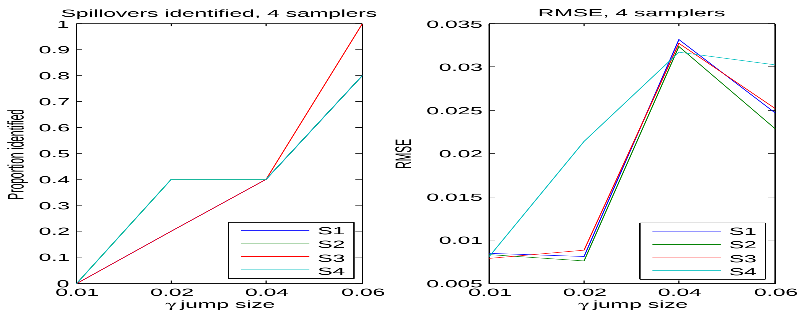

Figure 4.4.13: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (S).

Again, the posterior mean and the credible interval plots are not shown. Comparing the days identified in Table 17 to the results for the base case (K), the proportion of days identified is similar. This result indicates that small periods of spillover would also be detected by the samplers. For the simulated tests based on the Asian financial crisis parameters, all the 4 samplers perform similarly. In the presence of other noise, fewer number of spillover days are identified. The larger spillovers are identified better (similar to the results for the real data, 0.06 level is identified well). The RMSE does not change much between the tests, indicating that the posterior mean continues to be identified well.

In the next group of tests, the simulated data is based on parameters results from the Euro crisis. This is used as base case and then cases with higher variance of the regression parameters θ, increased variances of the volatility parameters $h_{1,t}, h_{2,t}$, increased variance of the world factor and then the spillover reduced from 10 days to 5 days are used.

Parameter	μ	ϕ	σ^2
θ_1	0.60	1.0	1 x 10^{-3}
θ_2	1.20	1.0	0.4 x 10^{-3}
θ_3	0.0	1.0	2 x 10^{-5}
θ_1	0.0	1.0	1 x 10^{-5}
h_1	-8.0	0.97	0.22^2
h_2	-14.0	0.98	0.55^2

Table 18: AR(1) process parameter values (L)

The daily returns for the world factor f_t are simulated from $\mathrm{N}(0, 0.0158^2)$. In this case, the spillover jump values $(+0.0025, \pm 0.005, \pm 0.01, \pm 0.02)$ are used for specific times and a background spillover level of ± 0.0005 at other times. These jump values are similar in size to the results from the Euro crisis. The spillover jumps are used for 10 days for each level of jump.

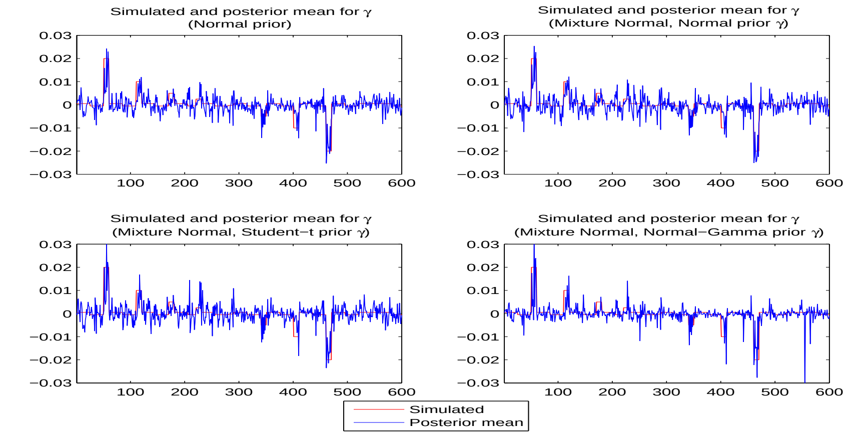

Figure 4.4.14: Simulated and posterior mean for γ using 4 samplers (L)

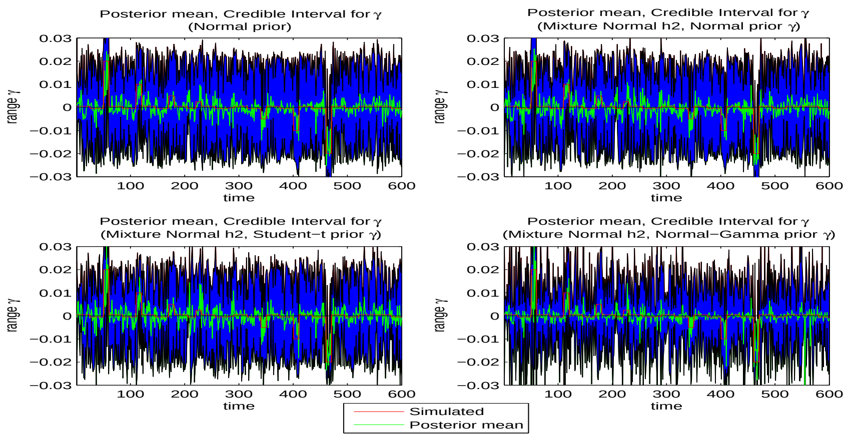

Figure 4.4.15: Posterior mean and 95 % credible interval for γ using 4 samplers (L)

γ	0.0025	0.005	0.01	0.02	-0.005	-0.01	-0.02
S1	1	0	2	4	2	1	6
S2	2	0	2	4	3	2	6
S3	1	0	4	4	3	1	6
S4	1	0	2	4	3	2	7

Table 19: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (L).

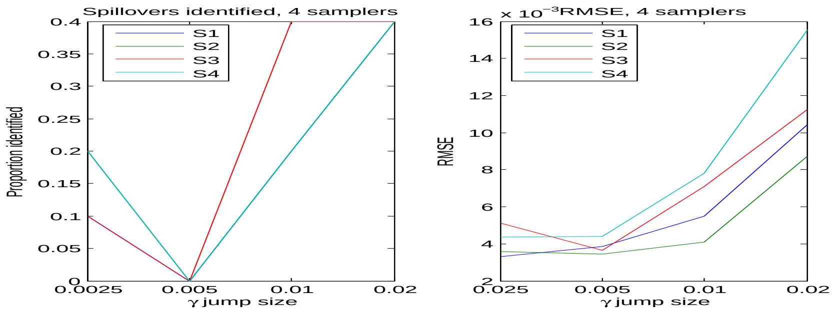

Figure 4.4.16: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (L).

In Figure 4.4.14, the posterior mean of the spillover is shown with the simulated values. The jump values are estimated well by all the 4 samplers. For the larger jumps (± 0.02), all the 4 samplers estimate the values well. In Figure 4.4.15, the 95% credible interval for the posterior mean of the spillover is shown. As before, the aim is to identify the cases when zero is not in the credible interval (that is, non zero values of the spillover). As the results in Table 19 show, even the larger spillover values are identified about half the time. The smaller values are poorly identified . This shows that the spillovers would be difficult to detect for the Euro crisis data. In Figure 4.4.16, the left hand plot shows the proportion of days identified for each spillover level (for the positive values of the spillover) and shows that the larger spillovers are better identified compared to the smaller spillovers. The right hand side plots show the RMSE results. Again, the error is smaller for the smaller spillover values. In this sense, the smaller values are better estimated. But for the smaller values, the non zero values of the spillover are not identified using the credible interval.

For case (N), the variance of the regression parameters is increased (for θ_1, increased from 1 x 10^{-3} to 2 x 10^{-3} and similarly for θ_2, increased from 0.4 x 10^{-3} to 0.8 x 10^{-3}). This would represent a more rapidly changing relationship with the world factor. The results from the 4 samplers are shown below (the posterior mean and credible interval plots are not shown, the results are summarised in the counts table and RMSE plots).

γ	0.0025	0.005	0.01	0.02	-0.005	-0.01	-0.02
S1	1	0	2	4	2	1	6
S2	2	0	2	4	2	1	7
S3	2	0	2	4	3	1	6
S4	1	0	2	5	3	2	7

Table 20: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (N).

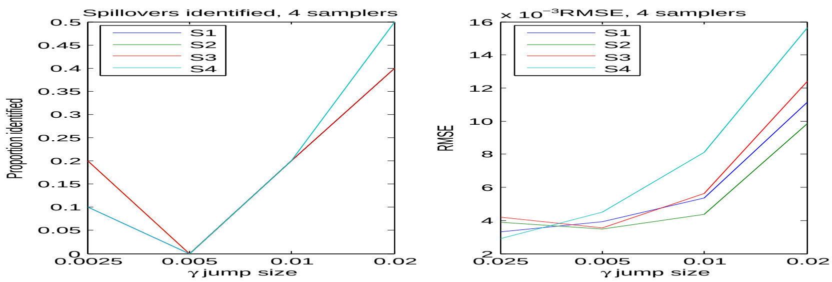

Figure 4.4.17: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (N).

Even the larger spillover values are identified about half the time and the smaller values are poorly identified (Table 20). In Figure 4.4.17, the left hand plot shows the proportion of days identified for each spillover level (for the positive values of the spillover) and shows that the larger spillovers are better identified compared to the smaller spillovers. The right hand side plot shows the RMSE results. Again, the RMSE is lower for the smaller spillover values. The results for case (N) with the higher variance for the regression parameters are very similar to the base case (L).

For case (P), the variance of the volatility parameters is increased, for h_1, increased from 0.22^2 to 0.4^2 and similarly for h_2, increased from 0.55^2 to 0.9^2. This would represent more volatile markets in the country where the contagion originates and the country where the where the volatility spills over to. The other parameters remain the same as for the base test (L). The results from the 4 samplers are shown below (the posterior mean and credible interval plots are not shown, the results are summarised in the counts table and RMSE plots).

γ	0.0025	0.005	0.01	0.02	-0.005	-0.01	-0.02
S1	1	0	1	4	2	0	6
S2	2	0	3	5	2	1	6
S3	2	0	4	4	2	1	6
S4	2	0	3	5	3	1	6

Table 21: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (P).

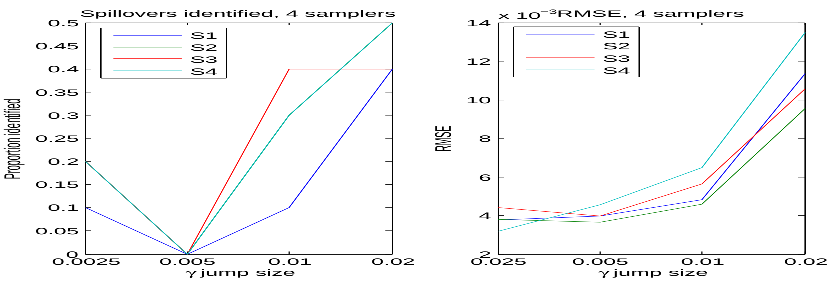

Figure 4.4.18: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (P).

The larger spillover values are identified about half the time and the smaller values are poorly identified (Table 21). Figure 4.4.18, the left hand plot shows the proportion of days identified for each spillover level (for the positive values of the spillover) and shows that the larger spillovers are better identified compared to the smaller spillovers. Again, the RMSE is lower for the smaller spillover values. For case P with the higher variance for the volatility parameters, the results are very similar to the base case (L).

For case (R), the variance of the world factor is increased from 0.0158^2 to 0.04^2. This would represent more volatile markets in the world factor (or large country in the region). The other parameters remain the same as for the base test (L). The results from the 4 samplers are shown below (the posterior mean and credible interval plots are not shown, the results are summarised in the counts table and RMSE plots).

γ	0.0025	0.005	0.01	0.02	-0.005	-0.01	-0.02
S1	1	0	2	4	2	1	6
S2	2	0	2	3	2	2	7
S3	2	0	2	4	3	1	7
S4	2	0	2	4	3	1	7

Table 22: Number of days volatility spillover identified from 10 days using 4 samplers and different jump levels (R)

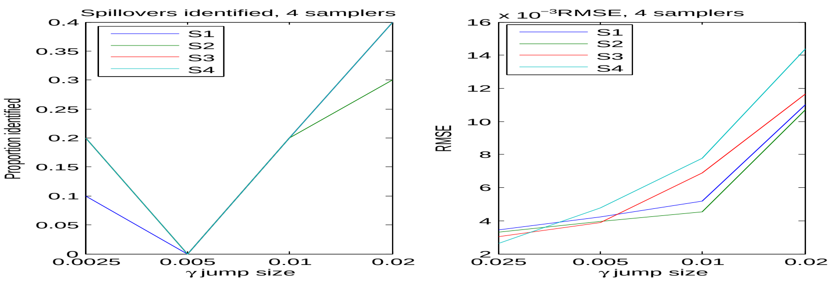

Figure 4.4.19: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (R).

The results remain similar to the base case (L). The larger spillover values are identified about half the time and the smaller values are poorly identified (Table 22). Figure 4.4.19, shows the proportion of days identified and the RMSE results.

For case (T), the spillover days are reduced from 10 to 5. This would represent short lived spillover effect. The other parameters remain the same as for the base test (L). The results from the 4 samplers are shown below.

γ	0.0025	0.005	0.01	0.02	-0.005	-0.01	-0.02
S1	0	0	1	1	2	0	4
S2	0	0	1	1	2	0	4
S3	0	0	2	1	2	0	4
S4	0	0	0	1	2	0	4

Table 23: Number of days volatility spillover identified from 5 days using 4 samplers and different jump levels (T).

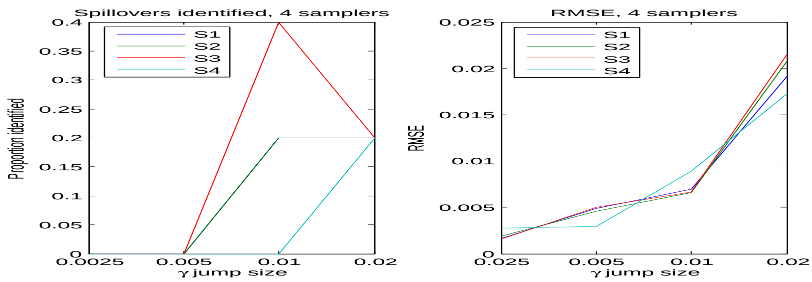

Figure 4.4.20: Proportion of spillover days identified and RMSE using 4 samplers and different jump levels (T).

Comparing the days identified in Table 23, to the results for the base case (L), the proportion of days identified for the spillover levels $0.02, 0.01$ and -0.01 is lower. The proportion of days for the -0.02 spillover is slightly higher. For the smaller spillovers, the proportion of days generally lower than the base case (L).

For the simulated data based on the Euro crisis parameters, the spillover jumps are lower compared to the simulated data based on the Asian financial crisis parameters and hence a lower proportion of the days are identified as non zero.

Even for the larger jumps based on the Euro crisis data, only about half the days are identified as spillover days. In the presence of other noise, the results do not change much. When fewer days of spillover are used (5 days), fewer days are identified compared to the simulated data based on the Asian financial crisis parameters.

4.4.3 Conclusion, simulated data

Using simulated data, all the 4 samplers identify the spillover days with the larger spillovers well. The simulated spillover value is estimated well by the posterior mean using all the 4 samplers.

When the simulated data is based on the Asian financial crisis parameters, for the larger spillover values, (Figure 4.4.5), the posterior mean is lower than the simulated values when the Normal prior is used for the spillover (for the spillover 0.06, samplers S1 and S2 use a Normal prior). When the heavy tailed distributions are used (samplers S3 and S4, using Student-t and Normal-Gamma priors), the posterior mean values are larger and estimate the larger simulated values better (for the spillover 0.06). But the number of days identified as volatility spillover days and the RMSE error for the 4 samplers are very similar when the spillover values are large. As the values of the spillover get smaller, a smaller proportion of the spillover days are identified in the simulation tests. Similarly, as the noise level from the other parameters increases, the spillover days are identified less well. Generally, samplers S1 and S2 (with the Normal priors) have the lower RMSE.

When the simulated data is based on the Euro crisis parameters (spillover jumps are smaller), the proportion of spillover days that are identified is lower than for the Asian financial crisis. As the jump values are small, the posterior mean values for all the 4 samplers are similar for even the larger jump (0.02). Generally, the samplers S1 and S2 (Normal prior) have lower RMSE compared to the heavy tailed priors.

The overall conclusion for the simulated data tests is that the 4 samplers perform similarly and the larger spillovers are identified better. In the presence of other noise (increased variance of other parameters), the number of days identified decreases. And when the spillover is for a shorter period, fewer days are identified, especially for the lower jump levels. The posterior mean of the spillover is identified well under all the test cases above.

4.5 Market data results

4.5.1 Asian financial crisis 1997-1998

This section uses the Hong Kong, Singapore and Japan returns data (1997-98). The results for the volatility and AR(1) parameters are similar to the results for the one at a time sampler (S1) and are not repeated here. In this section, the volatility spillover results for the 4 samplers are discussed.

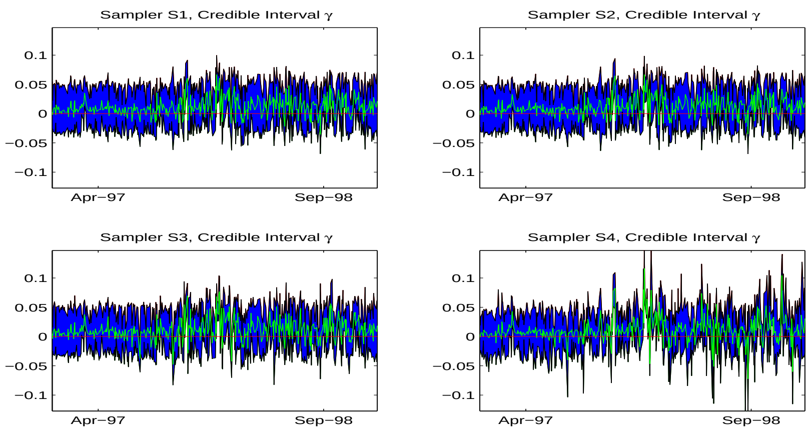

Figure 4.5.1: Volatility spillover γ, posterior mean and credible interval for the Asian financial crisis using 4 samplers.

All the 4 samplers identify the main crisis periods for the Asian financial crisis, when zero is not in the 95% credible interval for the posterior mean of the spillover parameter γ. For samplers S1 and S2 both using the Normal prior for the spillover, the results are very similar. When the heavy tailed prior distributions are used for γ, the posterior mean of γ is larger (for sampler S3 using a Student's-t prior, the values are larger than the Normal prior results) and when the sampler S4 (Normal-Gamma prior) is used, the posterior mean values are larger than the results for sampler S3.

In Figure 4.5.1, the results are shown for the full period of the data. The plot below, Figure 4.5.2, shows the posterior mean of γ with the spillover days marked with a red circle. These show other spillover days which are difficult to see in Figure 4.5.1. The detailed periods of volatility spillover are discussed below.

Posterior mean of γ, spillover days, 4 samplers

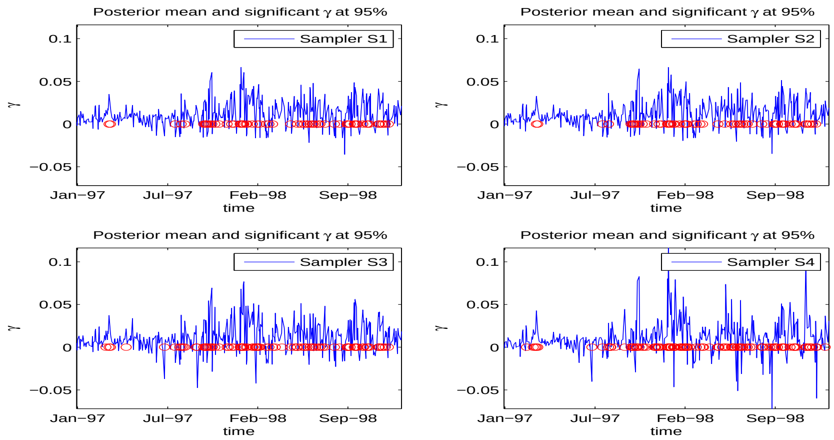

Figure 4.5.2: Volatility spillover days for the Asian financial crisis using 4 samplers.

Below, a few of the spillover periods are shown in more detail using smaller time periods on the plots, with a brief discussion of the economic events at the time.

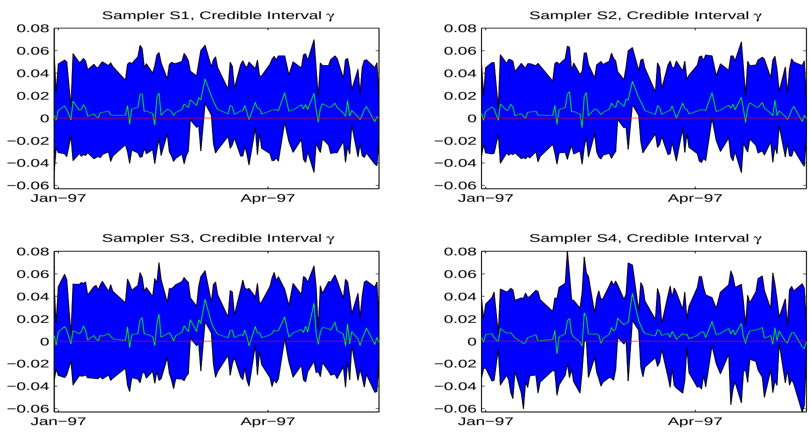

Figure 4.5.3: γ credible interval using 4 samplers.

In March 1997, Figure 4.5.3, the volatility spillover shows non zero values for 21, 22 March 1997 for 4 the samplers. In other studies, the Asian Financial Crisis 1997-1998 only the period July 1997 to October 1998 is included (which was the currency crisis period). Around 21 March 1997, there were fears of an interest rate rise and the Hong Kong index dropped 2.1% during the week and on the news of worse than expected export figures the Singapore index dropped 1.2% (The Financial Times, 21 March 1997). This may explain the the volatility spillover result (or a case of both the markets moving down for unrelated reasons).

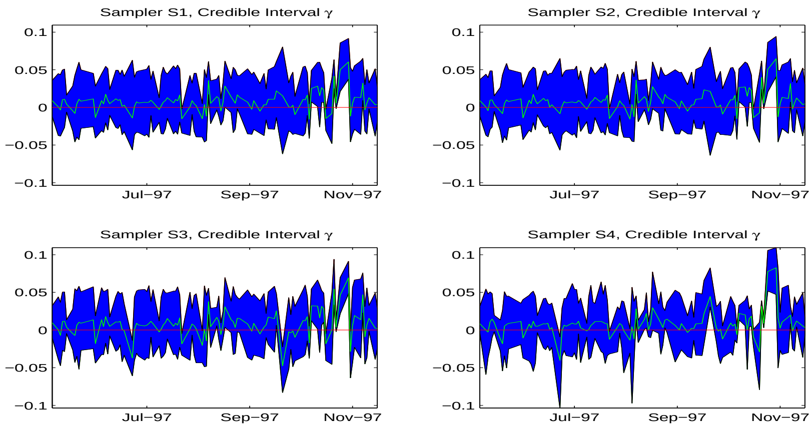

Figure 4.5.4: γ credible interval using 4 samplers.

July 1997 was the start of the Asian financial crises with the devaluation of the Thai baht in July 1997 and similar problems in Malaysia. The Japanese banks called in the loans from the countries affected by the crisis and this was a volatile period in Hong Kong and then the spillover to Singapore, Figure 4.5.4). The October 1997 period is Hong Kong crisis period, shown here more clearly (7 days in October, 3 days in November 1997), as described in the results for Figure 2.4.9. The heavy tailed distributions (samplers S3, S4) identify 2 more days in the October, November period.

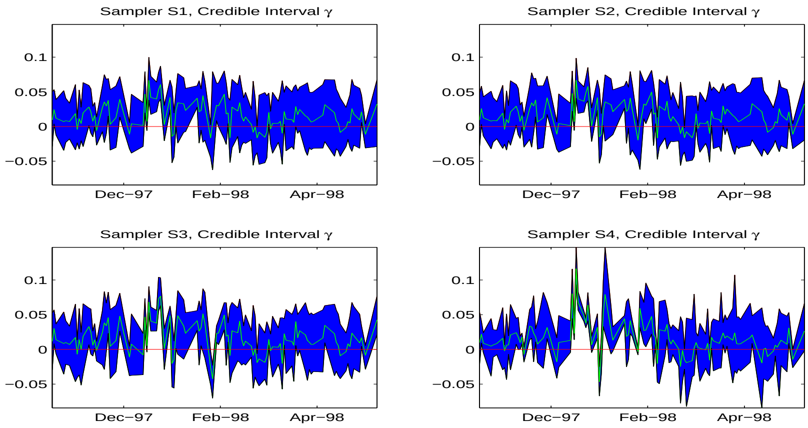

Figure 4.5.5: γ credible interval using 4 samplers.

The results in this case , Figure 4.5.5, identify the spillover periods in January 1998 when the Hang Seng index dropped over 8% and the volatility spread to Singapore. The results from the 4 samplers identify similar periods of volatility spillover during January, February and March 1999. For sampler S4, the (absolute) values of the spillover are larger when spillover is identified and show the contagion days more clearly. Samplers S1 and S2 identify 3 days, sampler S3 identifies 4 days and sampler S4 identifies 6 days.

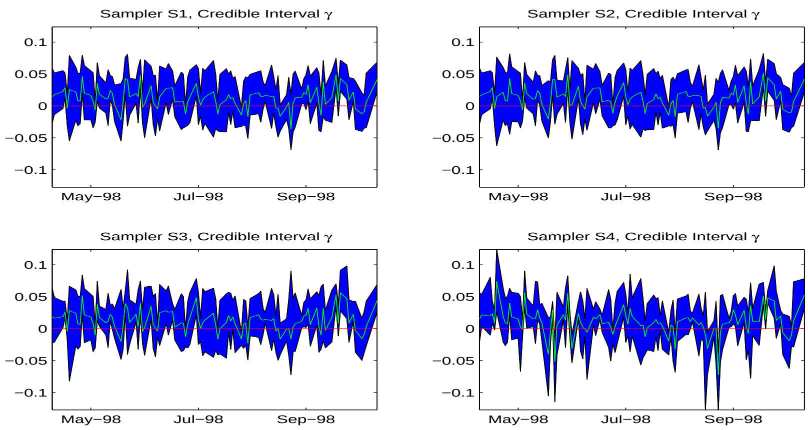

Figure 4.5.6: γ credible interval using 4 samplers.

September 17 1998, the Japanese stock market fell sharply and this resulted in sharp declines in Hong Kong, European and U.S markets. There was general panic in the financial markets with the collapse of Long Term Capital Management, a large hedge fund in the US (September 1998). September 28 1998, Japan Leasing Corporation, a large leasing company filed for bankruptcy protection. On October 3 1998, Japan announced a USD 30 billion aid package to support economies in the Southeast Asian region. There were no specific events in Singapore, hence spillover result in Figure 4.5.6 can be interpreted as the spillover of volatility from Hong Kong to Singapore (after allowing for the volatile returns in the world factor, in this case Japan). Samplers S1, S2 identify 7 days, samplers S3, S4 identify 8 days spillover during September 1998.

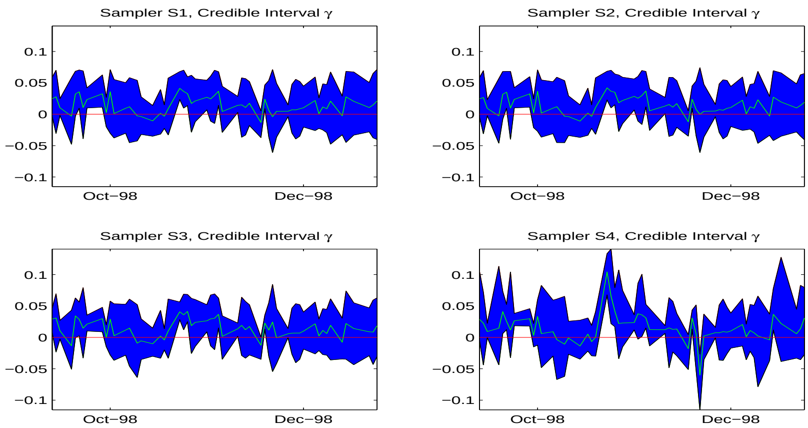

Figure 4.5.7: γ credible interval using 4 samplers.

Again, the 4 samplers show similar results, Figure 4.5.7. Sampler S4 identifies the spillover period more clearly (the posterior mean shows a larger value). In October 1998, there were fears of a global economic crisis. The US Federal Reserve cut interest rates to help the financial markets. The Russian central bank was required to print money to rescue the banks and Brazil announced austerity measures to secure IMF funding. There are no clear events directly related to Hong Kong or Singapore. The spillover may be due to Hong Kong links to the world markets and then the spread of the volatility from Hong Kong to Singapore. Samplers S1, S2 identify 4 days, samplers S3, S4 identify 5 days in October 1998.

4.5.2 European sovereign debt crisis 2007-2013

This section uses the Greece and Spain and Eurostoxx data (2007-2013). The results for the volatility and AR parameters are similar to the results for sampler S1 as in Section 2.4.4 and are not repeated here. In this section, the volatility spillover results using the 4 samplers are summarised.

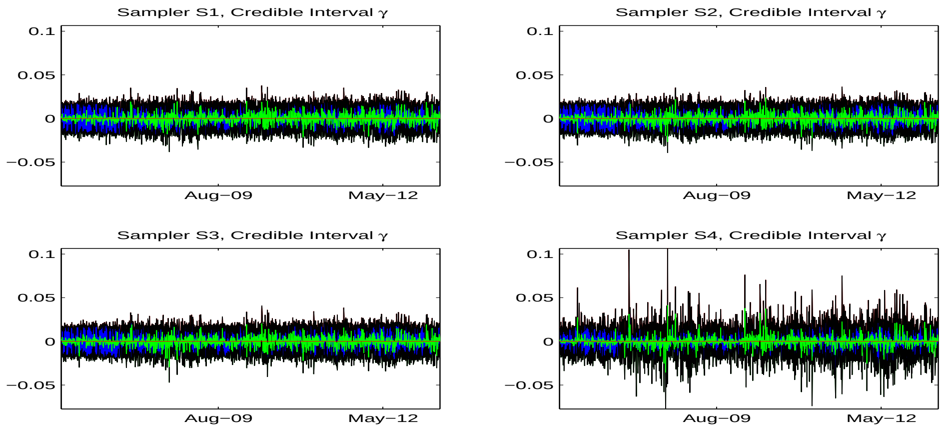

Figure 4.5.8: Volatility spillover γ posterior mean and credible interval for the Euro crisis using 4 samplers.

The 4 samplers show similar results for the posterior mean of the spillover parameter γ as in Figure 4.5.8. The 95% credible interval results are also similar for the 4 samplers. Sampler S4 (Normal-Gamma prior case) samples larger (absolute) values of the parameter but the credible intervals are similar to the other 3 samplers.

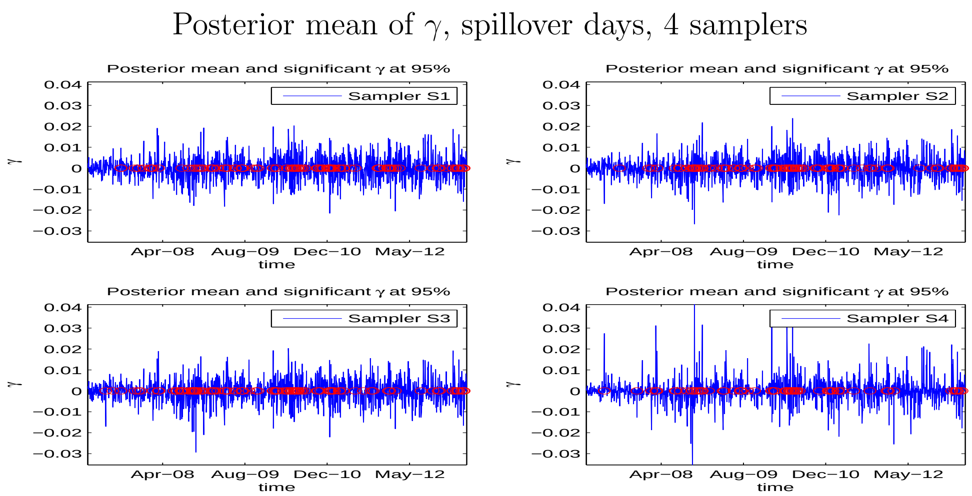

Figure 4.5.9: Volatility spillover days for the Euro crisis using 4 samplers.

Figure 4.5.9 shows the posterior mean of the spillover and the red circles indicate the spillover days. The main periods of the Greek crisis and spillover to Spain are identified well. This is discussed below in detailed plots for shorter intervals.

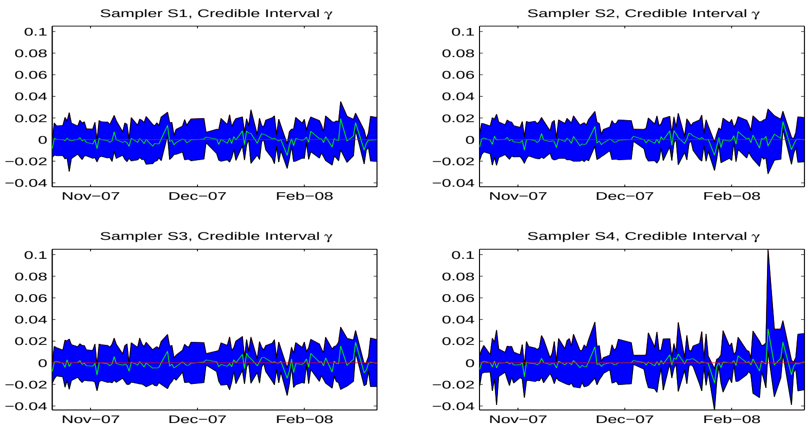

Figure 4.5.10: γ credible interval using 4 samplers.

A short period in March 2008 in Figure 4.5.10 is identified as a spillover period by the 4 samplers - this was the time of the pension reforms, mass protests, public sector strikes and volatile periods in the Greek stock market (reference BBC Greek debt time line). Samplers S1, S2 identify 2 days, samplers S3, S4 identify 3 days in the February-March 1998 period.

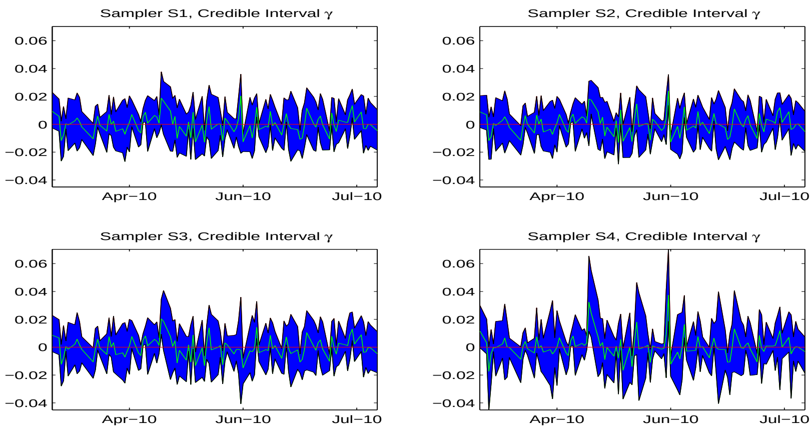

Figure 4.5.11: γ credible interval using 4 samplers.

The posterior mean results for γ in Figure 4.5.11 are similar for the 4 samplers and identify some volatility spillover days. During April-August 2010 there were fears of a possible default on Greek debt, trade unions had called for a general strike and the markets were volatile as described with Figure 2.4.15. The number of days identified by samplers S1, S2, S3 and S4 are 12, 16, 16 and 14.

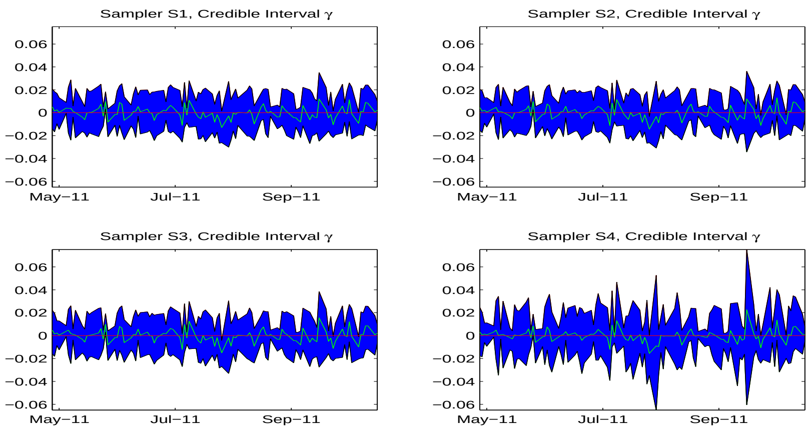

Figure 4.5.12: γ credible interval using 4 samplers.

The posterior mean results for γ, 4.5.12, are similar for the 4 samplers and identify some volatility spillover days and the posterior mean of γ is just non zero. In July 2011, all 3 main credit rating agencies cut Greek credit rating (details with 2.4.16), 109 billion Euro bailout fund was arranged through the European Financial Stability Fund and in October 2011, 50 % debt write off was agreed for Greece, and further austerity measures were announced). There was an announcement of a referendum on the austerity and reform package. This was a turbulent time in the Greek markets. The above plots show very small periods of volatility spillover, possibly because the bailout agreement had been anticipated had been under discussion for some time. Samplers S1, S2, S3 and S4 identify 2, 3, 3 and 1 day in the period May-September 2011.

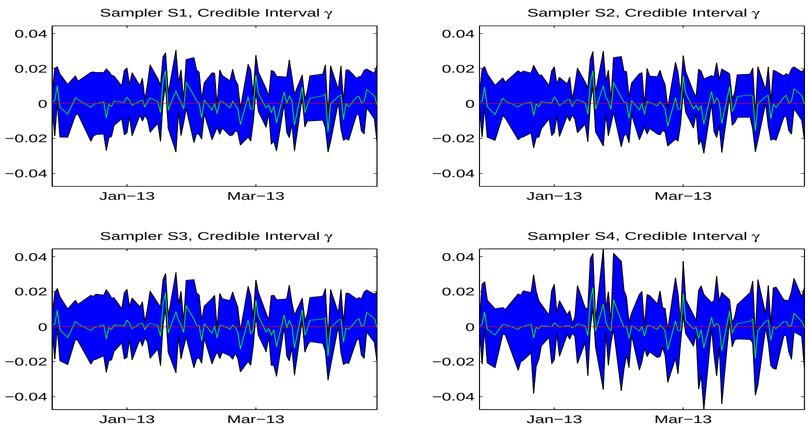

Figure 4.5.13: γ credible interval using 4 samplers.

Again, the 4 samplers perform similarly, Figure 4.5.13. In January 2013 the unemployment level in Greece reached nearly 27% and in April 2013, youth unemployment reached nearly 60%. It appears a continuation of the uncertain times for the Greek economy with volatility spillover to Eurozone countries. In the January-March 2013 period, samplers S1, S2, S3 and S4 identified 6, 5, 6 and 5 days.

4.5.3 Conclusion, market data

For the Asian financial crisis, when the real data is used, the spillover (and contagion) is identified quite well in the sense of comparing the spillover days to the economic events at the time. The periods that are identified for the Asian financial crisis are similar to those in other studies. The spillover parameter values are higher for the Asian financial crisis compared to the Euro crisis results. The number of days identified as volatility spillover is slightly higher when the heavy tailed priors (samplers S3, S4) are used.

The results for the Euro crisis match the times of financial difficulties in Greece. The values of the spillover parameter are lower and hence difficult to identify the spillover days. In this case, all the 4 samplers identify a similar number of days.

Chapter 5

Conclusions and further work

5.1 Conclusions

The work started with the question 'what is financial contagion'? And then it progressed to the question, can we use a method to identify financial contagion without the need to identify volatile and tranquil periods? A number of definitions have been used in the studies on financial contagion and the definitions are based on correlations and volatility spillover. We adopted the volatility spillover definition for this study. Models for statistical analysis of financial contagion relied on prior identification of the tranquil and volatile periods using external information and the use of some measure to identify a volatile period. The method would also need to include sufficiently long periods of data to estimate the parameters reliably. Our approach was to introduce time varying parameters and Bayesian methods with MCMC simulations to sample the posterior distributions of the parameters and use the significant values of the spillover parameter to identify the presence of financial contagion without the prior need to identify tranquil and volatile periods.

Dynamic Factor model, with time varying parameters allowed us to explain the daily log returns using time varying relationship with a world factor and then model the volatility spillover using the model described by Dungey et al. (2005), but using time varying parameters and without the need to identify tranquil and volatile periods. The volatility parameter for the first country was sampled one period at a time and this worked well for the volatility parameter. For the second country volatility parameter, we attempted block sampling using FFBS methods. This needed a step to approximate the log χ^2_1 density using a Normal Mixture density. Kim et al. (1998) use an 'offset parameter' c to make the approximation robust for small values of the residuals. In using this method, we realised that the residual may not have a Standard Normal density (especially in case where the regression step is used to relate the country returns and the world factor). Our results depended on the value of the offset parameter used in the MCMC simulations. We propose a method to standardise the residuals within the MCMC steps so that the results are much less dependent on the value of the offset parameter.

Use of Bayesian and MCMC methods (Forward Filtering Backward Sampling, Adaptive Metropolis-Hastings method to accept proposed samples) have worked well to sample the parameter values. Tests on simulated data using heavy tailed distributions (Student-t and Normal-Gamma priors) for the innovations in the spillover did not give significantly better results (in identifying the spillover days) compared to the Normal distributions case.

We used the model and the method successfully to identify the volatility spillover periods (contagion periods) for the Asian financial crisis and found similar periods to other previous research using classical statistics. Some periods during the financial turmoil in Greece (European sovereign debt crisis) identified the spillover of volatility from Greece to Spain. The periods identified are shorter, possibly because the markets are already very closely integrated, possibly because there was a period of banking related financial crisis in Spain for a period of the time.

5.2 Further work

In this study, a two country case with a world factor was used. This model can be extended to include multiple countries, possibly including an intermediate country as well. For the factor model, further parameters could be added to the world factor (for example interest rates, employment levels, investment flows in and out of a country) if the information is available at the same frequency as the daily returns.

We attempted to linearise the model (using a Taylor series expansion) and express the model in bivariate linear State Space form. This approximation was developed as an MCMC program but the results were not successful. Further work could be done to understand why this approach did not work well and possibly find a method to improve the results. When using this approximation, the acceptance rates for the proposed values were low. We attempted to sample for shorter periods (period length selected using a Poisson distribution) to improve the acceptance rates. The results of the volatility parameters when using this approach were not successful and there was not enough time to explain this or to improve the method and conclude the work.

This study used the country level market indices to study the contagion effect. The model and methods can also be used to study spread of volatility for other cases (for example, volatility spillover within a country, from property to the equity market; the spread of volatility from the property market in one country to the property market in another country).

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