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ScienceDirect

Energy Procedia 66 (2015) 245 – 248

Energy Procedia

The 12th International Conference on Combustion & Energy Utilisation – 12ICCEU

# A two-stage polynomial method for spectrum emissivity modeling

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**Abstract**

Spectral emissivity is a key in the temperature measurement by radiation methods, but not easy to determine in a combustion environment, due to the interrelated influence of temperature and wave length of the radiation. In multi-wavelength radiation thermometry, knowing the spectral emissivity of the material is a prerequisite. However in many circumstances such a property is a complex function of temperature and wavelength and reliable models are yet to be sought. In this study, a two stages partition low order polynomial fitting is proposed for multi-wavelength radiation thermometry. In the first stage a spectral emissivity model is established as a function of temperature; in the second stage a mathematical model is established to describe the dependence of the coefficients corresponding to the wavelength of the radiation. The new model is tested against the spectral emissivity data of tungsten, and good agreement was found with a maximum error of 0.64%



Peer-review under responsibility of the Engineering Department, Lancaster University

Keywords: Spectral Emissivity, measurement

## 1. A general model of Multi-spectral radiation emittance

The commonly used multi-wavelength thermometry in temperature measurement acquires the radiant intensity signal of an object being measured at a number of wave lengths, and determines the temperature and spectral radial emittance according to a certain model, traditionally the least squares regression. In doing so a relationship between the wave length and the emissivity is determined or adopted [1-3]. However, in many cases the emissivity of the material is an unknown function before the measurement takes place, this tends to cause either complexity or significant errors in the measurement. Currently there is yet a general model to suit many materials.

The authors in this study establish a method to tackle the above problem. First, The spectral emissivity of objects $\varepsilon(\lambda,T)$ is a function of temperature T and wave length , $\lambda$ defined by:

$$\varepsilon(\lambda,T)=M(\lambda,T)/M_b(\lambda,T) \tag{1}$$

where $M(\lambda,T)$ is spectral radiant emittance. According to Planck's radiation law and Wien approximation:

$$M_b(\lambda,T)=C_1\lambda^{-5}e^{-\frac{C_2}{\lambda T}} \tag{2}$$

Where $C_1, C_2$ are the first radiation constant and the second radiation constant respectively.

We define a general spectral emissivity with regards to wavelength as,

$$\ln\varepsilon(\lambda,T)=b_0+b_1\lambda+b_2\lambda^2+\cdots \tag{3}$$

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Peer-review under responsibility of the Engineering Department, Lancaster University
doi:10.1016/j.egypro.2015.02.042

Then, combining (1), (2) and (3), we derive our base model of multi-spectral radiation thermometry:

$$\ln M(\lambda,T)-\ln C_1+5\ln\lambda=-C_2/(T\lambda)+b_0+b_1\lambda+b_2\lambda^2+b_3\lambda^3+\cdots \tag{4}$$

## 2. The two stage piecewise polynomial fitting method for spectrum emissivity

Spectral emissivity is a function of both T and $\lambda$. However, corresponding to every fixed $\lambda$, $\varepsilon$ will be a function of only T. After examining a large number of emissivity curves of various materials, our intuitive thought is that the natural logarithm function can be a good approximation for a variety of spectral emissivity values. Therefore, for a fixed $\lambda$, the following model will apply:

$$\ln\varepsilon(\lambda,T)\approx\sum_{k=0}^{n}a_k(\lambda)T^{-k} \tag{5}$$

at fixed $\lambda$, where $a_0,a_1,\cdots,a_n$ can be obtained through solving the minimum norm of the following equation:

$$\ln\varepsilon(\lambda,T_i)=\sum_{k=0}^{n}a_k(\lambda)T_i^{-n},i=1,2,...,s \tag{6}$$

where s is the number of data samples. In high-temperature conditions, e.g. combustion, condition number of the coefficient matrix will be large, causing numerical instability. To improve the numerical stability,, $t=C_0/(T-T_0)$ is used for T, then model **Error! Reference source not found.** becomes:

$$\ln\varepsilon(\lambda,t)\approx\sum_{k=0}^{n}a_k(\lambda)t^k \tag{7}$$

Where $T_0$ and $C_0$ are constants.

The RHS of Eq.**Error! Reference source not found.** is a polynomial function of t, and the other coefficients $a_i(\lambda),\mathrm{i}=1,2,\cdots,\mathrm{n}$ in Eq.(7) can be solve by any appropriate regression method. After this, the remaining question of the relationship between $\varepsilon$ and $\lambda$ can be find through a similar process of regression.

With least squares fitting of polynomial function, the pathological degree of corresponding coefficient matrix of normal equation will increase with the order of polynomial. To improve the numerical stability, orthogonal polynomial is chosen as the base, and orthogonal polynomial fitting is performed, as described below.

In a linear space $P_n(t)=\left\{f(x)=\sum_{k=1}^{n}a_kt^{k-1}\mid a_1,a_2,...,a_n\in R\right\}$, corresponding to the points $t_1,t_2,...,t_n$, the inner product is defined as following:

$$(f,g)=\sum_{k=1}^{n}f(t_k)g(t_k) \tag{8}$$

Then the orthogonal basis for the inner product eq.8 can be obtained using the following three recursive formulae:

$$\begin{cases}\varphi_0(t)=1,\\ \varphi_1(t)=t-a_0 & k=1,2,...,n-2,\\ \varphi_{k+1}(t)=(t-a_k)\varphi_k(t)-b_k\varphi_{k-1}(t)\end{cases} \tag{9}$$

where

$$\begin{cases}a_k=(t\varphi_k,\varphi_k)/(\varphi_k,\varphi_k), & k=0,1,\cdots,n-2\\ b_k=(\varphi_k,\varphi_k)/(\varphi_{k-1},\varphi_{k-1}), & k=1,2,\cdots,n-2\end{cases} \tag{10}$$

Specially，when $t_k=t_1+(k-1)h,k=1,2,\cdots,n$,

By using the transform

$$x=(t+h-t_1)/h, \tag{11}$$

we can derive the following orthogonal polynomials from **Error! Reference source not found.** ~**Error! Reference source not found.** :

$$\begin{aligned}&\varphi_0(x)=1,\quad \varphi_1(x)=x-(n+1)/2\\ &\varphi_{m+1}(x)=(x-(n+1)/2)\varphi_m(x)-\varphi_{m-1}(x)\cdot m^2(n^2-m^2)/4(4m^2-1)\end{aligned} \tag{12}$$

## 3. Demonstration and discussions

To verify the new method, a demonstration is given for the surface emissivity of tungsten listed in Table 1. First, for fixed wavelengths, a quadratic form of Eq.(7) is used to correlate ε and T.

$$g(\lambda,T)=a_0(\lambda)+\frac{a_1(\lambda)}{(T-1500)}+\frac{a_2(\lambda)}{(T-1500)^2} \tag{13}$$

By regression $a_0$, $a_1$, $a_2$ and the maximum relative errors in different wavelength by (13) are found and listed in Table 2.

**Table 1.** Surface emissivity of tungsten

| | temperature /K | | | | |
|---|---|---|---|---|---|
| wavelength | 1600 | 1800 | 2000 | 2200 | 2400 |
| 0.34 | 0.4823 | 0.4798 | 0.4773 | 0.4748 | 0.4723 |
| 0.38 | 0.4775 | 0.4754 | 0.4733 | 0.4712 | 0.4691 |
| 0.42 | 0.4694 | 0.4678 | 0.4662 | 0.4646 | 0.463 |
| 0.46 | 0.462 | 0.4606 | 0.4592 | 0.4578 | 0.4564 |
| 0.5 | 0.4571 | 0.4552 | 0.4533 | 0.4514 | 0.4495 |
| 0.54 | 0.4539 | 0.4514 | 0.4489 | 0.4464 | 0.4439 |
| 0.58 | 0.4501 | 0.447 | 0.4439 | 0.4408 | 0.4377 |
| 0.62 | 0.445 | 0.4413 | 0.4376 | 0.4339 | 0.4302 |
| 0.66 | 0.4412 | 0.4369 | 0.4326 | 0.4283 | 0.424 |
| 0.7 | 0.4375 | 0.4331 | 0.4287 | 0.4243 | 0.4199 |
| 0.74 | 0.4304 | 0.4266 | 0.4228 | 0.419 | 0.4152 |

**Table 2.** Errors of the proposed model

| $\lambda$ | $a_0(\lambda)$ | $a_1(\lambda)$ | $a_2(\lambda)$ | $\max_{i=1,2,3,4,5}\left\|\frac{g(\lambda,T_i)-\ln\varepsilon(\lambda,T_i)}{\ln\varepsilon(\lambda,T_i)}\right\|$ |
|---|---|---|---|---|
| 0.34 | −0.75844 | 9.852553 | −693.181 | 0.002571 |
| 0.38 | −0.76397 | 8.342802 | −586.87 | 0.00215 |
| 0.42 | −0.77547 | 6.450046 | −453.636 | 0.001627 |
| 0.46 | −0.78922 | 5.728748 | −402.877 | 0.001416 |
| 0.5 | −0.80626 | 7.880335 | −554.312 | 0.001919 |
| 0.54 | −0.82098 | 10.47759 | −737.204 | 0.002526 |
| 0.58 | −0.83729 | 13.14773 | −925.329 | 0.003131 |
| 0.62 | −0.85691 | 15.92999 | −1121.46 | 0.003736 |
| 0.66 | −0.87378 | 18.74106 | −1319.74 | 0.004344 |
| 0.7 | −0.88402 | 19.35448 | −1363.02 | 0.00444 |
| 0.74 | −0.89325 | 16.93771 | −1192.53 | 0.003816 |

In order to improve precision, a piecewise polynomial with lower degree to fit $a_i(\lambda)$ can be used. We divided the wavelength in table2 into two sections $[0.34,0.54]$ and $[0.54,0.74]$. In each section we use orthogonal polynomials $\varphi_0(x),\varphi_1(x),\cdots,\varphi_m(x)$ given by (12)to fit $a_0(\lambda),a_1(\lambda),a_2(\lambda)$ respectively.

In section $[0.34,0.54]$, the polynomial is

$$a_i(\lambda)=a_{i0}+a_{i1}\varphi_1(x)+a_{i2}\varphi_2(x)+\cdots+a_{im}\varphi_m(x) \quad with\ x=(\lambda-0.30)/0.04, \tag{14}$$

and in section $[0.54,0.74]$, the polynomial is

$$a_i(\lambda)=\tilde{a}_{i0}+\tilde{a}_{i1}\varphi_1(x)+\tilde{a}_{i2}\varphi_2(x)+\mathrm{L}+\tilde{a}_{im}\varphi_m(x) \quad with\ x=(\lambda-0.50)/0.04, \tag{15}$$

Having obtained $a_i(\lambda)$, the second regression is carried out to find $a_i(\lambda)$ on $\lambda$. The final results are:

$$\begin{aligned} a_0(\lambda)&=-0.540550-1.086471\lambda+0.813839\lambda^2,\\ a_1(\lambda)&=-74.824185+373.387986\lambda-380.690960\lambda^2,\\ a_2(\lambda)&=5276.020568-26321.12089\lambda+26835.70313\lambda^2 \end{aligned} \tag{16}$$

In $[0.34,0.54]$, and in $[0.34,0.54]$, there are

$$\begin{aligned} \tilde{a}_0(\lambda)&=-0.8898513+0.6492\lambda-0.760156\lambda^2,\\ \tilde{a}_1(\lambda)&=175.485456-523.736824\lambda+409.736630\lambda^2,\\ \tilde{a}_2(\lambda)&=-12354.30804+36872.95571\lambda-28846.99777\lambda^2 \end{aligned} \tag{17}$$

With the coefficients determined, the model is used to generate the data for the temperatures and wavelengths listed in Table. Very good agreement is found with a maximum error below 0.64%.

## 4. Conclusion

A two-stage partition low order polynomial model was derived for multi-wavelength radiation thermometry. The new model takes account both temperature and wavelength of the radiant energy, thus is a more complete model than the

existing ones that do not make correlations against both variables simultaneously. The application of the model is demonstrated using the spectral emissivity data of tungsten with satisfactory accuracy.

As regression is not very difficult to carry out, and orthogonal polynomial is chosen as the base to improve the numerical stability, the model of this study will be ready to apply for a variety of materials, in including mixture of gases for combustion diagnosis, which is a distinct merit.

## 5. Acknowledgement

The project is supported by “Fundamental Research funds for the Central Universities”.

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