Longevity Indices and Pension Fund Risk

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Abstract

Pension fund longevity risk is becoming increasingly important. Longevity indices would allow the creation of liquid derivatives that could be used to hedge this risk. However, there are a number of criteria that such indices would need to fulfil to provide an optimal solution, as well as a number of forms that the derivatives could take. These features are discussed, together with the characteristics of some existing longevity indices.

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1 Pension Scheme Liabilities

The liabilities of defined benefit pension funds in the United Kingdom continue to grow. Even with the reduction in the level of benefits accruing due to the closure of pension schemes to new members or even future accrual as reported by the National Association of Pension Funds (2008) and others, falls in long-term interest rates have increased the value of benefits due. The increasingly guaranteed nature of pension liabilities over the last thirty years together with moves to mark-to-market valuations in the UK's Financial Reporting Standard (FRS) 17 and International Accounting Standard (IAS) 19 has also meant that the interest rate and inflation sensitivity of the liabilities has a more direct impact on pension scheme sponsors. This alone would mean that risk management had become more of an issue for pension schemes and their sponsors than ever before.

However, the rise in liabilities has occurred at the same time as the market values of pension scheme assets have fallen sharply. This has led to large deficits in pension schemes. Furthermore, the current financial situation also means that sponsors are less able to clear these deficits. These issues are not limited to UK pension schemes – they are internationally relevant. This all means that there is a global incentive to manage unrewarded risks – those with which no risk premium is associated – whilst maintaining market risk (for returns).

The first unrewarded risks to be dealt with are interest rate and inflation risks, through the use of liability driven investment (LDI). Once interest rate and inflation risks have been dealt with, one of the most important risks remaining becomes longevity risk. Broadly speaking, this is the risk to a financial institution that its beneficiaries live longer than expected. However, longevity risk is actually a number of separate risks. These are equivalent to three of the four mortality risks identified by the International Actuarial Association (2004), these being volatility, level and trend risk.

Volatility risk arises because any pension scheme provides benefits for only a finite number of individuals, each of whom will either die or survive over the period under investigation. The smaller the number of lives, the greater the risk that the actual number of deaths will differ significantly from the number expected.

The volatility of past mortality experience causes level risk. This is the risk that the current expected rates of mortality for a portfolio of lives are overestimated due to random variations in past experience.

Looking forward, there is the risk that mortality rates will fall more quickly than expected (or will not rise as quickly). This is trend risk.

The fourth risk discussed by the IAA is catastrophe risk. This applies to mortality in that there can be a large one-off increase in mortality due to an event such as a pandemic; however, similar one-off falls in mortality rates do not occur.

The first three risks lead to an interest in hedging longevity risk. There are a number of ways that this can be done and a number of decisions that need to be made. In particular:

- the extent to which the contract be collateralised;
- whether all variation will be hedged or just mortality improvements above a certain level;
- if improvements above a certain level are hedged, what that level is; and
- whether the experience will be hedged on an indemnity or an index basis.

The most extreme form of collateralisation is to buy an annuity or other fully collateralised investment. This involves paying over all assets in exchange for a series of longevity-related cash flows. A similar contract based on a standardised reference population, the EIB/BNP Paribas survivor bond, was designed but never launched. However, we have already seen that pension schemes would prefer to hold onto assets in order to generate additional returns. Annuities and survivor bonds would not allow them to do this. Other hypothetical contracts such as annuity futures reduce the level of collateralisation needed, but have a number of technical issues. The other extreme, where collateralisation requirements can be minimal, is to use a survivor swap, as discussed by Dowd et al (2006). This is a contract where one party undertakes to make periodic payments based on the expected mortality of a group of lives (the fixed leg) whilst the other party undertakes to make payments based on the actual mortality experience of that group of lives (the floating leg). It is also possible to have options on these swaps (swaptions) or to enter into mortality caps or floors to provide an asymmetric return profile. More information on the range of structures is given in Blake et al (2006).

A key question, however, is whether to hedge on an indemnity or an index basis. Using an indemnity basis means that the exact longevity experience of the portfolio of lives is hedged, as with an annuity. For the pension scheme in question this is superficially attractive since the longevity experience of the lives in question will be exactly replicated by the contract in force. However, such an approach may well be more expensive since from the point of view of the counterparty to the contract, the scope for using such risk to hedge, say, term assurance business is likely. Such implicit hedging does occur, as evidenced by Cox and Lin (2004). A scheme-specific contract is also less likely to be tradable, as investors' knowledge of the scheme is likely to be less than that of a widely-used reference population. This too would increase the cost. The alternative is a contract based on a standardised reference population: a longevity index. This gives the prospect of liquid, tradable mortality and longevity securities, swaps and derivatives.

2 Longevity Indices

In order to enter into a contract based on a standardised portfolio of lives, a suitable longevity index is needed. However, it is important to define what is meant by "suitable". A good starting point is the work of Bailey (1992a). In the context of the (un)suitability of peer-group benchmarks for investment performance measurement, his ideal benchmark can be summarised as being:

- unambiguous – components and constituents should be well-defined;
- investable – it should be possible to buy the components of a benchmark and track it;
- measurable – it should be possible to quantify the value of the benchmark on a reasonably frequent basis;
- appropriate – it should be consistent with an investor's style and objectives;
- reflective of current investment opinion – it should contain components about which the investor has opinions (positive, negative and neutral); and
- specified in advance – it should be known by all participants before the period of assessment has begun.

This list is subscribed to by Ansell et al (2003) who add a further criterion:

- observable – investors can observe the evolution of the benchmark across time, and how their decisions relate to the performanc of the benchmark.

The criteria of measurability and observability are similar, but subtly different. The difference is that observability implies a degree of immediacy. It also implies that an observable benchmark is actually available, rather than able to be calculated from its constituent parts.

Bailey (1992b) also considers other factors that can be used to assess the quality of a benchmark. In this case, there is an implicit assumption that the benchmark is more specific than a more general market portfolio containing all securities. There is obviously significant overlap between the criteria, but useful additional points are made. The full list of criteria can be summarised as being that:

- the benchmark should contain a high proportion of the securities held in the portfolio;
- the turnover of the benchmark's constituents should be low;
- benchmark allocations should be investable position sizes;
- an investor's active position should be given relative to the benchmark;
- the variability of the portfolio relative to the benchmark should be lower than its volatility relative to the market portfolio;
- the correlation between $r_p - r_m$ and $r_b - r_m$ should be strongly positive;
- the correlation between $r_p - r_b$ and $r_b - r_m$ should be close to zero; and
- the style exposure of the benchmark and the portfolio should be similar.

In this list r_m is the market return, r_b is the benchmark return and r_p is the portfolio return.

All of these features are aimed at producing a satisfactory benchmark against which performance can be measured. This is a different aim to the production of an index on which derivatives might be based, but many points are still relevant. Consider first the features listed by Bailey (1992a).

A lack of ambiguity is still key – it is important that the nature and timing of the payments on any longevity-related contract are well defined. This means specifying in detail the reference population on which the index is based, and the ways in which this population will change over time. It will change through the deaths of individuals, but also potentially as new lives join or leave for reasons other than death. For example, if the reference population is a national population, then there will be immigration and emigration to consider. The period over which calculations are made also needs to be specified, as does the calculation method including whether the index values will use smoothing.

Smoothing might be desirable (although not necessary) because of the random variation in mortality rates that occurs due to the fact that the number of lives in the reference population is finite. This can imply smoothing over time or across ages. This is sensible because it means that the index reflects the underlying mortality with these random effects removed. In particular, smoothing across ages allows information from a range of ages to be used to refine the results at a particular age.

This suggests that two new criteria are important when considering longevity indices. The first is objectivity – it is important that if mortality rates are smoothed, then whatever method is used is as objective as possible and leaves little or no scope for subjective opinion to be overlaid.

The second new criterion is transparency. Whatever smoothing approach is used, it must be available for scrutiny by investors.

Moving back to the list from Bailey (1992a), the concept of investibility is not directly transferable to longevity indices; however, measurability is. The longevity experience used to construct an index should be measurable. However, this raises several further issues. It is relatively straightforward to construct longevity indices in such a way that the data are available; however, ensuring that the data are produced in a timely manner and according to a pre-arranged timetable is less straightforward. Timely data are important and the issue is particularly pertinent for longevity indices where data collection and processing can take a long period of time. The longer it takes for data to be published, the greater the risk of something happening between the effective date of an index and the publication of the index value. A pre-arranged timetable is also key, since it is impossible to accurately value investments without this information. If marketable instruments are to be based on indices, it is important that these two conditions are met. This suggests that in addition to measurability, two more criteria are needed: for the indices to be produced in a timely manner and for indices to available according to a pre-arranged timetable.

At this point it is worth considering the additional criterion of observability given by Ansell et al (2003). Given the difficulty in obtaining timely and accurate information, an immediately observable index is possibly too much to ask. In particular, the collection and checking of data will always introduce some level of delay into the construction of a longevity index.

Appropriateness is also important. One measure of this is the extent to which an index follows the mortality experience of the portfolio of lives being hedged. The only investment that will exactly reflect this experience is an annuity, or some other product which provides an indemnity (rather than an index) solution. However, a compromise is needed between the extent to which an index reflects the experience of a particular pension scheme (with more indices providing more cover) and the number of schemes for which a particular index provides a reasonable reflection of experience. The advantage of many indices, each of which closely reflects experience is clear: each index provides a good hedge and reduces basis risk arising from differences between the experience in the index and in the pension scheme. Having fewer indices means exposing each pension scheme to more basis risk. However, for small pension schemes, even if the index correctly reflects the part of the population from which the scheme membership is drawn, volatility is likely to result in significant deviation between the experiences of the pension scheme and the index. More importantly, the trade in each of the few indices is likely to be greater than the trade in each of the many. This increased liquidity makes the few indices attractive to counterparties, thus reducing bid/offer spreads. Given an appropriate utility function, the increased basis risk can be valued, as can the bid/offer spread implicit in a contract, and the optimal granularity for a longevity index is that which minimises the sum of these costs. All of this suggests that appropriateness can therefore be recast as two competing characteristics: a need to reflect the nature of the portfolio of lives being hedge, and a need to encourage liquid securities and derivatives.

The issue of appropriateness as discussed above does not take into account volatility risk, only a combination of level and trend risk: it considers the appropriateness of an index in terms of whether it correctly represents the underlying mortality of the portfolio of lives being hedged. Taking into account the size of a pension scheme brings to mind the criteria of relevance from Bailey (1992b). These suggest the variability of the pension scheme liabilities relative to a specific index-based longevity swap should be significantly lower than their volatility relative to a population-based longevity swap. It also suggests that the correlation between $L_i - L_t$ and $L_p - L_t$ should be strongly positive, where L_t is the value of the floating leg of a total population-based longevity swap, L_i is the value of the floating leg of a more specific index-based longevity swap, and L_p is the value of a pension scheme's liabilities.

The penultimate requirement in the Bailey (1992a) list is that the benchmark should be reflective of current investment opinion. This has an analogy in longevity indices, in that the structure of an index should reflect the use to which the index will be put. This leads on to the discussion of index structure, covered in the next section, but it suggests that the original criterion of Bailey (1992a) could be left nearly untouched.

The final item in this list is that the benchmark should be specified in advance. This also applies to longevity indices, inasmuch as the composition of the index should be known in advance. However, the composition of the index in the distant future will often be an issue for derivatives priced off longevity indices, and defining the composition for such a long time scale is not straightforward given the potential changes in the availability of data. This suggests the need for an independent index committee to deal with any issues that arise.

The additional criterion of Ansell et al (2003) and three of the criteria of Bailey (1992b) relating to volatility- and correlation-relevance have already been addressed. Three others are covered by Bailey (1992a). This leaves two further features. Stability in terms of low turnover of constituents is important for performance measurement; however, it is also useful for the construction of longevity indices inasmuch as it is important that the criteria used to construct indices change only infrequently. Bailey (1992b) also recommends that active investment positions are given relative to the benchmark rather than some broader market-wide index. This criterion has no analogue in the construction of longevity indices.

This suggests the following criteria for longevity indices:

- unambiguous – the reference population on which the indices are based should be defined in detail, including details of how individuals can enter and leave the index (other than through death);

- transparent – the methods used to graduate mortality rates should be clear;
- objective – graduation methods should have as little subjective input as possible;
- measurable – the mortality experience of the reference population should be capable of being measured;
- timely – the mortality experience of the reference population should be available shortly after the effective date of that experience;
- regular – the indices should be produced in accordance with a pre-arranged timetable;
- appropriate – the indices should reflect the composition of the populations requiring hedging;
- popular – the indices should be few enough that securities, derivatives and swaps based on them will be liquid;
- relevant – the variability of the liabilities being hedged relative to the indices should be significantly lower than their volatility relative to population longevity;
- highly correlated – the correlation between $L_i - L_t$ and $L_p - L_t$ should be strongly positive, where L_t is the value of the floating leg a longevity swap based on the total population, L_i is the value of the floating leg of the more specific index-based longevity swap, and L_p is the value of a pension scheme's liabilities;
- reflective of current hedging needs – the structures of the indices should reflect the needs of those using them to hedge;
- stable – the criteria used to construct indices should change only infrequently; and
- specified in advance – the indices should be defined in advance as far as possible, and there should be an independent committee to deal with issues when this is not possible.

3 Index Metrics

The reflection of current hedging needs by longevity indices raises a number of interesting issues. The most important are around the number of indices to be quoted and the function of mortlaity rates to be used.

The question of how many indices to use is essentially one of index structure. When considering index structure, it is helpful to consider the similarity between mortality rates and interest rates. In particular, the rate of survival to a particular age is analogous to the rate of interest payable over different terms, shown in Figure 1.

Therefore, when considering the rate of survival for a group of individuals of different ages, this picture must be expanded into three dimensions, as shown in Figure 2, with the third dimension here being current age.

In this context, there are a number of potential structures for indices, but the easiest way is to consider the most fundamental building block, which is an index representing a single payment to a group of individuals of a single age at a single point in the future. If aggregated across a range of ages representing the age distribution of the reference population and across all future years, then the result would be a single index that could be used to hedge the longevity experience of a particular population. However, it might not match the cash flows exactly. In particular, the proportion of lives at each age assumed in the the index will not necessarily reflect the population of lives in an individual pension scheme.

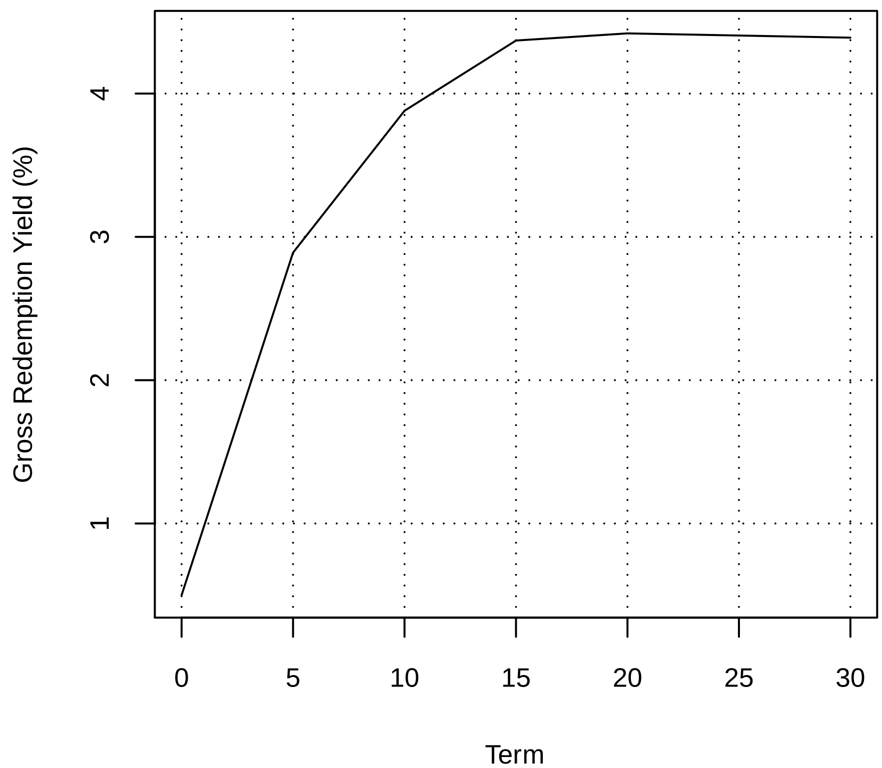
Better cash flow matching could be achieved by having a separate index at each age, but still aggregating over all future years for each age, since the swaps based on the indices could be bought at each age in line with the population's age distribution, the basis risk remaining being only between the experience of the reference population and the portfolio of lives rather than an additional mismatch between the age distributions. However, this approach leaves little flexibility to allow for longevity expectations that differ from those in the index. It also assumes that a cash flow matching approach is desirable, but cash flow matching is not the only way to manage risk.

If swaps for only a few ages were purchased, say current ages 60, 70, and 80, then the proportions of these swaps could be targeted so that the for a change in expected mortality rates over a given range of ages at a given point in the future, the value of the swaps and the value of the pension scheme liabilities would change by similar amounts. It is also possible to extend this to avoid having to worry about annual cash flow payments. Instead of aggregating the index building blocks across all future years, only a handful of years could be chosen, say 10, 20 and 30 years into the future. This would mean that pension scheme liabilities could be hedged by using single payments for each of current age 60, 70 and 80 for a future term of 10, 20 and 30 years. Even within this range not all combinations would be needed. In particular, the maximum combination of current age and future term could be limited to 90 years of age. This would leave the following swaps for which an index would be required:

Table 1: Swaps for which Indices Required

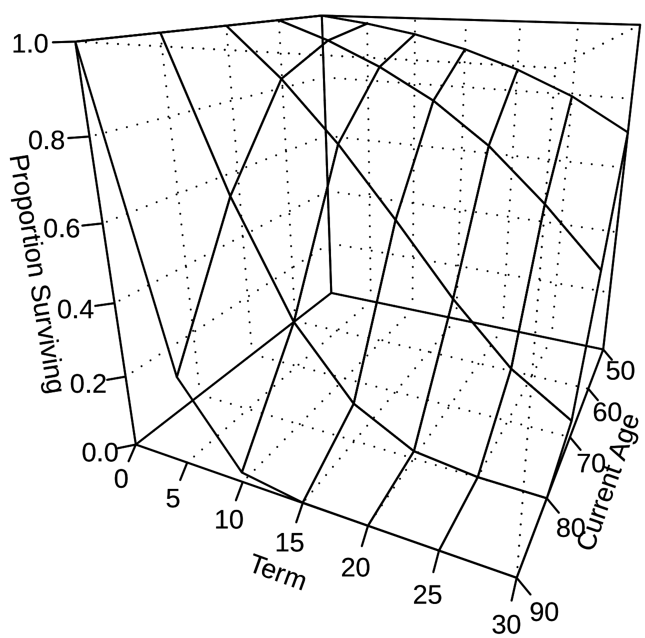
		Current Age (years)		
		60	70	80
Term (years)	10	✓	✓	✓
	20	✓	✓	
	30	✓		

Figure 1: UK Government Bond Yield Curve as at 5 February 2010



Source: Bloomberg

Figure 2: Proportion of Survivors for Given Current Ages, England and Wales Males, ONS 2008-based Principal Projections



Source: Office of National Statistics, Author's Calculations

The proportions of the swaps would be adjusted and combined with cash so that the sensitivity of the swap portfolio to changes in future expectations of life expectancy would be the same as that of the portfolio of lives. Having a small number of indices would also increase the liquidity in contracts based on these indices.

However, it is not entirely clear what the swaps should be based on. For example, a swap designed for pension schemes would be designed around single payments to an individual currently aged x years at a point n years in the future. This simply a multiple of the probability of survival to this point, ${}_np_x$.

It is, though, important to consider a broader range of options. Whilst it is pension scheme liabilities that are the main concern, life assurance companies might make natural counterparties. As Cox and Lin (2004) describe, there is an element of natural hedging between life assurance and pension liabilities within insurance companies, suggesting that life assurance business might be a good hedge for pension scheme liabilities. For a portfolio of life assurance policyholders it might be preferable to use the probability of survival for a an individual currently aged x for $n-1$ years, with death occurring in the nth year, ${}_{n-1}p_x \times q_{x+n-1}$.

It is also worth considering a simpler metric, the probability of mortality in year n for an individual currently aged x, in other words, q_{x+n-1}. These three approaches can be summarised as:

- pension-based swaps;
- life assurance-based swaps; and
- q forward-based swaps.

Swaps based on q forwards are described in more detail by Coughlan et al (2007).

As well as considering the effectiveness of these hedges for pension schemes, it is also worth looking at how well they perform when used to hedge life assurance liabilities. This aspect is considered only as an asise, so the lives assured are in teh same age range as the pensioners. In reality, a life assurance portfolio would probably have a much lower average age, but the model described below does work well for younger ages.

To assess the performance of these swaps I create a hypothetical swap portfolio based on each of the above measures and determine the optimal match for hypothetical portfolios of pensioners and life assurance policyholders. I do this using a stochastic mortality projection model. In particular, I use model M5 as described in Cairns et al (2009), parametrised with England and Wales Data from the years 1981 to 2004 inclusive and for ages 60 to 89. The model M5 can be summarised as:

$$\ln\left(\frac{q_{x+n}}{1-q_{x+n}}\right) = \alpha_n + \beta_n(x-\bar{x}), \qquad (1)$$

where $\bar{x}$ is the average age, in this case 74.5 years. The parameters α_n and β_n are both assumed to be random walks with drift, with the innovations in α_n being correlated with those in and β_n. The rates of drift are -0.0241 and .000502 respectively, and the correlation is 0.697. Whilst the model is parametrised using only ages 60 to 89, projections are obtained for ages up to 119 with a radix of 120 years of age being assumed. Projections for these extreme ages are not particularly robust, but are good enough for model fitting purposes.

The mortality rates in year 0 are taken from the 2006-2008 interim life tables for England and Wales. Rates are then projected 60 years into the future for ages 60 and above.

I produce 100 simulations of these 60 year projections. However, interest rates are taken to be deterministic, on the grounds that these can be hedged separately from mortality or longevity risk, although there are secondary interactions. The interest rates used are taken from the UK Government Bond Yields from 5 February 2010 as provided by Bloomberg. The raw data are used to calculate annual forward rates, with missing values being linearly interpolated. Forward rates beyond the end of the yield curve are assumed to be equal to the 30 year forward rate. These rates are then recombined into a 60-year spot interest rate curve. This is then used to calculate both portfolio and swap values.

The simulations are used to model the effects of variations in mortality rates away from a central estimate. The central estimate is derived by using model M5 with the volatilities set to zero. This means that instead of the innovations of α_n and β_n following correlated random walks, they simply change at fixed rates, these fixed rates being the drift parameters given above.

The hypothetical pension portfolio is created by assuming an initial population of pensioners that is stationary when calculated using the central estimates of mortality. The initial total payment at each age is in proportion to the proportion of lives at each age assuming that the population is stationary. The payments – which are non-increasing – are then run of as the population decreases in line with the central mortality estimate, and discounted back using the deterministic yield curve.

The same process is then applied to the life assurance liabilities, assuming that each member of the same population has a non-increasing sum assured payable on death rather than a non-increasing pension payable on survival. For both pensioners and lives assured, the final liability value is standardised to 100.

The values for the pension scheme and life assurance liabilities are then recalculated using the 100 simulated mortality paths, and the difference between these simulated results and the central estimates calculated. The results are two column vectors, $\boldsymbol{L}_P$ and $\boldsymbol{L}_L$, each containing 100 values representing the change in liabilities arising from a change in the mortality assumptions.

The swap contracts represent individual pension or life assurance payments or, in the case of q forwards, initial mortality rates at discrete future intervals. The fixed leg of each of these individual swaps – six for each of the three types – is therefore calculated as the present value of one of these payments using the central estimate of mortality. Each swap is standardised so that the fixed leg has a unit value. The 100 mortality scenarios are then applied to calculate the floating leg of the swap. The payout is then calculated as the fixed leg minus the floating leg. This gives three matrices, $\boldsymbol{X}_P$ for the pension-based swap, $\boldsymbol{X}_L$ for the life assurance-based swap and $\boldsymbol{X}_Q$ for the q forward-based swap, each with 100 rows representing the 100 scenarios and six columns representing the six combinations of age and term.

Another column vector can be taken to represent the holdings of each of the 6 swaps needed to best match the pension or life assurance portfolios. There will be six groups of parameters which can be generalised as , $\boldsymbol{\beta}_{UV}$ where U is equal to P for the pensioner portfolio and L for the life assurance portfolio, whereas V is equal to P for the pension-based swap, L for the life assurance-based swap and Q for the q forward based swap. For the pension scheme liabilities and the life assurance-based swap, for example, this means that the various matrices and vectors can be linked as follows:

$$\boldsymbol{L}_P = \boldsymbol{X}_L \boldsymbol{\beta}_{PL} + \boldsymbol{\epsilon}_{PL}. \tag{2}$$

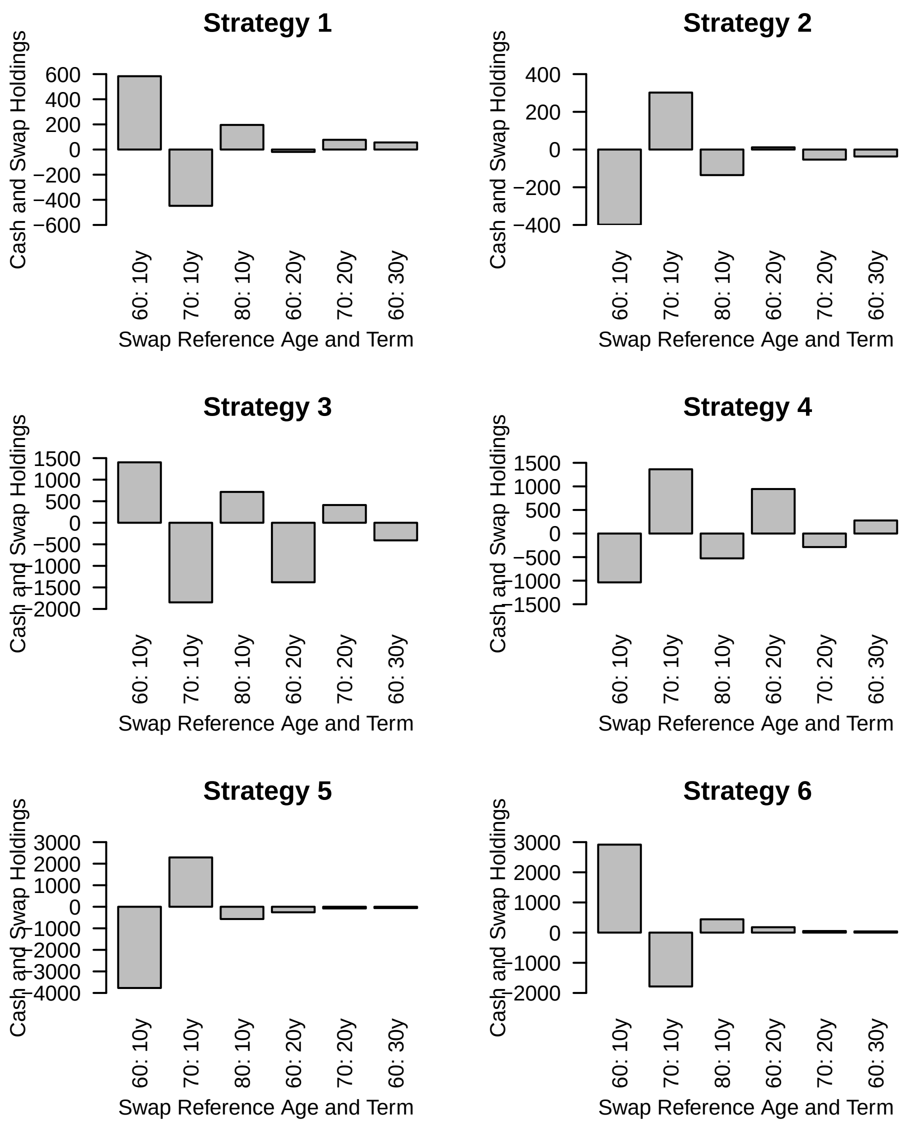
The vector $\boldsymbol{\epsilon}_{PL}$, whose subscripts mirror those for $\boldsymbol{X}_L \boldsymbol{\beta}_{PL}$, can be regarded as a vector showing the tracking error for each simulation. One way of finding the portfolio of swaps that best fits these scenarios is to define that portfolio as the one that minimises the sum squared differences. This means that the problem becomes the one of ordinary least squares estimation, and the vector $\boldsymbol{\beta_{PL}}$ can be estimated as:

$$\hat{\boldsymbol{\beta}}_{PL} = (\boldsymbol{X}'_L \boldsymbol{X}_L)^{-1} \boldsymbol{X}'_L \boldsymbol{L}_P. \tag{3}$$

In this analysis, $\boldsymbol{L}$ is redefined slightly if the swap being used is based on the portfolio being hedged, in particular if pension-based swaps are being used to hedge pension liabilities, or life assurance-based swaps are being used to hedge life assurance liabilities. In these cases, the liability payments due at the ages and times for which there are swaps (age 60 plus 10, 20 and 30 years, age 70 plus 10 and 20 years, and age 80 plus 10 years) are excluded from the calculation of liabilities in order to avoid endogeneity. This also means that the swap weights must be adjusted by adding the value of the liability excluded; in practice, however, this makes very little difference to the calculation results.

The volatility of the pension liabilities over the scenarios, measured as the standard deviation of the 100 liability values, is 2.250% of the value of the liabilities; for the life assurance liabilities, it is 1.589%. Using a pension-based swap portfolio can reduce these volatilities to 0.141% for the pensions liabilities and 0.098% for life assurance liabilities. Interestingly, using life assurance-based swaps can reduces the volatility only to 0.432% and 0.301% for pension and life assurance portfolios respectively. Using q forward-based swaps is marginally worse again, reducing the volatilities to 0.546% and 0.390% for pension and life assurance portfolios respectively. The weights of each swap are given in Figure 3.

Figure 3: Volatility-Minimising Swap Portfolios



Source: author's calculations

Strategies 1,3 and 5 – pension scheme liabilities
Strategies 2,4 and 6 – life assurance liabilities
Strategies 1 and 2 – pensions-based swap
Strategies 3 and 4 – life assurance-based swap
Strategies 5 and 6 – q forward-based swap

Clearly using a wider range of swaps would improve these figures, but the results remain interesting. In particular, the tracking error for life assurance liabilities is consistently lower than that of pensioner liabilities regardless of the nature of the swap contract.

The tracking errors of the pensions liabilities are consistently between 40% and 45% higher than those for the life assurance liabilities with and without hedging. This is due to the fact that the pension payments are initially higher than the life assurance payments. Combined with the effect of compound interest, this means that changes in mortality assumptions have a greater effect on the value of pension liabilities than they do on life assurance liabilities. The spread of payments is shown in Figure 4.

This is, though, slightly puzzling – it might be thought that the pension-based swap would perform better for pensions liabilities, the life assurance-based swap would perform better for life assurance liabilities, and q forwards would perform worse than both. However, whilst q forwards do provide the least effective match, the pension-based swap gives the best match for bost pensions and life assurance liabilities. The reason for this can be determined by considering what happens in the various scenarios. Consider a scenario where mortality rates fall uniformly. This will result in an increase in all of the components of the calculation of ${}_np_x$. However, these increases will be offset by the fall in q_x when calculating ${}_{n-1}p_x \times q_{x+n-1}$. Therefore whilst using a swap based on ${}_np_x$ will clearly give the best match for pension liabilities, it also gives an instrument that is more sensitive to changes in mortality rates and therefore better as a hedging instrument for life assurance liabilities.

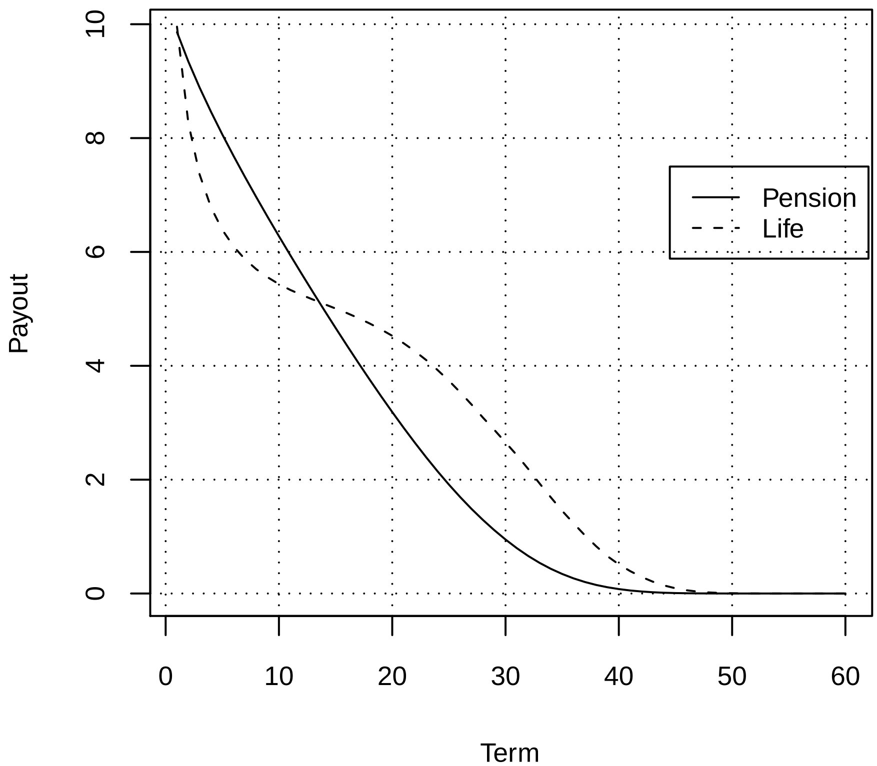
4 Existing Indices

Longevity indices – as distinct from mortality tables, whose main purpose is to value actuarial liabilities – do exist, and the characteristics of several are discussed below.

JP Morgan provides longevity indices for the United States, Germany, the Netherlands, and England and Wales in its LifeMetrics[1] suite, developed with the assistance of the Pensions Institute at Cass Business School and Watson Wyatt. For all of the regions mentioned, rates are based on mortality of the entire population. For England and Wales, LifeMetrics takes raw data from the UK Statistics Authority (formerly the Office for National Statistics) and applies a pre-defined smoothing algorithm. This index fulfils many of the criteria described above. However, since the population used is the entire England and Wales population, the relevance of this data to a specific group of lives such as a pension scheme or a book of annuitants is questionable. The Credit Suisse Longevity Index uses similar data and methods for the United States only. It therefore has similar advantages and disadvantages to the LifeMetrics data. There is also a product covering the populations of Germany and the

[1] www.lifemetrics.com

Figure 4: Expected Payouts from Pension and Life Assurance Liabilities



Netherlands from the Deutsche Börse[2]. Xpect produces monthly mortality data using an objective methodology.

Finally, Goldman Sachs developed the QxX index, which it offered until December 2009. This was a longevity index covering medically underwritten US lives. This had the advantage of being objectively calculated, as with the indices above, and was calculated more frequently (on a monthly basis). The number of lives covered was small (46,290 at outset), and the class of business was not necessarily relevant for the hedging of pensions (it covered the life settlements market), though it was perhaps more appropriate than an index based on a national population. However, as indicated above, Goldman Sachs decided in December 2009 to wind down its life settlements index[3].

5 Conclusion

Longevity indices could serve a useful role in facilitating the hedging of longevity risk in pension schemes. The characteristics of good indices are numerous, and whilst the criteria for good benchmarks as discussed by Bailey (1992a,b) and Ansell et al (2003) are useful, the nature of longevity indices means that additional considerations are needed. In particular, the scope for subjectivity and the construction of the indices could have a major impact on the success of indices. Longevity indices do exist, but since they are based largely on national population data, their relevance is perhaps limited. This suggests that new, more focussed indices would be a useful addition for those wishing to hedge longevity.

Good hedging results can be achieved using a relatively small number of swap contracts at key combinations of age and term. Such an approach would help to develop a liquid market in such swaps.

The most appropriate metric on which a swap should be based appears to be one using ${}_np_x$. This seems to be significantly better than other possible metrics.

The analysis here does not address an important issue – the extent to which these swaps would be worthwhile in the face of volatility risk. This is a particularly important issue for smaller pension schemes for whom volatlity risk can be the dominant longevity risk. In these cases, an indemnity-based cash flow matching solution, or even annuitisation, may be the only answer. However, for larger schemes the opportunities available for hedging the longevity risks would be significantly improved if indices were available that fulfilled the criteria in this paper, and swaps were created as described above.

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