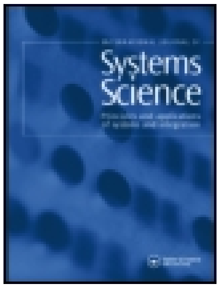

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Stabilisation of descriptor Markovian jump systems with partially unknown transition probabilities

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This paper is concerned with the stability and stabilisation problems for continuous-time descriptor Markovian jump systems with partially unknown transition probabilities. In terms of a set of coupled linear matrix inequalities (LMIs), a necessary and sufficient condition is firstly proposed, which ensures the systems to be regular, impulse-free and stochastically stable. Moreover, the corresponding necessary and sufficient condition on the existence of a mode-dependent state-feedback controller, which guarantees the closed-loop systems stochastically admissible by employing the LMI technique, is derived; the stabilizing state-feedback gain can also be expressed via solutions of the LMIs. Finally, numerical examples are given to demonstrate the validity of the proposed methods.

Keywords: stability and stabilisation; descriptor Markovian jump systems; partially unknown transition probabilities; linear matrix inqualities

1. Introduction

During the past decades, Markovian jump systems have been catching the attention of researchers and scholars all over the world, and many valuable results have been obtained. Markovian jump systems are modelled by many stochastic systems, which arise abrupt random changes in their structure, such as component failures or repairs, unexpected environmental changes, and so forth (Xu and Lam 2006; Boukas 2008). Stability and H_∞ control problems for standard state-space Markovian jump systems with partially unknown and fully known transition probabilities have been extensively explored in the past years (see Xu, Chen, and Lam 2003; Xiong, Lam, Gao, and Ho 2005; de Souza 2006; Xu and Mao 2007; Zhang, Boukas, and Lam 2008; Wang, Zhang, and Sreeram 2009, 2010; Zhang 2009; Zhang and Boukas 2009a, 2009b; Feng, Lam, and Shu 2010; Zhang and Lam 2010; Wang, Zhang, and Yang 2011; Zhang, He, Wu, and Zhang 2011; Guo and Zhu 2012).

Descriptor systems are also referred to as singular systems, semi-state systems, which are a natural representation of dynamical systems and describe a more wide range of systems than standard state-space systems (Dai 1989; Xu and Lam 2006). Stability and stabilisation of descriptor systems have been extensively explored in Dai (1989), Ishihara and Terra (2002), Xu and Lam (2004), Xu and Lam (2006) and references therein. More recently, stability criterions for descriptor Markovian jump systems, whose transition probabilities are fully known, have been widely proposed by Xu and Lam (2006), Boukas (2008), Xia, Zhang, and Boukas (2008), Xia, Boukas, Shi, and Zhang (2009) and Wu, Su, and Chu (2010). However, for descriptor Markovian jump systems with partially unknown transition probabilities, which contain the fully known and fully unknown as special cases, there have been not too much literatures to address (Sheng and Yang 2010; Chang, Fang, Lou, and Chen 2012).

The preceding facts motivate us to explore stability and stabilisation problems for continuous-time descriptor Markovian jump systems with partially unknown transition probabilities. In this paper, we shall first present a necessary and sufficient condition for the systems with partially unknown transition probabilities to be regular, impulse-free and stochastically stable, by prescribing a lower bound for the unknown diagonal elements in transition rate matrix. Then when the lower bound is unavailable, a sufficient condition for the stochastic admissibility is derived. All the results are formulated in terms of linear matrix inequalities (LMIs). Based on these statements, we then propose a necessary and sufficient condition and a sufficient condition for the closed-loop systems to be stochastically admissible by virtue of LMI technique. The stabilising state-feedback controller gain can be expressed by solutions of a set of LMIs. Compared with the existing results for standard state-space Markovian jump systems with partially unknown transition probabilities, this paper can be regarded as an extension to descriptor Markovian jump systems case.

The rest of this paper is organised as follows. Section 2 gives problem description and some lemmas. Section 3

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focuses on stochastic stability and stochastic stabilisation. Necessary and sufficient conditions and sufficient conditions for stochastic stability analysis and stochastic stabilisation synthesis problems are proposed. Section 4 provides numerical examples to illustrate the effectiveness of our methods, and Section 5 concludes the paper.

1.1. Notation

In this paper, $\mathbb{R}^n$ stands for the n-dimensional Euclidean space and $\mathbb{R}^{m\times n}$ represents the set of all $m \times n$ real matrices. The superscript T stands for matrix transposition; $(\Omega, \mathcal{F}, \mathcal{P})$ is the probability space. $\mathcal{E}\{\cdot\}$ is the expectation operator with respect to some probability measure. $\mathbb{N}^+$ represents the set of positive integers. The notation $P > 0$ ($P \geq 0$) implies that P is a real symmetric and positive definite (semi-positive definite) matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. For simplicity, sometimes we use M_i to denote $M(i)$.

2. Problem formulation and preliminaries

Fix the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following continuous-time descriptor Markovian jump systems described by

$$E\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t), \tag{1}$$

where $A(r_t) \in \mathbb{R}^{n\times n}$ and $B(r_t) \in \mathbb{R}^{n\times m}$ are known real constant matrices, $x \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the input vector. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular, without loss of generality, we can assume that $\mathrm{rank}(E) = r \leq n$. The jumping process $\{r_t, t \geq 0\}$, which takes values in a finite set $\ell \triangleq \{1, 2, \ldots, N\}$, is a right-continuous Markov process with the following mode transition probabilities:

$$P(r_{t+h} = j \mid r_t = i) = \begin{cases} \lambda_{ij} + o(h), & if \;\; j \neq i \\ 1 + \lambda_{ij}h + o(h), & if \;\; j = i, \end{cases}$$

where $h > 0$, $\lim_{t\to 0}(o(h)/h) = 0$ and $\lambda_{ij} \geq 0$ ($i, j \in \ell, j \neq i$), satisfying $\lambda_{ii} = -\sum_{j\in \ell, j\neq i}\lambda_{ij}$, represents the switching rate from mode i at time t to mode j at time $t + h$ for all $i \in \ell$. For the sake of simplicity, when $r_t = i \in \ell$, the system matrices of the ith mode ($A(r_t)$, $B(r_t)$) are denoted by (A_i, B_i). In this paper, not all the transition rates are considered to be available, namely some elements in transition rate matrix are unknown. More specifically, the transition rate matrix Λ can be expressed as

$$\Lambda = \begin{bmatrix} \lambda_{11} & ? & \cdots & ? & \lambda_{1N} \\ ? & \lambda_{22} & \cdots & ? & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ ? & ? & \cdots & ? & ? \\ \lambda_{N1} & \lambda_{N2} & \cdots & \lambda_{N(N-1)} & ? \end{bmatrix}, \tag{2}$$

where '?' denotes the unknown elements. For notational clarity, $\forall i \in \ell$, we denote $\ell = \ell_k^i + \ell_{uk}^i$ with $\ell_k^i \triangleq \{j : \lambda_{ij}$ is known$\}$, and $\ell_{uk}^i \triangleq \{j : \lambda_{ij}$ is unknown$\}$, besides, we denote $\lambda_k^i \triangleq \sum_{j\in\ell_k^i} \lambda_{ij}$ throughout the paper. Generally, when λ_{ii} is not exactly known, it is required that we give a lower bound $\underline{\lambda_i}$ for it.

Now we recall the following definition for the continuous-time descriptor Markovian jump system (1), which will be used in the rest of the paper.

Definition 2.1 (Xu and Lam 2006)

(1) The continuous-time descriptor Markovian jump system (1) is said to be regular if $\det(sE - A_i)$ is not identically zero for every $i \in \ell$.
(2) The continuous-time descriptor Markovian jump system (1) is said to be impulse-free if $\deg(\det(sE - A_i)) = \mathrm{rank}(E)$ for every $i \in \ell$.
(3) The continuous-time descriptor Markovian jump system (1) is said to be stochastically stable if for any $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{S}$, there exists a scalar $M(x_0, r_0) > 0$ such that

$$\lim_{t\to\infty} \mathcal{E}\left\{ \int_0^t x^T(s, x_0, r_0)x(s, x_0, r_0)ds | x_0, r_0 \right\} \leq M(x_0, r_0),$$

where $x(t, x_0, r_0)$ denotes the solution to system (1) at time t under the initial conditions x_0 and r_0.
(4) The continuous-time descriptor Markovian jump system (1) is said to be stochastically admissible if it is regular, impulse-free and stochastically stable.

In this paper, our aim is to develop a necessary and sufficient condition such that the unforced continuous-time descriptor Markovian jump system (1) with partially unknown transition rates (2) is stochastically admissible and to design a linear state-feedback controller for system (1) with partially unknown transition rates (2) such that the resulting closed-loop system is stochastically admissible.

The following lemma presents a necessary and sufficient condition for the unforce system (1) to be stochastically admissible.

Lemma 2.2 (Xu and Lam 2006): *The continuous-time descriptor Markovian jump system (1) is stochastically* *admissible if and only if there exist a set of matrices* $Y_i \in \mathbb{R}^{n\times n}$, $i \in \ell$, *such that*

$$EY_i = Y_i^T E^T \geq 0 \tag{3}$$

$$A_i Y_i + Y_i^T A_i^T + \sum_{j\in\ell} \lambda_{ij} EY_j < 0. \tag{4}$$

It is noted that the conditions in Lemma 2.2 contain equality constraints, which may suffer from numerical problems readily. Therefore, strict LMIs conditions are more desirable to be obtained. The following lemma, which gives a necessary and sufficient condition for system (1) to be stochastically admissible in terms of strict LMIs, can be regarded as the dual form of lemma 2 proposed by Wu et al. (2010). Considering its important role in this paper, we prove it with other different methods.

Lemma 2.3: *The unforced system (1) ($u(t) = 0$) is stochastically admissible if and only if there exist a set of positive definite matrices* $P_i \in \mathbb{R}^{n\times n}$ *and matrices* $\Phi_i \in \mathbb{R}^{(n-r)\times n}$, $i \in \ell$, *which satisfy for each* $i \in \ell$:

$$A_i(P_i E^T + V\Phi_i) + (P_i E^T + V\Phi_i)^T A_i^T + \sum_{j\in\ell} \lambda_{ij} EP_j E^T < 0, \tag{5}$$

where $V \in \mathbb{R}^{n\times(n-r)}$ *is of full column rank and satisfies* $EV = 0$.

Proof: (Sufficiency) Assume that there exist a set of positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and matrices $\Phi_i \in \mathbb{R}^{(n-r)\times n}$, $i \in \ell$ such that (5) holds. Set

$$Y_i = P_i E^T + V\Phi_i.$$

Then, by (5), it is easy to show

$$EY_i = Y_i^T E^T = EP_i E^T \geq 0$$

$$A_i Y_i + Y_i^T A_i^T + \sum_{j\in\ell} \lambda_{ij} EY_j < 0.$$

Therefore, by Lemma 2.2, we have that the unforced system (1) is stochastically admissible.

(Necessity) Assume that the unforced system (1) is stochastically admissible, then there exist nonsingular matrices M and N such that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, MA_i N = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix},$$

where A_{i4} is nonsingular for any $i \in \ell$. Then, we can select

$$M_i = \begin{bmatrix} I & -A_{i2}A_{i4}^{-1} \\ 0 & I \end{bmatrix} M.$$

Then, it is easy to see

$$\hat{E} = M_i EN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \hat{A}_i = M_i A_i N = \begin{bmatrix} \hat{A}_{i1} & 0 \\ A_{i3} & A_{i4} \end{bmatrix}, \tag{6}$$

where $\hat{A}_{i1} = A_{i1} - A_{i2}A_{i4}^{-1}A_{i3}$. It is easy to see that the stochastic stability of the unforced system (1) implies the following standard state-space Markovian jump system is stochastically stable

$$\dot{\xi} = \hat{A}_{i1}\xi,$$

where $\xi \in \mathbb{R}^r$. Therefore, from Xu and Lam (2006), we can readily have that there exist a set of positive definite matrices $\hat{P}_i \in \mathbb{R}^{r\times r}$, $i \in \ell$, such that

$$\hat{A}_{i1}\hat{P}_i + \hat{P}_i\hat{A}_{i1}^T + \sum_{j\in\ell} \lambda_{ij}\hat{P}_j < 0.$$

Thus, we can always find a sufficiently large scalar $\rho > 0$ such that for any $i \in \ell$

$$\hat{A}_i \begin{bmatrix} \hat{P}_i & 0 \\ 0 & -\rho A_{i4}^{-1} \end{bmatrix} + \begin{bmatrix} \hat{P}_i & 0 \\ 0 & -\rho A_{i4}^{-1} \end{bmatrix}^T \hat{A}_i^T + \sum_{j=1}^{N} \lambda_{ij}\hat{E}\begin{bmatrix} \hat{P}_j & 0 \\ 0 & \rho I \end{bmatrix}\hat{E}^T$$
$$= \begin{bmatrix} \hat{A}_{i1}\hat{P}_i + \hat{P}_i\hat{A}_{i1}^T + \sum_{j\in\ell}\lambda_{ij}\hat{P}_j & \hat{P}_i A_{i3}^T \\ A_{i3}\hat{P}_i & -2\rho I \end{bmatrix} < 0. \tag{7}$$

It is easy to see that

$$\begin{bmatrix} \hat{P}_i & 0 \\ 0 & -\rho A_{i4}^{-1} \end{bmatrix} = \begin{bmatrix} \hat{P}_i & 0 \\ 0 & \rho I \end{bmatrix}\hat{E}^T + \begin{bmatrix} 0 \\ -I \end{bmatrix}\begin{bmatrix} 0 & \rho A_{i4}^{-1} \end{bmatrix}. \tag{8}$$

Substituting (8) into (7) and pre- and post-multiplying (7) by M_i and M_i^T, we have

$$M_i\hat{A}_i N\left(N^{-1}\begin{bmatrix} \hat{P}_i & 0 \\ 0 & \rho I \end{bmatrix}N^{-T}N^T\hat{E}^T M_i^T + N^{-1}\begin{bmatrix} 0 \\ -I \end{bmatrix}HH^{-1}\begin{bmatrix} 0 & \rho A_{i4}^{-1} \end{bmatrix}M_i^T\right)$$
$$\left(N^{-1}\begin{bmatrix} \hat{P}_i & 0 \\ 0 & \rho I \end{bmatrix}N^{-T}N^T\hat{E}^T M_i^T + N^{-1}\begin{bmatrix} 0 \\ -I \end{bmatrix} \times HH^{-1}\begin{bmatrix} 0 & \rho A_{i4}^{-1} \end{bmatrix}M_i^T\right)^T N^T\hat{A}_i^T M_i^T$$

$$+\sum_{j=1}^{N}\lambda_{ij}M_i\hat{E}NN^{-1}\begin{bmatrix}\hat{P}_j & 0\\ 0 & \rho I\end{bmatrix}N^{-T}N^T\hat{E}^TM_i^T < 0, \tag{9}$$

where $H \in \mathbb{R}^{(n-r)\times(n-r)}$ is an arbitrary nonsingular matrix. Now define

$$P_i = N^{-1}\begin{bmatrix}\hat{P}_i & 0\\ 0 & \rho I\end{bmatrix}N^{-T}, \Phi_i = H^{-1}\begin{bmatrix}0 & \rho A_{i4}^{-1}\end{bmatrix}M_i^T,$$
$$V = N^{-1}\begin{bmatrix}0\\ -I\end{bmatrix}H. \tag{10}$$

Then, it follows from (6), (9) and (10) that a set of positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and matrices $\Phi_i \in \mathbb{R}^{(n-r)\times n}$, $i \in \ell$ defined in (10) satisfy (5). This completes the proof.

Remark 1: Compared with lemma 2 in Wu et al. (2010), Lemma 2.3 in this paper provides a much easier method to prove the stochastic admissibility and is more profitable to design the state-feedback controller, which will be encountered in the following.

3. Main Results

In this section, motivated by Zhang and Lam (2010), based on Lemma 2.3, we first present a necessary and sufficient criterion which ensures the unforced system (1) to be stochastically admissible.

Theorem 3.1: *The unforced continuous-time descriptor Markovian jump system (1) with partially unknown transition rates (2) and bounded diagonal elements is stochastically admissible if and only if there exist a set of positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and matrices $\Phi_i \in \mathbb{R}^{(n-r)\times n}$, $i \in \ell$, such that the following LMIs hold for each $i \in \ell$:*

If $i \notin \ell_k^i$,

$$\Xi_i + \mathcal{P}_k^i + \underline{\lambda_i}EP_iE^T - \underline{\lambda_i}EP_jE^T - \lambda_k^iEP_jE^T < 0,\ \forall j \in \ell_{uk}^i. \tag{11}$$

If $i \in \ell_k^i$,

$$\Xi_i + \mathcal{P}_k^i - \lambda_k^iEP_jE^T < 0,\ \forall j \in \ell_{uk}^i, \tag{12}$$

where $\mathcal{P}_k^i = \sum_{j\in\ell_k^i}\lambda_{ij}EP_jE^T$ *and* $\Xi_i = A_i(P_iE^T + V\Phi_i) + (P_iE^T + V\Phi_i)^TA_i^T$.

Proof: *Case 1* ($i \notin \ell_k^i$): Note that in this case, λ_{ii} is unknown and $\lambda_{ii} \le -\lambda_k^i$. Assume that $\lambda_{ii} = -\lambda_k^i$, this together with $\sum_{j\in\ell}\lambda_{ij} = 0$, implies that all the transition rates in the ith row are fully known, which contradicts with the condition that λ_{ii} is unknown. Therefore, we have $\lambda_{ii} < -\lambda_k^i$. The left side of (5) can be rewritten as

$$\begin{aligned}\Theta_i &= \Xi_i + \mathcal{P}_k^i + \lambda_{ii}EP_iE^T + \sum_{j\in\ell_{uk}^i, j\ne i}\lambda_{ij}EP_jE^T\\ &= \Xi_i + \mathcal{P}_k^i + \lambda_{ii}EP_iE^T + (-\lambda_{ii} - \lambda_k^i)\\ &\quad\times \sum_{j\in\ell_{uk}^i, j\ne i}\frac{\lambda_{ij}}{(-\lambda_{ii} - \lambda_k^i)}EP_jE^T.\end{aligned} \tag{13}$$

Since $\lambda_{ij} \ge 0, \forall j \in \ell_{uk}^i, j \ne i$, and $\sum_{j\in\ell_{uk}^i, j\ne i}\lambda_{ij} = -\lambda_{ii} - \lambda_k^i > 0$, we have $0 \le \lambda_{ij}/(-\lambda_{ii} - \lambda_k^i) \le 1$, $\forall j \in \ell_{uk}^i, j \ne i$ and $\sum_{j\in\ell_{uk}^i, j\ne i}\lambda_{ij}/(-\lambda_{ii} - \lambda_k^i) = 1$, then we can transform Θ_i to

$$\Theta_i = \sum_{j\in\ell_{uk}^i, j\ne i}\frac{\lambda_{ij}}{(-\lambda_{ii} - \lambda_k^i)}\big(\Xi_i + \mathcal{P}_k^i + \lambda_{ii}EP_iE^T - \lambda_{ii}EP_jE^T - \lambda_k^iEP_jE^T\big).$$

When $\lambda_{ij} \ge 0, \forall j \in \ell_{uk}^i, j \ne i$ is unknown, we can know that $\lambda_{ij}/(-\lambda_{ii} - \lambda_k^i), \forall j \in \ell_{uk}^i, j \ne i$ can achieve the arbitrary value in [0, 1]; then by virtue of properties of convex combination, it is clear that $\Theta_i < 0$ holds if and only if

$$\Xi_i + \mathcal{P}_k^i + \lambda_{ii}EP_iE^T - \lambda_{ii}EP_jE^T - \lambda_k^iEP_jE^T < 0, \forall j \in \ell_{uk}^i, j \ne i. \tag{14}$$

Since $\underline{\lambda_i}$ is the lower bound of λ_{ii}, we have

$$\underline{\lambda_i} \le \lambda_{ii} < -\lambda_k^i,$$

which means that there exists a sufficiently small scalar $\varepsilon > 0$ such that λ_{ii} takes value in $[\underline{\lambda_i}, -\lambda_k^i - \varepsilon]$; then λ_{ii} can be expressed by a convex combination of $\underline{\lambda_i}$ and $-\lambda_k^i - \varepsilon$. Consequently, (14) is equivalent to the following two LMIs hold simultaneously;

$$\Xi_i + \mathcal{P}_k^i + \underline{\lambda_i}EP_iE^T - \underline{\lambda_i}EP_jE^T - \lambda_k^iEP_jE^T < 0, \forall j \in \ell_{uk}^i, j \ne i \tag{15}$$

and

$$\Xi_i + \mathcal{P}_k^i - \lambda_k^iEP_iE^T - \epsilon(P_i - P_j) < 0, \forall j \in \ell_{uk}^i, j \ne i. \tag{16}$$

Since ε is arbitrary, (16) is equivalent to

$$\Xi_i + \mathcal{P}_k^i - \lambda_k^iEP_iE^T < 0.$$

Therefore, when $i \notin \ell_k^i$, we have the equivalency between (5) and (11).

Case 2 ($i \in \ell_k^i$): Note that in this case, λ_{ii} is known, then $\lambda_k^i < 0$. Since when $\lambda_k^i = 0$, transition rates turn to be the fully known case. Thus, the left side of (5) can be rewritten as

$$\Theta_i = \Xi_i + \mathcal{P}_k^i + \sum_{j \in \ell_{uk}^i} \lambda_{ij} E P_j E^T$$
$$= \Xi_i + \mathcal{P}_k^i - \lambda_k^i \sum_{j \in \ell_{uk}^i} \frac{\lambda_{ij}}{-\lambda_k^i} E P_j E^T.$$

Due to $0 \leq \lambda_{ij}/-\lambda_k^i \leq 1$ and $\sum_{j \in \ell_{uk}^i} \lambda_{ij}/-\lambda_k^i = 1$, then Θ_i can be transformed to

$$\Theta_i = \sum_{j \in \ell_{uk}^i} \frac{\lambda_{ij}}{-\lambda_k^i} (\Xi_i + \mathcal{P}_k^i - \lambda_k^i E P_j E^T).$$

Similarly, by properties of convex combination, it is clear that $\Theta_i < 0$ is equivalent to

$$\Xi_i + \mathcal{P}_k^i - \lambda_k^i E P_j E^T < 0.$$

This completes the proof. □

Remark 1: This theorem provides a necessary and sufficient criterion for continuous-time descriptor Markovian jump systems to be stochastically admissible. Noting that if $\ell_{uk}^i = \phi$, all the elements in transition rate matrix (2) are known, then system (1) becomes the one with fully known transition probabilities; conditions in Theorem 3.1 can also be transformed to those in Lemma 2.3. In the case when $E = I$, after some matrix manipulations, we can readily have that Theorem 3.1 reduces to theorem 1 in Zhang and Lam (2010).

When $\underline{\lambda_i}$ is unavailable, that is, the unknown diagonal λ_{ii} may take value in $(-\infty, 0]$, then we can obtain a sufficient condition for system (1) with arbitrarily reasonable λ_{ii}.

Corollary 3.2: *The unforced continuous-time descriptor Markovian jump system (1) with partially unknown transition rates (2) is stochastically admissible if there exist a set of positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and matrices $\Phi_i \in \mathbb{R}^{(n-r)\times n}$, $i \in \ell$, such that the following LMIs hold for each $i \in \ell$:*

If $i \notin \ell_k^i$,

$$\Xi_i + \mathcal{P}_k^i - \lambda_k^i E P_i E^T < 0, \qquad (17)$$

$$E P_i E^T \geq E P_j E^T, \quad \forall j \in \ell_{uk}^i, \ j \neq i. \qquad (18)$$

If $i \in \ell_k^i$,

$$\Xi_i + \mathcal{P}_k^i - \lambda_k^i E P_j E^T < 0, \ \forall j \in \ell_{uk}^i, \qquad (19)$$

where $\mathcal{P}_k^i = \sum_{j \in \ell_k^i} \lambda_{ij} E P_j E^T$ *and* $\Xi_i = A_i(P_i E^T + V\Phi_i) + (P_i E^T + V\Phi_i)^T A_i^T$.

Proof: Assume that there exist a set of positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and matrices $\Phi_i \in \mathbb{R}^{(n-r)\times n}$, $i \in \ell$ such that (17), (18) and (19) hold. It is not hard to see that (19) and (12) are the same; thus, it suffices to prove that (17) and (18) imply (11).

In Theorem 3.1, we can have that the unforced system (1) is stochastically admissible if and only if (13) holds. Here we recall (13) again and consider (17) and (18), then we can have

$$\Theta_i = \Xi_i + \mathcal{P}_k^i + \lambda_{ii} E P_i E^T + (-\lambda_{ii} - \lambda_k^i) \times \sum_{j \in \ell_{uk}^i, j\neq i} \frac{\lambda_{ij}}{(-\lambda_{ii} - \lambda_k^i)} E P_j E^T$$
$$\leq \Xi_i + \mathcal{P}_k^i + \lambda_{ii} E P_i E^T + (-\lambda_{ii} - \lambda_k^i) E P_i E^T$$
$$= \Xi_i + \mathcal{P}_k^i - \lambda_k^i E P_i E^T < 0. \qquad (20)$$

Therefore, the unforced system (1) is stochastically admissible. This completes the proof. □

Remark 2: In the case when $E = I$, it is easy to show that $V = 0$, then Corollary 3.2 reduces to theorem 3.1 in Guo and Zhu (2012). Therefore, Corollary 3.2 can be regarded as an extension of stability criterion for standard state-space Markovian jump systems to that for descriptor Markovian jump systems.

In the following, the stabilisation problem of system (1) with control input $u(t)$ is considered. The mode-dependent controller with the following form is designed:

$$u(t) = K(r_t)x(t), \qquad (21)$$

where $K(r_t)$ for all $r_t \in \ell$ are the controller gains to be determined. Using (1), the closed-loop system is represented as

$$E\dot{x}(t) = [A(r_t) + B(r_t)K(r_t)]x(t). \qquad (22)$$

According to Theorem 3.1, in the next, we will design the mode-dependent controller of form (21) such that the closed-loop system (22) is stochastically admissible.

Theorem 3.3: *The closed-loop system (22) with partially unknown transition rates (2) and bounded diagonal elements is stochastically admissible if and only if there exist a set of positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, matrices $\Phi_i \in \mathbb{R}^{(n-r)\times n}$ and $H_i \in \mathbb{R}^{m\times n}$, $\forall i \in \ell$, such that the following LMIs hold for each $i \in \ell$:*

If $i \notin \ell_k^i$,

$$\Xi_i + F_i + \mathcal{P}_k^i + \underline{\lambda_i} E P_i E^T - \underline{\lambda_i} E P_j E^T - \lambda_k^i E P_j E^T < 0, \ \forall j \in \ell_{uk}^i. \qquad (23)$$

If $i \in \ell_k^i$,

$$\Xi_i + F_i + \mathcal{P}_k^i - \lambda_k^i E P_j E^T < 0, \ \forall j \in \ell_{uk}^i, \qquad (24)$$

where $\mathcal{P}_k^i = \sum_{j\in\ell_k^i} \lambda_{ij} E P_j E^T$, $\Xi_i = A_i(P_i E^T + V\Phi_i) + (P_i E^T + V\Phi_i)^T A_i^T$ *and* $\Gamma_i = B_i H_i + H_i^T B_i^T$. *Moreover, if the above LMIs are true, the stabilising controller gain is given by* $K(i) = H_i(P_i E^T + V\Phi_i)^{-1}$.

Proof: Considering the closed-loop system (22), based on Theorem 3.1, it is easy to see that the closed-loop system (22) is stochastically admissible if and only if the following LMIs hold.

If $i \notin \ell_k^i$,

$$\overline{\Xi_i} + \mathcal{P}_k^i + \underline{\lambda_i} E P_i E^T - \underline{\lambda_i} E P_j E^T - \lambda_k^i E P_j E^T < 0, \ \forall j \in \ell_{uk}^i. \tag{25}$$

If $i \in \ell_k^i$,

$$\overline{\Xi_i} + \mathcal{P}_k^i - \lambda_k^i E P_j E^T < 0, \ \forall j \in \ell_{uk}^i, \tag{26}$$

where $\overline{\Xi_i} = (A_i + B_i K_i)(P_i E^T + V\Phi_i) + (P_i E^T + V\Phi_i)^T (A_i + B_i K_i)^T$. Now we prove that (25) and (26) guarantee that (23) and (24) hold and vice versa.

(Sufficiency) Substituting $K(i) = H_i(P_i E^T + V\Phi_i)^{-1}$ into the closed-loop system (22), we have

$$E\dot{x}(t) = [A_i + B_i H_i (P_i E^T + V\Phi_i)^{-1}]x(t).$$

It is easy to see that (23) and (24) can be rewritten as follows.

If $i \notin \ell_k^i$,

$$\underline{\Xi_i} + \underline{\Xi_i}^T + \mathcal{P}_k^i + \underline{\lambda_i} E P_i E^T - \underline{\lambda_i} E P_j E^T - \lambda_k^i E P_j E^T < 0, \ \forall j \in \ell_{uk}^i.$$

If $i \in \ell_k^i$,

$$\underline{\Xi_i} + \underline{\Xi_i}^T + \mathcal{P}_k^i - \lambda_k^i E P_j E^T < 0, \ \forall j \in \ell_{uk}^i,$$

where $\underline{\Xi_i} = (A_i + B_i H_i (P_i E^T + V\Phi_i)^{-1})(P_i E^T + V\Phi_i)$, then we have (25) and (26) hold. Thus, the closed-loop system (22) is stochastically admissible.

(Necessity) Setting $K(i) = H_i(P_i E^T + V\Phi_i)^{-1}$, from (25) and (26), it is straightforward that (23) and (24) hold.

This completes the proof. □

When we cannot have a lower bound for the unknown diagonal elements in the transition rate matrix (2), similarly to corollary 3.2, then we have the following corollary.

Corollary 3.4: *The closed-loop system (22) with partially unknown transition rates (2) is stochastically admissible if there exist a set of positive definite matrices* $P_i \in \mathbb{R}^{n\times n}$, *matrices* $\Phi_i \in \mathbb{R}^{(n-r)\times n}$ *and* $H_i \in \mathbb{R}^{m\times n}$, $\forall i \in \ell$, *such that the following LMIs hold for each* $i \in \ell$.

If $i \notin \ell_k^i$,

$$\Xi_i + \Gamma_i + \mathcal{P}_k^i - \lambda_k^i E^T P_i E < 0 \tag{27}$$

$$E^T P_i E \geq E^T P_j E, \quad \forall j \in \ell_{uk}^i, \ j \neq i. \tag{28}$$

If $i \in \ell_k^i$,

$$\Xi_i + \Gamma_i + \mathcal{P}_k^i - \lambda_k^i E^T P_j E < 0, \ \forall j \in \ell_{uk}^i, \tag{29}$$

where $\mathcal{P}_k^i = \sum_{j\in\ell_k^i} \lambda_{ij} E P_j E^T$, $\Xi_i = A_i(P_i E^T + V\Phi_i) + (P_i E^T + V\Phi_i)^T A_i^T$ *and* $\Gamma_i = B_i H_i + H_i^T B_i^T$. *Moreover, if the above LMIs are true, the stabilising controller gain is given by* $K(i) = H_i(P_i E^T + V\Phi_i)^{-1}$.

In the case when $\ell_{uk}^i = \phi$, system (22) becomes the one with fully known transition rates, and then Theorem 3.3 reduces to the following corollary.

Corollary 3.5: *The closed-loop system (22) with fully known transition rates is stochastically admissible if and only if there exist a set of positive definite matrices* $P_i \in \mathbb{R}^{n\times n}$, *matrices* $\Phi_i \in \mathbb{R}^{(n-r)\times n}$ *and* $H_i \in \mathbb{R}^{m\times n}$, $\forall i \in \ell$, *such that the following LMIs holds for each* $i \in \ell$:

$$\Xi_i + \Gamma_i + \mathcal{P}^i < 0 \tag{30}$$

where $\mathcal{P}^i = \sum_{j\in\ell} \lambda_{ij} E P_j E^T$, $\Xi_i = A_i(P_i E^T + V\Phi_i) + (P_i E^T + V\Phi_i)^T A_i^T$ *and* $\Gamma_i = B_i H_i + H_i^T B_i^T$. *Moreover, if the above LMIs are true, the stabilising controller gain is given by* $K(i) = H_i(P_i E^T + V\Phi_i)^{-1}$.

Assume that the transition probabilities are fully known. It can be verified that Corollary 3.5 can be obtained from Theorem 3.3; thus, the proof is omitted here.

Remark 3: It is noted that Corollary 3.5 can be obtained as a special case of Theorem 3.3, when $\ell_{uk}^i = \phi$. Our results do not involve any matrix decomposition, correspondingly numerical problems, which may arise in matrix decomposition, can be avoided. In the case when $\ell_{uk}^i \neq \phi$, we are not able to deal with such stabilising problems by theorem 6 in Xia et al. (2009), while we can also have a feasible solution by using the method presented in Theorem 3.3 or Corollary 3.4. Hence, our methods are more effective and much easier to be accomplished.

4. Numerical examples

In this section, two numerical examples are provided to show the validity of our results.

Example 4.1: (Xia et al. 2009) Consider the following continuous-time descriptor Markovian jump system (1) with two operation modes and the following data:

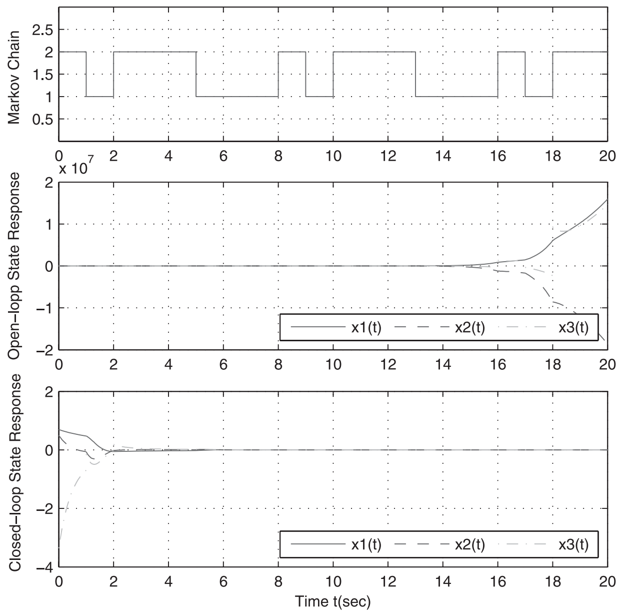

Figure 1. System response with transition rate matrix in Case 1.

$$E = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A(1) = \begin{bmatrix} 1.5 & -1.4 & 1.6 \\ -2.5 & -0.6 & -1.1 \\ 0.4 & 0.5 & -0.8 \end{bmatrix},$$

$$A(2) = \begin{bmatrix} 0.5 & 0.7 & 1.3 \\ 0.1 & 0.9 & 0.7 \\ 0.4 & 0.5 & 0.2 \end{bmatrix}$$

$$B(1) = \begin{bmatrix} 2.4 & 0.5 \\ 0 & 0.6 \\ 0.5 & 0 \end{bmatrix}, B(2) = \begin{bmatrix} 0 & 0.9 \\ -0.7 & 0.6 \\ 0.1 & 0 \end{bmatrix}, S = \begin{bmatrix} 0 \\ 0 \\ 0.8 \end{bmatrix}.$$

The transition rate matrices that we consider are shown in Table 1.

Our purpose here is to design a mode-dependent stabilising controller of the form of (21) such that the resulting closed-loop system (22) is stochastically admissible with the transition rates in Table 1. Using the LMI toolbox in MATLAB, solving (27)–(29) in Corollary 3.4 in this paper, the controller gains are calculated, which are shown in Table 2. With these controller, system responses for Case 1 and Case 2 are shown in Figure 1 and Figure 2, respectively.

As we can see from Figure 2 , open-loop system is diverging. After applying the controller in Case 2, trajectory simulation for the closed-loop system shown in the third subfigure is stochastically admissible with the same Markovian jump process under the given initial condition $x_0 = \begin{bmatrix} 0.7, & 0.5, & -3.5 \end{bmatrix}^T$. Compared with theorem 7 in Xia et al. (2009), Corollary 3.4 can not only deal with the fully known case, but can also cope with the partially unknown case. Therefore, our methods are more effective.

Table 1. Two different transition rate matrices.

Case 1			Case 2		
Mode	1	2	Mode	1	2
1	−0.8	0.8	1	−0.8	0.8
2	−0.5	0.5	2	?	?

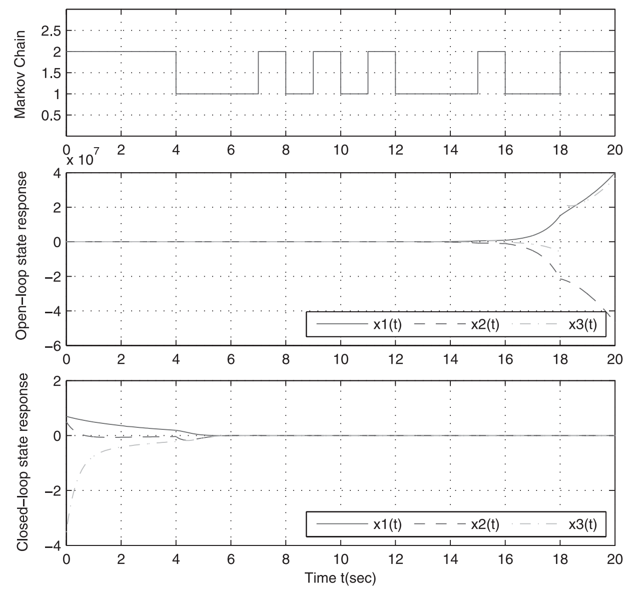

Figure 2. System response with transition rate matrix in Case 2.

Remark 1: As we have noted, our method can deal with continuous-time descriptor jump systems with partially known transition matrix effectively. In this example, for the ease of programming, the non-strict LMIs in Corollary 3.4 have been taken as strict ones to deal with. Through this example, it is obvious that Corollary 3.4 is much easier to be achieved and no matrix decomposition is involved.

Next, we will provide another example with bounded diagonal elements in transition rate matrix.

Example 4.2: Consider continuous-time descriptor Markovian jump systems with three modes and the following system matrices:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A(1) = \begin{bmatrix} 0.5 & -0.75 \\ 1 & 2 \end{bmatrix}, A(2) = \begin{bmatrix} 3.4 & -2 \\ 1 & -3 \end{bmatrix},$$

$$A(3) = \begin{bmatrix} 0.2 & 1 \\ 1 & -0.5 \end{bmatrix}$$

$$B(1) = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, B(2) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, B(3) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The transition rate matrix (2) is shown in Table 3 .

It is noted that the (2,2) block of transition rate matrix in Table 3 is unknown; thus, we can assume that it has a lower bound as 2, namely $\underline{\lambda_2} = 2$. Applying Theorem 3.3,

Table 2. Controllers for the two cases.

Case 1:	$K(1) = \begin{bmatrix} -1.16 & 3.99 & -1.68 \\ -0.44 & 2.60 & -1.41 \end{bmatrix}$	$K(2) = \begin{bmatrix} -0.43 & 2.14 & -0.19 \\ -0.80 & -0.24 & -1.18 \end{bmatrix}$
Case 2:	$K(1) = \begin{bmatrix} -0.96 & 2.09 & -1.32 \\ -0.86 & 2.53 & -2.64 \end{bmatrix}$	$K(2) = \begin{bmatrix} -0.51 & 2.85 & 0.19 \\ -0.91 & -0.17 & -1.25 \end{bmatrix}$

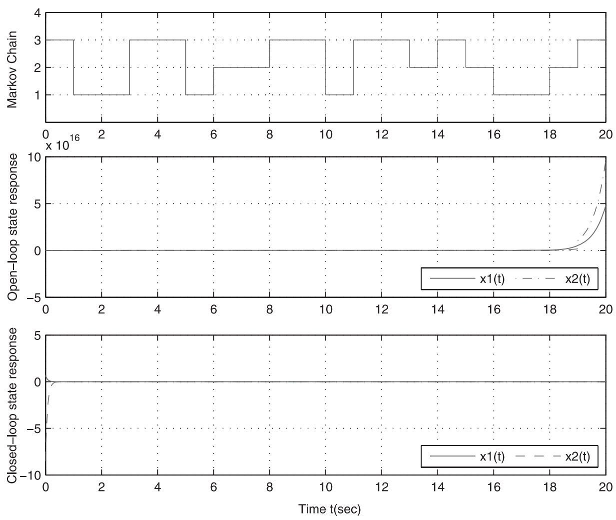

Figure 3. System response with transition rate matrix in Table 3.

a stabilising controller gain can be solved as

$$K(1) = \begin{bmatrix} -1.58 & 2.08 \end{bmatrix}, K(2) = \begin{bmatrix} -34.80 & 12.12 \end{bmatrix},$$
$$K(3) = \begin{bmatrix} 1.55 & 0.32 \end{bmatrix}.$$

Figure 3 shows the state response for the open-loop and closed-loop systems with the same Markovian jump process under the given initial condition $x_0 = \begin{bmatrix} 0.7, & -8.52 \end{bmatrix}^T$. As shown in Figure 3, the open-loop system is unstable. After applying the controller to the open-loop system, we can see from the third subfigure that the closed-loop systems is stochastically admissible. Therefore, it is clear that the designed controller in this paper is feasible and effective.

Table 3. Transition rate matrix.

Mode	1	2	3
1	−1.3	?	?
2	?	?	1.1
3	0.2	0.3	−0.5

5. Conclusion

This paper has considered the stochastic stability and stabilisation problems for continuous-time descriptor Markovian jump systems. A new necessary and sufficient condition and a sufficient condition for stochastic admissibility have been proposed by virtue of LMI technique. Based on those stability results, the stabilising state-feedback controllers are constructed through the explicit solutions of LMIs. At last, numerical examples are given to illustrate the effectiveness of our results.

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