K-RACi

Brandano, Sergio

Department of Computer Science

Technical Report No. 763

QUEEN MARY
AND WESTFIELD COLLEGE
UNIVERSITY OF LONDON

ISSN 1369-1961

K-RACi

Sergio Brandano

May 1999

In *Nonmonotonic Reasoning, Action, and Change.* Workshop at IJCAI-99, Stockholm, Sweden, August 1999.

$\mathcal{K}$-RACi

Sergio Brandano

Queen Mary and Westfield College
London, U.K.

Abstract

This paper formally defines the class of all problems of reasoning about actions and change where accurate and complete information about actions, together with strict inertia in continuous time, continuous change and alternative results of possibly concurrent and independent actions are the assumed properties. The intended model set for each member in the class is defined in terms of a model-theoretic trajectory semantics. The case is designated, in the *Features and Fluents* framework, with the $\mathcal{K}$-**RACi** family of reasoning problems.

1 Introduction

The analysis of the reality using models, even intended ones, admits a gap among the reality and models: the solution of a problem always is, as a matter of fact, the solution of the representation we built of the real problem. The model will always be a very limited description of the reality, however it must represent with reasonable accuracy those aspects of interest for the solution of the decisional problem with which we are dealing. Such analysis is then affected by one intrinsic methodological difficulty, namely the provability of the soundness and completeness relations for a given representation. The soundness refers to the accuracy of the representation, namely whether the real problem has been successfully represented within the chosen formal language. The completeness refers instead to the reproducibility of the real problem from the given formal problem, as well as to the recognizability whether the given formal problem is a real problem at all. The search for non-monotonic temporal logics, within Cognitive Robotics, has been affected by such difficulty from its very first result, which appeared in [Sandewall, 1972], up to the present day developments. An additional difficulty consisted in admitting several formal languages when dealing with problems that are not necessarily language-independent. The attempt to extend the language expressiveness of those logics, so to accurately represent the knowledge that is involved in Cognitive Robotics, was first made by Sandewall [1989a; 1989b], who proposed to combine non-monotonic temporal logic with differential equations and triggered the grand goal of developing a coherent theory for temporal reasoning, knowledge based planning and qualitative reasoning. Nowadays, as the result of such investigation, the *Features and Fluents* meta-theory of actions and change [Sandewall, 1994] poses itself between the reality and the models, by providing an explicit and formal account of (1) what are the epistemological and ontological assumptions we implicitly involved when representing problems, and (2) what is the relation between the semantics of each logic and the underlying semantics that formally defines the chosen assumptions. The underlying semantics consists of an ego-world game semantics that captures our intuitions or simplifying assumptions. The meta-theory provides a taxonomy of those assumptions, and logics for reasoning about actions and change can be gathered in congruence classes modulus the particular assumptions they satisfy, in determining their *range of applicability.* Of course it is not possible to prove that the underlying semantics captures our intuitions correctly, or that the simplifying assumptions are appropriate, however the difficulty of proving the main soundness and completeness relations for a given representation is delayed by one non trivial step, where the additional difficulty of admitting several formal languages finds an adequate and elegant solution. The underlying semantics is worthwhile iff it is considerably more transparent and intuitive than the logics themselves and if it can be related in a mathematically precise way to both the physical description of the world in which the robot is operating and to the physical characteristics of the robot itself, including its sensory and actuatory systems.

The underlying semantics that formally defines the broadest class in the Features and Fluents taxonomy, defines the knowledge representation method and the intended model set for each reasoning problem where accurate and complete information about actions ($\mathcal{K}$), together with strict inertia in *integer time* and alternative results of actions (**IA**) are the assumed simplifying assumptions. In order to make an explicit distinction between the reality and the models, two levels of description for the world were assumed: the *material level* and the *image level.* The material-level consists in the description of the physical system, or can be even identified with it, and uses continuous time and continuous-valued state variables that change according to the laws of physics. The image-level consists in the abstract representation of the material-level and models the way the world is perceived by the agent; it is obtained from the material-level by means of a perception (abstraction) function, and uses discrete-valued state variables.

In this paper we extended the image-level to the case of continuous-valued state variables. The assumed simplifying assumptions are: accurate and complete information about actions ($\mathcal{K}$), strict inertia in *continuous time*, *continuous change* and alternative results of possibly concurrent and independent actions (**RACi**). In the genuine style of the underlying semantics for $\mathcal{K}$-**IA**, the set of all the intended answers for each reasoning problem in $\mathcal{K}$-**RACi** is built simulating infinite game developments between a trajectory-semantics ego and the trajectory-semantics world. The new underlying semantics subsumes the underlying semantics for $\mathcal{K}$-**IA** and admits a trajectory-semantics world with explicit continuous time and a combination of encapsulated and/or non-encapsulated actions with discrete and/or continuous change, depending on the need.

2 Common-sense Reasoning

Common-sense reasoning is understood as a board game between an ego and the image-level world.

The board consists of the five-tuple $\langle \mathcal{B}, M, H, \mathcal{A}, \mathcal{C} \rangle$, named finite development. The component $\mathcal{B}$ is a subset of the temporal domain $\mathcal{T}$, which in turn is the set of all natural numbers; the smallest and the largest elements in $\mathcal{B}$ are respectively 0 (also "origo") and $n_{\mathcal{B}}$ (the point in time expressing the current "now"). M is a mapping which assigns values to temporal and object constants. H is the most important element of the board; it is a *history* of the image-level world from the origo of time-points up to $n_{\mathcal{B}}$, defined as the mapping from $\mathcal{B}$ to a set of states. A state is a function that, given a point in time, maps features to corresponding values. $\mathcal{A}$ is the *past-action set*, a set of tuples $\langle s, t, E \rangle$, $s, t \in \mathcal{B}$, $s < t < n_{\mathcal{B}}$, where s and t represent respectively the start time and the end time of the action E. Finally, $\mathcal{C}$ is the *current-action set*, a set of tuples $\langle s, E \rangle$ where $s \leq n_{\mathcal{B}}$ is the start time of the action E; $\mathcal{C}$ is the set of those actions which have been started but not yet terminated at time $n_{\mathcal{B}}$.

If J and J' are two finite developments, J' is said to be a correct revision of J iff the following conditions hold. $\mathcal{B} \subseteq \mathcal{B}'$ and if $b \in \mathcal{B}' - \mathcal{B}$ then $b > n_{\mathcal{B}}$. $M \subseteq M'$. The restriction of H' to $[0, n_{\mathcal{B}}]$ is equal to H. $\mathcal{A} \subseteq \mathcal{A}'$ and if $\langle s, t, E \rangle \in \mathcal{A}' - \mathcal{A}$ then $t = n_{\mathcal{B}'}$ and $\langle s, E \rangle \in \mathcal{C} - \mathcal{C}'$. Finally, if $\langle s, E \rangle \in \mathcal{C} - \mathcal{C}'$ then either $s = n_{\mathcal{B}}$ or $\langle s, n_{\mathcal{B}'}, E \rangle \in \mathcal{A}'$, and if $\langle s, E \rangle \in \mathcal{C}' - \mathcal{C}$ then is $s = n_{\mathcal{B}}$.

Each correct revision reports an update of the history, and the above condition on H represents the solution to the frame problem. The history is updated as follows. If H is an history over $\mathcal{B} = [0, t]$ and $h = \langle \sigma'_1, \sigma'_2, \ldots, \sigma'_k \rangle$ is a trajectory of E, then $H \rhd h = H'$ is the updated (extended) history over $\mathcal{B}' = [0, t + k]$ such that:

$$H'(s) = \begin{cases} H(t) \oplus \sigma'_i & \text{if } s = t + i > t \\ H(s) & \text{if } s \leq t \end{cases}$$

The operator $\oplus: \mathcal{S} \times \mathcal{S} \to \mathcal{S}$ is such that $\sigma \oplus \sigma'$ is the state σ^* where $\sigma^*(f) = \sigma'(f)$ when $\sigma'(f)$ is defined, and $\sigma^*(f) = \sigma(f)$ otherwise. In particular $H \rhd \langle \varnothing \rangle$, where $\langle \varnothing \rangle$ is the null trajectory, extends H from t to $t+1$ such that $H(t+1) = H(t)$.

The game starts at time 0 with the board in an initial configuration. $H(0)$ is the initial state of the world. Then the players take turns, correctly revising the board. During the game, the ego can do the following on its turn: (1) start an activity by adding a tuple $\langle s, E \rangle$ to $\mathcal{C}$, where $s = n_{\mathcal{B}}$; (2) end an activity by removing $\langle s, E \rangle$ from $\mathcal{C}$ and adding $\langle s, t, E \rangle$ to $\mathcal{A}$, where $t = n_{\mathcal{B}}$. The world can do the following on its turn: (1) add one member n' to $\mathcal{B}$, where $n' > max\{n : n \in \mathcal{B}\}$, and construct a new finite development as a *correct revision* up to n'; (2) leave $\mathcal{B}$ unchanged [1], and extend the history up to $n_{\mathcal{B}}$.

The precise account on world moves is given by the definition of *trajectory-semantics world*, where: (1) if $\langle n_{\mathcal{B}}, E \rangle \in \mathcal{C}$, then $\mathcal{B}' = \mathcal{B} \cup \{n_{\mathcal{B}} + k\}$, $H' = H \rhd h$, where $h = \langle \sigma'_1, \sigma'_2, \ldots, \sigma'_k \rangle$, $h \in Trajs(E, H(n_{\mathcal{B}}))$, and $\mathcal{A}' = \mathcal{A} \cup \{\langle n_{\mathcal{B}}, n_{\mathcal{B}} + k, E \rangle\}$; (2) if $\mathcal{C} = \varnothing$, then $\mathcal{B}' = \mathcal{B} \cup \{n_{\mathcal{B}} + 1\}$, $H' = H \rhd \langle \varnothing \rangle$, and $\mathcal{A}' = \mathcal{A}$.

3 The Synchronous Game

The western philosophy dates back to Aristotle the notion of time as measurable order of the movement. According to Aristotle (*Physics*, IV, 11, 219 b 1), time is not change in itself, but *the measure of change according to a before and an after*. Since time evolves because of change, the absence of change determines the end of the world. Aristotle assumed the world eternal. The emphasis on the irreversibility of change, due to the second principle of thermodynamics was later made by Reichenbach (*The Direction of Time*, 1956) for which time has a growth direction.

The notion of time for the $\mathcal{K}$-**IA** world fully respects both the above requirements, in fact: (1) when an action is initiated by the ego, that is the current action set is not empty, then the world increases the current *now* by k time-points, where k is the length of the invoked action; (2) when no action is initiated by the ego, that is the current-action set is empty, then the world initiates, executes and terminates a null action which trajectory is the null trajectory $\langle \varnothing \rangle$ of length 1, so that the current *now* is increased by one time-point. Therefore, executed actions are the image-level justification of flowing time, and determine, de facto, the way the image-level world beats time. Concerning the direction of time, actions may also have reversible effects on feature values, in fact, due to the correct revision, if $b \in \mathcal{B}' - \mathcal{B}$ then is $b > n_{\mathcal{B}}$.

In pursuing the aim of extending the given notion of time to the continuum case, we meet two problems.

[1] Sandewall ruled this out in the definition of trajectory-semantics world, by imposing $n' = n_{\mathcal{B}} + 1$.

The first problem is that no action can be physically initiated and terminated such that its duration is infinitesimal and its repeated execution is the image-level justification to continuous time. The world humans perceive is, in fact, fully discrete, and time and continuity are abstractions of the human mind. At the image-level, the problem with beating the continuous time *formally* consists in the well known non-existence of the successor function for real numbers, in fact the image-level world could just advance time by increasing the current *now* by an infinitesimal quantity ε, and lose a continuum of time-points between the *now* $n_{\mathcal{B}}$ and $n' = n_{\mathcal{B}} + \varepsilon$.

The second problem is tightly connected with the absence of a successor function for real numbers, and occurs as follows, with some preliminaries. The image-level world has a set-theoretic description $\langle Infl, Trajs\rangle$ given in terms of influenced features and corresponding trajectories. If E is an action and σ_s is a state, $Infl(E, \sigma_s)$ represents the set of those features which may be affected if the action E is initiated in the state σ_s, while $Trajs(E, \sigma_s)$ represents the set of possible trajectories of the action E if the action E is initiated in σ_s, and consists of a set of finite nonempty sequences of partial states each of which assigns values to those features appearing in $Infl(E, \sigma_s)$ but not to other features. Each single trajectory defines an action alternative. The set-theoretic description then consists of a *propositional* table where the description of each trajectory is given in a detailed case by case: if σ_s is the state of the world at time s, a trajectory in $Trajs(E, \sigma_s)$ is a finite nonempty sequence $\langle\sigma_{s+1}, \ldots, \sigma_t\rangle$ of tuples in $\mathcal{F} \times \mathcal{V}$ (states), from $s + 1$ up to a certain time-point t, $t \neq s + 1$, where the trajectory ends. Now, if the value domain for features is a continuous domain, we obtain a continuum of possible starting states, together with a tuple $\langle Infl(E, \sigma), Trajs(E, \sigma)\rangle$ for each specific starting state σ and action E, and a continuum of ordered partial states for each trajectory in $Trajs(E, \sigma)$. Such set-theoretic description could never fit the above mentioned propositional table.

The given ontology falls beyond $\mathcal{K}$-**IA**.

3.1 Continuous Time

From the standpoint of a human being, since humans may generally perceive the world in different manners, the time itself is susceptible of different understandings. The time in fact flows in a *natural fashion* according to our peculiar common sense, and logical approaches to temporal reasoning are plenty of (at all natural) time structures. Let then assume the existence of a certain machinery capable to describe the flux of time, let say a "master clock", and get rid of that by giving freedom for its definition. We simply assume per default a strictly increasing evolution of the *now*, and rely to the master clock the task to beat time. At the image-level, the master clock is a strictly monotonic rising function, and its function symbol *clock* is either associated to an "internal" or an "external" function. An internal function is, per definition, a computable function; in being internal, the agent is rationally aware of the flux of time, so to know exactly how the time is beaten. An external function, instead, has no internal formal definition, it may also be a non computable function, it is simply invoked and its actual value is obtained for magic. The agent is still aware of the flux of time, but has no *given* understanding of how the time is beaten. An example of internal clock may be the successor function of Peano's arithmetic, recursively defined as $clock = 0$, $clock = clock + 1$, where 0 is the origo of time-points and 1 is the length of the null trajectory. An external clock function may be the function which reads the signal of an atomic clock; in that case the resulting underlying semantics is *hybrid*, since at least this extra logical component is involved. The function beating the continuous time is clearly not computable, and therefore external.

We now fix an order relation and operations on members of the time-point domain. The *basic time structure* is $\langle\mathcal{T}; \{+^2, clock^0\}, \{=^2, \sqsubseteq^2\}\rangle$, defined with the obvious first order language $\mathcal{L}_\Sigma$ for time-points, built on top of the signature $\Sigma = \{\{+^2, clock^0\}, \{=^2, \sqsubseteq^2\}\}$. Depending on the need, additional axioms will characterize the temporal structure we like to deal with.

- Axioms for the sum function. For all pair of time-points s and t in $\mathcal{T}$ is defined a third time-point in $\mathcal{T}$, written $s + t$, such that:

  S1 : $\forall r, s, t \in \mathcal{T} \,.\, (r + s) + t = r + (s + t)$

  S2 : $\forall s, t \in \mathcal{T} \,.\, s + t = t + s$

  S3 : $\forall t \in \mathcal{T} \,.\, \exists\, t_0 \in \mathcal{T} : t + t_0 = t$

  S1 is the usual axiom for the associativity, *S2* for the commutativity and *S3* for the neuter element.

- Axiom for the *clock* function. For all pairs of subsequent calls of *clock*, ν and ν' in $\mathcal{T}$,

  C1 : $\nu \sqsubset \nu'$

- The equality is a symmetric relation, that is

  E1 : $\forall s, t \in \mathcal{T} \,.\, (s = t) \rightarrow (t = s)$

- For all pair of time-points s and t, it is possible to establish whether s precedes t ($s \sqsubseteq t$) or t precedes s ($t \sqsubseteq s$) according to the following axioms for the order predicate:

  O1 : $\forall t \in \mathcal{T} \,.\, t \sqsubseteq t$

  O2 : $\forall t \in \mathcal{T} \,.\, t_0 \sqsubseteq t$

  O3 : $\forall s, t \in \mathcal{T} \,.\, s \sqsubseteq t \wedge t \sqsubseteq s \Rightarrow s = t$

  O4 : $\forall r, s, t \in \mathcal{T} \,.\, r \sqsubseteq s \wedge s \sqsubseteq t \Rightarrow r \sqsubseteq t$

  O5 : $\forall s, t \in \mathcal{T} \,.\, s \sqsubseteq t \vee t \sqsubseteq s$

  O6 : $\forall r, s, t \in \mathcal{T} \,.\, r \sqsubseteq s \rightarrow r + t \sqsubseteq s + t$

  O7 : Let A and B be non empty subsets of $\mathcal{T}$ such that $a \sqsubseteq b$ for all $a \in A$ and $b \in B$. Then exists $\xi \in \mathcal{T}$ such that $a \sqsubseteq \xi \sqsubseteq b$ for all $a \in A$ and $b \in B$.

  O1 is the usual axiom for the reflexivity, *O3* for the anti-symmetry, *O4* for the transitivity, *O5* (optional) for the totality and *O7* (optional) is the axiom of completeness.

The neuter element t_0 is the *origo* of time-points. The relation $s \sqsubset t$ is an abbreviation for $s \sqsubseteq t \wedge s \neq t$. The relation $s \sqsubseteq t$ ($s \sqsubset t$) is equivalent to $t \sqsupseteq s$ ($t \sqsupset s$).

If *clock* is the successor function of Peano's arithmetic, we re-obtain the time structure for $\mathcal{K}$-**IA**. In the sequel we assume *O7* and *O5*, that is $\mathcal{T}$ continuous and linearly ordered. The resulting time structure is the classical structure $\mathcal{R}^+$ for the positive real numbers augmented with the axiom *C*1 for the master clock function.

3.2 Continuous Change

The following assumptions are made. Each action must influence a finite number of features. For all f_i and f_j in $Infl(E, \sigma)$ is $f_i \neq f_j$. Each action has a finite number of alternatives and a finite number of starting states σ such that $Infl(E, \sigma) \neq \varnothing$.

From the apriorism, a finite number of ordered partial states defines a discrete trajectory, that can be also explicitly written, when reasonably small. Clearly a continuum of ordered partial states cannot be explicitly written, unless with a characteristic property that *implicitly* defines them by selection over a common domain. Because of the nonexistence of a "next partial state", the order relation between partial states must necessarily be the order relation between time-points, where each partial state consists of a set of tuples $\langle t, f, v\rangle$ in $\mathcal{T}\times\mathcal{F}\times\mathcal{V}$. Let $\mathcal{H}$ be the set of all tuples (named *observations*) $\langle t, f, v\rangle$, varying t in $\mathcal{T}$, f in $\mathcal{F}$ and v in $\mathcal{V}$. The order relation $\sqsubseteq$ applies as follows on members of $\mathcal{H}$: $\langle t_1, f_1, v_1\rangle \sqsubseteq \langle t_2, f_2, v_2\rangle$ if and only if $t_1 \sqsubseteq t_2$, for all $\langle t_1, f_1, v_1\rangle$ and $\langle t_2, f_2, v_2\rangle$ in $\mathcal{H}$. Since we assumed *O5*, the set $(\mathcal{H}; \sqsubseteq)$ is a totally ordered set.

How do we describe trajectories?

Definition 3.1 *A* partial fluent *is a function* $\varphi: \mathcal{T}^3 \to \mathcal{V}$ *which definitional domain is* $Dom(\varphi) = \{\langle x, y, z\rangle \in \mathcal{T}^3 : x \sqsubseteq y \sqsubseteq z\}$. *We define* length *of* $\varphi(s, \tau, t)$, *where* $\varphi(s, \tau, t)$ *is the invocation of a partial fluent, that unique member* l *of* $\mathcal{T}$ *such that* $s + l = t$.

Partial fluents are *single-feature trajectory descriptors.* Partial fluents may be *internal* and static, *internal* and dynamic, or *external* of two different species.

An internal static partial fluent is a computable function which definition is known and does not change passing time. For example, an internal static partial fluent could be a mathematical function $\varphi : \mathbb{R}^{3+} \to \mathbb{R}$, or a function $\varphi : \mathbb{N}^3 \to \{true, false\}$. For a given feature f_j such that several actions are concurrently running and influencing f_j, instead of separately consider all involved partial fluents we can just consider their combined function φ_j, like in Newtonian mechanics. An internal static partial fluent allows also to simulate lack of knowledge when complete knowledge about the change is not required, or the change is not exactly known; its definition may be as follows:

$$\varphi_j(s, \tau, t) = \begin{cases} w & \text{if } \tau = s \\ unknown & \text{if } \tau \in (s, t) \\ v & \text{if } \tau = t \end{cases}$$

where w is the expected value of a precondition involving the feature f_j, *unknown* represents $\varkappa f_j$ (read "occluded f_j"), and v is the terminal value of the postcondition. Clearly the tuple $\langle t, f, unknown\rangle$ is an abbreviation for $\bigvee_i \langle t, f, v_i\rangle$, varying i over all possible tuples $\langle t, f, v_i\rangle$ in $\mathcal{H}$ (assuming a continuous value domain for features, i is necessarily a continuous index), and express the *don't know* non-determinism.

An internal dynamic partial fluent is a computable function which definition is known and is allowed to change, passing time, according to internal criteria. Its definition may be updated, for instance, during a learning process.

An external partial fluent of first specie is a computable function which definition is known and is allowed to change passing time according to external criteria. An example of external partial fluent of first specie is the following. Suppose the function $\varphi_i : \mathbb{R}^3 \to \mathbb{R}^3$ represents the i-th airway within a certain airspace. The invocation $\varphi_i(s, \tau, t)$, varying τ in $[s, t]$, represents the trajectory of an airplane, and is generated by an air-traffic scheduler, which is supposed to be "external" to the airplane co-pilot system, so that for all τ in $[s, t]$ no other airplane is using the i-th airway. The co-pilot system "knows" φ_i at time τ, and may drive the airplane accordingly.

An external partial fluent of second specie is a partial function which definition is not necessarily known, and may also be a non computable function. A simple example of external partial fluent of second specie is the non-predictable trajectory (position in space) of another agent inhabiting the same dynamical system.

Under the $\mathcal{K}$ epistemological assumption, external partial fluents of the second specie are not allowed, in fact their definition is not known and the information they provide can not be predicted. Prediction problems could not be solved, unless by means of statistical reasoning.

Definition 3.2 *Let E be an action and σ a state of the world at time s such that $Infl(E, \sigma) \neq \varnothing$. Let n be the number of alternatives of the action E initiated in σ and let $\varphi_{1j}, \ldots, \varphi_{ij}, \ldots, \varphi_{nj}$ be the partial fluents that are associated to each feature f_j in $Infl(E, \sigma)$. Furthermore, for all φ_{ih} and φ_{ik} associated to f_h and f_k in $Infl(E, \sigma)$, where $i = 1, \ldots, n$, is $s \notin Dom(\varphi_{ih})$, $s \notin Dom(\varphi_{ik})$ and $length(\varphi_{ih}) = length(\varphi_{ik})$. The i-th trajectory of E initiated in σ is then defined as the totally ordered set $(T_i(E, \sigma); \sqsubseteq)$, where $T_i(E, \sigma)$ is as follows:*

$$T_i(E, \sigma) = \{\langle \tau, f_j, \varphi_{ij}(s, \tau, t)\rangle \in \mathcal{H} : \tau \in Dom(\varphi_{ij}) \text{ and } f_j \in Infl(E, \sigma)\}$$

Given a trajectory $T_i(E, \sigma)$, the set

$$h(\tau) = \{\langle t, f, v\rangle \in T_i(E, \sigma) : t = \tau\}$$

represents the partial state *of the world at time τ according to $T_i(E, \sigma)$. If $s = Min\{\tau : \langle \tau, f, v\rangle \in T_i(E, \sigma)\}$ and $t = Max\{\tau : \langle \tau, f, v\rangle \in T_i(E, \sigma)\}$, the* length *of the trajectory $T_i(E, \sigma)$ is defined as the unique member l of $\mathcal{T}$ such that $s + l = t$. Let E be an action and σ a starting state. The set $Trajs(E, \sigma)$ is the set of all possible trajectories of E initiated in σ.*

Let H be a finite history over $[t_0, t]$, let E be an action, σ a state of the world at time t, and let $T_i(E, \sigma)$ be a trajectory such that $l = length(T_i(E, \sigma)) \neq 0$. Then $H \triangleright T_i(E, \sigma)$ is the finite history H' over $[t_0, t+l]$ defined as follows:

$$H'(s) = \begin{cases} H(t) \oplus h(s) & \text{if } t \sqsubset s \sqsubseteq t + l \\ H(s) & \text{if } s \sqsubseteq t \end{cases}$$

where $h(s)$ is the partial state of the world at time s, according to $T_i(E, \sigma)$.

Definition 3.3 *A* trajectory-semantics world *is an image-level world which corresponds to some set-theoretic world description. A world W is the* corresponding world *of a set-theoretic world description $\langle Infl, Trajs\rangle$ if and only if it satisfies the following conditions. If $J = \langle \mathcal{P}, M, H, \mathcal{A}, \mathcal{C}\rangle$, ν is the largest member of $\mathcal{P}$ and H is defined over $[t_0, \nu]$, then $W(J, J')$ if and only if $J' = \langle \mathcal{P} \cup (\nu, clock], M, H', \mathcal{A}', \varnothing\rangle$, and either of the followings hold:*

- *there exists an unique $E \in \mathcal{E}$ such that $\mathcal{C} = \{\langle \nu, E\rangle\}$, $H' = H \triangleright T_i(E, \sigma)$ with $T_i(E, \sigma) \in Trajs(E, H(\nu))$, and $\mathcal{A}'$ becomes $\mathcal{A} \cup \{\langle \nu, \mu, E\rangle\}$ as soon as clock is equal to μ, where $\mu = \nu + length(T_i(E, \sigma))$;*
- *$\mathcal{C} = \varnothing$, $H' = H \triangleright \varnothing$, and $\mathcal{A}' = \mathcal{A}$.*

The definition 3.3 is equivalent to the definition of trajectory-semantics world for $\mathcal{K}$-**IA** if we assume $\mathcal{T} = \mathbb{N}$ and the master clock is defined as follows: *clock* is initially zero, then is $clock = clock + 1$ if $\mathcal{C} = \varnothing$ and $clock = clock + length(T)$ if $\mathcal{C} = \{\langle \nu, E\rangle\}$ and $T \in Trajs(E, H(\nu))$, where $\mathcal{C}$ is the current-action set at $\nu = clock$.

3.3 The Game

The ego-world game involving continuous time and continuous change is as follows:

- At time ν the ego communicates its decision to initiate a certain action or activity E, by adding the tuple $\langle \nu, E\rangle$ to $\mathcal{C}$ (we assume synchronous communication between the ego and the world). Then the move of the world consists in modifying the finite development as follows:
    1. At time ν, the world non-deterministically selects a trajectory T_i from the trajectory set $Trajs(E, H(\nu))$.
    2. From ν to $\nu' = \nu + l$, $l = length(T_i(E, H(\nu)))$, the world's state changes according to the chosen T_i, that is $H(\rho, f_j) = \varphi_{ij}(\nu, \rho, l)$ for all ρ in $Dom(\varphi_{ij}(\nu, \rho, l))$.
    3. At time ν' the world adds the tuple $\langle \nu, \nu', E\rangle$ to $\mathcal{A}$ and resets $\mathcal{C}$ to the empty set (this communicates a "terminated" message to the ego).

  The ego can also terminate E earlier than ν', let say at the present time $\nu'' = clock$, by removing $\langle \nu, E\rangle$ from $\mathcal{C}$ and adding $\langle \nu, \nu'', E\rangle$ to $\mathcal{A}$. In that case the world persists in its values at ν'' and at subsequent time-points until another initialization message is received from the ego.
- The ego has not decided to initiate an action at time ν, so that the current-action set $\mathcal{C}$ is empty. In this case the world's state simply persists in its values at ν and at subsequent new time-points until an initialization message is received from the ego.
- No other rules apply.

3.4 The Scenario Description

A *scenario* for a world W is a set of games where W was the world player. A *scenario description* consists in the description of the world itself and of how its state has changed over time.

Definition 3.4 *A* scenario description *in $\mathcal{K}$-**RA** is defined as a five-tuple $\langle \mathcal{K}, \mathcal{O}, \langle \mathbf{RA}, LAW\rangle, SCD, OBS\rangle$ where $\mathcal{K}$ is the assumed epistemology, $\mathcal{O}$ is a finite set of objects, SCD is a set of action occurrences, and OBS is a set of observation statements. The tuple $\langle \mathbf{RA}, LAW\rangle$, where **RA** is the assumed ontology and LAW is a set of action laws in Full Trajectory Normal Form with internal static partial fluents, is the corresponding logical description of the set theoretic description $\langle Infl, Trajs\rangle$ of the world, for some $\mathcal{K}$-**RA** world.* □

From the set-theoretic description of a world as a pair $\langle Infl, Trajs\rangle$ we shall now construct a corresponding logical formula, called the *Full Trajectory Normal Form* (FTNF) for the action laws.

Definition 3.5 *A* trajectory formula *for a given feature f_j in $\mathcal{F}$ is the first-order formula $\forall\, \tau \in [s, t] \subset \mathcal{T}\,.\, [\tau] f_j \doteq \varphi_j(\tau)$ where the descriptor φ_j is a partial fluent defined over $D \subseteq [s, t] \subset \mathcal{T}$, and $s \neq t$.*

Definition 3.6 *Let M be a mapping and $\theta = \{s/M(\mathbf{s}), t/M(\mathbf{t})\}$ a valuation. The meaning of a trajectory formula $\forall\, \tau \in [s, t] \subset \mathcal{T}\,.\, [\tau] f_j \doteq \varphi_j(s, \tau, t)$ is the lattice $(S_j(\theta); \sqsubseteq)$, where the domain $S_j(\theta)$ is defined as follows:*

$$S_j(\theta) = \{\langle \tau, f_j, \varphi_j(s\theta, \tau, t\theta)\rangle \in \mathcal{H} : \tau \in Dom(\varphi_j)\}$$

In the sequel, S will represent a trajectory formula and $S(\theta)$, for a given valuation θ, will represent its meaning.

Definition 3.7 (FTNF) *An action law in Full Trajectory Normal Form with no alternatives is a mapping of an action statement to a conjunction of trajectory formulas and is written in the form*

$$[s, t]A \Rrightarrow \bigwedge_{j=1}^{m} S_j$$

where $A \in \mathcal{E}$, the conjunction $\bigwedge_{j=1}^{m} S_j$ represents a trajectory set, and each S_j is the corresponding trajectory formula for the feature f_j. The Full Trajectory Normal Form for actions with alternative results is the mapping

$$[s, t]A \Rrightarrow \bigvee_{i=1}^{n} (\bigwedge_{j=1}^{m} S_{ij})$$

where the action occurrence $[s, t]A$ is expanded into a formula in Full Disjunctive Normal Form, *that is into* *a disjunction of conjunctions of trajectory formulas S_{ij}, each of which corresponds to the feature f_j in the alternative i.* □

The following conventions are imposed. Within the same action definition, alternatives involve the same set of features and partial fluents have the same length, which defines the length of the action itself. Each action type has influence over those features occurring in the action definition, while all other feature values remain unchanged, and there is no change during those periods when there is no action.

We now show the above FTNF is an equivalent and compact version of the FTNF for the discrete case.

Let Pre be a conjunction of statements like $[s]f_n = v_n$, and let $Post$ be a conjunction of statements like $[s,t]f_m := v_m$; the former represents the initial expected values for those features involved in a precondition, while the latter describes the trajectory for those features involved in a postcondition.

1. $[s,t]A \Rrightarrow [s,t]f := v$
2. $[s,t]A \Rrightarrow [s,t]f := v \vee [s,t]f := \neg v$
3. $[s,t]A \Rrightarrow Pre \Rightarrow Post$
4. $[s,t]A \Rrightarrow (Pre_1 \vee \ldots \vee Pre_k) \Rightarrow Post$
5. $[s,t]A \Rrightarrow Pre \Rightarrow (Post_1 \vee \ldots \vee Post_k)$
6. $[s,t]A \Rrightarrow C_1 \vee \ldots \vee C_k$

The basic case (1) is trivial. The generic alternative (6) is a disjunction of C_i statements, each of which is like the body $Pre \Rightarrow Post$ of a simple conditional (3), where Pre may also be empty, while an empty set of postconditions is not allowed. For each Pre_i in C_i and Pre_j in C_j, although with possibly different values, they involve the same set of features. The random effect (2) is that particular instance of the generic alternative for which two or more distinct values are non-deterministically assigned to the same feature. In order to not compromise scenario description's consistency, the disjunction is intended to be exclusive. The conditional with disjunctive preconditions (4) is an alternative writing for a generic alternative where each C_i has the same postcondition $Post$. The conditional with disjunctive postconditions (5) is an alternative writing for a generic alternative where each C_i has the same precondition Pre. The simple conditional, by exploiting partial fluents expressiveness, may be rewritten as follows: (1) write a partial fluent φ_n for each element $[s]f_n = v_n$ in Pre; (2) write a partial fluent ψ_m for each element $[s,t]f_m := v_m$ in Post; (3) for each partial fluent ψ_m such that there exists a φ_n for the same feature, *complete* ψ_m with the definition of φ_n and eliminate φ_n; (4) make a conjunction of all obtained partial fluents. For example, the famous action law $[s,t]Fire \Rrightarrow [s]l \Rightarrow ([s,t]l := false \wedge [s,t]a := false)$ is rewritten as $[s,t]Fire \Rrightarrow \forall\tau \in [s,t].[\tau]a \doteq to\_die(s,\tau,t) \wedge [\tau]l \doteq to\_fire(s,\tau,t)$ where the partial fluents to_die and to_fire are defined as follows:

$$to\_die(s,\tau,t) = \begin{cases} unknown & \text{if } \tau \in (s,t) \\ false & \text{if } \tau = t \end{cases}$$

$$to\_fire(s,\tau,t) = \begin{cases} true & \text{if } \tau = s \\ unknown & \text{if } \tau \in (s,t) \\ false & \text{if } \tau = t \end{cases}$$

The case of alternative results of actions, which assumption is represented with the **A** ontological assumption, covers the following cases: random effect, simple conditional (that corresponds to the **Ad** ontological assumption), conditional with disjunctive postconditions, and the generic alternative. Then **A** covers everything of the above list, except the case of disjunctive preconditions, in fact it does not allow alternative results of actions according to the chosen direction of time.

3.5 Intended models

The definitions of complete development set, intended model set and classical model set for a scenario description in $\mathcal{K}$-**RA** are substantially identical to the original ones for $\mathcal{K}$-**IA**.

Definition 3.8 *Let Υ be a scenario description in $\mathcal{K}$-**RA**. The* set of complete developments *of a game is obtained as follows: (1) construct the unique trajectory-semantics world W that is precisely specified by $\langle \mathbf{RA}, LAW\rangle$, then select an arbitrary trajectory-semantics ego and an arbitrary initial state; (2) generate all possible developments which can be obtained in games between them; (3) add an arbitrary M component to each development; (4) restrict the set of developments to those where all formulas in $SCD \cup OBS$ are satisfied, and where there is a one-to-one correspondence between actions in the development and action statements in SCD. This subset is $Mod(\Upsilon)$.*

Definition 3.9 *The* intended model set *for a scenario description Υ in $\mathcal{K}$-**RA** is defined as follows:*

$$\Sigma_{\mathcal{K}\text{-}\mathbf{RA}}(\Upsilon) = \{\langle M, H\rangle \mid \langle \mathcal{P}, M, H, \mathcal{A}, \mathcal{C}\rangle \in Mod(\Upsilon)\}$$

Definition 3.10 *The* classical model set *for a scenario description Υ in $\mathcal{K}$-**RA** is defined as follows:*

$$[\![\Upsilon]\!] = [\![LAW[SCD] \cup OBS]\!]$$

where $LAW[SCD]$ is the expansion of all formulae in SCD using their corresponding definitions in LAW.

The following proposition holds as an immediate generalization of the analogous result for $\mathcal{K}$-**IA**:

Proposition 3.1 $\Sigma_{\mathcal{K}\text{-}\mathbf{IA}}(\Upsilon) \subseteq \Sigma_{\mathcal{K}\text{-}\mathbf{RA}}(\Upsilon) \subseteq [\![\Upsilon]\!]$

Proof. The model set $\Sigma_{\mathcal{K}\text{-}\mathbf{RA}}(\Upsilon)$ equals $\Sigma_{\mathcal{K}\text{-}\mathbf{IA}}(\Upsilon)$ in case of discrete worlds, while $\Sigma_{\mathcal{K}\text{-}\mathbf{IA}}(\Upsilon)$ is empty in case of continuous or almost-continuous worlds, so that

$$\Sigma_{\mathcal{K}\text{-}\mathbf{IA}}(\Upsilon) \subseteq \Sigma_{\mathcal{K}\text{-}\mathbf{RA}}(\Upsilon)$$

The set $[\![\Upsilon]\!]$ allows models which do not satisfy inertia and models where there are additional actions besides those specified in SCD, so that $\Sigma_{\mathcal{K}\text{-}\mathbf{RA}}(\Upsilon) \subseteq [\![\Upsilon]\!]$. □

The case of "concurrency of actions" is designated, in the *Features and Fluents* framework, with the **C** ontological assumption. The simpler case of "concurrency of independent actions" is obtained by specializing **C** with the **i** ontological sub-characteristics.

- Two or more actions are said to be *concurrent* iff time intervals in which they are respectively performed they overlap in at least one time point.
- Two or more actions are said to *influence* each other iff they share at least one feature, otherwise they are said to be *independent.*
- A feature is *shared* among a group of actions over a certain time interval iff the involved actions *depends* on the feature value during the given time interval.

No structural extensions are required for the underlying semantics of a world with concurrent independent actions. When playing the game, if the ego initiates several independent actions at the same time τ, then the world simply chooses one trajectory function non-deterministically. If $\varphi_j^1, \ldots, \varphi_j^n$ partial fluents influence the same feature f_j during $[\tau, \mu]$, each one independently on the other, then only one partial fluent will be considered for that feature as the composed partial fluent $\varphi_j^1 \circ \ldots \circ \varphi_j^n$ of the given partial fluents, and its definitional domain will be the closed and limited interval in time $[\tau, \mu]$. The symbol "$\circ$" represents the composition operator. When such composed function is a constant function c, then persistence arise for the influenced feature if and only if exists a small positive $\varepsilon \in \mathcal{T}$ such that $H(\tau - \epsilon) = c(\xi)$, for all ξ in $[\tau, \mu]$.

Scenario descriptions in $\mathcal{K}$-**RACi** are substantially identical to scenario descriptions in $\mathcal{K}$-**RA**, except for action occurrences which are now allowed to overlap in at least one time-point. The intended and classical model set are defined in the usual fashion, and the relation between $\mathcal{K}$-**RACi** models and $\mathcal{K}$-**RA** models is described by the following theorem:

Theorem 3.1 *Given are* $\Upsilon_1, \ldots, \Upsilon_n$ *scenario descriptions, each of which is of the form* $\Upsilon_i = \langle \mathcal{K}, \mathcal{O}, \langle \mathbf{RA}, LAW_i \rangle, SCD_i, OBS \rangle$. *Assume also that* $\bigcup_{i=1}^n SCD_i$ *is a schedule appropriate for the* **RACi** *ontological family. The following relation holds:*

$$\Sigma_{\mathcal{K}\text{-}\mathbf{RACi}} \left(\bigcup_{i=1}^{n} \Upsilon_i \right) = \bigcup_{i=1}^{n} \Sigma_{\mathcal{K}\text{-}\mathbf{RA}} (\Upsilon_i)$$

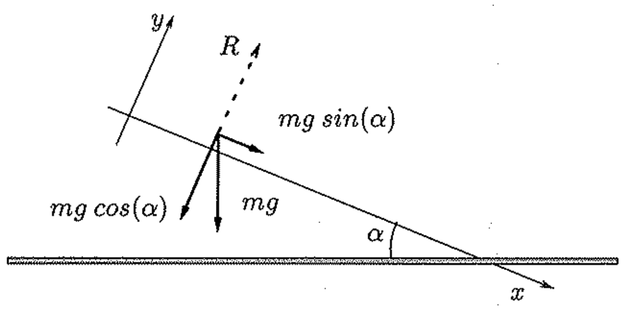
Proof. *Independent concurrent actions are compositional, in fact their joint effect is the sum of their individual effects.* □

The proposition 3.1 holds for $\mathcal{K}$-**RACi** too.

The following is an example of reasoning problem in this class.

Scenario 3.1 (Galileo's inclined plane) *Given is an inclined plane with no friction, and let* α *be the angle it makes with respect to the horizontal plane. We place an object of mass* m *on the inclined plane, then we release the object.*

Will the object move along the inclined plane? What are the involved actions at a given point in time τ and what are their results?



According to the given description, the mass of the object is m and the angle is α, then $[0]mass \hat{=} m$ and $[0]angle \hat{=} \alpha$ are in the OBS part of the scenario description. Still according to the given description, as well as classical Newtonian mechanics, two forces are always exerted on the object: the weight force W and the reaction R of the plane. No other forces apply. Then the SCD part of the scenario description consists of $[\square]W(obj)$ and $[\square]R(obj)$, where obj is an object constant. Finally, the action laws for $W()$ and $R()$ are given in the LAW part of the scenario description:

$$[s,t]W(object) \Rrightarrow \forall \tau \in [s,t].[\tau]force(object) \dot{=} \varphi_1(s,\tau,t)$$

$$[s,t]R(object) \Rrightarrow \forall \tau \in [s,t].[\tau]force(object) \dot{=} \varphi_2(s,\tau,t)$$

where s and t are temporal variables, $object$ is an object variable, $force$ is the feature we are interested in. If $\circ$ is the symbol for the vectorial sum, then the partial fluent φ_1 is defined as the vectorial sum of $[0]mass * g * sin([0]angle)$ and $[0]mass * g * cos([0]angle)$, while φ_2 is defined as $-[0]mass * g * cos([0]angle)$. According to the ego-world game, the force exerted on the object at time τ is

$$H(\tau, force(obj)) = \varphi_1(s,\tau,t) \circ \varphi_2(s,\tau,t) \dot{=} mg\ sin(\alpha)$$

Since the scenario was not explicit on α, a number of possible situations may arise from it; for example:

$$mg\ sin(\alpha) = \begin{cases} > 0 & \text{if } 0 < \alpha < \frac{\pi}{2} \\ < 0 & \text{if } \frac{\pi}{2} < \alpha < \pi \\ = 0 & \text{if } \alpha = 0 \text{ or } \alpha = \pi \\ = mg & \text{if } \alpha = \frac{\pi}{2} \end{cases}$$

If $0 < \alpha < \frac{\pi}{2}$ or $\frac{\pi}{2} < \alpha < \pi$, then the object moves along the inclined plane. If $\alpha = 0$, then the object is lying on a horizontal plane, and we obtain the case of inertia with influencing actions. If $\alpha = \frac{\pi}{2}$, then the inclined plane is a vertical plane and $\varphi_2(s,\tau,t) = 0.0$.

Passing time, α may change under the direct effect of a lifting action. The underlying semantics is tolerant in that respect since the action laws are parametric in α.

4 Some assessment results

In [Sandewall, 1994], the PMON entailment (Pointwise Minimization of Occlusion with No-change Premises) has been designed and proven sound and complete with respect to the full $\mathcal{K}$-**IA** class of reasoning problems. PMON is the best entailment method defined in [Sandewall, 1994]; it is equivalent to the Chronological Assignment and Minimization of Occlusion and Change (CAMOC) and subsumes the Prototypical Global Minimization (PGM, $\mathcal{K}$**r-IsAz**), the Original Chronological Minimization (OCM, $\mathcal{K}$**sp-IsAd**), the Prototypical Chronological Minimization (PCM, $\mathcal{K}$**p-IAex**), the Prototypical Chronological Minimization with Filtering (PCMF, $\mathcal{K}$-**IAex**), the Global Minimization of Occlusion with No-change Premises (GMOC, $\mathcal{K}$**r-IsA**), and the Chronological Minimization of Occlusion and Change (CMOC, $\mathcal{K}$-**IAe**). In [Yi, 1995], the PCM entailment has been proven correctly applicable also for $\mathcal{K}$**p-IsAnCi**.

In [Thielscher, 1994], the Action Description Language $\mathcal{A}$ of Gelfond and Lifschitz [1993] has been proven sound and complete with respect to the $\mathcal{K}$-**IbsAd** class.

In [Denecker, 1993, section 6.3], the Incomplete Situation Calculus (ISC) has been proven sound and complete with respect to the language $\mathcal{A}$. The result consists of a sound and complete transformation from $\mathcal{A}$ domain descriptions to incomplete logic programs with integrity constraints, where the reasoning procedure adopted for the resulting programs is the SLDNFA Resolution Rule. Per transitivity on the result of Thielscher, ISC is sound and complete with respect to the $\mathcal{K}$-**IbsAd** class.

All of the above classes are subclasses of $\mathcal{K}$-**RACi**.

In [Brandano, 1998], a simulative Algebraic Semantics, variant of the classical Fixed Point Semantics for the Horn Clause Logic [VanEmden and Kowalski, 1976], has been designed and proven sound and complete with respect to the $\mathcal{K}$**sp-RdAdCi** class of reasoning problems. In [Brandano, 1998], a non-simulative Algebraic Semantics has been designed and proven sound and complete with respect to the full $\mathcal{K}$-**RACi** class.

5 Conclusion

The trajectory semantics for $\mathcal{K}$-**IA** is extended to the case of continuous change, and the *Features and Fluents* framework is augmented with the definition of the $\mathcal{K}$-**RACi** class of reasoning problems. The class describes the knowledge representation method and the intended model set for each reasoning problem where "accurate and complete information about actions" is the epistemological assumption, represented by the designator $\mathcal{K}$, and "strict inertia in continuous time, continuous change and alternative results of possibly concurrent and independent actions" are the epistemological assumptions, represented by the designator **RACi**.

The result of this work implies that Sandewall's systematic approach applies also to logics for continuous change, and this work gives a base for analyzing their range of applicability.

References

[Brandano, 1998] S. Brandano. A Logic-Based Calculus of Fluents. Technical Report TR-01/98, Department of Computer Science, University of Pisa, 1998.

[Denecker, 1993] M. Denecker. *Knowledge Representation and Reasoning in Incomplete Logic Programming.* PhD thesis, University of Leuven, 1993.

[Gelfond and Lifschitz, 1993] M. Gelfond and V. Lifschitz. Representing Actions and Change by Logic Programs. *Journal of Logic Programming*, 17, 1993.

[Sandewall, 1972] E. Sandewall. An approach to the frame problem, and its implementation. In *Machine Intelligence*, volume 7, pages 195–204. Edinburgh University Press, 1972.

[Sandewall, 1989a] E. Sandewall. Combining logic and differential equations for describing real-world systems. In *Principles of Knowledge Representation and Reasoning (KR)*, pages 412–420, 1989.

[Sandewall, 1989b] E. Sandewall. Filter preferential entailment for the logic of action in almost continuous worlds. In *International Joint Conference on Artificial Intelligence*, pages 894–899, Detroit, USA, 1989.

[Sandewall, 1994] E. Sandewall. *Features and Fluents. The Representation of Knowledge about Dynamical Systems.* Oxford University Press, 1994.

[Thielscher, 1994] M. Thielscher. An analysis of systematic approaches to reasoning about actions and change. In *International Conference on AIMSA*, 1994.

[VanEmden and Kowalski, 1976] M. H. VanEmden and R. A. Kowalski. The Semantics of Predicate Logic as a Programming Language. *Journal of the Association for Computing Machinery*, 23(4):733–742, 1976.

[Yi, 1995] Choong-Ho Yi. Reasoning about Concurrent Actions in the Trajectory Semantics. Licentiate thesis, Linköping University, Sweden, 1995.