Approximation and visualization of experimental data with B-spline surfaces

Vassilev, T.I.; Smrikarov, A. S.

765

Approximation and visualization of experimental data with B-spline surfaces

T. I. Vassilev

Dept. of Computer Science, Queen Mary and Westfield College, Mile End road, London E1 4NS, UK

A. S. Smrikarov

Dept. of Computer Systems, Technical University, 8 Studentska St, 7017 Rousse, Bulgaria

Abstract

This paper describes a method for constructing a fair B-spline surface that approximates a data set of a non-rectangular topology. The mathematical model $z = f(x, y)$ is visualized by a map showing lines of equal z.

Keywords: B-spline Surface, Fair Approximation, Bezier Patch.

1 Introduction

The performance of many industrial objects is evaluated by a function of two variables $z = f(x, y)$. Very often it has to be modeled from a noisy data set of a semi-rectangular topology. This means that the data points are organized in m rows but the number of points in every row is different. Jahns et al.[1] suggested a two dimensional polynomial for the approximation of such a data set. Although this choice may serve reasonably well in lots of cases, it is definitely not appropriate for the case shown in Fig. 1 where the surface has three local minimums. It means that the degree of the polynomial should be at least six which will, as proved by de Boor[2], inevitably lead to unwanted oscillations and failure to preserve the shape of the data. A much better choice is a piecewise polynomial function[2].

The objective of this paper is to develop a general method for modeling and visualizing a function of two variables from a noisy semi-rectangular data set where direct B-spline approximation cannot be applied. According to the various standards the surface has to be shown as a map of lines with equal z (see Fig. 1).

Preprint submitted to Elsevier Science *5 September 1995*

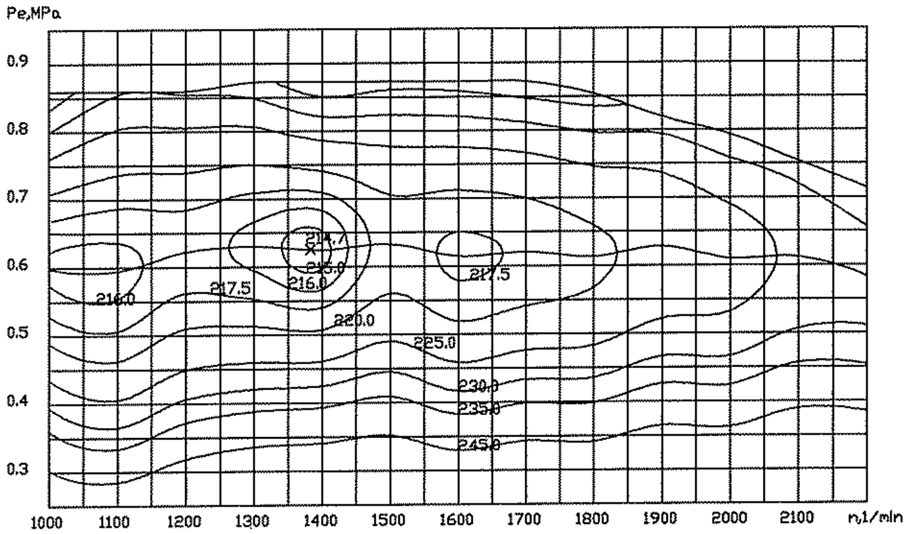

Fig. 1. Diesel Engine Performance Map

2 B-spline approximation

The equation of a tensor product B-spline surface is

$$\mathbf{w}(u,v) = \sum_i \sum_j B_i(u) B_j(v) \mathbf{v}_{ij}, \tag{1}$$

where u and v are parameters, $B_i(u)$ and $B_j(v)$ are the B-splines blending functions and $\mathbf{v}_{ij}$ are the surface control points. Let $\mathbf{D}$ be a $(m \times n)$ matrix containing 3D points. The problem of interpolation of this kind of data set with a B-spline surface is described by Farin[3]. Its solution leads to a system of the type $\mathbf{AVB} = \mathbf{D}$ which can be solved in two steps:

(i) Solve $\mathbf{AY} = \mathbf{D}$ which is applying a number of curve interpolations to the initial data points.

(ii) Solve $\mathbf{VB} = \mathbf{Y}$ which is applying a number of curve interpolations to the control points, computed in the previous step.

The points in a semi-rectangular data set are organized in m rows but the number of points in each row is n_i, $i = 1, \ldots, m$. Let k be the order of the B-splines and let l_u and l_v be the numbers of the polynomial segments in direction of parameters u and v respectively. Then the approximation of a semi-rectangular data set with a B-spline surface can be done as follows:

(i) Perform m least squares B-spline curve approximations[2] to the m rows of data points with l_v polynomial segments for each one. Although the curves are with a different number of data points after this step each of them will have the same number of control points $(l_v + k)$.

(ii) Perform $(l_v + k)$ curve approximations to the control points obtained in the previous step with l_u polynomial segments for each one. After this step we will have the matrix $\mathbf{V}$ of $(l_u + k) \times (l_v + k)$ control points.

Note that the method of Jahns et al.[1] is only a special case of the above described technique with $l_u = l_v = 1$.

In many practical cases the measurement of data points in all intervals is not possible for physical reasons, e.g. unstable work of the tested object. This lack of data results in an approximating surface with unwanted bumps and wiggles. In order to obtain a fair surface we apply the approach of Vassilev[4]. Its main idea is to insert additional data points that minimize the following integral

$$E = \iint\limits_{surface} [c_1 \mathbf{w}_{uv}^2 + c_2 \mathbf{w}_{u^2v}^2 + c_3 \mathbf{w}_{uv^2}^2 + c_4 \mathbf{w}_{u^2v^2}^2] \, du \, dv, \tag{2}$$

where the suffixes mean partial derivatives. If the data set is as shown in Fig. 1 the approximation can also be applied with a non-parametric surface. Then we will have a mathematical model $z = f(x, y)$ of the object that can be used for predicting one of the values x, y, z if given the other two.

3 Visualization as a map of equal z lines

The problem here is to find the intersection of the surface $z = f(x, y)$ with a plane $z = a$. One can use any of the numerical methods for solving equations[5] but due to the properties of a Bezier patch, this can be done in a much more efficient way. Farin[3] describes a method for computing the intersection of a Bezier curve with a line $x = b$ using a recursive subdivision technique. This can be easily extended to Bezier patches. Due to the convex hull property of the Bezier curves and patches we can check if the plane $z = a$ intersects the patch or not. Evaluating a point on a Bezier patch results in four adjacent sub-patches. Applying the procedure recursively we can find (with a given accuracy) all the points that occur on the intersection of the plane with the patch. Since a B-spline surface is a set of Bezier patches one can compute the intersection of the plane with the B-spline surface.

4 Results

The results of applying the algorithms are shown in Fig. 1 and 2. Fig. 1 illustrates the visualization of the surface as a map using the recursive subdivision approach.

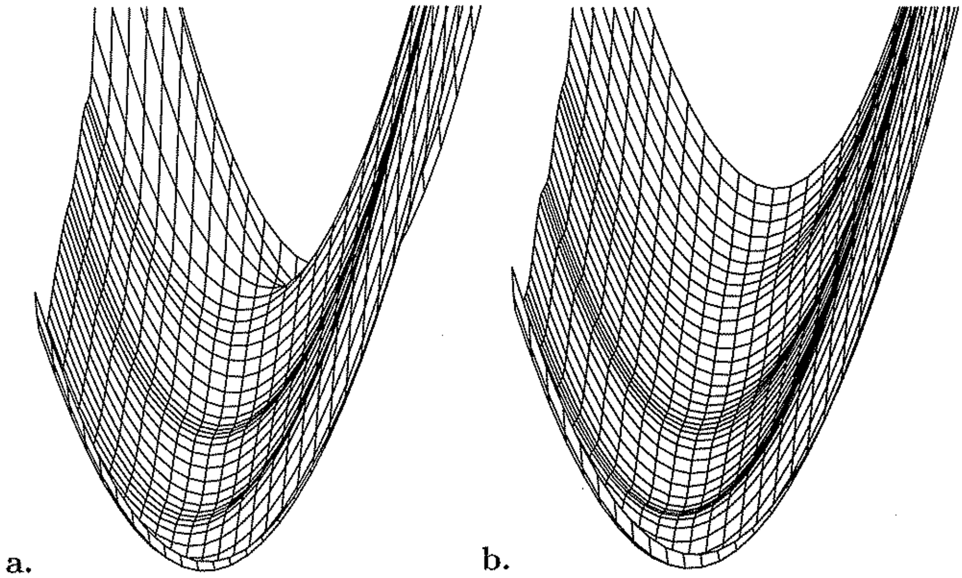

Fig. 2. B-spline surface: **a.** without fairing; **b.** with fairing

Fig. 2 shows the effect of the fairing. The unwanted undulation in Fig. 2a is removed by the fairing (Fig. 2b) inserting several new data points.

References

[1] Jahns, G., Förster K. and Claar II P., Modeling Diesel Engine Performance, *SAE Technical paper series*, SAE 871615, 1987.

[2] de Boor, C., *A Practical Guide to Splines*, Springer-Verlag, New York, 1978.

[3] Farin, G., *Curves and Surfaces for Computer Aided Geometric Design*, Academic Press, Inc. 1993.

[4] Vassilev, T., A Method for Fair B-spline Interpolation and Approximation by Insertion of Additional Data Points, Submitted for publication.

[5] Press, W. H., Flannery, B. P., Teukolsky, S. A. and Vetterling W. T., *Numerical Recipes, The Art of Scientific Computations*, Cambridge University Press, New York, 1989.