Insurance Risk Classification

How much is socially optimal?

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Insurance Risk Classification

Insurance – Pooling of risk

- Everybody is exposed to **risks**, but only a few suffer a **loss**.
- **Insurers** compensate **insureds** if a particular loss occurs.
- Insurers charge a **premium** for their services.
- Why is insurance possible at an affordable premium? **Pooling**.

Insurance risk classification

- Homogeneous risk: Charge the same premium for all.
- Heterogeneous risk: Contentious issue of **adverse selection**.

Adverse Selection

What is adverse selection?

No commonly accepted standard definition of *adverse selection*.

Definition (Actuarial perspective)

Insurer faces **loss** due to risk not factored in at the time of sale due to **asymmetric information** between the insurer and the insured.

Definition (Economic perspective)

An individual's **demand for insurance** (the propensity to buy insurance and the quantity purchased) is **positively correlated** with the individual's **risk of loss** (higher risks buy more insurance).

Question:

Why is this a **bad outcome** and **for whom**?

Theory and Practice

Traditional theory:

If insurers cannot charge **risk-differentiated** premiums, then:

- higher risks buy more insurance, lower risks buy less insurance,
- raising the **pooled** price of insurance,
- lowering the demand for insurance,

usually portrayed as a bad outcome, both for **insurers** and for **society**.

In practice:

Policymakers often see merit in restricting insurance risk classification

- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

Question:

How can we reconcile theory with practice?

Agenda

We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance (with and without risk classification)?
- **Which** regime is most beneficial to society?

We find:

Social welfare is maximised by maximising **loss coverage.**

Definition (Loss coverage)

Expected population losses compensated by insurance.

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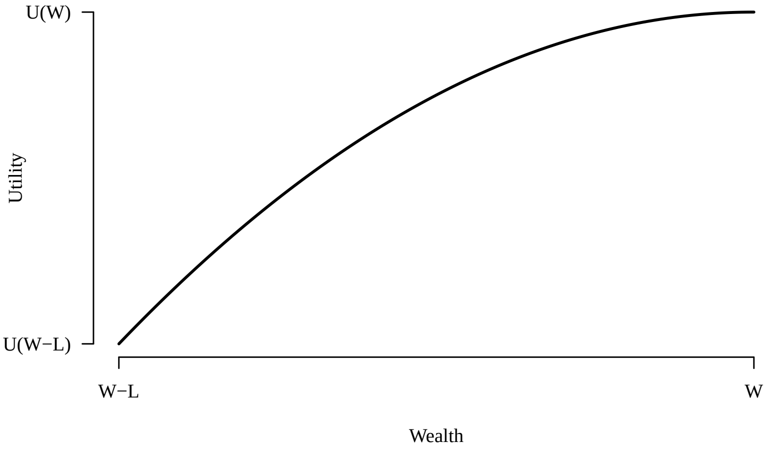
Why do people buy insurance?

Assumptions

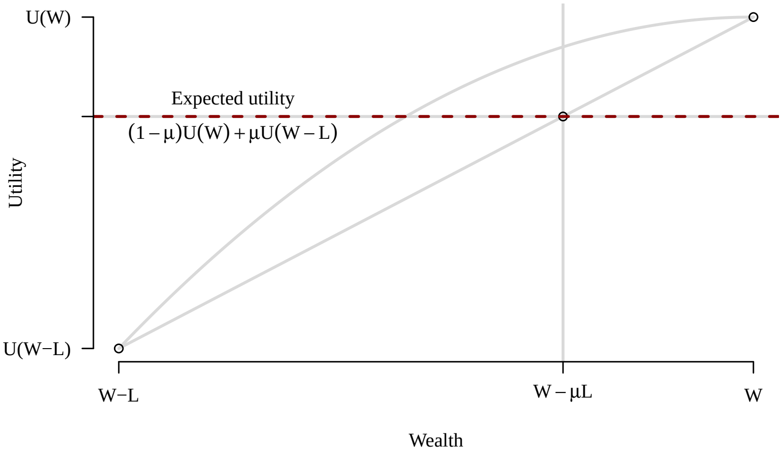
Consider an individual with

- an initial wealth W,
- exposed to the risk of loss L,
- with probability μ,
- utility of wealth $U(w)$, with $U'(w) > 0$ and $U''(w) < 0$,
- an opportunity to insure at premium rate π.

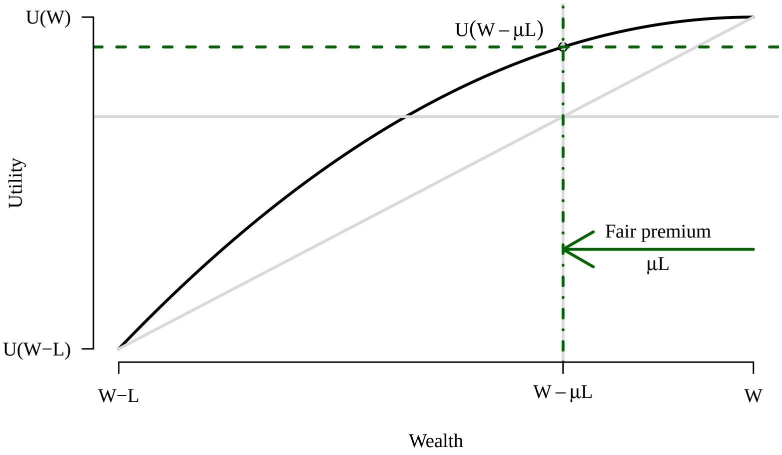
Utility of wealth



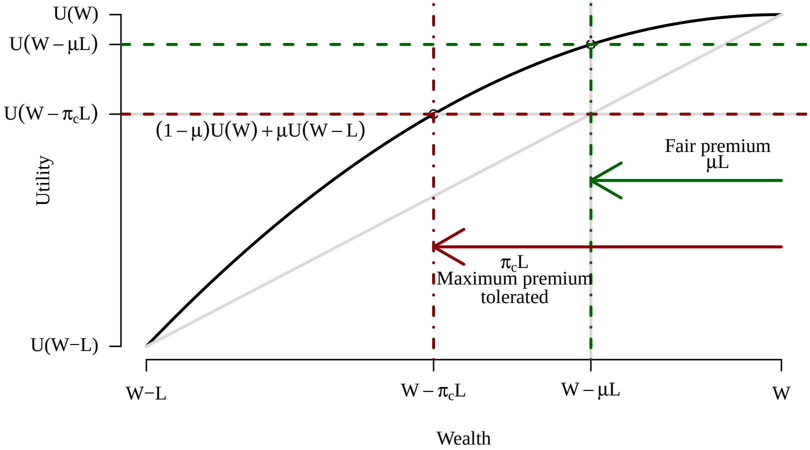
Expected utility: Without insurance



Expected utility: Insured at fair actuarial premium



Maximum premium tolerated: π_c



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Modelling demand for insurance

Simplest model:

Based on the given set-up:

- All will buy insurance if $\pi < \pi_c$;
- None will buy insurance if $\pi > \pi_c$.

Reality: Not all will buy insurance even at fair premium. **Why?**

Heterogeneity:

- Individuals are homogeneous in terms of underlying risk.
- Individuals can be heterogeneous in terms of **attitude to risk**.

Utility as a Random Variable

$[U(w)](v)$ is utility of wealth w for an individual v chosen at random.

- $[U(w)](v)$: (non-random) utility function of wealth for individual v.
- $\widetilde{U(w)}$ is a random variable for a specific wealth w.

Demand is a survival function

Condition for buying insurance:

Given premium π, individual v chosen at random will buy insurance if:

$$\underbrace{[U(W - \pi L)](v)}_{\text{With insurance}} > \underbrace{(1-\mu) \times [U(W)](v) + \mu \times [U(W-L)](v)}_{\text{Without insurance}}.$$

Standardisation

Suppose all individuals within the risk-group are standardised so that:

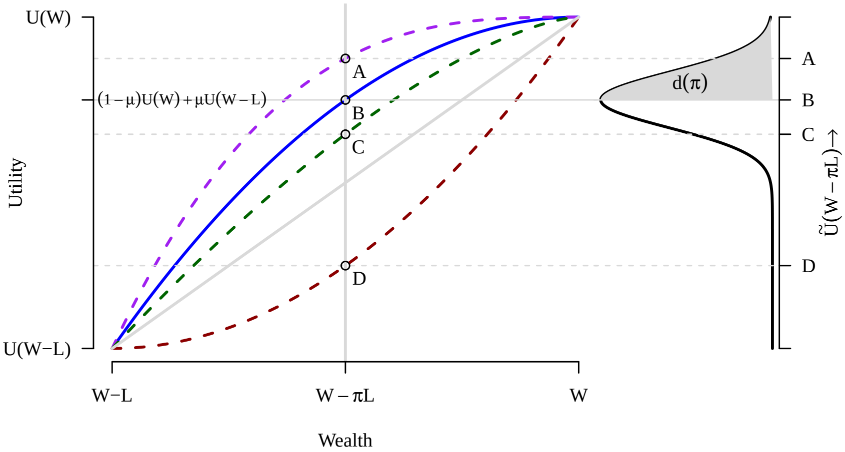
$$[U(W)](v) = 1,$$
$$[U(W-L)](v) = 0.$$

Demand as a survival function:

Given premium π, insurance demand, $d(\pi)$, is the survival function:

$$d(\pi) = \mathrm{Prob}[U(W - \pi L) > (1-\mu)U(W) + \mu\, U(W-L)].$$

Demand is a survival function



Illustrative example: $W = L = 1$

Power utility function:

$$\widetilde{U(w)} = w^{\widetilde{\gamma}}.$$

Heterogeneity in risk preferences: Distribution of $\widetilde{\gamma}$:

$$\text{Prob}[\widetilde{\gamma} \leq x] = \begin{cases} 0 & \text{if } x < 0 \\ k\, x^{\lambda} & \text{if } 0 \leq x \leq (1/k)^{1/\lambda},\ k > 0, \lambda > 0, \\ 1 & \text{if } x > (1/k)^{1/\lambda}. \end{cases}$$

Demand for insurance:

$$d(\pi) = \text{Prob}[\widetilde{U\,(W - \pi L)} > (1-\mu)U(W) + \mu\, U(W - L)],$$

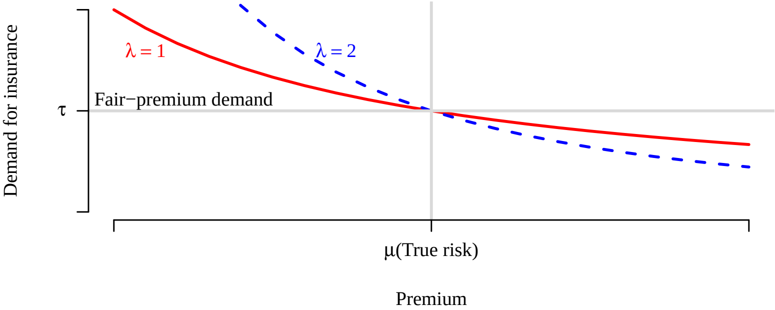
$$d(\pi) \approx \text{Prob}[\widetilde{\gamma} < \frac{\mu}{\pi}] = k\left(\frac{\mu}{\pi}\right)^{\lambda}.$$

$$d(\pi) \propto \pi^{-\lambda}.$$

Illustrative example: $W = L = 1$

Demand elasticity (Iso-elastic demand):

$$d(\pi) \propto \pi^{-\lambda} \Rightarrow \epsilon(\pi) = \left| \frac{\frac{\partial d(\pi)}{d(\pi)}}{\frac{\partial \pi}{\pi}} \right| = \lambda.$$



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Risk classification

Consider a population of individuals with the same:

- initial wealth $W = 1$;
- potential loss $L = 1$;
- form of iso-elastic demand function $d(\pi) \propto \pi^{-\lambda}$; and
- demand elasticity λ.

Suppose the population can be divided into 2 risk-groups, with:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: p_1 and p_2;
- fair premium demand: $d_1(\mu_1) = \tau_1$ and $d_2(\mu_2) = \tau_2$, i.e.

$$d_i(\pi) = \tau_i \left(\frac{\pi}{\mu_i}\right)^{-\lambda}, \quad i = 1, 2.$$

Risk-differentiated premium

Equilibrium:

If risk-differentiated premiums π_1 and π_2 are allowed,

- Total premium: $\sum_i p_i \; d_i(\pi_i) \; \pi_i$.
- Total claims: $\sum_i p_i \; d_i(\pi_i) \; \mu_i$.

Equilibrium is achieved when insurers break even, i.e. $\pi_i = \mu_i$.

Adverse Selection:

No losses for insurers. No (actuarial/economic) adverse selection.

Loss coverage (Population losses compensated by insurance):

Loss coverage = $\sum_i p_i \; d_i(\mu_i) \; \mu_i = \sum_i p_i \; \tau_i \; \mu_i$.

Pooled premium

Equilibrium:

If only a pooled premium π_0 is allowed,

- Total premium: $\sum_i p_i \, d_i(\pi_0) \, \pi_0$.
- Total claims: $\sum_i p_i \, d_i(\pi_0) \, \mu_i$.

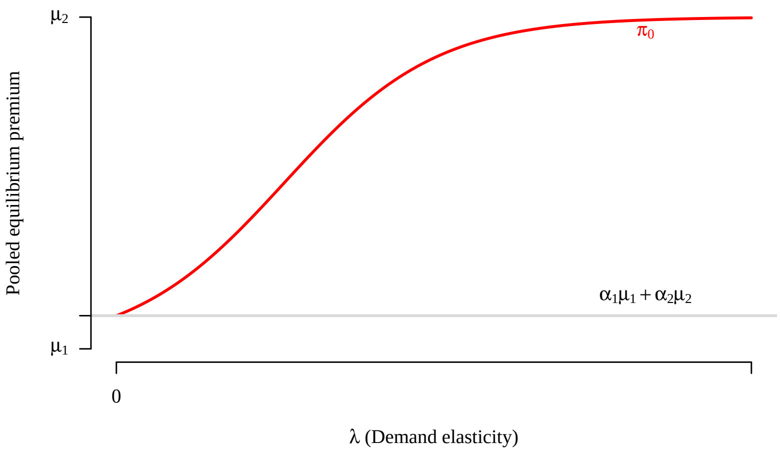
Equilibrium is achieved when insurers break even, i.e.

$$\text{Total premium} = \text{Total claims},$$

$$\Rightarrow \sum_i p_i \, d_i(\pi_0)\pi_0 = \sum_i p_i \, d_i(\pi_0) \, \mu_i,$$

$$\Rightarrow \pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}, \text{ where } \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}.$$

Pooled premium: Adverse selection



Pooled premium: Adverse selection

Adverse selection: Summary

- The pooled equilibrium is greater than the average premium charged under full risk classification:

$$\pi_0 > \alpha_1 \mu_1 + \alpha_2 \mu_2 \Rightarrow \text{(Economic) adverse selection.}$$

- No losses for insurers! $\Rightarrow$ No (actuarial) adverse selection.

Adverse selection is not useful to measure social efficacy of insurance.

Loss coverage ratio

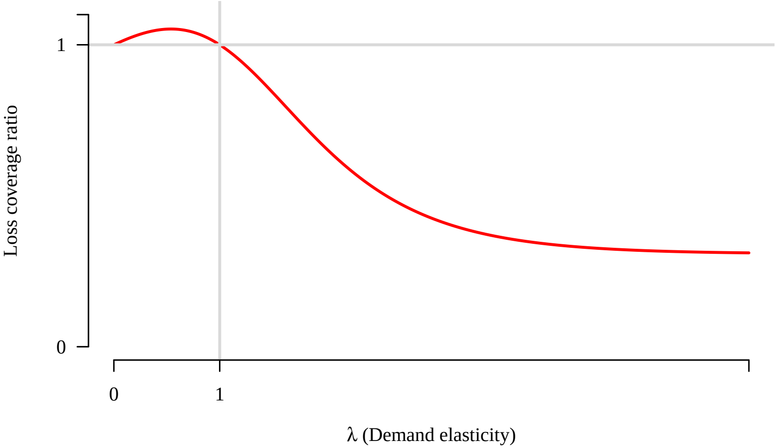
Loss coverage (Population losses compensated by insurance):

Loss coverage = $\sum_i p_i \; d_i(\pi_0) \; \mu_i$.

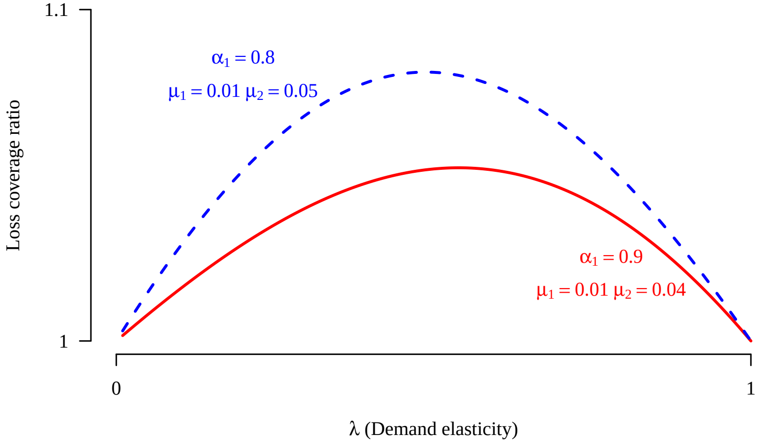
Loss coverage ratio:

$$C = \frac{\text{Loss coverage for pooled premium}}{\text{Loss coverage for risk-differentiated premium}},$$
$$= \frac{\sum_i p_i \; d_i(\pi_0) \; \mu_i}{\sum_i p_i \; d_i(\mu_i) \; \mu_i},$$
$$= \frac{1}{\pi_0^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}.$$

Loss coverage ratio



Loss coverage ratio



Loss coverage ratio: Summary

Summary

- $\lambda < 1 \Rightarrow$ Loss coverage is more when risk classification is banned.
- $\lambda = 1 \Rightarrow$ Loss coverage is the same in both risk classification regimes.
- $\lambda > 1 \Rightarrow$ Loss coverage is more when full risk classification is used.
- Empirical evidence suggests $\lambda < 1$, providing justification for restricting risk classification.
- The maximum value of loss coverage ratio depends on the relative risk and relative size of the risk groups.
- A pooled premium might be highly beneficial in the presence of a small group with very high risk exposure.

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Social welfare

Definition (Social welfare)

Social welfare, G, is the sum of all individuals' expected utilities:

$$G = \sum_i p_i \Big[\underbrace{d_i(\pi_i) U^*(W - L\pi_i)}_{\text{Insured population}} + \underbrace{(1 - d_i(\pi_i)) \{(1 - \mu_i) U(W) + \mu_i U(W - L)\}}_{\text{Uninsured population}} \Big],$$

where $U^*(W - L\pi_i)$ is the expected utility of the insured population.

Linking social welfare to loss coverage

Setting $U(W - L) = 0$ and assuming $L\pi_i \approx 0$ gives:

$$\begin{aligned} G &= U(W) \sum_i p_i d_i(\pi_i) \mu_i + \text{Constant}, \\ &= \text{Positive multiplier} \times \textbf{Loss coverage} + \text{Constant}. \end{aligned}$$

Loss coverage provides a good proxy (which depends only on observable data) for social welfare (which depends on unobservable utilities).

Result: Maximising loss coverage maximises social welfare.

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Conclusions

Adverse selection need not be adverse

Restricting risk classification

- will always increase adverse selection;
- increases loss coverage if $\lambda < 1$.

Summary

Loss coverage provides a better metric than adverse selection in measuring social welfare.

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