INSURANCE LOSS COVERAGE AND SOCIAL WELFARE

By MingJie Hao†, Angus S. Macdonald‡, Pradip Tapadar†and R. Guy Thomas†

ABSTRACT

Loss coverage, defined as expected losses compensated by insurance, has recently been suggested as a metric for evaluation of different insurance risk classification schemes. This paper makes connections between this approach and utility-based approaches, in two main areas. First, previous work on loss coverage has used an aggregate insurance demand function; we provide a micro-foundation in variations across individuals in utility of wealth. Second, we reconcile loss coverage to a utilitarian concept of social welfare, defined as the sum of individuals' expected utilities over the entire population. Specifically, we show that if insurance premiums are negligible relative to wealth, maximising loss coverage maximises social welfare. From a policy perspective, this may be a useful result because maximising loss coverage does not require knowledge of individuals' (unobservable) utility functions; loss coverage is based solely on observable quantities.

KEYWORDS

Loss coverage; social welfare. J.E.L Classification: D82, G22.

CONTACT ADDRESS

†School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK.
‡Department of Actuarial Mathematics and Statistics, and the Maxwell Institute for Mathematical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, UK.
Corresponding author: Pradip Tapadar, P.Tapadar@kent.ac.uk.

1. Introduction

Regulatory restrictions on insurance risk classification are pervasive in life insurance and other personal insurance markets. For example, gender classification in insurance pricing has been banned in the European Union since 2012; Obamacare allows classification only by age, location, family size and smoking status; and many countries have restricted insurers' use of genetic test results. Such restrictions are usually seen by economists as potentially increasing equity, but also reducing efficiency.

A simple version of the usual efficiency argument is as follows. If insurers are not permitted to charge risk-differentiated prices, they have to pool different risks at a common pooled price.[1] This pooled price is cheap for higher risks and expensive for lower risks; so more insurance is bought by higher risks, and less insurance is bought by lower risks. The equilbirum pooled price of insurance is higher than a population-weighted average of true risk premiums. Also, in most markets the number of higher risks is smaller than the number of lower risks, so the total number of risks insured falls. The usual efficiency argument focuses on this reduction in coverage, e.g. "This reduced pool of insured individuals reflects a decrease in the efficiency of the insurance market" (Dionne and Rothschild, 2014, p185).

However, in some scenarios there is a counter-argument to this perception of reduced efficiency. Thomas (2008) pointed out that the rise in equilibrium price under pooling reflects a shift in coverage towards higher risks. If the shift in coverage is large enough, it can more than outweigh the fall in numbers insured. In this scenario, despite fewer risks being insured under pooling, expected losses compensated by insurance ('loss coverage') can be higher. It is not obvious that a risk classification regime under which more risks are voluntarily transferred and more losses are compensated should be regarded as less efficient.

Another perspective on this argument is that a public policymaker designing risk classification policies normally faces a trade-off between insurance of the 'right' risks (those more likely to suffer loss), and insurance of a larger number of risks. The optimal trade-off depends on demand elasticity in higher and lower risk-groups. The concept of loss coverage quantifies this trade-off, and provides a metric for comparing the effects of different risk classification schemes.

1.1 Motivating example

The argument just given can be illustrated by heuristic examples of insurance market equilbria under two alternative risk classification regimes: risk-differentiated premiums and pooled premiums.

Suppose that in a population of 2,000 risks, 32 losses are expected every year. There are two risk-groups. Each person in the high risk-group of 400 individuals has a probability of loss 4 times higher than each person in the low risk-group of 1,600 individuals. This is summarised in Table 1.

We assume that probability of loss is not altered by the purchase of insurance, i.e. there is no moral hazard. An individual's risk-group is fully observable to insurers and all insurers are required to use the same risk classification regime. In a competitive market, the equilibrium price of insurance is determined as the price at which insurers make zero

[1] In this paper we ignore the possibility that insurers banned from classifying risks induce separation of risk-groups by alternative contracts offering different levels of cover priced at different rates (e.g. Rothschild and Stiglitz, 1976). This approach is not feasible in markets with non-exclusive contracting, such as life insurance; and as far as we are aware, it is not common in practice in other markets where some restrictions on risk classification apply, for example in auto insurance in the European Union.

profit.

Under our first risk classification regime, insurers charge risk-differentiated premiums, which are actuarially fair to members of each risk-group. We assume that the proportion of each risk-group which buys insurance under these conditions, i.e. the 'fair-premium proportional demand', is 50%.[2] Table 1 shows the outcome. Half the losses in the population are compensated by insurance. We heuristically characterise this as a 'loss coverage' of 0.5.

Table 1: Equilibrium under risk-differentiated premiums: lower loss coverage.

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	1600	400	2,000
Expected population losses	16	16	32
Break-even premiums (differentiated)	0.01	0.04	0.016
Numbers insured	800	200	1,000
Insured losses	8	8	16
Loss coverage			0.5

Now suppose that a new risk classification regime is introduced, where insurers have to charge a single 'pooled' price to members of both the low and high risk-groups. One possible outcome is shown in Table 2, which can be summarised as follows:

(a) The pooled premium of 0.0194 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: (600 x 0.01 + 275 x 0.04)/875 = 0.0194).

(b) The pooled premium is expensive for low risks, so fewer of them buy insurance (600, compared with 800 before). The pooled premium is cheap for high risks, so more of them buy insurance (275, compared with 200 before). Because there are 4 times as many low risks as high risks in the population, the total number of policies sold falls (875, compared with 1,000 before).

(c) The resulting loss coverage is 0.53125. The shift in coverage towards high risks more than outweighs the fall in number of policies sold: 17 of the 32 losses (53%) in the population as a whole are now compensated by insurance (compared with 16 of 32 before).

[2]This figure is representative for life insurance. The Life Insurance Market Research Organisation (LIMRA) states that 44% of US households have some individual life insurance (LIMRA (2013)). The American Council of Life Insurers states that 144m individual policies were in force in 2013 (ACLI, 2014, p72); the US adult population (aged 18 years and over) at 1 July 2013 as estimated by the US Census Bureau was 244m.

Table 2: Equilibrium under pooled premiums: higher loss coverage.

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	1600	400	2000
Expected population losses	16	16	32
Break-even premiums (pooled)	0.0194	0.0194	0.0194
Numbers insured	600	275	875
Insured losses	6	11	17
Loss coverage			0.53125

The occurence of the favourable outcome (higher loss coverage) under pooling in Table 2 depends on the demand elasticities for insurance in high and low risk groups. Later in this paper, we shall show that the required demand elasticities are plausible.

1.2 Literature Review

Previous papers on loss coverage (Hao et al (2016), Thomas (2008, 2009)) modelled an insurance market with two risk-groups with higher and lower probabilities of loss. Individuals' probabilities of loss were observable by insurers. Insurers were assumed to compete only on price (not on risk classification regimes or contract offers) and made zero profits in equilibrium. The outcomes of permitting or banning risk classification were then evaluated by loss coverage in equilibrium, with higher loss coverage being preferred on the rationale stated above. Insurance demand from each risk-group was expressed as a proportion between 0 and 1, to reflect the empirical observation that many individuals do not buy insurance. Variation in purchasing decisions across persons with the same probabilities of loss was characterised as stochastic; there was no micro-foundation in individual decision-making. In particular, no reference was made to individual utilities.

This approach contrasts with other literature on insurance risk classification, as summarised in surveys such as Hoy (2006), Einav & Finkelstein (2011) and Dionne & Rothschild (2014). Other literature typically takes a utility-based approach: individuals make purchasing decisions which maximise their expected utilities, and the outcomes of different risk classification schemes are then evaluated by a social welfare function which is a (possibly weighted) sum of expected utilities over the whole population. For example Hoy (2006) uses a utilitarian social welfare function which assigns equal weights to the utilities of all individuals. Einav & Finklestein (2011) use a deadweight-loss concept which appears equivalent to a social welfare function with utilities cardinalized so as to weight willingness-to-pay equally across all individuals.

The present paper connects the loss coverage approach with utility-based approaches in two ways. First, we provide micro-foundations for the aggregate proportional demand function, based on variations between individuals in their utility functions, which explain why only a proportion of the population buys insurance at each price. Second, we reconcile loss coverage to the utilitarian concept of social welfare described above. Specifically, we show that if insurance premiums are negligible relative to wealth, maximising loss coverage maximises social welfare. From a policy perspective, this may be a useful result because maximising loss coverage does not require knowledge of individuals' (generally unobservable) utility functions; loss coverage is based solely on observable quantities.

The rest of this paper is organised as follows. Section 2 develops a theory of insurance demand, starting from expected utility calculations by individuals in a population in which all individuals have the same risk of loss but may have different utility functions. We show that if the insurer can observe individuals' risks but not their utility functions, the demand for insurance appears to the insurer to be proportional; the insurer observes only the proportion of individuals with a given risk who buy insurance at a given premium. Section 3 defines a model for insurance market equilibrium, in a population where individuals are characterized by both risk and utility function. Section 4 establishes the link between loss coverage and social welfare. Section 5 offers brief conclusions.

2. A Theory of Demand for Insurance

2.1 Utility of Wealth and Certainty Equivalence

Consider an individual with an initial wealth W, who is exposed to the risk of losing an amount of L with probability μ. Suppose preference for wealth is driven by the utility function $U(w)$, which is increasing in wealth w, i.e. $U'(w) > 0$.

Individuals are typically also assumed to be risk-averse i.e. $U''(w) < 0$. This provides the motivation for insurance purchase at an actuarially fair price, and initially we shall discuss individuals for whom the assumption holds. But we shall see later in Section 2.2 that our theory of insurance demand does *not* require that *all* individuals are risk-averse. Figure 1 shows an example of a utility function $U(w)$ with $U'(w) > 0$ and $U''(w) < 0$:

If no insurance is bought, occurrence of the risk event will reduce the individual's wealth from W to $(W - L)$ with probability μ. Hence the individual's expected utility, without insurance, is given by:

$$(1-\mu)U(W) + \mu\, U(W-L). \tag{1}$$

If, however, the individual has the option to insure against the risk at premium rate π per unit of loss and chooses to buy insurance for full cover, the individual's expected utility is:

$$U(W - \pi L), \tag{2}$$

because the individual's wealth diminishes immediately by the amount of premium, but there is no further uncertainty as the loss is insured.

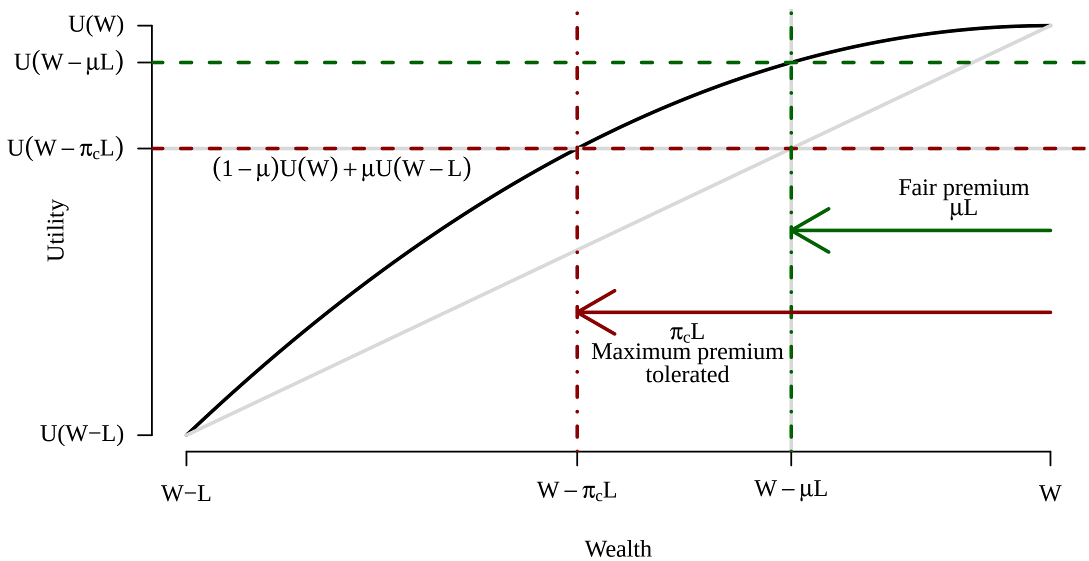

Figure 1: Insurance purchasing decision based on an individual's utility of wealth.

An individual will choose to buy insurance if the expected utility is higher with insurance than without it, i.e.

$$U(W - \pi L) > (1 - \mu)U(W) + \mu\, U(W - L). \tag{3}$$

In particular, individuals with concave utility functions will buy insurance at the actuarially fair premium $\pi = \mu$. Furthermore, these individuals will be prepared to purchase insurance up to the premium level π_c, where:

$$U(W - \pi_c L) = (1 - \mu)U(W) + \mu\, U(W - L), \tag{4}$$

which is also known as the certainty-equivalence principle. This is depicted in Figure 1.

2.2 Heterogeneity in Insurance Purchasing Behaviour

In the above model, all individuals with the same utility function and probability of loss either buy insurance or they do not, based on whether or not the premium being charged, π, exceeds π_c. However, in real insurance markets, we typically observe that not all individuals with the same probability of loss make the same purchasing decision. How can this variation in insurance purchasing decisions be explained?

One plausible explanation is that risk preferences vary between individuals. To formulate this variability, let us assume a population of individuals, all with the same risk μ but who may have different utility functions. Suppose for simplicity that utility functions belong to a family parameterized by a positive real number γ. So a particular individual's utility function can be denoted by $U_\gamma(w)$.

Further suppose that an individual's utility function parameter γ is sampled randomly from an underlying random variable Γ with distribution function $F_\Gamma(\gamma)$. So, a particular individual's utility function, $U_\gamma(w)$, is a random quantity[3], the randomness being induced by $F_\Gamma(\gamma)$.

Based on this formulation, an individual will choose to buy insurance if and only if the following condition is satisfied for the combination of the offered premium π and their particular utility function $U_\gamma(w)$:

$$U_\gamma(W - \pi L) > (1 - \mu)U_\gamma(W) + \mu\, U_\gamma(W - L), \tag{5}$$

Note that all individuals are behaving deterministically, given their knowledge.

Although utility functions of different individuals can have different origins and scales, certainty-equivalent decisions are independent of these choices. So without loss of generality, we will assume that all individuals have the same utility at the "end points" $W - L$ and W. And for clarity, we will suppress the subscript γ for the utility at the "end points" and write $U(W)$ and $U(W - L)$ as they are the same for all individuals. We can then write Equation (5) as:

$$U_\gamma(W - \pi L) > u_c \text{ where} \tag{6}$$

$$u_c = (1 - \mu)U(W) + \mu\, U(W - L) \text{ is a constant.} \tag{7}$$

This says that an individual insures if the utility from insurance exceeds a critical value u_c. Note that u_c is the same for all individuals who are exposed to the same probability of loss.

Figure 2 provides a graphical representation showing utility functions of four individuals with the same probability of loss μ. The concave utility curves, with points A, B and C, represent risk-averse individuals, where higher concavity represents higher risk-aversion. We also show a convex utility curve, with point D, which represents a risk-loving (or risk-neglecting) individual. (As mentioned previously, our model does not require that all individuals are risk-averse.) For the individual at point A, the utility with insurance, $U_{\gamma_A}(W - \pi L)$, exceeds the critical value u_c, where γ_A is the individual's utility function parameter. So the individual buys insurance. For the individuals at points C and D, the inverse applies, so they do not purchase insurance. The individual at point B is indifferent.

[3]We must be careful not to call the function $U_\gamma(w)$ a random variable. We shall have no need of any of the metric structure of spaces of functions that this would entail.

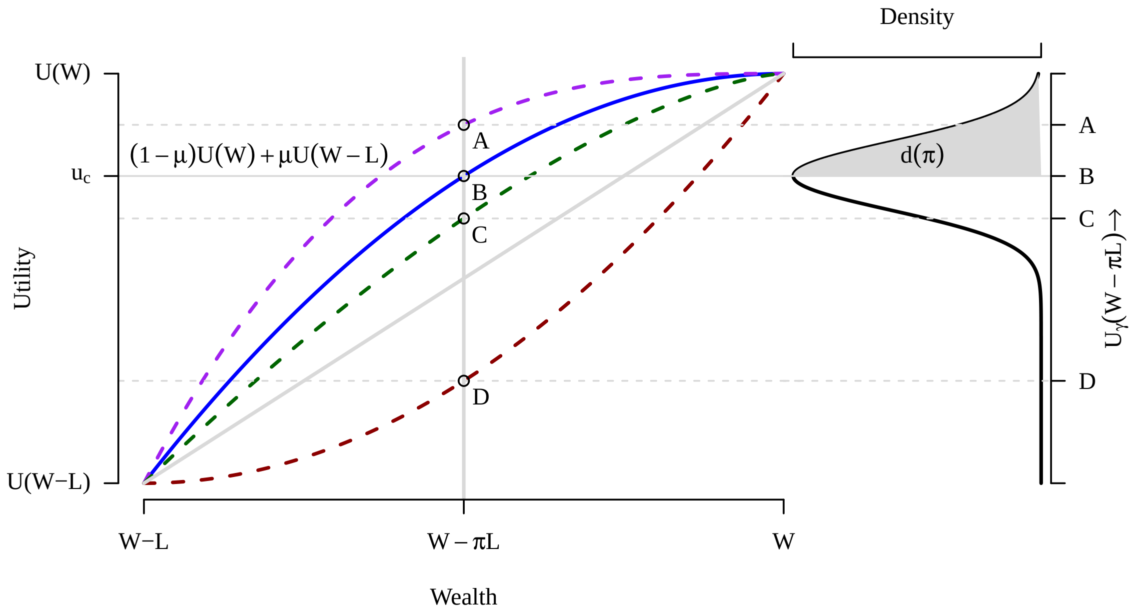

Figure 2: Heterogeneous utility functions within a risk-group, leading to proportional insurance demand.

The utility at the fixed wealth $(W - \pi L)$ is a random variable, that we denote by $U_\Gamma(W - \pi L)$. The distribution function of $U_\Gamma(W - \pi L)$ is induced by that of Γ and we denote it by $G_\Gamma(\gamma)$. The corresponding probability density function of the utilities at that level of wealth is shown in the rotated plot on the right-hand side of Figure 2.

Now assume that the insurer cannot observe individuals' utility functions. Then, for given offered premium π, all the insurer can observe of insurance purchasing behaviour is the proportion of individuals who buy insurance. We call this a demand function and denote it by $d(\pi)$. We have:

$$d(\pi) = \mathrm{P}\left[U_\Gamma(W - \pi L) > u_c\right] = 1 - G_\Gamma(u_c). \tag{8}$$

Insurance purchase is denoted by the shaded area, $d(\pi)$, under the density graph for $U_\Gamma(W - \pi L)$.

We note the following three properties of demand for insurance:

1. $d(\pi)$, denotes a proportion, as $0 \leq d(\pi) \leq 1$ is a valid probability.

2. $d(\pi)$ is non-increasing in π, i.e. demand for insurance cannot increase when premium increases. This can be shown as follows: For utility functions with $U'(w) > 0$, if

$\pi_1 < \pi_2$, the random variable $U_\Gamma(W - \pi_1 L)$ is statewise dominant[4] over the random variable $U_\Gamma(W - \pi_2 L)$. So,

$$\pi_1 < \pi_2 \Rightarrow \mathrm{P}\left[U_\Gamma(W - \pi_1 L) > u_c\right] \geq \mathrm{P}\left[U_\Gamma(W - \pi_2 L) > u_c\right] \Rightarrow d(\pi_1) \geq d(\pi_2). \quad (9)$$

3. Each individual's decision is completely deterministic, given what they know. But to the insurer it appears stochastic, given what the insurer knows. In respect of any individual chosen randomly, define the function Q to be $Q = 1$ if they buy insurance or $Q = 0$ if they do not. To the individual concerned, Q is a deterministic function. To the insurer, Q is a Bernoulli random variable with parameter $d(\pi)$. A full probabilistic model accounting for these different levels of information is given in Appendix A.

As noted earlier, certainty equivalent decisions do not depend on the origins and scales of utility functions, so we can standardise the utility functions such that all individuals have the same utilities $U(W)$ and $U(W - L)$ at the "end points" W and $W - L$. The following standardisation is convenient:

$$U(W) = 1, \quad (10)$$
$$U(W - L) = 0. \quad (11)$$

The constant u_c in Equation 8 then becomes $(1 - \mu)$, and so the demand for insurance is:

$$d(\pi) = \mathrm{P}[U_\Gamma(W - \pi\, L) > 1 - \mu]. \quad (12)$$

2.3 Example

This sub-section gives an illustrative example of the link from a specific distribution of risk preferences to a specific proportional demand for insurance where individuals are exposed to the same probability of loss.

Suppose $W = L = 1$ with a power utility function:

$$U_\gamma(w) = w^\gamma, \quad (13)$$

so that $U_\gamma(0) = 0$ and $U_\gamma(1) = 1$. This particular form of utility function leads to:

$$\text{relative risk aversion coefficient:} \;\; -w\frac{U''_\gamma(w)}{U'_\gamma(w)} = 1 - \gamma. \quad (14)$$

So the heterogeneity in preferences between individuals can be modelled through the randomness of the risk aversion parameter γ. As outlined in Section 2.2, we define a

[4] One random variable is statewise dominant over a second if the first is at least as high as the second in all states of nature, with strict inequality for at least one state. It is an absolute form of dominance.

positive random variable Γ, and individual risk preferences γ are then instances drawn from the distribution of Γ.

Demand for insurance at a given premium π is then:

$$\begin{aligned} d(\pi) &= \mathrm{P}\left[U_\Gamma(1-\pi) > 1-\mu\right], &(15)\\ &= \mathrm{P}\left[(1-\pi)^\Gamma > 1-\mu\right], &(16)\\ &= \mathrm{P}\left[\Gamma \log(1-\pi) > \log(1-\mu)\right], \text{ as log is monotonic,} &(17)\\ &= \mathrm{P}\left[\Gamma < \frac{\log(1-\mu)}{\log(1-\pi)}\right], \text{ as } \log(1-\pi) < 0, &(18)\\ &\approx \mathrm{P}\left[\Gamma < \frac{\mu}{\pi}\right], \text{ as } \log(1-x) \approx -x, \text{ for small } x. &(19) \end{aligned}$$

Now suppose Γ has the following distribution:

$$F_\Gamma(\gamma) = \mathrm{P}\left[\Gamma \le \gamma\right] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau\,\gamma^\lambda & \text{if } 0 \le \gamma \le (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}, \end{cases} \tag{20}$$

where τ and λ are positive parameters. Note that $\tau = \lambda = 1$ leads to a uniform distribution. λ controls the shape of the distribution function and τ controls the range over which Γ takes its values.[5]

Based on this distribution for Γ, the demand for insurance in Equation (19) takes the form:

$$d(\pi) = \tau\left(\frac{\mu}{\pi}\right)^\lambda, \tag{21}$$

which corresponds to iso-elastic demand, the constant demand elasticity being:

$$\epsilon(\pi) = -\frac{\partial \log(d(\pi))}{\partial \log \pi} = \lambda. \tag{22}$$

The parameter τ can also be interpreted as the *fair-premium demand*, that is the demand when an actuarially fair premium is charged. See Hao *et al.* (2016) for details.

The illustrative numerical example given in Section 1.1 can then be shown to correspond to this iso-elastic demand function, with fair-premium demand $\tau = 0.5$ and constant demand elasticity $\lambda = 0.435$ for both risk-groups. These are reasonable parameters.[6]

---

[5] This is a generalised version of the Kumaraswamy distribution, which in its standard form takes values only over [0,1] (Kumarasawamy (1980)).

[6] Approximately half the population has some life insurance (see footnote 2). For yearly renewable term insurance in the US, demand elasticity has been estimated at 0.4 to 0.5 (Pauly *et al.*, 2003). A questionnaire survey about life insurance purchasing decisions produced an estimate of 0.66 (Viswanathan *et al.*, 2007).

An important point to note here is that power utility function of the form given in Equation 13 is concave only if the risk aversion parameter γ is less than 1. Such a constraint can be imposed on random variable Γ by setting $\tau = 1$ in Equation (20). Then the third branch of Equation (20) implies that $d(\pi) = 1$ for $\pi < \mu$, which corresponds to the standard assumption in the economics literature that all individuals are risk-averse and hence will buy insurance for premiums not exceeding their probability of loss. By permitting some individuals to be ‘risk-lovers’, our model better represents the partial take-up of insurance which is observed in practice. Although ‘risk-loving’ or ‘risk-seeking’ are the usual descriptions, ‘risk-neglecting’ might be a more realistic one.

3. Insurance Risk Classification, Market Equilibrium and Loss Coverage

3.1 Framework for Insurance Risk Classification

In Section 2, we have developed a framework for insurance demand based on heterogeneous risk preferences of individuals who have the same wealth W and the same probabilities of loss amount L. In this section, we sketch a generalised framework to allow individuals to belong to different risk-groups having different loss probabilities. Full details are in Appendix A.

Suppose a population can be sub-divided into n distinct risk-groups with probabilities of loss given by $\mu_1, \mu_2, \ldots, \mu_n$. Without loss of generality, we assume the risk-groups are indexed in increasing order of risk, i.e. $0 < \mu_1 < \mu_2 < \ldots < \mu_n < 1$.

Let μ be a random variable denoting the probability of loss for an individual chosen at random from the whole population, such that $\mathrm{P}[\mu = \mu_i] = p_i$ for $i = 1, 2, \ldots, n$. In other words, the proportion of the population belonging to risk-group i is p_i.

Suppose insurers charge premiums $\pi_1, \pi_2, \ldots, \pi_n$ for the respective risk-groups. Initially we do not impose any constraints on the order or size of insurance premiums, so that the insurers are free to charge any premiums to any risk-group. Based on the framework developed in Section 2, we denote the demand for insurance for risk-group i, given offered premium π_i, by $d_i(\pi_i)$, where $0 \le d_i(\pi_i) \le 1$ and $d_i(\pi_i)$ is non-increasing in π_i.

Let the insurance purchasing decision of an individual chosen at random from the whole population be represented by the indicator random variable Q, taking the value of 1 if insurance is purchased; and 0 otherwise. Then conditional on the risk-group, Q is a Bernoulli random variable defined by:

$$\mathrm{E}[\, Q \mid \mu = \mu_i \,] = \mathrm{P}[\, Q = 1 \mid \mu = \mu_i \,] = d_i(\pi_i). \tag{23}$$

Then the expected population demand for insurance is the unconditional expected value of Q:

$$\mathrm{E}[\, Q \,] = \sum_{i=1}^{n} \mathrm{E}[\, Q \mid \mu = \mu_i \,]\, \mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} d_i(\pi_i) p_i. \tag{24}$$

Now suppose that the occurrence of a loss event for an individual chosen at random from the whole population is represented by the indicator random variable, X, taking the value of 1 if a loss event has occurred; and 0 otherwise. Then X is a Bernoulli random variable defined as:

$$\mathrm{E}[\,X \mid \mu = \mu_i\,] = \mathrm{P}[\,X = 1 \mid \mu = \mu_i\,] = \mu_i. \tag{25}$$

Then the expected population loss is the unconditional expected value of X:

$$\mathrm{E}[\,X\,] = \sum_{i=1}^{n} \mathrm{E}[\,X \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} \mu_i p_i. \tag{26}$$

We assume that Q and X are independent, conditional on $\mu = \mu_i$. That is, the level of risk may influence the decision to buy insurance, but there is no moral hazard; insured individuals in any risk-group are not more likely to suffer the loss event than uninsured individuals. Then the expected claims outgo for insurers is:

$$\begin{aligned} E[QX] &= \sum_{i=1}^{n} \mathrm{E}[\,QX \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i], \\ &= \sum_{i=1}^{n} \mathrm{E}[\,Q \mid \mu = \mu_i\,]\,\mathrm{E}[\,X \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i], \\ &= \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. \end{aligned} \tag{27}$$

Finally, for an individual chosen at random from the whole population, define random variable Π, as the premium paid by that individual. As premiums are only paid by individuals who purchase insurance, $\Pi = Q\Pi$. And since everybody in risk-group i is offered the same premium π_i, we have:

$$\mathrm{E}[\,\Pi \mid \mu = \mu_i\,] = \mathrm{E}[\,Q\Pi \mid \mu = \mu_i\,] = \mathrm{E}[\,Q \mid \mu = \mu_i\,]\pi_i = d_i(\pi_i)\pi_i. \tag{28}$$

Then the unconditional expected premium income is:

$$E[\Pi] = \sum_{i=1}^{n} \mathrm{E}[\,\Pi \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i. \tag{29}$$

The expected profit for insurers, as a function of risk-classification regime $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, is then :

$$\rho(\underline{\pi}) = E[\Pi] - E[QX] = \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. \tag{30}$$

3.2 Equilibrium in the Insurance Market

Equilibrium is achieved when the expected profit for insurers is zero. In other words, $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$ denotes an equilibrium, if it satisfies the *equilibrium condition*:

$$\rho(\underline{\pi}) = 0 \;\Leftrightarrow\; \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i = 0. \tag{31}$$

A full probabilistic model, of heterogeneity in insurance purchasing behaviour leading to a framework within which insurance risk classification and market equilibrium can be analysed, is provided in Appendix A.

In what follows, for brevity, we confine our attention to two obvious, and opposing, risk classification schemes, though, under suitable regulation, there are infinitely many possibilities.

3.2.1 *Full risk classification*

An obvious solution to Equation (31) is to set premiums equal to the respective loss probabilities, i.e. $\pi_i = \mu_i$ for $i = 1, 2, \ldots, n$. We call this particular equilibrium the *full risk classification* regime.

3.2.2 *No risk classification*

At the other end of the spectrum is the *pooled* equilibrium where risk classification is banned and so all risk-groups are charged the same premium π_0, i.e. $\pi_i = \pi_0$ for $i = 1, 2, \ldots, n$. The existence of a pooled equilibrium can be demonstrated as follows. Setting the pooled premium $\pi_0 = \mu_1$, the probability of loss for the lowest risk-group, leads to $\rho(\mu_1) \leq 0$.[7] Setting the pooled premium at the highest level of risk, i.e. $\pi_0 = \mu_n$, gives $\rho(\mu_n) \geq 0$. Assuming insurance demand to be a continuous function of premium, there exists at least one root $\pi_0 \in [\mu_1, \mu_n]$ which gives a pooled equilibrium, i.e. $\rho(\pi_0) = 0$.[8]

3.3 Loss coverage

We suggested in the motivating examples in Section 1.1, that loss coverage — heuristically characterised as the proportion of the population's losses compensated by insurance — can be used as a measure for social efficacy of insurance. Loss coverage can now be formally defined within our model framework as the expected insurance claims outgo, or expected population losses compensated by insurance, at equilibrium i.e. $E[QX]$ as defined in Equation (27). So:

$$\text{Loss coverage: } LC(\underline{\pi}) = E[QX], \tag{32}$$

[7] For notational convenience, we specify only one argument for multivariate functions if all arguments are equal, e.g. we write $\rho(\pi)$ for $\rho(\pi, \pi, \ldots, \pi)$.

[8] Uniqueness is not guaranteed, but the lowest of any multiple roots can arguably be regarded as the only true equilibrium. This is because any putative equilibrium above the lowest root can be broken by one insurer charging slightly more than the lowest root (Hoy & Polborn, 2000). In any event, the theory developed in this paper is applicable around any equilibrium.

where $\underline{\pi}$ satisfies the equilibrium condition in Equation (31).

To compare the relative merits of different risk classification regimes, we need to define a reference level of loss coverage. We use the level under risk-differentiated premiums, and so define the *loss coverage ratio*, as follows:

$$\text{Loss coverage ratio: } C = \frac{LC(\underline{\pi})}{LC(\underline{\mu})}. \tag{33}$$

3.4 Examples

Following Hao *et al.* (2016), we continue the example developed in Section 2.3. Suppose there are two risk-groups with population proportions p_1, p_2, probabilities of loss $\mu_1 < \mu_2$ and insurance demand modelled as per Equation (22):

$$d_i(\pi) = \tau_i \left(\frac{\mu_i}{\pi}\right)^{\lambda}, \quad i = 1, 2. \tag{34}$$

If the same premium π_0 is charged for both risk-groups, the equilibrium premium satisfying $\rho(\pi_0) = 0$ is unique and is given by:

$$\pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}, \quad \text{where} \quad \alpha_i = \frac{p_i \tau_i}{p_1 \tau_1 + p_2 \tau_2}, \; i = 1, 2 \tag{35}$$

that is, α_i is the *fair-premium demand-share*, that is the share of total demand represented by risk-group i when actuarially fair premiums are charged.

The loss coverage ratio, comparing loss coverage under pooled premiums to that under risk-differentiated premiums, is:

$$C = \frac{1}{\pi_0^{\lambda}} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}, \tag{36}$$

where π_0 is the pooled equilibrium premium given in Equation (35).

Figures 3 and 4 show the plots of pooled equilibrium premium, insurance demand (cover) and loss coverage ratio as a function of demand elasticity λ, for the risks $(\mu_1, \mu_2) = (0.01, 0.04)$ and fair-premium demand-shares $(\alpha_1, \alpha_2) = (0.9, 0.1)$. Compared with the result under risk-differentiated premiums, under pooling the premium is always higher, and demand (cover) is always lower. This reduction in cover is the perceived loss of efficiency arising from adverse selection. Loss coverage, on the other hand, is not always lower: for this iso-elastic demand function, it is higher than under risk-differentiated premiums if demand elasticity is less than 1. There is some empirical evidence that insurance demand elasticities are typically less than 1 in many markets (Pauly et al., 2003; Viswanathan et al., 2007; Chernew et al., 1997; Blumberg et al., 2001; Buchmueller & Ohri, 2006; Butler, 1999).

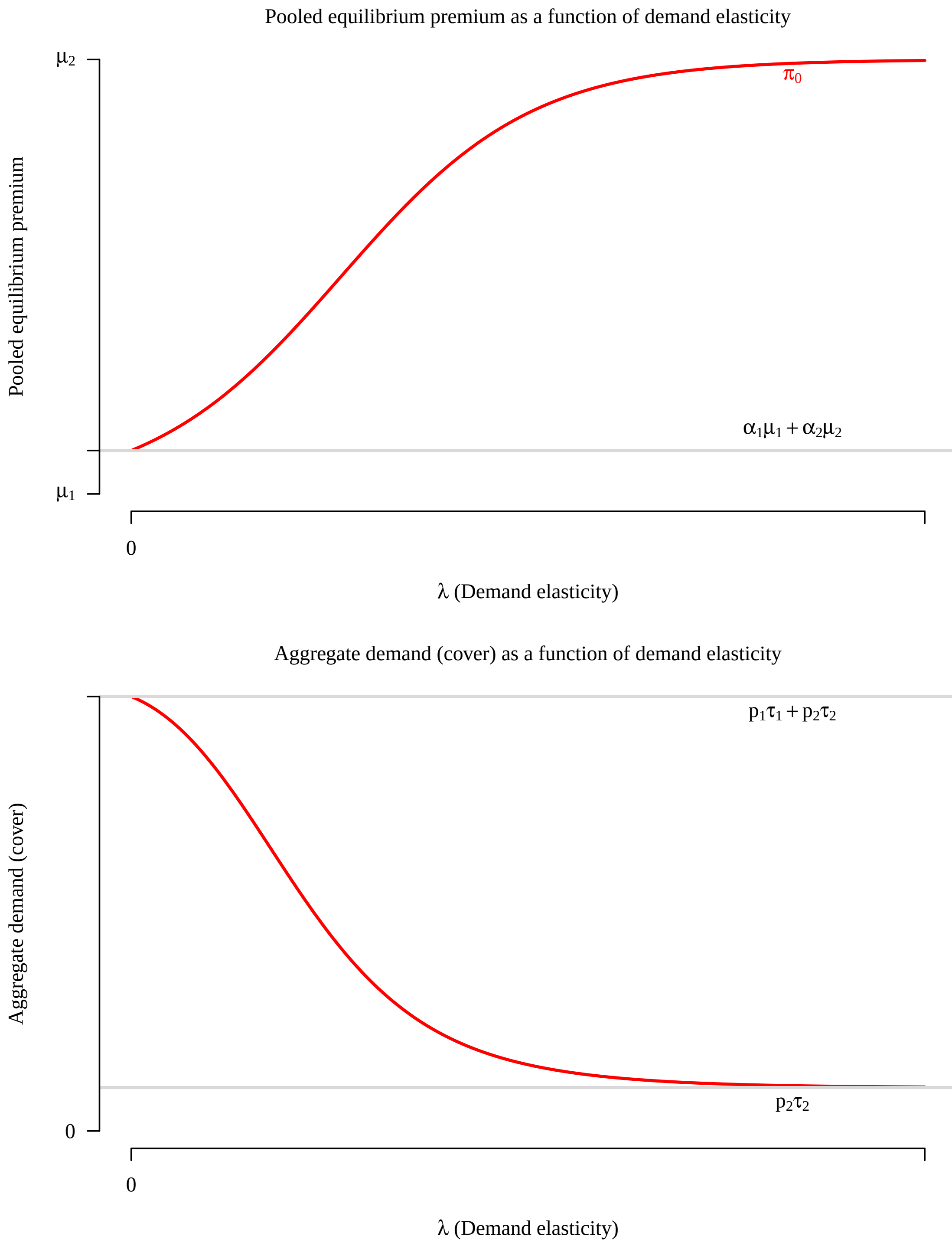

Figure 3: Pooled equilibrium premium (top panel) and aggregate demand (bottom panel) as functions of demand elasticity.

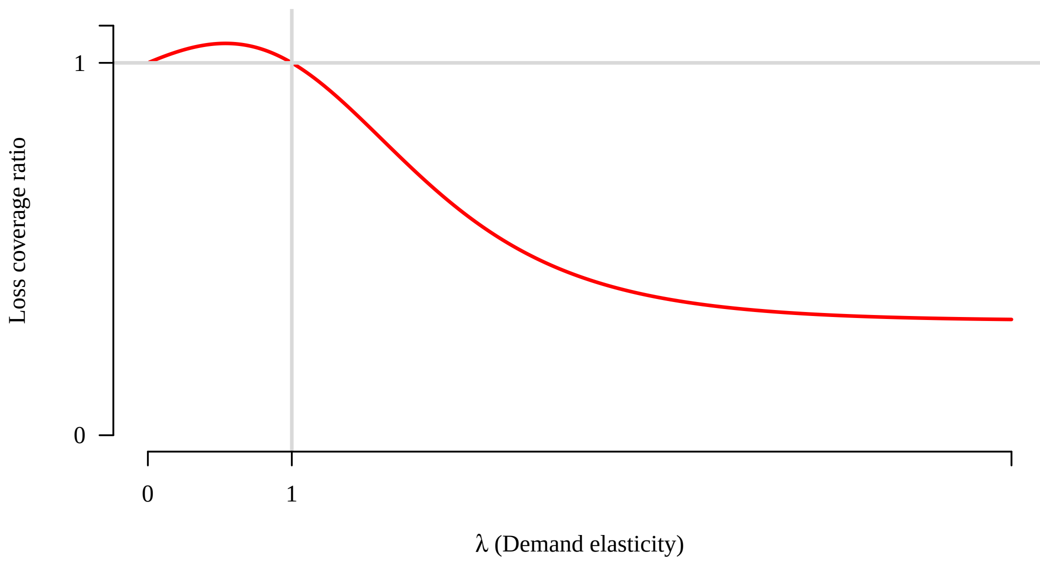

Figure 4: Loss coverage ratio as a function of demand elasticity.

The pattern shown in Figure 4 can be formalised by noting the following property of the loss coverage ratio given in Equation (36):

$$\lambda \lesseqgtr 1 \Rightarrow C \gtreqless 1. \tag{37}$$

This says that for iso-elastic demand, pooling produces higher loss coverage than risk-differentiated premiums if demand elasticity is less than 1. The proof of this result is provided in Appendix B.

4. Social Welfare and Loss Coverage

Our approach to social welfare is in the same spirit as Hoy (2006): we assume cardinal and interpersonally comparable utilities, and assign equal weights to the utilities of all individuals. This equal-weights approach is based on the Harsanyi (1955) 'veil of ignorance' argument: that is, behind the (hypothetical) 'veil of ignorance', where one does not know what position in society (e.g. high risk or low risk) one occupies, the appropriate probability to assign to being any individual is $1/n$, where n is the number of individuals in society. Alternative risk classification regimes can then be compared by comparing expected utility in each regime for the (hypothetical) individual utility-maximiser behind the 'veil of ignorance'.

In our model in Sections 2 and 3, suppose an individual is selected at random from the whole population. The individual's expected utility can be written as follows:

$$\text{Social Welfare} = \mathrm{E}\left[\, Q\, U_\Gamma(W - \Pi L) + (1-Q)\left[(1-X)\, U_\Gamma(W) + X\, U_\Gamma(W-L)\right]\right] \qquad (38)$$

where the first part represents random utility if insurance is purchased; and the second part is the random utility if insurance is not purchased.

As certainty equivalent decisions do not depend on the origins and scales of utility functions, in Section 2, we assumed without loss of generality, that utilities for all individuals are the same at the 'end-points', W and $W - L$. But, this argument cannot be directly extended to Equation (38), because individuals' utilities can differ for identical levels of wealth, which has direct implications for social welfare.

However, without any standardisation, Equation (38) is susceptible to being dominated by a 'utility monster' who derives more utility from a given level of wealth than all other individuals combined (see Bailey (1997), Nozick (1974)). So we propose to continue standardising utility functions so that all utilities are the same at 'end-points', W and $W - L$, as before. This standardisation implies that the same 'disutility of uninsured loss' $[U(W) - U(W - L)]$ is assigned to all individuals, but utility if insurance is purchased $U_\Gamma(W - \Pi L)$ differs between individuals. Under this standardisation, social welfare, denoted by S can be expressed as:

$$S = \mathrm{E}\left[\, Q\, U_\Gamma(W - \Pi L)\right] + (1-Q)\left[(1-X)\, U(W) + X\, U(W-L)\right]. \qquad (39)$$

To derive an expression for S, we consider the constituent parts of Equation (39) separately. Here we sketch the argument, the full probabilistic model is in Appendix A.

First:

$$
\begin{aligned}
&\mathrm{E}\,[\,Q\,U_\Gamma(W-\Pi L)\,] \\
&= \sum_{i=1}^{n} \mathrm{E}[\,Q\,U_\Gamma(W-\pi_i L) \mid \mu=\mu_i\,]\,\mathrm{P}[\mu=\mu_i], && (40)\\
&= \sum_{i=1}^{n} \mathrm{E}[\,U_\Gamma(W-\pi_i L) \mid U_\Gamma(W-\pi_i L) > u_{c_i}\,,\, \mu=\mu_i\,]\;\mathrm{P}\,[U_\Gamma(W-\pi_i L) > u_{c_i} \mid \mu=\mu_i\,]\;p_i, && (41)\\
&= \sum_{i=1}^{n} U_i^*(W-\pi_i L)\,d_i(\pi_i)\,p_i, \quad \text{using Equation (8)}, && (42)
\end{aligned}
$$

where $u_{c_i} = (1-\mu_i)U(W)+\mu_i U(W-L)$ (as defined in Equation (7)) and $U_i^*(W-\pi_i L) = \mathrm{E}[\,U_\Gamma(W-\pi_i L) \mid U_\Gamma(W-\pi_i L) > u_{c_i}\,,\, \mu=\mu_i\,]$ represents the expected utility of individuals purchasing insurance in risk-group i.

Using the assumption that all individuals have the same utilities $U(W)$ and $U(W-L)$ at wealth levels W and $W-L$, and that the random variables Q and X are independent given a risk-group, the second part of Equation (39) becomes:

$$
\begin{aligned}
&\mathrm{E}\,[\,(1-Q)\,[(1-X)\,U(W)+X\,U(W-L)]\,] \\
&= \sum_{i=1}^{n} \mathrm{E}[\,(1-Q)\,[(1-X)\,U(W)+X\,U(W-L)] \mid \mu=\mu_i\,]\,\mathrm{P}[\mu=\mu_i], && (43)\\
&= \sum_{i=1}^{n} [(1-d_i(\pi_i))\,\{(1-\mu_i)U(W)+\mu_i\,U(W-L)\}]\,p_i && (44)
\end{aligned}
$$

Combining Equations (42) and (44), we get the following expression for social welfare:

$$S = \sum_{i=1}^{n} \Big[\underbrace{d_i(\pi_i)\, U_i^*(W - \pi_i\, L)}_{\text{Insured population}} + \underbrace{(1 - d_i(\pi_i)) \left\{(1-\mu_i)U(W) + \mu_i\, U(W-L)\right\}}_{\text{Uninsured population}} \Big] p_i, \quad (45)$$

$$\begin{aligned} &= \underbrace{\sum_{i=1}^{n} \left[(1-\mu_i)U(W) + \mu_i\, U(W-L)\right]\, p_i}_{\text{Constant as a function of } \pi_i} \\ &+ \underbrace{\left(\sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i \right) \times \left[U(W) - U(W-L)\right]}_{\text{Loss coverage} \times \text{Positive multiplier}} \\ &- \underbrace{\sum_{i=1}^{n} d_i(\pi_i) \left[U(W) - U_i^*(W - \pi_i L)\right]\, p_i}_{\text{Adjustment factor to account for premiums}}\,. \end{aligned} \quad (46)$$

A regulator or a policymaker aiming to maximise social welfare, will be interested in choosing a risk-classification regime $\underline{\pi}$ which maximises S.

In most insurance contracts, the premium amount $\pi_i\, L$ will account for a small proportional of the initial wealth W, so that $U(W) \approx U_i^*(W - \pi_i L)$. Under this approximation, Equation (46) simplifies to:

$$S \approx \text{Constant} + \textbf{Loss Coverage} \times \text{Positive multiplier}. \quad (47)$$

In other words, in most cases maximising social welfare becomes approximately equivalent to maximising loss coverage.

This may be a useful approximation, because social welfare depends on (unobservable) utility functions, but loss coverage depends solely on observable quantities. Hence a regulator or policymaker may wish to use loss coverage as a proxy for social welfare.

In cases where premiums cannot be assumed to be small compared to initial wealth W, unapproximated S needs to be maximised. To work with the full expression of social welfare, it might be more convenient to re-express Equation (46) as:

$$S = \text{Constant} + \sum_{i=1}^{n} d_i(\pi_i)\, \underbrace{\left[U_i^*(W - \pi_i L) - \left\{(1-\mu_i)U(W) + \mu_i\, U(W-L)\right\}\right]}_{\text{Excess expected utility of insurance over gamble of not insuring}}\, p_i. \quad (48)$$

Maximising social welfare in this context becomes equivalent to maximising the excess of aggregate expected utility of insurance over the gamble of not insuring.

5. Conclusions

We have proposed loss coverage as an intuitively appealing metric for evaluation of different insurance risk classification schemes. Loss coverage is defined as the expected population losses compensated by insurance at market equilibrium.

Bans on insurance risk classification typically induce adverse selection, leading to a fall in the number of insured individuals compared with that obtained under full risk classification. This reduction in coverage is usually seen as inefficient. However, adverse selection also typically leads to a shift in coverage towards higher risks. If this shift is large enough, it can more than outweigh the fall in numbers insured, so that loss coverage is increased. Since this implies that more risk is voluntarily traded and more losses are compensated, it is a counter-argument to the perception of reduced efficiency.

For coverage to shift towards higher risks when risk classification is banned, it must be the case that not all individuals choose to buy insurance at any given premium. This is an observable reality in many insurance markets. We have shown that it can be explained by heterogenous utility functions, which are unobservable by the insurer. Individuals make decisions completely deterministically on the basis of certainty-equivalent utility calculations, but the insurer observes apparently stochastic decision-making, resulting in a proportional insurance demand function.

We have also shown that loss coverage can be reconciled with (although it is not the same as) an equal-weights definition of utilitarian social welfare in an insurance market, in the spirit of Hoy (2006) or Dionne and Rothschild (2014). Specifically, if insurance premiums are negligible relative to wealth, then maximising loss coverage maximises social welfare. Notably, however, the calculation of social welfare requires utility functions to be observable, while the calculation of loss coverage does not.

References

American Council of Life Insurers (2014). 2014 Life Insurers Factbook. http://www.acli.org. (Accessed 3 September 2015).

Bailey, J.W. (1997). Utilitarianism, institutions and justice. Oxford University Press.

Blumberg, L., Nichold, L. & Banthin, J. (2001). Worker decisions to purchase health insurance. International Journal of Health Care Finance and Economics, **1**, 305–325.

Buchmueller, T.C. & Ohri, S. (2006). Health insurance take-up by the near-elderly. Health Services Research, **41**, 2054–2073.

Butler, J. R. (2002). Policy change and private health insurance: Did the cheapest policy do the trick?. Australian Health Review, **25**, 6.33–41

Cardon, J.H. & Hendel, I. (2001). Asymmetric information in health insurance: evidence from the National Medical Expenditure Survey. Rand Journal of Economics, **32**, 408–427.

Cawley, J. & Philipson, T. (1999). An empirical examination of information barriers to trade in insurance. American Economic Review, **89**, 827–846.

Chernew, M., Frick, K. & McLaughlin, C. (1997). The demand for health insurance coverage by low-income workers: Can reduced premiums achieve full coverage?. Health Services Research, **32**, 453–470.

Chiappori, P-A. & Salanie, B. (2000). Testing for asymmetric information in insurance markets. Journal of Political Economy, **108**, 56–78.

Cohen, A. & Siegelman, P. (2010). Testing for adverse selection in insurance markets. Journal of Risk and Insurance, **77**, 39–84.

De Jong, P. & Ferris, S (2006). Adverse selection spirals. ASTIN Bulletin, **36**, 589–628.

Dionne, G. & Rothschild, C.G. (2014). Economic effects of risk classification bans. Geneva Risk and Insurance Review, **39**, 184-221.

Einav, L. & Finkelstein, A. (2011). Selection in insurance markets: theory and empirics in pictures. Journal of Economic Perspectives, **25**, 115–138.

Finkelstein, A. & McGarry, K. (2006). Multiple dimensions of private information: evidence from the long-term care insurance market. American Economic Review, **96**, 938–958.

Finkelstein, A. & Poterba, J. (2004). Adverse selection in insurance markets: policyholder evidence from the UK annuity market. Journal of Political Economy, **112**, 183–208.

Hao, M., Macdonald, A.S., Tapadar, P. & Thomas, R.G. (2016). Insurance loss coverage under restricted risk classification: The case of iso-elastic demand. To appear in ASTIN Bulletin.

Hoy, M. (2006). Risk classification and social welfare. Geneva Papers on Risk and Insurance, **31**, 245–269.

Hoy, M. & Polborn, M. (2000). The value of genetic information in the life insurance market. Journal of Public Economics, **78**, 235–252.

Kumarasawamy, P. (1980). A generalized probability density function for double-bounded random processes. Journal of Hydrology, **46**, 79–88.

LIMRA (2013). Facts about life 2013. http://www.limra.com. (Accessed 3 September 2015).

Nozick, R. (1974). Anarchy, state and utopia. Basic Books, N.Y..

Pauly, M.V., Withers, K.H., Viswanathan, K.S., Lemaire, J., Hershey, J.C., Armstrong, K., & Asch, D.A. (2003). Price elasticity of demand for term life insurance and adverse selection. NBER Working Paper, **9925**, .

Rothschild, M. and Stiglitz, J. (1976). Equilibrium in competitive insurance markets: an essay on the economics of imperfect information. Quarterly Journal of Economics, **90(4)**, 630–649. Thomas, R.G. (2008). Loss coverage as a public policy objective for risk classification schemes. Journal of Risk and Insurance, **75**, 997–1018.

Viswanathan, K.S., Lemaire, J., K. Withers, K., Armstrong, K., Baumritter, A., Hershey, J., Pauly, M., & Asch, D.A. (2006). Adverse selection in term life insurance purchasing due to the BRCA 1/2 Genetic Test and elastic demand. Journal of Risk and Insurance, **74**, 65–86.

Appendices

A. Probabilistic Model of Heterogeneous Insurance Purchasers

We can construct a probabilistic model by supposing that any individual sampled at random posesses two attributes, risk of suffering a loss event (or just 'risk' for short) and a utility function.

- We suppose that 'risk' is defined as the probability μ of suffering a defined loss event. For simplicity, suppose the set of possible values of μ is the finite set $M = \{\mu_1, \mu_2, \ldots, \mu_n\}$, that $\mathcal{G}$ is the power set of M and that $\mathrm{P}[\mu = \mu_i] = p_i$.
- For simplicity, suppose that all utility functions belong to a family parameterized by a real number γ. Individuals' utility functions take values in R.

Then the idea of risk and utility being heterogeneous in a population may be modelled by the probability space $(\Omega, \mathcal{F}, P)$ where:

- The sample space is $\Omega = M \times R$.
- The sigma-algebra $\mathcal{F}$ is $\mathcal{G} \times \mathcal{B}$, where $\mathcal{B}$ is the Borel sigma-algebra on R.
- The probability measure P is assumed to be given by a probability function $F(\mu, \gamma)$, discrete in its first component and absolutely continuous in its second component.

An individual sampled at random has the attributes μ and γ given by the probability F. We must have the marginal distribution:

$$p_i = \mathrm{P}[\mu = \mu_i] = \int_{\{\mu_i\} \times R} dF(\mu, \gamma) = \int_R dF(\mu_i, \gamma) \tag{49}$$

where the first integral is Stieltjes, summing over the first component of F and integrating over the second component.

Two individuals with the same value μ_i of μ may be said to belong to the same risk group, for insurance purposes. The insurer is supposed able to observe μ and will offer the same premium π_i to everyone with risk μ_i. It is assumed that an individual with risk μ_i, offered premium π_i, will decide to buy insurance, or not, non-randomly, determined by their utility function. We suppose, however, that the insurer cannot observe γ. Since different individuals, sampled at random and allocated to the same risk-group, can have different utility functions, the insurer will observe heterogeneous behaviour within a risk-group. That is, even though all in the risk-group are offered the same premium rate, some will buy insurance and others will not. The purchasing decision, given the utility function, is non-random, but to the insurer it appears to be random because of the unobserved heterogeneity. At most, the insurer can observe the proportion of individuals in any risk-group that buy insurance. Thus the insurer may model the insurance-buying decision of an individual in a given risk-group as a Bernoulli random variable.

The insurer's premium strategy may be represented by a $\mathcal{G}-$measurable random variable on M, or by a $(\mathcal{G} \times \{\emptyset, \Omega\})-$measurable random variable on Ω. In either case, denote it by Π. The insurance purchasing decision may be represented by an indicator Q, taking the value 1 if insurance is purchased and 0 otherwise. For a given premium strategy Π on the insurer's part, Q is an $\mathcal{F}-$measurable random variable on Ω. Its restriction to a fixed value of the risk $\mu = \mu_i$ is the Bernoulli random variable that the insurer observes in that risk-group.

The proportion of risks with $\mu = \mu_i$ that buy insurance, which we may call a 'demand function' and denote by $d_i(\pi_i)$, is the conditional expected value of Q:

$$d_i(\pi_i) = \mathrm{P}[\, Q = 1 \mid \mu_i\,] = \mathrm{E}[\, Q \mid \mu_i\,] = \frac{\int_R Q(\mu_i, \gamma)\, dF(\mu_i, \gamma)}{\int_R dF(\mu_i, \gamma)} \tag{50}$$

and the expected population demand for insurance is the unconditional expected value of Q:

$$\begin{aligned}
\mathrm{E}[Q] &= \int_\Omega Q(\mu, \gamma)\, dF(\mu, \gamma) && (51)\\
&= \sum_{i \in M} \int_R Q(\mu_i, \gamma)\, dF(\mu_i, \gamma) && (52)\\
&= \sum_{i \in M} \left(\frac{\int_R Q(\mu_i, \gamma)\, dF(\mu_i, \gamma)}{\int_R dF(\mu_i, \gamma)} \times \int_R dF(\mu_i, \gamma) \right) && (53)\\
&= \sum_{i \in M} d_i(\pi_i)\, p_i. && (54)
\end{aligned}$$

Define X to be a Bernoulli random variable, indicating that a loss event occurs. Given μ_i, X has parameter μ_i, and does not depend on any utility function. Observation of X is new information, not part of the model above. Then:

$$\begin{aligned}
\mathrm{E}[X] &= \int_\Omega \mathrm{E}[\, X \mid \mu, \gamma]\, dF(\mu, \gamma) && (55)\\
&= \sum_{i \in M} \mathrm{E}[\, X \mid \mu_i\,] \int_R dF(\mu_i, \gamma) && (56)\\
&= \sum_{i \in M} \mu_i p_i. && (57)
\end{aligned}$$

Assume that Q and X are independent, conditional on μ_i. That is, the level of risk may influence the decision to buy insurance, but there is no moral hazard; insured individuals in any risk-group are not more likely to suffer the loss event than uninsured individuals. Then the expected claims outgo for the insurer is:

$$\begin{aligned}
\mathrm{E}[QX] &= \int_\Omega \mathrm{E}[\,QX \mid \mu,\gamma\,]\,dF(\mu,\gamma) && (58)\\
&= \int_\Omega Q(\mu,\gamma)\,\mathrm{E}[\,X \mid \mu,\gamma\,]\,dF(\mu,\gamma) \qquad (Q \text{ is } \mathcal{F}-\text{measurable}) && (59)\\
&= \sum_{i\in M} \mathrm{E}[\,X \mid \mu_i\,] \int_R Q(\mu_i,\gamma)\,dF(\mu_i,\gamma) && (60)\\
&= \sum_{i\in M} \mu_i\, d_i(\pi_i)\, p_i \qquad \text{(following Equation (53)).} && (61)
\end{aligned}$$

Finally, the expected premium income is:

$$\begin{aligned}
\mathrm{E}[Q\Pi] &= \int_\Omega \mathrm{E}[\,Q\Pi \mid \mu,\gamma\,]\,dF(\mu,\gamma) && (62)\\
&= \int_\Omega Q(\mu,\gamma)\,\mathrm{E}[\,\Pi \mid \mu,\gamma\,]\,dF(\mu,\gamma) && (63)\\
&= \sum_{i\in M} \mathrm{E}[\,\Pi \mid \mu_i\,] \int_R Q(\mu_i,\gamma)\,dF(\mu_i,\gamma) && (64)\\
&= \sum_{i\in M} \pi_i\, d_i(\pi_i)\, p_i \qquad \text{(following Equation (53)).} && (65)
\end{aligned}$$

Based on the formulation of expected premium income and claims outgo, the total expected profit for insurers, as a function of risk-classification regime $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, can be defined as:

$$\text{Expected profit for insurers:} \qquad \rho(\underline{\pi}) = E[Q\Pi] - E[QX] = \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. \tag{66}$$

Finally we define social welfare as expected utility of an individual chosen at random, i.e.

$$\text{Social Welfare} = \mathrm{E}\left[Q\,U_\Gamma(W-\pi L) + (1-Q)\big[X\,U_\Gamma(W-L) + (1-X)\,U_\Gamma(W)\big]\right]. \tag{67}$$

as in Equation (38). Let us review the measurability and dependencies of the quantities we will need.

μ	is	$\mathcal{G}-$measureable.
Γ	is	$\mathcal{B}-$measureable (Borel sigma-algebra on R).
Π	is	$\mathcal{G}-$measureable.
Q	is	$\mathcal{F}-$measureable, but not independent of Π.
X	is	neither $\mathcal{G}-$measureable nor $\mathcal{F}-$measureable, but it is independent of Π.

Note that $\mathrm{E}[\,X \mid \mathcal{F}\,] = \mathrm{E}[\,X \mid \mu_i\,] = \mu_i$. Considering S term by term.

$$\begin{aligned}
& \mathrm{E}[Q\,U_\Gamma(W - \pi L)] && (68)\\
= \;& \mathrm{E}[\,\mathrm{E}[Q\,U_\Gamma(W - \pi L) \mid \mathcal{F}\,]\,] && (69)\\
= \;& \sum_{i=1}^{n} p_i \int_R Q(\mu_i, \gamma)\, U_\gamma(W - \pi_i L)\, dF(\pi_i, \gamma) && (70)\\
= \;& \sum_{i=1}^{n} p_i\, d_i(\pi_i) \frac{\int_R Q(\mu_i, \gamma)\, U_\gamma(W - \pi_i L)\, dF(\pi_i, \gamma)}{d_i(\pi_i)} && (71)\\
= \;& \sum_{i=1}^{n} p_i\, d_i(\pi_i)\, \mathrm{E}\left[\int_R Q(\mu_i, \gamma)\, U_\gamma(W - \pi_i L)\, dF(\pi_i, \gamma) \,\middle|\, Q(\mu_i, \cdot) = 1\right] && (72)
\end{aligned}$$

where $Q(\mu_i, \cdot)$ denotes the restriction of Q to the ith risk-group. This is equivalent to Equation (42) in the main text. Next:

$$\begin{aligned}
& \mathrm{E}[\,(1 - Q)\, X\, U_\Gamma(W - L)\,] && (73)\\
= \;& \mathrm{E}\,\mathrm{E}[\,(1 - Q)\, X\, U_\Gamma(W - L) \mid \mathcal{F}\,] && (74)\\
= \;& \sum_{i=1}^{n} p_i \int_R (1 - Q(\mu_i, \gamma))\, U_\gamma(W - L)\, \mathrm{E}[\,X \mid \mathcal{F}\,]\, dF(\pi_i, \gamma) && (75)\\
= \;& \sum_{i=1}^{n} p_i\, \mu_i \int_R (1 - Q(\mu_i, \gamma))\, U_\gamma(W - L)\, dF(\pi_i, \gamma) && (76)\\
= \;& \sum_{i=1}^{n} p_i\, \mu_i\, (1 - d_i(\pi_i)) \frac{\int_R (1 - Q(\mu_i, \gamma))\, U_\gamma(W - L)\, dF(\pi_i, \gamma)}{1 - d_i(\pi_i)} && (77)\\
= \;& \sum_{i=1}^{n} p_i\, \mu_i\, (1 - d_i(\pi_i))\, \mathrm{E}\left[\int_R (1 - Q)\, U_\gamma(W - L)\, dF(\mu_i, \gamma) \,\middle|\, Q(\pi_i, \cdot) = 0\right] && (78)\\
= \;& \sum_{i=1}^{n} p_i\, \mu_i\, (1 - d_i(\pi_i)) U(W - L), \quad \text{if } U_\gamma(W - L) = U(W - L) \text{ for all } \gamma. && (79)
\end{aligned}$$

Similarly,

$$\begin{aligned}
& \mathrm{E}[\,(1 - Q)\,(1 - X)\, U_\Gamma(W)\,] && (80)\\
= \;& \sum_{i=1}^{n} p_i\, (1 - \mu_i)\, (1 - d_i(\pi_i))\, \mathrm{E}\left[\int_R (1 - Q)\, U_\gamma(W)\, dF(\mu_i, \gamma) \,\middle|\, Q(\pi_i, \cdot) = 0\right] && (81)\\
= \;& \sum_{i=1}^{n} p_i\, (1 - \mu_i)\, (1 - d_i(\pi_i)) U(W), \quad \text{if } U_\gamma(W) = U(W) \text{ for all } \gamma. && (82)
\end{aligned}$$

B. Loss Coverage Ratio

The argument given here follows Hao *et al.*, (2016). The loss coverage ratio for the case of equal demand elasticity is given in Equation (36) and can be expressed as follows:

$$C = \frac{1}{\pi_0^\lambda}\frac{\alpha_1\mu_1^{\lambda+1}+\alpha_2\mu_2^{\lambda+1}}{\alpha_1\mu_1+\alpha_2\mu_2}, \quad \text{where} \quad \pi_0 = \frac{\alpha_1\mu_1^{\lambda+1}+\alpha_2\mu_2^{\lambda+1}}{\alpha_1\mu_1^{\lambda}+\alpha_2\mu_2^{\lambda}}; \tag{83}$$

$$= \left[w\mu_1^{\lambda-1}+(1-w)\mu_2^{\lambda-1}\right]^\lambda \left[w\mu_1^{\lambda}+(1-w)\mu_2^{\lambda}\right]^{1-\lambda} \quad \text{where} \quad w = \frac{\alpha_1\mu_1}{\alpha_1\mu_1+\alpha_2\mu_2}; \tag{84}$$

$$= E_w\left[\mu^{\lambda-1}\right]^\lambda E_w\left[\mu^\lambda\right]^{1-\lambda}, \tag{85}$$

where E_w denotes expectation in this context and the random variable μ takes values μ_1 and μ_2 with probabilities w and $1-w$ respectively.

Result B.1. *For $\lambda > 0$,*

$$\lambda \lesseqqgtr 1 \Rightarrow C \gtreqqless 1. \tag{86}$$

Proof. **Case $\lambda = 1$:** It follows directly from Equation (85) that $C(1) = 1$.

Case $0 < \lambda < 1$: Holder's inequality states that, if $1 < p, q < \infty$ where $1/p + 1/q = 1$, for positive random variables X, Y with $E[X]^p, E[Y]^q < \infty$, $E\left[X^p\right]^{1/p} E\left[Y^q\right]^{1/q} \geq E[XY]$.

Setting $1/p = \lambda$, $1/q = 1-\lambda$, $X = \mu^{\lambda(\lambda-1)}$ and $Y = 1/X$, applying Holder's inequality to Equation (85) gives,

$$C = E_w\left[X^{1/\lambda}\right]^\lambda E_w\left[Y^{1/(1-\lambda)}\right]^{1-\lambda} \geq E_w[XY] = 1. \tag{87}$$

Case $\lambda > 1$: Lyapunov's inequality states that, for positive random variable μ and $0 < s < t$, $E[\mu^t]^{1/t} \geq E[\mu^s]^{1/s}$.

So Equation 85 gives:

$$C = \frac{E_w\left[\mu^{\lambda-1}\right]^\lambda}{E_w\left[\mu^\lambda\right]^{\lambda-1}} = \left[\frac{E_w\left[\mu^{\lambda-1}\right]^{1/(\lambda-1)}}{E_w\left[\mu^\lambda\right]^{1/\lambda}}\right]^{\lambda(\lambda-1)} \leq 1, \tag{88}$$

as $E_w\left[\mu^{\lambda-1}\right]^{1/(\lambda-1)} \leq E_w\left[\mu^\lambda\right]^{1/\lambda}$ for $\lambda > 1$ by Lyapunov's inequality.

□