Adaptive and Robust Fault-Tolerant Tracking Control of Contact force of Pantograph-Catenary for High-Speed Trains

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Abstract: This paper presents a modified multi-body dynamic model and a linear time-invariant model with actuator faults (loss of effectiveness faults, bias faults) and matched and unmatched uncertainties. Based on the fault model, a class of adaptive and robust tracking controllers are proposed which are adjusted online to tolerate the time-varying loss of effectiveness faults and bias faults, and compensate matched disturbances without the knowledge of bounds. For unmatched uncertainties, optimal control theory is added to the controller design processes. Simulations on a pantograph are shown to verify the efficiency of the proposed fault-tolerant design approach.

Keywords: high-speed train, pantograph-catenary, adaptive and robust, fault-tolerant, matched and unmatched uncertainties.

1. INTRODUCTION

The contact between pantograph and catenary is the most critical part to modern high-speed trains in the transmission of electrical energy. The force exerted by the pantograph on the contact wire may oscillates heavily, which can originate electric arcs to damage the mechanical structure and reduce the system performance Levant, A. (2001).

Because of the elastic character from a static perspective by its stiffness and from a dynamic one by its frequency, the pantograph-catenary system contains time-varying parameters in its system matrix which make the modeling difficult Benet, J. (2007), Park, T. J. (2002), Pisano, A. (2008). Some researchers have used different approaches to regulate the contact force to a pre-specified constant value, such as robust optimal control Lin, Y. C. (2007), H_∞ control Makino, T. (1997).

For instance, extreme environmental conditions lead to limitations to train operation and/or damage of components of the pantograph and catenary in the short or long run. Therefore, it is required to have the ability to tolerate the actuator faults and to guarantee the pantograph-catenary system has the desired performance.

There are many papers to deal with the fault-tolerant problems, which are divided into two categories: active fault-tolerant control and passive fault-tolerant control. Jiang, B. (2006) do some researches on the active fault tolerant control based on the adaptive fault diagnose and accommodation; Tao G. (2001), Ye D. (2006), Yang G. H. (2010) have done some work on the passive fault tolerant. Tao G. (2001) try to deal with the stuck fault based on the adaptive compensation mechanical. Ye D. (2006) proposes a classical control structure to deal with actuator loss of effectiveness faults, and Yang G. H. (2010) defines an adaptive H_∞ performance index and designs a class of adaptive controllers which satisfy the defined performance index.

Li X. J. (2012), Wu L. B. (2014), Zhang Y. (2009) try to use robust adaptive method to cop with multi types of actuator faults in presence of both matched and unmatched disturbances and uncertainties. Li X. J. (2012) mainly focus on actuator faults with matched uncertainties and disturbances, Wu L. B. (2014) concentrates on the specific unmatched uncertainties using the bounds of the uncertainties. Zhang Y. (2009) deals with the specific nonlinear uncertainties whose bounds are related to the system matrix.

In this paper, considering the characteristics of the pantograph-catenary system, the dynamic model with matched and unmatched uncertainties and disturbances is established with actuator faults (loss of effectiveness faults,

bias faults). Then a class of adaptive robust fault-tolerant compensation controller for the pantograph catenary fault system are proposed. It assumes that the bounds on the bias faults, and the matched uncertainties exist and are unknown. Then the adaptive laws are proposed to estimate these unknown bounds and it is capable of compensating faults, disturbances and uncertainties automatically. Finally, it is shown that the proposed scheme ensures the solution of the resulting adaptive closed-loop system being uniformly bounded. Without the unmatched disturbances, the states asymptotically converge to zero, that is, the tracking error asymptotically converges to zero.

The remain parts of this paper are organized as follows: In Section 2, the system model is established and preliminary formulation are presented. In Section 3, adaptive and robust fault-tolerant controllers are designed. Simulation results based on pantograph-catenary are shown to verify the effective of the designed controller in Section 4. Finally, some conclusions end this paper in Section 5.

2. SYSTEM MODELING AND PROBLEMS FORMULATION

2.1 Pantograph-Catenary Dynamic Modeling

In this section, a modified multi-body mathematical models for the pantograph-catenary is introduced. The pantograph model used for the initial control system designing is the two degree of freedom model shown in Figure 1. In particular, a linear system can approximate the pantograph dynamics in a suitable vicinity of the working configuration, and this simplified model of the over head suspended system is characterized by lumped time-varying parameters that have been shown to be sufficiently accurate for control analysis and design purposes Balestrino, A. (2000). The equivalent mechanical parameters of the catenary
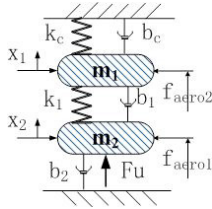

Fig. 1. Linear mass-spring-damper model

present a periodic behavior along each span Balestrino, A. (2000). We consider the parameters as follows:

$$
\begin{aligned}
b_c\left(t\right) &= b_{c0} + b_{c1}\sin\left(\frac{2\pi}{L}vt\right), \\
k_c\left(t\right) &= k_{c0} + k_{c1}\sin\left(\frac{2\pi}{L}vt\right).
\end{aligned} \quad (1)
$$

where v is the velocity of vehicle, L is the single span length, k_{c0} is the average stiffness of contact wire in a signal span, and b_{c0} is the average damp of contact wire.

The dynamic equations governing this system (movement of the pantograph) are described by

$$
\begin{aligned}
b_c\left(t\right)\dot{x}_1 &+ k_c\left(t\right)x_1 + m_1\ddot{x}_1 + b_1\left(\dot{x}_1 - \dot{x}_2\right) \\
&+ k_1\left(x_1 - x_2\right) = f_{aero2}, \\
m_2\ddot{x}_2 + b_2\dot{x}_2 &+ k_1\left(x_2 - x_1\right) + b_1\left(\dot{x}_2 - \dot{x}_1\right) \\
&= F_u + f_{aero1}.
\end{aligned} \quad (2)
$$

where m_1 and m_2 are the equivalent mass of pantograph head and frame, k_1 and d_1 are the stiffness and damp of pantograph head, d_2 is the equivalent damp between pantograph and vehicle, k_c and d_c are the equivalent stiffness and damp of contact wire at the contact point. The force f_{aero1} and f_{aero2} represent the aerodynamic force, which will be simulated by Gaussian-white-noise, and F_u refers to the static uplift force, which is produced by air pressure or spring loading. The parameters in the above will be given later.

The contact force between the pantograph and the contact wire has the following expression

$$
F_c = \max\left\{\, k_1\left(x_2 - x_1\right) + b_1\left(\dot{x}_2 - \dot{x}_1\right),\, 0\,\right\}. \quad (3)
$$

Therefore, letting $x = [\dot{x}_1,\ \dot{x}_2,\ x_1,\ x_2]$, $u = F_u$, $\omega_1 = f_{aero1}$, $\omega_2 = f_{aero2}$ and $y = F_c$, it follows that

$$
\begin{aligned}
\dot{x} &= Ax + \Delta g\left(x,t\right) + B_1\left(u + \omega_1\right) + B_2\omega_2, \\
y &= Cx,
\end{aligned} \quad (4)
$$

where

$$
\Delta g\left(x,t\right) = \Delta A_1\left(t\right)x + \Delta A_2(t)x, \quad C = \left[\,-b_1\ b_1\ -k_1\ k_1\,\right],
$$

$$
A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{m_2} \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} \frac{1}{m_1} \\ 0 \\ 0 \\ 0 \end{bmatrix},
$$

$$
\Delta A_1\left(t\right) = \begin{bmatrix} \Delta_1+\delta_1 & \delta_2 & \Delta_2+\delta_3 & \delta_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Delta A_2\left(t\right) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \delta_5 & \delta_6 & \delta_7 & \delta_8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

with $u = F_u$, $\omega_1 = f_{aero1}$, $\omega_2 = f_{aero2}$,

$$
a_{11} = -\frac{b_1+b_{c0}}{m_1}, a_{12} = \frac{b_1}{m_1}, a_{13} = -\frac{k_1+k_{c0}}{m_1}, a_{14} = \frac{k_1}{m_1},
$$

$$
a_{21} = \frac{b_1}{m_2}, a_{22} = -\frac{b_1+b_2}{m_2}a_{23} = \frac{k_1}{m_3}, a_{24} = -\frac{k_1}{m_3},
$$

$$
\Delta_1 = -\frac{b_{c1}}{m_1}sin(\frac{2\pi}{L}vt), \ \Delta_2 = -\frac{k_{c1}}{m_1}sin(\frac{2\pi}{L}vt).
$$

where δ_i, $i = 1\cdots 8$ represent parametric uncertainties of k_1, b_1 and b_2 with assumed to be bounded.

2.2 Fault Modeling

To formulate the fault-tolerant control problem, the fault model must be established firstly. Here, we consider actuator faults simultaneously including loss of effectiveness faults and bias faults. For the pantograph and catenary system, the following uniform actuator fault model is exploited

$$
u^F\left(t\right) = \rho(t)u + f(t). \quad (5)
$$

where $\rho(t)$ is the efficiency factor matrix and $f(t)$ is the bias value of the actuator. Then the dynamics of (4) with actuator faults (5) is described by

$$
\begin{aligned}
\dot{x} &= Ax + \Delta g\left(x,t\right) + B_1\rho(t)u + B_1(f(t)+\omega_1) + B_2\omega_2, \\
y &= Cx.
\end{aligned} \quad (6)
$$

2.3 Assumptions

Considering the actuator fault pantograph system described by (6), the problem under consideration is to develop an adaptive and robust controller such that: under normal operation, the corresponding closed-loop system is stable, and the output $Sy(t)$ tracks the reference signal y_r without steady-state error, $\lim\limits_{t\to\infty} e\left(t\right) = 0$, $e\left(t\right) = y_r\left(t\right) - Sy\left(t\right)$, where $S = 1$ in pantograph-catenary system; in the

case of actuator faults, all the signals of the closed-loop system are uniformly bounded and that the required output $Sy(t)$ tracks the reference signal $y_r(t)$ without steady-state error. It's well known that the tracking error integral action of a controller can efficiently eliminate the steady-state tracking error for the command signal. In order to obtain the adaptive tracking controller with tracking error integral, the following augmented systems are proposed by combining (5) and (6).

$$\dot{\bar{x}} = \bar{A}\bar{x} + \Delta\bar{g}(\bar{x},t) + \bar{B}_1\rho(t)u + \bar{B}_1(f(t)+\omega_1) + \bar{B}_2\bar{\omega}_2, \quad (7)$$

where $\chi(t) = \int_0^t e(\tau)\,d\tau$, $\Delta\bar{g}(\bar{x},t) = \begin{bmatrix} 0 \\ \Delta g(x,t) \end{bmatrix} = \Delta\bar{A}_1(t)\bar{x} + \Delta\bar{A}_2(t)\bar{x}$, $\bar{x} = \begin{bmatrix} \chi \\ x \end{bmatrix}$, $\bar{\omega}_2 = \begin{bmatrix} y_r \\ \omega_2 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} 0 & -SC \\ 0 & A \end{bmatrix}$, $\bar{B}_1 = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}$, $\bar{B}_2 = \begin{bmatrix} I & 0 \\ 0 & B_2 \end{bmatrix}$, $\Delta\bar{A}_1(t) = \begin{bmatrix} 0 \\ \Delta A_1 \end{bmatrix}$ and $\Delta\bar{A}_2(t) = \begin{bmatrix} 0 \\ \Delta A_2 \end{bmatrix}$.

Remark 1. According to the structure of $\{\bar{A}_1, \bar{B}_1, \bar{B}_2\}$, $\{\Delta\bar{A}_1, \bar{\omega}_1\}$ and $\{\Delta\bar{A}_2, \bar{\omega}_2\}$ are the matched and unmatched disturbances, respectively. The terms $\Delta\bar{A}_1\bar{x}$ and $\bar{\omega}_2$ are both the unmatched uncertainties which will be dealt by robust optimal technique based on the structure relation with $\bar{B}_1$, respectively. ∇

Lemma 1. Fuzhen Zhang (2011), For matrix $\bar{B}_1 \in C_r^{m\times n}$, ($r > 0$ represents matrix rank), then there exists decomposition

$$\bar{B}_1 = QR, \quad (8)$$

where $Q \in R^{m\times r}$ and decomposition $Q^TQ = I$, where $R \in R^{r\times n}$ and $RR^T > 0$.

Assumption 1. The pair $\{A, B_1\}$ is completely controllable.

Assumption 2. The loss of effectiveness factor $\rho(t) \in (0,1)$ is unknown time-varying actuator efficiency factor satisfying

$$|\dot{\rho}(t)| \leq \rho_0\rho(t). \quad (9)$$

Assumption 3. The bias fault vector $f(t)$ is assumed to be bounded, i.e., there exists an unknown constant $\bar{f}$ such that $\|f(t)\| \leq \bar{f}$; ω_1 is also assumed to be bounded, i.e., there exist an unknown constants $\bar{\omega}_1$ such that $\|\omega_1\| \leq \bar{\omega}_1$.

Assumption 4. Qu, Z. (1992) The nonlinear uncertainties $\Delta\bar{g}(\bar{x},t)$ satisfies

$$\|\bar{x}^T P\Delta\bar{g}(\bar{x},t)\| \leq \alpha(\bar{x},t) \quad (10)$$

for all $(\bar{x},t) \in R^n \times R$. Furthermore, $\alpha(\bar{x},t)$ has the property that the function $\frac{\alpha(\bar{x},t)}{\|\bar{x}^T P\bar{B}_1\|}$ is continuous, uniformly bounded with respect to t, and is locally uniformly bounded with respect to $\bar{x}$.

Remark 2. The uncertain term $\Delta\bar{A}_1(t)\bar{x}$ does not satisfy Assumption 4, which can be expressed as

$$\Delta\bar{A}_1(t) = HF(t)E \quad (11)$$

with $F^TF \leq I$ and H, E are the constant matrices.

For another uncertainties $\Delta\bar{A}_2(t)\bar{x}$, it can be decomposed as

$$\Delta\bar{A}_2(t) = Q\Sigma(t) \quad (12)$$

with $\|\Sigma(t)\| \leq l^*$, where l^* is a positive constant and Q is satisfies $\bar{B}_1 = QR$. Therefore,

$$\frac{\|\bar{x}^T P\Delta\bar{A}_2(t)\|}{\|\bar{x}^T P\bar{B}_1\|} \leq \frac{\|\bar{x}^T PQ\|\|\Sigma(t)\|}{\lambda_{\min}(RR^T)\|\bar{x}^T PQ\|} \leq \frac{l^*}{\lambda_{\min}}, \quad (13)$$

i.e., there is a positive constant ζ satisfying

$$\|\bar{x}^T P\Delta\bar{A}_2(t)\| \leq \zeta\|\bar{x}^T P\bar{B}_1\|, \quad (14)$$

where $\zeta = \frac{l^*}{\lambda_{\min}(RR^T)}$.

So $\Delta\bar{A}_2(t)\bar{x}$ satisfies the Assumption 4. ∇

Assumption 5. For any actuator failure mode, the equation

$$rank\{\bar{B}_1\rho(t)\} = rank\{\bar{B}_1\}. \quad (15)$$

holds.

Remark 3. In the pantograph-catenary system, the states are completely available and controllable. Assumption 5 introduces a condition of actuator redundancy of the system, which is necessary for completely compensating the loss of control fault. But in the pantograph-catenary system, because of there is just one actuator, it can not tolerate the actuator loss of control case. ∇

Proposition 1. The matrix rank relation of (15) is holds if and only if there exist matrix $K^*(t)$ satisfying

$$B\rho(t)K^*(t) = B. \quad (16)$$

Proof. The proof is omitted. □

2.4 Control Objective

The objective of pantograph controller is to minimize fluctuation in contact force around the desired value. Therefore, the main control objective is to construct an adaptive and robust memoryless state feedback fault-tolerant tracking controller for the actuator faulty system in the presence of disturbances and uncertainties to guarantee that the remaining actuators can still ensure that all the signals are bounded and the tracking error between the plant output $Sy(t)$ and the reference output signal y_r goes to zero asymptotically when $\omega_2 = 0$, i.e.,

$$\lim_{t\to\infty} e(t) = 0; \quad (17)$$

when $\omega_2 \neq 0$, the tracking error $e(t)$ satisfies a defined performance index.

3. ADAPTIVE AND ROBUST FAULT-TOLERANT TRACKING CONTROLLER DESIGN

In this section, to achieve the desired control objectives, the control structure is constructed as:

$$u = u_c + u_n \quad (18)$$

where u_c is designed to deal with the loss of effectiveness fault and to guarantee the performance index, simultaneously; u_n is designed to compensate the matched disturbances and part of unmatched uncertainties.

For the "healthy" system, the controller u_c is designed as $u_c = K_1\bar{x}$, where $u_2 = K_2(t)$, and $u_3 = K_3(t)$.

Applying u_c to the "healthy" system, it follows that

$$\dot{\bar{x}} = \bar{A}\bar{x} + \Delta\bar{A}_1(t)\bar{x} + \bar{B}_1 u + \bar{B}_2\bar{\omega}_2. \quad (19)$$

Then it is straightforward to obtain following closed-loop system

$$\dot{\bar{x}} = (\bar{A} + \Delta\bar{A}_1(t) + \bar{B}_1 K_1)\bar{x} + \bar{B}_2\bar{\omega}_2. \quad (20)$$

The objective now is to design the gain K_1 to guarantee the "healthy" system to be stable and satisfy the optimal

performance index. The following lemma 2 is used to calculate the gain K_1 and Layapunov matrix P.

Lemma 2. Ye D. (2006) Considering the closed-loop augmented normal system (20) and the performance index

$$J_t = \int_0^t \left[\chi^T Q_1 \chi + x^T Q_2 x + u^T R u \right] dt \qquad (21)$$

where Q_1 and Q_2 are positive semi-definite matrices, and R is a positive definite matrix. For the given positive constants γ and τ, if there exist symmetric matrices X, T and a matrix W satisfying the following linear matrix inequalities,

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\tau^{-1} I & 0 \\ * & * & * & * & * & -\tau I \end{bmatrix} < 0, \qquad (22)$$

$$\begin{bmatrix} T & I \\ I & X \end{bmatrix} > 0, \qquad (23)$$

where

$\Xi_{11}=He(\bar{A}X + \bar{B}_1 W)$, $\Xi_{12}=\bar{B}_2$, $\Xi_{13}=W^T R^{1/2}$, $\Xi_{14}=XQ^{1/2}$, $\Xi_{15}=XH$, $\Xi_{16}=XE^T$, $Q = diag[Q_1,\ Q_2]$,

then the following controller can stabilizes the closed-loop augmented normal system with $P = X^{-1}$ and $K_1 = WX^{-1}$.

Furthermore, an upper bound of the performance index (21) can be obtained

$$J_t \leq \gamma \int_0^t \bar{\omega}_2^T \bar{\omega}_2 dt + \bar{x}^T (0) T \bar{x} (0). \qquad (24)$$

The proposition 1 shows that there exists $K^*(t)$ guarantees that $\bar{B}_1 \rho(t) K^*(t) = \bar{B}_1$, i.e., there exist matrix $W_1 = K^*(t)W$ satisfying with (22) which W is replaced by W_1.

Because $\rho(t)$ is unknown for the unknown fault mode, we have to estimate the gain K_1. Then u_c becomes

$$u_c = \hat{K}_1(t)\bar{x}. \qquad (25)$$

Applying (18) and (25) to (7), the following closed-loop system is formed

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}_1 \rho(t)\hat{K}_1\bar{x} + \bar{B}_1 \rho(t) (K_2(t) + K_3(t)) + \\ + \Delta \bar{g}(x,t) + \bar{B}_1 f(t) + \bar{B}_1 \omega_1 + \bar{B}_2 \bar{\omega}_2. \qquad (26)$$

The following lemmas are needed for controller designing.

Lemma 3. Hao, L. Y. (2013), For the diagonal matrix $\rho(t)$ in (15), there exists a positive constant μ satisfying the following inequality

$$\bar{x}^T P \bar{B}_1 \rho(t) \bar{B}_1^T P \bar{x} \geq \mu \left\| \bar{x}^T P \bar{B}_1 \right\|^2. \qquad (27)$$

Lemma 4. For any matrix X and Y with appropriate dimensions, the following inequality holds

$$X^T Y + Y^T X \leq \alpha X^T X + \alpha^{-1} Y^T Y, \\ -X^T Y - Y^T X \leq \alpha X^T X + \alpha^{-1} Y^T Y. \qquad (28)$$

Denote $\hat{K} = \tilde{K} + K$ with $\{\hat{K},\ \tilde{K},\ K\} \in R^{m \times n}$, then based on the above lemma, the following inequality holds:

$$tr(\tilde{K}^T \Gamma \tilde{K}) - tr(K^T \Gamma K) \leq 2tr(\hat{K}^T \Gamma \tilde{K}). \qquad (29)$$

Based on Assumption 3 and Lemma 1, there exist positive constants k_4 and k_5 satisfying

$$\|f(t) + \omega_1(t)\| \leq \|f(t)\| + \|\omega_1(t)\| \leq \mu k_4, \\ \left\| \bar{x}^T P \Delta \bar{A}_2(t) \right\| \leq \mu k_5 \left\| \bar{x}^T P \bar{B}_1 \right\|. \qquad (30)$$

Here, it is worth pointing out that the constants μ, k_4 and k_5 are unknown which should be estimated.

Now consider the adaptive law of $\hat{K}_1$ as

$$\dot{\hat{K}}_1 = -\Gamma \bar{x} \bar{x}^T P \bar{B}_1 - \rho_0 \hat{K}_1 \qquad (31)$$

where Γ is a positive constant matrix. The control function $K_2(t)$ is designed as

$$K_2(t) = -\frac{\bar{B}_1^T P \bar{x} \hat{k}_4^2}{\left\| \bar{B}_1^T P \bar{x} \right\| \hat{k}_4 + \varepsilon e^{-\beta t}} \qquad (32)$$

where $\hat{k}_4$ is the estimation function of k_4, ε and β are adjustable positive parameters. The adaptive law of $\hat{k}_4$ is proposed as

$$\dot{\hat{k}}_4 = \gamma_1 \left\| \bar{B}_1^T P \bar{x} \right\| - \frac{1}{2}\gamma_1 \varepsilon e^{-\beta t} \hat{k}_4. \qquad (33)$$

The control function of $K_3(t)$ is designed as

$$K_3(t) = -\frac{1}{2}\eta \hat{k}_5 \bar{B}_1^T P \bar{x} \qquad (34)$$

in which $\hat{k}_5$ is updated by the adaptive law

$$\dot{\hat{k}}_5 = -\frac{1}{2}\gamma_2 \varepsilon e^{-\beta t} \hat{k}_5 + \frac{1}{2}\gamma_2 \eta \left\| \bar{B}_1^T P \bar{x} \right\|^2. \qquad (35)$$

Remark 4. From the expression of (33) and (35), it is obvious $\hat{k}_4 > 0$ and $\hat{k}_5 > 0$ can be ensured by choosing the appropriate initial value of $\hat{k}_4(0)$ and $\hat{k}_5(0)$. $\qquad \nabla$

Let $\tilde{K}_1 = \hat{K}_1 - K_1$, $\tilde{k}_4 = \hat{k}_4 - k_4$ and $\tilde{k}_5 = \hat{k}_5 - k_5$. Then it is ready to present the following theorem.

Theorem 1. Considering the closed-loop system formed by system (26) and the controllers (32) and (34) with adaptive laws (31), (33) and (35), in which P is provided by LMIs (22) and (23). Then under Assumption 1-5 the signal $\{\bar{x},\ \tilde{K}_1,\ \tilde{k}_4,\ \tilde{k}_5\}$ of the closed-system are uniformly bounded and the states $\bar{x}$ goes to zero asymptotically with $\bar{\omega}_2 = 0$, i.e.,

$$\lim_{t \to \infty} \bar{x} = 0. \qquad (36)$$

When $\bar{\omega}_2 \neq 0$,

$$J_t \leq \gamma \int_0^t \bar{\omega}_2^T \bar{\omega}_2 dt + \bar{x}^T (0) T \bar{x}(0). \qquad (37)$$

Proof. The proof is composed of two steps:
step 1. Choose a Lyapunov function for the loss of effectiveness faulty system as follows:

$$V_1 = \bar{x}^T P \bar{x} + tr(\rho(t)\tilde{K}_1^T \Gamma^{-1} \tilde{K}_1), \qquad (38)$$

The time derivative of V_1 with $u = u_c = \hat{K}_1 \bar{x}$ is given by

$$\dot{V}_1 \leq \bar{x}^T He(P(\bar{A} + \Delta \bar{A}_1 + \bar{B}_1 \rho(t) K_1)\bar{x} + 2\bar{x}^T P \bar{B} \rho(t)\tilde{K}_1 \bar{x} \\ + 2\bar{x}^T P \bar{B}_2 \bar{\omega}_2(t) + \rho_0 tr(\rho(t)\tilde{K}_1^T \Gamma^{-1} \tilde{K}_1) \\ + 2tr(\rho(t)\tilde{K}_1^T \Gamma^{-1} \dot{\tilde{K}}_1). \qquad (39)$$

Denote $-Q = He(P(\bar{A} + \Delta \bar{A}_1 + \bar{B}_1 \rho(t) K_1))$. For large enough Γ, based on (29), it has

$$\dot{V}_1 \leq -\bar{x}^T Q \bar{x} + 2\bar{x}^T P \bar{B} \rho(t) \tilde{K}_1 \bar{x} + 2\rho_0 tr(\rho(t) \tilde{K}_1^T \Gamma^{-1} \tilde{K}_1)$$
$$- \rho_0 tr(\rho(t) K_1^T \Gamma^{-1} K_1) + 2tr(\rho(t) \tilde{K}_1^T \Gamma^{-1} \dot{\tilde{K}}_1)$$
$$+ 2\bar{x}^T P \bar{B}_2 \bar{\omega}_2(t)$$
$$< -\bar{x}^T Q \bar{x} + 2\bar{x}^T P \bar{B}_2 \bar{\omega}_2(t). \tag{40}$$

step 2. Choose a Lyapunov function for system (26) as the following form:

$$V_2 = V_1 + \mu \gamma_1^{-1} \tilde{k}_4^2 + \mu \gamma_2^{-1} \tilde{k}_5^2. \tag{41}$$

The time derivative of V_2 with $u = u_c + u_n$ is

$$\dot{V}_2 \leq \dot{V}_1 + 2\bar{x}^T P \bar{B}_1 \rho(t) K_2(t) + 2\bar{x}^T P \bar{B}_1 \rho(t) K_3(t)$$
$$+ 2\bar{x}^T P \bar{B}_1 \|(\omega_1 + f(t))\| + 2 \|\bar{x}^T P Q\| \|\Sigma(t)\| \|\bar{x}\|$$
$$+ 2\mu \gamma_1^{-1} \tilde{k}_4 \dot{\tilde{k}}_4 + 2\mu \gamma_2^{-1} \tilde{k}_5 \dot{\tilde{k}}_5, \tag{42}$$

Based on Lemma 3, the control function (32) and the adaptive law (33),

$$2\bar{x}^T P \bar{B}_1 \rho(t) K_2(t) + 2\bar{x}^T P \bar{B}_1 \|\omega_1(t) + f(t)\| + 2\mu \gamma_1^{-1} \tilde{k}_4 \dot{\tilde{k}}_4$$
$$\leq -\frac{2\mu \|\bar{x}^T P \bar{B}_1\| \hat{k}_4^2}{\|\bar{x}^T P \bar{B}_1\| \hat{k}_4 + \varepsilon e^{-\beta t}} + 2\mu \|\bar{x}^T P \bar{B}_1\| k_4$$
$$+ 2\mu \gamma_1^{-1} \tilde{k}_4 \dot{\hat{k}}_4$$
$$\leq 2\mu \varepsilon e^{-\beta t} + \frac{1}{4} \mu \varepsilon e^{-\beta t} k_4^2. \tag{43}$$

Based on Young's inequality $2ab \leq \frac{1}{c}a^2 + cb^2, \ \forall a, \ b, \ c > 0$, it has

$$2 \|\bar{x}^T P Q\| \|\Sigma(t)\| \|\bar{x}\| \leq l^{*2} \eta \|\bar{x}^T P Q\|^2 + \frac{1}{\eta} \|\bar{x}\|^2$$
$$\leq k_5 \|\bar{x}^T P \bar{B}_1\|^2 + \frac{1}{\eta} \|\bar{x}\|^2 \tag{44}$$

where $k_5 = l^{*2} \eta \zeta / \mu$.

Based on Lemma 3, from the control gain (34) and the adaptive law (35), we have

$$2\bar{x}^T P \bar{B}_1 K_3(t) + \mu \|\bar{x}^T P \bar{B}_1\|^2 k_5 + 2\mu \gamma_2^{-1} \tilde{k}_5 \dot{\tilde{k}}_5 \leq \frac{1}{4} \mu \varepsilon e^{-\beta t} k_5^2. \tag{45}$$

In light of the inequality of the form $0 \leq \frac{ab}{a+b} \leq a, \ \forall a, \ b > 0$ and $-a^2 - ab < \frac{b^2}{4}$, it is concluded that

$$\dot{V}_2 \leq \dot{V}_1 + \frac{1}{\eta} \|\bar{x}\|^2 + \varepsilon e^{-\beta t} \kappa \tag{46}$$

where $\kappa = \mu \left(2 + \frac{1}{4}\left(k_4^2 + k_5^2\right)\right)$.

Denote $\sigma(t) = \int_0^t \varepsilon e^{-\beta \tau} dt = \frac{\varepsilon}{\beta}\left(1 - e^{-\beta t}\right) < \bar{\sigma}$, then $\sigma(\infty) = \frac{\varepsilon}{\beta}, \ \bar{\sigma} = \frac{\varepsilon}{\beta}$.

When $\bar{\omega}_2 = 0$, integrating both sides of (46) from 0 to t,

$$V_2(t) + \int_0^t \lambda_{min}(Q + \frac{1}{\eta}) \|\bar{x}\|^2 d\tau \leq V_2(0) + \kappa \bar{\sigma}, \tag{47}$$

which means that $\bar{x} \in L_2$ and $V_2(t) \in L_\infty$, and thus $\bar{x} \in L_\infty$ and $\{\hat{k}_4, \tilde{k}_4, \hat{k}_5, \tilde{k}_5\} \in L_\infty$. Note that the right hand side of (47) does not involve with time, $\{\bar{x}, \tilde{k}_4, \tilde{k}_5\}$ are uniform bounded.

Furthermore, (46) implies that

$$\lim_{t \to \infty} \int_0^t \lambda_{min}(Q + \frac{1}{\eta}) \|\bar{x}\|^2 d\tau \leq V_2(\infty) + \kappa \sigma(\infty). \tag{48}$$

Since $\{\bar{x}, \tilde{k}_4, \tilde{k}_5\}$ are uniform bounded, it follows that from (31), (33) and (35), $\{\bar{x}, \tilde{k}_4, \tilde{k}_5\}$ is continuous, which implies that it is uniformly continuous. Therefore $\lambda_{min}(Q + \frac{1}{\eta}) \|\bar{x}\|^2$ is also uniformly continuous. Applying the Barbalat lemma Ioannou, P. A. (2012) yields that $\lim_{t \to \infty} \int_0^t \lambda_{min}(Q + \frac{1}{\eta}) \|\bar{x}\|^2 d\tau = 0$, i.e.

$$\lim_{t \to \infty} \bar{x} = 0, \quad \lim_{t \to \infty} e = 0. \tag{49}$$

when $\bar{\omega}_2(t) \neq 0$, based on Ye D. (2006)

$$J_t \leq \gamma \int_0^t \bar{\omega}_2^T \bar{\omega}_2 dt + \bar{x}^T(0) T \bar{x}(0). \tag{50}$$

Hence, the results follow. $\qquad \square$

Remark 5. Let $y_0 = \chi$, then $\ddot{y}_0 = \ddot{\chi} = \dot{e} = \dot{y}_r - C\dot{x}$. If the y_r is the constant given reference signal, it can get that $\dot{e} = -C\dot{x}$. Based on the system structure (7), it's easy to see that y_r has no influence on x in the open system. Therefore, the designed controller can guarantee that $\dot{x} \in L_\infty$, $\dot{e} \in L_\infty$ when y_r is constant. Because of $e = y_r - Cx$, $e \in L_2 \cap L_\infty$, it can be concluded that $\lim_{t \to \infty} e = 0$.

4. SIMULATION

To show the effective of the presented adaptive approach, the parameters of the controller are given as follow

$$x(0) = [0, 0, 0, 0, 1], \ \hat{K}_1(0) = [3, 2, 0, 1, 0],$$
$$k_4(0) = 1, \ k_5(0) = 2,$$
$$\Gamma = 5I, \ \gamma_1 = \gamma_2 = 1000,$$
$$\eta = 500, \ \rho_0 = 3, \ \alpha = 2, \ \beta = 0.01.$$

The simulated speed is given by $V = 360 \ km/h$ and the aerodynamic force is simulated by gaussian-white-noise with power 100. The actuator fault mode is assumed to be as follows:

$$\rho(t) = \begin{cases} 1 & 0s < t < 10s, \\ 1 - 4(t-10) & 10s < t < 10.2s, \\ 0.2 & 10.2s < t. \end{cases}$$
$$f(t) = 20 \sin(3t)$$

The parameters for pantograph-catenary are summarized in table 4 as Xiaodong, Z. (2011), with component parameters uncertainty values in $\Delta A_1(t)$ and $\Delta A_2(t)$ being $\delta_i = 1$ for $i = 1 \cdots 8$, $k_{c1} = 7000$, and $b_{c1} = 240$.

Table 1. Pantograph-Catenary Parameters

Parameter	Notation	Value
Head	m_1	$12kg$
parameter	b_1	$70N \cdot s/m$
	k_1	$4740N/m$
Frame	m_2	$13kg$
parameter	b_2	$70N \cdot s/m$
Catenary	$k_{c1} = k_{c0}$	$7000N/s$
parameter	$b_{c1} = b_{c0}$	$240N \cdot s/m$

Figure 2,3 shows the evolution of vertical positions and their deviations during the period, resulted by actuator faults, the time-varying catenary's parameters and the aerodynamic force. Note that x_1 is the point of contact between the contact wire of the catenary and the pantograph and x_2 is the states represents variation in the

vertical positions of the mass m_2. These states reflect the vertical displacements of the pantograph-catenary.
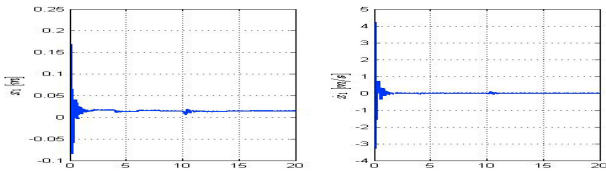

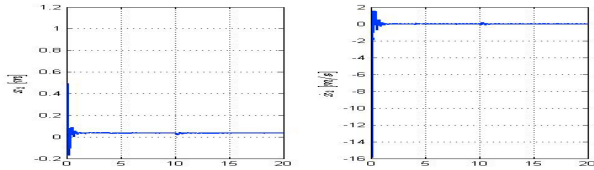Fig. 2. Simulation of displacement and velocities of the masses of the model 1

Fig. 3. Simulation of displacement and velocities of the masses of the model 2

Figure 4 shows that the contact force between pantograph and catenary tracking the given value asymptotically in the presence of the actuator fault, time-varying parameter uncertainties and aerodynamic influence.
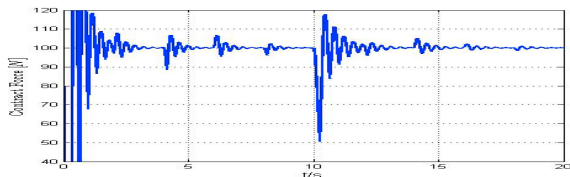

Fig. 4. Contact force for pantograph-catenary

The evolution of x_1 and x_2 indicates that the contact distance between the pantograph and the catenary is stable in a position. The observed oscillations show the vertical movements of the pantograph. The fault of violently loss of effectiveness on the air pressure results in fluctuation of the positions x_1 and x_2, but under the designed controller, the contact force back to the desired value $100N$ asymptotically.

5. CONCLUSION

This paper establishes the dynamic model of the pantograph-catenary and converts it into a linear-time invariant system with uncertainties. Considering the aerodynamic force, actuator time-varying loss of effectiveness faults and bias faults, an adaptive and robust controller is proposed to guarantee the system asymptotically tracking a pre-specified value. Simulation results show that the designed controllers are robust and effective.

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