Value at Risk models with long memory features and their economic performance

Evangelia Mitrodima and Jaideep Oberoi*
School of Mathematics, Statistics and Actuarial Science
and Kent Centre for Finance
University of Kent

This version: August 2015

Abstract

We study alternative dynamics for Value at Risk (VaR) that incorporate a slow moving component and information on recent aggregate returns in established quantile (auto) regression models. These models are compared on their economic performance, and also on metrics of first-order importance such as violation ratios. By better economic performance, we mean that changes in the VaR forecasts should have a lower variance to reduce transaction costs and should lead to lower exceedance sizes without raising the average level of the VaR. We find that, in combination with a targeted estimation strategy, our proposed models lead to improved performance in both statistical and economic terms.
Keywords: Long memory time series; Quantile forecasts; Conditional loss

1 Introduction

Value at Risk (VaR) is now a ubiquitous indicator of financial market risk. Its formal adoption in regulatory standards has ensured its integration into the everyday operation of financial institutions

*Correspondence to: Jaideep Oberoi, University of Kent, SMSAS Cornwallis Buildings, Canterbury CT2 7NF, United Kingdom. *Email*: j.s.oberoi@kent.ac.uk. *Tel*: +44 1227 82 3865. We are grateful for detailed comments from Jim Griffin, and for helpful discussions with Peter Christoffersen, Christian Dorion, Hitesh Doshi, John Galbraith, Ekaterini Panopoulou, Gareth Peters, David Veredas, and seminar participants at the Computational and Financial Econometrics conference (Dec 2013). We are grateful to the Engineering and Physical Sciences Research Council for financial support of Evangelia Mitrodima's PhD.

and regulators. A large literature has developed to find better ways of estimating VaR. The goal of this paper is to contribute to improvements in the time series forecasting of VaR by taking into account some well known features of risk dynamics in stock returns, and to do so while targeting the economic implications of the VaR model. The economic implications we refer to are the transaction costs from variation in VaR, and the extent that shocks exceed anticipated risk thresholds.

While we pay due attention to the statistical performance of the models, we are motivated by the potential for achieving relatively smoother VaR series without raising the overall level of VaR. Given the role of VaR in setting practical limits on trading positions, frequent changes could lead to unnecessary transaction costs or to induce conservatism leading to misallocation of capital. We hypothesize that gradual adjustments to VaR that capture low-frequency movements would help make the series smoother. At the same time, we anticipate that this approach will reduce the need for costly recapitalization of investment strategies.

It is widely recognized that clustering and asymmetry in return volatility imply that VaR increases when the value of investments falls. This in turn implies that a trader would be forced to reduce her position at an unprofitable juncture, effectively selling low and buying high, in order to bring capital in line with risk controls. Intuitively, this problem calls for adjustments to portfolio positions in a timely manner. If the VaR series includes small gradual adjustments that come closer to anticipating the severity of anticipated shocks, there could be a translated benefit in portfolio performance. By allowing for a gradually time-varying tendency for VaR, we hope to achieve this outcome.

There are several approaches to estimating VaR. As the VaR is a prespecified quantile of the return distribution, it can in principle be estimated from any time series model of returns where the distribution of the future return is known conditionally. Alternatively, one could use the history of past returns to non-parametrically estimate the relevant percentile. The two approaches could also be combined by using a model, e.g. from the ARCH (Engle, 1982) family, to obtain a filtered distribution. Christoffersen (2009) provides an overview of the types of methodologies available for VaR forecasting, and also a detailed analysis explaining the range of possibilities under the Filtered Historical Simulation approach. Under this system, it is possible to use a parametric filtering method to estimate and forecast volatility, and then use this in a simulated environment to determine the relevant quantile forecast of the distribution. In this paper, we focus on the Conditional Autoregressive Value at Risk (CAViaR) model proposed by Engle and Manganelli (2004). CAViaR is a semiparametric approach that involves modeling the desired quantile directly in a non-linear autoregressive framework. We examine ways to modify CAViaR specifications to incorporate insights from the volatility modeling literature.

We have two main reasons for focussing on CAViaR models. Firstly, though several existing studies have compared various types of VaR models, the potential of CAViaR models in their various specifications has not been fully explored. This is particularly true for CAViaR models with longer memory, although the volatility literature contains several analogous models. In addition, we hope to make an additional contribution by examining some of the empirical properties of CAViaR models, such as the types of issues that arise in their estimation.

We systematically evaluate three approaches proposed in the literature to improve the dynamics of the original CAViaR model specifications. First, we consider the role of the 'news impact function' - the term coined by Engle and Ng (1993) to represent the impact of the 'shock' in today's return on the next day's variance. Drawing parallels to the volatility literature, we consider whether the 'news' is best represented by today's return, or a conditionally scaled version of it. CAViaR models traditionally incorporate the size of the return as an input in the news impact function. We explore whether it is more effective to consider the news element to be the scaled return (scaled by the filtered quantile), so that large shocks are more meaningful on days when the scale of the return was expected to be low.

Previous authors have extended CAViaR models by accounting for autocorrelation in returns, see Kuester et al. (2006), and more generally by including regressors other than lags of VaR in their models. This approach has been found to work well for out-of-sample prediction: we hypothesize that this performance is partly explained by variation in the average return across sample periods. In view of this, the second new approach we examine involves incorporating past daily, weekly and monthly returns as regressors in the quantile specification, à la Corsi (2009), to capture possible horizon effects in a simple manner.

The notion that there is a time-varying serial correlation in returns could also be captured by regime-based or other nonlinear models. Gerlach et al. (2011) propose a threshold-based model whereby the VaR process entirely shifts based on whether the previous day's return was above or below an assumed threshold (usually zero). This approach also performs well in forecasting comparisons. We ask if we could achieve similar results while reducing the day to day variation in the VaR brought about by the threshold structure. To attempt this, we include in our model a slowly moving component of the VaR interpreted as generating a time-varying mean. By allowing for more gradual adjustments in the mean of the process, our aim is to reduce its overall variance relative to a model with discontinuities. Thus, the final element of our proposals is a two component model for VaR in the spirit of Engle and Lee (1999) or Ding and Granger (1996).

VaR models are evaluated on their performance in different ways. These include the out-of-sample hit ratio, or the proportion of actual exceedances (when the loss exceeds the VaR) relative to the expected number of exceedances. Danielsson (2002) showed that most models were not entirely reliable when examined for their hit ratios over different estimation windows, offering a motivation to seek improvements in VaR models. In addition, Kuester et al. (2006) found that most models tend to have more hits than expected out-of-sample, often failing the basic unconditional coverage test. More recently, Gerlach et al. (2011) have shown that CAViaR models perform well compared to other approaches in estimating VaR. They compare a wide range of models based on hit ratios, the unconditional coverage (UC) test from Kupiec (1995), the conditional coverage (CC) test of Christoffersen (1998), and the Dynamic Quantile (DQ) test of Engle and Manganelli (2004). The latter two tests are based on the idea that hits or exceedances should be independent of each other. In terms of comparing statistical performance, we report all the above criteria, along with the duration test of Christoffersen and Pelletier (2004) which Berkowitz et al. (2011) find to perform well in some cases.

Our other criteria for comparison are the mean and variance of the VaR time series, the realized conditional VaR (cVaR), and the mean excess loss. The realized cVar is the average loss conditional on exceedances. In isolation, this metric may favour models that have hits more often on low loss days, irrespective of the number of hits or the total realized loss. The mean excess loss is defined as the average of the amount by which the VaR was exceeded when hits occurred. The larger this number, the greater the amount of capital (in excess of the allocated loss bearing capacity) that needs to be replaced just as the portfolio value has fallen. This is similar to alternative economic criteria for VaR model selection proposed by Lopez (1997) and Sarma et al. (2003). Basak and Shapiro (2001) argue that VaR-based portfolio management induces systemic risk due to the tendency for liquidity feedback. This is because sales are induced by risk-adjustment requirements precisely when prices have fallen. A model that leads to lower conditional excess losses would plausibly help alleviate this risk. While it is well understood that the criteria above cannot be used in isolation for model selection, it is our goal to identify the model specifications that help meet more of them simultaneously.

Finally, another contribution of our paper is to show how an alternative estimation strategy can make a difference to the results. We simply select starting values for the parameters in an analogous way to the variance targeting approach used in GARCH model estimation (Christoffersen, 2003). This strategy leads to better estimates for comparable datasets in most cases, as we discuss later in the paper.

In the next section, we set out the various specifications we examine, including the component model. We then use the following sections to, in turn, explain our estimation strategy, describe the empirical exercise and finally provide conclusions.

2 The Models

We discuss below the various types of CAViaR models, extending their specification to meet our objectives of smoother and economically more desirable VaR forecasts. Throughout this paper, our quantity of interest is the quantile of returns, with the understanding that the VaR is typically reported as the additive inverse of the relevant return quantile.

2.1 Original four CAViaR models

For a return series $\{r_t\}$, Engle and Manganelli (2004) introduce CAViaR models in a generic format as

$$q_{\theta,t}(\beta) = \beta_{\theta,0} + \sum_{i=1}^{m} \beta_{\theta,i} q_{\theta,t-i}(\beta) + \sum_{j=1}^{n} \beta_{\theta,j} l\left(\mathbf{x}_{t-j}\right), \tag{1}$$

where the θ-quantile is modeled as a combination of autoregressive terms and a function of a finite number of lagged values of observables contained in $\mathbf{x}$ (including the history of returns $\{r_s\}_{s=0}^{t}$). It is clear that the parameter vector β is allowed to depend on the quantile being modeled. For the rest of this paper, we drop the θ subscript for ease of reading, though it is understood that any set of βs here refers exclusively to a specific quantile level θ. Also, as this is quite clearly understood in the literature, we will no longer point out the dependence of the series of quantile estimates on the parameters β. Hence, $q_{\theta,t}(\beta)$ will simply be referred to as q_t.

Engle and Manganelli (2004) estimate and test four specifications with well known counterparts in the volatility or variance-modeling literature. Although we list all four specifications, the rest of the paper will focus on only three of these, as is explained below. In each of the four cases, the number of lags of past information in Eqn. 1 is set at 1. These specifications are, for a given series of returns r_t,

- Adaptive CAViaR:

  $$q_t = q_{t-1} + \beta\left[\{1 + \exp\left(f\left(r_{t-1} - q_{t-1}\right)\right)\}^{-1} - \theta\right]$$

  This model has the property that it automatically raises the VaR after a hit or exceedance and gradually reduces it until the next hit - a simple way to address volatility clustering. It is found to perform much worse than other models, so we will not consider it further in this study. However, see McAleer et al. (2010) for how such a scheme appropriately modified may have appeal from a regulatory reporting perspective.

- Symmetric Absolute Value CAViaR(SAV):

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 |r_{t-1}| \tag{2}$$

- Asymmetric Slope CAViaR (AS):

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 r_{t-1}^{+} + \beta_3 r_{t-1}^{-} \tag{3}$$

  where the notation employed is: $x^+ = \max(x, 0)$ and $x^- = -\min(x, 0)$.

- Indirect GARCH(1,1) CAViaR, henceforth IG:

$$q_t = -\left(\beta_0 + \beta_1 q_{t-1}^2 + \beta_2 r_{t-1}^2\right)^{\frac{1}{2}} \tag{4}$$

While IG models are useful for modeling extreme quantiles like those required for VaR estimation, they do have the disadvantage that the quantiles need to be signed, so they cannot be used to model quantiles that could change signs over time.

2.2 Other CAViaR specifications

We now discuss various alternative specifications and extensions of the models in Equations 2 - 4.

2.2.1 The news impact function

The news impact function captures the response of the conditional quantile to today's information, and it may have several features such as non-linearity and asymmetry. For CAViaR models where $r_t = \varepsilon_t$ (i.e., the mean daily return is assumed to be zero and ε_t is a daily error term), the news impact is modeled as a function of the past return(s). However, one could also interpret the 'news' or the surprise element of the return as the size of the return relative to the conditional quantile based on the previous day's information. As q_{t-1} is known at time $t-2$, we could then think of scaling r_{t-1} by q_{t-1} to obtain this interpretation. In comparison to volatility models of the form $r_t = \sigma_t z_t$, this would be analogous to treating the standardized return z_t as news. Consider the case where the return generating process is conditionally iid (say normal) with mean zero. Then, the ratio r_t/q_t would represent on average a fixed proportion, based on θ, of a standard normal variable.

The original three models we consider can be adjusted with scaled news impact functions as follows:

- SAVscaled:

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 |r_{t-1}/q_{t-1}|. \tag{5}$$

- ASscaled:

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 (r_{t-1}/q_{t-1})^+ + \beta_3 (r_{t-1}/q_{t-1})^- , \tag{6}$$

  and

- IGscaled:

$$q_t = -\left(\beta_0 + \beta_1 q_{t-1}^2 + \beta_2 (r_{t-1}/q_{t-1})^2\right)^{\frac{1}{2}} . \tag{7}$$

The IG model may be further modified to display asymmetry in the news impact. Two of the most widely used GARCH news impact functions are Nonlinear Asymmetric (NA), due to Engle and Ng (1993), and GJR, following Glosten et al. (1993). The specifications for indirect GARCH CAViaR versions are:

- IG-NA:

$$q_t = -\left(\beta_0 + \beta_1 q_{t-1}^2 + \beta_2 (r_{t-1}/q_{t-1} - \beta_3)^2\right)^{\frac{1}{2}} \tag{8}$$

- IG-GJR:

$$q_t = -\left(\beta_0 + \beta_1 q_{t-1}^2 + \beta_2 r_{t-1}^2 + \beta_3 r_{t-1}^2 I_{r_{t-1}<0}\right)^{\frac{1}{2}} \tag{9}$$

Here, the IG-NA CAViaR naturally lends itself to the scaling approach used above.

2.2.2 Adjusting for serial correlation in returns

Kuester et al. (2006) show that adjusting for serial correlation in returns improves the forecasting performance of CAViaR models. Consider the CAViaR model with serial correlation in returns:

$$\begin{aligned} r_t &= \alpha r_{t-1} + \varepsilon_t \\ q_t &= \alpha r_{t-1} + q_t^{\varepsilon} \end{aligned} \tag{10}$$

where we are using the notation that q_t^{ε} is the θ-quantile of ε_t, just as q_t is the θ-quantile of r_t. We can now write any specification for q_t^{ε}, e.g. $q_t^{\varepsilon} = \beta_0 + \beta_1 q_{t-1}^{\varepsilon} + f(\varepsilon_{t-1})$. Kuester et al. (2006) propose the AR-IG-CAViaR specification as:

$$q_t = \alpha r_{t-1} - \left(\beta_0 + \beta_1 (q_{t-1} - \alpha r_{t-2})^2 + \beta_2 (\varepsilon_{t-1})^2\right)^{\frac{1}{2}} \tag{11}$$

where they write $VaR_t = -q_t$. The AR-SAV and AR-AS CAViaR models would then be written as:

- AR-SAV:

$$\begin{aligned} q_t &= \alpha r_{t-1} + \beta_0 + \beta_1 q_{t-1}^{\varepsilon} + \beta_2 |\varepsilon_{t-1}| \\ &= \alpha r_{t-1} + \beta_0 + \beta_1 (q_{t-1} - \alpha r_{t-2}) + \beta_2 |\varepsilon_{t-1}| \end{aligned} \tag{12}$$

- AR-AS:

$$\begin{aligned} q_t &= \alpha r_{t-1} + \beta_0 + \beta_1 q_{t-1}^{\varepsilon} + \beta_2 \varepsilon_{t-1}^{+} + \beta_3 \varepsilon_{t-1}^{-} \\ &= \alpha r_{t-1} + \beta_0 + \beta_1 (q_{t-1} - \alpha r_{t-2}) + \beta_2 (\varepsilon_{t-1})^{+} + \beta_3 (\varepsilon_{t-1})^{-} \end{aligned} \tag{13}$$

The corresponding versions to the IG-NA and IG-GJR models incorporating an AR parameter (in returns) would be:

- AR-IG-NA:

$$q_t = \alpha r_{t-1} - \left\{\beta_0 + \beta_1 (q_{t-1} - \alpha r_{t-2})^2 + \beta_2 \left(\frac{\varepsilon_{t-1}}{q_{t-1} - \alpha r_{t-2}} - \beta_3\right)^2\right\}^{\frac{1}{2}} \tag{14}$$

- AR-IG-GJR:

$$q_t = \alpha r_{t-1} - \left\{\beta_0 + \beta_1 (q_{t-1} - \alpha r_{t-2})^2 + \beta_2 (\varepsilon_{t-1})^2 + \beta_3 (\varepsilon_{t-1})^2 I_{r_{t-1} < \alpha r_{t-2}}\right\}^{\frac{1}{2}} \tag{15}$$

2.2.3 Incorporating multi-horizon returns as regressors

It is possible to expand the information set used to predict the quantiles by incorporating other regressors. For instance, Jeon and Taylor (2013) use information in the implied volatility (IV) of options to enhance CAViaR forecasts, both directly using a forecast combination strategy, and also by using the IV as a regressor. Rubia and Sanchis-Marco (2013) show that using regressors representing liquidity and financial conditions may improve the performance of a CAViaR model.

In a more direct approach, Schaumburg (2012) modifies the original IG-CAViaR model by the inclusion of the previous day's return as a regressor, further incorporating a threshold-based approach titled the AR-TGARCH CAViaR:

$$q_t = \beta_1 r_{t-1} - \left(\beta_2 + \beta_3 q_{t-1}^2 + \beta_4 r_{t-1}^2 + \beta_5 r_{t-1}^2 I_{r_{t-1}<0}\right)^{\frac{1}{2}} \tag{16}$$

Note that this model is the same as the IG-GJR specification in Eqn. 9, with the previous day's return as an added regressor. This is in fact different from the AR-IG-GJR model in Eqn. 15 because of how it is derived. It may also be worth examining whether incorporating past returns at lower frequencies as regressors would improve forecasting ability.

When we observe that the VaR from our model has not been exceeded for a long period of time, should we adjust our model? If so, should the adjustment lead to an increase or decrease in the VaR? This is a difficult question because it depends on your view of the return process and on the loss function. This also has significant economic implications. Historically, we have observed that large positive aggregate returns over a long period of time are followed by large negative shocks. This is not just a liberal interpretation of the adage "The higher they climb, the harder they fall." A large literature on bubbles is dedicated to identifying instances where asset prices have deviated persistently from their true value. Unfortunately, it is very difficult to precisely quantify how large is 'large' for returns and how long is the 'long' period before prices revert. Hence, our approach is simply to consider the possibility that past returns at lower frequencies contain important information for VaR estimation.

The notion of incorporating information about different horizons has significant support in the literature. Glosten et al. (1993) document that monthly return volatility dynamics are different from daily dynamics in that positive returns lead to a prediction of a decrease in volatility, whereas daily returns are negatively correlated with conditional volatility. Bianco et al. (2009) find that returns have dependence at different frequencies. Venetis and Peel (2005) find changes in the serial correlation in returns based on the volatility. Among those that consider the impact of different frequencies or horizons in asset pricing are Adrian and Rosenberg (2008) and Duffie et al. (2007). The latter find that future defaults are predicted by one year lagged stock return performance. Similarly, Doshi et al. (2013) use past returns among other variables to find credit default swap prices.

A simple approach to introducing long memory in volatility is employed by Corsi (2009). Following the reasoning in his HAR-RV model, we modify all the above models to include past returns as regressors. We begin by considering the following multi-horizon models:

- SAV-mh

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 |r_{t-1}| + \beta_3 r^w_{t-1} + \beta_4 r^m_{t-1}. \tag{17}$$

where r^w_t, r^m_t are average returns over the week and, respectively, the month until and including t. In the estimation, we apply the convention that a week is 5 working days and a month consists of 22 working days.

- AS-mh

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 r^+_{t-1} + \beta_3 r^-_{t-1} + \beta_4 r^w_{t-1} + \beta_5 r^m_{t-1} \tag{18}$$

- IG-mh

$$q_t = -\left(\beta_0 + \beta_1 q^2_{t-1} + \beta_2 r^2_{t-1}\right)^{\frac{1}{2}} + \beta_3 r_{t-1} + \beta_4 r^w_{t-1} + \beta_5 r^m_{t-1} \tag{19}$$

- IG-NA-mh

$$q_t = -\left(\beta_0 + \beta_1 q^2_{t-1} + \beta_2 \left(r_{t-1}/q_{t-1} - \beta_3\right)^2\right)^{\frac{1}{2}} + \beta_4 r_{t-1} + \beta_5 r^w_{t-1} + \beta_6 r^m_{t-1} \tag{20}$$

- IG-GJR-mh

$$q_t = -\left(\beta_0 + \beta_1 q^2_{t-1} + \beta_2 r^2_{t-1} + \beta_3 r^2_{t-1} I_{r_{t-1}<0}\right)^{\frac{1}{2}} + \beta_4 r_{t-1} + \beta_5 r^w_{t-1} + \beta_6 r^m_{t-1} \tag{21}$$

Later, in Section 2.3.2, we will consider a similar extension for long memory versions of the above models.

2.2.4 Another threshold-based nonlinear effects models for comparison

Gerlach et al. (2011) propose the threshold-CAViaR model as:

$$q_t = \begin{cases} \beta_1 + \beta_2 q_{t-1} + \beta_3 |r_{t-1}|, & r_{t-1} \leq 0 \\ \beta_4 + \beta_5 q_{t-1} + \beta_6 |r_{t-1}|, & r_{t-1} \geq 0 \end{cases} \tag{22}$$

The nonlinearities in this model imply that there would be both shifts in the level of VaR, and changes in the impact of today's return on VaR, depending on whether the return is greater or less than a threshold value (0 in Eqn. 22). In order to achieve these two effects without moving to a completely non-linear model, we could apply a slightly modified version of the news impact function borrowed from the E-GARCH model of Nelson (1991). The Shifted Asymmetric Slope or SAS-CAViaR model could be written as:

$$\text{SAS: } q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 (r_{t-1} + \beta_3) + \beta_4 (r_{t-1} + \beta_3)^- \tag{23}$$

However, the T-CAViaR specification also allows the autoregressive parameter in the quantile itself to change based on the sign of the return. This added flexibility performs well in tests, so it once again seems to suggest that there could be a change in the level and rate of mean reversion for the quantile process. In Section 2.3 we propose a two-component model in order to avoid the sudden shifts that are brought about by threshold crossing, and to achieve a smoother series that could potentially save transaction costs.To this end, we focus on extending the CAViaR models based on the approach of Engle and Lee (1999), which has been successfully applied to option pricing by Christoffersen et al. (2008) and Christoffersen et al. (2010).

2.3 Component CAViaR models

2.3.1 Using the GARCH structure to write component models

A standard GARCH(1,1) model (Bollerslev, 1986) is written as

$$\begin{aligned} r_t &= \varepsilon_t = \sigma_t z_t; \qquad z_t \sim iid \quad N(0,1) \\ \sigma_t^2 &= \omega + a\varepsilon_{t-1}^2 + b\sigma_{t-1}^2 \end{aligned}$$

For this model, if z_θ is the θ-quantile of the standard normal distribution, we have $q_t = \sigma_t z_\theta$, where q_t is the θ-quantile of ε_t. This implies

$$\begin{aligned} q_t^2 &= \omega z_\theta^2 + a z_\theta^2 \varepsilon_{t-1}^2 + b q_{t-1}^2 \\ &= \widetilde{\omega} + \widetilde{a}\varepsilon_{t-1}^2 + b q_{t-1}^2 \end{aligned}$$

which is the IG-CAViaR model of Eqn. 4.

Engle and Lee (1999) define the Component GARCH model as

$$\begin{aligned} \sigma_t^2 &= h_t + b\left(\sigma_{t-1}^2 - h_{t-1}\right) + a\left(\varepsilon_{t-1}^2 - h_{t-1}\right) \\ h_t &= \omega + \rho h_{t-1} + \varphi\left(\varepsilon_{t-1}^2 - \sigma_{t-1}^2\right) \end{aligned}$$

where h_t is now a slower-moving component around which the variance fluctuates.

By analogy to the IG relation above, the two-component IG version of the CAViaR model becomes

$$\begin{aligned} q_t^2 &= u_t^2 + \beta\left(q_{t-1}^2 - u_{t-1}^2\right) + a\left(z_\theta^2 \varepsilon_{t-1}^2 - u_{t-1}^2\right) \\ u_t^2 &= \omega z_\theta^2 + \rho u_{t-1}^2 + \varphi\left(z_\theta^2 \varepsilon_{t-1}^2 - q_{t-1}^2\right) \end{aligned}$$

Allowing z_θ to be subsumed in the respective parameters removes any parametric assumptions, giving

Component - IG (C-IG):

$$\begin{aligned} q_t &= -\left\{u_t^2 + \beta_1\left(q_{t-1}^2 - u_{t-1}^2\right) + \beta_2\left(\beta_3 r_{t-1}^2 - u_{t-1}^2\right)\right\}^{\frac{1}{2}} \\ u_t^2 &= \beta_4 + \beta_5 u_{t-1}^2 + \beta_6\left(\beta_7 r_{t-1}^2 - q_{t-1}^2\right) \end{aligned} \tag{24}$$

where we are estimating β_3 and β_7, though they could both be interpreted as the appropriate identical scaling factor that would set the 'shock' term to be zero in expectation if the underlying process was a Component GARCH model.

The Component N-GARCH model in Christoffersen et al. (2010) may be written as:

$$\begin{aligned} \sigma_t^2 &= h_t + b\left(\sigma_{t-1}^2 - h_{t-1}\right) + a\left(\varepsilon_{t-1} - c_1\sqrt{h_{t-1}}\right)^2 \\ h_t &= \omega + \rho h_{t-1} + \varphi\left(\varepsilon_{t-1} - c_2\sigma_{t-1}\right)^2 \end{aligned}$$

so the two-component version of the indirect N-GARCH CAViaR becomes

$$\begin{aligned} q_t^2 &= u_t^2 + \beta\left(q_{t-1}^2 - u_{t-1}^2\right) + a\left(z_\theta\varepsilon_{t-1} - c_1\sqrt{u_{t-1}^2}\right)^2 \\ u_t^2 &= \omega z_\theta^2 + \rho u_{t-1}^2 + \varphi\left(z_\theta\varepsilon_{t-1} - c_2 q_{t-1}\right)^2 \end{aligned}$$

Following from the scaling approach in the IG-NA model in Eqn. 8, and utilizing the flexibility of the CAViaR approach, we can simplify and reduce the number of parameters to get the following model (C-IG-NA):

$$q_t = -\left\{u_t^2 + \beta_1\left(q_{t-1}^2 - u_{t-1}^2\right) + \beta_2\left(\frac{\varepsilon_{t-1}}{u_{t-1}} - \beta_3\right)^2\right\}^{\frac{1}{2}}$$
$$u_t^2 = \beta_4 + \beta_5 u_{t-1}^2 + \beta_6\left(\frac{\varepsilon_{t-1}}{q_{t-1}} - \beta_7\right)^2 \tag{25}$$

Further, the GARCH model of Taylor (1986) and Schwert (1989) takes the form

$$\sigma_t = \omega + b\sigma_{t-1} + \alpha\left|\varepsilon_{t-1}\right| \tag{26}$$

giving us the SAV-CAViaR model as:

$$q_t = \widetilde{\omega} + bq_{t-1} + \widetilde{\alpha}\left|\varepsilon_{t-1}\right|$$

or the asymmetric form

$$\sigma_t = \omega + b\sigma_{t-1} + \alpha_1\varepsilon_{t-1}^{+} + \alpha_2\varepsilon_{t-1}^{-} \tag{27}$$

giving us the AS-CAViaR model:

$$q_t = \widetilde{\omega} + bq_{t-1} + \widetilde{\alpha}_1\varepsilon_{t-1}^{+} + \widetilde{\alpha}_2\varepsilon_{t-1}^{-}$$

Consider again the stationary form of Eqn. 26

$$\sigma_t = \overline{\sigma} + b\left(\sigma_{t-1} - \overline{\sigma}\right) + \alpha\overline{\sigma}\left(\left|z_{t-1}\right| - E\left|z\right|\right)$$

In component form, we could define

$$\sigma_t = \overline{\sigma}_t + b\left(\sigma_{t-1} - \overline{\sigma}_{t-1}\right) + \alpha\left(\left|\varepsilon_{t-1}\right| - \overline{\sigma}_{t-1}E\left|z\right|\right)$$
$$\overline{\sigma}_t = \omega + \rho\overline{\sigma}_{t-1} + \gamma\left(\left|\varepsilon_{t-1}\right| - \sigma_{t-1}E\left|z\right|\right)$$

The corresponding component version is then

$$q_t = u_t + b\left(q_{t-1} - u_{t-1}\right) + \alpha\left(z_\theta\left|\varepsilon_{t-1}\right| - u_{t-1}E\left|z\right|\right)$$
$$u_t = \widetilde{\omega} + \rho u_{t-1} + \gamma\left(z_\theta\left|\varepsilon_{t-1}\right| - q_{t-1}E\left|z\right|\right)$$

This is the same as writing the C-SAV model as:

$$q_t = u_t + \beta_1\left(q_{t-1} - u_{t-1}\right) + \beta_2\left(\left|\varepsilon_{t-1}\right| - \beta_3 u_{t-1}\right)$$
$$u_t = \beta_4 + \beta_5 u_{t-1} + \beta_6\left(\left|\varepsilon_{t-1}\right| - \beta_7 q_{t-1}\right) \tag{28}$$

A similar exercise for the AS-CAViaR model begins with rewriting the volatility equation (Eqn. 27) as

$$\sigma_t = \overline{\sigma} + b\left(\sigma_{t-1} - \overline{\sigma}\right) + \alpha_1 \overline{\sigma}\left(z_{t-1}^{+} - E\left[z^{+}\right]\right) + \alpha_2 \overline{\sigma}\left(z_{t-1}^{-} - E\left[z^{-}\right]\right)$$

where $\overline{\sigma}$ is replaced by its time-varying counterpart and the process is again rewritten as

$$\begin{aligned}\sigma_t &= \overline{\sigma}_t + b\left(\sigma_{t-1} - \overline{\sigma}_{t-1}\right) + \alpha_1\left(\varepsilon_{t-1} - \overline{\sigma}_{t-1}E\left[z^{+}\right]\right) I_{\varepsilon_{t-1}\geqslant 0} + \alpha_2\left(\left|\varepsilon_{t-1}\right| - \overline{\sigma}_{t-1}E\left[z^{-}\right]\right) I_{\varepsilon_{t-1}<0} \\ \overline{\sigma}_t &= \omega + \rho\overline{\sigma}_{t-1} + \gamma_1\left(\varepsilon_{t-1} - \sigma_{t-1}E\left[z^{+}\right]\right) I_{\varepsilon_{t-1}\geqslant 0} + \gamma_2\left(\left|\varepsilon_{t-1}\right| - \sigma_{t-1}E\left[z^{-}\right]\right) I_{\varepsilon_{t-1}<0}\end{aligned}$$

The CAViaR version then becomes

$$\begin{aligned}q_t &= u_t + b\left(q_{t-1} - u_{t-1}\right) + \alpha_1\left(z_\theta\left|\varepsilon_{t-1}\right| - u_{t-1}E\left[z^{+}\right]\right) I_{\varepsilon_{t-1}\geqslant 0} + \alpha_2\left(z_\theta\left|\varepsilon_{t-1}\right| - u_{t-1}E\left[z^{-}\right]\right) I_{\varepsilon_{t-1}<0} \\ u_t &= \widetilde{\omega} + \rho u_{t-1} + \widetilde{\gamma}_1\left(z_\theta\left|\varepsilon_{t-1}\right| - q_{t-1}E\left[z^{+}\right]\right) I_{\varepsilon_{t-1}\geqslant 0} + \widetilde{\gamma}_2\left(z_\theta\left|\varepsilon_{t-1}\right| - q_{t-1}E\left[z^{-}\right]\right) I_{\varepsilon_{t-1}<0}\end{aligned}$$

We can thus estimate the Component AS (C-AS) CAViaR model as:

$$\begin{aligned}q_t &= u_t + \beta_1\left(q_{t-1} - u_{t-1}\right) + \beta_2\left(\left|\varepsilon_{t-1}\right| - \beta_3 u_{t-1}\right) I_{\varepsilon_{t-1}\geqslant 0} + \beta_4\left(\left|\varepsilon_{t-1}\right| - \beta_5 u_{t-1}\right) I_{\varepsilon_{t-1}<0} \\ u_t &= \beta_6 + \beta_7 u_{t-1} + \beta_8\left(\left|\varepsilon_{t-1}\right| - \beta_9 q_{t-1}\right) I_{\varepsilon_{t-1}\geqslant 0} + \beta_{10}\left(\left|\varepsilon_{t-1}\right| - \beta_{11} q_{t-1}\right) I_{\varepsilon_{t-1}<0}\end{aligned} \tag{29}$$

2.3.2 Alternative flexible component structure

As Engle and Manganelli (2004) point out, though the original CAViaR models are exactly specified counterparts of corresponding GARCH models, CAViaR models are in fact more general than the assumed GARCH models. Thus, while we can derive CAViaR specifications from GARCH models, it is not necessary to do so, mainly because we do not assume the parametric form of the distribution of residuals. In order to introduce a long-memory component, we do not necessarily need to work with the GARCH component structure. An alternative direct approach is to replace the intercept parameter β_0 with a time-varying process that induces a long memory property to the VaR. Consider the AS-CAViaR model in Eqn. 3. We could rewrite it as:

$$q_t = u + \beta_1(q_{t-1} - u) + \beta_2 r_{t-1}^{+} + \beta_3 r_{t-1}^{-} \tag{30}$$

Taking the unconditional expectation, we get

$$u = \overline{q} - \frac{\beta_2 E\left[r^{+}\right] + \beta_3 E\left[r^{-}\right]}{1 - \beta_1} \tag{31}$$

Unlike the traditional approach to writing such models, here $u \neq \overline{q}$. Substituting u from Eqn. 31 back into Eqn. 30, we see that it is the same as writing

$$q_t = \overline{q} + \beta_1(q_{t-1} - \overline{q}) + \beta_2\left(r^+_{t-1} - E\left[r^+\right]\right) + \beta_3\left(r^-_{t-1} - E\left[r^-\right]\right) \tag{32}$$

By allowing u to vary over time, we write the general form of the Component CAViaR model as:

$$\begin{aligned} q_t &= u_t + \sum_i \beta_i\left(q_{t-i} - u_{t-i}\right) + \sum_i \gamma_i f(r_{t-i}) \\ u_t &= \omega + \sum_i \delta_i u_{t-i} + \sum_i \phi_i g\left(r_{t-i}\right) \end{aligned}$$

This shows that we need not write the model as a combination of two processes with zero-mean shocks. As a result, we can consider estimating the following component versions of the original models as:

Flexible Component - SAV (FC-SAV):

$$\begin{aligned} q_t &= u_t + \beta_1(q_{t-1} - u_{t-1}) + \beta_2\left|r_{t-1}\right| \\ u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5\left(r_{t-1}\right) \end{aligned} \tag{33}$$

Flexible Component - SAV multi horizon (FC-SAVmh):

$$\begin{aligned} q_t &= u_t + \beta_1(q_{t-1} - u_{t-1}) + \beta_2\left|r_{t-1}\right| \\ u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5 r_{t-1} + \beta_6 r^w_{t-1} + \beta_7 r^m_{t-1} \end{aligned} \tag{34}$$

Flexible Component - AS (FC-AS):

$$\begin{aligned} q_t &= u_t + \beta_1(q_{t-1} - u_{t-1}) + \beta_2 r^+_{t-1} + \beta_3 r^-_{t-1} \\ u_t &= \beta_4 + \beta_5 u_{t-1} + \beta_6\left(r_{t-1}\right) \end{aligned} \tag{35}$$

Flexible Component - AS multi horizon (FC-ASmh):

$$\begin{aligned} q_t &= u_t + \beta_1(q_{t-1} - u_{t-1}) + \beta_2 r^+_{t-1} + \beta_3 r^-_{t-1} \\ u_t &= \beta_4 + \beta_5 u_{t-1} + \beta_6 r_{t-1} + \beta_7 r^w_{t-1} + \beta_8 r^m_{t-1} \end{aligned} \tag{36}$$

Flexible Component IG (FC-IG):

$$
\begin{aligned}
q_t &= -\left\{u_t^2 + \beta_1\left(q_{t-1}^2 - u_{t-1}^2\right) + \beta_2 r_{t-1}^2\right\}^{\frac{1}{2}} \\
u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5 r_{t-1}
\end{aligned} \tag{37}
$$

Flexible Component IG multi horizon (FC-IGmh)

$$
\begin{aligned}
q_t &= -\left\{u_t^2 + \beta_1\left(q_{t-1}^2 - u_{t-1}^2\right) + \beta_2 r_{t-1}^2\right\}^{\frac{1}{2}} \\
u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5 r_{t-1} + \beta_6 r_{t-1}^w + \beta_7 r_{t-1}^m
\end{aligned} \tag{38}
$$

2.3.3 Interpreting the alternative component structure

Consider again the FC-AS model in Eqn. 35. Substituting the relationship in Eqn. 31 for u_t,we see that

$$
\overline{q}_t = \left[\beta_4 + \left(\frac{1-\beta_5}{1-\beta_1}\right)\left(\beta_2 E\left[r^+\right] + \beta_3 E\left[r^-\right]\right)\right] + \beta_5 \overline{q}_{t-1} + \beta_6\left(r_{t-1}/q_{t-1}\right)
$$

and that

$$
\begin{aligned}
\overline{q} &= \frac{\beta_4}{1-\beta_5} + \frac{\beta_2 E\left[r^+\right] + \beta_3 E\left[r^-\right]}{1-\beta_1} \\
\text{or } \overline{u} = u &= \frac{\beta_4}{1-\beta_5}
\end{aligned}
$$

In these alternative component models, the deviation $q_t - u_t$ is the component that represents an adjusted distance from the unconditional mean $\overline{q}$ and the dynamics of u_t capture the time dependence in $\overline{q}_t$, albeit with an adjusted mean level. We have introduced a long memory feature in the quantile process. As long as the persistence of the component u_t is higher than that of $q_t - u_t$ we are able to interpret it as a ‘long-term’ component; with the caveat that q_t tends to a quantity different from u_t in the long run.

3 Estimation

CAViaR parameters (β) are estimated by the regression quantile (RQ) criterion:

$$
\min_{\beta} \frac{1}{T} \sum_{t=1}^{T} \left[\theta - I\left(r_t < q_t(\beta)\right)\right]\left[r_t - q_t(\beta)\right]
$$

The optimization surface for this problem is known to be multimodal and thus problematic to minimize over. Engle and Manganelli (2004) estimate the parameters by a genetic algorithm approach, proposing a large number of uniform random parameter vectors, picking a proportion of them and then inputting these in a simplex based search algorithm (fminsearch) in MATLAB. We find that it is important to get the relative scale of the parameters right to improve the chances of finding a global minimum. To this end, we apply a procedure based on targeting of the empirical (unconditional) quantile of the returns series by assuming stationarity of the quantile time series, but only as an input to the optimizer. We start the minimizer (fminsearch) at a grid of initial parameter values, in each case setting the starting value for β_0 to be consistent with the other parameters and the appropriate in-sample quantile of the data. To clarify, we are not reducing one parameter as is done in parametric GARCH models when applying variance targeting - our approach is merely designed to obtain helpful starting values for the optimizer. As an example, for the SAV model (Eqn. 2), for each starting value pair of $\beta_1 \in \{0.5, 0.65, 0.8, 0.95\}$ and $\beta_2 \in \{-0.25, 0, 0.25\}$, we set

$$\beta_0 = \hat{q}\,(1 - \beta_1) - \beta_2 \hat{E}\,[|r|]$$

where the ˆ symbol represents an unconditional estimate of the quantity from the data (in-sample). In the case of the scaled models, we make a further simplification, substituting in quantities for the normal distribution where appropriate. For instance, $E\,|z| = \sqrt{2/\pi}$ for $z \sim N\,(0, 1)$. As an example, for the 1% quantile, $\hat{E}\,[|r/q|]$ is substituted with $\left[1/\phi^{-1}(0.01)\right]\sqrt{2/\pi}$, where $\phi^{-1}\,(\cdot)$ is the inverse of the standard normal distribution.

In our comparison of the realized quantile function with the same data and models as in Engle and Manganelli (2004), we find that we tend to obtain a lower RQ using this approach. This approach appears to become more important as the number of parameters increases.

4 Empirical exercise

In this section, we provide details of the comparative performance of the various models. We begin first by describing the data and then in each case the criteria for comparison.

4.1 Data

Given our twofold objective of providing model comparisons while also highlighting estimation differences, we carry out the empirical exercise on the same dataset as in Engle and Manganelli

(2004). This dataset, which is downloadable from the website of Professor Manganelli (www.simonemanganelli.org), consists of two stocks - General Motors (GM) and IBM - and the S&P 500 index for the period April 7, 1986 to April 7, 1999. Of this, the last 500 days (corresponding to two years) is left for out-of-sample testing.

The models we consider are (with equation numbers in parentheses):

- SAV (2), AS (3), IG (4), IG-NA (8), IG-GJR (9)
- AR (11 - 15) and multi-horizon (17 - 21)versions of the above models
- SAVscaled (5), ASscaled (6), IGscaled (7)
- SAS (23), C-SAV (28), C-AS (29), C-IG (4), C-IG-NA (25)
- Flexible component counterparts of SAV, AS, IG, along with their multi-horizon versions (33 - 38).

The comparisons are tabulated in Tables 1 - 14 at the end of the paper.

4.2 Estimation strategy

Before proceeding to the performance comparisons, we focus briefly on the importance of the estimation strategy. As mentioned before, rather than use a large matrix of uniform random numbers as the basis for starting the optimization procedure, we choose a grid of starting values in each case to be consistent with each other and with a stationary quantile process. The most direct evidence of the effectiveness of this strategy is the improvement in the estimation of the three original models relative to the original paper. Even for the simplest model, the 1% SAV in Table 3, we see that the objective function is lower for all the assets and the proportion of hits both in-sample and out-of-sample has been improved. For example, in the case of S&P500 we obtain 0.0080 proportion of hits out-of-sample, while in the original paper the out-of-sample proportion is 0.0180. The only exception is the case of the AS model at 5% for S&P500. Although the RQ and proportion of hits in-sample are similar to the original algorithm, the proportion of hits out-of-sample is higher using our algorithm (0.0700¿0.0640). This is an example where we can see the tension between the news impact and the level of the autoregressive parameter. In our estimates the autoregressive parameter of the model is lower (0.8757¡0.9025), but the parameter on the lagged value of the negative return is higher (0.3372¿0.2871) than that reported by Engle and Manganelli (2004) in their Table 1.

4.3 Out-of-sample results

Our out-of-sample forecasting exercises are based on one-day-ahead rolling forecasts, but not rolling estimates. We initially estimate the model for the in-sample period, and produce a one-day ahead estimate of the VaR. Further updates over the course of the out-of-sample period are based on rolling the return data ahead one day, using the estimated parameters from the in-sample period. Banks typically use one-day ahead 1% forecasts as described in Berkowitz and O'Brien (2002) and Berkowitz et al. (2011). However, they would naturally update their model estimates regularly, not least because the composition of their portfolios would change from day to day. We replicate the approach of Engle and Manganelli (2004), which tests the models' ability to capture long-term underlying dynamics in the quantiles.

The out-of-sample forecasts are compared first on their hit ratios. Although the hit ratio is a metric with widely known and well documented flaws, it remains a first-order criterion for regulators. Also, several authors in succession have shown that many VaR models tend to produce higher than expected hit ratios out-of-sample and also fail the basic unconditional coverage test. Following Danielsson (2002), we consider a hit ratio in the range of 0.8 to 1.2 to be acceptable. In other words, for the $\theta = 5\%$, a proportion of hits ranging between 0.04 and 0.06 would be acceptable. Throughout the analysis in the tables provided, we highlight in bold the hit rates that lie within the range 0.8 to 1.2.

We also provide the p-values for the out-of-sample Dynamic Quantile test (DQout) from Engle and Manganelli (2004), the Likelihood Ratio tests of unconditional coverage, independence and conditional coverage (respectively LR-uc, LR-i, LR-cc) from Christoffersen (1998), and the duration tests (D and Di) from Christoffersen and Pelletier (2004).

We also report the economic criteria for smoothness (mean and variance of the VaR series), realized c-VaR (ES), and mean excess loss (MEL), as defined in the introduction. The mean level of the quantile series is used as a check to ensure that the variance has not decreased at the cost of a raised average level of capital. Other approaches proposed in the literature include Komunjer and Giacomini (2005) and Gaglianone et al. (2011).

4.3.1 Overall model performance

As VaR is used for the determination of the capital requirements of the firm, the risk manger should carefully choose the internal model with respect to all the criteria considered here. One reason for considering the hit ratio is that an excess number of exceedances would require that the manager increase the constant factor in the formula for setting the capital level and thus get penalized. On the other hand, overestimation of VaR would again have negative results in the face of competition.

Turning to the proportion of hits out-of-sample at the 1% quantile level, we find that nearly all the models produce acceptable hit ratios for GM. On the other hand, nearly all the models fail in producing accurate out-of-sample hits for IBM. For S&P 500, there is an even mix of models that generate a hit ratio in the acceptable range. At the 5% probability level, the results are similar for GM, while more models are within the acceptable range for IBM and S&P500.

For the test statistics, we report by * the assets for which the test statistic does not support the specific form of dependence being tested given the obtained p-value at 1% significance level. In other words, we highlight the models that are not supported, or where the null hypothesis of lack of (the respective type of) dependence in hit sequences is rejected.

Models for GM are not rejected by any test at either significance level. For IBM, IG-GJR and AR-IG-GJR are both rejected by the DQ test at 1% and 5%. All the other models pass the tests although they do not produce accurate hits out-of-sample. Some accurate forecasts are produced for IBM at 5% and those models are not rejected by the DQ test. On the other hand, the models that are able to deliver accurate forecasts or perform well on other criteria for S&P500 are rejected by the DQ test and vice versa: those that pass the test produce biased forecasts out-of-sample. We are unable to explain the reason for this contradiction.

Most models share a similar average level of VaR at each asset and level. However, some models perform slightly better than others. In the case of GM, the Flexible Component models are generally superior to all others at the 1% level, while the more complex models (based on IG and AS rather than SAV) are marginally better at the 5% level. In the case of s&P500, the Flexible Component models outperform at the 5% level, while there is no clear pattern at the 1% level. In general, longer memory and more complex dynamics are useful in keeping the mean of the VaR series low despite an improvement in hit ratios.

When comparing smoothness through variance, as expected, the introduction of long memory features lowers the variance of the overall series. Even incorporating scaled news impact functions has the effect of significantly reducing the variance, as it pushes up the persistence of the series. Threshold effects, on the other hand, do increase variance as feared. This is illustrated visually in Figure 1 where the VaR series estimated using SAS (green) and FC-AS (black) models are plotted for S&P500 at 1%.

In terms of reducing cVaR and MEL, the introduction of asymmetry and long memory, as well as multi horizon regressors, all improve the excess losses.

4.3.2 Effect of adjusting the news impact function

Scaling the news impact function uniformly improves upon the original models, which in any case perform better than expected thanks to the estimation procedure. The main feature of the models with scaled news impact functions is a higher autoregressive parameter on past values of q, much closer to 1.

4.3.3 Effect of incorporating an AR parameter for returns

The AR models perform exceedingly well, once again improving upon the original models. One reason why the AR models may perform better is the difference in the average return in and out-of-sample. In the estimation sample, the mean of daily returns is 0.01%, while in the out-of-sample period it is 0.1% for GM. Also, IBM has an in-sample mean of 0.000289, whereas the mean of the out-of-sample period is 0.0021. While the in-sample means differ from the out-of-sample ones, there is no evidence that this is not just a result of randomness of the sample, given the datasets in question have been widely studied and no breaks have been identified.

4.3.4 Effect of introducing a component structure

The Flexible Component models are found to be promising for capturing the behaviour of the time-varying quantile and provide us with accurate ratio of violations across the assets. At 1% all the three models deliver accurate forecasts for GM and S&P500. We are also able to obtain improved forecasts for IBM using the F-C-AS model, although in general most models perform poorly on hit ratios with IBM. All the models pass the tests for producing uncorrelated hits out-of-sample. However, we notice one rejection from the DQ test - but not from the other tests - while the forecasting performance of the model is acceptable in terms of the ratio of hits (C-IG for S&P500 at 1% probability level).

Besides being able to produce accurate ratio of hits for both the in-sample period and the out-of-sample, the models also perform well on smoothness criteria.

On the other hand, the increased number of parameters and the increased structure imposed by the Component models directly derived from GARCH models does not appear to offer any advantage in improving hit ratios, but there is a slight improvement in the economic loss criteria.

4.3.5 Effect of incorporating multiple horizon returns

The introduction of multi horizon returns is effective not only in improving the original CAViaR models, but also in improving the overall performance with respect to IBM. However, once we account for long memory in the Flexible Component models, the introduction of multi horizon regressors only offers marginal gains at best. It appears that there is some information in the past returns that helps introduce conservatism in the VaR series, without raising the average level of capital required.

5 Conclusion

This paper presents improvements in the economic performance of existing CAViaR models through the introduction of long memory properties, the use of an alternative estimation strategy, and finally by including information on multi-horizon returns among the predictors for VaR.

The modifications to the models are motivated by the need to allow VaR to change smoothly while taking into account variation in the underlying mean level of VaR. In addition to controlling transaction costs, the effect of such improvements may also be to reduce the severity of unexpected components of extreme losses when the VaR is exceeded.

Our empirical exercise used a very long series of data, in part to allow comparison with the original paper, but also to examine the ability of the models to capture long-term underlying dynamics of the quantile process. The semiparametric nature of the model makes this difficult because a slight shift in the parameters could cause a significant change in the number of exceedances. As a result, the trade-off between introducing complexity or smoothness and the predictive performance of the model is noticeable. However, the models perform very well out-of-sample, suggesting that CAViaR models in general are good at capturing the underlying quantile process. However, the results from the introduction of alternative dynamics all point to the need for longer memory features in CAViaR models.

Some questions are left to future work. We have attempted to evaluate the VaR models using a range of criteria rather than a single metric. similar to c-VaR, but it would be of interest if we were to estimate the models using altered loss functions to get a handle on c-VaR directly.Taylor (2008) offers an alternative approach, where he uses the asymmetric least squares (ALS) regression to estimate expectiles rather than using a quantile regression. Such a direct approach would be useful to further examine some of the arguments in the present paper, including the link between severity of losses and aggregate past returns.

References

Adrian, T., Rosenberg, J., 2008. Stock returns and volatility: Pricing the short-run and long-run components of market risk. The Journal of Finance 63 (6), 2997–3030.

Basak, S., Shapiro, A., 2001. Value-at-risk-based risk management: optimal policies and asset prices. Review of Financial Studies 14 (2), 371–405.

Berkowitz, J., Christoffersen, P., Pelletier, D., 2011. Evaluating value-at-risk models with desk-level data. Management Science 57 (12), 2213–2227.

Berkowitz, J., O'Brien, J., 2002. How accurate are value-at-risk models at commercial banks? The Journal of Finance 57 (3), 1093–1111.

Bianco, S., Corsi, F., Ren, R., 2009. Intraday lebaron effects. Proceedings of the National Academy of Sciences 106 (28), 11439–11443.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics 31, 307–327.

Christoffersen, P., 1998. Evaluating interval forecasts. International Economic Review 39 (4), 841–862.

Christoffersen, P., 2003. Elements of Financial Risk Management. Academic Press, London.

Christoffersen, P., 2009. Value–at–risk models. In: Handbook of Financial Time Series. Springer, pp. 753–766.

Christoffersen, P., Dorion, C., Jacobs, K., Wang, Y., 2010. Volatility components, affine restrictions, and nonnormal innovations. Journal of Business & Economic Statistics 28 (4), 483–502.

Christoffersen, P., Jacobs, K., Ornthanalai, C., Wang, Y., 2008. Option valuation with long-run and short-run volatility components. Journal of Financial Economics 90 (3), 272–297.

Christoffersen, P., Pelletier, D., 2004. Backtesting value- at- risk: A duration- based approach. Journal of financial econometrics 2, 84–108.

Corsi, F., 2009. A simple approximate long- memory model of realized volatility. Journal of Financial Econometrics 7 (2), 174–196.

Danielsson, J., 2002. The emperor has no clothes: limits to risk modelling. Journal of banking and finance 26, 1273–1296.

Ding, Z., Granger, C. W., 1996. Modeling volatility persistence of speculative returns: A new approach. Journal of Econometrics 73 (1), 185 – 215.
URL http://www.sciencedirect.com/science/article/pii/0304407695017372

Doshi, H., Ericsson, J., Jacobs, K., Turnbull, S. M., 2013. Pricing credit default swaps with observable covariates. Review of Financial Studies, hht015.

Duffie, D., Saita, L., Wang, K., 2007. Multi-period corporate default prediction with stochastic covariates. Journal of Financial Economics 83 (3), 635–665.

Engle, R. F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of uk inflation. Econometrica 50, 987–1008.

Engle, R. F., Lee, G., 1999. A permanent and transitory component model of stock return volatility. In: Cointegration, Causality and Forecasting: A festschrift in honor of Clive W.J. Granger. Oxford University Press, pp. 475–497.

Engle, R. F., Manganelli, S., 2004. CAViaR: Conditional autoregressive value at risk by regression quantiles. Journal of Business & Economic Statistics 22, 367–381.

Engle, R. F., Ng, V. K., 1993. Measuring and testing the impact of news on volatility. The Journal of Finance 48 (5), 1749–1778.

Gaglianone, W. P., Lima, L. R., Linton, O., Smith, D. R., 2011. Evaluating value-at-risk models via quantile regression. Journal of Business & Economic Statistics 29 (1).

Gerlach, R. H., Chen, C. W., Chan, N. Y., 2011. Bayesian time-varying quantile forecasting for value-at-risk in financial markets. Journal of Business & Economic Statistics 29 (4).

Glosten, L. R., Jagannathan, R., Runkle, D. E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance 48 (5), 1779–1801.

Jeon, J., Taylor, J. W., 2013. Using caviar models with implied volatility for value-at-risk estimation. Journal of Forecasting 32 (1), 62–74.

Komunjer, I., Giacomini, R., 2005. Evaluation and combination of conditional quantile forecasts. Journal of Business & Economic Statistics 23.

Kuester, K., Mittnik, S., Paolella, M., 2006. Value-at-risk prediction: A comparison of alternative strategies. Journal of Financial Econometrics 4(1), 53–89.

Kupiec, P. H., 1995. Techniques for verifying the accuracy of risk measurement models. THE J. OF DERIVATIVES 3 (2).

Lopez, J. A., 1997. Regulatory evaluation of value-at-risk models. Tech. rep., Federal Reserve Bank of New York.

McAleer, M., Jimenez-Martin, J.-A., Prez-Amaral, T., 2010. A decision rule to minimize daily capital charges in forecasting value-at-risk. Journal of Forecasting 29 (7), 617–634.

Nelson, D. B., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica 59 (2), pp. 347–370.

Rubia, A., Sanchis-Marco, L., 2013. On downside risk predictability through liquidity and trading activity: A dynamic quantile approach. International Journal of Forecasting 29 (1), 202–219.

Sarma, M., Thomas, S., Shah, A., 2003. Selection of value-at-risk models. Journal of Forecasting 22 (4), 337–358.

Schaumburg, J., 2012. Predicting extreme value at risk: Nonparametric quantile regression with refinements from extreme value theory. Computational Statistics & Data Analysis 56 (12), 4081 – 4096.

Schwert, G. W., 1989. Why does stock market volatility change over time? The Journal of Finance 44, 1115–1153.

Taylor, J. W., 2008. Estimating Value at Risk and Expected Shortfall using expectiles. Journal of Financial Econometrics 6 (2), 231–252.

Taylor, S., 1986. Modelling Financial Time Series. John Wiley, West Sussex, United Kingdom.

Venetis, I. A., Peel, D., 2005. Non-linearity in stock index returns: the volatility and serial correlation relationship. Economic Modelling 22 (1), 1–19.

Tables and Figures follow

	SAV	**SAV scaled**	**AR -SAV**	**AS**	**AS scaled**	**AR -AS**	**IG**	**IG scaled**	**AR -IG**	**IG -NA**	**AR -IG-NA**	**IG -GJR**	**AR -IG-GJR**	**SAS**
RQ	1.5847	1.5841	1.5779	1.5749	1.5822	1.5732	1.5846	1.6383	1.5725	1.5758	1.5604	1.5707	1.5699	1.5476
Hit-In	**0.0097**	**0.0101**	**0.0105**	**0.0097**	**0.0097**	**0.0101**	**0.0101**	**0.0105**	**0.0101**	**0.0101**	**0.0101**	**0.0097**	**0.0097**	**0.0097**
Hit-Out	**0.0120**	**0.0120**	**0.0100**	0.0140	**0.0120**	**0.0120**	**0.0100**	**0.0100**	**0.0100**	**0.0120**	**0.0120**	**0.0100**	**0.0100**	**0.0120**
DQout	0.9487	0.9416	0.9971	0.9615	0.9718	0.9767	0.9973	0.9983	0.9987	0.9485	0.9656	0.9994	0.9994	0.9892
LRuc	0.6596	0.6596	0.9964	0.3939	0.6596	0.6596	0.9964	0.9964	0.9964	0.6596	0.6596	0.9964	0.9964	0.6596
LRi	-	-	-	-	-	-	-	-	-	-	-	-	-	-
LRcc	-	-	-	-	-	-	-	-	-	-	-	-	-	-
D	0.8845	0.9942	0.8844	0.9631	0.9786	0.5887	0.8089	0.8017	0.9538	0.7482	0.8140	0.8618	0.7326	0.8404
Di	0.6319	0.9253	0.6223	0.7879	0.8690	0.3033	0.5229	0.5076	0.7787	0.4474	0.5228	0.6535	0.5212	0.5646
Mean	-0.0445	-0.0443	-0.0442	-0.0429	-0.0440	-0.0440	-0.0449	-0.0461	-0.0449	-0.0431	-0.0441	-0.0454	-0.0454	-0.0433
100*var	0.0088	0.0065	0.0066	0.0107	0.0075	0.0166	0.0128	0.0574	0.0120	0.0096	0.0097	0.0168	0.0168	0.0162
MEL	-0.0601	-0.0594	-0.0599	-0.0590	-0.0600	-0.0588	-0.0600	-0.0612	-0.0595	-0.0593	-0.0593	-0.0607	-0.0603	-0.0591
ES	-0.0161	-0.0163	-0.0160	-0.0173	-0.0164	-0.0163	-0.0157	-0.0162	-0.0151	-0.0173	-0.0155	-0.0143	-0.0143	-0.0158

Table 1: Performance of the original CAViaR models and extensions based on scaling the news impact function or adding an autoregressive term in returns. Series: **GM**. Quantile level: **1**%

	SAV	**SAV scaled**	**AR -SAV**	**AS**	**AS scaled**	**AR -AS**	**IG**	**IG scaled**	**AR -IG**	**IG -NA**	**AR -IG-NA**	**IG -GJR**	**AR -IG-GJR**	**SAS**
RQ	1.7304	1.7305	1.7211	1.7031	1.7038	1.7016	1.7372	1.7608	1.7321	1.7209	1.7445	1.7033	1.6990	1.6886
Hit-In	**0.0098**	**0.0098**	**0.0098**	**0.0101**	**0.0098**	**0.0101**	**0.0105**	**0.0101**	**0.0105**	**0.0098**	**0.0101**	**0.0105**	**0.0094**	**0.0105**
Hit-Out	0.0160	0.0160	0.0140	0.0180	0.0160	0.0140	0.0180	0.0200	0.0160	0.0160	0.0160	0.0200	0.0200	0.0160
DQout	0.0321	0.0342	0.0292	0.0305	0.0431	0.0295	0.0399	0.0404	0.0419	0.0424	0.0491	0.0000*	0.0000*	0.0432
LRuc	0.2131	0.2131	0.3939	0.1049	0.2131	0.3939	0.1049	0.0473	0.2131	0.2131	0.2131	0.0473	0.0473	0.2131
LRi	-	-	-	-	-	-	-	-	-	-	-	-	-	-
LRcc	-	-	-	-	-	-	-	-	-	-	-	-	-	-
D	0.9565	0.9422	0.9949	0.7860	0.7867	0.9037	0.7081	0.6520	0.7220	0.8469	0.7499	0.6460	0.2051	0.7481
Di	0.8655	0.8084	0.9422	0.7179	0.5171	0.9175	0.7435	0.6019	0.5851	0.6019	0.5282	0.9568	0.4495	0.6321
Mean	-0.0410	-0.0408	-0.0433	-0.0404	-0.0408	-0.0416	-0.0424	-0.0423	-0.0427	-0.0410	-0.0422	-0.0420	-0.0418	-0.0422
100*var	0.0056	0.0059	0.0073	0.0124	0.0087	0.0094	0.0069	0.0173	0.0055	0.0078	0.0030	0.0188	0.0191	0.0139
MEL	-0.0660	-0.0655	-0.0660	-0.0635	-0.0649	-0.0646	-0.0632	-0.0640	-0.0650	-0.0651	-0.0649	-0.0620	-0.0639	-0.0640
ES	-0.0264	-0.0266	-0.0237	-0.0263	-0.0258	-0.0249	-0.0250	-0.0261	-0.0247	-0.0262	-0.0256	-0.0250	-0.0250	-0.0238

Table 2: Performance of the original CAViaR models and extensions based on scaling the news impact function or adding an autoregressive term in returns. Series: **IBM**. Quantile level: **1**%

	SAV	**SAV scaled**	**AR -SAV**	**AS**	**AS scaled**	**AR -AS**	**IG**	**IG scaled**	**AR -IG**	**IG -NA**	**AR -IG-NA**	**IG -GJR**	**AR -IG-GJR**	**SAS**
RQ	1.0482	1.0590	1.0230	1.0293	1.1353	1.0172	1.0539	1.0623	1.0204	1.0612	1.0275	1.0409	1.0210	0.9484
Hit-In	**0.0101**	**0.0097**	**0.0097**	**0.0097**	**0.0097**	**0.0097**	**0.0101**	**0.0093**	**0.0104**	**0.0086**	**0.0090**	**0.0104**	**0.0097**	**0.0097**
Hit-Out	**0.0080**	**0.0100**	**0.0100**	0.0160	**0.0080**	0.0180	0.0160	**0.0100**	**0.0120**	0.0140	**0.0120**	0.0160	**0.0120**	0.0240
DQout	0.0002*	0.0031*	0.0027*	0.0476	0.0000*	0.0329	0.0311	0.0021*	0.0000*	0.0279	0.0000*	0.0496	0.0000*	0.0036*
LRuc	0.6445	0.9964	0.9964	0.2131	0.6445	0.1049	0.2131	0.9964	0.6596	0.3939	0.6596	0.2131	0.6596	0.0075*
LRi	-	-	-	-	-	-	-	-	-	-	-	-	-	-
LRcc	-	-	-	-	-	-	-	-	-	-	-	-	-	-
D	0.8833	0.8915	0.3800	0.8742	0.0124	0.3069	0.9287	0.8305	0.4806	0.8783	0.3797	0.5505	0.2357	0.0394
Di	0.7008	0.7194	0.1755	0.6348	0.0035*	0.3622	0.9526	0.7234	0.2332	0.9075	0.1933	0.3451	0.0891	0.0171
Mean	-0.0247	-0.0253	-0.0241	-0.0230	-0.0255	-0.0229	-0.0254	-0.0252	-0.0255	-0.0238	-0.0252	-0.0238	-0.0255	-0.0257
100*var	0.0114	0.0105	0.0139	0.0097	0.0047	0.0113	0.0194	0.0109	0.0186	0.0105	0.0141	0.0206	0.0212	0.0733
MEL	-0.0391	-0.0406	-0.0389	-0.0372	-0.0429	-0.0369	-0.0382	-0.0422	-0.0373	-0.0425	-0.0415	-0.0380	-0.0389	-0.0332
ES	-0.0147	-0.0141	-0.0139	-0.0148	-0.0170	-0.0145	-0.0138	-0.0146	-0.0129	-0.0156	-0.0129	-0.0147	-0.0127	-0.0102

Table 3: Performance of the original CAViaR models and extensions based on scaling the news impact function or adding an autoregressive term in returns. Series: **S&P500**. Quantile level: **1%**

	GM					IBM					SP500				
	ASmh	**IGmh**	**SAVmh**	**IG-NAmh**	**IG-GJRmh**	**ASmh**	**IGmh**	**SAVmh**	**IG-NAmh**	**IG-GJRmh**	**ASmh**	**IGmh**	**SAVmh**	**IG-NAmh**	**IG-GJRmh**
RQ	1.5656	1.5646	1.5709	1.5723	1.5571	1.6938	1.7107	1.6944	1.7089	1.6832	1.0204	1.0349	1.0351	1.0273	1.0205
Hit-In	**0.0102**	**0.0094**	**0.0106**	**0.0094**	**0.0110**	**0.0098**	**0.0098**	**0.0095**	**0.0102**	**0.0106**	**0.0105**	**0.0098**	**0.0101**	**0.0120**	**0.0112**
Hit-Out	0.0140	**0.0100**	**0.0100**	**0.0080**	**0.0100**	0.0160	0.0160	0.0160	0.0180	0.0180	0.0160	**0.0100**	**0.0080**	0.0220	0.0160
DQout	0.9602	0.9998	0.8003	0.9992	0.9975	0.0429	0.0402	0.0445	0.0000^*	0.0000^*	0.0446	0.0025^*	0.0002^*	0.0301	0.0340
LRuc	0.3939	0.9964	0.9964	0.6445	0.9964	0.2131	0.2131	0.2131	0.1049	0.1049	0.2131	0.9964	0.6445	0.0196	0.2131
LRi	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
LRcc	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
D	0.8257	0.7326	0.8676	0.6882	0.9267	0.6180	0.6180	0.7445	0.7209	0.6810	0.5796	0.5706	0.9486	0.0999	0.5394
Di	0.6619	0.5212	0.5996	0.5646	0.7792	0.6214	0.6214	0.4468	0.6125	0.7235	0.5418	0.3398	0.8701	0.2075	0.5364
Mean	-0.0429	-0.0458	-0.0448	-0.0461	-0.0449	-0.0407	-0.0428	-0.0424	-0.0407	-0.0430	-0.0231	-0.0249	-0.0231	-0.0260	-0.0226
100*var	0.0113	0.0161	0.0126	0.0087	0.0177	0.0084	0.0146	0.0149	0.0047	0.0222	0.0094	0.0153	0.0080	0.0293	0.0119
MEL	-0.0585	-0.0611	-0.0585	-0.0616	-0.0587	-0.0650	-0.0641	-0.0657	-0.0636	-0.0620	-0.0374	-0.0397	-0.0393	-0.0293	-0.0372
ES	-0.0174	-0.0143	-0.0160	-0.0143	-0.0152	-0.0260	-0.0244	-0.0241	-0.0267	-0.0235	-0.0146	-0.0143	-0.0159	-0.0127	-0.0151

Table 4: Performance of the multi horizon CAViaR models at 1%

	GM				**IBM**				**SP500**			
	C-AS	**C-IG**	**C-SAV**	**C-IG-NA**	**C-AS**	**C-IG**	**C-SAV**	**C-IG-NA**	**C-AS**	**C-IG**	**C-SAV**	**C-IG-NA**
RQ	1.5147	1.5642	1.5560	1.5487	1.6340	1.7307	1.7193	1.7194	0.9339	1.0316	1.0344	1.0526
Hit-In	**0.0105**	**0.0097**	**0.0101**	**0.0105**	**0.0105**	**0.0101**	**0.0101**	**0.0101**	**0.0104**	**0.0093**	**0.0101**	**0.0104**
Hit-Out	**0.0100**	**0.0080**	0.0580	**0.0120**	0.0200	0.0160	0.0160	0.0180	0.0160	**0.0120**	0.0220	0.0160
DQout	0.9997	0.0001*	0.0000*	0.9777	0.0141	0.0393	0.0466	0.0452	0.6678	0.0020*	0.0140	0.0321
LRuc	0.9964	0.6445	0.0000*	0.6596	0.0473	0.2131	0.2131	0.1049	0.2131	0.6596	0.0196	0.2131
LRi	-	0.0194	0.7639	-	-	-	-	-	-	-	-	-
LRcc	-	0.0586	0.0000*	-	-	-	-	-	-	-	-	-
D	0.2497	0.4429	0.0000*	0.8879	0.2649	0.6180	0.8752	0.3892	0.4414	0.4469	0.5443	0.7793
Di	0.1093	0.4021	0.0008*	0.6684	0.8061	0.6214	0.7637	0.2152	0.3383	0.2044	0.5107	0.6579
Mean	-0.0451	-0.0455	-0.0410	-0.0452	-0.0424	-0.0424	-0.0433	-0.0404	-0.0263	-0.0254	-0.0248	-0.0245
100*var	0.0108	0.0162	0.0139	0.0234	0.0220	0.0062	0.0085	0.0053	0.0338	0.0155	0.0126	0.0111
MEL	-0.0580	-0.0601	-0.0463	-0.0590	-0.0581	-0.0646	-0.0649	-0.0643	-0.0307	-0.0396	-0.0367	-0.0375
ES	-0.0123	-0.0144	-0.0257	-0.0142	-0.0222	-0.0249	-0.0231	-0.0267	-0.0075	-0.0139	-0.0139	-0.0147

Table 5: Performance of the Component models at 1%

	GM			**IBM**			**SP500**		
	FC-AS	**FC-IG**	**FC-SAV**	**FC-AS**	**FC-IG**	**FC-SAV**	**FC-AS**	**FC-IG**	**FC-SAV**
RQ	1.5693	1.5803	1.5847	1.6951	1.7277	1.7329	1.0201	1.0332	1.0488
Hit-In	**0.0097**	**0.0093**	**0.0097**	**0.0098**	**0.0101**	**0.0101**	**0.0101**	**0.0101**	**0.0097**
Hit-Out	**0.0120**	**0.0120**	**0.0120**	**0.0120**	0.0160	0.0180	**0.0120**	**0.0100**	**0.0120**
DQout	0.9602	0.9706	0.9507	0.0118	0.0268	0.0385	0.0138	0.0010*	0.0115
LRuc	0.6596	0.6596	0.6596	0.6596	0.2131	0.1049	0.6596	0.9964	0.6596
LRi	-	-	-	-	-	-	-	-	-
LRcc	-	-	-	-	-	-	-	-	-
D	0.8037	0.5887	0.8845	0.9612	0.6180	0.7206	0.9476	0.5706	0.9706
Di	0.5086	0.3033	0.6319	0.7786	0.6214	0.5813	0.7429	0.3398	0.8416
Mean	-0.0434	-0.0457	-0.0447	-0.0426	-0.0424	-0.0436	-0.0240	-0.0256	-0.0240
100*var	0.0125	0.0165	0.0095	0.0174	0.0068	0.0102	0.0138	0.0172	0.0097
MEL	-0.0593	-0.0600	-0.0601	-0.0660	-0.0654	-0.0643	-0.0378	-0.0387	-0.0395
ES	-0.0167	-0.0148	-0.0159	-0.0233	-0.0249	-0.0234	-0.0130	-0.0136	-0.0153

Table 6: Performance of the Flexible Component models at 1%

	GM			IBM			SP500		
	FC-ASmh	**FC-IGmh**	**FC-SAVmh**	**FC-ASmh**	**FC-IGmh**	**FC-SAVmh**	**FC-ASmh**	**FC-IGmh**	**FC-SAVmh**
RQ	1.5508	1.6276	1.5661	1.6875	1.6965	1.6887	0.9877	0.9679	1.0119
Hit-In	**0.0102**	**0.0098**	**0.0098**	**0.0106**	**0.0098**	**0.0106**	**0.0101**	**0.0094**	**0.0101**
Hit-Out	**0.0120**	**0.0120**	**0.0120**	0.0140	0.0160	0.0140	0.0140	**0.0120**	0.0140
DQout	0.9527	0.0147	0.9970	0.0294	0.0000*	0.0307	0.0309	0.0117	0.0232
LRuc	0.6596	0.6596	0.6596	0.3939	0.1049	0.2131	0.3939	0.6596	0.3939
LRi	-	-	-	-	-	-	-	-	-
LRcc	-	-	-	-	-	-	-	-	-
D	0.8037	0.8586	0.9720	0.9037	0.4855	0.7768	0.9045	0.8011	0.3839
Di	0.5086	0.5809	0.8249	0.9175	0.3940	0.5867	0.9248	0.5054	0.1733
Mean	-0.0445	-0.0472	-0.0431	-0.0406	-0.0405	-0.0423	-0.0240	-0.0247	-0.0238
100*var	0.0114	0.0044	0.0107	0.0128	0.0111	0.0146	0.0177	0.0201	0.0093
MEL	-0.0583	-0.0618	-0.0597	-0.0640	-0.0649	-0.0637	-0.0361	-0.0383	-0.0383
ES	-0.0153	-0.0149	-0.0172	-0.0259	-0.0265	-0.0239	-0.0120	-0.0108	-0.0134

Table 7: Performance of the Flexible Component models with multi horizon regressors at 1%

	SAV	**SAV scaled**	**AR -SAV**	**AS**	**AS scaled**	**AR -AS**	**IG**	**IG scaled**	**AR -IG**	**IG -NA**	**AR-IG-NA**	**IG -GJR**	**AR -IG-GJR**	**SAS**	**AR -SAS**
RQ	5.0739	5.0721	5.0715	5.0477	5.0480	5.0457	5.0817	5.0589	5.0805	5.0449	5.0415	5.0757	5.0740	5.0493	5.0457
Hit-In	**0.0498**	**0.0502**	**0.0502**	**0.0502**	**0.0498**	**0.0498**	**0.0498**	**0.0502**	**0.0494**	**0.0498**	**0.0502**	**0.0502**	**0.0506**	**0.0494**	**0.0494**
Hit-Out	**0.0420**	**0.0460**	**0.0420**	**0.0480**	**0.0500**	**0.0500**	**0.0480**	**0.0480**	**0.0460**	**0.0520**	**0.0500**	**0.0480**	**0.0520**	**0.0480**	**0.0500**
DQout	0.8877	0.9626	0.8890	0.9209	0.9179	0.9198	0.9096	0.9449	0.8781	0.8965	0.9051	0.9203	0.8786	0.9196	0.9200
LRuc	0.4048	0.6850	0.4048	0.8444	0.9918	0.9918	0.8444	0.8444	0.6850	0.8303	0.9918	0.8444	0.8303	0.8444	0.9918
LRi	0.8610	0.9883	0.8610	0.9143	0.8419	0.8419	0.9143	0.9143	0.9883	0.7717	0.8419	0.9143	0.7717	0.9143	0.8419
LRcc	0.6960	0.9209	0.6960	0.9753	0.9803	0.9803	0.9753	0.9753	0.9209	0.9370	0.9803	0.9753	0.9370	0.9753	0.9803
D	0.9039	0.9577	0.9351	0.4795	0.4318	0.4835	0.8132	0.9874	0.9321	0.7312	0.4876	0.7200	0.8885	0.2635	0.2436
Di	0.9586	0.8309	0.9508	0.2277	0.1970	0.2339	0.5415	0.9176	0.9192	0.4294	0.2309	0.4228	0.6269	0.1077	0.0951
Mean	-0.0279	-0.0284	-0.0279	-0.0277	-0.0278	-0.0276	-0.0279	-0.0279	-0.0280	-0.0280	-0.0280	-0.0277	-0.0276	-0.0280	-0.0277
100*var	0.0025	0.0023	0.0025	0.0026	0.0024	0.0029	0.0034	0.0024	0.0036	0.0031	0.0030	0.0038	0.0034	0.0026	0.0028
MEL	-0.0396	-0.0392	-0.0396	-0.0392	-0.0391	-0.0391	-0.0392	-0.0391	-0.0394	-0.0391	-0.0391	-0.0391	-0.0390	-0.0393	-0.0391
ES	-0.0113	-0.0108	-0.0113	-0.0113	-0.0113	-0.0114	-0.0114	-0.0113	-0.0114	-0.0110	-0.0110	-0.0116	-0.0117	-0.0110	-0.0114

Table 8: Performance of the original CAViaR models and extensions based on scaling the news impact function or adding an autoregressive term in returns. Series: **GM**. Quantile level: **5**%

	SAV	**SAV scaled**	**AR -SAV**	**AS**	**AS scaled**	**AR -AS**	**IG**	**IG scaled**	**AR -IG**	**IG -NA**	**AR-IG-NA**	**IG -GJR**	**AR -IG-GJR**	**SAS**	**AR -SAS**
RQ	4.9208	4.9206	4.8811	4.8727	4.8750	4.8579	4.9706	4.9556	4.9101	4.9020	4.9368	4.9026	4.8867	4.8635	4.8416
Hit-In	**0.0500**	**0.0492**	**0.0496**	**0.0500**	**0.0500**	**0.0500**	**0.0503**	**0.0492**	**0.0507**	**0.0492**	**0.0507**	**0.0503**	**0.0507**	**0.0500**	**0.0485**
Hit-Out	**0.0500**	**0.0520**	**0.0580**	0.0700	0.0680	0.0660	0.0720	0.0620	0.0680	0.0640	0.0700	0.0740	0.0760	0.0680	**0.0600**
DQout	0.4847	0.5491	0.0924	0.0568	0.0831	0.0946	0.1684	0.1106	0.3059	0.0756	0.2399	0.0026*	0.0052*	0.0963	0.2897
LRuc	0.9918	0.8303	0.4169	0.0510	0.0774	0.1143	0.0328	0.2304	0.0774	0.1645	0.0510	0.0205	0.0125	0.0774	0.3141
LRi	0.5140	0.5822	-	0.7480	0.8197	0.3437	0.6790	0.9548	0.8197	0.9689	0.2642	0.6129	0.5502	0.8197	-
LRcc	0.8082	0.8401	-	0.1415	0.2049	0.1836	0.0940	0.4863	0.2049	0.3802	0.0799	0.0601	0.0370	0.2049	-
D	0.7359	0.6462	0.8190	0.7498	0.8203	0.2306	0.2165	0.5963	0.4586	0.9418	0.6202	0.6356	0.0306	0.8160	0.7844
Di	0.4362	0.3576	0.5407	0.7796	0.9581	0.2928	0.1278	0.3224	0.3324	0.8821	0.6493	0.8749	0.1664	0.9083	0.4858
Mean	-0.0258	-0.0256	-0.0253	-0.0243	-0.0245	-0.0250	-0.0241	-0.0248	-0.0250	-0.0251	-0.0248	-0.0250	-0.0248	-0.0250	-0.0253
100*var	0.0052	0.0040	0.0046	0.0052	0.0042	0.0056	0.0033	0.0035	0.0054	0.0042	0.0047	0.0081	0.0075	0.0051	0.0063
MEL	-0.0382	-0.0384	-0.0377	-0.0372	-0.0374	-0.0373	-0.0376	-0.0383	-0.0373	-0.0379	-0.0374	-0.0369	-0.0367	-0.0373	-0.0375
ES	-0.0127	-0.0128	-0.0129	-0.0139	-0.0136	-0.0131	-0.0147	-0.0138	-0.0134	-0.0132	-0.0138	-0.0134	-0.0135	-0.0131	-0.0126

Table 9: Performance of the original CAViaR models and extensions based on scaling the news impact function or adding an autoregressive term in returns. Series: **IBM**. Quantile level: **5%**

	SAV	**SAV scaled**	**AR -SAV**	**AS**	**AS scaled**	**AR -AS**	**IG**	**IG scaled**	**AR -IG**	**IG -NA**	**AR-IG-NA**	**IG -GJR**	**AR -IG-GJR**	**SAS**	**AR -SAS**
RQ	2.9807	2.9821	2.9208	2.9188	2.9470	2.8920	2.9733	2.9848	2.9152	2.9644	2.9312	2.9618	2.9156	2.9406	2.9512
Hit-In	**0.0496**	**0.0496**	**0.0496**	**0.0500**	**0.0496**	**0.0507**	**0.0507**	**0.0493**	**0.0507**	**0.0500**	**0.0489**	**0.0503**	**0.0507**	**0.0500**	**0.0500**
Hit-Out	**0.0540**	**0.0500**	0.0620	0.0700	**0.0600**	0.0740	**0.0580**	0.0640	0.0700	0.0740	0.0800	0.0700	0.0720	0.0660	0.0680
DQout	0.0033*	0.0039*	0.0009*	0.0002*	0.0014*	0.0031*	0.0001*	0.0002*	0.0000*	0.0009*	0.0006*	0.0035*	0.0001*	0.0000*	0.0010*
LRuc	0.6776	0.9918	0.2304	0.0510	0.3141	0.0205	0.4169	0.1645	0.0510	0.0205	0.0043*	0.0510	0.0328	0.1143	0.0774
LRi	0.6708	0.8062	-	0.2642	0.4909	0.1989	0.5474	0.3890	-	0.1989	0.1250	0.2642	-	0.3437	0.3021
LRcc	0.8379	0.9703	-	0.0799	0.4752	0.0299	0.6002	0.2625	-	0.0299	0.0053*	0.0799	-	0.1836	0.1234
D	0.9412	0.4679	0.9135	0.1180	0.7204	0.0690	0.9185	0.5913	0.6768	0.5541	0.6947	0.6967	0.1659	0.6711	0.1273
Di	0.7277	0.2215	0.7751	0.0516	0.4380	0.3058	0.8903	0.3287	0.8194	0.4981	0.9976	0.7237	0.9952	0.4781	0.0534
Mean	-0.0140	-0.0140	-0.0138	-0.0139	-0.0138	-0.0138	-0.0140	-0.0137	-0.0136	-0.0138	-0.0138	-0.0139	-0.0135	-0.0139	-0.0140
100*var	0.0033	0.0028	0.0047	0.0052	0.0028	0.0059	0.0049	0.0023	0.0058	0.0027	0.0030	0.0059	0.0057	0.0041	0.0049
MEL	-0.0219	-0.0221	-0.0213	-0.0212	-0.0216	-0.0204	-0.0213	-0.0221	-0.0211	-0.0216	-0.0217	-0.0212	-0.0209	-0.0214	-0.0213
ES	-0.0081	-0.0081	-0.0079	-0.0078	-0.0081	-0.0077	-0.0081	-0.0084	-0.0082	-0.0083	-0.0079	-0.0082	-0.0082	-0.0080	-0.0080

Table 10: Performance of the original CAViaR models and extensions based on scaling the news impact function or adding an autoregressive term in returns. Series: **S&P500**. Quantile level: **5**%

	GM					IBM					SP500				
	ASmh	**IGmh**	**SAVmh**	**IG-NAmh**	**IG-GJRmh**	**ASmh**	**IGmh**	**SAVmh**	**IG-NAmh**	**IG-GJRmh**	**ASmh**	**IGmh**	**SAVmh**	**IG-NAmh**	**IG-GJRmh**
RQ	5.0131	5.0406	5.0110	5.0315	5.0221	4.8290	4.8898	4.8638	4.8645	4.8675	2.8423	2.9025	2.9061	2.9306	2.8737
Hit-In	**0.0486**	**0.0490**	**0.0506**	**0.0498**	**0.0498**	**0.0492**	**0.0511**	**0.0504**	**0.0496**	**0.0507**	**0.0500**	**0.0511**	**0.0500**	**0.0489**	**0.0496**
Hit-Out	**0.0520**	**0.0560**	**0.0500**	**0.0540**	**0.0480**	0.0640	0.0760	**0.0520**	0.0660	0.0700	**0.0540**	0.0820	**0.0480**	0.0980	0.0760
DQout	0.8921	0.8178	0.9041	0.8559	0.9129	0.1624	0.0006*	0.6831	0.0921	0.0235	0.0237	0.0006*	0.0015*	0.0000*	0.0001*
LRuc	0.8303	0.5386	0.9918	0.6776	0.8444	0.1645	0.0125	0.8303	0.1143	0.0510	0.6776	0.0024*	0.8444	0.0000*	0.0125
LRi	0.7717	0.6895	0.8419	0.7039	0.9143	0.3890	0.9464	0.5822	0.8935	0.7480	-	0.1059	0.8775	0.1152	0.1713
LRcc	0.9370	0.7642	0.9803	0.8533	0.9753	0.2625	0.0441	0.8401	0.2849	0.1415	-	0.0027*	0.9693	0.0000*	0.0173
D	0.3969	0.9211	0.4901	0.6187	0.9484	0.3147	0.0563	0.6163	0.7699	0.6538	0.7790	0.0075*	0.9569	0.2961	0.5201
Di	0.1740	0.9832	0.2327	0.3276	0.7967	0.3294	0.4043	0.3268	0.5860	0.7560	0.4995	0.1449	0.7993	0.8026	0.4485
Mean	-0.0278	-0.0277	-0.0278	-0.0278	-0.0281	-0.0248	-0.0248	-0.0252	-0.0253	-0.0251	-0.0139	-0.0136	-0.0142	-0.0136	-0.0139
100*var	0.0030	0.0024	0.0031	0.0026	0.0027	0.0048	0.0069	0.0047	0.0048	0.0078	0.0055	0.0047	0.0043	0.0035	0.0059
MEL	-0.0392	-0.0391	-0.0389	-0.0392	-0.0393	-0.0375	-0.0368	-0.0380	-0.0378	-0.0369	-0.0210	-0.0210	-0.0216	-0.0210	-0.0208
ES	-0.0112	-0.0116	-0.0112	-0.0114	-0.0111	-0.0132	-0.0138	-0.0131	-0.0130	-0.0133	-0.0074	-0.0082	-0.0077	-0.0084	-0.0077

Table 11: Performance of the multi horizon CAViaR models at 5%

	GM				IBM				SP500			
	C-AS	**C-IG**	**C-SAV**	**C-IG-NA**	**C-AS**	**C-IG**	**C-SAV**	**C-IG-NA**	**C-AS**	**C-IG**	**C-SAV**	**C-IG-NA**
RQ	4.9909	5.0786	5.0683	5.0471	4.7919	4.9281	4.9030	4.9094	2.8271	2.9637	2.9568	2.9645
Hit	0.0502	0.0498	0.0502	0.0494	0.0526	0.0507	0.0503	0.0485	0.0500	0.0503	0.0496	0.0485
Hit-Out	0.0520	0.0480	0.0400	0.0480	0.0060	0.0600	0.0900	0.0680	0.0480	0.0560	0.0760	0.0540
DQout	0.0000*	0.0000*	0.0000*	0.0000*	0.9546	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*
LRuc	0.0000*	0.0000*	0.0000*	0.0000*	0.3336	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*
LRi	0.7717	0.9143	0.7854	0.9143	-	0.8782	0.5482	0.8197	-	0.6074	0.1713	0.6708
LRcc	0.0000*	0.0000*	0.0000*	0.*	-	0.0000*	0.0000*	0.0000*	-	0.0000*	0.0000*	0.0000*
D	0.0000*	0.0000*	0.0000*	0.0000*	0.1425	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*
Di	0.2122	0.7029	0.5166	0.4444	0.2096	0.7942	0.0612	0.1345	0.9512	0.3522	0.8211	0.5434
Mean	-0.0278	-0.0277	-0.0284	-0.0277	-0.0366	-0.0251	-0.0243	-0.0248	-0.0139	-0.0141	-0.0135	-0.0140
100*var	0.0028	0.0026	0.0025	0.0022	0.2081	0.0043	0.0037	0.0032	0.0049	0.0049	0.0025	0.0032
MEL	-0.0387	-0.0393	-0.0396	-0.0396	-0.0366	-0.0378	-0.0365	-0.0382	-0.0211	-0.0216	-0.0216	-0.0222
ES	-0.0550	-0.0586	-0.0541	-0.0567	-0.0538	-0.0663	-0.0732	-0.0673	-0.0357	-0.0400	-0.0428	-0.0399

Table 12: Performance of the Component models at 5%

	GM			**IBM**			**SP500**		
	FC-AS	**FC-IG**	**FC-SAV**	**FC-AS**	**FC-IG**	**FC-SAV**	**FC-AS**	**FC-IG**	**FC-SAV**
RQ	5.0471	5.0767	5.0739	4.8464	4.9345	4.9208	2.8782	2.9682	2.9808
Hit-In	**0.0498**	**0.0506**	**0.0498**	**0.0496**	**0.0503**	**0.0500**	**0.0500**	**0.0511**	**0.0500**
Hit-Out	**0.0500**	**0.0500**	**0.0420**	**0.0520**	**0.0580**	**0.0500**	**0.0460**	**0.0560**	**0.0520**
DQout	0.9203	0.7962	0.8877	0.5050	0.3664	0.4847	0.0027*	0.0000*	0.0056*
LRuc	0.9918	0.9918	0.4048	0.8303	0.4169	0.9918	0.6850	0.5386	0.8303
LRi	0.8419	0.8419	0.8610	0.7371	0.8020	0.5140	0.9508	0.6074	0.7371
LRcc	0.9803	0.9803	0.6960	0.9238	0.6970	0.8082	0.9193	0.7254	0.9238
D	0.4835	0.9770	0.9039	0.8190	0.7564	0.7359	0.8279	0.5976	0.8897
Di	0.2339	0.9206	0.9586	0.5275	0.6549	0.4362	0.9047	0.3522	0.6288
Mean	-0.0277	-0.0279	-0.0279	-0.0250	-0.0252	-0.0258	-0.0142	-0.0140	-0.0138
100*var	0.0029	0.0024	0.0025	0.0060	0.0046	0.0052	0.0051	0.0050	0.0030
MEL	-0.0390	-0.0391	-0.0396	-0.0376	-0.0378	-0.0382	-0.0213	-0.0216	-0.0219
ES	-0.0114	-0.0114	-0.0113	-0.0129	-0.0132	-0.0127	-0.0072	-0.0081	-0.0083

Table 13: Performance of the Flexible Component models at 5%

	GM			**IBM**			**SP500**		
	FC-ASmh	**FC-IGmh**	**FC-SAVmh**	**FC-ASmh**	**FC-IGmh**	**FC-SAVmh**	**FC-ASmh**	**FC-IGmh**	**FC-SAVmh**
RQ	4.9997	5.1116	5.0059	4.8124	4.8712	4.8223	2.8588	2.8503	2.8552
Hit-In	**0.0502**	**0.0490**	**0.0502**	**0.0511**	**0.0496**	**0.0500**	**0.0507**	**0.0525**	**0.0478**
Hit-Out	**0.0480**	**0.0500**	**0.0440**	**0.0580**	0.0700	**0.0440**	**0.0440**	0.0640	**0.0580**
DQout	0.9196	0.2126	0.8641	0.0654	0.3699	0.2931	0.0013*	0.0834	0.0542
LR-uc	0.8444	0.9918	0.5367	0.4169	0.0510	0.5367	0.5367	0.1645	0.4169
LR-i	0.9143	0.8419	0.9366	-	0.7480	0.9746	-	0.3890	0.5474
LR-cc	0.9753	0.9803	0.8236	-	0.8258	0.2049	-	0.2625	0.6002
D	0.7921	0.6067	0.3983	0.5835	0.1154	0.9203	0.6801	0.3707	0.8228
Di	0.5779	0.3265	0.1843	0.3965	0.2411	0.8012	0.7405	0.4297	0.5619
Mean	-0.0281	-0.0278	-0.0277	-0.0252	-0.0246	-0.0253	-0.0144	-0.0138	-0.0136
100*var	0.0029	0.0009	0.0029	0.0066	0.0045	0.0051	0.0055	0.0042	0.0037
MEL	-0.0388	-0.0400	-0.0390	-0.0369	-0.0374	-0.0382	-0.0213	-0.0208	-0.0221
ES	-0.0109	-0.0120	-0.0113	-0.0129	-0.0138	-0.0127	-0.0070	-0.0075	-0.0078

Table 14: Performance of the Flexible Component models with multi horizon regressors at 5%

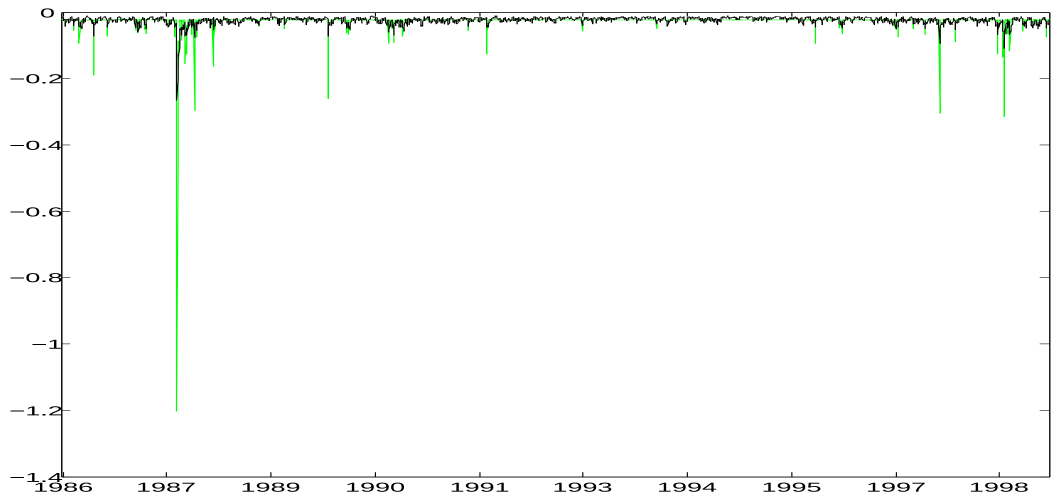

Figure 1: VaR for daily SP500 returns for SAS (green) and C-AS (black) models at 1%.