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OPTIMAL TAXATION POLICY FOR A PREY-PREDATOR FISHERY MODEL WITH RESERVES*

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Abstract: This paper is concerned with optimal control problem for a prey-predator fishery model with prey dispersal in two areas, one of which is free fishing area and the other is reserved area. Selective harvesting effort on prey population is considered. To conserve resource in the ecosystem, the regulatory agency manages exploitation of biological resources by imposing a taxation. The existence of its nonnegative equilibria and their qualitative analysis are discussed. The objective is to maximize the monetary economic interests, by virtue of Pontryagin's maximum principle, the optimal control problem is solved and an optimal taxation policy is obtained to keep the sustainable development of the ecosystem as well as achieve the economic interests of harvesting effort at an ideal level. A simulation example is carried out to show the consistency with the obtained theoretical results.

Key words: *prey-predator, fishery, reserved area, harvesting effort, optimal taxation policy*

Mathematics Subject Classification: *37N35, 37N40*

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1 Introduction

Ecological resources, like fishery and forest, are very useful renewable resources for human, however, with the rapid growth of population, the competition between human and some ecological resources are increasingly serious for limited space and food. Facing the dwindling resource stocks and deteriorating environment, the importance of protection of ecological resources becomes more and more highlighted.

Over the past decades, many researchers devoted themselves to the effective management of ecological resources. Some available regulating measures are advocated to control the exploitation, such as, reserves, taxation, lease of property rights, license fees, seasonal harvesting and so on. Among all of these, reserves are deemed to a feasible instrument to protect the sustainability of ecological resources. As have suggested by [9], reserves not only protect species inside the reserved area, but they can also increase the species enrichment and diversities. [4] proposed a fishery model with reserve area in a aquatic environment and elaborated the effect of the reserve area on the fishery model. [1] described a prey-predator type fishery model with prey dispersal in a two-patch environment and investigated how to maximize the net economic revenue earn from the fishery through implementing the sustainable properties of the fishery to keep the ecological balance. [13] established a Holling

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II functional response prey-predator fishery model with marine reserves and pointed out that so long as the prey population in the reserved zone did not extinct, the both prey always existed. By virtue of singular systems theory [17], a singular ecological-economic model with harvesting and migration was proposed by [18], they utilized a variable structure method to eliminate the singular induced bifurcation (for further understanding, please refer to [12, 15, 16, 19, 20] and the references therein) and design a stabilized controller.

As the exploitation of ecological resources are often companied by the economic interests. The incentive driven by the profits may destroy the ecosystem, ultimately, may lead to a crisis of humans. Consequently, there is widespread concern upon how to keep the ecological balance and maximize the revenue from the harvesting of ecological resources. As pointed out in [2], economists are extremely interested in taxation, partly owing to its flexibility and partly attributing to the fact that a competitive economic system can be better maintained under taxation than other regulatory measures. Recently, considerable attentions are paid to the modeling of harvesting of ecological resources. In these models, the harvesting effort is supposed to be a differential variable, several kinds of harvesting policies are utilized to study the dynamical behavior of the model system. Moreover, the optimal taxation policy problems are also discussed. A dynamic reaction model of a fishery consisting of two competing species with nonselective harvesting were considered in [14], where taxation was a regulatory policy to control exploitation. [7, 8] dealt with the selective harvesting in a ratio-dependent prey-predator fishery model and the optimal taxation policy problems were considered. A prey-predator model with stage structure for prey and selective harvest effort on predator was proposed in [10], in which taxation was a control instrument to protect the population from overexploitation, and the impact of variation of gestation delay on the stability switch of the model system was also analyzed. In [21], a prey-predator model with gestation delay, stage structure for predator, and selective harvesting effort on mature predator was considered, the effects of taxation on the existence, stability behavior and trade-offs between profits and ecological balance were also discussed elaborately. The marine reserves and taxation were jointly introduced in [6], the optimal taxation policy was provided. [11] established an ecoepidemiological prey-predator model with selective harvesting effort on predator population, in which the control variable were chosed to be vaccination and taxation, Bendixson criterion was used to discuss the global stability behavior. By taking the crowding effect into consideration, a dynamical model was proposed and analyzed in [3] to discuss the effect of population on a resource biomass, and taxation was also taken as a control variable. [5] dealt with a prey-predator model system in the presence of some alternative food to predator and selective harvesting on prey species, where the maximum sustainable yield level was not considered as a reasonable method to prevent the model system from extinction, and taxation was utilized to overcome this defect, and was deemed to be superior.

As we have seen from the above, the prey-predator fishery model with both reserves and taxation have not been considered, which is the incentive of this paper. In this paper, we establish a prey-predator fishery model system with prey population dispersal in two aquatic environments, a free fishing area and a reserved area where predation and harvesting are prohibited. The prey population obeys the logistic growth law and the migrations between the free fishing area and the reserved area are considered to be stochastic. The predator consumes the prey in the free fishing area in proportion to the predator population and grows with logistic law and the capacity proportional to the prey population in the free fishing area. By taking the harvesting effort as the dynamical variable and utilizing the taxation as a control instrument, the dynamical behavior of the model system around the nonnegative equilibria are analyzed elaborately, particularly, only the global stability of interior equilibrium is discussed. To keep the trade-offs between ecological protection and commercial exploitation, the optimal taxation policy problem is formulated, then, by using Pontryagin's maximum principle, the optimal control problem is solved and the optimal taxation policy is provided. Finally, the effect of marine reserve is also illustrated by analyzing the dynamical response of our model system.

The organization of our paper is stated as follows: The next section is devoted to the model formulation. The qualitative analysis of the nonnegative equilibria will be performed in the third section, specially, just the global stability behavior of interior equilibrium is studied. The optimal taxation policy problem is formulated and solved in the fourth section. A simulation example is provided to support the analytical findings in this paper. Finally, this paper ends with a conclusion.

2 Model Formulation

Consider the model proposed by [9] which is shown as follows:

$$\begin{cases} \dfrac{dx(t)}{dt} = rx(t)\left(1-\dfrac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - mx(t)z(t) - qE(t)x(t) \\ \dfrac{dy(t)}{dt} = sy(t)\left(1-\dfrac{y(t)}{L}\right) + \sigma_1 x(t) - \sigma_2 y(t) \\ \dfrac{dz(t)}{dt} = \alpha z(t)\left(1-\dfrac{z(t)}{\gamma x(t)}\right) \end{cases} \tag{2.1}$$

where $x(t)$ and $y(t)$ are the respective densities of prey population inside the free fishing zone and reserved zone at time t. $z(t)$ is the density of predator population at time t. Prey population migrate from the free fishing zone to the reserved zone at a rate σ_1 and the reserved zone to the free fishing zone at a rate σ_2, respectively. $r(K)$ and $s(L)$ are the intrinsic growth rates (carrying capacities) of prey population inside the free fishing zone and reserved zone, respectively. m is the predation rate, and γ is the equilibrium ratio of prey-to-predator population. $E(t)$ is the harvesting effort at time t.

As we know, a competitive system can be better maintained by taxation rather than other regulatory methods. In order to protect the fishery resources from overexploitation, the regulatory agency often imposes a taxation $\tau > 0$ per unit of the harvested prey population($\tau < 0$ implies the subsides paid to the fishermen). Herein, we take $E(t)$ as a dynamic variable, following [21], the following dynamic reaction model described by differential equations can be obtained:

$$\begin{cases} \dfrac{dx(t)}{dt} = rx(t)\left(1-\dfrac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - mx(t)z(t) - qE(t)x(t) \\ \dfrac{dy(t)}{dt} = sy(t)\left(1-\dfrac{y(t)}{L}\right) + \sigma_1 x(t) - \sigma_2 y(t) \\ \dfrac{dz(t)}{dt} = \alpha z(t)\left(1-\dfrac{z(t)}{\gamma x(t)}\right) \\ \dfrac{dE(t)}{dt} = \alpha_0 E(t)\left(\left(p-\tau\right)qx(t) - c\right) \end{cases} \tag{2.2}$$

with the initial conditions

$$x(0) \geq 0, y(0) \geq 0, z(0) \geq 0, E(0) \geq 0 \tag{2.3}$$

where some of the parameters share the same meanings with model (2.1), p is the fixed price per unit of the harvested prey population, c is the fixed cost of harvesting per unit of effort, and α_0 is the stiffness parameter measuring the strength of reaction of harvesting effort.

For convenience of the subsequent analysis, it is assumed that

$$r - \sigma_1 - qE > 0, \quad s - \sigma_2 > 0 \tag{2.4}$$

Remark 2.1. It should be noted that the model in the absence of predator has been studied in [6]. Compared with the model proposed by [6], the model (2.2) by incorporating a predator into the ecosystems in this paper seems to be more realistic. Correspondingly, it can also show the more complex biological phenomenon.

3 Qualitative Analysis of Model System

This section aims to analyze the existence of nonnegative equilibria, discuss their local stability and global stability. Specially, interior equilibrium is the key focus of our discussion.

3.1 Existence of Equilibria

By equating the left-hand sides of differential equations to zero, after a little manipulation, we find five equilibria: $P_0\,(0,0,0,0)$, $P_1\,(x_1,y_1,0,0)$, $P_2\,(x_2,y_2,z_2,0)$, $P_3\,(x_3,y_3,0,E_3)$, $P^*\,(x^*,y^*,z^*,E^*)$.

It is obvious that $P_0\,(0,0,0,0)$ always exists. Thereby, we first verify the existence of equilibrium $P_1\,(x_1,y_1,0,0)$ where x_1, y_1 are the positive solutions of the equations below:

$$\begin{aligned} rx\left(1-\frac{x}{K}\right) - \sigma_1 x + \sigma_2 y &= 0 \\ sy\left(1-\frac{y}{L}\right) + \sigma_1 x - \sigma_2 y &= 0 \end{aligned} \tag{3.1}$$

Expressing y with regard to x leads to

$$y_1 = \frac{1}{\sigma_2}\left(\frac{rx_1^2}{K} - (r-\sigma_1)\,x_1\right) \tag{3.2}$$

where x_1 satisfies the following cubic equation

$$a_1x^3 + b_1x^2 + c_1x + d_1 = 0 \tag{3.3}$$

with

$$\begin{aligned} a_1 &= \frac{sr^2}{L\sigma_2^2K^2} \\ b_1 &= -\frac{2sr\,(r-\sigma_1)}{L\sigma_2^2K} \\ c_1 &= \frac{s\,(r-\sigma_1)^2}{L\sigma_2^2} - \frac{r\,(s-\sigma_2)}{K\sigma_2} \\ d_1 &= \frac{(s-\sigma_2)}{\sigma_2}\,(r-\sigma_1) - \sigma_1 \end{aligned} \tag{3.4}$$

The existence of a positive solution to equation (3.3) can be assured if the conditions below are satisfied

$$\begin{aligned} &\frac{s\left(r-\sigma_1\right)^2}{L\sigma_2^2} > \frac{r\left(s-\sigma_2\right)}{K\sigma_2} \\ &\left(s-\sigma_2\right)\left(r-\sigma_1\right) < \sigma_1\sigma_2 \end{aligned} \tag{3.5}$$

In order to make the equilibrium biologically meaningful, component y_1 should be positive, that is to say, an additional inequality

$$\frac{rx_1}{K} > \left(r-\sigma_1\right) \tag{3.6}$$

must holds.

Consequently, we have the following theorem.

Theorem 3.1. *Provided that* (3.5) *and* (3.6) *hold, then dynamic system* (2.2) *has a non-negative equilibrium* $P_1\left(x_1, y_1, 0, 0\right)$.

Now, we discuss the existence of equilibrium $P_2\left(x_2, y_2, z_2, 0\right)$. x_2, y_2 and z_2 are positive and satisfy the following equation

$$\begin{aligned} &rx(t)\left(1-\frac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - mx(t)z(t) = 0 \\ &sy(t)\left(1-\frac{y(t)}{L}\right) + \sigma_1 x(t) - \sigma_2 y(t) = 0 \\ &z(t) = \gamma x(t) \end{aligned} \tag{3.7}$$

Solving y_2 and z_2 from the first equation and the third equation of (3.7) respectively, we can obtain

$$\begin{aligned} &y_2 = \frac{1}{\sigma_2}\left[\left(\frac{r}{K}+m\gamma\right)x_2^2 - \left(r-\sigma_1\right)x_2\right] \\ &z_2 = \gamma x_2 \end{aligned} \tag{3.8}$$

substituting y_2 and z_2 into the second equation of (3.7), we obtain a equation in regard to x

$$a_2x^3 + b_2x^2 + c_2x + d_2 = 0 \tag{3.9}$$

where

$$\begin{aligned} &a_2 = \frac{s}{L\sigma_2^2}\left(\frac{r}{K}+m\gamma\right)^2 \\ &b_2 = -\frac{2s\left(r-\sigma_1\right)}{L\sigma_2^2}\left(\frac{r}{K}+m\gamma\right) \\ &c_2 = \frac{s\left(r-\sigma_1\right)^2}{L\sigma_2^2} - \frac{\left(s-\sigma_2\right)}{\sigma_2}\left(\frac{r}{K}+m\gamma\right) \\ &d_2 = \frac{\left(s-\sigma_2\right)}{\sigma_2}\left(r-\sigma_1\right) - \sigma_1 \end{aligned} \tag{3.10}$$

After little manipulations, there exist a positive solution x_2 to equation (3.9) if the following inequalities hold

$$\begin{aligned} &\frac{s\left(r-\sigma_1\right)^2}{L\sigma_2} < \left(s-\sigma_2\right)\left(\frac{r}{K}+m\gamma\right) \\ &\left(s-\sigma_2\right)\left(r-\sigma_1\right) < \sigma_1\sigma_2 \end{aligned} \tag{3.11}$$

To guarantee the positiveness of y_2, the following condition must be met

$$x_2 > (r - \sigma_1) / \left(\frac{r}{K} + m\gamma\right) \tag{3.12}$$

Therefore, we can come to a conclusion.

Theorem 3.2. *Provided that* (3.11) *and* (3.12) *hold, then dynamic system* (2.2) *has a non-negative equilibrium* $P_2\left(x_2, y_2, z_2, 0\right)$.

Next, we are prepared to show that equilibrium $P_3\left(x_3, y_3, 0, E_3\right)$ is existent under certain conditions. Since x_3, y_3 and E_3 are the positive solution of the equations below

$$\begin{aligned} & rx(t)\left(1 - \frac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - qE(t)x(t) = 0 \\ & sy(t)\left(1 - \frac{y(t)}{L}\right) + \sigma_1 x(t) - \sigma_2 y(t) = 0 \\ & \alpha_0 E(t)\left(\left(p - \tau\right) qx(t) - c\right) = 0 \end{aligned} \tag{3.13}$$

After solving the system of equations, the elements in equilibrium P_3 are given as

$$\begin{aligned} x_3 &= \frac{c}{\left(p - \tau\right) q}, \\ y_3 &= \frac{\left(s - \sigma_2\right) + \sqrt{\left(s - \sigma_2\right)^2 + 4\left(s/L\right)\sigma_1 x_3}}{2s/L}, \\ E_3 &= \frac{1}{qx_3}\left(\left(r - \frac{rx_3}{K} - \sigma_1\right) x_3 + \sigma_2 y_3\right) \end{aligned} \tag{3.14}$$

On account of the positiveness of x_3, y_3 and E_3, the following inequality is supposed to be satisfied

$$\tau < p - \frac{Kc}{\left(r - \sigma_1\right) rq} \tag{3.15}$$

Thus, according to the aforesaid discussion, the next theorem can be given.

Theorem 3.3. *Provided that inequality* (3.15) *holds, then dynamic system* (2.2) *has a non-negative equilibrium* $P_3\left(x_2, y_2, 0, E_3\right)$.

Finally, let us end up this subsection with the analysis of the existence of the interior equilibrium or positive equilibrium $P^*\left(x^*, y^*, z^*, E^*\right)$. Actually, x^*, y^*, z^* and E^* are the positive solution of

$$\begin{aligned} 0 &= rx(t)\left(1 - \frac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - mx(t)z(t) - qE(t)x(t) \\ 0 &= sy(t)\left(1 - \frac{y(t)}{L}\right) + \sigma_1 x(t) - \sigma_2 y(t) \\ 0 &= \alpha z(t)\left(1 - \frac{z(t)}{\gamma x(t)}\right) \\ 0 &= \alpha_0 E(t)\left(\left(p - \tau\right) qx(t) - c\right) \end{aligned} \tag{3.16}$$

from which we have

$$x^* = \frac{c}{(p-\tau)\,q}, \quad z^* = \frac{\gamma c}{(p-\tau)\,q}, \quad y^* = \frac{(s-\sigma_2) + \sqrt{(s-\sigma_2)^2 + \frac{4\sigma_1 sc}{(p-\tau)qL}}}{2s/L},$$
$$E^* = \frac{1}{qx^*}\left[(r-\sigma_1)\,x^* + \sigma_2 y^* - \left(\gamma m + \frac{r}{K}\right)x^{*2}\right] \tag{3.17}$$

Since the biological meaning of interior equilibrium, the following condition needs to be satisfied to guarantee the positivity of all the components of interior equilibrium.

$$0 < \tau < p - \frac{c\,(\gamma mK + r)}{qK\,(r-\sigma_1)} \tag{3.18}$$

Theorem 3.4. *Provided that inequality* (3.18) *holds, then dynamic system* (2.2) *has a non-negative equilibrium* $P^*(x^*, y^*, z^*, E^*)$.

3.2 Local Stability

The local stability behavior can be analyzed by computing the variational matrix

$$J = \begin{bmatrix} r - \frac{2rx}{K} - \sigma_1 - mz - qE & \sigma_2 & -mx & -qx \\ \sigma_1 & s - \frac{2sy}{L} - \sigma_2 & 0 & 0 \\ \frac{\alpha z^2}{\gamma x^2} & 0 & \alpha - \frac{2\alpha z}{\gamma x} & 0 \\ \alpha_0 qE\,(p-\tau) & 0 & 0 & \alpha_0\{(p-\tau)\,qx - c\} \end{bmatrix} \tag{3.19}$$

As shown in [9], even if equilibrium $P_0(0,0,0,0)$ is defined for the dynamic system (2.2), its corresponding linearized system do not exist, which implies that the local stability can not be investigated.

To determine the local stability of $P_1(x_1, y_1, 0, 0)$, the Jaccobian of dynamic system (2.2) around $P_1(x_1, y_1, 0, 0)$ is

$$J(P_1) = \begin{bmatrix} -\frac{rx_1}{K} - \frac{\sigma_2 y_1}{x_1} & \sigma_2 & -mx_1 & -qx_1 \\ \sigma_1 & -\frac{sy_1}{L} - \frac{\sigma_1 x_1}{y_1} & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha_0\{(p-\tau)\,qx_1 - c\} \end{bmatrix} \tag{3.20}$$

which gives the characteristic equation at P_1

$$det \begin{bmatrix} -\left(\frac{rx_1}{K} + \frac{\sigma_2 y_1}{x_1}\right) - \lambda & \sigma_2 & -mx & -qx \\ \sigma_1 & -\left(\frac{sy_1}{L} + \frac{\sigma_1 x_1}{y_1}\right) - \lambda & 0 & 0 \\ 0 & 0 & \alpha - \lambda & 0 \\ 0 & 0 & 0 & \alpha_0\{(p-\tau)\,qx - c\} - \lambda \end{bmatrix} = 0 \tag{3.21}$$

From (3.21), it is easily observed that there exists a positive eigenvalue $\lambda = \alpha$, accordingly, it is obvious that P_1 is unstable.

At $P_2(x_2, y_2, z_2, 0)$, the Jaccobian of dynamic system (2.2) reduces to

$$J(P_2) = \begin{bmatrix} -\frac{rx_2}{K} - \frac{\sigma_2 y_2}{x_2} & \sigma_2 & -mx_2 & -qx_2 \\ \sigma_1 & -\frac{sy_2}{L} - \frac{\sigma_1 x_2}{y_2} & 0 & 0 \\ \alpha\gamma x_2 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & \alpha_0\{(p-\tau)\,qx_2 - c\} \end{bmatrix} \tag{3.22}$$

thus, we can acquire the following characteristic equation about P_2

$$\left(\lambda-\alpha_0\left(\left(p-\tau\right)qx_2-c\right)\right)\left(\lambda^3+n_1\lambda^2+n_2\lambda+n_3\right)=0 \tag{3.23}$$

where

$$\begin{aligned}
n_1&=\alpha+\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}+\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\\
n_2&=\alpha\left(\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}+\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)+\left(\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}\right)\left(\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)-\sigma_1\sigma_2+m\alpha\gamma x_2^2\\
n_3&=\alpha\left(\left(\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}\right)\left(\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)-\sigma_1\sigma_2\right)+m\alpha\gamma x_2^2\left(\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)
\end{aligned} \tag{3.24}$$

It is straightforward to check that

$$\begin{aligned}
&n_1>0,n_2>0,n_3>0,\\
&n_1n_2-n_3=\alpha\left(\alpha+\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}+\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)\left(\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}+\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)\\
&+\left(\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}+\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)\left(\left(\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}\right)\left(\frac{sy_2}{L}+\frac{\sigma_1x_2}{y_2}\right)-\sigma_1\sigma_2\right)\\
&+m\alpha\gamma x_2^2\left(\alpha+\frac{rx_2}{K}+\frac{\sigma_2y_2}{x_2}\right)>0
\end{aligned} \tag{3.25}$$

In the light of Routh-Hurwitz criterion, it can be found that the local stability of P_2 lies in the sign of $(p-\tau)\,qx_2-c$, that is, if $(p-\tau)\,qx_2-c>0$, equilibrium P_2 is unstable, while $(p-\tau)\,qx_2-c<0$, equilibrium P_2 is locally asymptotically stable.

Around $P_3\left(x_3,y_3,0,E_3\right)$, the Jacobian of dynamic system (2.2) becomes

$$J\left(P_3\right)=\begin{bmatrix}-\frac{rx_3}{K}-\frac{\sigma_2y_3}{x_3} & \sigma_2 & -mx_3 & -qx_3\\ \sigma_1 & -\frac{sy_3}{L}-\frac{\sigma_1x_3}{y_3} & 0 & 0\\ 0 & 0 & \alpha & 0\\ \alpha_0qE_3\left(p-\tau\right) & 0 & 0 & 0\end{bmatrix} \tag{3.26}$$

and its corresponding characteristic equation is

$$det\begin{bmatrix}-\left(\frac{rx_3}{K}+\frac{\sigma_2y_3}{x_3}\right)-\lambda & \sigma_2 & -mx_3 & -qx_3\\ \sigma_1 & -\left(\frac{sy_3}{L}-\frac{\sigma_1x_3}{y_3}\right)-\lambda & 0 & 0\\ 0 & 0 & \alpha-\lambda & 0\\ \alpha_0qE_3\left(p-\tau\right) & 0 & 0 & -\lambda\end{bmatrix}=0 \tag{3.27}$$

from which we find that a positive solution $\lambda=\alpha$ appears, thus, P_3 is unstable.

At $P^*\left(x^*,y^*,z^*,E^*\right)$, the Jacobian of dynamic system (2.2) can be further reduced to be

$$J=\begin{bmatrix}-\frac{rx^*}{K}-\frac{\sigma_2y^*}{x^*} & \sigma_2 & -mx^* & -qx^*\\ \sigma_1 & -\frac{sy^*}{L}-\frac{\sigma_1x^*}{y^*} & 0 & 0\\ \alpha\gamma & 0 & -\alpha & 0\\ \alpha_0qE^*\left(p-\tau\right) & 0 & 0 & 0\end{bmatrix} \tag{3.28}$$

It follows that the characteristic equation of $P^*\left(x^*,y^*,z^*,E^*\right)$ is

$$\lambda^4+m_1\lambda^3+m_2\lambda^2+m_3\lambda+m_4=0 \tag{3.29}$$

where

$$
\begin{aligned}
m_1 =& \alpha + \frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*} \\
m_2 =& \left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right)\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) - \sigma_1\sigma_2 + \alpha\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \\
&+ \alpha_0 q^2 E^* x^* (p-\tau) + \alpha\gamma m x^* \\
m_3 =& \alpha\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right)\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) - \alpha\sigma_1\sigma_2 + \alpha_0 q^2 E^* x^* (p-\tau)\left(\alpha + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \\
&+ \alpha\gamma m x^* \left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \\
m_4 =& \alpha\alpha_0 q^2 E^* x^* (p-\tau)\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right)
\end{aligned}
\tag{3.30}
$$

It can be checked that

$$
\begin{aligned}
&m_1 > 0, \qquad m_2 > 0, \qquad m_3 > 0, \qquad m_4 > 0, \\
m_1 m_2 - m_3 =& \left(\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right)\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) - \sigma_1\sigma_2\right)\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \\
&+ \alpha\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right)\left(\alpha + \frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \\
&+ \alpha_0 q^2 E^* x^* (p-\tau)\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right) + \alpha\gamma m x^*\left(\alpha + \frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right) > 0,
\end{aligned}
\tag{3.31}
$$

$$
\begin{aligned}
& m_1 m_2 m_3 - m_1^2 m_4 - m_3^2 \\
&= \Bigg\{\left(\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right)\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) - \sigma_1\sigma_2\right)\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \\
&+ \alpha_0 q^2 E^* x^* (p-\tau)\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right) + \alpha\gamma m x^*\left(\alpha + \frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right)\Bigg\} m_3 \\
&+ \alpha^2\left\{\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right)\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) - \sigma_1\sigma_2 + \gamma m x^*\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right)\right\} \\
& \left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right)\left(\alpha + \frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \\
&+ \alpha^2 \alpha_0 q^2 E^* x^* (p-\tau)\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right)\left(\alpha + \frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*} + \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right)
\end{aligned}
\tag{3.32}
$$

From the aforesaid computation, along with Routh-Hurwitz criteria, we can know that the roots of (3.29) lie in the left-hand side of the complex plane. Accordingly, we prove the locally asymptotic stability of the interior equilibrium P^*.

From above all, we summarize these discussions as the following theorem.

Theorem 3.5. *The local stability of nonnegative equilibria are shown as follows:*

1. *Although $P_0(0,0,0,0)$ is defined for dynamic system (2.2), it can not be linearized, which leads to the fact that we have no ability to determine whether P_0 is stable.*

2. *As long as* $P_1(x_1, y_1, 0, 0)$ *exits, it is unstable.*

3. *If* $(p-\tau)\,qx_2 - c < 0$, $P_2(x_2, y_2, z_2, 0)$ *is locally asymptotically stable, whereas, when* $(p-\tau)\,qx_2 - c > 0$, P_2 *is unstable.*

4. $P_3(x_3, y_3, 0, E_3)$ *is always unstable provided that it satisfies* (3.15)*, that is, its existent condition.*

5. $P^*(x^*, y^*, z^*, E^*)$ *is locally asymptotically stable, as long as its existence can be guaranteed.*

3.3 Global Stability

Since interior equilibrium is biologically meaningful, this subsection mainly studies the globally asymptotic stability of interior equilibrium.

Theorem 3.6. *If* $\mu_1 < x < \mu_2$*, then* $P^*(x^*, y^*, z^*, E^*)$ *is globally asymptotically stable, where* $\mu_1 = 1 + \frac{2r}{K\gamma m} - \sqrt{\frac{4r}{K\gamma m} + \frac{4r^2}{K^2\gamma^2 m^2}}$ *and* $\mu_2 = 1 + \frac{2r}{K\gamma m} + \sqrt{\frac{4r}{K\gamma m} + \frac{4r^2}{K^2\gamma^2 m^2}}$ *.*

Proof. Construct the Lyapunov functional candidate as

$$\begin{aligned} V(x,y,z,E) = &\left(x - x^* - x^* ln\left(\frac{x}{x^*}\right)\right) + \rho_1\left(y - y^* - y^* ln\left(\frac{y}{y^*}\right)\right) \\ &+ \rho_2\left(z - z^* - z^* ln\left(\frac{z}{z^*}\right)\right) + \rho_3\left(E - E^* - E^* ln\left(\frac{E}{E^*}\right)\right) \end{aligned} \tag{3.33}$$

where ρ_1, ρ_2 and ρ_3 are unknown positive parameter to be determined.

Differentiating $V(x,y,z,E)$ with respect to t along with the trajectory of dynamic system (2.2), it can be shown as

$$\frac{dV}{dt} = \frac{x - x^*}{x}\frac{dx}{dt} + \rho_1\frac{y - y^*}{y}\frac{dy}{dt} + \rho_2\frac{z - z^*}{z}\frac{dz}{dt} + \rho_3\frac{E - E^*}{E}\frac{dE}{dt} \tag{3.34}$$

Let $\rho_1 = \frac{\sigma_2 y^*}{\sigma_1 x^*}$, $\rho_2 = \frac{1}{\alpha}$ and $\rho_3 = \frac{1}{\alpha_0(p-\tau)}$, substituting (3.16) in to (3.34), after a little skillful computation, the equation (3.34) can be further expressed as

$$\begin{aligned} \frac{dV}{dt} = &-\frac{s\sigma_2 y^*}{L\sigma_1 x^*}(y - y^*)^2 - \frac{\sigma_2}{x^* xy}(x^* y - xy^*)^2 \\ &-\left\{\frac{r}{K}(x - x^*)^2 - \left(\frac{1}{x} - m\right)(x - x^*)(z - z^*) + \frac{1}{\gamma x}(z - z^*)^2\right\} \end{aligned} \tag{3.35}$$

It can be easily observed that provided $\left(\frac{1}{x} - m\right) < \frac{4r}{K\gamma x}$, that is, $\mu_1 < x < \mu_2$, $\frac{dV}{dt} \le 0$ holds, and the equality does not always hold for any $(x,y,z,E) \ne (x^*, y^*, z^*, E^*)$.

In light of Lyapunov stability theory, we can draw a conclusion that $P^*(x^*, y^*, z^*, E^*)$ is globally asymptotically stable.

□

4 Optimal Control of Model System

In what follows, we will determine the optimal taxation policy to achieve the maximum net revenues from the harvested population.

With the purpose of planning harvesting and keeping the sustainable development of ecosystem, we design an optimal harvesting policy to maximize the total discounted net revenue from the harvesting by regarding taxation as a control instrument.

As we know, Net economic revenue to the society $\pi(x(t), y(t), z(t), E(t))$ = Net economic revenue from harvesting + Net economic revenue to the regulatory agency = $(p-\tau)\,qx(t)E(t) - cE(t) + \tau qx(t)E(t) = (pqx(t) - c)\,E(t)$. Thus, the optimal performance index can be formulated as:

$$J = \int_0^{+\infty} e^{-\delta t}\left(pqx(t) - c\right)E(t) \tag{4.1}$$

where δ is the instantaneous annual rate of discount.

For the given model system (2.2) with initial condition (2.3), the optimal control problem is to seek an admissible taxation policy taking value in $[\tau_{min}, \tau_{max}]$ to maximize the performance index (4.1). τ_{min} and τ_{max} are the feasible upper and lower limit of the taxation for harvested effort, respectively. Specifically, $\tau_{min} < 0$ means that subsidies have an effect on the evolution of the model system.

By virtue of Pontryagin's Maximum Principle [2], the Hamiltonian function of this control problem can be constructed as:

$$\begin{aligned} &H\left(x(t), y(t), E(t), \sigma(t), t\right) \\ &= e^{-\delta t}\left[pqx(t) - c\right]E(t) \\ &\quad + \lambda_1(t)\left[rx(t)\left(1 - \frac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - mx(t)z(t) - qE(t)x(t)\right] \\ &\quad + \lambda_2(t)\left[sy(t)\left(1 - \frac{y(t)}{L}\right) + \sigma_1 x(t) - \sigma_2 y(t)\right] \\ &\quad + \lambda_3(t)\left[\alpha z(t)\left(1 - \frac{z(t)}{\gamma x(t)}\right)\right] + \lambda_4(t)\left[\alpha_0 E(t)\left((p-\tau)\,qx(t) - c\right)\right] \end{aligned} \tag{4.2}$$

where $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$ and $\lambda_4(t)$ are adjoint variables.

Assume that the optimal solution does not occur at the extreme point, then the necessary condition for singular control τ^* to be optimal is given by

$$\frac{\partial H}{\partial \tau} = -\lambda_4 \alpha_0 Eqx = 0, \quad \text{which implies } \lambda_4 = 0 \tag{4.3}$$

and the adjoint equations are

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x} = -\left[e^{-\delta t} pqE + \lambda_1\left(r - \frac{2rx}{K} - \sigma_1 - mz - qE\right) + \lambda_2\sigma_1 + \lambda_3\frac{\alpha z^2}{\gamma x^2}\right] \tag{4.4}$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y} = -\left[\lambda_1\sigma_2 + \lambda_2\left(s - \frac{2sy}{L} - \sigma_2\right)\right] \tag{4.5}$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial z} = -\left[-\lambda_1 mx + \lambda_3\left(\alpha - \frac{2\alpha z}{\gamma x}\right)\right] \tag{4.6}$$

$$\frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial E} = -\left[e^{-\delta t}\left(pqx - c\right) - \lambda_1 qx\right] \tag{4.7}$$

By considering the interior equilibrium P^*, the above differential equations can be simplified to be

$$\frac{d\lambda_1}{dt} = -\left[e^{-\delta t} pqE^* - \lambda_1\left(\frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}\right) + \sigma_1\lambda_2 + \alpha\gamma\lambda_3\right] \tag{4.8}$$

$$\frac{d\lambda_2}{dt} = -\lambda_1\sigma_2 + \lambda_2\left(\frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}\right) \tag{4.9}$$

$$\frac{d\lambda_3}{dt} = mx^*\lambda_1 + \alpha\lambda_3 \tag{4.10}$$

$$\lambda_1 = \left(p - \frac{c}{qx*}\right)e^{-\delta t} \tag{4.11}$$

Combined (4.11) with (4.10), we have

$$\lambda_3(t) = -\frac{A_1}{\alpha+\delta}e^{-\delta t} \tag{4.12}$$

where $A_1 = \left(p - \frac{c}{qx*}\right)mx^*$.

Likewise, we can obtain

$$\lambda_2(t) = -\frac{A_3}{A_2+\delta}e^{-\delta t} \tag{4.13}$$

where $A_2 = \frac{sy^*}{L} + \frac{\sigma_1 x^*}{y^*}$ and $A_3 = \sigma_2\left(p - \frac{c}{qx*}\right)$.

$$\lambda_1(t) = -\frac{B_2}{B_1+\delta}e^{-\delta t} \tag{4.14}$$

where $B_1 = \frac{rx^*}{K} + \frac{\sigma_2 y^*}{x^*}$ and $B_2 = \frac{\sigma_1 A_3}{A_2+\delta} + \frac{\alpha\gamma A_1}{\alpha+\delta} - pqE^*$.

It follows from (4.11) and (4.14) that

$$\left(p - \frac{c}{qx*}\right) = -\frac{B_2}{B_1+\delta} \tag{4.15}$$

which provides a way to solve optimal control τ. Consequently, the optimal path can be given by $x^* = x_\delta, y^* = y_\delta, z^* = z_\delta, E^* = E_\delta$.

Remark 4.1. It can be observed from (4.12), (4.13) and (4.14) that shadow prices $\lambda_i(t)e^{\delta t}$ $(i = 1,2,3,4)$ remain constant over time in an optimum equilibrium which implies that they strictly satisfy the transversality condition at ∞, thus, they remain bounded as $t \to \infty$. Taking the interior equilibrium $P^*(x^*, y^*, z^*, E^*)$ into account, (4.11) can be rewritten as

$$\lambda_1 qx^* = e^{-\delta t}(pqx^* - c) = e^{-\delta t}\frac{\partial \pi}{\partial E} \tag{4.16}$$

which means that the user's total cost of harvesting per unit effort is equal to the discounted values of the future price at the steady state effort level.

5 Simulation Example

With the aid of MATLAB 2010a, a simulation example is provided to show the effectiveness of our theoretical result.

For testifying the validity of our result, we take the Zhoushan fishery which is located in Zhejiang province and is the biggest fishery in China. As given by [18], the total size of sea area is approximately more than 10800km^2. The size of the inshore area is about 3700km^2, and the size of the offshore area is about 7100km^2. One kind of fish in Zhoushan fishery is the coiliaspp and the total population is 1099 million in the whole sea area. To prevent

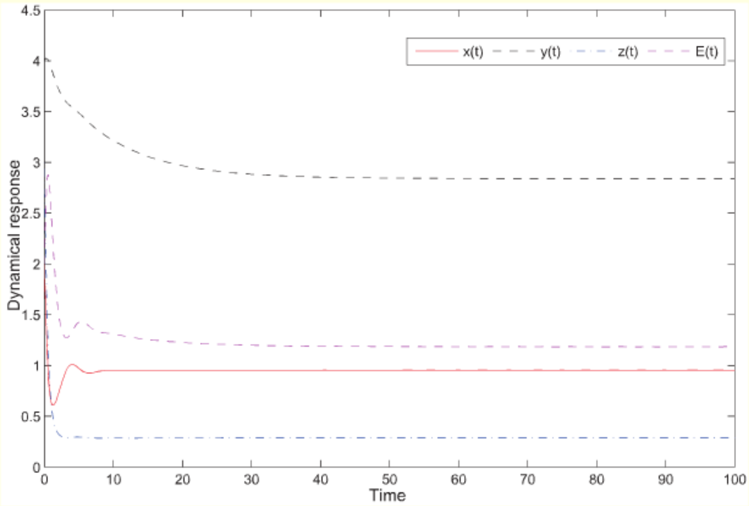

Figure 1: Dynamical responses of model system (2.2) with the optimal taxation.

over exploitation, only the coiliaspp in the inshore area is permitted to be harvested, while that in the offshore area is prohibited. On the other hand, the eel is thrown in the inshore area so as to promoting the growing of the coiliaspp in the inshore area. According to the data in [18] and [22], the carrying capacity of the inshore area is about 423 million and that of the offshore area is about 676 million, the equilibrium ratio γ is approximately 0.3. The intrinsic growth rates in inshore area and offshore area are respectively assumed to be $r = 0.6$ and $s = 0.2$. The coiliaspp migrates between the inshore area and the offshore area at a same rate $\sigma_1 = \sigma_2 = 0.3$. The eel captures the coiliaspp at a rate $m = 0.02$, and the intrinsic rate of the ell is $\alpha = 0.5$. The stiffness parameter is $\alpha_0 = 0.24$. It is also supposed that the capture coefficient q is 1 and the coiliaspp is sold at the average unit price $p = 11$ and its unit cost $c = 6$. All the parameters are set in appropriate units.

For model system (2.2), we can evaluate the range of the taxation for which the interior equilibrium is existent and asymptotically stable. As we have analyzed in the foregoing section, theoretically, the desirable range of the taxation is $(0, \tau_{max})$, numerically, we can find that the model system (2.2) is asymptotically stable for any $\tau \in (0, 10.8516)$. Thus, in the following, we are to find out the optimal control τ in the interval $(0, 10.8516)$ with instantaneously annual rate of discount $\delta = 0.03$. Solving the equation (4.15), the optimal taxation τ_δ which takes value in the interval $(0, 10.8516)$ is about 4.7160. Consequently, the optimal equilibrium levels of biological population and harvesting effort can be also shown as $(x_\delta, y_\delta, z_\delta, E_\delta) = (0.9548, 2.8406, 0.4774, 1.1881)$.

For the optimal taxation, the time trajectories of prey population $x(t)$ in free fishing zone, prey population $y(t)$ inside protected zone, predator population $z(t)$ and harvesting effort $E(t)$ is depicted in Figure 1, from which we can observe that the model system (2.2) is globally asymptotically stable around the interior equilibrium. Actually, with the optimal taxation, the net economic revenue to the society can be achieved, what is important is that there is a balance between commercial exploitation and sustainable development of ecosystem.

As depicted in Figure 2, Figure 3, Figure 4 and Figure 5, the coiliaspp inside the inshore

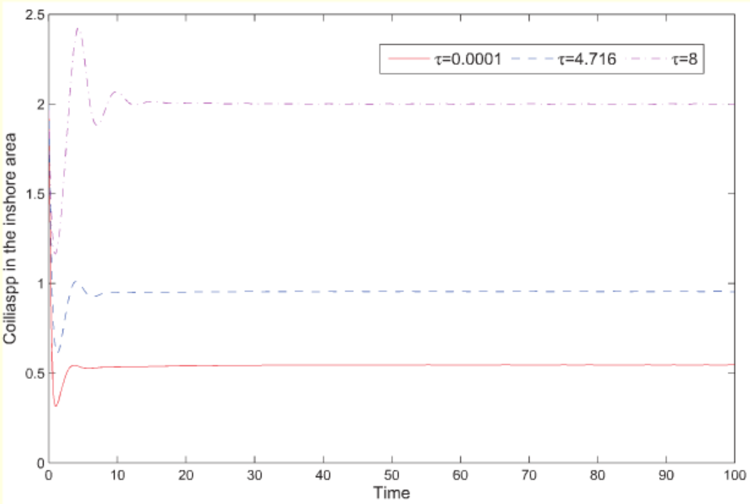

Figure 2: Dynamical responses of the coiliaspp in the inshore area with different tax.

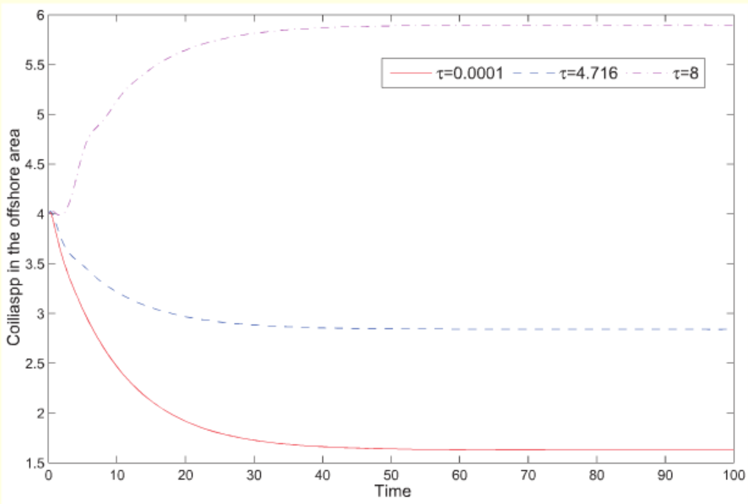

Figure 3: Dynamical responses of the coiliaspp in the offshore area with different tax.

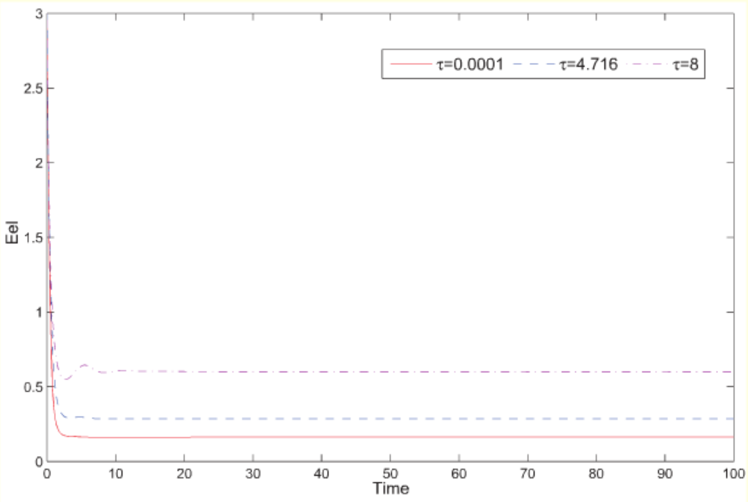

Figure 4: Dynamical responses of the eel with different tax.

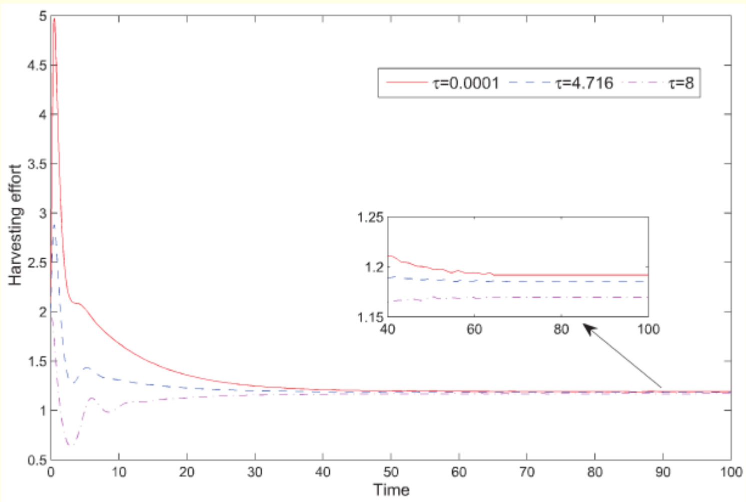

Figure 5: Dynamical responses of the harvesting effort with different tax.

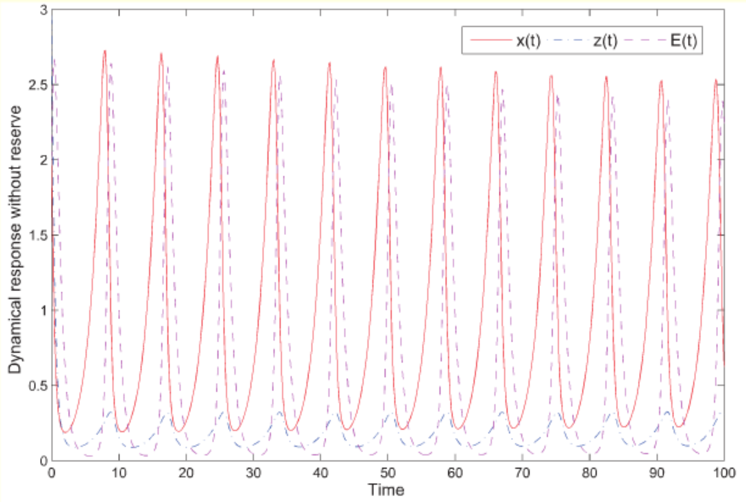

Figure 6: Dynamical responses of model system (2.2) without reserves and the optimal taxation $\tau = 4.7160$.

area and offshore area and the eel are increasingly growing, and the harvesting effort is progressively decreasing with the increase of the tax. It reflects the fact that the tax can regulate the exploitation on the coiliaspp, when little tax is needed to pay for the coiliaspp, the exploitation is profitable, in this case, the fishermen will make a large number of effort to harvest the coiliaspp, while, by raising the tax, the profit is reduced, then the fishermen will not pay too much effort on the harvesting of the coiliaspp, which leads to the increase of the coiliaspp and eel. Furthermore, the maximum economic benefits are achieved when the tax takes its optimal value.

From the Figure 6, it is shown that the dynamical response of the model system (2.2) without reserves reveals that if there is no refuge for the prey, the oscillation of the ecological system is extraordinarily noteworthy, which may results in the difficulty of convergence, eventually, lead to unbalance of the ecological system. Whereas, the rapidity and convergence of our model system illustrate the ability of marine reserves to protect abundance and diversities. Consequently, the marine reserves have a direct implications for the potential benefits to prey population adjacent to the reserves.

6 Conclusion

In this paper, a bioeconomic model system is established. The prey grows with the logistic law and lives in a two-patch zone, a free fishing zone and a reserved zone where all the exploitations and predation are permitted, the migrations between the free fishing zone and the reserved zone are supposed to be stochastic. The predator consumes the prey in the free fishing zone and obeys the logistic growing law with capacity proportional to the population of the prey in the free fishing zone. In this biological system, only the prey in the free fishing zone is available to be harvested. It is well known that biological resources have a strongly economic benefits, thus, most of people have an imperious desire to exploit the profitable resources, instead of receiving the maximum of the economic profits, unplanned exploitation has led to a terrible impact on ecosystems and the people driven by the interests which are reflected by the dwindling biological resources and the difficulty to harvest the enough food supply. Consequently, to overcome this dilemma, a control instrument, taxation, is proposed. By regarding the harvesting effort as the differential variable, a differential equation in relation to economic respect is introduced.

For the aforementioned bioeconomic model system, the existence of the nonnegative equilibria are firstly discussed, then the local stabilities of all the nonnegative equilibria are analyzed in detail and conditions to guarantee the locally asymptotic stability or instability are proposed. Particularly, in view of the biological meanings of interior equilibrium, the globally asymptotic stability of interior equilibrium are studied. To prevent the ecosystem from overexploitation and achieve the maximum economic benefits simultaneously, the optimal control problem is formulated by taking the net economic revenue to the society. Using the Pontryagin's Maximum Principle, the optimal control is solved, and the optimal equilibrium levels are shown. From the discussion in this paper, we obtain the optimal control which can ensure the economic benefits be maximum and all the biotic population always exist. Biologically, this result implies that the optimal taxation can guarantee the sustainable development of ecosystem and the maximization of economic interests. The effect of marine reserve on the ecosystem is also highlighted, as have been expected, marine reserve plays an indispensable role in maintaining the enrichment and diversities of species. Consequently, the method proposed in this paper can be regarded as a consultancy for management agency to govern the ecosystem reasonably.

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