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Inventories and the Stockout Constraint in General

Equilibrium

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Abstract

We study the implications of a stockout constraint in a dynamic general equilibrium model, which can explain both RBC and inventory facts well. Under the stockout constraint, inventories and demand are complements in generating sales, and hence the optimal level of inventories increases in expected demand. We also show that the inventory to sales ratio is both persistent and countercyclical because the cost of carrying inventories is mainly determined by the interest rate. We use this model to disentangle output and sales, by matching the key inventory moments, and find that preference and productivity shocks are equally important in data. Finally, we assess whether improvements in inventory management can explain the Great Moderation. We find that, although improvements in inventory management can reduce the need for inventory holdings, which decreases output volatility relative to sales volatility, lower levels of inventories actually increases sales volatility. Because these two effects offset each other, a change in inventory management does not change output volatility to any great extent.

KEYWORDS: Inventory investment, Inventory cycles, Stockout constraint, Great Moderation.

JEL CLASSIFICATION: E12, E20, E32.

1   Introduction

Inventories represent the difference between production and sales and thus, broadly speaking, place a wedge between the demand and supply sides of the economy and this wedge actually continues to be of great importance in the analysis of business cycles.[1] Table 1 shows the contribution of inventory investment to the output decline in the most recent, and largest, postwar recession in 2007-9 was some 33% and, although smaller than the average postwar contribution, can hardly be said to be insignificant. Accordingly inventory accumulation has been placed at the centre of the production and sales adjustment process but despite an extensive literature on inventories, most existing theoretical studies of inventories focus only on a firm or industry level analysis and there are relatively few general equilibrium analyses.[2] The motivation of our work is thus to investigate, within a flexible price micro-founded dynamic stochastic general equilibrium model, the role of inventories in the presence of a stockout constraint, in which no intermediate seller can sell more products to a final goods producer than the inventories they hold, whilst focussing on interplay between output, demand and stockout probability.[3]

Our work shows that the introduction of this constraint leads to the model satisfying the well-known stylized facts on inventories: production (output) is more volatile than sales; inventory investment is procyclical; and the inventory to sales (I/S) ratio is countercyclical and persistent.[4] The mechanism that our model mimics these facts has already been revealed by Kahn (1987, 1992), and, in this sense, our model can be regarded as the general equilibrium extension of his work. Under the stockout constraint, the key trade-off in the inventory management is that having low levels of inventories risks losing sales opportunities but having excess inventories imposes a cost of carry. We briefly review how this trade-off generates the inventory facts in our model.

First, the probability of losing sales opportunity is an increasing function of *expected* demand, which means that producers – sellers of intermediate goods in our model –

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[1]Much early quantitative work was directed towards understanding the nature and causes of the inventory cycle. Schumpeter's (1939) analysis of the business cycle placed considerable weight on Kitchin's (1923) observation of cycles being associated with unintended changes in inventories. This point was followed by Metzler (1941) who formulated a model of the duration of inventory cycle and as an accelerator mechanism in final output. And inventories continue to account for a large share of GDP fluctuations, particularly in recessions. For example, Fitzgerald (1997) reports that 'changes in inventory investment are, on average, more than one-third the size of quarterly changes in real GDP over the post-war period.' See also Blinder and Maccini (1991).

[2]To name a few, Hornstein and Sarte (2001), Boileau and Letendre (2004) and Jung and Yun (2005) for inventory analysis in a sticky price environment, and Fisher and Hornstein (2000) for the (S,s) model. See below for Khan and Thomas (2007a and 2007b), Wen (2011) and Wang and Wen (2009).

[3]Note also we undertake our analysis without any recourse of costs of investment, as in much of the extant literature.

[4]Theoretical studies of inventories started with production smoothing and buffer stock inventories in which strong demand implies that buyers take inventories from sellers (thus, inventories decrease when demand is strong) and, hence, these inventory facts had been considered to be puzzling.

need a higher level of inventories in booms, and this explains the first two stylized facts. Looking at this more closely, consider a positive demand shock,[5] in which an increase in demand naturally leads to the same increase in output, in absence of any inventories. But with the addition of inventories, under a stockout constraint, producers produce more to accumulate inventories to avoid too high a stockout probability. In this respect, we can treat inventories and demand as 'inputs in producing sales', and so these two inputs are complements to each other. In this paper, we focus on this nature of inventories – sellers' inventory management in 'producing sales'. Note that, even without the stockout constraint, inventories can work as a buffer against *unanticipated* demand shocks. Hence, inventories and sales are positively correlated to each other at business cycle frequencies but negatively correlated at higher frequencies, as reported by Ramey and West (1999) and Wen (2005).

Second, the cost of holding inventories, or the cost of carry, is the opportunity cost of carrying inventories from one period to the next, i.e., the interest income that would be obtained by holding savings instead. Hence, the optimal level of inventories (relative to sales) is decreasing in the real interest rate, or the external financing cost of holding inventories paid by producers, as pointed out by Bernanke and Gertler (1995). This explains the third inventory fact, that the I/S ratio is countercyclical and persistent, simply because the interest rate is procyclical and persistent, see also Bils and Kahn (2000) and Bils (2004).

Producers hold inventories in order to overcome the stockout constraint that would otherwise hinder trade between sellers and buyers and hence we refer to the inventory holdings due to this motivation as distributors' demand. Because distributors' demand amplifies a demand shock, it might be tempting to infer that distributors' demand also amplifies overall output volatility and, in our model, production is indeed more volatile than sales. This idea has been taken up by others, such as Kahn, McConnell, and Perez-Quiros (2002), as an explanation for the Great Moderation. Our numerical exercises however show that having more inventories crowds out capital investment and consumption, given the resource constraint. This means that, as inventory management improves distributors' demand for inventories decreases, which in turn leads to a weaker crowding-out for investment and consumption. As a result, the volatility of demand increases along with improvements in inventory management. In sum, relaxing the stockout constraint gradually, we find that, certainly output volatility relative to sales decreases, but the volatility of sales per se increases. And so, in our model, improving inventory management cannot solely explain any Great Moderation.

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[5]Note that, with supply shocks (technology shocks), it is not surprising for above two inventory facts to hold; production is more volatile than sales simply because the source of shocks is on the production side, and hence inventory investment is procyclical simply because an increase in sales is not enough to absorb the increase in production (see Blinder (1986) for example). What is important in this article is that, even if the source of shocks lies on the demand side, production is still more volatile than sales.

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Our research is closely related to that of Khan and Thomas (2007b). Their (S,s) inventory management model has some success in replicating inventory facts qualitatively and quantitatively, while their stockout constraint generates almost no inventories in the steady state; see also Khan and Thomas (2007a). Different from their stockout constraint model, our model can mimic the inventory behavior both in the steady state and fluctuation around it. This is mainly because we assume price posting and positive profit margin. If prices are perfectly flexible, the demand for goods will then adjust until it is equated to the inventories to be sold. Also, if the net profit margin is zero, to avoid a positive cost of inventory carry, firms will optimally choose zero inventories. That is, producers must be compensated by a positive profit margin when their goods are sold; otherwise, they do not want to take the risk of the cost of carry when their goods are unsold; see the discussion of return dominance in Khan and Thomas (2007a). In Khan and Thomas (2007a), only if marginal costs are expected to increase to a sufficiently large degree, producers may choose to hold inventories to exploit a *negative cost of carry*. We argue, however, that such inventories are held not because of the stockout avoidance but because of a production smoothing motive, where producers want to produce their products when their production cost is low and store them in the form of inventories. Note that we do not intend to claim that the stockout model is superior to (S,s) model; rather, in our opinion, the (S,s) ordering model is more suitable in explaining *buyers'* inventory management, while our stockout constraint model is actually also suitable for the analysis of *sellers'* inventory management as well.[6]

In parallel work, Wen (2011) and Wang and Wen (2009) develop a 'refrigerator' model that assumes the stockout takes place on the buyers side. That is, buyers buy goods, keep them in their refrigerator (or warehouses) and when an unanticipated preference shock takes place they face stockouts for their favorite goods and carry less preferred goods as inventory investments into the next period. This device essentially study the stockout constraint as buyers' problem by conflating sellers and buyers, while it allow them to replicate the inventory stylized facts. Unlike these works, our model however takes the stockout constraint as the sellers' problem and by doing so we can explicitly study the double-sided nature of the stockout constraint. While the inventory management is primarily of sellers under the stockout constraint, the stockout also affects the buyers' side as well. That is, if a seller face a stockout, it means that there must be at least

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[6]We do not believe that, as often claimed, the (S,s) and the stockout constraint models are limited to retailers' inventories and producers' final goods inventories, respectively. Certainly, some evidence such as Blinder and Maccini (1991) show that inventories of intermediate goods and raw material explain the majority of inventory investment. But, it is plausible that, within a manufacturing company, a manager of a division, which is at the middle of the production line, employs an (S,s) rule to order intermediate goods to an upstream division, and at the same time he holds the output of his division to avoid stockout when he gets an order from the downstream division. The point is, as long as there are frictions and some degree of uncertainty in the demand for and/or the supply to a division at the timing of decision making, there is a non-trivial optimisation problem even in pipeline inventory management.

one buyer who also faces a stockout. Having explicit interaction between sellers and buyers, we can explicitly discuss, for example, the stockout probabilities for sellers and buyers separately. In addition, in our model, we can separate output and sales, and hence we can explicitly experiment the model to see the relative importance of demand and supply shocks. Also, Kryvtsov and Midrigan (2010) extend their work (2008) and consider a sticky wage and price model where firms hold inventories to avoid a stockout in the face of a demand shock. Their main interest is the role of inventories in explaining impulse response functions (IRFs) to a monetary policy shock and so mostly the countercyclicality of the I/S ratio is examined. In addition, a new aspect of our research is that we explicitly use moment matching to explore the importance of both preference and technology shock and so can study the stockout probability as generating gaps between output and sales.

The paper is organized in the following manner: Section 2 describes the model; Section 3 shows its simulation results; and finally Section 4 concludes.

2   The Model

This section first outlines the assumptions specific to our model, and then describes the optimization problems of a representative household, final goods firms, and intermediate goods firms in this order. Finally, we discuss the problems of aggregation and equilibrium determination.

2.1   Overview and Model Specific Assumptions

This subsection gives an overview of the model. There are three types of agents; a representative household, final goods producers (final firms) and intermediate goods producers (intermediate firms). The household consumes, invests and supplies labour and capital to intermediate firms.[7] Intermediate firms use labour and capital to produce intermediate goods, which is sold to the final firms in the intermediate goods markets. Final goods firms can be thought of as retailers as they simply convert different types of intermediate goods into identical final goods.

It is the intermediate goods markets that are subject to the stockout constraint; that is, no intermediate firm can sell more goods than they have on their shelf, even if more buyers than expected appear. Given double-sided nature of the stockout constraint,

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[7]Being precise, the distribution of households must have the same dimension as that of final and intermediate firms. Otherwise we would lose consistency; e.g., output (supplied by high dimension agents) cannot be equated to consumption and investment (demanded by a low dimension agent). For our model, this issue can be particularly important, because the dimensionality of agents' distribution is non-standard. However, under the conventional assumptions, such as perfect risk-sharing, all households behave identically. Hence, throughout this paper, we treat households collectively as a representative household to keep our exposition simple.

the optimization problems of these two types of firms are shaped accordingly, while the household's optimization is quite standard. Throughout this paper, a 'buyer' and a 'seller' are always an intermediate firm and a final firm, respectively.[8] Also, unless confusing, we use 'inventories' to signify unsold goods or goods on shelf depending on the context, while 'inventory investment' always means the change in unsold goods. Before discussing the agents' optimizations, the rest of this subsection explains two model specific assumptions; (1) distribution of sellers and buyers, which facilitates aggregation, and (2) timing assumption for sellers' decision making, which makes the stockout constraint meaningful.

2.1.1   Distribution of Sellers and Buyers

As discussed, in the intermediate goods markets, intermediate goods firms (sellers) sell intermediate goods to final goods firms (buyers), but the key friction here is the stockout constraint. Here, we discuss some additional assumptions that make the stockout constraint sensible.

First, the intermediate goods are differentiated á la Dixit-Stiglitz. A positive profit margin is necessary to encourage sellers to have inventories. As discussed further below, the stockout constraint is costly for sellers, because there is always the risk that some goods are unsold. However, if sellers are rewarded by zero profit margin when they sell their goods, there is no incentive for them to hold inventories; see the discussion on the return dominance found in Khan and Thomas (2007b) in this relation.[9]

Second, to capture the production differentiation, the variety of intermediate goods distributes over a line segment $[0, 1]$; or equivalently, we can understand this as there are infinitely many markets for different types of intermediate goods. To facilitate aggregation, we further assume that there is a unit mass of sellers for each variety. In total, there are a continuum of sellers who distribute over *a rectangle* $[0, 1] \times [0, 1]$. For a certain variety of goods (i.e., within a single intermediate good market), intermediate goods producers sell the identical goods.

Third, to keep consistency, we assume that final goods producers also distribute over $[0, 1] \times [0, 1]$; i.e., a buyer is represented by a point on rectangle $[0, 1] \times [0, 1]$. Also, a buyer (i) visits all markets but (ii) visits only one seller in each market. The former means that *on average* a seller sees a unit mass of buyers $[0, 1]$. The latter means that there are always buyers who cannot buy goods due to the stockout. That is, assumption (ii) is necessary because, if instead we allow these unlucky buyers to search around other sellers in the same market, they will find goods at the end of the day, meaning that

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[8]Furthermore we shall, on occasion, refer to he as the intermediate firm, or seller, and to she as the final goods firm, or buyer.

[9]See Appendix A.2, which describes the limit that monopolistic competition approaches to perfect competition.

there must be no unsold goods or stockout. This further implies that, under Dixit-Stigliz monopolistic competition, the stockout is costly not only for sellers but also for buyers. Because the lost varieties cannot be perfectly substituted by other varieties, to produce a certain level of final goods, they have to buy more quantities for available varieties, which leads to an increase in the production cost of final firms.

Given distributions of agents over a rectangle, there are two measures corresponding to two aggregation modes; aggregation within a market and aggregation over the markets. Roughly speaking, a continuum of intermediate firms in each market allows us to aggregate sellers' side, while having infinitely many varieties we can aggregate buyers behavior. For the former, however, the more important assumption for aggregation is constant returns to scale production technology, which we discuss in detail in Section 2.5. For the latter, intuitively, different buyers can buy different sets of varieties, but the measure of available varieties is the same for all buyers.

2.1.2 Idiosyncratic Shocks and Timing Assumption

We think that sellers hold inventories because of the demand uncertainty and in this paper that is captured by an idiosyncratic shock; specifically, the number of buyers N_t^{ji} who visit a sellers is stochastic and hence is different among sellers. Superscript ji indicates seller j in market i; e.g., N_t^{ji} reads the number of buyers who seller j in market i meets at time t. To make the stockout constraint meaningful, we imposes two restrictions on both buyers' and sellers' sides. As already discussed, given double-sided nature of the stockout constraint, the stockout constraint not only affects sellers but also buyers.

On the sellers' side, we assume that each seller must decide both production and price levels before observing his demand shock. More specifically, we have to assume that sellers determine their production before observing N_t^{ji}, because otherwise they can adjust their production level to avoid stockout and unsold goods. Similarly, we have to assume that sellers decide their sales price before observing N_t^{ji}, which we call 'price posting', because otherwise they can adjust their price level so that demand equals goods that they have on shelf. For example, if a seller faces a lot of buyers, he can increase his sales price to exploit his strong demand shock. This increase in price reduces the demand per buyer, but he does not care, because anyway he has too many customers. Similarly, if a seller receives only a few buyers, he can stimulate the demand per buyer by discounting his sales price to avoid the cost of carrying unsold goods to the next period. As a result, no stockout or unsold goods take place without price posting; see our discussion about Khan and Thomas (2007b) in Introduction.

On the buyers' side, as discussed in the previous subsection, when a buyer cannot buy a variety of goods, we do not allow her to visit other shops; that is, if seller j has too

many buyers N_t^{ji}, some lucky buyers (say, those on the forepart of the queue) can buy as much as they want, but some unlucky buyers cannot buy the goods at all.[10] This implies that buyers also suffer from the stockout constraint, which leads to an increase in the marginal cost of producing final goods under product differentiation among intermediate goods.

In addition, in our numerical experiments, we also assume that the household decides its labour supply before observing the current period aggregate shocks. This assumption is perhaps reasonable given infrequent nature of labour contract. Even without this, the model captures the inventory behavior at the business cycle frequencies almost equally well. But without this informational assumption, given weak convexity of the cost function of the intermediate firms, our model cannot capture the high frequency behavior of inventory investment in response to unanticipated shocks.[11] In this sense, it seems that the capital adjustment cost in Wen (2011) plays the similar role to our informational imperfection for the labour decision, while having capital adjustment cost tends to yield too low investment volatility; see Table 3 in Wen (2011).

2.2 Household

The household optimization is quite standard. The infinitely-lived representative household maximizes expected lifetime utility. The household supplies capital and labour, while it demands final goods for consumption and investment:

$$\max_{\{C_t, H_t, B_t, I_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ (1 - \psi) \frac{C_t^{1-\gamma}}{1-\gamma} \xi_t^C + \psi (1 - H_t) \right\} \right], \tag{1}$$

s.t.

$$Y_t^F + B_{t+1} = R_t^B B_t + \left(R_t^K - 1 \right) K_t + W_t H_t + D_t, \tag{2a}$$

$$C_t + I_t = Y_t^F, \tag{2b}$$

$$K_{t+1} = (1 - \delta) K_t + I_t. \tag{2c}$$

The household maximizes the present value of time separable period utility $U[.,.]$ with subjective discount factor β^t. For the period utility, we assume that the cross partial is zero $\partial^2 U / \partial C_t \partial H_t = 0$. Parameter ψ governs the relative importance of leisure $1 - H_t$, where H_t is hours worked in period t, while γ is the coefficient of relative risk aversion. We follow Khan and Thomas (2007a) and introduce a preference shock ξ_t^C, which follows

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[10]Another possible setup is *pro rata* allocation of goods to all buyers. However, in this case, anticipating the possibility of stockout, buyers have an incentive to overstate their demand, which complicates the analysis. Also, in this case, the amount that a buyer buys may differ across varieties, which triggers a further complication in aggregation.

[11]In our simulation results presented in Table 9, we allow for changes in these assumptions.

an AR(1) process. Later, we interpret it as a demand shock. As mentioned above, we assume that households cannot respond to current period aggregate shocks in their labour supply decision.

The first constraint shows the period budget constraint. While the period expense comprises the purchase of final, or retail, goods Y_t^F and bond purchases B_{t+1}, the period revenue is the sum of the gross return on bonds purchased in the previous period $R_t^B B_t$, the *net* return on capital $\left(R_t^K - 1\right) K_t$, labour income $W_t H_t$ and dividends D_t. The household takes gross bond return R_t^B, gross capital return R_t^K and wage W_t as givens. Final goods purchased Y_t^F can be used as consumption C_t and investment I_t. The third constraint shows the evolution of capital, where δ is its depreciation rate.

The household's first order conditions (FOCs) are all standard:

$$E_t\left[\Lambda_{t,t+1} R_{t+1}^B\right] = 1, \tag{3a}$$

$$\frac{\partial U_t/\partial L_t}{\partial U_t/\partial C_t} = W_t, \tag{3b}$$

$$E_t\left[\Lambda_{t,t+1}\left\{R_{t+1}^K - \delta\right\}\right] = 1, \tag{3c}$$

where $\Lambda_{s,\tau} = \beta\frac{\partial U_\tau/\partial C_\tau}{\partial U_s/\partial C_s}$ for $\tau \geq s \geq 0$ is the stochastic discount factor, SDF.

2.3 Final Goods Firms

The role of final goods producers is to convert intermediate goods into final goods using the standard Dixit-Stiglitz aggregator:

$$Y_t^F = \left[\int_0^1 Q_t^i\left(M_t^{b,i}\right)^{\frac{\theta-1}{\theta}} \mathrm{d}i\right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

where $M_t^{b,i}$ is the i-th variety of intermediate goods, which is measured in physical units, and Q_t^i is the indicator variable, which is 1 if this final firm has access to variety i and 0 otherwise. Intuitively, Q_t^i picks up the varieties that she actually buys. Parameter $\theta > 1$ is the elasticity of substitution among intermediate goods. For the final firms' optimization, we drop superscript j because final goods firms do not care the identity of sellers in each market; also, we do not need to track the identity of buyers, because our economic environment ensures that final firms' behavior is symmetric. Given (4), each final goods producer minimizes her expenditure on intermediate goods:

$$\min \int_0^1 Q_t^i P_t^{M,i} M_t^{b,i}\mathrm{d}i, \tag{5}$$

where $P_t^{M,i}$ is the price of the i-th intermediate goods.

There are a number of issues worth considering. First, we define the measure of the

available variety for each final firm as

$$Q_t = \int_0^1 Q_t^i \mathrm{d}i. \tag{6}$$

Because idiosyncratic shocks determine whether a final firm can buy a variety or not, Q_t^i differs among buyers. However, the law of large numbers (LLN) guarantees that the value of Q_t must be the same for all buyers (final firms). At the same time, Q_t also defines the probability that each buyer can buy a certain type of intermediate goods, and hence $1 - Q_t$ is the stockout probability for buyers.[12]

Second, (4) can be regarded as a quantity index, and *one of possible intermediate goods price indices* is:

$$P_t^M = \left[\int_0^1 \frac{Q_t^i}{Q_t} \left(P_t^{M,i} \right)^{1-\theta} \mathrm{d}i \right]^{\frac{1}{1-\theta}} = Q_t^{\frac{1}{\theta-1}} \lambda_t^{F,MC}, \tag{7}$$

where $\lambda_t^{F,MC}$ is the marginal cost of final goods production. We have chosen this price index, because this is the 'average' price of available intermediate goods,[13] which is perhaps the most closest concept to the actual price indices such as the producer price index. As a result of the stockout constraint, the price index is not equal to the marginal cost: instead, the effective intermediate price $Q_t^{\frac{-1}{\theta-1}} P_t^M$ is equated to $\lambda_t^{F,MC}$ at optimum. Since $Q_t \leq 1$ in general, $Q_t^{\frac{-1}{\theta-1}} P_t^M \geq P_t^M$, i.e. the effective intermediate goods price is higher than the price index, which is one representation of the cost of stockout on the buyers' side.

Third, as we assume a competitive final goods market, from the zero profit condition, we obtain $\lambda_t^{F,MC} = 1$, where the price of final goods is normalized to be 1. From (7), the effective intermediate price must be given by

$$Q_t^{\frac{-1}{\theta-1}} P_t^M = 1. \tag{8}$$

Clearly, the intermediate goods price is lower than final goods price; $P_t^M \leq 1$. Intuitively, because intermediate goods markets are not efficient due to the stockout constraint, if the average intermediate goods price is the same as the final goods price, final goods firms suffer from the business losses. To have zero profit, the intermediate goods price must be lower than the final goods price. Term $Q_t^{\frac{-1}{\theta-1}} > 1$ in (8) represents the cost of losing varieties.

Fourth, the first order condition with respect to $M_t^{b,i}$ leads to the following demand

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[12] As we need some additional notation, we derive this result formally in Section 2.4.1 again.

[13] To see this point, consider $\theta \to 0$ (extremely inelastic, hence fixed demand composite); then, $P_t^M = \int_0^1 Q_t^i P_t^{M,i} di / \int_0^1 Q_t^i di$. On the contrary, our quantity index (4) is a sort of 'summation'. To see this point, consider $\theta \to \infty$ (perfect substitution); then, $Y_t^F = \int_0^1 Q_t^i M_t^{b,i} di$.

function, which will appear as a constraint in the intermediate goods firms' optimization problem in the next subsection:

$$M_t^{b,i} = Q_t^{\frac{-\theta}{\theta-1}} \left(\frac{P_t^{M,i}}{P_t^M} \right)^{-\theta} Y_t^F. \tag{9}$$

Note that, unlike the standard demand function, which depends on relative prices alone, there is an additional term $Q_t^{-\theta/(\theta-1)}$, which is another expression of the inefficiency arising from the stockout constraint. To understand this argument, consider the symmetric equilibrium, in which $P_t^{M,i}/P_t^M = 1$,[14] and then $M_t^{b,i} = Q_t^{\frac{-\theta}{\theta-1}} Y_t^F$. This implies $Y_t^F < Q_t^{\frac{-1}{\theta-1}} Y_t^F = Q_t M_t^{b,i}$; i.e., output Y_t^F is smaller than the total inputs. Here, gap $Q_t^{\frac{-1}{\theta-1}}$ is another representation of the cost of losing varieties. Note that $Q_t M_t^{b,i}$ is the physical amount that a final firm buys, because it is the number of available varieties Q_t times the quantity $M_t^{b,i}$ of each variety actually purchased. Intuitively, as a result of product differentiation, the lack of access to some varieties leads to some inefficiency in production (4), and hence more inputs are required to produce a certain level of Y_t^F.

Fifth and finally, the following two-stage budgeting holds:

$$\int_0^1 Q_t^i P_t^{M,i} M_t^{b,i} \mathrm{d}i = Q_t^{\frac{-1}{\theta-1}} P_t^M Y_t^F. \tag{10}$$

Again, the multiplicative term $Q_t^{\frac{-1}{\theta-1}} > 1$ in (10) represents the cost of losing varieties.[15] Note that we have shown the cost of stockout on the buyers' side in (7), (8), (9) and (10), but all of them point to the same inefficiency from different angles. Of course, if final firms have access to all types of intermediate goods ($Q_t = 1$), or if there is no product differentiation ($\theta = 0$), there is no cost of stockout; i.e., $Q_t^{\frac{-1}{\theta-1}} = 1$.

2.4 Intermediate Goods Firms

Intermediate firms are also subject to the stockout constraint. They take the final firms' demand curve as a constraint; compare (9) and (12d).

$$\max_{\left\{ P_t^{M,ji}, S_t^{ji}, H_t^{ji}, K_t^{ji}, U_{t+1}^{ji}, Y_t^{M,ji} \right\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \Lambda_{0,t} \left\{ P_t^{M,ji} S_t^{ji} - W_t H_t^{ji} - \left(R_t^K - 1 \right) K_t^{ji} \right\} \right], \tag{11}$$

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[14]In Section 2.5 below, we show that our equilibrium is indeed symmetric.

[15]Note that $Q_t^{\frac{-1}{\theta-1}} P_t^M Y_t^F = P_t^M (Q_t M_t^b)$ in the symmetric equilibrium, where $M_t^b = M_t^{b,i}$. The right hand side shows 'price' times 'quantity actually purchased'.

s.t.

$$S_t^{ji} = \min\left\{G_t^{ji}, M_t^{ji}\right\}, \tag{12a}$$

$$U_{t+1}^{ji} = G_t^{ji} - S_t^{ji}, \tag{12b}$$

$$G_t^{ji} = U_t^{ji} + Y_t^{M,ji}, \tag{12c}$$

$$M_t^{ji} = M_t^{b,ji} N_t^{ji} = Q_t^{\frac{-\theta}{\theta-1}} \left(\frac{P_t^{M,ji}}{P_t^M}\right)^{-\theta} Y_t^F N_t^{ji}, \tag{12d}$$

$$Y_t^{M,ji} = \xi_t^M \left(K_t^{ji}\right)^\alpha \left(H_t^{ji}\right)^{1-\alpha}. \tag{12e}$$

The j-th intermediate firm in the i-th market maximizes the present value of current and future net cash inflow. It takes wage W_t and return on capital R_t^K as given (hence no ji superscripts). Section 2.2 defines the stochastic discount factor $\Lambda_{0,t}$.

The first constraint shows the stockout constraint for sales; S_t^{ji} is the minimum of goods on shelf G_t^{ji} and demand for intermediate goods M_t^{ji}. The second and third constraints are for the evolution of unsold goods U_{t+1}^{ji} and the definition of goods on shelf G_t^{ji}. We assume intermediate firms can place all of today's output $Y_t^{M,ji}$ in today's market but, if demand is weak, unsold goods are carried into the next period. The fourth constraint states total demand M_t^{ji} as demand per buyer $M_t^{b,ji}$ times number of buyers N_t^{ji} and, as a result of monopolistic competition, demand per buyer $M_t^{b,ji}$ decreases in its relative sales price $P_t^{M,ji}/P_t^M$.[16] The last constraint shows that the production function is Cobb-Douglas with capital share α and productivity shock ξ_t^M. We assume that ξ_t^M follows an AR(1) process.

2.4.1 Deriving Some Key Expressions

Before considering the optimization, we can obtain some key expressions, which are essentially alternative representations of some of the constraints. The discussion in this subsection relies only on (12a) and (12d). To keep our discussion simple, we introduce additional notation:

$$n_t^{ji} = \ln N_t^{ji}, \tag{13a}$$

$$g_t^{ji} = \ln(G_t^{ji}/M_t^{b,ji}), \tag{13b}$$

$$n_t^{ji} \sim \phi\left(\psi\right), \tag{13c}$$

where ψ is the vector of parameters of the probability density function ϕ of n_t^{ji}. Later, we limit ψ so that the expected value of N_t^{ji}, which must be the total measure of buyers in each market, is normalized to be one. Since the number of buyers cannot be negative,

---

[16] Recall that Q_t appears in the demand curve because of the cost of losing varieties; see (9).

it is natural to take its logarithm. Given this notation, we can rewrite (12a) as

$$S_t^{ji} = \begin{cases} G_t^{ji} & \text{if } n_t^{ji} > g_t^{ji} \text{ (stockout)} \\ M_t^{b,ji} e^{n_t^{ji}} & \text{otherwise} \end{cases}. \tag{14}$$

Given these, we define the following three variables: (a) sellers' stockout probability π_t^{ji}, (b) buyers' cost of stockout $\tilde{\pi}_t^{ji}$ and (c) expected sales for each seller.[17]

$$\pi_t^{ji}(g_t^{ji}) = \Pr[n_t^{ji} > g_t^{ji}] = 1 - \Phi(g_t^{ji}; \psi), \tag{15a}$$

$$\tilde{\pi}_t^{ji}(g_t^{ji}) = \hat{E}[N_t^{ji}|n_t^{ji} > g_t^{ji}]\pi_t^{ji}(g_t^{ji}), \tag{15b}$$

$$\hat{E}_t[S_t^{ji}] = M_t^{b,ji}\hat{E}[N_t^{ji}|n_t^{ji} < g_t^{ji}]\left(1 - \pi_t^{ji}(g_t^{ji})\right) + G_t^{ji}\pi_t^{ji}(g_t^{ji})$$

$$= M_t^{b,ji}\left(1 - \tilde{\pi}_t^{ji}(g_t^{ji})\right) + G_t^{ji}\pi_t^{ji}(g_t^{ji}), \tag{15c}$$

where Φ is the cumulative distribution function (*cdf*) of n_t^{ji}, and the hat notation on $\hat{E}_t[\ ]$ indicates that the information set includes all information up to time t except for idiosyncratic shock n_t^{ji} at t.[18] For $\tilde{\pi}_t^{ji}$, $\hat{E}[N_t^{ji}|n_t^{ji} > g_t^{ji}]$ is the expected number of buyers conditional that the stockout takes place. That is, in the sellers expectation formation, $\hat{E}[N_t^{ji}|n_t^{ji} > g_t^{ji}] - G_t^{ji}$ shows the number of excess buyers who he loses due to the stockout constraint, if it is binding.[19] Note that both π_t^{ji} and $\tilde{\pi}_t^{ji}$ are strictly decreasing in g_t^{ji}. Ratio $\hat{E}_t[S_t^{ji}]/G_t^{ji}$ is also a function of only g_t^{ji} (as well as parameters ψ) and is increasing in g_t^{ji}.

In our numerical experiments below, we assume a log-normal distribution for N_t^{ji}. That is, $\Phi(\cdot\ ; -0.5\sigma_N^2, \sigma_N)$ is the *cdf* of the normal distribution with mean $-\sigma_N^2/2$ and variance σ_N^2, so that $\hat{E}[N_t^{ji}] = 1$, where σ_N governs the size of the idiosyncratic shock. In this case, it is clear that π_t^{ji} and $\tilde{\pi}_t^{ji}$ move closely to each other.[20]

$$\pi_t^{ji}(g_t^{ji}) = 1 - \Phi(g_t^{ji}; -0.5\sigma_N^2, \sigma_N) = \frac{1}{\sqrt{2\pi}\sigma_N}\int_{g_t^{ji}}^{\infty} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji} + \frac{\sigma_N^2}{2}\right)^2} \mathrm{d}n_t^{ji}, \tag{16a}$$

$$\tilde{\pi}_t^{ji}(g_t^{ji}) = \hat{E}[N_t^{ji}|n_t^{ji} > g_t^{ji}]\pi_t^{ji}(g_t^{ji}) = \frac{1}{\sqrt{2\pi}\sigma_N}\int_{g_t^{ji}}^{-\infty} N_t^{ji} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji} - \frac{\sigma_N^2}{2}\right)^2} \mathrm{d}n_t^{ji}$$

$$= 1 - \Phi(g_t^{ji}; +0.5\sigma_N^2, \sigma_N), \tag{16b}$$

$$\hat{E}_t[S_t^{ji}] = M_t^{b,ji}\left(1 - \tilde{\pi}_t^{ji}\right) + G_t^{ji}\pi_t^{ji}. \tag{16c}$$

---

[17]Corresponding to these expressions, we have already obtained (a) buyers' stockout probability, (b) buyers' cost of stockout and (c) varieties available to each buyer in Section 2.3.

[18]Here, to derive the second line of (15c), we use the normalization assumption: $1 = \hat{E}_t[N_t^{ji}]$ $= \hat{E}[N_t^{ji}|n_t^{ji} < g_t^{ji}]\left(1 - \pi_t^{ji}(g_t^{ji})\right) + \hat{E}[N_t^{ji}|n_t^{ji} > g_t^{ji}]\pi_t^{ji}(g_t^{ji})$.

[19]Hence, perhaps it is more natural to define buyers' cost of stockout as the expected number of lost buyers; $(\hat{E}[N_t^{ji}|n_t^{ji} > g_t^{ji}] - G_t^{ji})\pi_t^{ji}(g_t^{ji})$. But, we find the above definition significantly reduces notations.

[20]Here, to derive $\tilde{\pi}_t^{ji}$ and $\hat{E}_t[S_t^{ji}]$, we use the completion of square: $\exp\{n_t^{ji}\}\exp\{\frac{-1}{2\sigma_N^2}(n_t^{ji} + \sigma_N^2/2)^2\}$ $= \exp\{\frac{-1}{2\sigma_N^2}(n_t^{ji} - \sigma_N^2/2)^2\}$.

Finally, as mentioned in Section 2.3, we can now show Q_t also means the probability that a buyer *does not* face stockout. To show this, let N_t^{ji*} be the number of buyers who can buy variety i without facing stockout at the j-th seller's shop,

$$N_t^{ji*} = \begin{cases} G_t^{ji}/M_t^b & \text{if } n_t^{ji} > g_t^{ji} \quad \text{(stockout)} \\ N_t^{ji} & \text{otherwise} \quad \text{(not stockout)} \end{cases}.$$

Due to LLN, the buyers' probability of not facing stockout Q_t equals the number of buyers who can buy the variety in each market, which is the aggregation of N_t^{ji*} over j:

$$Q_t = \frac{1}{\sqrt{2\pi}\sigma_N} \left[\int_{-\infty}^{g_t^{ji}} e^{n_t^{ji}} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji} + \frac{\sigma_N^2}{2}\right)^2} dn_t^{ji} + \int_{g_t^{ji}}^{\infty} \frac{G_t^{ji}}{M_t^{b,ji}} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji} + \frac{\sigma_N^2}{2}\right)^2} dn_t^{ji} \right]. \quad (17)$$

Previewing our results, in equilibrium, (i) demand per buyer is the same for all buyers; $M_t^{b,ji} = M_t^b$ (see Section A.1), and (ii) aggregate sales equals expected sales by LLN; $S_t = \hat{E}_t[S_t^{ji}]$. Hence, (17) and (16c) imply that the aggregate sales is the number of available varieties to each buyer times the amount that each buyer actually purchases:[21]

$$S_t = Q_t M_t^b. \quad (18)$$

Now, having above expressions, we are able to show some key differentiations. Assuming that sellers (intermediate goods firms) decide their production and price before observing N_t^{ji}, their optimization requires the knowledge of the derivatives of $\hat{E}_t[S_t^{ji}]$ with respect to their choice variables. We assume that ϕ is continuously differentiable (here, we do not rely on the log-normal assumption). Now, $\hat{E}_t[S_t^{ji}]$ is differentiable, although S_t^{ji} is not.[22] Hence,

$$\begin{aligned} \frac{\partial \hat{E}[S_t^{ji}]}{\partial G_t^{ji}} &= \frac{\partial}{\partial G_t^{ji}} \int_{-\infty}^{\infty} \min\left\{ G_t^{ji}, M_t^{b,ji} e^{n_t^{ji}} \right\} \phi(dn_t^{ji}), \\ &= \int_{g_t^{ji}}^{\infty} \frac{\partial}{\partial G_t^{ji}} G_t^{ji} \phi(dn_t^{ji}) + \int_{-\infty}^{g_t^{ji}} \frac{\partial}{\partial G_t^{ji}} \left(M_t^{b,ji} e^{n_t^{ji}} \right) \phi(dn_t^{ji}), \\ &= \int_{g_t^{ji}}^{\infty} \phi(dn_t^{ji}) = 1 - \Pr[M_t^{ji} < G_t^{ji}] = \pi_t^{ji}, \end{aligned} \quad (19)$$

where note that $\partial(M_t^{b,ji} e^{n_t^{ji}})/\partial G_t^{ji} = 0$. Similarly, since $\partial G_t^{ji}/\partial M_t^{b,ji} = 0$,

$$\frac{\partial \hat{E}[S_t^{ji}]}{\partial M_t^{b,ji}} = \int_{-\infty}^{g_t^{ji}} e^{n_t^{ji}} \phi(dn_t^{ji}) = \hat{E}[N_t^{ji}|n_t^{ji} < g_t^{ji}]\left(1 - \pi_t^{ji}(g_t^{ji})\right) = 1 - \tilde{\pi}_t^{ji}. \quad (20)$$

Here, we can offer an important intuition that goods on shelf G_t^{ji} and demand $M_t^{b,ji}$ are

---

[21] See also Section 2.3 for the discussion about (8).

[22] Hence, our idiosyncratic shock allows us to avoid dealing with the kinked constraint (12a) directly.

compliments in the sense that, as shown in (19), the 'marginal sales' generated by an additional G_t^{ji} is increasing in $M_t^{b,ji}$ since π_t^{ji} is strictly increasing in $M_t^{b,ji}$. The same sort of complementarity between G_t^{ji} and $M_t^{b,ji}$ can be also found in (20). Because of this, sellers want to have more inventories to capture strong demand and hence we refer to this accumulation of G_t^{ji} following strong demand as distributors' demand, which we discuss further in the following sections.

We also use the following results to derive the FOCs of intermediate goods firms. Note that (12c) implies $\partial G_t^{ji}/\partial U_t^{ji} = \partial G_t^{ji}/\partial Y_t^{M,ji} = 1$, and (12d) means $\partial M_t^{b,ji}/\partial P_t^{M,ji}$ $= -\theta M_t^{ji}/P_t^{M,ji} = -\theta Q_t^{\frac{-\theta}{1-\theta}}(P_t^{M,ji}/P_t^M)^{-\theta}M_t^F/P_t^{M,ji}$. By the chain rule,

$$
\begin{aligned}
\frac{\partial \hat{E}[S_t^{ji}]}{\partial P_t^{M,ji}} &= \frac{\partial M_t^{b,ji}}{\partial P_t^{M,ji}}\frac{\partial \hat{E}[S_t^{ji}]}{\partial M_t^{b,ji}} = -\theta\left(1-\tilde{\pi}_t^{ji}\right)Q_t^{\frac{-\theta}{1-\theta}}\left(\frac{P_t^{M,ji}}{P_t^M}\right)^{-\theta}\frac{M_t^F}{P_t^{M,ji}}, \\
\frac{\partial \hat{E}[S_t^{ji}]}{\partial U_t^{ji}} &= \frac{\partial \hat{E}[S_t^{ji}]}{\partial Y_t^{M,ji}} = \pi_t^{ji}.
\end{aligned}
$$

2.4.2 FOCs for Intermediate Goods Firms

Now, we can solve the optimization problem (11) and (12). In (21), the last two FOCs are standard; wage equals marginal product of labour and the net rental rate of capital equals the marginal product of capital where the values of marginal products are evaluated in terms of marginal cost λ_t^{MC}. The first three FOCs are, however, peculiar to our model.[23]

$$
\begin{aligned}
P_t^{M,ji} &= \theta\frac{1-\tilde{\pi}_t^{ji}}{Q_t}\left(P_t^{M,ji} - \lambda_t^{U,ji}\right), && \text{(21a)} \\
0 &= E_t\left[\Lambda_{t,t+1}\left\{P_{t+1}^{M,ji}\pi_t^{ji} + \lambda_{t+1}^{U,ji}\left(1-\pi_t^{ji}\right)\right\}\right] - \lambda_t^{U,ji}, && \text{(21b)} \\
0 &= P_t^{M,ji}\pi_t^{ji} + \lambda_t^{U,ji}\left(1-\pi_t^{ji}\right) - \lambda_t^{MC,ji}, && \text{(21c)} \\
0 &= \lambda_t^{MC,ji}\frac{\partial Y_t^{M,ji}}{\partial H_t^{ji}} - W_t, && \text{(21d)} \\
0 &= \lambda_t^{MC,ji}\frac{\partial Y_t^{M,ji}}{\partial K_t^{ji}} - \left(R_t^K - 1\right). && \text{(21e)}
\end{aligned}
$$

The first condition (21a) is with respect to the price of intermediate goods $P_t^{M,ji}$. Rearranging, we obtain the time-varying markup formula, where effective elasticity of substitution $\tilde{\theta}_t$ is a function of π_t^{ji} and Q_t:

$$
P_t^{M,ji} = \frac{\tilde{\theta}_t^{ji}}{\tilde{\theta}_t^{ji}-1}\lambda_t^{U,ji} \quad , \text{ where } \quad \tilde{\theta}_t^{ji} = \theta\frac{1-\tilde{\pi}_t^{ji}}{Q_t}. \tag{22}
$$

---

[23] The results in Section 2.4.1 are used to derive them.

The second condition (21b) is with respect to unsold goods U_{t+1}^{ji} and makes clear that $\lambda_t^{U,ji}$ is the shadow price of U_{t+1}^{ji} not U_t^{ji}, where $\lambda_t^{U,ji}$ can also be regarded as the cost of sales in accounting. The terms inside the curly bracket of (21b) mean that *marginal* U_{t+1}^{ji} can be sold with probability π_{t+1}^{ji} or left unsold with probability $1 - \pi_{t+1}^{ji}$ at $t+1$; if it is sold it generates revenue $P_{t+1}^{M,ji}$, but, if not, it is carried to the next period $t+2$ and its value is $\lambda_{t+1}^{U,ji}$ at the end of $t+1$. Hence, this condition says that, at the optimum, today's shadow price $\lambda_t^{U,ji}$ is equal to the present value of U_{t+1}^{ji}. In addition, if sellers' stockout probability π_t^{ji} goes to zero, (21b) and (21c) imply that $E_t\left[\Lambda_{t,t+1}\lambda_{t+1}^{MC,ji}\right] = \lambda_t^{MC,ji}$, which shows the production smoothing; in this sense, our model incorporates a buffer stock model as a special case. Note that (21b) can be rewritten in Jorgenson's user cost representation:

$$E_t\left[\Lambda_{t,t+1}\left(P_{t+1}^{M,ji} - \lambda_{t+1}^{U,ji}\right)\Pr[M_{t+1}^{ji} > G_{t+1}^{ji}]\right] = \lambda_t^{U,ji} - E_t\left[\Lambda_{t,t+1}\lambda_{t+1}^{U,ji}\right]. \qquad (23)$$

The third condition (21c) is with respect to output $Y_t^{M,j}$. In a similar manner to (21b), if sold production generates revenue $P_t^{M,ji}$, but if not it is carried to the next period and its value is $\lambda_t^{U,ji}$. Hence, (21c) says that, at optimum, marginal cost $\lambda_t^{MC,ji}$ is equal to the value of the marginal unit of $Y_t^{M,ji}$.

2.5   Aggregation

Aggregation is non-trivial in this model because an individual seller's unsold goods U_t^{ji} differs across the set of intermediate firms due to the idiosyncratic shock N_{t-1}^{ji}. However, we show that, given different U_t^{ji}, intermediate firms choose different $Y_t^{M,ji}$ so that they have the same final choice of $G_t^{ji} = G_t$; in other words, all intermediate firms choose identical G_t^{ji} regardless of U_t^{ji} by choosing different $Y_t^{M,ji}$.[24]   This crucially depends on the assumption of constant returns to scale (CRS) production, which guarantees that λ_t^{MC} is common to all sellers; $\lambda_{t+1}^{MC,ji} = \lambda_{t+1}^{MC}$. Hence, (21b) and (21c) imply that $\lambda_t^{U,ji} = \lambda_t^U$ (note that SDF $\Lambda_{t,t+1}$ is common to all firms). From (21a) and (21c),

$$\pi_t^{ji}\lambda_t^U + \left(\lambda_t^{MC} - \lambda_t^U\right) = \theta\frac{1 - \tilde{\pi}_t^{ji}}{Q_t}\left(\lambda_t^{MC} - \lambda_t^U\right).$$

Because π_t^{ji} and $\tilde{\pi}_t^{ji}$ are both *strictly* increasing in g_t^{ji} under log-normal N_t^{ji}, the both sides of this expression are strictly increasing and decreasing in g_t^{ji} respectively (note that $\lambda_t^{MC} - \lambda_t^U > 0$), which means this expression uniquely pins down g_t^{ji} as a function

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[24]Related to this, we would like to note that, although aggregate output is always positive, there is a possibility of having negative production at individual intermediate goods producers' level. Certainly, allowing negative production is counter-intuitive, but there is no inconsistency. Under our timing assumption that they cannot have negative production after observing the demand shock, such a negative production cannot negate the stockout constraint; see Appendix 2.5 for further discussion.

of variables that are independent from ji. Hence, $\pi_t^{ji} = \pi_t$ and $\tilde{\pi}_t^{ji} = \tilde{\pi}_t$ for all ji. Given these, (21a) implies $P_t^{M,ji} = P_t^M$, which in turn leads to $M_t^{b,ji} = M_t^b$ because of (9). Since g_t^{ji} does not depend on ji, G_t^{ji} is also the same for all ji. In sum, we do not impose symmetricity assumption, but the equilibrium must be symmetric.[25] Intuitively, while CRS guarantees that all intermediate firms face same marginal cost λ_t^{MC}, intermediate firms' first order conditions depend on $\hat{E}_t[S_t^{ji}]$, which is ex ante the same for all intermediate firms because they do not observe idiosyncratic shock N_t^{ji} when they make their decision. Given common $\hat{E}_t[S_t^{ji}]$, G_t^{ji} and λ_t^{MC}, the probability of stockout and its associated costs (π_t and $\tilde{\pi}_t$) are the same for all intermediate firms.

The only variables that differ among intermediate firms are U_{t+1}^{ji}, S_t^{ji}, K_t^{ji}, H_t^{ji} and $Y_t^{M,ji}$. It is straightforward to aggregate K_t^{ji}, H_t^{ji} and $Y_t^{M,ji}$ under CRS. Again, appealing to LLN, aggregate sales S_t is equal to the expected sales of a seller $\hat{E}_t[S_t^{ji}]$; see (16c). Hence, $M_t^{b,ji} = M_t^b$ implies:

$$S_t = M_t^b (1 - \tilde{\pi}_t) + G_t \pi_t, \tag{24}$$

We know that aggregate G_t and aggregate U_{t+1} is simply given by their definitions (12c) and (12b), which are linear. For the other variables, due to the above symmetricity, we can obtain aggregated variables simply by dropping superscripts j and i.

2.6 Equilibrium

The core part of the model has 19 endogenous variables and 19 equations. Because all agents behave symmetrically, we drop off superscript ji in the following. In our model, given the initial condition $\{U_0, K_0\}$, the proper transversality (non-explosive) conditions and exogenous shocks $\{\xi_t^M, \xi_t^g\}_{t=0}^\infty$, the equilibrium is defined as the set of variables $\{R_t^B, R_t^K, W_t, P_t^M, Q_t, \pi_t, \tilde{\pi}_t, \lambda_t^U, \lambda_t^{MC}, C_t, H_t, M_t^F, Y_t^M, I_t, S_t, M_t^b, G_t, U_{t+1}, K_{t+1}\}_{t=0}^\infty$ that satisfies the following equilibrium conditions:

- Household constraints (2b-c) and its FOCs (3);
- Final goods firms' FOCs (8);
- Intermediate goods firms' constraints (12b-e) and their FOCs (21); and
- Definitions and aggregation of variables (16a), (16b), (18) and (24).

3 Numerical Results

This section describes the quantitative properties of the model. The model developed in Section 2 is numerically simulated by linearizing the equilibrium equations around the non-stochastic steady state; see Section 2.6. Note that aggregate sales S_t is a

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[25]This symmetric result is similar to that in Lagos and Wright (2005). We thank the Editor for pointing this out.

smooth function (though individual sales S_t^{ji} are not) and hence it can be linearized. We have two sources of shocks: productivity and preferences. We interpret the former as a supply shock and the latter as a demand shock, although we must be cautious about such labelling.[26] We compare our model performance with U.S. data and a no-inventory version of the model, which is obtained simply by setting $\sigma_N = 0$. Though there are several minor differences from the standard RBC model, such as imperfect substitution among varieties, the no-inventory version can be regarded as a variant of the standard RBC model comparable to the benchmark experiments. Finally, note that, in aggregate, our model falls into the class of the models with representative agents and flexible prices.[27] One period in our model is one quarter, and we mainly focus on the business cycle frequencies by using the Baxter-King band-pass filter in obtaining the second moments.

3.1 Basic Inventory Facts

Before examining numerical results, let us remind ourselves of the key inventory facts.

1. Inventory investment is procyclical at business cycle frequencies.

Following Khan and Thomas (2007b), we measure inventory investment by dU_{t+1}/Y_t rather than $dU_{t+1} = U_{t+1} - U_t$. Note that we cannot take the logarithm of dU_{t+1} since it can be negative, while, using dU_{t+1} as it is, the resultant moments are affected by the measurement unit. As the first column of Table 5 shows, if procyclicality is measured by its correlation with output, it is 0.64 at business cycle frequencies and is 0.49 at high frequencies. However, $\text{cor}\{dU_{t+1}/Y_t, S_t\}$ at business cycle frequencies is 0.41 (not shown) but -0.42 at high frequencies. As Wen (2005) discussed, this is because inventories work as a buffer at high frequencies. Inventory investment is positively correlated to both output and sales at business cycle frequencies but these correlations differ between output and sales at high frequencies.

2. Output Y_t is more volatile than sales S_t.

As shown in Table 5, the ratio of volatilities between output and sales is less than one; $\sigma(S_t)/\sigma(Y_t) = 0.83$. This is the same at high frequencies but a lesser extent: $\sigma(S_t)/\sigma(Y_t) = 0.93$ (not shown).

3. I/S ratio is countercyclical and persistent.

This fact is reported by Ramey and West (1999) and Wen (2005). In our notation, inventory-to-sales (I/S) ratio is U_t/S_t. In our data set, $\text{cor}\{U_t/S_t, Y_t\} = -0.52$ and $\text{cor}\{U_t/S_t, U_{t-1}/S_{t-1}\} = 0.88$. This is the same fact that Blinder and Maccini (1991) considered implausible, when implementing a reduced form regression and finding that

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[26] For example, even the technology shock stimulates demand through wage and capital return.

[27] Although we assume price posting (see Section 2.1.2), meaning that intermediate goods prices $P_t^{M,ji}$ cannot react to idiosyncratic shock N_t^{ji}, they (and their index P_t^M) can react to all aggregate shocks. Hence, in our model, there is no nominal rigidity in aggregate.

the adjustment speed of the inventory is quite slow, given the fact that 'even the widest swings in inventory stocks amount to no more than a few days of production' (Blinder and Maccini, 1991, p.81).

We note that, given the high correlation between output and sales in the data, the traditional inventory facts (1) and (2) at the business cycle frequency essentially restate the same fact from two different angles. This is evident form the law of motion of inventories (25a), taking variances after moving terms,

$$Var\left(S_t\right) = Var\left(Y_t^M\right) + Var\left(\mathrm{d}U_{t+1}\right) - 2Cov\left(\mathrm{d}U_{t+1}, Y_t^M\right),$$

where $\mathrm{d}U_{t+1} = U_{t+1} - U_t$. This means that $Cov\left(\mathrm{d}U_{t+1}, Y_t^M\right) > 0$ is a necessary condition of $Var\left(S_t\right) < Var\left(Y_t^M\right)$. A similar manipulation shows that $Cov\left(\mathrm{d}U_{t+1}, S_t\right) > 0$ is a sufficient condition of $Var\left(S_t\right) < Var\left(Y_t^M\right)$. Note that output and sales are very closely correlated to each other in data, implying that measuring procyclicality of inventory investment by correlation with output or sales does not matter very much; i.e., $Cov\left(\mathrm{d}U_{t+1}, Y_t^M\right) > 0$ is almost equivalent to $Cov\left(\mathrm{d}U_{t+1}, S_t\right) > 0$. Hence, procyclical inventory investment implies more volatile output than sales, and vice versa.

3.2   Parameter Selection and Steady State

For the standard RBC parameters, we follow conventional values to facilitate the comparison (see Table 2), which generate reasonable steady state values (see Table 3). In the steady state, consumption and investment are around 80% and 15% of output, respectively. Relative weight for leisure in the utility function is set so that working hours are roughly 1/3 of time endowment. Capital depreciation rate is matched to capital stock/annual GDP ratio, which is around 2.6. For the elasticity of substitution among varieties θ, we set it to be 7.5, which is rather common in the standard new Keynesian models. Steady state stockout probability π_{ss} is mainly affected by θ, and $\theta = 7.5$ generates a plausible stockout probability 8.1% (see Bils (2004)).[28]

To see the effects of inventories, we experiment with several sizes of idiosyncratic shocks $\sigma_N = \{0.00, 0.40, 1.73\}$, where $\sigma_N = 0.0$ is essentially the RBC model, and $\sigma_N = 0.4$, which leads to inventory-to-sales ratio $U_{ss}/S_{ss} = 0.66$ (around two months) in the steady state; see Table 6 for the results with $\sigma_N = 0.4$. In data, inventory-to-sales ratio is roughly two months; see Bils (2004) for example. We have chosen $\sigma_N = 1.73$ by matching the key inventory moments, which we discuss further in the next subsection. In our numerical experiments, we mainly consider the model behavior with $\sigma_N = 1.73$ rather than targeting a particular value of U_{ss}/S_{ss}, because (i) there is no service sector

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[28]The key determinant of π_{ss} is the net profit margin. For example, we can have the same stockout probability by instead adding annual convenience yield 1.3% of inventories with $\theta = 10.0$, which generates the almost same quantitative results as our benchmark model.

in this model and (ii) there are aspects of the stockout problem that we do not model such as the reputation cost of a stockout. For $\sigma_N = 1.73$, U_{ss}/S_{ss} is 4.2 quarters; as σ_N becomes higher, the unsold goods in the steady state becomes greater. Because of the cost of losing varieties ($Q_{ss} = 0.63$, i.e., the buyers' stockout probability is 37%), the intermediate goods price is strictly lower than the final goods price in the steady state; $P_{ss}^M = 0.93$. The effects of changing σ_N are discussed in Section 3.5.1.

In terms of the exogenous shock processes, because sales are not necessarily equal to output in our model, our model has relative advantage to investigate the relative importance of demand and supply shocks.[29] In this respect, we employ a simple moment matching to fix the parameters of exogenous shock processes together with the size of idiosyncratic shock σ_N in Section 3.3.

Finally, as discussed in Section 2.1.2, we assume that labour supply cannot respond to the current period aggregate shocks.[30] This is because otherwise if production can respond to all aggregate shocks, the model cannot generate a sudden decline in the inventory holdings right after a demand shock at aggregate level.[31] Even if all information is available for labour supply decision, inventory facts are satisfied in aggregate almost equally well at business cycle frequencies, but inventories work little as a buffer stock. To see its importance quantitatively, Section 3.5.2 discusses the effect of changing this information assumption.

[Table 2: Parameters around here]
[Table 3: Steady State around here]

3.3   Moment Matching

To pin down the parameters of aggregate supply, aggregate demand and idiosyncratic shocks, we conduct a simple moment matching, in which we seek the parameter values of these shock processes to minimize the (weighted) sum of squared gaps of the seven key moments between the model and the US data. Note that, in our stochastic simulations, the generated second moments are also stochastic; hence, we put more weights on the moments that are simulated more precisely in the stochastic simulations, see Appendix A.3 for more details. This method is in spirit similar to Crucini Residual method as employed in Khan and Thomas (2007a).

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[29] The exogenous shock processes do not affect the non-stochastic steady state.

[30] This plays a similar role of the capital adjustment cost in Wen (2011). However, given strong crowding out effect that we are going to discuss in Section 3.4, it leads to too low variability of capital investment; see Table 3 in Wen (2011). Setting aside the model performance, we also believe, given infrequent labor contract, assuming information imperfection in labour supply seems to be relatively safe choice in adding an aggregate real rigidity.

[31] The idiosyncratic shock is unanticipated but it is integrated out in aggregation. Note also that information assumption does not affect the non-stochastic steady state.

The four key parameters to be 'estimated' are: the AR(1) coefficient on the demand shock ρ_C, the AR(1) coefficient on the supply shock ρ_M, the ratio of the standard deviations of the innovations to both shocks σ_C/σ_M and the size of the idiosyncratic shocks σ_N. In terms of the target moments to be matched, we have two groups. First, as we are interested in the relative importance of demand and supply shocks, we found that it is useful to target (i) the contemporaneous correlation between wages and output $\text{cor}\{W, Y\}$ and (ii) the ratio of the standard deviation of consumption relative to that of output $\text{sd}(C)/\text{sd}(Y)$. It is well-known, for example, that the correlation of wage to output is near $+1$ if the shock is on the supply side, while it is almost -1 for the demand shock; see Bencivanga 1992 for example. Analogously, it is well-known that the variance of consumption to output is less in an RCB-type model when supply shocks dominate demand. Examine the columns under the label of Benchmark ($\sigma_N = 1.73$) in Table 5 to assess the difference between the model based on the demand or supply shocks alone.[32]

The second group is simply motivated by the following inventory facts; output is more volatile than sales; inventory investment is procyclical; inventory-to-sales ratio is countercyclical and persistent. For the correlation between inventory investment (as percentage of GDP) and output, we focus on $\text{cor}\{dU/Y, Y\}$ at higher frequencies because, as Wen (2005) reported, the cyclicality of the inventory investment is expected to be quite different between demand and supply shocks at high frequencies. On the other hand, at business cycle frequencies, $\text{cor}\{dU/Y, Y\}$ and $\text{cor}\{dU/Y, S\}$ are almost same, given very high correlation between output Y and sales S.

The resultant parameter values and moments are listed in Table 4. First, we find that the idiosyncratic shock is relatively large ($\sigma_N = 1.73$); compare Tables 5 and 6 to see its effects. As discussed above, this large value is necessary to generate sufficiently large inventory fluctuations in our model. Second, both shocks are fairly persistent. Third, the ratio of the innovations to the supply and demand shocks is $0.504 : 1$. Also, since supply shock is less persistent, the ratio of supply and demand shock volatilities is $0.228 : 1$. In this naïve comparison, the supply shock is much less volatile than the demand shock.[33]

For the third point above, however, it is premature to conclude that the supply shock is less important than the demand shock. Actually, as shown in our numerical results in Table 5, output volatility is almost one half with demand shock only than with supply shock only. This is not surprising because the supply shock directly affects output, and indeed if instead we focus on the consumption volatility it is greater for the demand shock than for the supply shock. In terms of $\text{cor}\{W, Y\}$ and $\text{sd}(C)/\text{sd}(Y)$, the data moments are both near the simple average between supply shock only and demand shock

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[32]Note that Table 1 is not directly comparable with Table 5 because the former employs a filter, while the latter uses growth rates.

[33]Note that $0.23 : 1 = 0.504/(1 - 0.876) : 1/(1 - 0.944)$.

only cases. All in all, in our model, it seems that the supply and demand shocks are almost equally important.

[Table 4: Moment Matching around here]

3.4 Simulation Results

We find that, while adding the stockout constraint does not deteriorate the model performance in mimicking the RBC facts, it can explain the inventory facts fairly well. To capture the essence, we would like to introduce simplified, though not exact, demand equations. Since the effects via Q_t is quantitatively small, ignoring the cost of losing varieties (i.e., keeping $Q_t = 1$ so that $S_t = Y_t^F$), we can rewrite (2b), (12) and (16c) as:

$$Y_t^M = C_t + I_t + (U_{t+1} - U_t), \tag{25a}$$

$$S_t = C_t + I_t \tag{25b}$$

$$S_t = S\left[Y_t^M + U_t, \ M_t^b\right] \tag{25c}$$

where note that (25c) schematically captures the fact that goods on shelf $Y_t^M + U_t$ is complimentary to demand M_t^b in generating sales, as discussed in Section 2.4.1.[34]

3.4.1 RBC Facts

In terms of working hours, our model performs similarly to the standard RBC model. Hours are less volatile than output for the supply shock and vice versa for the demand shock. Wage and labour productivity are almost perfectly positively correlated to output Y_t^M for the supply shock. In contrast, with the demand shock, it is almost perfectly negatively correlated to Y_t^M, as found in Christiano and Eichenbaum (1992) for government expenditure shock and Bencivenga (1992) for preference shock. For both shocks, capital investment is as volatile as the data,[35] although it is slightly less volatile for the demand shock, because it is crowded out by consumption. The volatility of investment is too high in the no-inventory case because inventories compete with capital in the sense that the former generates sales while the latter generates output. In a sense, inventory and capital investments also crowd out each other to some extent.

To see these crowding-out among capital investment I_t, inventory investment $U_{t+1} - U_t$ and consumption C_t, see Figure 1. Under $\rho_C = 0.944$, capital investment goes below the steady state level after a a positive preference (demand) shock, while, at the date when the shock hits, inventory investment works as a buffer. On the quantity side,

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[34]Actually, (25c) is equivalent to (24), in which π_t and $\tilde{\pi}_t$ are functions of only $G_t^{ji} = Y_t^M + U_t$ and M_t^b. Also, (9) and (18) imply $S_t = Q_t^{\frac{-1}{\theta-1}} Y_t^F$.

[35]In Table 5, following convention, for the US data, durable goods consumption is included in capital investment, not in consumption.

(25a) implies that, given output, a higher consumption implies either lower capital or inventory investments or both. On the price side, we can see the increase in interest rate from the mid left panel of Figure 1, implying that the cost of borrowing for investment and the Jorgenson's user costs for inventories both increase after a positive preference shock. Hence, from these two aspects, the crowding-out can be explained. The key parameter here is ρ_C; if we set, say, $\rho_C = 0.990$, capital investment increases significantly after a positive preference shock. In this case, although the real interest increases more sharply, because the consumption is expected to be strong for a longer period, capital investment increases by increasing output Y_t^M sharply. Hence, the shape of IRF of I_t is quite sensitive to ρ_C, which holds even in the no-inventory case.

[Figure 1: IRFs to Preference (Demand) Shock around here]

[Figure 2: IRFs to Productivity (Supply) Shock around here]

3.4.2 Traditional Two Inventory Facts

For both supply and demand shocks, production is more volatile than sales, and inventory investment is procyclical (see Table 5). With the supply shock, it is hardly surprising because the source of shocks lies in the production sector. In our model, however, even with the demand shock, production is more volatile than sales. To see this, consider the upper left panel of Figure 1. First, right after a positive preference (demand) shock, the inventory investment decreases, simply because intermediate goods producers use inventories as a buffer to accommodate a sudden increase in demand. As discussed in Section 2.4.2, inventories also work as a buffer stock in our model, even without the idiosyncratic shock ($\sigma_N = 0$).[36] Subsequently, however, inventory investment increases, because, in generating sales, inventories and demand are compliments as discussed in Section 2.4.1. To capture a strong demand, intermediate goods firms want to accumulate inventories (which we call distributors' demand), they must produce more than what they sell; see also (25a).[37]

As a result, as previously reported and discussed by Wen (2005), if we apply a high-frequency filter, we find $\text{cor}\{dU/Y, S\} = -0.42$ and $\text{cor}\{dU/Y, Y\} = 0.49$ in the US data; see Table 5. This captures the buffer stock behavior of inventories. However, at the business cycle frequencies, the distributors' demand plays a more important

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[36]More closely looking into the inventory behavior, we find that inventories work as a buffer mainly because of our information assumption that labour supply cannot react to the current period aggregate shocks. Under our function and parameter assumptions, the effects of intertemporal substitution on the production side are quite weak. In other words, though it is surely working, the production smoothing due to a convex cost is quantitatively very weak in our model.

[37]Actually, distributors' demand works even for a positive productivity shock, as long as it leads to an increase in demand, although productivity shocks can mimic the two traditional inventory facts without the help of this mechanism. In addition, as shown in the upper left panel of Figure 2, mainly because capital investment increases sharply, to accommodate this initial strong demand, inventory investment decreases slightly right after a positive supply shock.

role, which adds demands on top of consumption and capital investment; see (25b). Hence, as reported in Table 5, cor$\{dU/Y, Y\}$ is 0.98 and 0.47 for supply and demand shocks respectively; i.e., inventory investment is procyclical in business cycle frequencies for both shocks. At first glance, these finding leads us to postulate that inventories suppresses the effects of demand shocks at high frequencies but amplifies them at business cycle frequencies. However, we actually find not, which we discuss in Sections 3.5.1 and 3.5.2.

[Table 5: Key Second Moments around here]
[Table 6: Key Second Moments for Different Idiosyncratic Shock around here]

3.4.3 Inventory to Sales Ratio

The inventory to sales ratio (I/S ratio) is countercyclical and persistent in the calibrated and estimated versions of our model (see Tables 5 and 6). The behavior of I/S ratio is mainly governed by the cost of carry of unsold goods $\lambda_t^{U,j} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,j}]$ as shown in (23), and hence is governed by interest rate R_t^B. Intuitively, when the economy booms, high R_t^B discourages intermediate firms from having inventories relative to expected sales. Hence, roughly speaking, the countercyclicality and persistence of I/S ratio are due to the procyclicality and persistence of the interest rate in our model.[38] Quantitatively, compared to the data, our model generates the persistence in the I/S ratio similar to the data but its volatility is relatively low and its correlation with output is too large negative for both shocks. In Table 5, we see that cor$\{U/S, Y\}$ (correlation between I/S ratio and output) is -0.52 in the US data, while it is -0.91 in our benchmark simulation ($\sigma_N = 1.73$), while the first autocorrelation of I/S ratio is 0.88 in the data and is 0.92 in the simulation.

3.4.4 Intermediate Goods Price, Markups and Elasticity of Substitution

In this model, we have two markup concepts P_t^M/λ_t^U and P_t^M/λ_t^{MC}[39] that are both positively correlated with output (see Table 5) with their impulse response functions almost identical shapes to each other though different magnitude (see Figures 1 and 2). This is because the effective elasticity of substitution $\tilde{\theta}_t$ is countercyclical for both shocks; see (22) for the definition of $\tilde{\theta}_t$. The intuition of this is closely related to that of I/S ratio. That is, interest rate R_t^B tends to be higher in booms, which leads to a higher cost of carry of unsold goods U_{t+1}, meaning that the optimal inventory holdings relative to

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[38] However, there is a mechanism that generates a small degree of endogenous persistence; see Section 3.5.3.

[39] The latter (sales price/marginal cost) is the standard definition of markup in the absence of inventories. But, with inventories, (21b) and (21c) imply that it is the shadow price of unsold goods λ_t^U that corresponds to the concept of the 'cost of sales' in accounting. In this sense, P_t^M/λ_t^U is the proper definition of markup in our model, but for the comparison sake we show P_t^M/λ_t^{MC} as well.

sales becomes lower. To reduce the risk of having their goods unsold, intermediate goods firms accept a high stockout probability π_t and a concomitant loss of sales opportunities $\tilde{\pi}_t$. Because the effects of Q_t, which is low in booms, are quantitatively small under our parameter setting, the behavior of $\tilde{\theta}_t$ is dominated by $\tilde{\pi}_t$. In data, the evidence on the mark-up is inconclusive. For example, Martins and Scarpetta (1999) are supportive of a procyclical markup,[40] while Small (1997) and Nishimura, Ohkusa, and Ariga (1999) find some evidence of a countercyclical markup; others such as Marchetti (2002) draw an indefinite conclusion. See also Rotemberg and Woodford (1999) among others for the importance of the cyclicality of markup in optimal monetary policy settings.

3.5   Effects of Changing Parameters

In this subsection, we investigate the effects of changing three parameters: (i) the standard deviation of idiosyncratic shocks σ_N; (ii) the share of observable component in the aggregate shocks η; and (iii) the persistence of the exogenous shocks ρ_C and ρ_M. We investigate the effects of changing σ_N and η, not only to check the robustness but also to draw some implications on the causes of the *Great Moderation.* In addition to the two leading explanations – good monetary policy and good luck – Kahn, McConnell, and Perez-Quiros (2002) suggest that the increase in output stability observed since around 1980 in the U.S. may be due to the improvement in inventory management, which may have been induced by new IT technologies. In our model, improved inventory management can be interpreted as a lower σ_N and more information available at the timing of labour supply decision.

In accounting for the reduction of the volatility of total output, Kahn, McConnell, and Perez-Quiros (2002) reports the following key observations started at around early 1980s; (i) output volatility has decreased, which can be partly explained by the reduction in sales volatility; (ii) in light of the evolution of inventories, the reduction of output volatility relative to sales volatility is mainly accounted for by the decreases in both inventory investment volatility and correlation between inventory investment and sales; (iii) the level and the fluctuation of I/S ratio have declined. Our Tables 7, 8 and 9 effectively correspond to their variance decomposition using the equations equivalent to (25a) and (25b); see Appendix A.5.

[Table 7: Decomposition of Output Variance: US data around here]

3.5.1   Size of Idiosyncratic Shocks σ_N

Table 8 shows that changes in σ_N have little effect on output volatility in our model under either shock. On the one hand, given demand volatility, lower σ_N reduces the volatility

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[40]See also Bils and Kahn (2000).

of inventory investment and its correlation with sales; hence, for both shocks, output volatility *relative to sales volatility* increases as σ_N increases. In this sense, our model captures the intuition that Kahn, McConnell, and Perez-Quiros (2002) suggested. On the other hand, however, the volatility of sales increases when σ_N is low. This is because of the crowding out as discussed above. In booms, inventory investment increases (to exploit strong demand), which suppresses consumption and capital investment given resource constraint (25a). However, as σ_N decreases, the effects of this crowding out decreases and as a result the volatility of consumption and capital investment increases. Because of these two offsetting effects, the total effect is not monotone. See also Table 6 for other aspects of changing σ_N.

[Table 8: Decomposition of Output Variance: Different Idio Shock around here]

3.5.2 Information Available to Labour Supply Decision

Let us also consider the information available at the timing of the labour supply decision. It is straightforward to decompose shock ξ_t into observable component ξ_t^{ob} and unobservable component ξ_t^{un}; if for example $(1 - \eta)\,\%$ of ξ_t is observable, let $\xi_t = \sqrt{1 - \eta}\xi_t^{ob} + \sqrt{\eta}\xi_t^{un}$ for $0 \leq \eta \leq 1$.[41] Then, we allow labour supply H_t can react only to ξ_t^{ob}. In Table 9, we set η as 0 (full information), 0.5 (50% aggregate shocks are observable for labour supply decision) and 1.0 (labour supply is decided before observing the current period aggregate shocks, benchmark). As is clear from Table 9, there is little impact on the variance decomposition. In our model setting, this information affects mainly the high frequency inventory behavior and not the model behavior at business cycle frequencies. Note that, since the information assumption is irrelevant to the *non-stochastic* steady state, I/S ratio at the steady state is not affected by η.

Closer investigation tells us a bit more story. First, a lack of information has a direct effect on output volatility; since labour supply cannot react to unobservable shocks, the lack of information directly suppresses output volatility. Second, however, for example, right after a positive demand shock (which is captured by high-frequency filtered moments), less responsive labour supply causes a drop in inventory investment. Hence, in the subsequent periods, production must increase so that inventories catch up with sales. This distributors' demand increases the procyclicality of inventory investment in the business cycle frequencies. These two effects offset each other, leading to little effects in total.

[Table 9: Decomposition of Output Variance: Different Information around here]

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[41] Here, ξ_t^{ob} and ξ_t^{un} both follow a normal distribution with the same variance as ξ_t.

3.5.3 Non-Persistent Shocks

This subsection examines the model behavior with $i.i.d.$ aggregate shocks. All in all, the model performance is poor with $i.i.d.$ shocks. But, this exercise allows us to eliminate the effects of distributors' demand (since there is no predictable components in shocks), allowing inventories to play only the role of a buffer stock. Table 10 shows the result when (i) capital stock is fixed by fixing capital investment at the level of steady state depreciation, (ii) labor supply is determined after observing all aggregate shocks up to the current period (perfect information). The former means that we shut down the crowding out that we discussed above. For the latter, note that, if labor is determined after observing the demand shock, since capital is predetermined, output cannot react to the demand shock, meaning that the demand shock has no effects on quantities such as consumption and output. Also, since $i.i.d.$ implies that the most fluctuations concentrate on high frequencies, we use (iii) band pass filter for 2 to 40 quarters. Under these setup, we know that production smoothing is solely due to the convex cost function. The main finding is that buffer stock inventories generate persistence in some small degree. For $i.i.d.$ demand shocks, on the one hand, output is persistent, because of production smoothing. Intermediate firms optimally choose to accommodate unexpected strong demand by reducing inventories right after a positive demand shock, not by increasing production. In subsequent periods, intermediate firms increase their production to recover their lost inventories. Hence, inventories as a buffer stock generate persistent output from $i.i.d.$ demand shocks; in this version of the model, the first autocorrelation of Y_t^M is 0.44. For $i.i.d.$ supply shocks, on the other hand, sales is persistent, because of consumption smoothing. Rather than consuming a sudden increase in output at one time, such an increase in output is stored in the form of inventories. Hence, inventories as a buffer stock generate persistent sales from $i.i.d.$ supply shocks. These exercises show that buffer stock inventories not only insulate production from demand shocks but also insulate demand from supply shocks in general equilibrium. Finally, with $i.i.d.$ shocks, the correlation between output and sales is much lower, and, because inventories gradually return back to the steady state level, the I/S ratio is persistent.

[Table 10: Persistence under of iid Aggregate Shocks around here]

3.6   Summary of Numerical Results

In terms of the standard RBC facts, our model inherits most of the features from the standard RBC model. The only difference is a less volatile capital investment than a standard RBC model, because inventory investment competes with capital investment and crowd outs in some degree, allowing a better, though slightly, fit with the US data.

With specific reference to the basic inventory facts, the model performs well. The key intuition is distributors' demand. Since the target level of inventories is increasing

in demand, if there is one unit of increase in demand, the target level of inventories becomes higher. Hence, after a positive demand shock, inventory investment becomes positive (procyclical inventory investment), and output must increase more than sales to accumulate inventories. Because of this, the inventory behavior is strongly affected by the expected demand. The behavior of inventories relative to sales is mainly affected by interest rate (through the cost of carry of inventories). We find that in booms with high interest rates, sellers optimally choose a lower I/S ratio by accepting high stockout probability, which leads to both countercyclical and persistent I/S ratio.

In our discussion, the existence of inventories (due to the stockout constraint) may seem to amplify shocks at first glance, which is true only given size of the demand fluctuations. However, in general equilibrium, where demand is also endogenous, inventory investment crowds out capital investment, and to a lesser decree consumption as well. Hence, an increase in inventory investment in boom suppresses the increases in capital investment and consumption; as a result, inventory investment suppresses the volatility of sales (demand). In this respect, it is fair to say that, certainly, our model captures most of the mechanism discussed by Kahn, McConnell, and Perez-Quiros (2002), our model does not provide strong support for the hypothesis that improvements in inventory management have reduced GDP volatility.

4   Conclusion

In this paper, we investigate a dynamic general equilibrium model with a stockout constraint faced by intermediate goods producers. The stockout constraint means that, even if demand is strong, sellers cannot sell more than the goods on shelf that they have. Because of this, to generate sales, sellers need inventories; i.e., sellers hold inventories to overcome the constraint when they distribute their goods. The key trade-off is that (a) having too few inventories is costly because it leads to too high a stockout probability, while (b) having too much inventories is also costly because it leads to too high a cost of carrying inventories. The former implies that the optimal level of inventories is increasing in demand, which explains why inventory investment is procyclical. In booms, sellers have to produce not only to accommodate strong demand but also to accumulate inventories to generate sales, which we call distributors' demand because sellers need inventories to distribute their products to buyers. In the presence of the stockout constraint, hence, one unit of increase in demand leads to more than one unit of increase in output. The latter implies that the optimal level of inventories relative to sales is strongly affected by interest rate (cost of carry), which explain why inventory to sales ratio is persistent and countercyclical; see Bernanke and Gertler (1995).

Note that the above observations hold mainly at business cycle frequencies. For high frequency behavior, our model also naturally incorporates the production smooth

motivation of inventories, where sellers want to avoid volatile production paths given convex cost function. That is, sellers allow inventories to decline right after an unanticipated demand shock, which is captured by the negative correlation between inventory investment and sales at high frequencies; see Wen (2005). Also, at first glance, it might be tempted to conclude that the distributors' demand discussed above may amplify the output volatility. In our model, certainly, *given demand fluctuations*, the stockout constraint amplifies it; production is more volatile than sales. The mechanism behind it is almost the same as what Kahn, McConnell, and Perez-Quiros (2002) argue in explaining the Great Moderation. However, because of the resource constraint, having more inventories crowds out capital investment and consumption. Hence, as shown in our numerical experiment, as we reduce the effects of the stockout constraint, the lower crowding out effect raises the demand volatility. In total, we see (a) output volatility relative to sales decreases, but (b) the volatility of sales increases. All in all, under our model setup and parameters, improving inventory management cannot explain the Great Moderation.

The most closely related work to our model is Wen (2011), where he treats the stockout constraint as buyers' problem by conflating demand and supply sides, which keeps his model simple. In contrast, we explicitly consider the double-sided nature of the stockout constraint from the perspective of both sellers and buyers. Here, we emphasize that the stockout constraint is the inventory management problem of sellers. In our view, while an (S,s) model may be suitable to study inventories on the buyers' side, the stockout model allows us to focus on the sellers' inventory management problem as well. Because of this, for example, we can explicitly investigate the stockout probabilities of sellers and buyers separately. More importantly, however, our analysis explains why Khan and Thomas (2007a) find very different results from ours as well as Wen's (2011). In their seminal paper, they find that, while their (S,s) ordering model is successful in explaining inventory facts, their version of the stockout constraint model fails to generate a sufficiently high average inventory level, which means that it cannot explain business cycle fluctuations, as opposed to our numerical analysis of the contribution of inventories to business cycle fluctuations. By explicitly analyzing sellers and buyers separately, we show that it is important to assume (1) a small degree of price inflexibility,[42] (iii) production decision before observing demand shocks and (ii) positive net profit margin. For (i) and (ii), as discussed in Section 2.1.2, the stockout constraint is negated by either price adjustment or production adjustment after observing demand shocks; sellers have to hold inventories because of demand uncertainty, but such uncertainty has no effects if either of them is possible. For (iii), while to accept the risk of incurring the cost

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[42] Note that as discussed in Section 2.1.2, this price rigidity, which we call price posting is only within one period and hence in aggregate there is no price stickiness over the period. In this sense, our model falls into the class of flexible price model.

of carrying inventories, sellers must be compensated by positive profit when goods are sold; zero profit margin means that the return on inventory investment is negative in expectation.

Finally, as shown in our analytical and numerical results, the interest rate plays a key roll in determining the behavior of inventories and markups (via the user cost of inventories). Although monetary policy is absent in our model, as Bernanke and Gertler (1995) suggest, inventories may have an important interaction with monetary policy.[43] Indeed, given the extensive recent development of dynamic macroeconomic models to incorporate financial spreads, it has not escaped our attention that inventory management may play an important role in the monetary policy transmission but we leave this question to future research.

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[43] Many New Keynesian authors have suggested reasons for the continuing importance of inventories: Gertler and Gilchrist (1994) and Kashyap, Lamont, and Stein (1994), point to inventories as collateral for external finance, while Bernanke and Gertler (1995) show that inventory investment responds quickly to a monetary policy shock. Also, Kahn, McConnell, and Perez-Quiros (2002) suggest that improved inventory management techniques may provide a clue understanding the long 1990s expansion, which continued until 2008, and the so-called Great Moderation. We extend our analysis to the sticky price dimension in future work.

A   Appendix

A.1   Negative Production

In this Appendix, we would like to raise an attention that there is a counterintuitive behavior at the level of individual intermediate goods producers.[44] Under the assumption of the log-normal idiosyncratic demand shock, roughly speaking, the worst possible demand is such that a seller or producer observes zero buyers. In this case, this producer is forced to carry all of his products to the next period as unsold goods. If hypothetically the optimal target of the goods on shelf is unchanged, the unsold goods carried from the previous period already meet this target in the next period as well, meaning that his optimal production is zero at $t+1$. In stochastic simulations, however, the optimal goods on shelf is a function of mainly aggregate expected sales and interest rate (via the cost of carry), and hence it changes over time. Thus, there is a possibility of having negative production at individual producer's level, especially when the aggregate demand is lower than the previous period. However, we argue that this is not a significant problem, especially for the aggregate behavior of the model. First, production never becomes negative in aggregate. Second, given our persistent aggregate shocks, such negative production at individual producer level is small in magnitude. Third, it is possible to eliminate such negative production by assuming a distribution function with some positive lower bound (such as uniform) for the idiosyncratic shock. Indeed, the earlier version of this paper employed a uniform distribution for the idiosyncratic shocks, but there was no sensible difference given linearization technique in our simulations, although the algebraic expression becomes messier with uniform distribution. Finally, certainly, allowing negative production is counter-intuitive, but there is no internal inconsistency. Under our timing assumption that production decision is made before observing the idiosyncratic shock, they cannot liquidate their unsold goods by producing a negative amount, meaning that some sellers still have to carry their unsold goods to the next period.

A.2   Perfect Substitution

This appendix sketches the proof that the model economy reduces to the standard RBC model when $\theta \rightarrow \infty$ (perfect substitute). Though it is rather intuitive, the exact derivation proves that limiting case shows some complicated behavior, which could be potentially interesting. First, dividing (21a) by θ, it is clear that, as $\theta \rightarrow \infty$, $(P_t^{M,ji} - \lambda_t^{U,ji}) \rightarrow 0$ and/or $\tilde{\pi}_t^{ji} \rightarrow 1$ must hold. However, the former cannot be true, because it implies that (21b) would not satisfy a transversality condition. Actually, $\tilde{\pi}_t^{ji} \rightarrow 1$ and, at the limit, (21b) reduces to $E_t[\Lambda_{t,t+1}\{P_{t+1}^{M,ji} - \lambda_{t+1}^{U,ji}\}] = \lambda_t^{U,ji} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,ji}]$

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[44]We thank the Editor for pointing this out.

(the expected gross profit margin just covers the cost of carrying inventories) and $\lambda_t^{U,ji} = E_t[\Lambda_{t,t+1}\lambda_{t+1}^{MC,ji}] \neq 0$ (interestingly we can calculate the shadow price of U_{t+1}^{ji}, even though U_{t+1}^{ji} does not exist). Also, because $P_t^{M,ji} \to \lambda_t^{MC,ji}$ as $\theta \to \infty$ and constant returns to scale guarantees $\lambda_t^{MC,ji}$ is the same for all ji, (21c) directly shows that $P_t^{M,ji} \to \lambda_t^{MC}$. Furthermore, (8) implies $P_t^M \to 1$ (= final goods price), and hence $\lambda_t^{MC} \to 1$.

Second, (16a) implies that $g_t^{ji} \to g_l$ and hence $\pi_t^{ji} \to 1$, as $\tilde{\pi}_t^{ji} \to 1$, where g_l is the lower support of g_t^{ji} (under our log-normal distribution assumption, $g_l = -\infty$). Intuitively, as $\theta \to \infty$, the net profit margin (gross profit margin minus the cost of carry) becomes zero, meaning that (i) even if the marginal U_{t+1}^{ji} is sold in the next period, sellers does not appreciate such a sales very much since the net profit is zero, but (ii) if U_{t+1}^{ji} is unsold, sellers simply have to pay its cost of carry $\lambda_t^{U,j} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,j}]$. That is, having unsold goods is a one-sided unfair betting; i.e., get zero if win, but pay some if lose. Because sellers do not care about the loss of sales opportunity in this case, sellers optimally choose G_t^{ji} as if they see the minimum possible number of buyers N_l ($N_t^{ji} = N_l$ for all ji). Hence, the stockout always takes place, which means that *effectively* sellers do not care about demand uncertainty. Thus, $U_{t+1}^{ji} \to 0$ for all ji, unless the cost of carry is negative; i.e., unless sellers expect a sharp increase in the marginal cost of production in the future.

Third, (9) implies that $M_t^{b,ji} \to M_t^F/Q_t$, which means that, if a buyer has an access to $Q_t\%$ of varieties, he buys each available variety by M_t^F/Q_t, and his total purchase is just M_t^F (not affected by Q_t or N_t). Since $Q_t \to N_l$ (intuitively because $1 - Q_t$ is the stockout probability for buyers and such probability must be $(1 - N_l)/1$, where 1 is the total number, or measure, of buyers), $M_t^{ji} \to M_t^F$. Since neither N_l nor g_l affects M_t^{ji}, $G_t^{ji} \to M_t^F$ for all ji. At the limit, $M_t^F = Y_t^M = G_t^{ji}$ since $U_{t+1}^{ji} = 0$ (again, unless $E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,j}] - \lambda_t^{U,j} > 0$). Note that, under our log-normality assumption, $N_l = 0$, and hence $Q_t \to 0$ and $M_t^{b,ji} \to \infty$, which may sound strange. But, the above results still hold, because they approach to 0 and ∞ at balanced speeds; $Q_t M_t^{b,ji} = M_t^F$. Roughly speaking, this is the situation in which each buyer can buy an infinitesimally small number of varieties, say, one out of a million, and buy a huge amount of this single variety of goods. This does not cause any problem; since all goods are perfect substitute, buyers do not need to visit more than one market.

In sum, unless the cost of carry becomes negative, at the limit that $\theta \to \infty$, (i) there is no unsold goods $U_{t+1}^{ji} = 0$ and hence no inventory investment $U_{t+1}^{ji} - U_t^{ji} = 0$, (ii) sales equals output; $G_t^{ji} = Y_t^{M,ji} = S_t^{ji} = M_t^F$, (iii) buyers can achieve their purchasing index M_t^F without suffering from the cost of losing varieties, (iv) sellers choose the marginal cost pricing; $P_t^{M,ji} = \lambda_t^{MC} = 1$ (= final goods price), and (v) although stockout always takes place, stockout does not have any importance for both sellers and buyers. Since all other parts of the equilibrium are the same as the standard RBC model, this completes

the sketch of the proof. These results hold for a general class of distribution functions of the idiosyncratic shock.

A.3 Moment Matching

We implement our moment matching procedure as follows. First, let $\boldsymbol{\theta}$ be the vector of parameters to be pinned down. Then, depending on the actual value of $\boldsymbol{\theta}$, the model generates a given set of moments $m(\boldsymbol{\theta})$, where we explicitly write the moments as functions of parameters. In the stochastic simulation with a finite simulation period (142 quarters, which is the same as our data length), $m(\boldsymbol{\theta})$ has some distribution. Letting $H^{-1}(\boldsymbol{\theta})$ be the variance and covariance of $m(\boldsymbol{\theta})$ in this stochastic simulation, we use its inverse as weights to allocate the relative importance of the target moments. If, for example, hypothetically $H^{-1}(\boldsymbol{\theta})$ is diagonal (that is where there is no correlation among moments), then the weights are simply equivalent to precision of each estimate; i.e., we put a higher weight on a moment that is more precisely measured. Hence, given $H(\bar{\boldsymbol{\theta}})$, we choose $\boldsymbol{\theta}$ that minimizes the following quadratic form

$$\min_{\boldsymbol{\theta}} (m(\boldsymbol{\theta}) - m_{data})^T H(\bar{\boldsymbol{\theta}}) (m(\boldsymbol{\theta}) - m_{data}) \tag{26}$$

where m_{data} are the data moments. Note that, in the minimization problem, $H(\bar{\boldsymbol{\theta}})$ is fixed; otherwise, minimization would seek a high precision instead of a small distance between the data and the simulated moments. As $H(\bar{\boldsymbol{\theta}})$ takes a different values for each $\bar{\boldsymbol{\theta}}$, we need to employ a sequence of iterations to ensure $\boldsymbol{\theta} = \bar{\boldsymbol{\theta}}$. That is, once this minimization problem is solved for given $\bar{\boldsymbol{\theta}}$, we update $H(\bar{\boldsymbol{\theta}})$ by $H(\boldsymbol{\theta})$ and solve the minimization problem until $\boldsymbol{\theta} = \bar{\boldsymbol{\theta}}$.

A.4 Inventories as Options to Sell

This subsection compares the first order conditions with respect to unsold goods U_{t+1}^{ji} with the Black-Scholes option pricing formula. That is, we claim that having inventories is having options to sell. For comparison sake, we reproduce Jorgenson's user cost representation () of the FOC of U_{t+1}^{ji}.

$$E_t \left[\Lambda_{t,t+1} \left(P_{t+1}^{M,ji} - \lambda_{t+1}^{U,ji} \right) \Pr[M_{t+1}^{ji} > G_{t+1}^{ji}] \right] = \lambda_t^{U,ji} - E_t \left[\Lambda_{t,t+1} \lambda_{t+1}^{U,ji} \right].$$

Next, remember that Black-Scholes formula[45] of a call option can be rewritten as

$$E_t^{\mathbf{Q}} \left[e^{-r(T-t)} \left(S_T - K \right) \mathbf{1}(S_T > K) \right] = V_t^{call},$$

---

[45] For this representation, see equation (12.7) (and p.90 for notation) in Bjork (2004) among others.

where S_T is the price of underlying stock at expiration date T and K is the strike price. Also, indicator function $\mathbf{1}(S_T > K)$ is 1 if $S_T > K$ but is 0 otherwise, and $E_t^{\mathbf{Q}}[\cdot]$ is the expectation operator under the risk-neutral probability. This expression simply says that the cost of purchasing a call option V_t^{call} is equal to the present value of $S_T - K$ conditional that the call is in-the-money (i.e., $S_T > K$) under the risk-neutral measure with respect to S_T.

There are clear one-to-one relationships:[46] (i) cost of holding option: $\lambda_t^{U,ji} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,ji}]$ vs. V_t^{call}; (ii) discount factor: $\Lambda_{t,t+1}$ vs. $e^{-r(T-t)}$; (iii) profit margin: $P_{t+1}^{M,ji} - \lambda_{t+1}^{U,ji}$ if $G_{t+1}^{ji} > M_{t+1}^{ji}$ vs. $S_T - K$ if $S_T > K$. Note that, since intermediate firms are risk neutral by constant returns to scale, the difference between $E_t[\cdot]$ and $E_t^{\mathbf{Q}}[\cdot]$ does not really matter in this comparison. Finally, note that the option payoff is kinked on the maturity date but is differentiable (and hence an option delta exists) before the maturity date, because of the uncertainty in the stock price S_T, which is exactly parallel to the reason why we can differentiate the expected sales, but not sales itself.

A.5 Variance Decomposition of Output

We summarize the demand side equations as follows:

$$\text{law of motion for inventories:} \quad Y_t = (U_{t+1} - U_t) + S_t, \tag{27a}$$

$$\text{sales and final goods:} \quad S_t = Q_t M_t^b = Q_t^{\frac{-1}{\theta-1}} Y_t, \tag{27b}$$

$$\text{goods market clearing:} \quad Y_t = C_t + I_t. \tag{27c}$$

where we use (12b) and (12c) for (27a), (18) and (9) for (27b), and (2b) for (27c). Hence, noting that $Y_{ss}^M = S_{ss}$,

$$
\begin{aligned}
V\left[Y_t^M/Y_{ss}^M\right] &= V\left[\mathrm{d}U_{t+1}/Y_{ss}^M\right] + 2Cov\left[\mathrm{d}U_{t+1}/Y_{ss}^M, S_t/S_{ss}\right] + V\left[S_t/S_{ss}\right], &(28)\\
V\left[S_t/S_{ss}\right] &\simeq V\left[Y_t^F/Y_{ss}^F\right] - \frac{2}{\theta-1}Cov\left[Y_t^F/Y_{ss}^F, Q_t/Q_{ss}\right] + \left(\frac{-1}{\theta-1}\right)^2 V\left[Q_t/Q_{ss}\right], \\
&= \frac{C_{ss}^2}{S_{ss}^2}V\left[C_t/C_{ss}\right] + \frac{I_{ss}^2}{S_{ss}^2}V\left[I_t/I_{ss}\right] + 2\frac{C_{ss}I_{ss}}{S_{ss}^2}Cov\left[C_t/C_{ss}, I_t/I_{ss}\right] \\
&\quad - \frac{2}{\theta-1}Cov\left[Y_t^F/Y_{ss}^F, Q_t/Q_{ss}\right] + \left(\frac{-1}{\theta-1}\right)^2 V\left[Q_t/Q_{ss}\right]. &(29)
\end{aligned}
$$

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[46] In addition, the difference between $N(d_2)$ and $N(d_1)$ in the standard Black-Scholes formula (see any textbook for these notations) is almost exactly the same as the difference between π_t and $\tilde{\pi}_t$. This is not by chance; we can interpret $N(d_2)$ as the probability that $S_T > K$ under the risk-neutral measure, while it can be shown that $N(d_1) = E_t^Q[S_T/S_t \mid S_T > K]N(d_2)$; compare this with (16b).

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B Tables

Table 1: Peak-to-Trough Inventory Investment Change as a % of GDP Drop

Peak	Trough	A	B (= D/C)	C GDP Growth	D Contribution
1948.4	1949.4	210.8	204.3	-7.2	-14.8
1953.2	1954.2	65.5	66.4	-10.7	-7.1
1957.3	1958.2	98.2	98.3	-13.5	-13.3
1960.2	1961.1	161.1	160.1	-2.7	-4.2
1969.4	1970.4	592.3	585.3	-0.6	-3.8
1973.4	1975.1	69.3	70.4	-13.0	-9.1
1980.1	1980.3	69.9	70.6	-8.6	-6.1
1981.3	1982.4	98.9	98.9	-2.6	-2.6
1990.3	1991.1	46.3	46.3	-1.4	-0.6
2001.1	2001.4	-65.3	-64.1	3.0	-1.9
2007.4	2009.2	31.7	33.1	-15.6	-5.2

Notes: The peaks and troughs show NBER business cycle dates (as of January 2013).
$A = \frac{\text{change in inventory investment at trough} - \text{that at peak}}{\text{GDP at trough} - \text{GDP at peak}}$
$D = \frac{\text{change in inventory investment at trough} - \text{that at peak}}{\text{GDP at peak}}$, which is the contribution of inventory investment to GDP growth rate.

Table 2: Parameters

Symbol	Name	Benchmark
β	subjective discount factor	0.986
γ	relative risk aversion	1.000
ψ	relative weight for leisure	0.650
θ	elasticity of substitution of intermediate goods	7.500
α	capital share in production	0.350
δ	depreciation rate of capital	0.015
σ_N	size parameter of the idio shock*	1.725
ρ_C	AR(1) coef of preference shock*	0.944
ρ_M	AR(1) coef of productivity shock*	0.876
σ_C	sd of innov to preference shock*	0.011
σ_M	sd of innov to productivity shock*	0.005

Note: Items with * (ρ_M, ρ_U, σ_C/σ_M and σ_N) are estimated by the moment matching to inventory related second moments. The other parameters are determined so that the steady state values match to the data. Note that $\eta = 1$ means that labour supply is determined before observing all aggregate shocks.

Table 3: Steady State Values

	Name	No Invent. ($\sigma_N = 0.00$)	Low Invent. ($\sigma_N = 0.40$)	Benchmark ($\sigma_N = 1.73$)
R^B	gross return on bonds	1.015	1.015	1.015
R^K	net return on capital	1.030	1.030	1.030
W	wage	1.970	1.928	1.611
P^M	price index of intermediate goods	1.000	0.996	0.931
Q	measure of available variety	1.000	0.972	0.629
π	Pr[Stockout] for sellers	-	0.081	0.061
$\tilde{\pi}$	cost of losing sales opportunity	-	0.158	0.570
λ^U	shadow price of unsold goods	-	0.842	0.750
λ^{MC}	marginal cost of intermediate production	-	0.855	0.761
C	consumption	1.061	1.038	0.868
H	hours	0.358	0.354	0.334
Y^F	output of final goods	1.253	1.224	1.014
Y^M	output of intermediate goods	1.253	1.229	1.089
I	capital investment	0.192	0.186	0.147
S	sales of intermediate goods	1.253	1.229	1.089
U	unsold goods	0.000	0.816	4.578
K	capital	12.804	12.392	9.774

Note: See the footnote on Table 2.

Table 4: Moment Matching

	Data US	Matched Param & Mom	low 5%	up 95%	Dem Shk Only	Sup Shk Only
- Estimated Parameters						
ρ_C	$-$	0.944	$-$	$-$	0.944	$-$
ρ_M	$-$	0.876	$-$	$-$	$-$	0.876
σ_M/σ_C	$-$	0.504	$-$	$-$	0.000	∞
σ_N	$-$	1.725	$-$	$-$	1.725	1.725
- Target Moments						
$cor\left\{\frac{dU}{Y}, Y\right\}$ *	0.495	0.717	0.666	0.765	0.912	0.482
$cor\left\{\frac{dU}{Y}, S\right\}$ *	-0.425	-0.690	-0.757	-0.611	-0.846	-0.619
$cor\left\{W, Y\right\}$	-0.155	-0.025	-0.344	0.306	-0.998	0.810
$cor\left\{\frac{U}{S}, Y\right\}$	-0.516	-0.911	-0.938	-0.876	-0.965	-0.910
$sd(C)/sd(Y)$	0.565	0.623	0.468	0.809	1.347	0.164
$sd(S)/sd(Y)$	0.831	0.685	0.640	0.734	0.907	0.616
$cor\left\{\frac{U_t}{S_t}, \frac{U_{t-1}}{S_{t-1}}\right\}$	0.881	0.923	0.898	0.942	0.927	0.920

Note: * indicates that the high band-pass filter of 2-4 quarters is applied and, for the others, the business cycle band-pass filter of 8-40 quarters is applied.

Table 5: Key Second Moments

| | US data | | Benchmark ($\sigma_N = 1.73$) | | | | | |
| | | | Both | | Dem Shock | | Sup Shock | |
	rel sd	corr	rel sd	corr	rel sd	corr	rel sd	corr
- Band pass filter with 8 to 40 quarters								
Y^M	1.18	1.00	1.30	1.00	0.58	1.00	1.16	1.00
S	0.83	0.96	0.68	0.97	0.90	0.96	0.62	0.99
H	1.08	0.84	1.01	0.94	1.57	1.00	0.81	0.99
C	0.56	0.86	0.63	0.54	1.35	0.93	0.16	0.67
I	2.99	0.96	3.25	0.71	2.35	-0.74	3.41	0.99
$\frac{U}{S}$	1.35	-0.52	0.77	-0.91	1.03	-0.96	0.69	-0.91
$\frac{dU_{t+1}}{Y^M}$	0.23	0.64	0.37	0.90	0.27	0.47	0.40	0.98
Q	-	-	0.27	-0.88	0.38	-0.95	0.23	-0.89
π	-	-	0.96	0.88	1.39	0.95	0.83	0.89
$\tilde{\pi}$	-	-	0.34	0.88	0.48	0.95	0.29	0.89
$\frac{P^M}{\lambda^U}$	-	-	0.04	0.88	0.06	0.95	0.04	0.89
$\frac{P^M}{\lambda^{MC}}$	-	-	0.03	0.88	0.04	0.95	0.02	0.89
$\tilde{\theta}$	-	-	0.18	-0.88	0.26	-0.95	0.15	-0.89
W	0.61	-0.15	0.35	-0.03	0.67	-1.00	0.19	0.81
$\frac{Y^M}{H}$	0.59	0.16	0.34	0.14	0.58	-1.00	0.23	0.87
$cor\left\{\frac{U_t}{S_t}, \frac{U_{t-1}}{S_{t-1}}\right\}$	-	0.88	-	0.92	-	0.92	-	0.92
$cor\left\{\frac{U_t}{S_t}, \frac{U_{t-2}}{S_{t-2}}\right\}$	-	0.61	-	0.70	-	0.72	-	0.70
- Band pass filter with 2 to 4 quarters								
$cor\left\{\frac{dU}{Y}, Y\right\}$	-	0.49	-	0.71	-	0.91	-	0.48
$cor\left\{\frac{dU}{Y}, S\right\}$	-	-0.42	-	-0.68	-	-0.85	-	-0.62

Notes: The rel sd and corr are standard deviation relative to that of output (total intermediate production) and correlation with GDP, respectively, except for rel sd of Y^M which shows the sd of intermediate production. The main sources of US data are US NIPA and current employment statistics from 1975Q1 to 2010Q3. R^B is effective Fed Funds rate (FRED2), and P^M is PPI for final goods. Note that U/S is taken from M3, US Census Bureau, while the inventory investment in dU/Y^M is from NIPA. The band-pass filter (8 to 40 quarters) is applied with a maximum lag length of K=12.

Table 6: Key Second Moments for Different Idiosyncratic Shock

| | No-Inventories ($\sigma_N = 0.00$) | | | | | | Low Inventories ($\sigma_N = 0.40$) | | | | | |
| | Both | | Dem Shock | | Sup Shock | | Both | | Dem Shock | | Sup Shock | |
	rel sd	corr	rel sd	corr	rel sd	corr	rel sd	corr	rel sd	corr	rel sd	corr
- Band pass filter with 8 to 40 quarters												
Y^M	1.30	1.00	0.63	1.00	1.14	1.00	1.29	1.00	0.62	1.00	1.13	1.00
S	1.00	1.00	1.00	1.00	1.00	1.00	0.91	1.00	0.92	1.00	0.91	1.00
H	1.04	0.94	1.60	1.00	0.80	0.99	1.04	0.94	1.60	1.00	0.79	0.99
C	0.65	0.63	1.26	0.92	0.22	0.79	0.65	0.62	1.28	0.93	0.21	0.78
I	5.10	0.84	2.69	0.00	5.61	0.99	4.62	0.80	2.73	-0.23	5.02	0.99
$\frac{U}{S}$	-	-	-	-	-	-	0.89	-0.90	0.98	-0.95	0.86	-0.89
$\frac{dU_{t+1}}{Y^M}$	-	-	-	-	-	-	0.11	0.84	0.10	0.79	0.11	0.86
Q	-	-	-	-	-	-	0.05	-0.90	0.05	-0.94	0.04	-0.89
π	-	-	-	-	-	-	1.57	0.90	1.85	0.94	1.48	0.89
$\tilde{\pi}$	-	-	-	-	-	-	1.29	0.90	1.52	0.94	1.22	0.89
$\frac{P^M}{\lambda^U}$	-	-	-	-	-	-	0.04	0.90	0.04	0.94	0.03	0.89
$\frac{P^M}{\lambda^{MC}}$	-	-	-	-	-	-	0.01	0.90	0.01	0.94	0.01	0.89
$\tilde{\theta}$	-	-	-	-	-	-	0.20	-0.90	0.23	-0.94	0.19	-0.89
W	0.37	0.07	0.60	-1.00	0.25	0.87	0.37	0.04	0.61	-1.00	0.24	0.86
$\frac{Y^M}{H}$	0.37	0.07	0.60	-1.00	0.25	0.87	0.37	0.09	0.60	-1.00	0.25	0.88
$cor\left\{\frac{U_t}{S_t}, \frac{U_{t-1}}{S_{t-1}}\right\}$	-	-	-	-	-	-	-	0.91	-	0.92	-	0.91
$cor\left\{\frac{U_t}{S_t}, \frac{U_{t-2}}{S_{t-2}}\right\}$	-	-	-	-	-	-	-	0.67	-	0.69	-	0.67
- Band pass filter with 2 to 4 quarters												
$cor\left\{\frac{dU}{Y}, Y\right\}$	-	-	-	-	-	-	-	0.76	-	0.97	-	0.50
$cor\left\{\frac{dU}{Y}, S\right\}$	-	-	-	-	-	-	-	-0.12	-	0.27	-	-0.31

Note: See the notes on 5.

Table 7: Decomposition of Output Variance: US data

		US Data					
		1975Q1-2010Q4	%	**1984Q1-2010Q4**	%	**1975Q1-1983Q4**	%
$Var\left[\hat{Y}_t^M\right]$	A	2.12	100.0	0.83	100.0	4.84	100.0
$Var\left[\frac{dU_{t+1}}{Y_{ss}^M}\right]$	B	0.09	4.5	0.07	8.3	0.20	4.0
$2Cov\left[\frac{dU_{t+1}}{Y_{ss}^M}, \hat{S}_t\right]$	C	0.36	16.9	0.09	10.9	0.90	18.6
$Var\left[\hat{S}_t\right]$	D	1.45	68.6	0.62	74.9	3.19	65.9
SS adjustment	E	0.21	10.1	0.05	5.9	0.56	11.4
$\frac{C_{ss}^2}{(Y_{ss}^F)^2}Var\left[\hat{C}_t\right]$	F	0.23	16.0	0.12	18.6	0.41	12.8
$\frac{I_{ss}^2}{(Y_{ss}^F)^2}Var\left[\hat{I}_t\right]$	G	1.22	84.2	0.60	95.5	2.09	65.3
$\frac{2C_{ss}I_{ss}}{(Y_{ss}^F)^2}Cov\left[\hat{I}_t, \hat{C}_t\right]$	H	0.96	66.1	0.47	74.7	1.68	52.5
Other terms	I	-0.96	-66.3	-0.55	-88.7	-0.98	-30.6

Notes: This table shows the contribution of each demand component to output variance at business cycle frequencies (8 to 40 quarters) and for sales, A=B+C+D+E and D=F+G+H+I; see equations (28) and (29). The SS adjustment (E) shows the adjustment for the trend changes (lack of the steady state) in dU/Y^M over time in the data, which is eliminated by the band-pass filter. Other terms (I) include terms related to Q_t for the model simulations and terms related to net exports and government expenditure for the US data. Finally note that durable consumption is included in capital investment I_t, rather than consumption C_t.

Table 8: Decomposition of Output Variance: Different Idio Shock

Type of Shocks:		No Inventories ($\sigma_N = 0.00$)						Low Inventories ($\sigma_N = 0.40$)						Benchmark ($\sigma_N = 1.73$)					
		Both	%	Demand	%	Supply	%	Both	%	Demand	%	Supply	%	Both	%	Demand	%	Supply	%
$Var\left[\hat{Y}_t^M\right]$	A	1.73	100.0	0.40	100.0	1.33	100.0	1.68	100.0	0.39	100	1.30	100	1.72	100.0	0.35	100.0	1.38	100.0
$Var\left[\frac{dU_{t+1}}{Y_{ss}^M}\right]$	B	0.00	0.0	0.00	0.0	0.00	0.0	0.02	1.2	0.00	1.0	0.02	1.3	0.24	13.9	0.02	7.1	0.22	15.7
$2Cov\left[\frac{dU_{t+1}}{Y_{ss}^M}, \hat{S}_t\right]$	C	0.00	0.0	0.00	0.0	0.00	0.0	0.27	16.0	0.05	13.8	0.22	16.7	0.68	39.3	0.04	10.9	0.64	46.5
$Var\left[\hat{S}_t\right]$	D	1.73	100	0.40	100.0	1.33	100.0	1.39	82.8	0.33	85.2	1.07	82.0	0.80	46.8	0.28	81.9	0.52	37.9
SS adjustment	E	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
$\frac{C_{ss}^2}{(Y_{ss}^F)^2} Var\left[\hat{C}_t\right]$	F	0.51	29.4	0.46	114.0	0.05	3.4	0.50	36.0	0.46	138.3	0.04	3.9	0.49	60.7	0.46	161.8	0.03	5.1
$\frac{I_{ss}^2}{(Y_{ss}^F)^2} Var\left[\hat{I}_t\right]$	G	1.05	60.7	0.07	16.8	0.99	74.0	0.82	58.9	0.07	20.0	0.76	71.1	0.37	46.4	0.04	14.1	0.33	64.0
$\frac{2C_{ss}I_{ss}}{(Y_{ss}^F)^2} Cov\left[\hat{I}_t, \hat{C}_t\right]$	H	0.17	9.9	-0.12	-30.8	0.30	22.5	0.05	3.7	-0.20	-60.0	0.25	23.8	-0.14	-17.7	-0.25	-88.3	0.11	21.3
Other terms	I	0.00	0.0	0.00	0.0	0.00	0.0	0.02	1.3	0.00	1.7	0.01	1.2	0.08	10.6	0.04	12.5	0.05	9.5
U_{ss}/S_{ss}		0.000						0.663						4.202					

Note: See notes on Table 7.

Table 9: Decomposition of Output Variance: Different Information

Share of unobservable component in the aggregate shocks at current period that labor supply can react:

Type of Shocks:		η = 0% (full info)						η = 0% (mid case)						η = 100% (benchmark)					
		Both	%	Demand	%	Supply	%	Both	%	Demand	%	Supply	%	Both	%	Demand	%	Supply	%
$Var\left[\hat{Y}_t^M\right]$	A	1.96	100.0	0.37	100.0	1.59	100.0	1.85	100.0	0.36	100.0	1.49	100.0	1.72	100.0	0.35	100.0	1.38	100.0
$Var\left[\frac{dU_{t+1}}{Y_{ss}^M}\right]$	B	0.23	12.0	0.00	0.5	0.23	14.6	0.24	12.9	0.01	3.7	0.23	15.1	0.24	13.9	0.02	7.1	0.22	15.7
$2Cov\left[\frac{dU_{t+1}}{Y_{ss}^M}, \hat{S}_t\right]$	C	0.72	36.7	0.01	2.8	0.71	44.6	0.70	37.9	0.02	6.9	0.68	45.5	0.68	39.3	0.04	10.9	0.64	46.5
$Var\left[\hat{S}_t\right]$	D	1.00	51.3	0.36	96.7	0.65	40.8	0.91	49.2	0.32	89.4	0.59	39.4	0.80	46.8	0.28	81.9	0.52	37.9
SS adjustment	E	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
$\frac{C_{ss}^2}{(Y_{ss}^F)^2}Var\left[\hat{C}_t\right]$	F	0.52	51.7	0.49	135.8	0.03	5.2	0.51	55.8	0.48	147.3	0.03	5.1	0.49	60.7	0.46	161.8	0.03	5.1
$\frac{I_{ss}^2}{(Y_{ss}^F)^2}Var\left[\hat{I}_t\right]$	G	0.44	43.6	0.02	5.4	0.42	64.3	0.41	44.7	0.03	9.2	0.38	64.2	0.37	46.4	0.04	14.1	0.33	64.0
$\frac{2C_{ss}I_{ss}}{(Y_{ss}^F)^2}Cov\left[\hat{I}_t, \hat{C}_t\right]$	H	-0.04	-4.4	-0.19	-51.7	0.14	22.2	-0.09	-10.2	-0.22	-67.9	0.13	21.8	-0.14	-17.7	-0.25	-88.3	0.11	21.3
Other terms	I	0.09	9.1	0.04	10.6	0.05	8.3	0.09	9.7	0.04	11.4	0.05	8.8	0.08	10.6	0.04	12.5	0.05	9.5
U_{ss}/S_{ss}		4.202		4.202				4.202		4.202				4.202		4.202			

Notes: See the notes on Table 7.and U_{ss}/S_{ss} (I/S ratio in the steady state) is invariant against the change in the information assumptions.

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Table 10: Persistence under of iid Aggregate Shocks (band pass filtered for 2 to 40 quarters)

	US data	No-Inventory Case (σ_N=0.0)						Benchmark Case (σ_N=1.73)					
		Both Shocks		Demand Shock		Supply Shock		Both Shocks		Demand Shock		Supply Shock	
		persist	iid	persist	iid	persist	iid	persist	iid	persist	iid	persist	iid
$Cor(Y_t^M, Y_{t-1}^M)$	0.88	0.70	-0.06	0.72	-0.05	0.68	-0.06	0.67	-0.08	0.72	0.44	0.65	-0.08
$Cor(Y_t^M, Y_{t-2}^M)$	0.72	0.45	-0.06	0.47	-0.06	0.41	-0.06	0.40	-0.07	0.48	0.26	0.38	-0.07
$Cor(S_t, S_{t-1})$	0.87	0.70	-0.06	0.72	-0.05	0.68	-0.06	0.72	-0.06	0.71	-0.06	0.77	0.68
$Cor(S_t, S_{t-2})$	0.75	0.45	-0.07	0.47	-0.06	0.41	-0.06	0.48	-0.07	0.47	-0.07	0.55	0.43
$Cor(\frac{U_t}{S_t}, \frac{U_{t-1}}{S_{t-1}})$	0.79	0.52	-0.07	0.49	0.03	0.52	-0.10	0.77	0.15	0.72	0.05	0.85	0.58
$Cor(\frac{U_t}{S_t}, \frac{U_{t-2}}{S_{t-2}})$	0.51	0.32	-0.06	0.31	-0.07	0.32	-0.06	0.55	0.07	0.48	0.01	0.67	0.36
$sd(S_t)/sd(Y_t^M)$	0.85	1.00	1.00	1.00	1.00	1.00	1.00	0.58	0.58	1.08	8.04	0.24	0.05
$Cor(Y_t^M, S_t)$	0.94	1.00	1.00	1.00	1.00	1.00	1.00	0.73	0.08	1.00	0.73	0.86	0.44

Notes: Label "persist" means $\rho_M = 0.876$ and $\rho_C = 0.944$ as in Table 2. In this experiment, capital stock is fixed at its steady state level and investment is equal to capital depreciation. Also, the band pass filter picks up the fluctuations at 2 to 40 quarters. Hence, some autocorrelations are not the same as in Table 5.
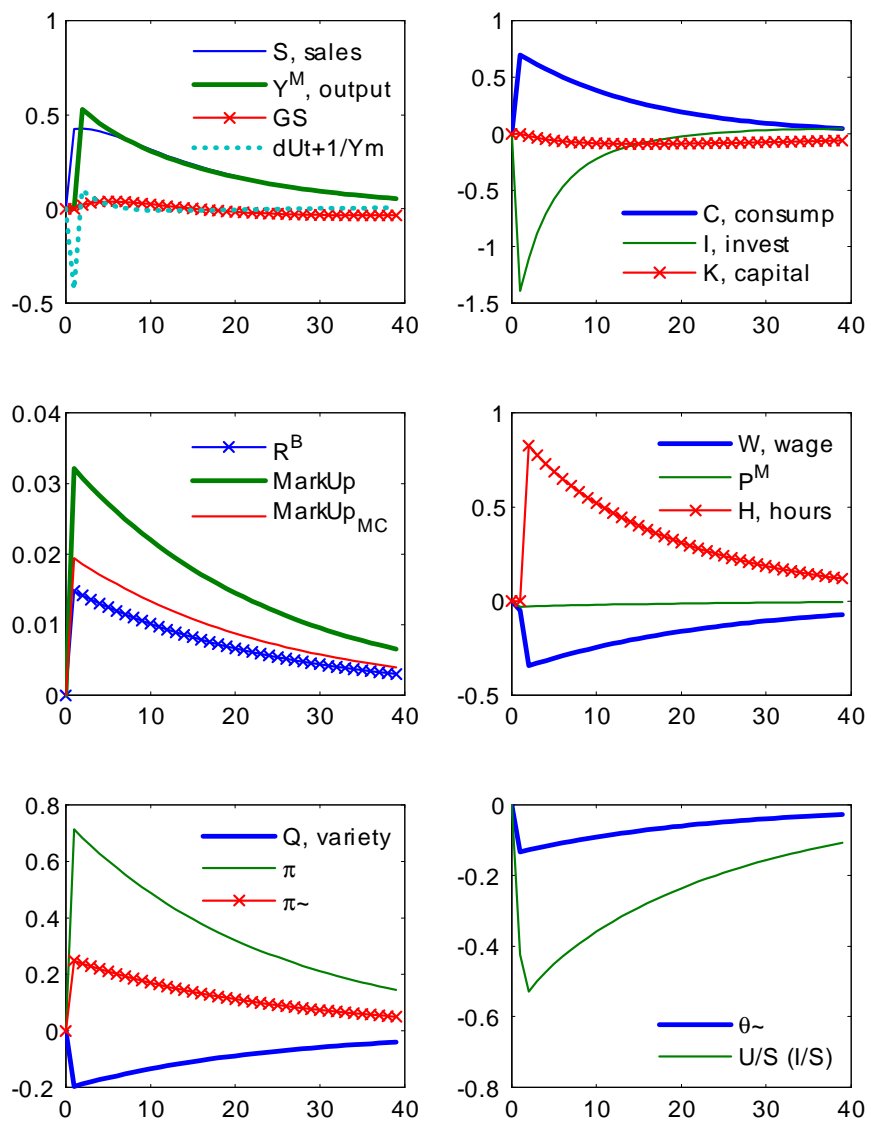
C Figures

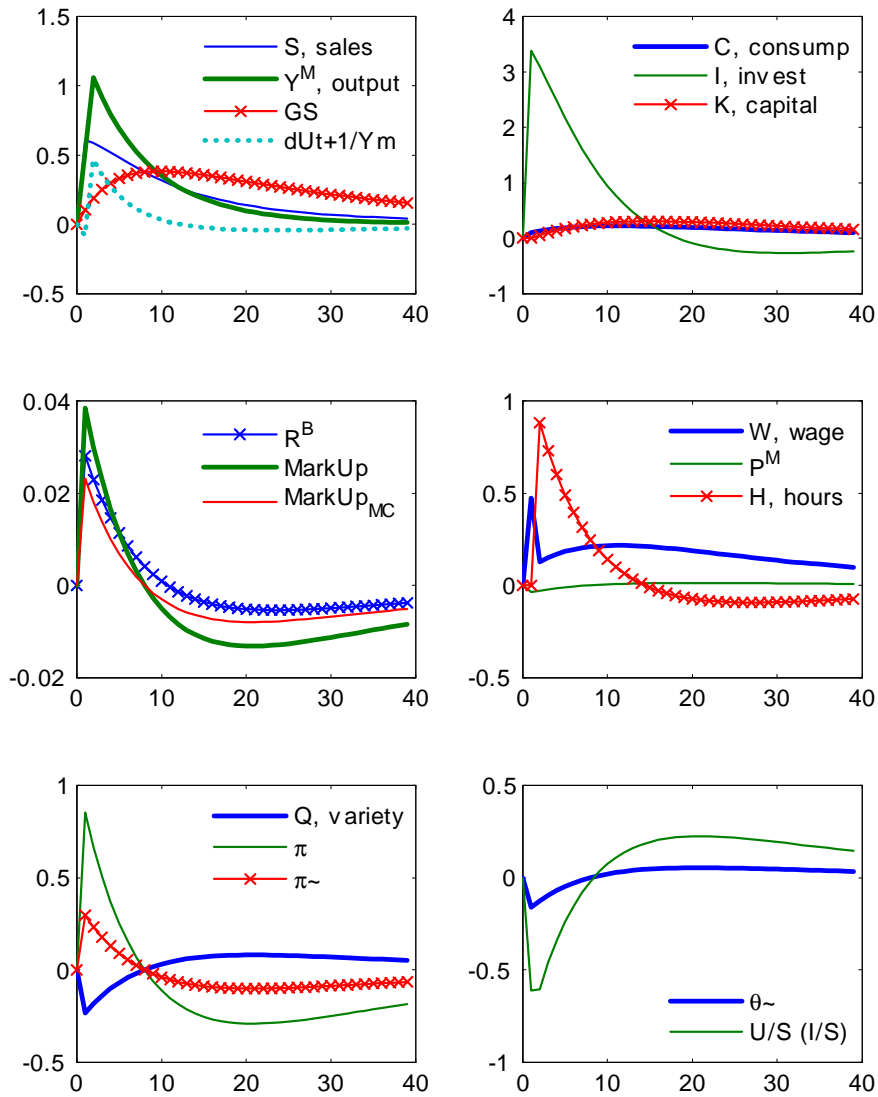Figure 1: Selected impulse response functions to 1% positive preference shock.

Figure 2: Selected impulse response functions to 1% positive supply shock.