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H_∞ filtering for stochastic singular fuzzy systems with time-varying delay

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Abstract This paper considers the H_∞ filtering problem for stochastic singular fuzzy systems with time-varying delay. We assume that the state and measurement are corrupted by stochastic uncertain exogenous disturbance and that the system dynamic is modeled by Ito-type stochastic differential equations. Based on an auxiliary vector and an integral inequality, a set of delay-dependent sufficient conditions is established, which ensures that the filtering error system is $e^{\lambda t}$-weighted integral input-to-state stable in mean (iISSiM). A fuzzy filter is designed such that the filtering error system is impulse-free, $e^{\lambda t}$-weighted iISSiM and the H_∞ attenuation level from disturbance to estimation error is below a prescribed scalar. A set of sufficient conditions for the solvability of the H_∞ filtering problem is obtained in terms of a new type of Lyapunov function and a set of linear matrix inequalities. Simulation examples are provided to illustrate the effectiveness of the proposed filtering approach developed in this paper.

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1 Introduction

In recent years, there has been increasing research interest in state estimation due to its theoretical and practical significance in control design and signal processing. One of the most celebrated estimation methods is Kalman filtering, which provides an optimal estimation of the state variables. However, it should be pointed out that one main shortcoming of Kalman filtering is that the priori statistical information of the external noise on the considered systems must be known. In view of this, an alternative estimation method based on H_∞ filtering technique has been proposed recently [1–4]. One of the main advantages of H_∞ filtering is that it is not necessary to exactly know the statistical properties of the external disturbance, but the external disturbance is assumed to have bounded energy. The objective of this paper was to design a filter such that the associated filtering error system satisfies a prescribed disturbance attenuation level.

On the other hand, with the growing complexity of dynamic systems, nonlinear systems have become popular research topics and have gained extensive attention. In nonlinear control theory, the T–S fuzzy approach that was first proposed by Takagi and Sugeno [5] has received increasing attention because the T–S

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fuzzy system is a systematic representation for nonlinear systems. It is well-known that, by using the T–S fuzzy model, the global behavior of a nonlinear system can be represented by a weighted sum of some locally simple linear subsystems. Many issues related to the stability analysis and control synthesis of T–S fuzzy systems have been studied [6–11] in the past decades. Recently, a class of fuzzy systems which is described by the singular form have been considered [12–14], where the model is an extension of T–S fuzzy model. A singular system is also called a descriptor system, differential algebraic system or a generalized state-space system, which arises from a convenient and natural modeling process in characterizing a class of practical systems. In [12], the problem of stability analysis for nonlinear singular systems with Markovian jumping parameters and mode-dependent interval time-varying delay was studied. New delay-dependent stability conditions were derived by constructing a mode-dependent Lyapunov function and using integral inequalities. A robust observer-based output feedback control for singular fuzzy systems in the presence of immeasurable states, approximation errors and uncertainty was proposed in [14]. Time-delay phenomena were first discovered in biological systems [40] and were later found in various practical systems, such as networked systems [41,42], population dynamics [43] and communication systems. They are often a source of instability and poor control performance. In the above actual systems, there is a phenomenon that the random and time-delay are presented at the same time. In order to accurately describe the dynamic characteristics of the systems, the stochastic time-delay systems are established and have attracted the attention of many researchers [34–37]. A large variety of important and interesting methods have been proposed for the analysis and design of singular fuzzy systems with time-delay [15,16].

The H_∞ filtering problem for singular systems has been of continuous interest because of its wide applications [17,18]. Based on the fact that T–S fuzzy model is a powerful tool to describe a nonlinear system, some authors used fuzzy approach to investigate the filtering problem for nonlinear singular systems with time-delay [19]. By applying T–S fuzzy model, a nonlinear dynamic system can be transformed to a set of linear subsystems via fuzzy rules. In this type of fuzzy model, local dynamics in different state-space regions are represented by linear models. So we can study the filtering problem of nonlinear systems by employing these methods which are used to deal with the filtering problem for linear systems. It should be noted that the H_∞ filtering problem of linear systems has been studied, and a great number of results have been reported (see [20–22]). However, compared with linear systems, the H_∞ filtering problem for nonlinear systems has not been fully investigated although it is important in control design and signal processing [23,24].

The term of input-to-state stable (ISS) was proposed by Sontag [25]. It plays an important role in stability analysis and controller design of deterministic nonlinear systems. The ISS of control systems has been widely studied, and many results have been obtained. Meanwhile, there have been various extensions for ISS, such as integral input-to-state stable (iISS), input-to-state stable in mean (ISSiM), $e^{\lambda t}$-weighted iISSiM and so on [26,27].

In addition, there is H_∞ filtering problem reported for a class of special nonlinear systems [38,39]. In [38], the H_∞ filtering problem was considered for a class of stochastic nonlinear systems with time-delay and the nonlinear term satisfying a Lipschitz constraint. We know that a T–S fuzzy system is a systematic representation for general nonlinear systems. Different from the special nonlinear systems, the H_∞ filtering problem is considered for general nonlinear systems which are descried by T–S fuzzy method in this paper. The focus is on the design of a fuzzy filter such that the corresponding filtering error system is $e^{\lambda t}$-weighted iISSiM and the H_∞ attenuation level from noise to estimation error is below a prescribed scalar. Based on an auxiliary vector, an integral inequality and a linear matrix inequalities (LMIs) technique, a set of sufficient conditions is proposed to ensure that the filtering error system is $e^{\lambda t}$-weighted iISSiM. The desired fuzzy filter is established in terms of a set of linear matrix inequalities. Three examples are provided to illustrate the effectiveness of the proposed fuzzy filter design method.

Notations The symbols $\mathbb{R}$ and $\mathbb{R}^+$ denote the set of real numbers and the set of nonnegative real numbers, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The superscript 'T' stands for matrix transposition. $\varepsilon\{\cdot\}$ denotes the expectation. The expression $\alpha \in K_\infty$ denotes that α is a K_∞ function. $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. I denotes the identity matrix. The expression $A < B$ means that the matrix $B - A$ is positive definite. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are used to denote the maximum and minimum eigenvalue of A, respectively. In symmetric block matrices, '$*$' is used as an ellipsis for terms induced by symmetry. $\|x\|$ denotes the Euclidean norm defined by $\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2}$ for every $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$. L_∞ denotes the set of all functions endowed with the essential supremum norm defined by $|u| = \sup\{\|u\|, t \geq 0, u \in \mathbb{R}^q\} < \infty$.

2 Problem formulation

Consider the following continuous-time T–S model with time-varying delay:
Plant rule i: IF $\mu_1(t)$ is F_{i1}, and $\mu_2(t)$ is F_{i2}, and ... and $\mu_p(t)$ is F_{ip}, THEN

$$\begin{aligned} E\mathrm{d}x(t) &= (A_i x(t) + A_{di} x(t - d(t)) + B_i u(t))\mathrm{d}t \\ &\quad + M_i x(t)\mathrm{d}\omega \\ \mathrm{d}y(t) &= (C_i x(t) + D_i u(t))\mathrm{d}t \\ z(t) &= L_i x(t) \\ x(t) &= \varphi(t), \qquad t \in [-\bar{d}, 0], \quad i = 1, 2, \ldots, r \end{aligned} \tag{1}$$

where $\mu_1(t), \mu_2(t), \ldots, \mu_p(t)$ are the premise variables and measurable, $F_{i1}, \ldots, F_{ip}$ are the fuzzy sets, r is the number of IF-THEN rules, $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in L_\infty$ is the input, $y(t) \in \mathbb{R}^m$ is the measurement output, $\omega(t)$ is a one-dimensional Brownian motion satisfying $\varepsilon\{\mathrm{d}\omega\} = 0$ and $\varepsilon\{\mathrm{d}\omega^2\} = \mathrm{d}t$, $z(t) \in \mathbb{R}^q$ is a linear combination of state variables to be estimated, $\varphi(t)$ is the initial condition relating to the time-varying delay $\mathrm{d}(t)$, which satisfies for all $t \geq 0$

$$0 < \mathrm{d}(t) \leq \bar{d}$$

where $\bar{d}$ is a scalar. The matrix $E \in \mathbb{R}^{n \times n}$ may be singular. The matrices A_i, A_{di}, B_i, M_i, C_i, D_i and L_i are known constant matrices with compatible dimensions.

By using a center-average defuzzifier, product inference and singleton fuzzier, the dynamic fuzzy model (1) can be represented by

$$\begin{aligned} E\mathrm{d}x(t) &= \sum_{i=1}^{r} h_i(\mu(t))[(A_i x(t) + A_{di} x(t - \mathrm{d}(t)) \\ &\quad + B_i u(t))\mathrm{d}t + M_i x(t)\mathrm{d}\omega] \\ \mathrm{d}y(t) &= \sum_{i=1}^{r} h_i(\mu(t))[(C_i x(t) + D_i u(t))\mathrm{d}t] \\ z(t) &= \sum_{i=1}^{r} h_i(\mu(t)) L_i x(t) \\ x(t) &= \varphi(t), \qquad t \in [-\bar{d}, 0] \end{aligned} \tag{2}$$

where $\mu(t) = [\mu_1(t), \mu_2(t), \ldots, \mu_p(t)]$, for $i = 1, 2, \ldots, r$,

$$\begin{aligned} h_i(\mu(t)) &= \frac{\bar{\vartheta}_i(\mu(t))}{\sum_{j=1}^{r} \bar{\vartheta}_j(\mu(t))}, \\ \bar{\vartheta}_i(\mu(t)) &= \prod_{j=1}^{p} F_{ij}(\mu_j(t)) \end{aligned} \tag{3}$$

and $F_{ij}(\mu_j(t))$ is the grade of membership function of $\mu_j(t)$ in F_{ij}. It is assumed that

$$\bar{\vartheta}_i(\mu(t)) \geq 0, \qquad \sum_{i=1}^{r} \bar{\vartheta}_i(\mu(t)) > 0$$

Therefore,

$$h_i(\mu(t)) \geq 0, \quad \sum_{i=1}^{r} h_i(\mu(t)) = 1 \tag{4}$$

Consider the following singular fuzzy filter:
Filter rule i: IF $\mu_1(t)$ is F_{i1}, and $\mu_2(t)$ is F_{i2}, and ... and $\mu_p(t)$ is F_{ip}, THEN

$$\begin{aligned} Edx_f(t) &= A_{fi} x_f(t)\mathrm{d}t + B_{fi}\mathrm{d}y(t) \\ z_f(t) &= L_{fi} x_f(t) \end{aligned}$$

where $x_f(t) \in \mathbb{R}^n$, $z_f(t) \in \mathbb{R}^q$, and A_{fi}, B_{fi} and L_{fi} are matrices to be determined. Then, the overall fuzzy filter can be inferred by

$$\begin{aligned} Edx_f(t) &= \sum_{i=1}^{r} h_i(\mu(t))[A_{fi} x_f(t)\mathrm{d}t + B_{fi}\mathrm{d}y(t)] \\ z_f(t) &= \sum_{i=1}^{r} h_i(\mu(t)) L_{fi} x_f(t) \end{aligned} \tag{5}$$

From (2) and (5), the filtering error system can be obtained as follows:

$$\begin{aligned} \bar{E}d\xi(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mu(t))h_j(\mu(t))[(\bar{A}_{ij}\xi(t) \\ &\quad + \bar{A}_{dij}\xi(t - \mathrm{d}(t)) + \bar{B}_{ij}u(t))\mathrm{d}t + \bar{M}_{ij}\xi(t)\mathrm{d}\omega] \\ \bar{z}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mu(t))h_j(\mu(t))\bar{L}_{ij}\xi(t) \end{aligned} \tag{6}$$

where

$$\xi(t)=\begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \quad \bar{z}(t)=z(t)-z_f(t), \quad \bar{E}=\begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}$$
$$\bar{A}_{ij}=\begin{bmatrix} A_i & 0 \\ B_{fj}C_i & A_{fj} \end{bmatrix}, \quad \bar{A}_{dij}=\begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{B}_{ij}=\begin{bmatrix} B_i \\ B_{fj}D_i \end{bmatrix} \bar{M}_{ij}=\begin{bmatrix} M_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{L}_{ij}=\begin{bmatrix} L_i & -L_{fj} \end{bmatrix}$$

Introduce an auxiliary vector function $\eta(t)$ such that

$$d\eta(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))[(\bar{A}_{ij}\xi(t) + \bar{A}_{dij}\xi(t-\mathrm{d}(t))+\bar{B}_{ij}u(t))\mathrm{d}t] \tag{7}$$

Using Eq. (6), the following equality is obtained:

$$\bar{E}d\xi(t)=d\eta(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\bar{M}_{ij}\xi(t)\mathrm{d}\omega$$

that is

$$\bar{E}\xi(t)-\bar{E}\xi(t-\mathrm{d}(t))=\eta(t)-\eta(t-\mathrm{d}(t)) + \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\left[\int_{t-\mathrm{d}(t)}^{t}\bar{M}_{ij}\xi(t)\mathrm{d}\omega\right] \tag{8}$$

Definition 1 [26] System (2) is $\mathrm{e}^{\lambda t}$-weighted iISSiM for some $\lambda>0$ if there exist functions $\alpha_1,\alpha_2,\bar{\gamma}\in K_\infty$ such that for any $u(t)\in L_\infty$, $x_0\in\mathbb{R}^n$,

$$\mathrm{e}^{\lambda t}\varepsilon[\alpha_1(\|x(t)\|)]\le\alpha_2(\|x_0\|)+\int_0^t \mathrm{e}^{\lambda s}\bar{\gamma}(|u(s)|)\mathrm{d}s \tag{9}$$

Remark 1 The iISS property that was introduced by Sontag [28] is weaker than the ISS. The iISS property has been shown to be a natural extension of ISS, and it is as useful as ISS in analysis of nonlinear control systems. The concept named as iISSiM was introduced in [29].

Assumption 1 $rank(\bar{E}\quad\bar{M})=rank(\bar{E})$ and the pair $(\bar{E},\bar{A})$ is regular, where $\bar{A}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\bar{A}_{ij}$ and $\bar{M}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\bar{M}_{ij}$.

Remark 2 Under the assumption above, the Ito stochastic term does not affect the system structure. It should be noted that the pair $(\bar{E},\bar{A})$ is regular if $\det(s\bar{E}-\bar{A})$ is not identically zero, which can guarantee the existence of solution to the singular system (6). At the same time, impulsive behavior may exist at initial time which may damage the singular system. It is necessary to deal with the impulsive behavior when a singular system is considered.

Lemma 1 [30] *The pair* $(\bar{E},\bar{A})$ *is impulse-free if and only if* $\bar{A}_4$ *is nonsingular, where there are nonsingular matrices* M_2 *and* N_2 *such that*

$$M_2\bar{E}N_2=\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad M_2\bar{A}N_2=\begin{bmatrix} \bar{A}_1 & \bar{A}_2 \\ \bar{A}_3 & \bar{A}_4 \end{bmatrix}$$

Lemma 2 [2] *If the following conditions hold:*

$$\Omega_{ii}<0, \quad i=1,2,\ldots,r$$
$$\frac{1}{r-1}\Omega_{ii}+\frac{1}{2}(\Omega_{ij}+\Omega_{ji})<0, \quad i\neq j, i, j=1,2,\ldots,r$$

then the following inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\Omega_{ij}<0$$

where Ω_{ij} *is an affine matrix-valued function,* $h_i(\mu(t))$ *satisfy* (3) *and* (4), $i, j=1,2,\ldots,r$.

The fuzzy H_∞ filtering problem addressed in this paper can be formulated as follows: given the fuzzy singular system (2) and a prescribed level of noise attenuation $\gamma>0$, determine a filter in form (5) such that the following requirements are satisfied:

(a) the filtering error system (6) is impulse-free and $\mathrm{e}^{\lambda t}$-weighted iISSiM;
(b) under the zero initial condition, the filtering error system (6) satisfies

$$\|\bar{z}\|_{\varepsilon 2}\le\gamma\|u\|_2$$

where

$$\|\bar{z}\|_{\varepsilon 2}=\left(\varepsilon\left\{\int_0^\infty|\bar{z}|^2\mathrm{d}t\right\}\right)^{\frac{1}{2}}$$

for all nonzero $u(t)\in L_2[0,\infty)$.

3 Main results

In this section, an LMI approach will be proposed to solve the fuzzy H_∞ filtering problem for system (6).

Theorem 1 *Given a scalar $\gamma > 0$, the filtering error system (6) is impulse-free and* $e^{\lambda t}$*-weighted iISSiM with the H_∞ performance $\gamma > 0$, if there exist matrices $P, Q_1 > 0, R > 0, H > 0, S_{1ij}, S_{2ij}, S_{3ij}, S_{4ij}$ and S_{5ij} for $i, j = 1, 2, \ldots, r$, such that the following inequalities hold:*

$$\bar{E}^T P = P^T \bar{E} \geq 0 \tag{10}$$

$$\Pi_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{11}$$

$$\frac{1}{r-1}\Pi_{ii} + \frac{1}{2}(\Pi_{ij} + \Pi_{ji}) < 0, \quad i \neq j,\ i, j = 1, 2, \ldots, r \tag{12}$$

where

$$\Pi_{ij} = \begin{bmatrix} \Phi_{1ij} & \Phi_{2ij} \\ * & \Phi_{3ij} \end{bmatrix}$$

$$\Phi_{1ij} = \begin{bmatrix} \varphi_{11ij} & \varphi_{12ij} & \bar{E}^T S_{3ij} & P^T \bar{B}_{ij} \\ * & \varphi_{22ij} & -\bar{E}^T S_{3ij} & S_{5ij}^T \bar{B}_{ij} \\ * & * & -Q_1 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Phi_{2ij} = \begin{bmatrix} -S_{1ij}^T & \bar{A}_{ij}^T S_{4ij} & \bar{L}_{ij}^T \\ -S_{2ij}^T & \varphi_{26ij} & 0 \\ -S_{3ij}^T & 0 & 0 \\ 0 & \bar{B}_{ij}^T S_{4ij} & 0 \end{bmatrix}, \Phi_{3ij} = \begin{bmatrix} -R & 0 & 0 \\ * & \varphi_{66ij} & 0 \\ * & * & -I \end{bmatrix}$$

$$\varphi_{11ij} = P^T \bar{A}_{ij} + \bar{A}_{ij}^T P + \bar{M}_{ij}^T (\bar{E}^+)^T \bar{E}^T P \bar{E}^+ \bar{M}_{ij} + Q_1 + S_{1ij}^T \bar{E} + \bar{E}^T S_{1ij} + H$$

$$\varphi_{12ij} = P^T \bar{A}_{dij} + \bar{A}_{ij}^T S_{5ij} - S_{1ij}^T \bar{E} + \bar{E}^T S_{2ij}$$

$$\varphi_{22ij} = S_{5ij}^T \bar{A}_{dij} + \bar{A}_{dij}^T S_{5ij} - S_{2ij}^T \bar{E} - \bar{E}^T S_{2ij}$$

$$\varphi_{26ij} = \bar{A}_{dij}^T S_{4ij} - S_{5ij}^T$$

$$\varphi_{66ij} = \bar{d}^2 R - S_{4ij} - S_{4ij}^T$$

Proof We first show that the system (6) is impulse-free. Under the Assumption 1, the pair $(\bar{E}, \bar{A})$ is regular. Then, there are nonsingular matrices G and K such that

$$G\bar{E}K = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad G\bar{A}K = \begin{bmatrix} \bar{A}_1 & \bar{A}_2 \\ \bar{A}_3 & \bar{A}_4 \end{bmatrix}$$

$$G^{-T}PK = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

From (10), it follows that

$$K^T \bar{E}^T G^T G^{-T} P K = K^T P^T G^{-1} G \bar{E} K \geq 0$$

that is

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11}^T & P_{21}^T \\ P_{12}^T & P_{22}^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \geq 0$$

then $P_{11} = P_{11}^T$, $P_{12} = 0$.

From (11) and (12), the following inequality holds:

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\mu(t)) h_j(\mu(t)) \{P^T \bar{A}_{ij} + \bar{A}_{ij}^T P + Q_1 + H + \bar{M}_{ij}^T (\bar{E}^+)^T \bar{E}^T P \bar{E}^+ \bar{M}_{ij} + \bar{E}^T S_{1ij} + S_{1ij}^T \bar{E}\} < 0$$

It implies that

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\mu(t)) h_j(\mu(t)) \{P^T \bar{A}_{ij} + \bar{A}_{ij}^T P + \bar{M}_{ij}^T (\bar{E}^+)^T \bar{E}^T P \bar{E}^+ \bar{M}_{ij} + \bar{E}^T S_{1ij} + S_{1ij}^T \bar{E}\} < 0$$

Thus,

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\mu(t)) h_j(\mu(t)) \{K^T P^T G^{-1} G \bar{A}_{ij} K + K^T \bar{A}_{ij}^T G^T G^{-T} P K + K^T (\bar{M}_{ij}^T (\bar{E}^+)^T \bar{E}^T P \bar{E}^+ \bar{M}_{ij} + \bar{E}^T S_{1ij} + S_{1ij}^T \bar{E}) K\} < 0$$

that is

$$\begin{bmatrix} * & * \\ * & P_{22}^T \bar{A}_4 + \bar{A}_4^T P_{22} \end{bmatrix} < 0$$

It is easy to see that

$$P_{22}^T \bar{A}_4 + \bar{A}_4^T P_{22} < 0 \tag{13}$$

which implies that $\bar{A}_4$ is nonsingular. According to Lemma 1 1, it is easy to find that the pair $(\bar{E}, \bar{A})$ is impulse-free.

Next we show that the system (6) is $e^{\lambda t}$-weighted iISSiM. Choose a Lyapunov function as follows:

$$V = V_1 + V_2 + V_3 \tag{14}$$

where

$$V_1 = \xi^T(t) \bar{E}^T P \xi(t) \tag{15}$$

$$V_2 = \int_{t-\bar{d}}^{t} \xi^T(s) Q_1 \xi(s) \mathrm{d}s \tag{16}$$

$$V_3 = \bar{d} \int_{-\bar{d}}^{0} \int_{t+\theta}^{t} \dot{\eta}^T(s) R \dot{\eta}(s) \mathrm{d}s \mathrm{d}\theta \tag{17}$$

Let Λ be the weak infinitesimal operator. Using the Ito differential formula and the following inequality

$$-\int_{t-\bar{d}}^{t}\dot{\eta}^{\mathrm{T}}(s)R\dot{\eta}(s)\mathrm{d}s \le -\frac{1}{\bar{d}}\int_{t-\mathrm{d}(t)}^{t}\dot{\eta}^{\mathrm{T}}(s)\mathrm{d}s R\int_{t-\mathrm{d}(t)}^{t}\dot{\eta}(s)\mathrm{d}s$$

then we can obtain

$$\begin{aligned}\mathit{\Lambda}V_1 =& 2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\left\{\xi^{\mathrm{T}}(t)P^{\mathrm{T}}(\bar{A}_{ij}\xi(t)\right.\\ &\left.+\bar{A}_{dij}\xi(t-\mathrm{d}(t))+\bar{B}_{ij}u(t))\right\}\\ &+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}h_i(\mu(t))h_j(\mu(t))h_m(\mu(t))h_n(\mu(t))\\ &\times\xi^{\mathrm{T}}(t)\bar{M}_{ij}(\bar{E}^{+})^{\mathrm{T}}\bar{E}^{\mathrm{T}}P\bar{E}^{+}\bar{M}_{mn}\xi(t)\\ \le& \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\left\{\xi^{\mathrm{T}}(t)(\bar{A}_{ij}^{\mathrm{T}}P+P^{\mathrm{T}}\bar{A}_{ij}\right.\\ &+\bar{M}_{ij}(\bar{E}^{+})^{\mathrm{T}}\bar{E}^{\mathrm{T}}P^{\mathrm{T}}\bar{E}^{+}\bar{M}_{ij})\xi(t)\\ &\left.+2\xi^{\mathrm{T}}(t)P^{\mathrm{T}}\bar{A}_{dij}\xi(t-\mathrm{d}(t))+2\xi^{\mathrm{T}}(t)P^{\mathrm{T}}\bar{B}_{ij}u(t)\right\}\end{aligned}\tag{18}$$

$$\mathit{\Lambda}V_2 = \xi^{\mathrm{T}}(t)Q_1\xi(t)-\xi^{\mathrm{T}}(t-\bar{d})Q_1\xi(t-\bar{d})\tag{19}$$

$$\begin{aligned}\mathit{\Lambda}V_3 \le& \bar{d}^2\dot{\eta}^{\mathrm{T}}(t)R\dot{\eta}(t)\\ &-(\eta(t)-\eta(t-\mathrm{d}(t)))^{\mathrm{T}}R(\eta(t)-\eta(t-\mathrm{d}(t)))\end{aligned}\tag{20}$$

where

$$\begin{aligned}\zeta(t) =& \big[\,\xi^{\mathrm{T}}(t)\ \xi^{\mathrm{T}}(t-\mathrm{d}(t))\ \xi^{\mathrm{T}}(t-\bar{d})\ u^{\mathrm{T}}(t)\\ &\dot{\eta}^{\mathrm{T}}(t)\ \eta^{\mathrm{T}}(t)-\eta^{\mathrm{T}}(t-\mathrm{d}(t))\,\big]^{\mathrm{T}}\end{aligned}$$

From (7) and (8), for appropriate matrices $S_{kij}, k = 1, 2, \ldots, 5$,

$$\begin{aligned}&2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\zeta^{\mathrm{T}}(t)\bar{S}_{ij}^{\mathrm{T}}[\bar{E}\xi(t)-\bar{E}\xi(t-\mathrm{d}(t))\\ &-(\eta(t)-\eta(t-\mathrm{d}(t)))-\int_{t-\mathrm{d}(t)}^{t}\bar{M}_{ij}\xi(s)\mathrm{d}\omega]=0\end{aligned}\tag{21}$$

$$\begin{aligned}&2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\zeta^{\mathrm{T}}(t)\tilde{S}_{ij}^{\mathrm{T}}[\bar{A}_{ij}\xi(t)\\ &+\bar{A}_{dij}\xi(t-\mathrm{d}(t))+\bar{B}_{ij}u(t)-\dot{\eta}(t)]=0\end{aligned}\tag{22}$$

where

$$\bar{S}_{ij} = \begin{bmatrix} S_{1ij} & S_{2ij} & S_{3ij} & 0 & 0 & 0\end{bmatrix}$$
$$\tilde{S}_{ij} = \begin{bmatrix} 0 & S_{5ij} & 0 & 0 & S_{4ij} & 0\end{bmatrix}$$

By direct calculation, it is easy to obtain

$$\begin{aligned}\mathit{\Lambda}V \le& \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\zeta^{\mathrm{T}}(t)\Pi_{1ij}\zeta(t)\\ &+\rho u^{\mathrm{T}}(t)u(t)-\xi^{\mathrm{T}}(t)H\xi(t)\end{aligned}\tag{23}$$

where

$$\mathit{\Pi}_{1ij} = \begin{bmatrix}\bar{\Phi}_{1ij} & \bar{\Phi}_{2ij}\\ * & \bar{\Phi}_{3ij}\end{bmatrix}$$

$$\bar{\Phi}_{1ij} = \begin{bmatrix}\varphi_{11ij} & \varphi_{12ij} & \bar{E}^{\mathrm{T}}S_{3ij} & P^{\mathrm{T}}\bar{B}_{ij}\\ * & \varphi_{22ij} & -\bar{E}^{\mathrm{T}}S_{3ij} & S_{5ij}^{\mathrm{T}}\bar{B}_{ij}\\ * & * & -Q_1 & 0\\ * & * & * & -\rho I\end{bmatrix}$$

$$\bar{\Phi}_{2ij} = \begin{bmatrix}-S_{1ij}^{\mathrm{T}} & \bar{A}_{ij}^{\mathrm{T}}S_{4ij}\\ -S_{2ij}^{\mathrm{T}} & \varphi_{26ij}\\ -S_{3ij}^{\mathrm{T}} & 0\\ 0 & \bar{B}_{ij}^{\mathrm{T}}S_{4ij}\end{bmatrix},\quad \bar{\Phi}_{3ij} = \begin{bmatrix}-R & 0\\ * & \varphi_{66ij}\end{bmatrix}$$

Let $\rho = \gamma^2$. Using (11)–(12),

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\mu(t))h_j(\mu(t))\mathit{\Pi}_{1ij} < 0$$

Then,

$$\mathit{\Lambda}V \le \rho u^{\mathrm{T}}(t)u(t)-\xi^{\mathrm{T}}(t)H\xi(t)\tag{24}$$

By the definition of Lyapunov function V in (14), there exist scalars λ_1, λ_2 and λ_3 satisfying

$$\begin{aligned}V \le& \lambda_1\xi^{\mathrm{T}}(t)\xi(t)+\lambda_2\int_{t-\bar{d}}^{t}\xi^{\mathrm{T}}(s)\xi(s)\mathrm{d}s\\ &+\lambda_3\int_{t-\bar{d}}^{t}\xi^{\mathrm{T}}(s-\mathrm{d}(s))\xi(s-\mathrm{d}(s))\mathrm{d}s+\bar{\rho}u^{\mathrm{T}}(t)u(t)\end{aligned}$$

Define a new function

$$W(t) = \mathrm{e}^{\lambda t}V(\xi(t))$$

where $\lambda > 0$ is a scalar.
Then, the following relationship is true:

$$\begin{aligned}\varepsilon\left\{\mathrm{e}^{\lambda t}V\right\} =& \varepsilon\{V_0\}+\varepsilon\left\{\int_0^t \mathrm{e}^{\lambda s}(\lambda V+\mathit{\Lambda}V)\mathrm{d}s\right\}\\ \le& \varepsilon\{V_0\}+\varepsilon\left\{\int_0^t \mathrm{e}^{\lambda s}[\lambda(\lambda_1\xi^{\mathrm{T}}(s)\xi(s)\right.\\ &+\lambda_2\int_{s-\bar{d}}^{s}\xi^{\mathrm{T}}(\theta)\xi(\theta)d\theta+\bar{\rho}u^{\mathrm{T}}(t)u(t)\\ &\left.+\lambda_3\int_{s-\bar{d}}^{s}\xi^{\mathrm{T}}(\theta-\mathrm{d}(\theta))\xi(\theta-\mathrm{d}(\theta))\mathrm{d}\theta)+\mathit{\Lambda}V]\mathrm{d}s\right\}\end{aligned}\tag{25}$$

Taking into account the following integral inequality:

$$\int_0^t \int_{s-\bar{d}}^s e^{\lambda s}\xi^T(\theta)\xi(\theta)d\theta ds \le \bar{d}e^{\lambda\bar{d}}\int_0^t e^{\lambda s}\xi^T(s)\xi(s)ds$$

it follows that

$$\begin{aligned}\varepsilon\left\{e^{\lambda t}V\right\} &\le \varepsilon\{V_0\} \\ &\quad + \varepsilon\left\{\int_0^t \lambda(\lambda_1+\lambda_2\bar{d}e^{\lambda\bar{d}}+\lambda_3\bar{d}e^{2\lambda\bar{d}})e^{\lambda s}\xi^T(s)\xi(s)ds \right.\\ &\quad \left. + \int_0^t e^{\lambda s}(\lambda\bar{\rho}u^T(s)u(s)+\Lambda V)ds\right\} \qquad (26)\\ &= \varepsilon\{V_0\} \\ &\quad + \varepsilon\left\{\int_0^t e^{\lambda s}(\hat{\lambda}\xi^T(s)\xi(s)+\lambda\bar{\rho}u^T(s)u(s)+\Lambda V)ds\right\}\end{aligned}$$

where $\hat{\lambda} = \lambda(\lambda_1+\lambda_2\bar{d}e^{\lambda\bar{d}}+\lambda_3\bar{d}e^{2\lambda\bar{d}})$.

Using the (24), it is easy to see that

$$\Lambda V + \xi^T(t)H\xi(t) \le \rho u^T(t)u(t)$$

It is clear that there exists a scalar $\hat{\lambda} > 0$, such that $\hat{\lambda} = \lambda_{\min}(H)$. Then

$$\begin{aligned}\varepsilon\left\{e^{\lambda t}V\right\} &\le \varepsilon\{V_0\} + \varepsilon\left\{\int_0^t e^{\lambda s}(\rho+\lambda\bar{\rho})u^T(s)u(s)ds\right\}\\ &= \varepsilon\{V_0\} + \varepsilon\left\{\int_0^t e^{\lambda s}\bar{\beta}u^T(s)u(s)ds\right\}\\ &\le \varepsilon\{\bar{\alpha}(\|x_0\|)\} + \varepsilon\left\{\int_0^t e^{\lambda s}\bar{\beta}u^T(s)u(s)ds\right\}\end{aligned}$$

which implies that

$$e^{\lambda t}\varepsilon\left\{\hat{\alpha}(\|x\|)\right\} \le \bar{\alpha}(\|x_0\|) + \int_0^t e^{\lambda s}\bar{\beta}(|u(s)|)ds$$

where $\hat{\alpha}, \bar{\alpha}, \bar{\beta} \in K_\infty$.

From Definition 1, the system (6) is $e^{\lambda t}$-weighted iISSiM.

Finally, consider the H_∞ performance of the system (6). Define the following index for system (6):

$$\begin{aligned}J &= \varepsilon\left\{\int_0^t (\bar{z}^T(s)\bar{z}(s)-\gamma^2u^T(s)u(s))ds\right\}\\ &= \varepsilon\left\{\int_0^t (\bar{z}^T(s)\bar{z}(s)-\gamma^2u^T(s)u(s)+\Lambda V)ds\right\}\\ &\quad - \varepsilon(V)\\ &\le \varepsilon\left\{\int_0^t (\bar{z}^T(s)\bar{z}(s)-\gamma^2u^T(s)u(s)+\Lambda V)ds\right\} \qquad (27)\\ &\le \varepsilon\left\{\sum_{i=1}^r\sum_{j=1}^r h_i(\mu(t))h_j(\mu(t))\right.\\ &\quad \left.\times\int_0^t (\zeta^T(s)\Pi_{ij}\zeta(s)-\xi^T(s)H\xi(s))ds\right\}\end{aligned}$$

By inequalities (11)–(12),

$$\sum_{i=1}^r\sum_{j=1}^r h_i(\mu(t))h_j(\mu(t))\Pi_{ij} < 0$$

Then,

$$J \le \varepsilon\left\{\int_0^t (-\xi^T(s)H\xi(s))ds\right\} < 0$$

Therefore, under zero initial condition, $\|\bar{z}\|_{\varepsilon_2} \le \gamma\|u\|_2$ for any nonzero $u(t) \in L_\infty$. This completes the proof.

Remark 3 Theorem 1 provides a set of sufficient conditions under which the filtering error system (6) is impulse-free and $e^{\lambda t}$-weighted iISSiM based on the auxiliary vector method. When the Ito stochastic term is zero, the system (6) reduces to a deterministic singular system with time-varying delay, and in this case, the auxiliary vector $d\eta(t) = d\xi(t)$.

Remark 4 Many of the results existing in the literature usually demand the upper bound of the derivative of time-delay to be known. However, in this paper, the time-delay $\mathrm{d}(t)$ need not to be differentiable or although $\mathrm{d}(t)$ is differentiable, the upper bound of derivative of time-delay need not to be known that is the time-delay $\mathrm{d}(t)$ only satisfies the condition $0 < \mathrm{d}(t) \le \bar{d}$. So this criterion includes some existing results as its special cases. Furthermore, in the proof of Theorem 1, the fuzzy-rule-dependent matrices S_{kij} are introduced, which make the result be less conservative.

The main result on the solvability of the fuzzy filtering problem is ready to be presented.

Theorem 2 *Consider the singular fuzzy system (2). Let $\gamma > 0$ be a constant scalar. The H_∞ fuzzy filtering problem is solvable, if there exist matrices X, Y, $Q_{11} > 0$, $H_1 > 0$, $H_2 > 0$, $R_{11} > 0$, A_{Fi}, B_{Fi}, L_{Fi}, S_{11ij}, S_{21ij}, S_{31ij}, S_{41ij} and S_{51ij} for $i, j = 1, 2, \ldots, r$ such that the following inequalities hold:*

$$E^TX = X^TE \ge 0 \qquad (28)$$

$$E^TY = Y^TE \ge 0 \qquad (29)$$

$$E^T(X-Y) \ge 0 \qquad (30)$$

$$\Xi_{ii} < 0, \quad i = 1, 2, \ldots, r \qquad (31)$$

$$\frac{1}{r-1}\Xi_{ii} + \frac{1}{2}(\Xi_{ij}+\Xi_{ji}) < 0,\quad i \ne j,\ i, j = 1, 2, \ldots, r \qquad (32)$$

where

$$\Xi_{ii} = \begin{bmatrix} \Xi_{1ii} & \Xi_{2ii} \\ * & \Xi_{3ii} \end{bmatrix}, \quad \Xi_{ij} = \begin{bmatrix} \Xi_{1ij} & \Xi_{2ij} \\ * & \Xi_{3ij} \end{bmatrix}$$

$$\Xi_{1ij} = \begin{bmatrix} \Omega_{1ij} & \Omega_{2ij} & \Omega_{3ij} & E^T S_{31ij} \\ * & \Omega_{4ij} & \Omega_{5ij} & E^T S_{31ij} \\ * & * & \Omega_{6ij} & -E^T S_{31ij} \\ * & * & * & -Q_{11} \end{bmatrix}$$

$$\Xi_{2ij} = \begin{bmatrix} Y^T B_i & -S_{11ij}^T & A_i^T S_{41ij} & L_i^T - L_{Fj}^T \\ \Omega_{7ij} & -S_{11ij}^T & A_i^T S_{41ij} & L_i^T \\ S_{51ij}^T B_i & -S_{21ij}^T & \Omega_{8ij} & 0 \\ 0 & -S_{31ij}^T & 0 & 0 \end{bmatrix}$$

$$\Xi_{3ij} = \begin{bmatrix} -\gamma^2 I & 0 & B_i^T S_{41ij} & 0 \\ * & -R_{11} & 0 & 0 \\ * & * & \Omega_{9ij} & 0 \\ * & * & * & -I \end{bmatrix}$$

$$\begin{aligned}
\Omega_{1ij} &= Y^T A_i + A_i^T Y + M_i^T (E^+)^T E^T X E^+ M_i \\
&\quad + Q_{11} + S_{11ij}^T E + E^T S_{11ij} + H_1 + H_2 \\
\Omega_{2ij} &= Y^T A_i + A_i^T X + C_i^T B_{Fj} + A_{Fj} + Q_{11} + S_{11ij}^T E \\
&\quad + E^T S_{11ij} + H_1 + M_i^T (E^+)^T E^T X E^+ M_i \\
\Omega_{3ij} &= Y^T A_{di} + A_i^T S_{51ij} - S_{11ij}^T E + E^T S_{21ij} \\
\Omega_{4ij} &= X^T A_i + B_{Fj}^T C_i + C_i^T B_{Fj} + A_i^T X + Q_{11} + H_1 \\
&\quad + E^T S_{11ij} + S_{11ij}^T E + M_i^T (E^+)^T E^T X E^+ M_i \\
\Omega_{5ij} &= X^T A_{di} + A_i^T S_{51ij} - S_{11ij}^T E + E^T S_{21ij} \\
\Omega_{6ij} &= S_{51ij}^T A_{di} + A_{di}^T S_{51ij} - S_{21ij}^T E - E^T S_{21ij} \\
\Omega_{7ij} &= X^T B_i + B_{Fj} D_i \\
\Omega_{8ij} &= - S_{51ij}^T + A_{di}^T S_{41ij} \\
\Omega_{9ij} &= \bar{d}^2 R_{11} - S_{41ij}^T - S_{41ij}
\end{aligned}$$

In this case, there exist nonsingular matrices U, $\bar{U}$, W, $\bar{W}$ such that

$$E^{\mathrm{T}} \bar{U} = U^{\mathrm{T}} E, \quad E W = \bar{W}^{\mathrm{T}} E^{\mathrm{T}}$$
$$X Y^{-1} = I - \bar{U} W, \quad Y^{-1} X = I - \bar{W} U$$

Then, the desired filter can be chosen as in (5) with the following parameters:

$$A_{fi}^{\mathrm{T}} = W^{-T} Y^{-T} A_{Fi} U^{-1}, \quad B_{fi}^{\mathrm{T}} = B_{Fi} U^{-1}$$
$$L_{fi} = L_{Fi} Y^{-1} W^{-1} \tag{33}$$

Proof Define

$$\Delta_1 = \begin{bmatrix} Y^{-1} & I \\ W & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} I & X \\ 0 & U \end{bmatrix}, \quad \Delta_3 = \begin{bmatrix} Y & 0 \\ 0 & I \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} Q_{11} & 0 \\ 0 & \delta Q_{12} \end{bmatrix}, R = \begin{bmatrix} R_{11} & 0 \\ 0 & \delta R_{12} \end{bmatrix}, H = \begin{bmatrix} H_1 & 0 \\ 0 & \bar{H}_2 \end{bmatrix}$$

$$P = \Delta_2 \Delta_1^{-1} = \begin{bmatrix} X & \bar{U} \\ U & -U Y^{-1} W^{-1} \end{bmatrix}$$

$$S_{kij} = \begin{bmatrix} S_{k1ij} & 0 \\ 0 & \delta S_{k2ij} \end{bmatrix}, \quad k = 1, 2, \ldots, 5$$

Using the method in [31] and substituting P into (10), it is straight forward to see that (28)–(30) hold.
At the same time, set

$$\Delta = diag \left\{ \Delta_1 \Delta_3, I, I, I, I, I, I \right\} \tag{34}$$

$$A_{Fi} = Y^{\mathrm{T}} W^{\mathrm{T}} A_{fi} U, \quad B_{Fi} = B_{fi}^{\mathrm{T}} U, \quad L_{Fi} = L_{fi} W Y$$
$$H_2 = Y^{\mathrm{T}} W^{\mathrm{T}} \bar{H}_2 W Y$$

Assuming that δ approaches 0. Using the Schur complement, and pre- and post-multiplying (11) and (12) by Δ^{T} and Δ, respectively, then (31) and (32) hold. This completes the proof.

In the case when $rank(E) = n$ in the singular system (2), from Theorem 2, the following fuzzy H_∞ filtering result can be obtained directly.

Corollary 1 *Consider fuzzy system (2) with $E = I$. Let $\gamma > 0$ be a constant scalar. The H_∞ fuzzy filtering problem is solvable, if there exist matrices $X > 0$, $Y > 0$, $Q_{11} > 0$, $H_1 > 0$, $H_2 > 0$, $R_{11} > 0$, A_{Fi}, B_{Fi}, L_{Fi}, S_{11ij}, S_{21ij}, S_{31ij}, S_{41ij} and S_{51ij} for $i, j = 1, 2, \ldots, r$ such that the following inequalities hold:*

$$\Xi_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{35}$$

$$\frac{1}{r-1} \Xi_{ii} + \frac{1}{2} (\Xi_{ij} + \Xi_{ji}) < 0, \quad i \neq j, \ i, j = 1, 2, \ldots, r \tag{36}$$

where

$$\Xi_{ii} = \begin{bmatrix} \Xi_{1ii} & \Xi_{2ii} \\ * & \Xi_{3ii} \end{bmatrix}, \Xi_{ij} = \begin{bmatrix} \Xi_{1ij} & \Xi_{2ij} \\ * & \Xi_{3ij} \end{bmatrix}$$

$$\Xi_{1ij} = \begin{bmatrix} \Omega_{1ij} & \Omega_{2ij} & \Omega_{3ij} & S_{31ij} \\ * & \Omega_{4ij} & \Omega_{5ij} & S_{31ij} \\ * & * & \Omega_{6ij} & -S_{31ij} \\ * & * & * & -Q_{11} \end{bmatrix}$$

$$\Xi_{2ij} = \begin{bmatrix} Y^T B_i & -S_{11ij}^T & A_i^T S_{4ij} & L_i^T - L_{Fj}^T \\ \Omega_{7ij} & -S_{11ij}^T & A_i^T S_{4ij} & L_i^T \\ S_{51ij}^T B_i & -S_{21ij}^T & \Omega_{8ij} & 0 \\ 0 & -S_{31ij}^T & 0 & 0 \end{bmatrix}$$

$$\Xi_{3ij} = \begin{bmatrix} -\gamma^2 I & 0 & B_i^T S_{41ij} & 0 \\ * & -R_{11} & 0 & 0 \\ * & * & \Omega_{9ij} & 0 \\ * & * & * & -I \end{bmatrix}$$

$$\begin{aligned}
\Omega_{1ij} =& Y^T A_i + A_i^T Y + M_i^T X M_i + Q_{11} + S_{11ij}^T \\
&+ S_{11ij} + H_1 + H_2 \\
\Omega_{2ij} =& Y^T A_i + A_i^T X + C_i^T B_{Fj} + A_{Fj} + Q_{11} + S_{11ij}^T \\
&+ S_{11ij} + H_1 + M_i^T X M_i \\
\Omega_{3ij} =& Y^T A_{di} + A_i^T S_{51ij} - S_{11ij}^T + S_{21ij} \\
\Omega_{4ij} =& X^T A_i + B_{Fj}^T C_i + C_i^T B_{Fj} + A_i^T X + Q_{11} \\
&+ S_{11ij}^T + S_{11ij} + H_1 + M_i^T X M_i \\
\Omega_{5ij} =& X^T A_{di} + A_i^T S_{51ij} - S_{11ij}^T + S_{21ij} \\
\Omega_{6ij} =& S_{51ij}^T A_{di} + A_{di}^T S_{51ij} - S_{21ij}^T - S_{21ij} \\
\Omega_{7ij} =& X^T B_i + B_{Fj} D_i \\
\Omega_{8ij} =& - S_{51ij}^T + A_{di}^T S_{41ij} \\
\Omega_{9ij} =& \bar{d}^2 R_{11} - S_{41ij}^T - S_{41ij}
\end{aligned}$$

In this case, there exist nonsingular matrices U and W such that

$$U^{\mathrm{T}} W = I - XY^{-1}$$

Then, the designed filter can be chosen as in (5) with the following parameters:

$$\begin{aligned} A_{fi}^{\mathrm{T}} &= W^{-T} Y^{-T} A_{Fi} U^{-1}, \quad B_{fi}^{\mathrm{T}} = B_{Fi} U^{-1} \\ L_{fi} &= L_{Fi} Y^{-1} W^{-1} \end{aligned} \tag{37}$$

4 Simulation examples

In this section, three examples are to be presented to illustrate the effectiveness of the proposed filter design method.

Example 1 Consider the stochastic singular T–S fuzzy system (2) with the following parameters:

Subsystems 1:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -6.3 & 0.2 & 0.4 \\ 0.3 & -3.4 & 1.2 \\ 0.2 & 0.5 & -4.5 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.2 & 0 & 0.2 \\ 0.1 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, M_1 = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & 0.1 & 0.2 \\ 0 & 0.1 & 0.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -2.1 & 0.6 & 1.3 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.7 & 0.8 & 1.5 \end{bmatrix}, D_1 = 0.3$$

Subsystems 2:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -5.5 & 0.3 & 0.6 \\ 0.2 & -4.6 & 0.5 \\ 0.3 & 0.8 & -3.9 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0.3 & 0.2 & 0.1 \\ 0 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix}, M_2 = \begin{bmatrix} 0.2 & 0.0 & 0.1 \\ 0.1 & 0.1 & 0.2 \\ 0 & 0.1 & 0.2 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1.5 \\ 0.6 \\ 0.2 \end{bmatrix}, C_2 = \begin{bmatrix} 0.6 & 0.3 & 0.7 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.5 & 0.2 & 0.6 \end{bmatrix}, D_2 = 0.2$$

The delay is assumed as $\mathrm{d}(t) = 1.2\,|\sin t|$, and a straightforward calculation gives $\bar{d} = 1.2$. The membership functions are selected as follows:

$$h_1 = \frac{1 - \sin(x_1)}{2}, \qquad h_2 = \frac{1 + \sin(x_1)}{2}$$

The disturbance attenuation level is chosen to be $\gamma = 0.9$. By using the Matlab LMI Control Toolbox in Theorem 2, the filter parameters can be obtained as follows:

$$A_{f1} = \begin{bmatrix} -87.2088 & -85.6361 & 16.4673 \\ -30.1708 & -44.9864 & 7.7489 \\ -10.8685 & -36.6665 & -7.5161 \end{bmatrix}$$

$$B_{f1} = \begin{bmatrix} 4.7756 \\ 2.2538 \\ -0.5894 \end{bmatrix}$$

$$L_{f1} = \begin{bmatrix} 10.4019 & 25.1335 & 3.0337 \end{bmatrix}$$

$$A_{f2} = \begin{bmatrix} -36.7414 & -25.8057 & 5.5603 \\ -14.5264 & -14.0861 & 2.6494 \\ -32.0722 & -28.7182 & -5.1419 \end{bmatrix}$$

$$B_{f2} = \begin{bmatrix} -3.0647 \\ -2.2366 \\ -1.1250 \end{bmatrix}$$

$$L_{f2} = \begin{bmatrix} 3.1201 & -2.0513 & 1.3006 \end{bmatrix}$$

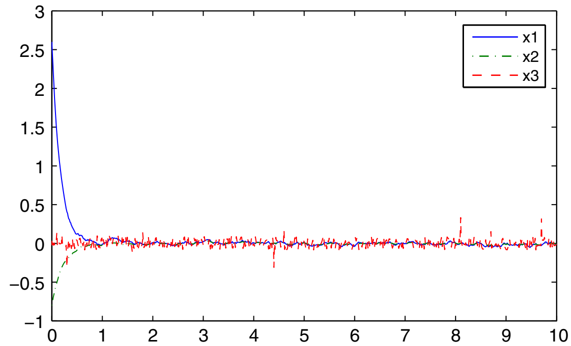

Fig. 1 State responses of the plant (2)

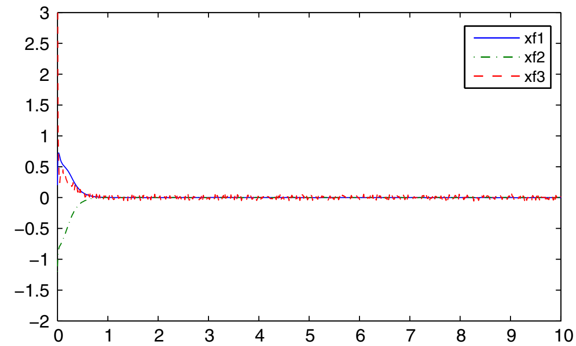

Fig. 2 State responses of the filter (5)

The simulation results of the state responses of the plant (2) and filter (5) are given in Figs. 1 and 2, respectively, which implied that the filtering error system is stable. Figure 3 gives the signals $z(t)$ and $z_f(t)$. Figure 4 shows the simulation result of the filtering error $\bar{z}(t) = z(t) - z_f(t)$. From the Figs. 1, 2, 3 and 4, it follows that the designed H_∞ filter has the desired performance.

In many literatures about H_∞ filtering for time-delay systems, the time-delay usually satisfies the following condition:

$$d_1 \leq \dot{d}(t) \leq d_2$$

This means that the time-delay function must be differentiable. But in many practical applications, the time-delay function is often not differentiable. This condition limits the application for estimation methods of many literatures. In this paper, we do not need to consider whether the time-delay function is differentiable or not. In this example, we can find that the time-delay $\mathrm{d}(t) = 1.2\,|\sin t|$ is not differentiable at the point 0. So the approaches that have been introduced in [17,24] are unavailable. This example shows that signal $z(t)$ can be well estimated by the filter and the filtering error system is impulse-free, $e^{\lambda t}$-weighted iISSiM and the H_∞ attenuation level from disturbance to estimation error is below a prescribed scalar.

Example 2 Consider the following stochastic system borrowed from [32]:

$$\begin{aligned} \mathrm{d}x(t) &= (Ax(t) + A_d x(t-d) + Bu(t))\mathrm{d}t + Mx(t)\mathrm{d}\omega \\ \mathrm{d}y(t) &= (Cx(t) + Du(t))\mathrm{d}t \\ z(t) &= Lx(t) \end{aligned}$$

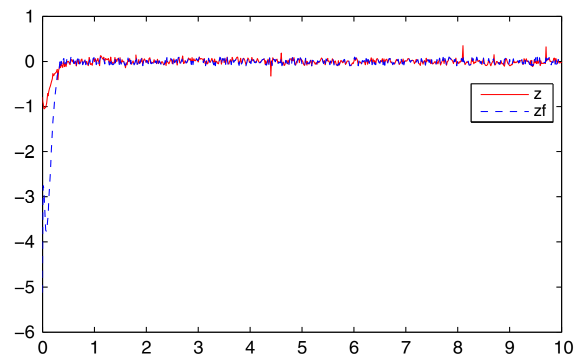

Fig. 3 Signals $z(t)$ and $z_f(t)$

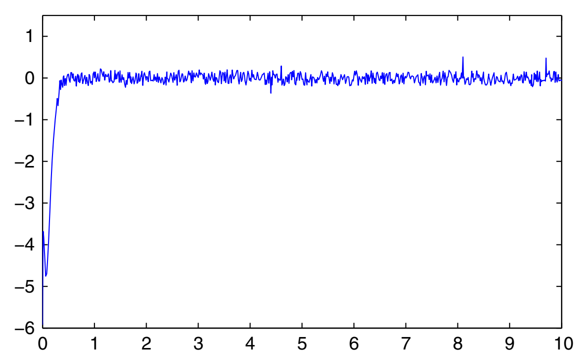

Fig. 4 Response of filtering error $\bar{z}$

which is the special case $r = 1$ of stochastic T–S fuzzy system (2). where

$$A = \begin{bmatrix} -5 & 0 \\ 1 & -10 \end{bmatrix}, A_d = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix}, C = \begin{bmatrix} -0.2 & 0.2 \\ 0.1 & -0.3 \end{bmatrix}$$

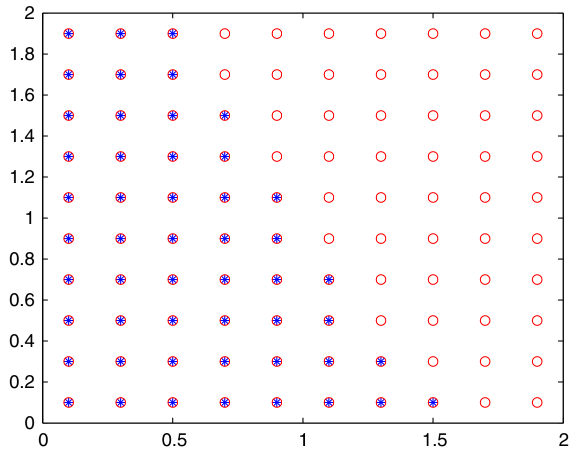

Fig. 5 Range of the feasible solutions

$$D = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.03 \end{bmatrix}, L = \begin{bmatrix} 1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}$$

The delay is assumed as $\bar{d} = 1.1$. The disturbance attenuation level is chosen as $\gamma = 0.8$. Using Corollary 1, the filter that is the special case $r = 1$ of system (5) can be obtained. The parameters of the filter are as follows:

$$A_f = \begin{bmatrix} -45.5682 & 10.5861 \\ -179.3650 & 44.8778 \end{bmatrix},$$

$$B_f = \begin{bmatrix} 2.5608 & 45.7382 \\ -9.2081 & 123.8466 \end{bmatrix} C_f = \begin{bmatrix} 8.3546 & -2.9553 \\ 1.9372 & -0.7882 \end{bmatrix}$$

In addition, when the range of parameters is expanded, that is

$$A = \begin{bmatrix} -5 & 0 \\ i & -10 \end{bmatrix}, \quad A_d = \begin{bmatrix} 2 & 0.5 \\ 1 & j \end{bmatrix}$$

where $i \in [0.1, 2]$, $j \in [0.1, 2]$, Fig. 5 shows the range of the feasible solutions for Corollary 1 of this paper and Theorem 2 of [32]. In Fig. 5, 'o' represents the range of the feasible solutions using Corollary 1 of this paper, and '$*$' represents the range of the feasible solutions using Theorem 2 of [32]. It is clear to see from Fig. 5 that the range of feasible solutions for Corollary 1 of this paper is wider than the result in [32]. Through this example, we can find that our results can be also used in the case of constant time-delay and the range of feasible solutions is wider than the result of [32].

Example 3 Consider a continuous stirred tank reactor (CSTR) in which the first-order irreversible exothermic reaction **A** → **B** occurs. Similar as the discussion in [33], the model of CSTR can be described by the following ordinary differential dynamic equation

$$\begin{aligned}
\hat{V}\frac{\mathrm{d}\hat{A}}{\mathrm{d}t} &= \lambda q \hat{A}_0 + q(1-\lambda)\hat{A}(t-\alpha) - q\hat{A}(t) \\
&\quad - \hat{V}K_0 \exp\left[\frac{-\hat{E}}{\hat{R}T(t)}\right]\hat{A}(t) \\
\hat{V}\hat{C}\hat{\rho}\frac{\mathrm{d}T}{\mathrm{d}t} &= q\hat{C}\hat{\rho}\,[\lambda T_0 + (1-\lambda)T(t-\alpha) - T(t)] \\
&\quad + \hat{V}(-\Delta\hat{H}(t))K_0 \exp\left[\frac{-\hat{E}}{\hat{R}T(t)}\right] \\
&\quad \times \hat{A}(t) - U(T(t) - T_\omega)
\end{aligned}$$

where $\hat{A}(t)$ is the concentration of chemical **A**, $T(t)$ is reactor temperature, α is recycle delay time, $\hat{V}$ is reactor volume, λ is coefficient of recirculation, q is feed flow rate, $\hat{A}_0$ is feed concentration, K_0 is reaction velocity constant, $\hat{E}/\hat{R}$ is ratio of Arrhenius activation energy to the gas constant, $\hat{\rho}$ is density, $\hat{l}$ is specific heat, $-\Delta\hat{H}(t)$ is heat of reaction, U is heat transfer coefficient times the surface area of reactor, T_0 is feed temperature, and T_ω is average coolant temperature in reactor cooling coil. When calorimeter is used to measure the heat of reaction, suppose that it is affected by the environment. In this case, the state-space representation of this model is given by

$$\begin{aligned}
dx_1 &= (\frac{-1}{\lambda}x_1(t) + D_\alpha(1 - x_1(t))\exp\left(\frac{x_2(t)}{\frac{1+x_2(t)}{\gamma_0}}\right) \\
&\quad + \left(\frac{1}{\lambda} - 1\right)x_1(t-\alpha))\mathrm{d}t \\
dx_2 &= \left(\left(\frac{1}{\lambda}+\beta\right)x_2(t) + \left(\frac{1}{\lambda}-1\right)x_2(t-\alpha) + \beta u(t)\right. \\
&\quad \left. + H_0 D_\alpha(1-x_1(t))\exp\left(\frac{x_2(t)}{\frac{1+x_2(t)}{\gamma_0}}\right)\right)\mathrm{d}t + Mx\,d\omega
\end{aligned}$$

where $\omega(t)$ is a standard one-dimensional Wiener process.

Assume that only the temperature can be measured on line, that is

$$\mathrm{d}y(t) = [\,0 \quad 1\,]x\,\mathrm{d}t$$

Now, taking the IF-THEN rules as follows:

Rule 1: IF x_2 is F_{12}, THEN

$$\begin{aligned}dx(t) =&(A_1x(t) + A_{d1}x(t-\alpha) + B_1u(t))dt\\ &+ M_1x(t)d\omega\\ dy(t) =&C_1xdt\\ z(t) =& L_1x(t)\end{aligned}$$

Rule 2: IF x_2 is F_{22}, THEN

$$\begin{aligned}dx(t) =&(A_2x(t) + A_{d2}x(t-\alpha) + B_2u(t))dt\\ &+ M_2x(t)d\omega\\ dy(t) =&C_2xdt\\ z(t) =& L_2x(t)\end{aligned}$$

Rule 3: IF x_2 is F_{32}, THEN

$$\begin{aligned}dx(t) =&(A_3x(t) + A_{d3}x(t-\alpha) + B_3u(t))dt\\ &+ M_3x(t)d\omega\\ dy(t) =&C_3xdt\\ z(t) =& L_3x(t)\end{aligned}$$

where

$$A_1 = \begin{bmatrix}-1.4274 & 0.0757\\ -1.4189 & -0.9442\end{bmatrix}, A_{d1} = \begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix}$$
$$B_1 = \begin{bmatrix}0\\ 0.3\end{bmatrix}, M_1 = \begin{bmatrix}0 & 0\\ 0.0350 & 0.0150\end{bmatrix}$$
$$C_1 = \begin{bmatrix}0 & 1\end{bmatrix}, L_1 = \begin{bmatrix}1 & 0\end{bmatrix}$$
$$A_2 = \begin{bmatrix}-2.0508 & 0.3958\\ -6.4066 & -1.6268\end{bmatrix}, A_{d2} = \begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix}$$
$$B_2 = \begin{bmatrix}0\\ 0.3\end{bmatrix}, M_2 = \begin{bmatrix}0 & 0\\ -0.0316 & 0.0131\end{bmatrix}$$
$$C_2 = \begin{bmatrix}0 & 1\end{bmatrix}, L_2 = \begin{bmatrix}1 & 0\end{bmatrix}$$

$$A_3 = \begin{bmatrix}-4.5279 & 0.3167\\ -26.2228 & -0.9387\end{bmatrix}, A_{d3} = \begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix}$$
$$B_3 = \begin{bmatrix}0\\ 0.3\end{bmatrix}, \quad M_3 = \begin{bmatrix}0 & 0\\ -0.6556 & 0.0633\end{bmatrix}$$
$$C_3 = \begin{bmatrix}0 & 1\end{bmatrix}, \quad L_3 = \begin{bmatrix}1 & 0\end{bmatrix}$$

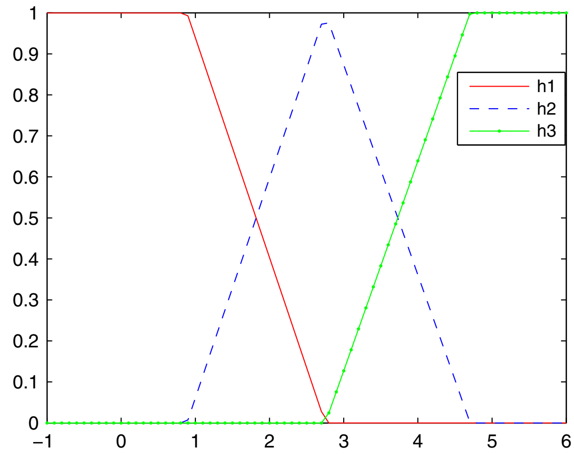

Fig. 6 Membership functions

Three corresponding membership functions (Fig. 6) are, respectively

$$h_1(x_2) = \begin{cases}1, & x_2 \le 0.8862\\ 1 - \frac{x_2 - 0.8862}{2.7520 - 0.8862}, & 0.8862 < x_2 < 2.7520\\ 0, & x_2 \ge 2.7520\end{cases}$$
$$h_2(x_2) = \begin{cases}1 - h_1(x_2), & x_2 < 2.7520\\ 1 - h_3(x_2), & x_2 \ge 2.7520\end{cases}$$
$$h_3(x_2) = \begin{cases}0, & x_2 \le 2.7520\\ 1 - \frac{x_2 - 2.7520}{4.7052 - 2.7520}, & 2.7520 < x_2 < 4.7052\\ 1, & x_2 \ge 4.7052\end{cases}$$

F_{12}, F_{22} and F_{32} are fuzzy sets; the corresponding membership functions are h_1, h_2 and h_3, respectively. The delay is assumed as $\alpha = 0.5$. The disturbance attenuation level is chosen as $\gamma = 0.5$. Using Corollary 1 with $D_i = 0$, the H_∞ filter (5) can be obtained with the parameters as follows:

$$A_{f1} = \begin{bmatrix}-3.0704 & -0.1743\\ -5.9197 & -17.4737\end{bmatrix}, \quad B_{f1} = \begin{bmatrix}-0.3583\\ -4.2071\end{bmatrix}$$
$$L_{f1} = \begin{bmatrix}-0.6021 & 0.7930\end{bmatrix}$$
$$A_{f2} = \begin{bmatrix}0.9161 & 3.8561\\ 14.5598 & -12.4149\end{bmatrix}, \quad B_{f2} = \begin{bmatrix}-0.1935\\ -3.7545\end{bmatrix}$$
$$L_{f2} = \begin{bmatrix}-0.6025 & 0.7931\end{bmatrix}$$
$$A_{f3} = \begin{bmatrix}8.7156 & 17.4908\\ 27.1725 & -10.4298\end{bmatrix}, \quad B_{f3} = \begin{bmatrix}1.0837\\ -2.4210\end{bmatrix}$$
$$L_{f3} = \begin{bmatrix}-0.5890 & 0.8088\end{bmatrix}$$

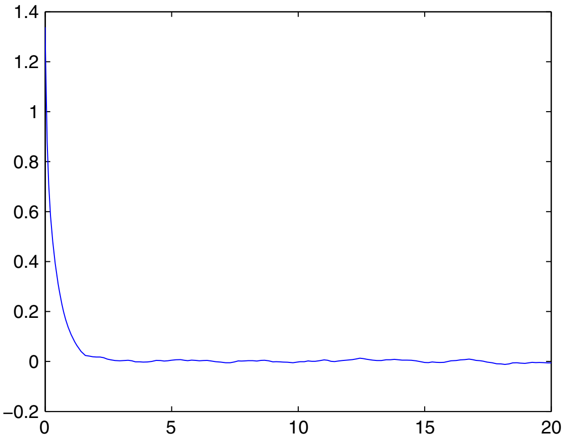

Fig. 7 Response of the estimation error

The response of the filtering error is denoted as $\bar{z} = z - z_f$ which is shown in Fig. 7. This example gives the application of H_∞ filtering for a CSTR and shows the effectiveness of the proposed approach.

5 Conclusions

In this paper, the H_∞ filtering problem for Ito stochastic singular fuzzy systems with time-varying delay has been studied. By using an auxiliary vector and an integral inequality, a delay-dependent sufficient condition has been proposed to guarantee $e^{\lambda t}$-weighted iISSiM and the H_∞ attenuation level for filtering error system. Then, the corresponding solvability condition for the fuzzy H_∞ filtering problem has been established by LMI, and fuzzy-rule-independent filter has been designed. For the time-varying delay in this paper, there is no limit on the bound of delay derivative that is the delay $d(t)$ need not to be differentiable or although $d(t)$ is differentiable, the upper bound of derivative need not to be known. Three examples have been provided to illustrate the effectiveness of the proposed fuzzy filter design method.

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